Positivity Bounds on Higgs-Portal Dark Matter

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Summary of our Work

- New Physics exist! (i.e. Dark Matter…)
  Though we don't know its complete elements and description.
- We can write down 'effective theory' for New physics that valid up to some energy scale, instead of full theory.
- Effective theory is generally constrained by Positivity condition. This strongly restricts the validity of theory.
- We consider typical Higgs-portal scalar Dark Matter Model. And we combine experimental bound and positivity constraint.

Effective Theory for Higgs portal DM

Theory of DM(φ)-Higgs(H) Interaction up to 8-Dimension

Dim-4 operators

$$\mathscr{L}_{\text{dim}-4} \supset -\frac{1}{6\Lambda^4}c_1 m_\varphi^4 \varphi^4 - \frac{4}{6\Lambda^4}c_2 m_H^4 |H|^4 - \frac{4}{6\Lambda^4}c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2$$

Dim-6 operators

$$\mathscr{L}_{\text{dim}-6} \supset -\frac{8}{6\Lambda^4}c_2' \lambda_H m_H^2 |H|^6 - \frac{4}{6\Lambda^4}c_3' \lambda_H m_\varphi^2 \varphi^2 |H|^4$$ (Non-derivative operators)

$$+\frac{1}{6\Lambda^4}d_1 m_\varphi^2 \varphi^2 (\partial_\mu \varphi)^2 + \frac{4}{6\Lambda^4}d_2 m_H^2 |H|^2 |D_\mu H|^2$$
$$+\frac{2}{6\Lambda^4}d_3 m_\varphi^2 \varphi^2 |D_\mu H|^2 + \frac{2}{6\Lambda^4}d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2$$ (2-derivative operators)

Effective Theory for Higgs portal DM

Theory of DM(φ)-Higgs(H) Interaction up to 8-Dimension

Dim-8 operators

$$\mathscr{L}_{\rm dim-8} \supset -\frac{4}{6\Lambda^4} c_2'' \lambda_H |H|^8$$ (Non-derivative operators)

$$+\frac{4}{6\Lambda^4} d_2' \lambda_H |H|^4 (\partial_\mu \varphi)^2 + \frac{2}{6\Lambda^4} d_4' \lambda_H |H|^4 |\partial_\mu H|^2$$ (2-derivative operators)

$$\mathcal{L}_2 = \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} O^{(1)}_{H^2\varphi^2} + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} O^{(2)}_{H^2\varphi^2} + \frac{C_{\varphi^4}}{\Lambda^4} O_{\varphi^4} + \frac{C^{(1)}_{H^4}}{\Lambda^4} O^{(1)}_{H^4} + \frac{C^{(2)}_{H^4}}{\Lambda^4} O^{(2)}_{H^4} + \frac{C^{(3)}_{H^4}}{\Lambda^4} O^{(3)}_{H^4}$$ (4-derivative operators)

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi) \qquad O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

$$O_{\varphi^4} = \partial_\mu \varphi \partial^\mu \varphi \partial_\nu \varphi \partial^\nu \varphi$$

$$O^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H) \qquad O^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$

$$O^{(3)}_{H^4} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$$

Effective Theory for Higgs portal DM

Determining Coefficients

$$\mathscr{L}_{\text{dim}-8} \supset -\frac{4}{6\Lambda^4} c_2'' \lambda_H |H|^8$$ (Non-derivative operators)

$$+\frac{4}{6\Lambda^4} d_2' \lambda_H |H|^4 (\partial_\mu \varphi)^2 + \frac{2}{6\Lambda^4} d_4' \lambda_H |H|^4 |\partial_\mu H|^2$$ (2-derivative operators)

$$\mathcal{L}_2 = \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} O^{(1)}_{H^2\varphi^2} + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} O^{(2)}_{H^2\varphi^2} + \frac{C_{\varphi^4}}{\Lambda^4} O_{\varphi^4} + \frac{C^{(1)}_{H^4}}{\Lambda^4} O^{(1)}_{H^4} + \frac{C^{(2)}_{H^4}}{\Lambda^4} O^{(2)}_{H^4} + \frac{C^{(3)}_{H^4}}{\Lambda^4} O^{(3)}_{H^4}$$ (4-derivative operators)

Coefficients are determined by experiment.
In theoretical sides, they determined by UV complete model

Positivity Bound

Not every UV complete models are allowed.

And we can evaluate validity of theory at the EFT level

$$\mathcal{L}_2 = \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} O^{(1)}_{H^2\varphi^2} + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} O^{(2)}_{H^2\varphi^2} + \frac{C_{\varphi^4}}{\Lambda^4} O_{\varphi^4} + \frac{C^{(1)}_{H^4}}{\Lambda^4} O^{(1)}_{H^4} + \frac{C^{(2)}_{H^4}}{\Lambda^4} O^{(2)}_{H^4} + \frac{C^{(3)}_{H^4}}{\Lambda^4} O^{(3)}_{H^4}$$

$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$	$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$
$O_{\varphi^4} = \partial_\mu \varphi \partial^\mu \varphi \partial_\nu \varphi \partial^\nu \varphi$	
$O^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$	$O^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$O^{(3)}_{H^4} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$	

Any Model that reduced to
positivity-violating coefficient are dangerous

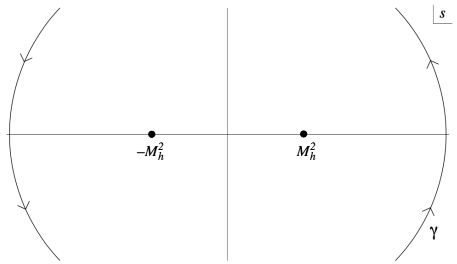
1. Positivity from Cross-section

Example) Forward Scattering Limit

Effective Lagrangian $\mathcal{L} = \partial^\mu \pi \partial_\mu \pi + \frac{c_3}{\Lambda^4}(\partial_\mu \pi \partial^\mu \pi)^2 + \ldots$ (2)

Integral of forward scattering ($t = 0$) amplitude for $\pi\pi \to \pi\pi$ process

$$0 = I = \frac{c_3}{\Lambda^4} + 2\,\frac{\mathrm{res}\mathcal{A}(s = M_h^2)}{(M_h^2)^3} = \oint_\gamma \frac{ds}{2\pi i}\frac{\mathcal{A}(s)}{s^3}$$



(M_h : Mass of heavy particle which completes UV theory)

Optical theorem (from unitary) implies $\frac{c_3}{\Lambda^4} = \frac{2}{\pi}\int ds \frac{s\sigma(s)}{s^3} \quad \to c_3 \geq 0$

Ref : Nima Arkani-Hamed et al, arXiv:hep-th/0602178v2

Positive property of cross-section argue positive coefficient

2. Positivity from Superluminality

Example) 4-derivative operator

$$\mathcal{L} \supset \frac{1}{2}(\partial\varphi)^2 + \frac{c}{\Lambda^4}(\partial_\mu\varphi\partial^\mu\varphi)^2 \quad \rightarrow \quad \text{e.o.m} \;\; \partial^2\varphi + \frac{4c}{\Lambda^4}(\partial^\mu\varphi)(\partial^\nu\varphi)\partial_\mu\partial_\nu\varphi + \cdots = 0$$

$$\rightarrow \quad \text{Fourier Transform} \quad \left(k^2 - \frac{4c}{\Lambda^4}(C\cdot k)^2 + \cdots\right)\varphi = 0\;, \quad C^\mu \equiv \partial^\mu\varphi_0$$

Background Solution of φ

Physical states is only able to propagate vector k with ($k^2 \leq 0$). *Thus* $c \geq 0$ *satisfied.*

(Disperse relation suggests $\omega^2 = v^2(k)|\vec{k}|^2$, $\frac{\omega^2}{|\vec{k}|^2} = v^2 \leq 1$, $\frac{\omega^2 - |\vec{k}|^2}{|\vec{k}|^2} = \frac{k^2}{|\vec{k}|^2} \leq 0$)

Ref : Nima Arkani-Hamed et al, arXiv:hep-th/0602178v2

Another perspective of positivity.

Different viewpoint, Same bound.

So Question :
How Positivity constrains
Dark Matter Physics?

We thus investigate Positivity bounds for
Higgs-Portal Dark Matter Model.
Combining with experimental constraints

Positivity Analysis of the model

Superpose State Analysis

Bound comes from Following Equations

$$|a\rangle = \sum_{i=1}^{5} u_i |i\rangle, \ |b\rangle = \sum_{i=1}^{5} v_i |i\rangle \qquad u_i v_j u_k^* v_l^* \frac{d^2}{ds^2} M(ij \to kl)(s, t=0)\Big|_{s\to 0} \geq 0$$

$$|H\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} |1\rangle + i|2\rangle \\ |3\rangle + i|4\rangle \end{pmatrix} \qquad |\varphi(\mathrm{DM})\rangle = |5\rangle$$

Results

$$C^{(1)}_{H^4} + C^{(2)}_{H^4} \geq 0, \qquad C^{(1)}_{H^2\varphi^2} \geq 0,$$

$$C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4} \geq 0, \qquad C_{\varphi^4} \geq 0,$$

$$C^{(2)}_{H^4} \geq 0, \qquad 4\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4})C_{\varphi^4}} \geq \left| C^{(1)}_{H^2\varphi^2} + 2C^{(2)}_{H^2\varphi^2} \right| - C^{(1)}_{H^2\varphi^2}.$$

Example - EFTheorize UV Completion

Extra Dimension model

Model Ref : arXiv:1306.4107v2

5D Metric

$$ds^2 = w(z)^2 \left(e^{-2r}(\eta_{\mu\nu} + G_{\mu\nu}) - (1+2r)^2 dz^2\right)$$

z : 5th dimension / $G_{\mu\nu}$: Fluctuation of 4D component of 5D metric

r : Radion. Fluctuation of Extra dimension size. This couples to Energy-momentum tensor

5D Action

$$\mathcal{S} \supset \int d^d x \sqrt{-g}\mathcal{L} \supset \int d^d x \sqrt{-g}\, w^2(z) \left(2rT - G_{\mu\nu}T^{\mu\nu}\right)$$

$T_{\mu\nu}$: Energy-momentum tensor of fields, including Dark Matter and Higgs. / $T = T^{\mu}_{\ \mu}$

$$\begin{aligned} T^H_{\mu\nu} &= (D_\mu H)^\dagger D_\nu H + (D_\nu H)^\dagger D_\mu H - g_{\mu\nu}[g^{\rho\sigma}(D_\rho H)^\dagger D_\sigma H] \\ &\quad + g_{\mu\nu}(m_H^2 |H|^2 + \lambda_H |H|^4), \end{aligned}$$

$$T^\varphi_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}(g^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi) + \frac{1}{2}g_{\mu\nu}m_\varphi^2\varphi^2$$

In this example, SM Higgs and Dark section connected via graviton-like $G_{\mu\nu}$ and radion r

Example - EFTheorize UV Completion

$$\mathcal{S} \supset \int d^d x \sqrt{-g}\mathcal{L} \supset \int d^d x \sqrt{-g}\, w^2(z) \left(2rT - G_{\mu\nu}T^{\mu\nu}\right)$$

Integrate out 5th dimension

$$\mathcal{L}_G = -\frac{c_H}{M} G^{\mu\nu} T^H_{\mu\nu} - \frac{c_\varphi}{M} G^{\mu\nu} T^\varphi_{\mu\nu}$$

$$\mathcal{L}_r = \frac{c^r_H}{\sqrt{6}M} r\, T^H + \frac{c^r_\varphi}{\sqrt{6}M} r\, T^\varphi$$

Integrate out $G_{\mu\nu}$

$$\mathcal{L}_{G,\text{eff}} = \frac{1}{4m_G^2 M^2}\left(2T_{\mu\nu}T^{\mu\nu} - \frac{2}{3}T^2\right)$$

$$\mathcal{L}_{r,\text{eff}} = \frac{1}{12m_r^2 M^2} T^2$$

$$T_{\mu\nu} = c_H T^H_{\mu\nu} + c_\varphi T^\varphi_{\mu\nu} \qquad T = c_H T^H + c_\varphi T^\varphi$$

Graviton-like Interaction

Radion-like Interaction

Positivity Analysis for the Example

i.e.) Effective Theory came from
Graviton-like & Radion-like interaction Bounded by…

$$C^{(1)}_{H^4} + C^{(2)}_{H^4} = \frac{2c_H^2\Lambda^4}{m_G^2 M^2} \geq 0,$$

$$C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4} = \frac{4c_H^2\Lambda^4}{3m_G^2 M^2} + \frac{(c_H^r)^2\Lambda^4}{3m_r^2 M^2} \geq 0,$$

$$C^{(2)}_{H^4} = \frac{c_H^2\Lambda^4}{m_G^2 M^2} \geq 0,$$

$$C^{(1)}_{H^2\varphi^2} = \frac{2c_H c_\varphi\Lambda^4}{m_G^2 M^2} \geq 0, \quad \text{for} \quad c_H c_\varphi \geq 0,$$

$$C_{\varphi^4} = \frac{c_\varphi^2\Lambda^4}{3m_G^2 M^2} + \frac{(c_\varphi^r)^2\Lambda^4}{12m_r^2 M^2} \geq 0,$$

$$2\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4})C_{\varphi^4}} \geq -\left(C^{(1)}_{H^2\varphi^2} + C^{(2)}_{H^2\varphi^2}\right) = -\frac{4c_H c_\varphi\Lambda^4}{3m_G^2 M^2} - \frac{c_H^r c_\varphi^r\Lambda^4}{3m_r^2 M^2},$$

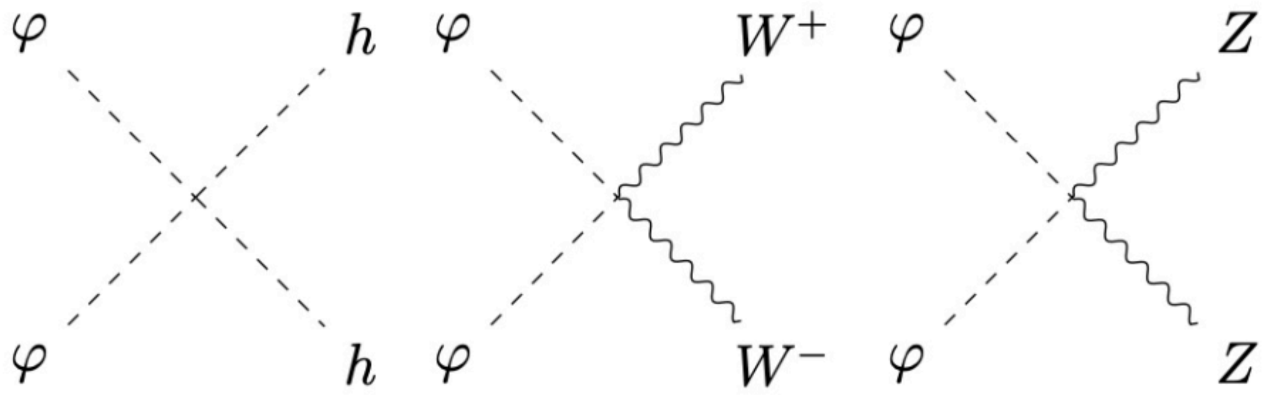
$$2\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4})C_{\varphi^4}} \geq C^{(2)}_{H^2\varphi^2} = -\frac{2c_H c_\varphi\Lambda^4}{3m_G^2 M^2} + \frac{c_H^r c_\varphi^r\Lambda^4}{3m_r^2 M^2}.$$

Positivity Bounds are almost automatically satisfied.

Required condition : $c_H c_\varphi \geq 0$

DM Analysis : Relic Abundance

3 main DM Annihilation Channels



In principle, $\varphi\varphi \to f\bar{f}$ processes are allowed.

However, they are highly constrained by experimental bounds.

Universal Coupling:
$c_3 = c_3', d_4 = d_4'$ required to Avoid Direct Detection Bound

$$+4c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2 + 4c_3' \lambda_H m_\varphi^2 \varphi^2 |H|^4 \Big)$$

$$2d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2 + 2d_4' \lambda_H |H|^4 (\partial_\mu \varphi)^2$$

$$|\mathcal{M}_{\varphi\varphi\to f\bar{f}}|^2 = \frac{4m_f^2 m_h^4 m_\varphi^4 (m_\varphi^2 - m_f^2)}{3\Lambda^8 (m_h^2 - 4m_\varphi^2)^2} \cdot \Big(2(c_3 - c_3') + d_4 - d_4'\Big)^2$$

DM Analysis : Relic Abundance

Boltzmann Equations for DM relic

$$\dot{n}_\varphi + 3Hn_\varphi = -\langle \sigma v_{\rm rel} \rangle_{\rm eff} \left(n_\varphi^2 - (n_\varphi^{\rm eq})^2 \right)$$

$$\langle \sigma v_{\rm rel} \rangle_{\rm eff} \quad = 2\langle \sigma v_{\rm rel} \rangle_{\varphi\varphi\to hh} + 2\langle \sigma v_{\rm rel} \rangle_{\varphi\varphi\to W^+W^-} + 2\langle \sigma v_{\rm rel} \rangle_{\varphi\varphi\to ZZ} + 2\langle \sigma v_{\rm rel} \rangle_{\varphi\varphi\to f\bar{f}},$$

$$\langle \sigma v_{\rm rel} \rangle_{\varphi\varphi\to ij} = \frac{|\mathcal{M}_{\varphi\varphi\to ij}|^2}{32\pi m_\varphi^2} \sqrt{1 - \frac{m_i^2}{m_\varphi^2}}$$

$$|\mathcal{M}_{\varphi\varphi\to f\bar{f}}|^2 = \frac{4m_f^2 m_h^4 m_\varphi^4 (m_\varphi^2 - m_f^2)}{3\Lambda^8 (m_h^2 - 4m_\varphi^2)^2} \cdot \Big(2(c_3 - c_3') + d_4 - d_4' \Big)^2$$

$$|\mathcal{M}_{\varphi\varphi\to ZZ}|^2 = \frac{1}{2} |\mathcal{M}_{\varphi\varphi\to W^+W^-}|^2 (m_W \to m_Z, s_W \to s_W c_W)$$

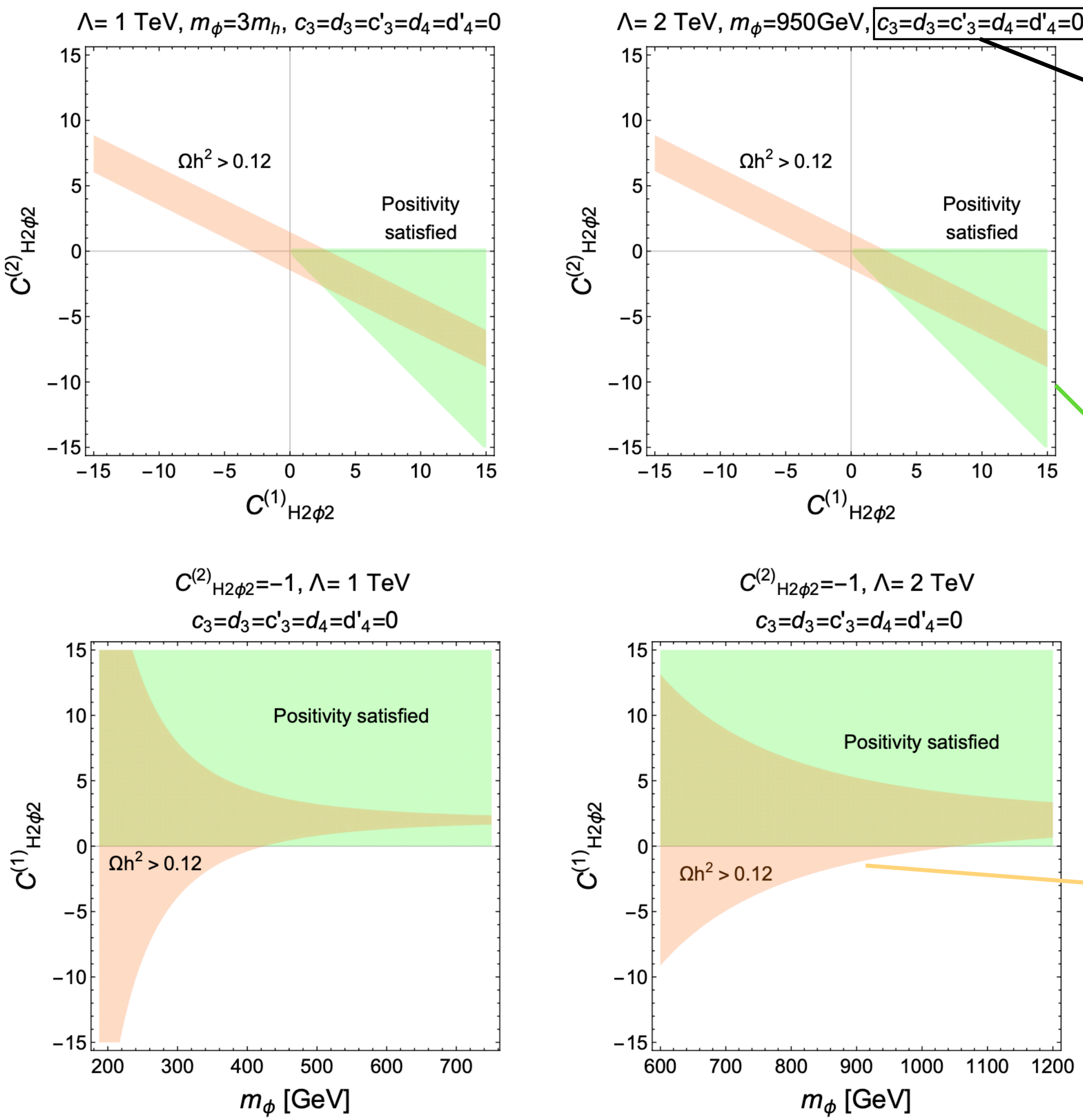
$s_w, c_w = \sin\theta_w, \cos\theta_w$, θ_w : Weinberg angle

$$\begin{aligned} |\mathcal{M}_{\varphi\varphi\to W^+W^-}|^2 \;=\; & \frac{2\pi^2\alpha^2 m_\varphi^4 v^4}{9\Lambda^8 m_W^4 s_W^4} \Bigg[9(C^{(1)}_{H^2\varphi^2})^2 (m_\varphi^2 - m_W^2)^2 - \frac{6C^{(1)}_{H^2\varphi^2}}{m_h^2 - 4m_\varphi^2} (m_\varphi^2 - m_W^2)(2m_\varphi^2 - m_W^2) \\ & \times \Big\{ \Big(2(c_3 - c_3') + d_3 + d_4 - d_4' - 3C^{(2)}_{H^2\varphi^2} \Big) m_h^2 + 4\Big(-d_3 + 3C^{(2)}_{H^2\varphi^2} \Big) m_\varphi^2 \Big\} \\ & + \frac{1}{(m_h^2 - 4m_\varphi^2)^2} (4m_\varphi^4 - 4m_\varphi^2 m_W^2 + 3m_W^4) \\ & \times \Big\{ \Big(2(c_3 - c_3') + d_3 + d_4 - d_4' - 3C^{(2)}_{H^2\varphi^2} \Big) m_h^2 + 4\Big(-d_3 + 3C^{(2)}_{H^2\varphi^2} \Big) m_\varphi^2 \Big\}^2 \Bigg], \end{aligned}$$

$$\begin{aligned} |\mathcal{M}_{\varphi\varphi\to hh}|^2 \;=\; & \frac{m_\varphi^4}{9\Lambda^8 (m_h^2 - 4m_\varphi^2)^2} \Bigg[\Big(2c_3 - d_3 + d_4 + 3C^{(2)}_{H^2\varphi^2} \Big) m_h^4 \\ & + \Big(4c_3 - 12c_3' + 6d_3 + 2d_4 - 6d_4' - 3C^{(1)}_{H^2\varphi^2} - 18C^{(2)}_{H^2\varphi^2} \Big) m_h^2 m_\varphi^2 \\ & + 4\Big(-2d_3 + 3C^{(1)}_{H^2\varphi^2} + 6C^{(2)}_{H^2\varphi^2} \Big) m_\varphi^4 \Bigg]^2, \end{aligned}$$

DM Analysis : Relic Abundance

Analysis for simplest case : Turning off all 6-dimension operators



Positivity Bound mainly comes from 8-dimension operators. Our paper includes the result for non-zero 6-dim coefficients

This Positivity Bound from

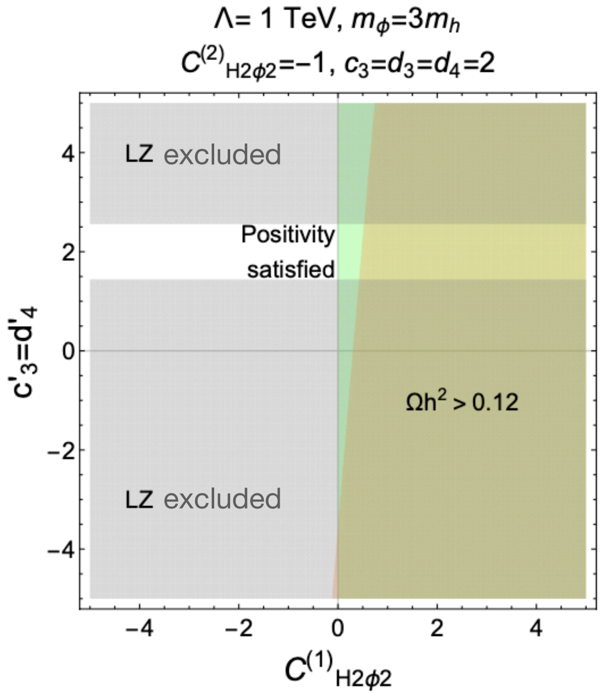
$$4\sqrt{(C^{(1)}_{H^4}+C^{(2)}_{H^4}+C^{(3)}_{H^4})C_{\varphi^4}} \geq \left|C^{(1)}_{H^2\varphi^2}+2C^{(2)}_{H^2\varphi^2}\right| - C^{(1)}_{H^2\varphi^2}.$$

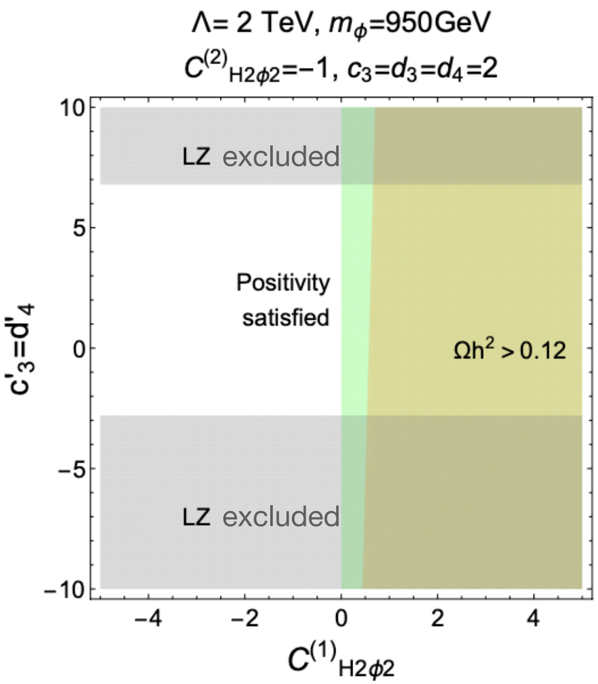
Boundary of the region reproduces proper DM relic

DM Analysis : Direct Detection

Bound from LUX-ZEPLIN Experiment

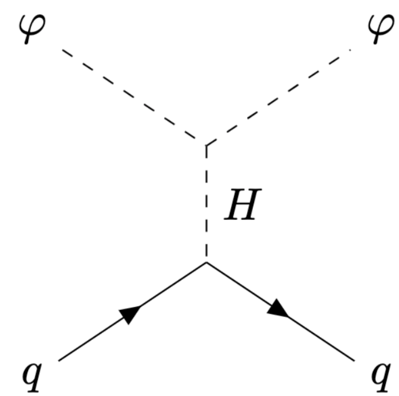
LZ : LUX-ZEPLIN Experiment
arXiv:2207.03764





$$\sigma_{\varphi-X} = \frac{\mu_N^2}{\pi m_\varphi^2 A^2}\Big(Z f_p + (A-Z) f_n\Big)^2$$

$$f_{p,n} = -\frac{(2(c_3 - c_3') - d_4 + d_4')m_{p,n} m_\varphi^2}{6\Lambda^4}\left(\sum_{q=u,d,s} f_{Tq}^{p,n} + \frac{2}{9} f_{TG}^{p,n}\right)$$

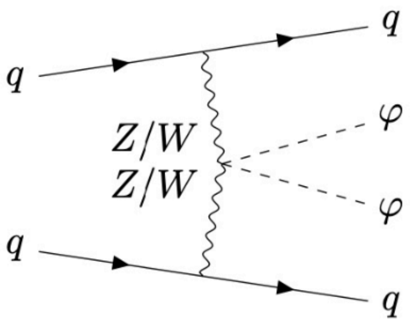


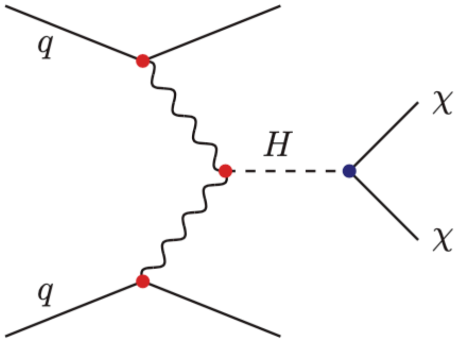
Bound are compatible with remaining parameter spaces!

Universal coupling : $c_3 = d_4 = d_4'$ assumed.

DM Analysis : Collider Searches

Diagram for LHC Detection





With Background Process
Mainly from $pp \to \nu\bar{\nu}jj$

Bound from LHC data

Ref : CMS Collaboration
arXiv:1905.07445

No bound for $C^{(1)}_{H^4}$ yet $\quad C^{(2)}_{H^4}/\Lambda^4 = [-2.7, 2.7]\,\text{TeV}^{-4} \quad C^{(3)}_{H^4}/\Lambda^4 = [-3.4, 3.4]\,\text{TeV}^{-4}$

$\sqrt{s} = 13$ TeV LHC, $L_{\text{int}} = 139\ \text{fb}^{-1}$	$\sigma^{\text{VBF}} \times B_{\text{inv}} = 0.11$ pb ($m_H = 1$ TeV)
$\Lambda = 1$ TeV, $m_\varphi = 375$ GeV	cross section from EFT operators
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (40, 40)$	0.28 pb
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (32, 32)$	0.11 pb
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (40, 0)$	0.012 pb
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (0, 40)$	0.097 pb

Conclusion (Returns)

- New Physics exist! (i.e. Dark Matter…)
  Though we don’t know its complete elements and description.
- Still, we can write down ‘effective theory’ for New physics, that valid up to some energy scale, instead of full theory.
- Effective theory is generally constrained by Positivity condition. This strongly restricts the validity of theory.
- We consider Higgs-portal scalar Dark Matter Model. And we combine experimental bound and positivity constraint.

Backup Slides

Extra Dimension Model

Model Ref : arXiv:13064107v2

$$ds^2 = w(z)^2\left(e^{-2r}(\eta_{\mu\nu} + G_{\mu\nu}) - (1+2r)^2 dz^2\right)$$

Massless 5D Metric fluctuation h_{MN} : from $ds^2 = (\eta_{MN} + h_{MN})dx^M dx^N$

h_{MN} can be decomposed to $h_{\mu\nu}$, $h_{\mu 5}$, and h_{55}. h_{55} correspond to the radion

r : Radion, Trace part of extra dimension metric

- Represents the size of compact dimension
- Couples to 4D Energy-momentum tensor

$w(z)^2 e^{-2r}$: Warp factor

- Similar to scale factor of Flat FRW metric, only filled with vacuum Energy

Backup Slides

Coefficient for Dim-8 operators

$$\mathcal{L}_{\text{Higgs-portal}} = \mathcal{L}_1 + \mathcal{L}_2$$

$$\begin{aligned}\mathcal{L}_1 \;=\; & -\frac{1}{6\Lambda^4}\Big(c_1 m_\varphi^4 \varphi^4 + 4c_2 m_H^4 |H|^4 + 8c_2' \lambda_H m_H^2 |H|^6 + 4c_2'' \lambda_H^2 |H|^8 \\ & + 4c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2 + 4c_3' \lambda_H m_\varphi^2 \varphi^2 |H|^4\Big) \\ & + \frac{1}{6\Lambda^4}\Big(d_1 m_\varphi^2 \varphi^2 (\partial_\mu \varphi)^2 + 4d_2 m_H^2 |H|^2 |D_\mu H|^2 + 4d_2' \lambda_H |H|^4 |D_\mu H|^2 \\ & + 2d_3 m_\varphi^2 \varphi^2 |D_\mu H|^2 + 2d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2 + 2d_4' \lambda_H |H|^4 (\partial_\mu \varphi)^2\Big),\end{aligned}$$

$$\begin{aligned}\mathcal{L}_2 \;=\; & \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} O^{(1)}_{H^2\varphi^2} + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} O^{(2)}_{H^2\varphi^2} \\ & + \frac{C_{\varphi^4}}{\Lambda^4} O_{\varphi^4} + \frac{C^{(1)}_{H^4}}{\Lambda^4} O^{(1)}_{H^4} + \frac{C^{(2)}_{H^4}}{\Lambda^4} O^{(2)}_{H^4} + \frac{C^{(3)}_{H^4}}{\Lambda^4} O^{(3)}_{H^4}\end{aligned}$$

$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$	$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$
$O_{\varphi^4} = \partial_\mu \varphi \partial^\mu \varphi \partial_\nu \varphi \partial^\nu \varphi$	
$O^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$	$O^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$O^{(3)}_{H^4} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$	

$$\frac{C_{\varphi^4}}{\Lambda^4} = \frac{c_\varphi^2}{3m_G^2 M^2} + \frac{(c_\varphi^r)^2}{12 m_r^2 M^2},$$

$$\frac{C^{(1)}_{H^4}}{\Lambda^4} = \frac{C^{(2)}_{H^4}}{\Lambda^4} = \frac{c_H^2}{m_G^2 M^2}, \quad \frac{C^{(3)}_{H^4}}{\Lambda^4} = -\frac{2c_H^2}{3m_G^2 M^2} + \frac{(c_H^r)^2}{3m_r^2 M^2},$$

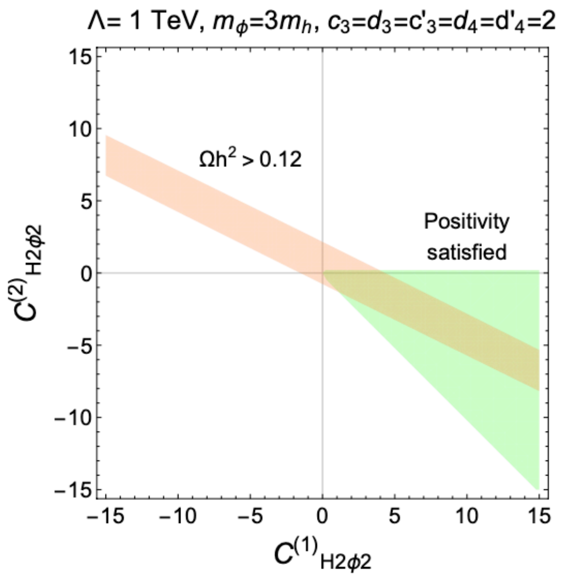
$$\frac{c_1}{\Lambda^4} = \frac{d_1}{\Lambda^4} = \frac{c_\varphi^2}{m_G^2 M^2} - \frac{2(c_\varphi^r)^2}{m_r^2 M^2},$$

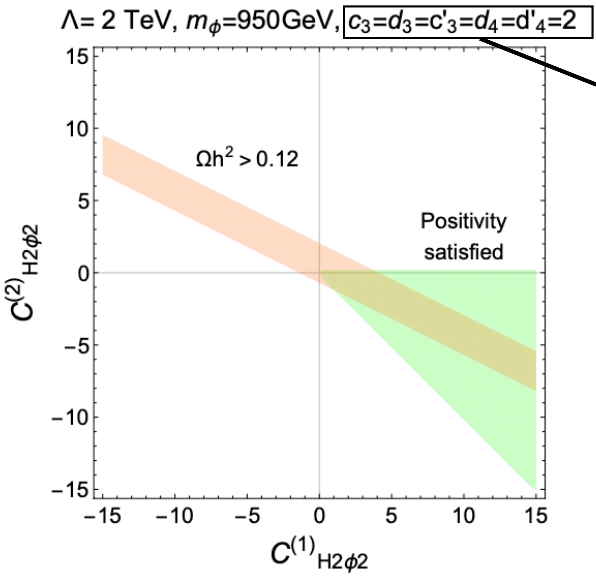
$$\frac{c_2^{(\prime,\prime\prime)}}{\Lambda^4} = \frac{d_2^{(\prime)}}{\Lambda^4} = \frac{c_H^2}{m_G^2 M^2} - \frac{2(c_H^r)^2}{m_r^2 M^2} = -3\frac{C^{(1)}_{H^4}}{\Lambda^4} - 6\frac{C^{(3)}_{H^4}}{\Lambda^4}.$$

$$\frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} = \frac{2c_H c_\varphi}{m_G^2 M^2}, \quad \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} = -\frac{2c_H c_\varphi}{3m_G^2 M^2} + \frac{c_H^r c_\varphi^r}{3m_r^2 M^2},$$

$$\frac{c_3^{(\prime)}}{\Lambda^4} = \frac{d_3}{\Lambda^4} = \frac{d_4^{(\prime)}}{\Lambda^4} = \frac{c_H c_\varphi}{m_G^2 M^2} - \frac{2c_H^r c_\varphi^r}{m_r^2 M^2} = -\frac{3}{2}\frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} - 6\frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4}$$

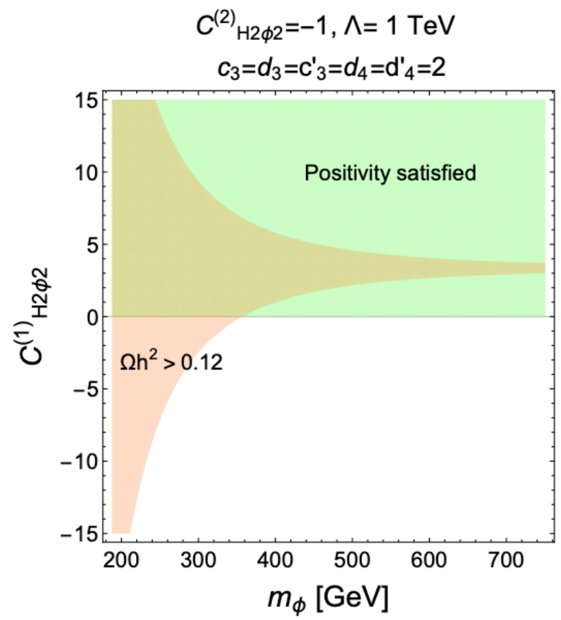
Backup Slides

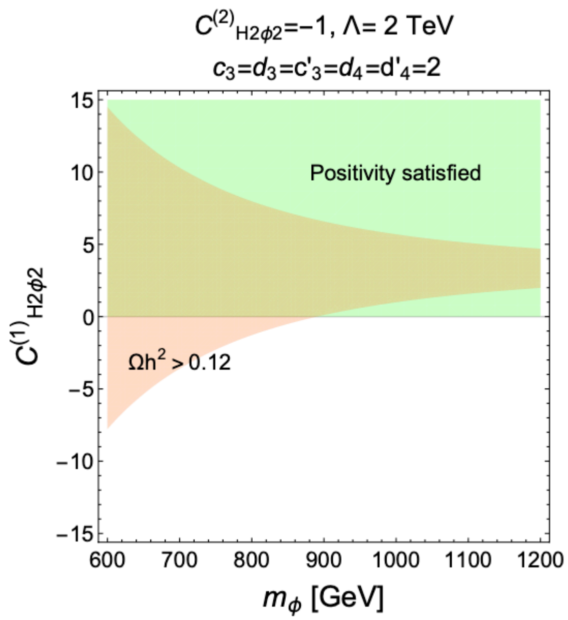




Of course we may turn on 6-dimension operators.
This changes relic abundance.

Positivity may affected if UV completion relates coefficients of 6-dim operators and 8-dim operators.





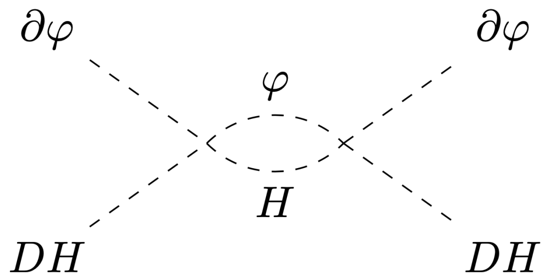
Backup Slides

Loop Contribution - How dim-6 operator correct dim-8 coefficient

Example : dim-8 operator of

$$\mathcal{L}_{\rm dim-8} = \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4}(D_\mu H)^\dagger (D_\nu H)\partial^\mu\varphi\partial^\nu\varphi + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4}(D_\mu H)^\dagger (D^\mu H)\partial_\nu\varphi\partial^\nu\varphi.$$

Additional contribution from these dim-6



$$\mathcal{L}_{\rm dim-6} = \frac{1}{3\Lambda^4}\left(\tilde{d}_3\varphi^2|D_\mu H|^2 + \tilde{d}_4|H|^2(\partial_\mu\varphi)^2\right)$$

Improved coefficient

$$\hat{C}^{(1)}_{H^2\varphi^2} = C^{(1)}_{H^2\varphi^2} + \frac{1}{9(4\pi)^2\Lambda^4}\left(\frac{26}{9}(\tilde{d}_3^2 + \tilde{d}_4^2) + \frac{40}{9}\tilde{d}_3\tilde{d}_4\right) + \frac{1}{9(4\pi)^2\Lambda^4}\frac{4}{3}(\tilde{d}_3 + \tilde{d}_4)^2 \ln\frac{\mu^2}{|s|},$$

$$\hat{C}^{(2)}_{H^2\varphi^2} = C^{(2)}_{H^2\varphi^2} - \frac{1}{9(4\pi)^2\Lambda^4}\frac{5}{9}(\tilde{d}_3 + \tilde{d}_4)^2 - \frac{1}{9(4\pi)^2\Lambda^4}\frac{1}{3}(\tilde{d}_3 + \tilde{d}_4)^2 \ln\frac{\mu^2}{|s|}.$$

We obtain minor contribution from dim-6 1-loop correction