Halo-independent bounds of WIMP-nucleon couplings from direct detection and neutrino observations in non-relativistic effective theory

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Based on JCAP03(2023)011(arXiv: 2212.05774)

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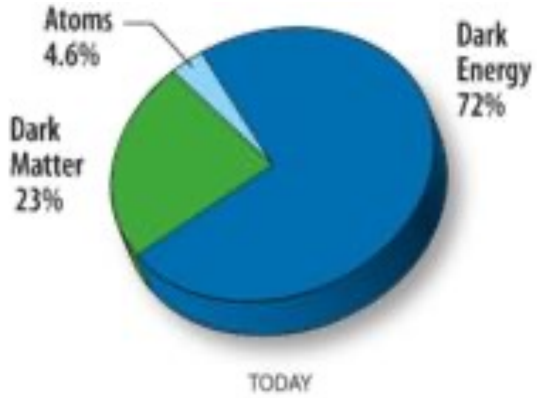
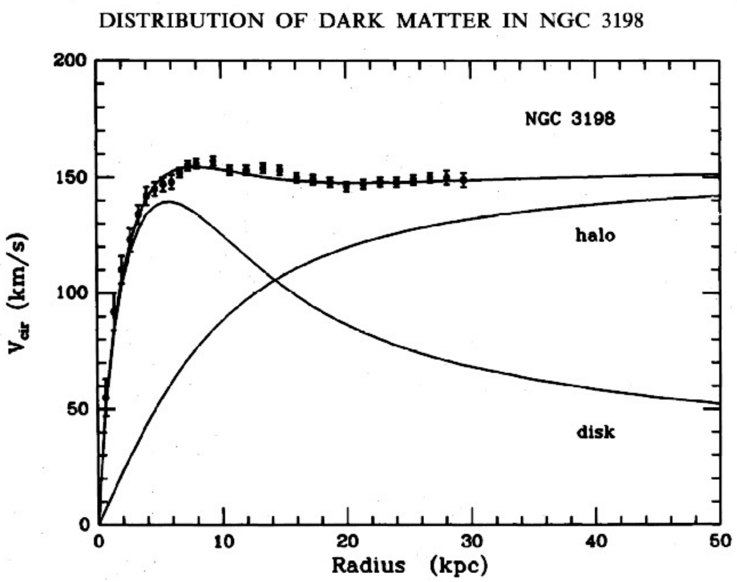
Dark Matter



"Content of the Universe - Pie Chart"
Wilkinson Microwave Anisotropy Probe.
National Aeronautics and Space
Administration. Retrieved 9 January 2018.

- Many evidences of Dark Matter
  - Galaxy rotational curve
  - CMB
  - Lensing effect

- Many candidates
  - Neutrino
  - Cold Dark Matter (CDM)
  - Weakly Interacting Massive Particle (WIMP)



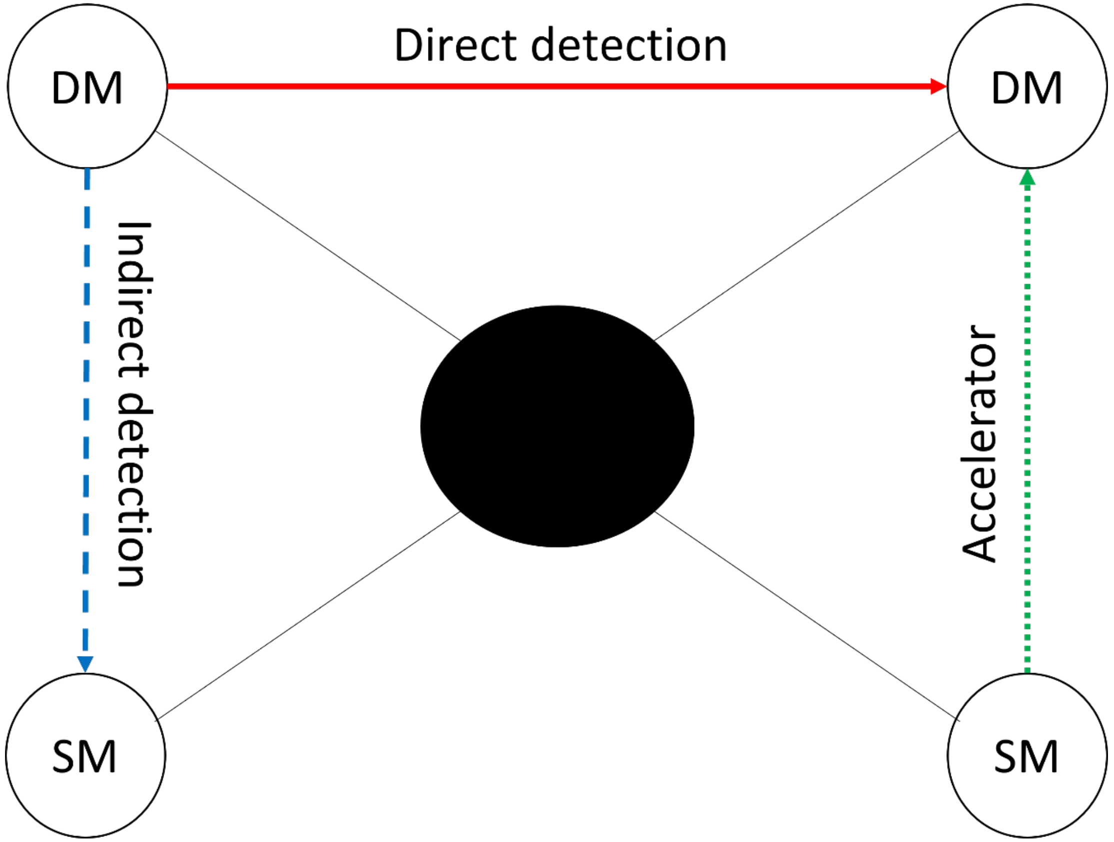
Galaxy rotational curve
K.G. Begeman et al. Extended rotation curves of spiral galaxies: dark haloes and modified dynamics, *Monthly Notices of the Royal Astronomical Society*, Volume 249, Issue 3, April 1991, Pages 523–537

WIMPs

Weakly Interacting Massive Particle (WIMP)

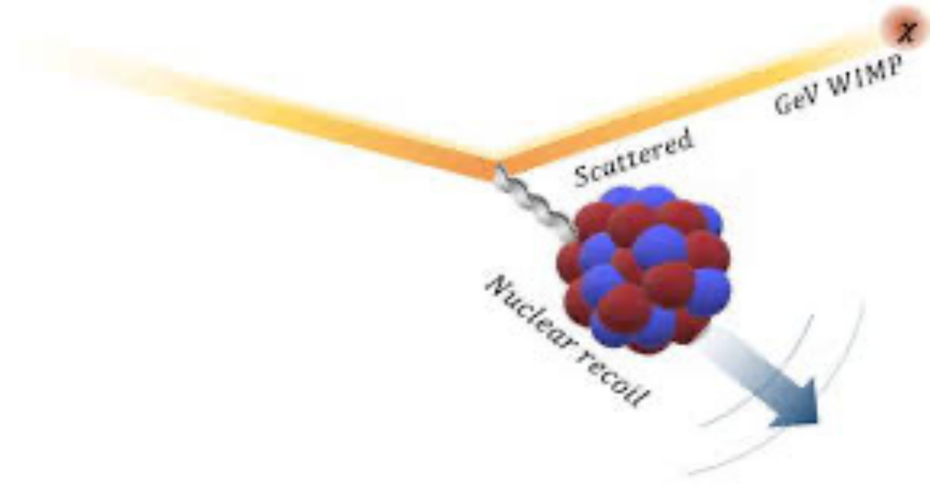
- Weak-type interaction
  - no electric charge, no color
- Mass range in GeV-TeV range
- WIMP miracle
  - correct relic abundance is obtained at WIMP $<\sigma v> = weak\ scale$
  - most extensions of SM are proposed independently at that scale.

Introduction: detection strategies



- Direct detection: DM interacts with SM particles (left to right)
- Indirect detection: DM annihilation (top to bottom)
- Accelerator: DM creation (bottom to top)

Direct Detection (DD)

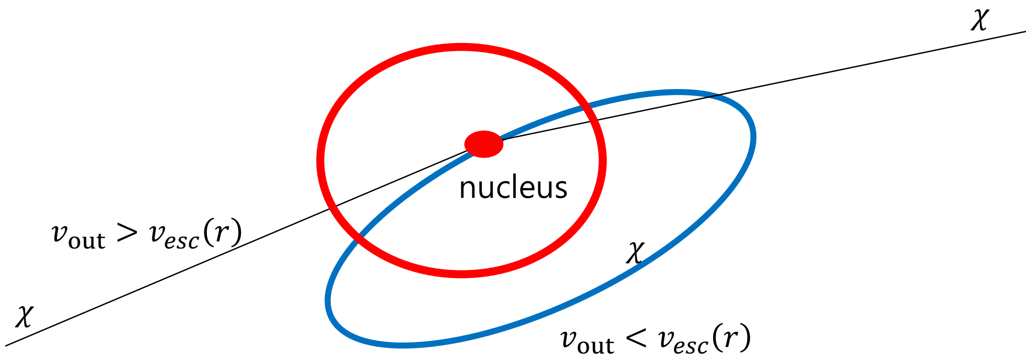


- The signals are WIMP-nucleus recoil events

- Low probability requires high exposure
- Underground to avoid background

- Different nuclear targets and background subtraction:
  - COSINE-100, ANAIS, DAMA, LZ, PandaX-4T, XENON-1T, PICO-60 and ect.

Indirect Detection



- WIMP scatters off nucleus at distance r inside celestial body
  - same interaction probed by DD

- If its outgoing speed v_{out} is below the escape velocity $v_{esc}(r)$, it gets locked into gravitationally bound orbit and keeps scattering again and again

- Capture in the Sun is favored for low (even vanishing) WIMP speeds

Non-Relativistic Effective Theory (NREFT)

- WIMP is slow, so that the recoil events are non-relativistic
- NREFT provides a general and efficient way to characterize results with mass of WIMP and coupling constants

Operators spin up to 1/2

- Hamiltonian: $\Sigma_{i=1}^{N}\left(c_i^n \mathcal{O}_i^n + c_i^p \mathcal{O}_i^p\right)$
- Non-relativistic process
  - all operators must be invariant by Galilean transformations ($v \sim 10^{-3}c$ in galactic halo)
- Building operators using: $i\frac{\vec{q}}{m_N},\ \vec{v}^{\perp},\ \vec{S}_\chi,\ \vec{S}_N$

$$\mathcal{O}_1 = 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp); \quad \mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}); \quad \mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$$

$$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}),$$

Non-Relativistic Effective Theory (NREFT)

- Each operators have distinct couplings to proton and neutron:

$$\Sigma_{\alpha=p,n}\Sigma_{i=1}^{15} c_i^\alpha \mathcal{O}_i^\alpha, \qquad c_2^\alpha \equiv 0$$

- Equivalent form using isospin:

$$\Sigma_{i=1}^{15}\left(c_i^0 1 + c_i^1 \tau_3\right)\mathcal{O}_i = \Sigma_{\tau=0,1}\Sigma_{i=1}^{15} c_i^\tau \mathcal{O}_i t^\tau, \qquad c_2^0 = c_2^1 \equiv 0$$

$$|p\rangle = \binom{1}{0} \quad |n\rangle = \binom{0}{1} \quad 1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \tau_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$c_i^0 = \frac{1}{2}\left(c_i^p + c_i^n\right) \quad c_i^1 = \frac{1}{2}\left(c_i^p - c_i^n\right)$$

$$t^0 \equiv 1 \quad t^1 \equiv \tau_3$$

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:
$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M|^2 \equiv \Sigma_k \Sigma_{\tau=0,1}\Sigma_{\tau'=0,1} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}, \left\{c_i^\tau, c_j^{\tau'}\right\}\right) W_k^{\tau\tau'}(y)$$

- Differential cross section : $$\frac{d\sigma}{dE_R} = \frac{1}{10^6}\frac{2m_N}{4\pi}\frac{c^2}{v^2}\left[\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\sum_{spin}|\mathcal{M}|^2\right]$$

- Differential rate : $$\frac{dR}{dE_R} = N_T \int_{v_{min}}^{v_{esc}} \frac{\rho_\chi}{m_\chi} v \frac{d\sigma}{dE_R} f(v)dv$$

- With $$E_R = \frac{\mu_{\chi N}^2 v^2}{m_N} \qquad v_{min} = \frac{1}{\sqrt{2m_N E_R}}\left|\frac{m_N E_R}{\mu_{\chi N}} + \delta\right|$$

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:

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- $R_k^{\tau\tau'}$: WIMP response function
  - Velocity dependence: $\mathcal{R}_k^{\tau\tau'} = \mathcal{R}_{k,0}^{\tau\tau'} + \mathcal{R}_{k,1}^{\tau\tau'}(v^2 - v_{min}^2)$
- $W_k^{\tau\tau'}$: nuclear response function
  - $y = (qb/2)$^2
  - b: harmonic oscillator size parameter
  - $k = M,\ \Delta,\ \Sigma',\ \Sigma'',\ \tilde{\Phi}'$ and Φ''
  - allowed responses assuming nuclear ground state is a good approximation of P and T

DD event rate

- DD event rate

$$R_{DD} = M\tau_{exp}\frac{\rho_\chi}{m_\chi}\int du\, f(u)u\,\Sigma_T\, N_T\int_{E_{R,th}}^{2\mu_{\chi T}^2 u^2/m_T} dE_R\,\zeta_{exp}\frac{d\sigma_T}{dE_R}$$

  - $M\tau_{exp}$: exposure
  - $E_{R,th}$: experimental energy threshold
  - ζ_{exp} : experimental features such as quenching, resolution, efficiency, etc.

- $R_{DD} = \int_0^{u_{esc}} f(u)\, H_{DD}(u)$

$$H_{DD}(u) = u\, M\tau_{exp}\frac{\rho_\chi}{m_\chi}\ \Sigma_T\, N_T\int_{E_{R,th}}^{2\mu_{\chi T}^2 u^2/m_T} dE_R\,\zeta_{exp}\frac{d\sigma_T}{dE_R}$$

Capture rate

- Capture rate

$$C_{\odot} = \frac{\rho_\chi}{m_\chi} \int du\, f(u) \frac{1}{u} \int_0^{R_\odot} dr\, 4\pi r^2\, w^2\ \Sigma_T\, \rho_T(r)\, \Theta\!\left(u_T^{C-max}\ - u\right) \int_{m_\chi\, u^2\,/2}^{2\mu_{\chi T}^2\, w^2\,/m_T} dE_R \frac{d\sigma_T}{dE_R}$$

  - ρ_T: the number of density of target
  - r: distance from the center of the Sun for Standard Solar Model AGSS09ph
  - u: DM velocity asymptotically far away from the Sun
  - $v_{esc}(r)$: escape velocity at distance r
  - $w^2(r) = u^2 + v_{esc}^2(r)$
- Neutrino Telescope (NT):
  - the neutrino flux from the annihilation of WIMPs captured in the Sun
  - DM annihilations into $b\bar{b}$

Capture rate

- with assumption of equilibrium between capture and annihilation: $\Gamma_\odot = C_\odot/2$

- $C_\odot = \int_0^{u^{c-max}} du\, f(u)\, H_C(u)$

$$H_C = \frac{\rho_\chi}{m_\chi}\frac{1}{u}\int_0^{R_\odot} dr\, 4\pi r^2\, w^2\ \Sigma_T\, \rho_T(r)\, \Theta\left(u_T^{C-max} - u\right) \int_{m_\chi u^2/2}^{2\mu_{\chi T}^2 w^2/m_T} dE_R \frac{d\sigma_T}{dE_R}$$

  - $u_T^{C-max} = v_{esc}(r)\sqrt{\frac{4m_\chi m_T}{(m_\chi - m_T)^2}}$: maximum WIMP speed for capture possible

Halo independent approach

- Scattering count rate:

$$R \sim \int dv\, H(v)\, f(v)$$

- Two parts of interaction and velocity distribution
  - needs to avoid uncertainty
  - interaction: include all possible interaction types
  - velocity distribution: halo independent approaches

- Model independent method: the most general scenarios

Halo independent approach

- The only constraint: $\int_{u=0}^{u_{max}} f(u)\, du = 1$
  - $f(u)$: arbitrary speed distribution

- Direct detection experiments have a threshold $u > u_{th}^{DD}$
  - Due to the energy threshold of experimental detectors

- Capture in the Sun is favored for low WIMP speeds
  - $u < u_T^{C-max}$

- In order to cover full speed range: combine DD and capture

F. Ferrer, A. Ibarra, S. Wild A novel approach to derive halo-independent limits on dark matter properties, JCAP09(2015)052

Halo independent approach

- Considering one effective coupling (c_i) at a time:
  - $R_{exp}\left(c_i^2\right) = \int du\, f(u)\, H_{exp}\left(c_i^2, u\right) \leq R_{max}$
  - R_{max} : corresponding experimental bound

- Using relation : $H\left(c_i^2, u\right) = c_i^2 H(c_i = 1, u)$
  - $H\left(c_{i,max}^2(u), u\right) = R_{max}$
  - $c_{i,max}^2(u) = \frac{R_{max}}{H(c_i=1,u)}$
  - $c_{i,max}(u)$: upper limit on c_i at single speed stream u

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Halo independent approach

- $R_{exp}(c_i^2) = \int du\, f(u)\, H_{exp}(c_i^2, u) \leq R_{max}$

$$\begin{aligned} R(c_i^2) &= \int_0^{u_{\max}} du\, f(u)\, H(c_i^2, u) \\ &= \int_0^{u_{\max}} du\, f(u)\, \frac{c_i^2}{c_{i\,\max}^2(u)} H(c_{i\,\max}^2(u), u) \\ &= \int_0^{u_{\max}} du\, f(u)\, \frac{c_i^2}{c_{i\,\max}^2(u)} R_{\max} \leq R_{\max} \end{aligned}$$

- upper limit on i-th coupling c_i:

$$c_i^2 \leq \left[\int_0^{u_{max}} du\ \frac{f(u)}{c_{i,max}^2(u)} \right]^{-1}$$

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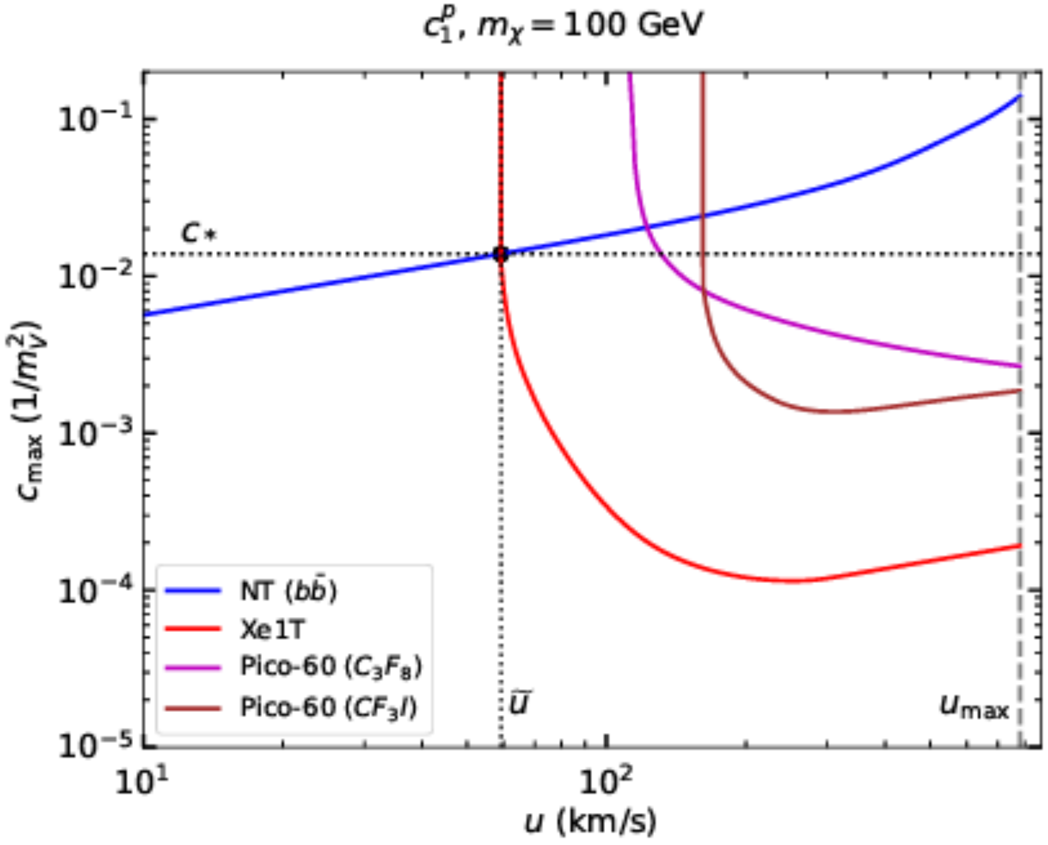
Halo independent approach

- $c_* = c_{max}^{NT}(\tilde{u}) = c_{max}^{DD}(\tilde{u})$: halo independent limit
- $\tilde{u}$: intersection speed of NT and DD



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- To cover whole speed range, one may combine DD and NT
  - $u_T^{C-max} = v_{esc}(r)\sqrt{\frac{4m_\chi m_T}{(m_\chi - m_T)^2}}$
  - $\left(u_{th}^{DD}\right)^2 = \frac{m_T}{2\mu_{\chi T}^2} E_{R,th}$

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Halo independent approach

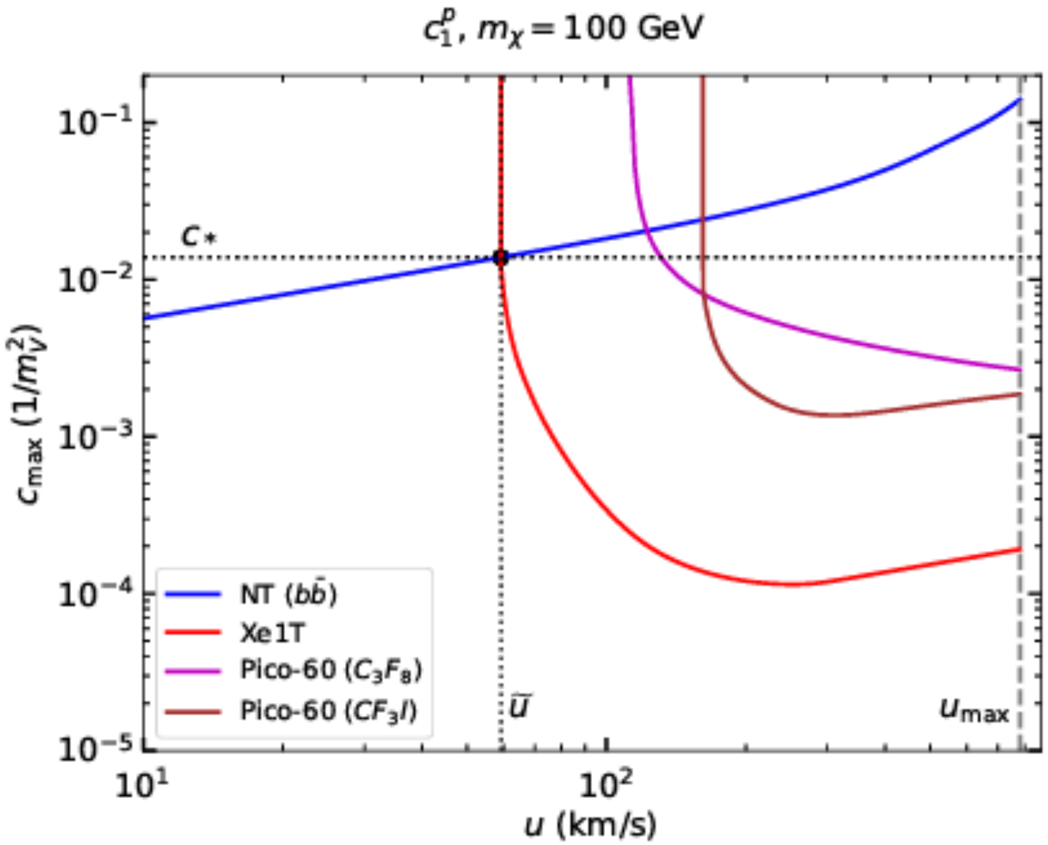
- Intersection:

$$(c^{\mathrm{NT}})^2{}_{\max}(u) \leq c_*^2 \qquad \text{for } \; 0 \leq u \leq \tilde{u}$$
$$(c^{\mathrm{DD}})^2{}_{\max}(u) \leq c_*^2 \qquad \text{for } \; \tilde{u} \leq u \leq u_{\max}$$

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du\, f(u)\right]^{-1} = \frac{c_*^2}{\delta},$$
$$c^2 \leq c_*^2 \left[\int_{\tilde{u}}^{u_{\max}} du\, f(u)\right]^{-1} = \frac{c_*^2}{1-\delta}.$$

$$\delta = \int_0^{\tilde{u}} du\, f(u)$$

$$c^2 \leq 2\, c_*^2.$$



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Halo independent approach

- If $(c^{DD})^2_{max}(u) > c_*^2$ at $u = u_{max}$

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du f(u)\right]^{-1} = \frac{c_*^2}{\delta},$$

$$c^2 \leq (c^{\mathrm{DD}})^2{}_{\mathrm{max}}(u_{\mathrm{max}}) \left[\int_{\tilde{u}}^{u_{\mathrm{max}}} du f(u)\right]^{-1} = \frac{(c^{\mathrm{DD}})^2{}_{\mathrm{max}}(u_{\mathrm{max}})}{1-\delta}.$$

$$c^2 \leq (c^{\mathrm{DD}})^2{}_{\mathrm{max}}(u_{\mathrm{max}}) + c_*^2.$$

- If $u_{th}^{DD} > u_{max}$

$$c^2 \leq (c^{\mathrm{NT}})^2 (u_{\mathrm{max}}) .$$

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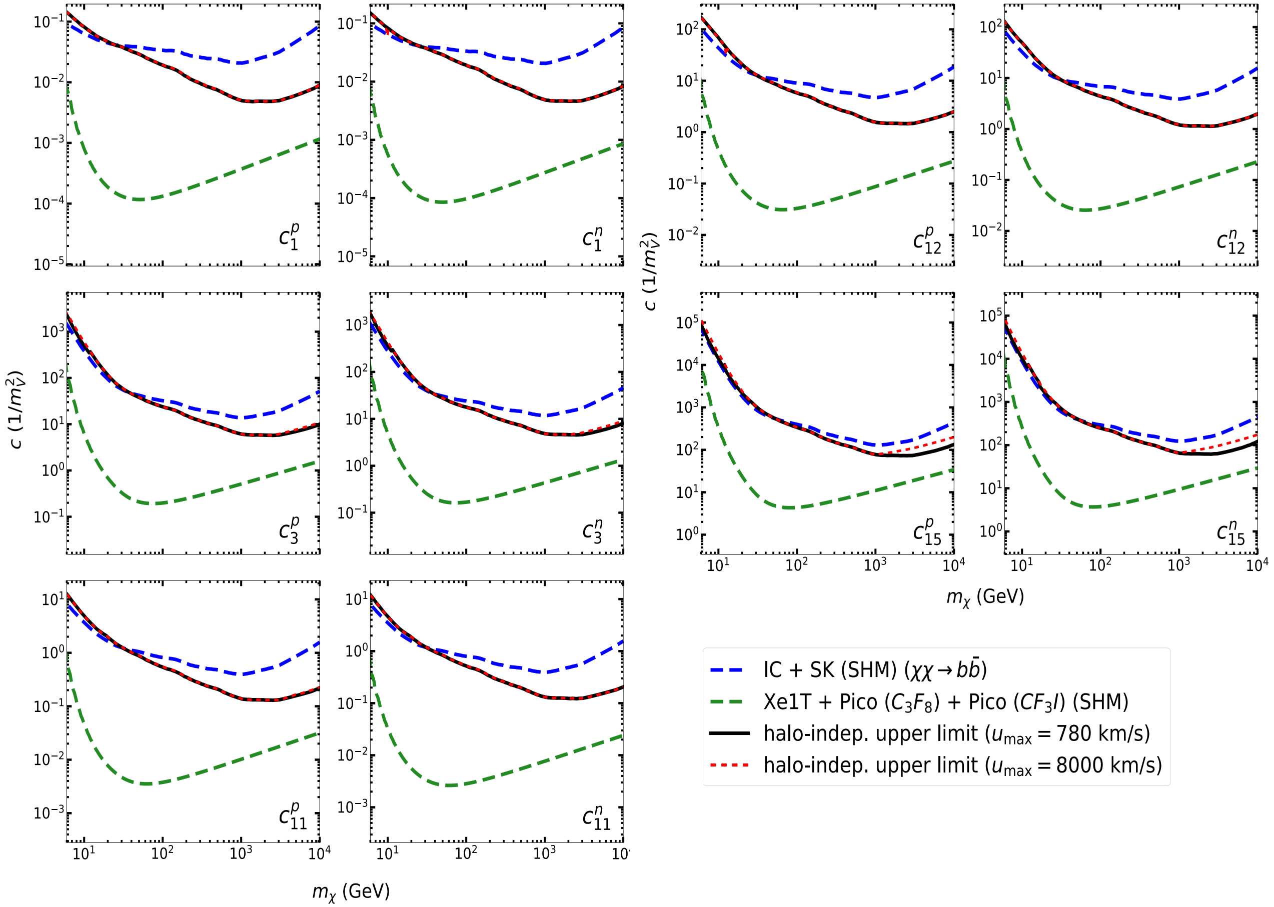
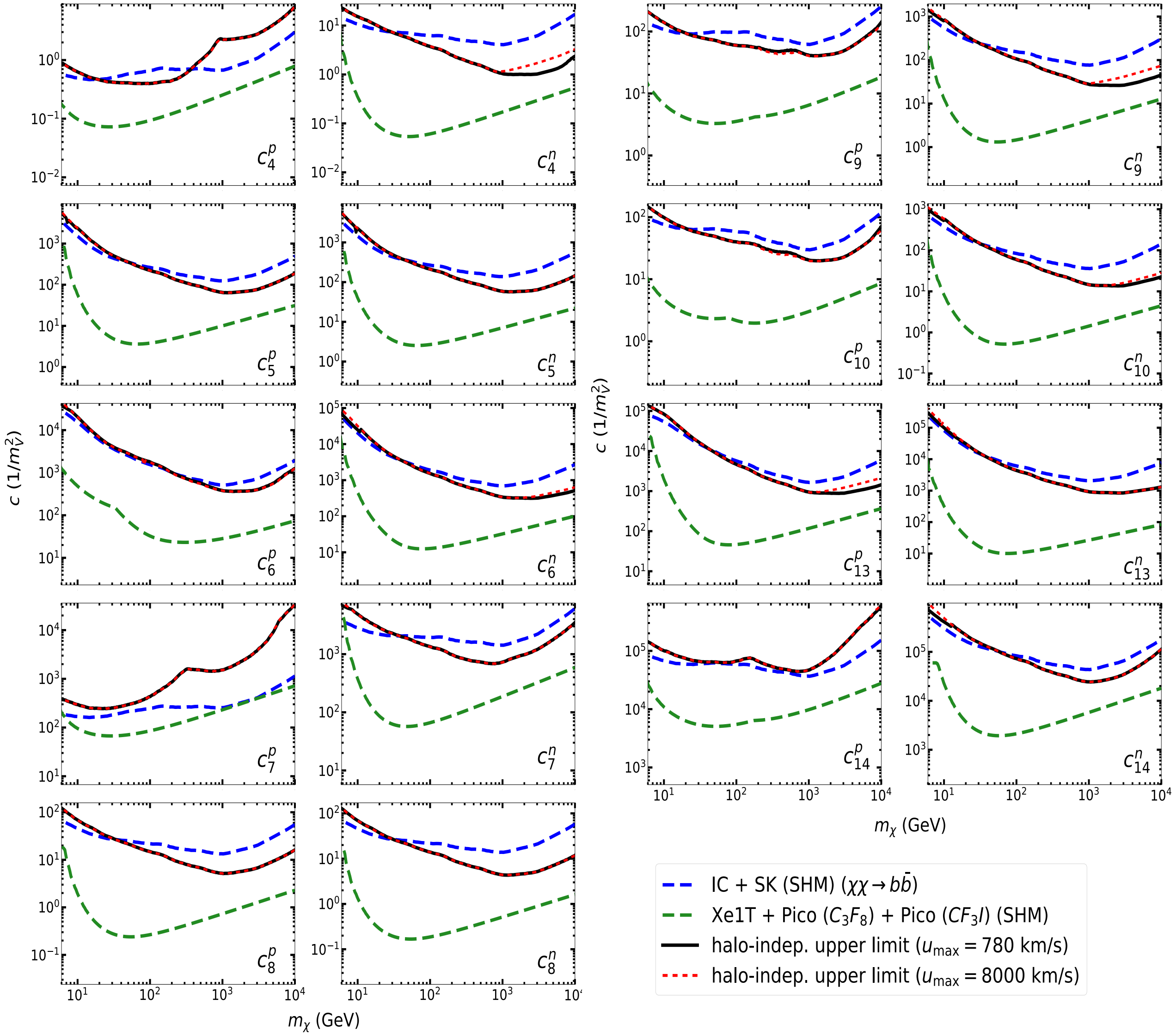
Halo independent approach





- Effect of large u_{max} is mild
  - factor less than 2

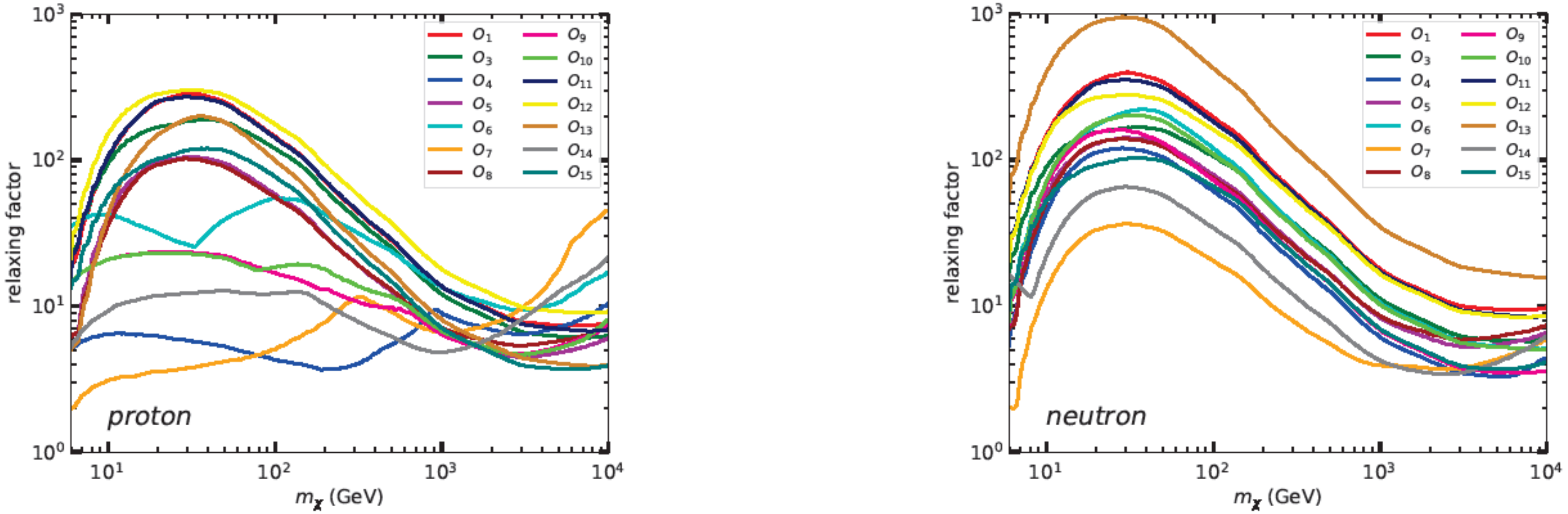
S. Kang, A. Kar, S. Scopel, Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering from direct detection and neutrino observations, JCAP03(2023)011 (arXiv:2212.05774)

Halo independent approach

- Relaxing factor

$$r_f^2 = \frac{2c_*^2}{(c_{\mathrm{SHM}}^{\mathrm{exp}})^2} = 2c_*^2 \int_0^{u_{\max}} du \frac{f_M(u)}{(c^{\mathrm{exp}})^2_{\max}(u)} = 2c_*^2 \left\langle \frac{1}{(c^{\mathrm{exp}})^2_{\max}} \right\rangle \simeq 2c_*^2 \left\langle \frac{1}{(c^{\mathrm{exp}})^2_{\max}} \right\rangle_{\mathrm{bulk}},$$

  - $\int_{bulk} du\, f_M(u) \approx 0.8$



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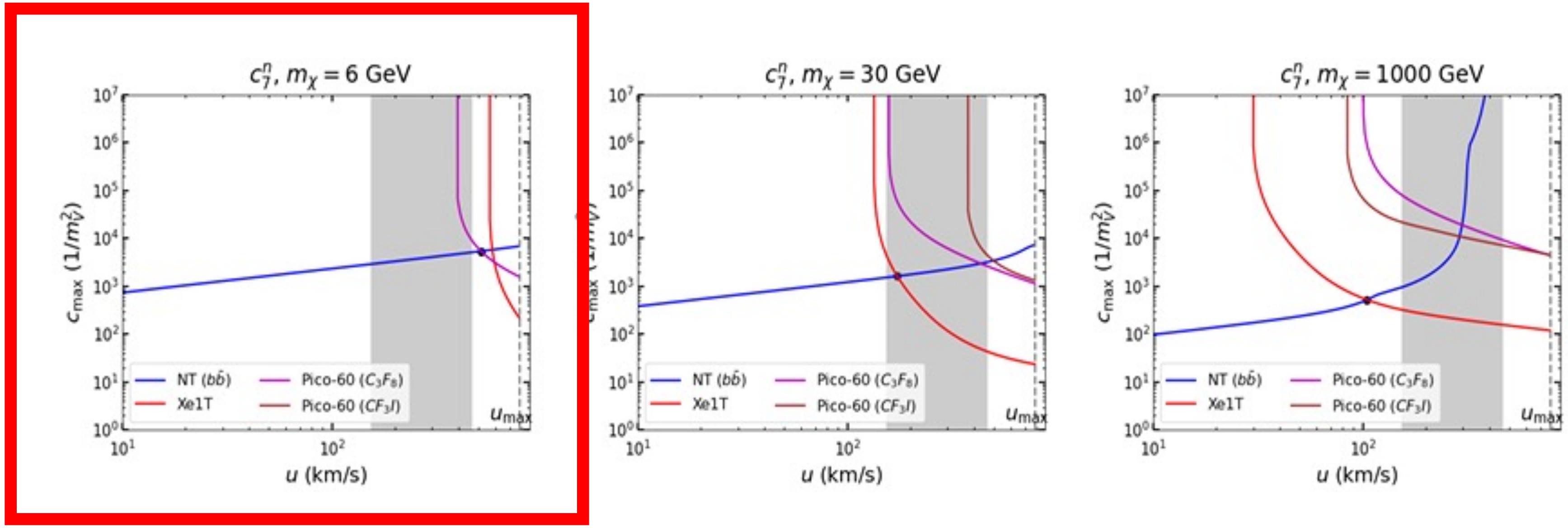
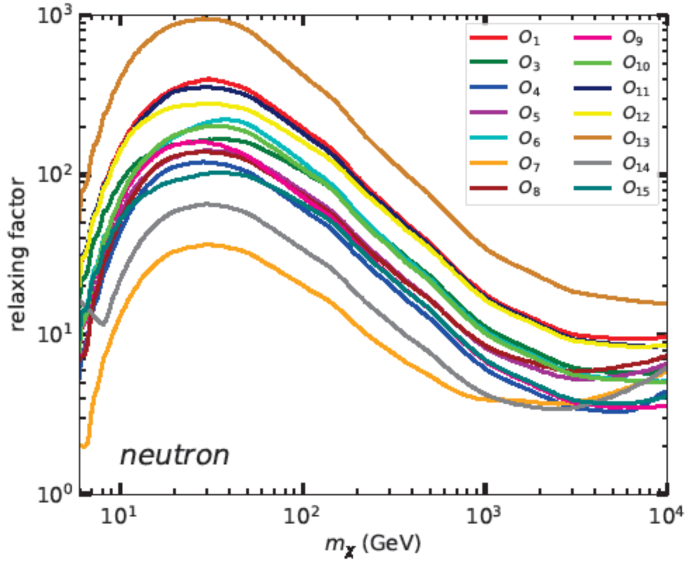


- small or large mass range
  - outside the bulk of Maxwellian
  - smooth dependence on u
- intermediate range (10 ~ 200 GeV)
  - inside the bulk of Maxwellian
  - steep dependence on u



$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\exp})^2_{\max}} \right\rangle_{\text{bulk}},$$

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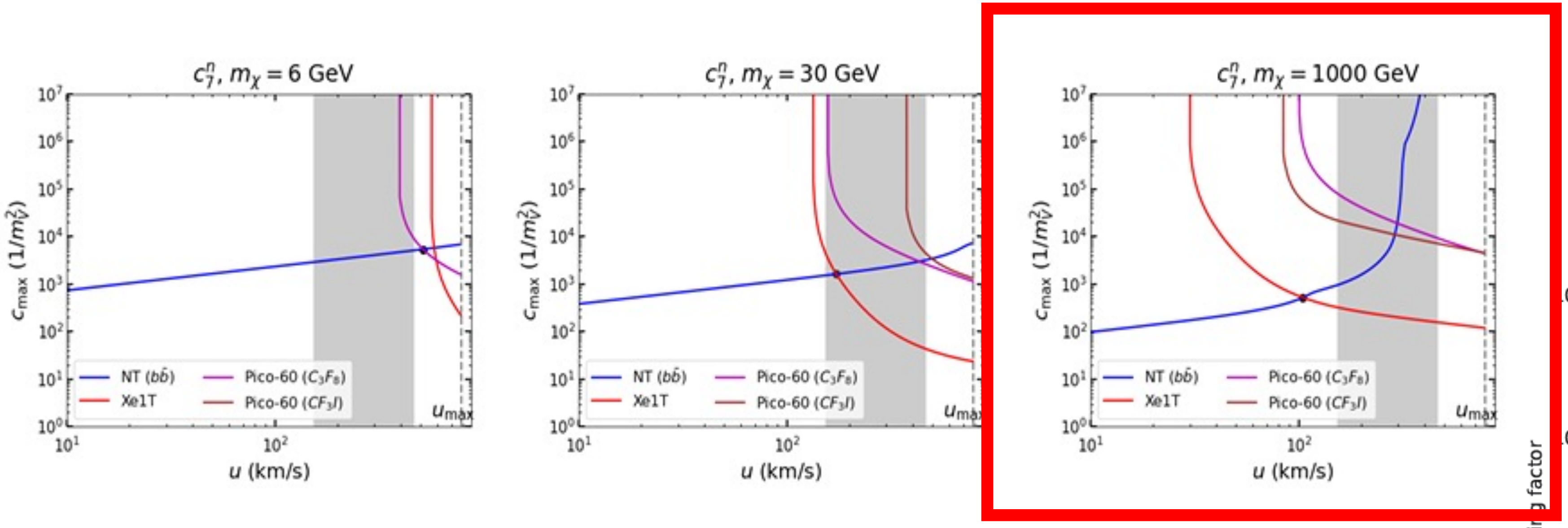

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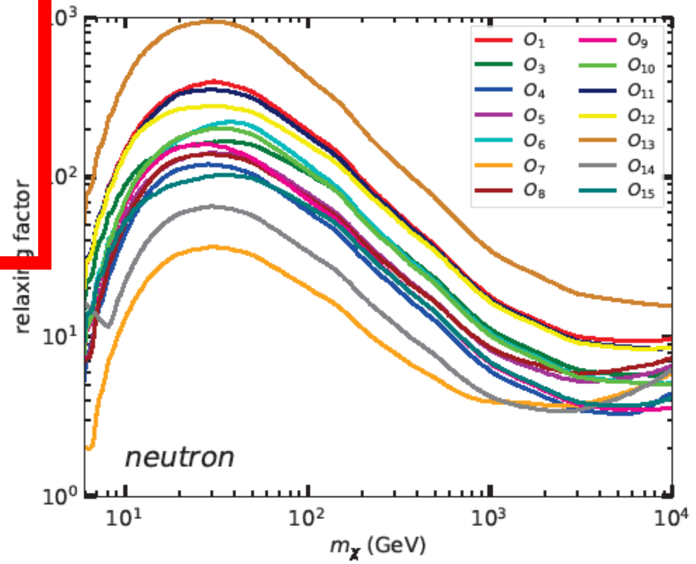

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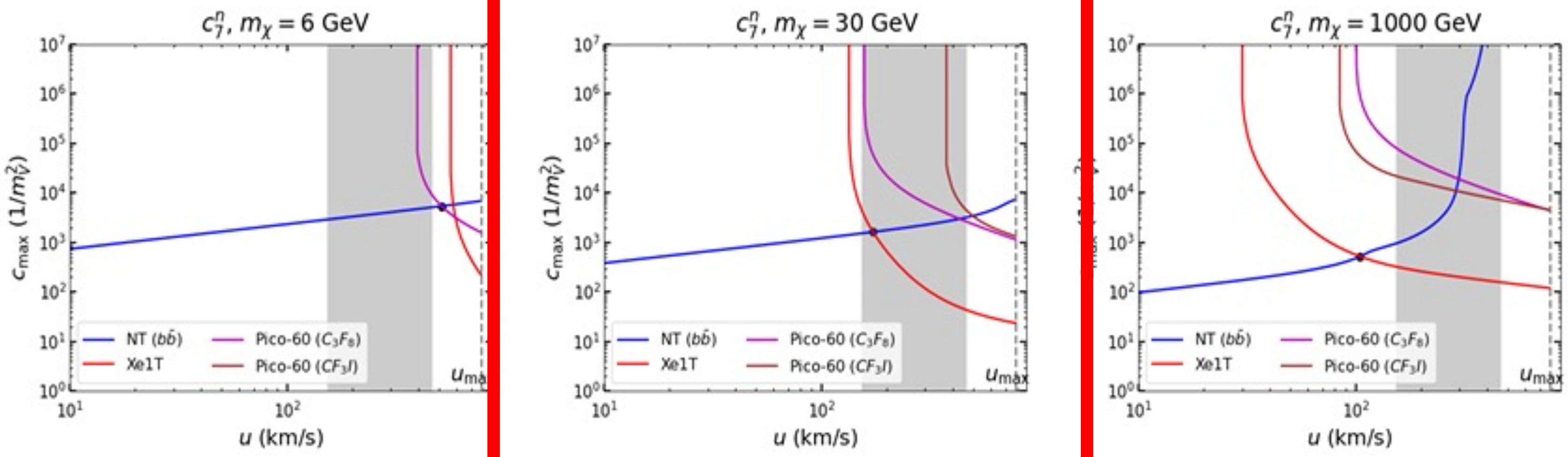

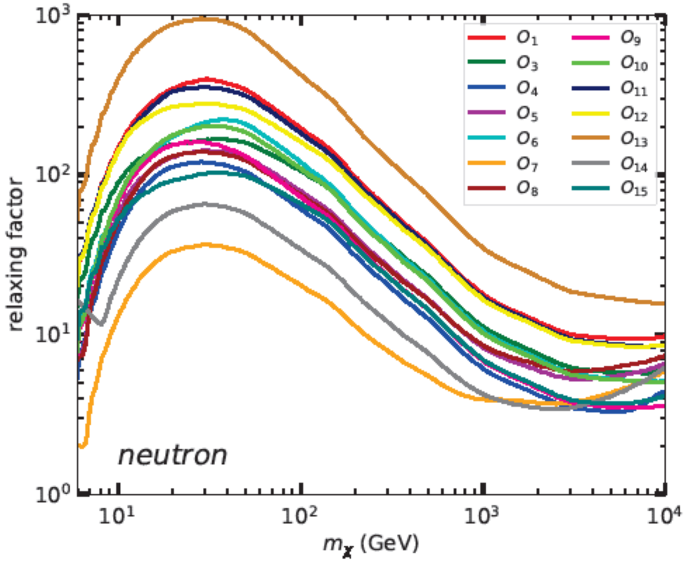

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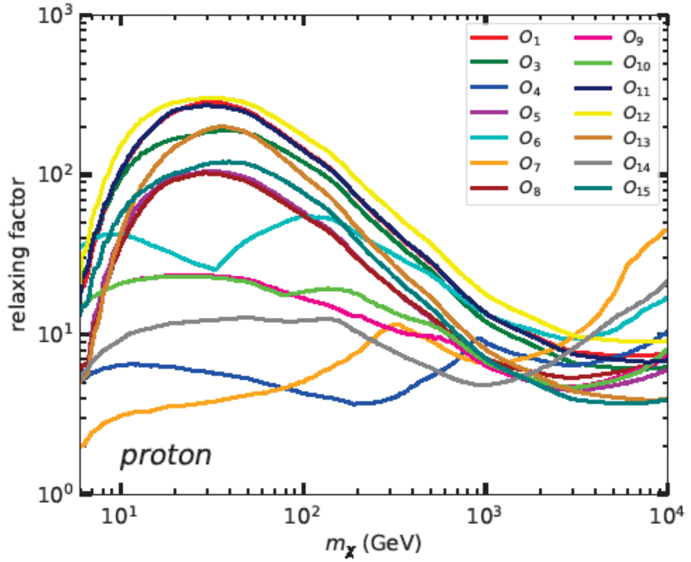
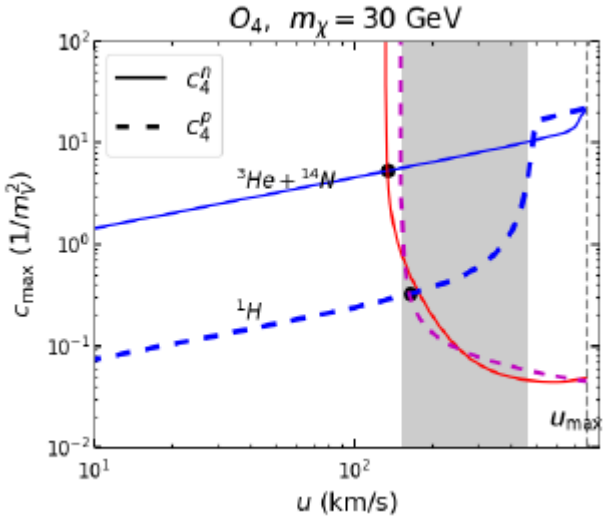
Halo independent approach

- small relaxing factors
  - $O_{4,7}$: SD with no q suppression
  - $O_{9,10,14}$: SD with q^2 suppression
  - O_6 : SD with q^4 suppression



operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$



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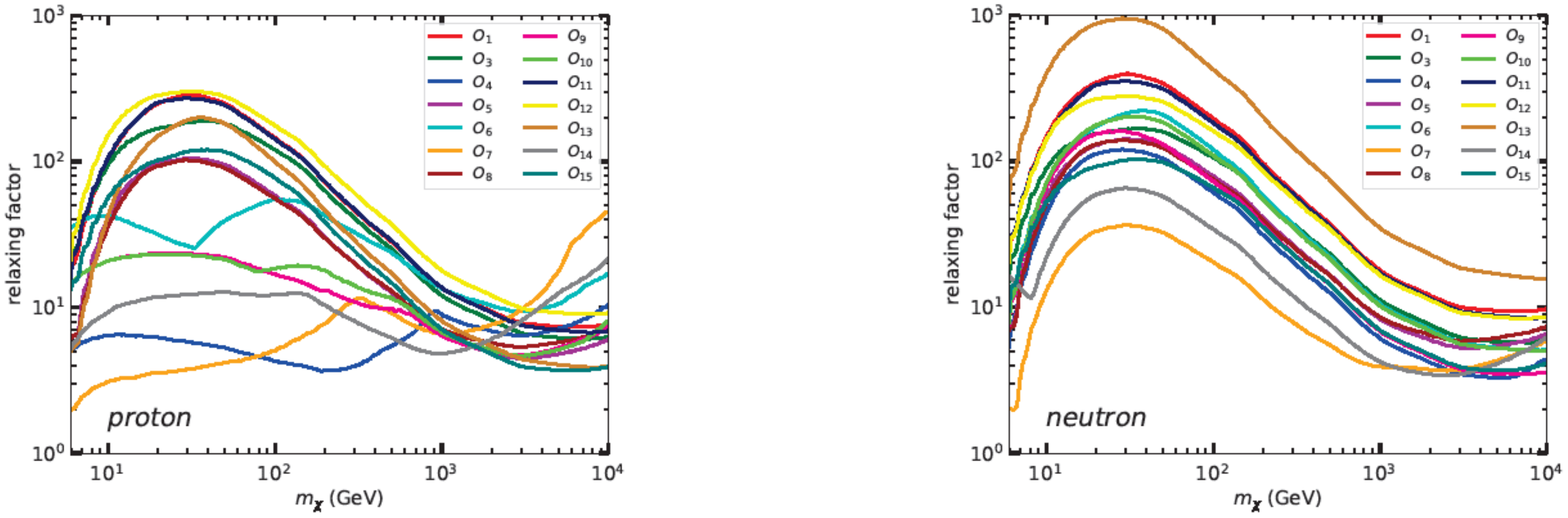
Halo independent approach

- High relaxing factor:
  the halo-independent method can weaken the bound

$$r_f^2 = \frac{2c_*^2}{(c_{\rm SHM}^{\rm exp})^2} = 2c_*^2 \int_0^{u_{\rm max}} du \frac{f_M(u)}{(c^{\rm exp})^2_{\rm max}(u)} = 2c_*^2 \left\langle \frac{1}{(c^{\rm exp})^2_{\rm max}} \right\rangle \simeq 2c_*^2 \left\langle \frac{1}{(c^{\rm exp})^2_{\rm max}} \right\rangle_{\rm bulk},$$



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Summary

- Halo-independent method can be applied to any speed distribution
- Combining results of direct detection experiments and capture in the Sun may provide halo-independent bounds on each couplings
- In most cases the relaxation of halo-independent bounds is moderate in low and high m_χ
- More moderate values of the relaxation is obtained with c_{SD}^p
- With the high relaxing factor, halo-independent method weaken the bounds; sensitive on speed distribution