Revisiting the gravitational lensing effect using radio wave polarization

arXiv:2306.07157

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XVI International Conference on Interconnections between Particle Physics and Cosmology
IBS, June 13, 2023

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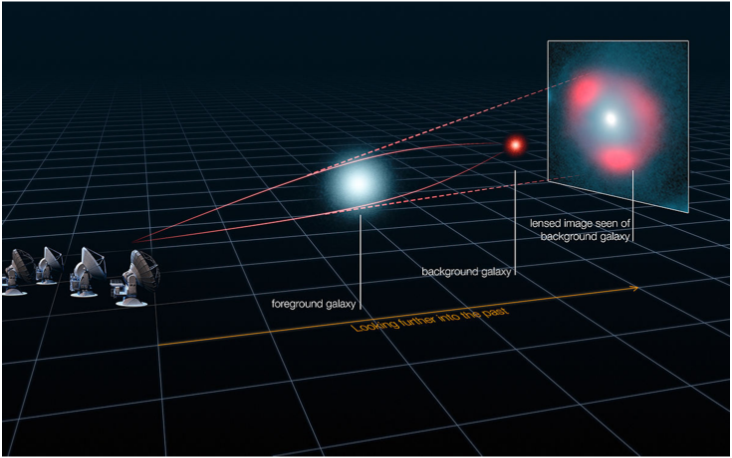
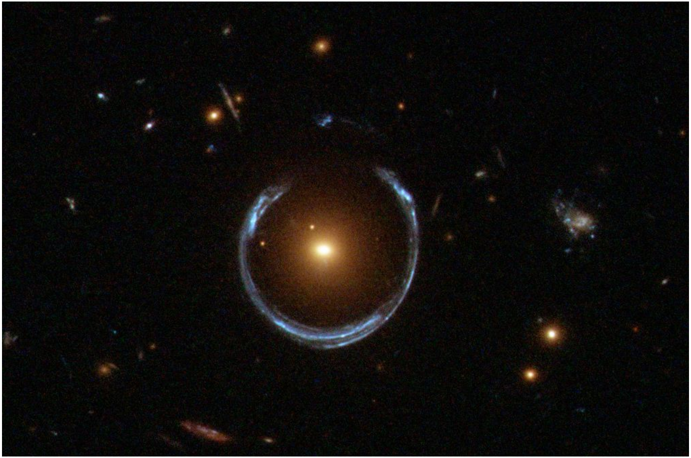
Gravitational lensing



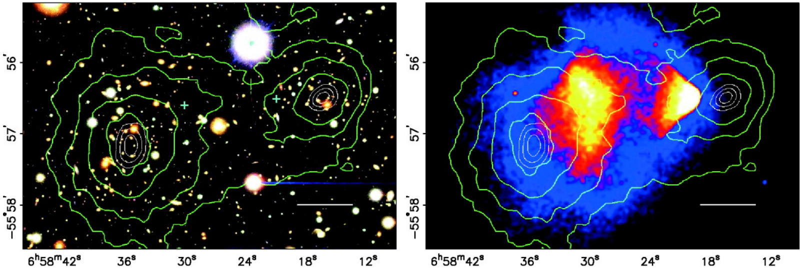
https://www.eso.org/public/images/eso1313b/

Gravitational lensing



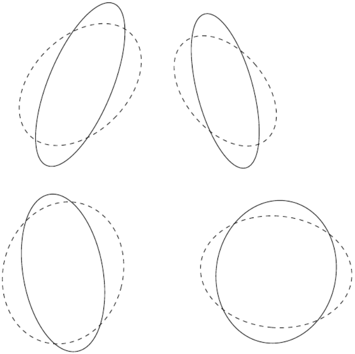
A Horseshoe Einstein Ring from Hubble Space Telescope.
https://commons.wikimedia.org/wiki/File:A_Horseshoe_Einstein_Ring_from_Hubble.JPG

Bullet Cluster, a direct empirical proof for dark matter



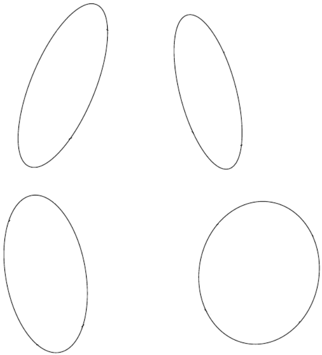
D. Clowe et al., “A direct empirical proof of the existence of dark matter,” [arXiv:astro-ph/0608407].

Galaxy lensing analysis: the traditional method



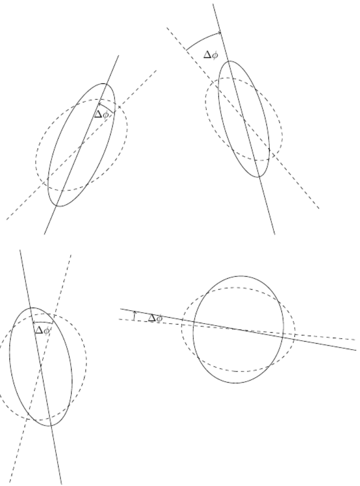
-Solid line: observed galactic image
-Dotted line: original galactic image

Galaxy lensing analysis: the traditional method



-Solid line: observed galactic image
-The original galactic images are not visible.

Galaxy lensing analysis: the new method



-straight solid line: orientation of galaxy after gravitational lensing

-straight dotted line: orientation of galaxy before gravitational lensing

Convergence and shear

θ_1, θ_2: the observed positions in terms of the two orthogonal coordinates on the sky. β_1, β_2: the actual positions

$$A_{ij} \equiv \frac{\partial \beta_i}{\partial \theta_j}$$

$$A = \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix}$$

κ:convergence (size change) γ: shear (shape change)
Eigenvalues ($\gamma = \sqrt{\gamma_1^2+\gamma_2^2}$)

$$a_+ = 1-\kappa+\gamma, \quad a_- = 1-\kappa-\gamma$$

$$a_+ a_- = (1-\kappa)^2 - \gamma^2 = 1 - 2\kappa + \mathcal{O}(\kappa^2) + \mathcal{O}(\gamma^2)$$

size change: not directly observable
shape change: directly observable

Reduced shear

Reduced shear

$$g_\alpha \equiv \frac{\gamma_\alpha}{1-\kappa}$$

$$A = \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{pmatrix}$$

From g_1, g_2 to κ, Clowe et al. used

$$\nabla \ln(1-\kappa) = \frac{1}{1-g_1^2-g_2^2}\begin{pmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{pmatrix}\begin{pmatrix} g_{1,1}+g_{2,2} \\ g_{2,1}-g_{1,2} \end{pmatrix}$$

Traditional method to estimate reduced shear

$I(\theta)$: the brightness distribution.
$\bar{\theta}_i$: is the light-intensity weighted center of the galaxy image

$$\int d^2\theta I(\theta)(\theta_i - \bar{\theta}_i) = 0$$

The quadrupole moment:

$$Q_{ij} = \frac{\int d^2\theta I(\theta)(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta I(\theta)}$$

The eigenvalues of this 2×2 matrix:

$$a^2 = \frac{T + \sqrt{Q_1^2 + Q_2^2}}{2}, \quad b^2 = \frac{T - \sqrt{Q_1^2 + Q_2^2}}{2}$$

where

$$Q_1 \equiv Q_{11} - Q_{22}, \quad Q_2 \equiv 2Q_{12}, \quad T = Q_{11} + Q_{22}$$

a: the semi-major axis, b: the semi-minor axis

Traditional method to estimate reduced shear

The eigenvalues of this 2×2 matrix:

$$a^2 = \frac{T + \sqrt{Q_1^2 + Q_2^2}}{2}, \quad b^2 = \frac{T - \sqrt{Q_1^2 + Q_2^2}}{2}$$

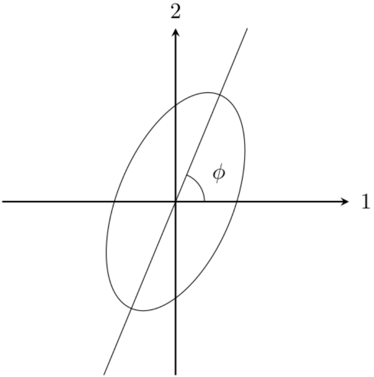
$$a^2 - b^2 = \sqrt{Q_1^2 + Q_2^2}, \quad a^2 + b^2 = T$$

$$\frac{a^2 - b^2}{a^2 + b^2} = \frac{\sqrt{Q_1^2 + Q_2^2}}{T} = \left| \frac{Q_1 + iQ_2}{T} \right|$$

Complex ellipticity: $e_1 + ie_2 \equiv \dfrac{Q_1 + iQ_2}{T}$

Traditional method to estimate reduced shear

ϕ: the angle between the axis 1 and the major axis of the observed elliptical image of galaxy



$$e_1 + ie_2 = \left(\frac{a^2 - b^2}{a^2 + b^2}\right) e^{2i\phi}, \quad \text{i.e.,} \quad \tan(2\phi) = \frac{e_2}{e_1} = \frac{2Q_{12}}{Q_{11} - Q_{22}}$$

Traditional method to estimate reduced shear

Gravitational lensing changes e_α

$$\delta e_\alpha = P^\gamma_{\alpha\beta} g_\beta$$

$P^\gamma_{\alpha\beta} = 2(\delta_{\alpha\beta} - e_\alpha e_\beta)$: the shear susceptibility tensor
g_β (g_1 and g_2): the reduced shear.

$$g_\beta = (P^\gamma_{\alpha\beta})^{-1}(e_\alpha - e^{(s)}_\alpha)$$

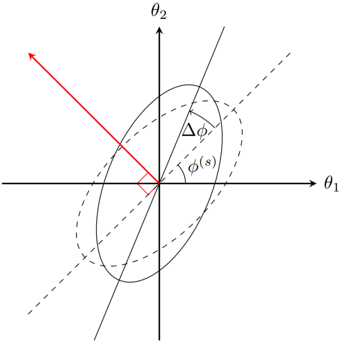
(s): "source."
The randomness of the original orientation of galaxies:

$$\langle e^{(s)}_\alpha \rangle = 0$$

Average of $(P^\gamma_{\alpha\beta})^{-1} e_\alpha$ is g_β.

Image rotation from lensing

Francfort et al., “Image rotation from lensing,” [arXiv:2106.08631]



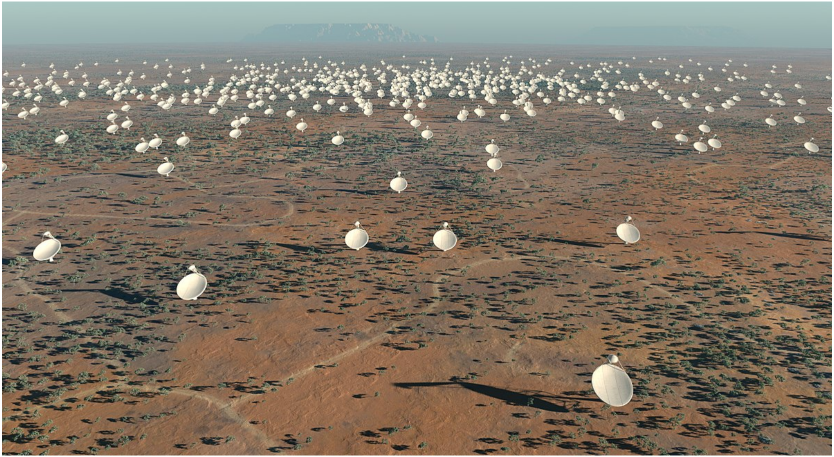
-Red arrow: polarization direction of radio waves

$$\theta_{\mathrm{pol}} = \phi^{(s)} + 90^\circ$$

$$\gamma_2 \cos 2\phi^{(s)} - \gamma_1 \sin 2\phi^{(s)} = \frac{\varepsilon^2}{2-\varepsilon^2}\Delta\phi$$

$\varepsilon = \sqrt{1 - \dfrac{b^2}{a^2}}$, the usual ellipticity

Square Kilometer Array (SKA)



https://www.skatelescope.org/copyright/

Not shear, but reduced shear

Francfort et al. deliberately ignored the size change (shear).

$$D = D_s \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \exp\begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} \tag{1}$$

The exponential is given by

$$\exp\begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} = \begin{pmatrix} 1+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\gamma_1 \end{pmatrix} \tag{2}$$

$$A = \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{pmatrix}$$

$\log(a_+/a_-)/2 = \log((1+\gamma)/(1-\gamma))/2 = \gamma + \mathcal{O}(\gamma^3)$

$$\frac{1}{2}\log\left(\frac{1-\kappa+\gamma}{1-\kappa-\gamma}\right) = \frac{1}{2}\log\left(\frac{1+g}{1-g}\right) = g + \mathcal{O}(g^3)$$

Not the shear, but the reduced shear

- If they considered the size change, the shear in their formula must be replaced by the reduced shear.

$$g_2 \cos 2\phi^{(s)} - g_1 \sin 2\phi^{(s)} = \frac{\varepsilon^2}{2-\varepsilon^2}\Delta\phi$$

“Linear regression”

- $$\varepsilon^2 = 1 - \frac{b^2}{a^2}, \qquad \frac{\varepsilon^2}{2-\varepsilon^2} = \frac{a^2-b^2}{a^2+b^2} = |e_1 + ie_2|$$

- $$g_2 \cos 2\phi^{(s)} - g_1 \sin 2\phi^{(s)} = |e_1 + ie_2|\Delta\phi$$

Required sensitivity of polarization

- How accurately do we need to measure the polarization direction for the new method to yield more accurate results than the traditional method?
- The traditional method

$$\sigma_g \approx \frac{0.27}{\sqrt{N}}$$

- The new method

$$\sigma_g \approx \frac{1.53\ \delta(\Delta\phi)}{\sqrt{N}}$$

- $\Delta\phi$ has to be measured better than 10°.

Conclusion

- Future radio wave surveys such as SKA will measure the polarization direction of the radio waves from galaxies.
- From this polarization direction, we can deduce the original orientation of each galaxy.
- By comparing this with the observed orientation of each galactic image, we can know how much angle the galactic image is rotated due to gravitational lensing.
- Then, we can analyze the gravitational lensing effect more accurately, provided that the polarization direction can be measured within an error of 10°.
- If the gravitational lensing effect in the Bullet Cluster is reanalyzed, using the polarization data of radio wave, we will be more sure where the dark matter exists in the Bullet Cluster.

Supplemental Materials

The reason for the polarization of the radio waves

- The magnetic field in a galaxy dominantly in the galactic plane
- Synchrotron radiation in radio waves by the electrons moving in the magnetic field of a galaxy

Change in the polarization direction

- The change in the polarization direction due to gravitational lensing: order of the deflection angle.
  Hou, Fan, Zhu, [arXiv:1907.07486 [gr-qc]]
- The deflection angle less than a half arc-minutes for clusters.
  P. Schneider, [arXiv:astro-ph/0306465]