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BSM Theories and Flavor Physics

Quark and lepton hierarchies from S4' modular flavor symmetry

Junichiro Kawamura

Institute for Basic Science, CTPU

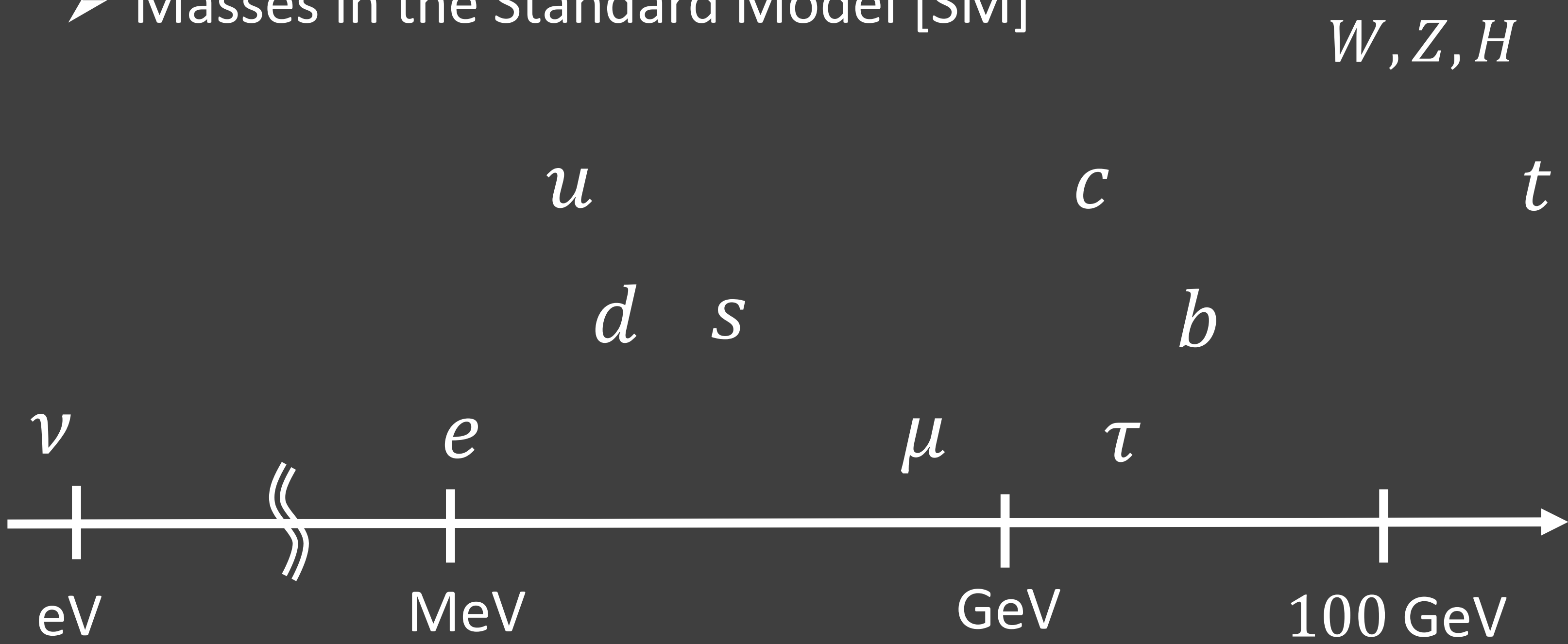
based on arXiv:2301.07439, 2302.11183 [PLB]

in collaboration with

Y.Abe (U. of Wisconsin), T.Higaki (Keio U.), T.Kobayashi (Hokkaido U.)

Mass hierarchies

- Masses in the Standard Model [SM]



Why so hierarchical ?

Quark hierarchies

- Cabibbo-Kobayashi-Maskawa [CKM] matrix

$$M_u \qquad\qquad U_L^\dagger M_u U_R = \mathrm{diag}(m_u, m_c, m_t)$$

$$M_d \quad \rightarrow \quad V_L^\dagger M_d V_R = \mathrm{diag}(m_d, m_s, m_b)$$

$$\frac{\mathrm{g}}{\sqrt{2}} W_\mu^+ \overline{u_i}\, \delta_{ij} d_j \qquad\qquad \frac{\mathrm{g}}{\sqrt{2}} W_\mu^+ \overline{u_i}\, V_{ij}^{CKM} d_j$$

diagonal W-couplings — diagonal masses

CKM matrix $$V^{CKM} = U_L^\dagger V_L \sim \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.041 \\ 0.009 & 0.040 & 0.99 \end{pmatrix}$$

CKM also has hierarchical structure

Lepton hierarchies

- charged lepton masses are hierarchial

$$\left(m_e, m_\mu, m_\tau\right) \sim (0.5, 106, 776)\ \mathrm{MeV}$$

- neutrino masses are not so hirarchial

$$\Delta m_{21}^2 \sim 10^{-5}\mathrm{eV}^2,\ \Delta m_{32}^2 \sim 10^{-3}\mathrm{eV}^2$$

→ $m_2 \sim 0.1^{-2.5}, m_3 \sim 0.1^{-1.5}$ eV for $m_1 < m_{2,3}$

- PMNS matrix has large mixing angles

$$V^{PMNS} \sim \begin{pmatrix} 0.8 & 0.6 & 0.2 \\ 0.3 & 0.6 & 0.7 \\ 0.5 & 0.6 & 0.7 \end{pmatrix}$$

Aim of this work

Understand the hierarchies in quark and lepton hierarchies

- Modular flavor symmetry

what if Yukawa couplings (masses) are modular form ?

Altarelli, Feruglio, 2010

$$Y = Y(\tau) \to (c\tau + d)^k \rho(r) Y(\tau)$$

- non-Abelian discrete flavor symmetry
- → Froggatt-Nielsen [FN] mechanism by residual symmetry
- modular symmetry appears in string models

T.Kobayashi, S.Nagamoto et.al. ‘17 ’18 ‘20
J.Lauer, J.Mas, H.P.Nilless ‘89, ‘91,
S.Ferrara, D.Lust, S.Theisen, ‘89
A.Baur, H.P.Nilles, A.Trautner,
PKS.Vaudrevange S.Ramos-Sanches, ‘19, ‘20

→ explain the quark and lepton hierarchies

Outline

1. Introduction
2. Modular flavor symmetry
3. S_4' model for quark and lepton hierarchies
4. Summary

Modular group

- modular group $\Gamma \Leftrightarrow$ special linear group

$$\Gamma := SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \ a,b,c,d \in \mathbb{Z}\,, ad-bc=1 \right\}$$

generators

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT$$

- action to modulus τ : complex scalar with $\mathrm{Im}\,\tau > 0$

$$\tau \xrightarrow{\Gamma} \frac{a\tau + b}{c\tau + d} \qquad \tau \xrightarrow{S} -1/\tau \qquad \tau \xrightarrow{T} \tau + 1 \qquad \tau \xrightarrow{R} \tau$$

Finite modular group Γ_N

- Congruence group $\Gamma(N)$ level $N \in \mathbb{N}$

$$\Gamma(N) \coloneqq \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \;\middle|\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \bmod N \right\}$$

ex) $T^N = \begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in \Gamma(N) \Leftrightarrow \tau \to \tau + N$

- finite modular group $\Gamma_N \coloneqq \Gamma/\Gamma(N)$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT, \quad T^N = 1$$

→ isomorphic to non-Abelian discrete symmetries for $N \leq 5$

$$\Gamma_2 \simeq S_3', \quad \Gamma_3 \simeq A_4', \quad \Gamma_4 \simeq S_4', \quad \Gamma_5 \simeq A_5'$$

*e.g. $\Gamma_4/\mathbb{Z}_2^R \simeq S_4$

$\Gamma_4 \simeq S_4'$ modular symmetry Novichkov, Penedo, Petkov, Titov, 18'

- Representations under $S_4 = S_4'/\mathbb{Z}_2^R$
  - two singlets $1, 1'$, one doublet 2 and two triplets $3, 3'$
  - there are $\mathbb{Z}_2^R$-odd representations denoted by $\hat{r}$ under S_4'

- Modular form of rep. r and weight $k \in \mathbb{N}$

  is a holomorphic function of τ transforms as

$$Y_r^{(k)} = Y_r^{(k)}(\tau) \to (c\tau + d)^k \rho(r) Y_r^{(k)}(\tau) \qquad \tau \xrightarrow{\Gamma} \frac{a\tau + b}{c\tau + d}$$

$\rho(r)$: representation matrix of r

  - the number of rep. is fixed for a given weight k
  - one $\hat{3}$ at $k = 1$, one 2 and one $3'$ at $k = 2$ and so on

Residual $\mathbb{Z}_4^T$ symmetry

Novichkov, Penedo, Petkov, 21’

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad T^4 = 1$$

- At $\tau \sim i\infty$

  τ is insensitive to $\tau \xrightarrow{T} \tau + 1$ → $\mathbb{Z}_4^T$ symmetry is unbroken

- Modular forms at $\mathrm{Im}\tau \gg 1$

$$Y_{\hat{3}}^{(1)}(\tau) \sim \begin{pmatrix} \sqrt{2}\epsilon(\tau) \\ \epsilon(\tau)^2 \\ -1 \end{pmatrix} \quad \begin{matrix} \mathbb{Z}_4^T\text{-charge} \\ 1 \\ 2 \\ 0 \end{matrix} \qquad \epsilon(\tau) \sim 2\exp\left(\frac{2\pi i \tau}{4}\right) \ll 1$$

powers of $\epsilon \ll 1$ is controlled by $\mathbb{Z}_4^T$ charge

→ Froggatt-Nielsen [FN] mechanism $\left(\frac{\langle\phi\rangle}{\Lambda}\right)^n \Leftrightarrow \epsilon(\tau)^n$

natural and predictive realization of FN mech.

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Representations of S_4'

1st example for both Q and L from single modular sym.

- Quark sector

$\mathbb{Z}_4^T$-charge

$$\overset{0}{}\quad\overset{0}{}\quad\overset{1}{}$$

$$u^c = 1 \oplus 1 \oplus \hat{1}'$$

RH up quark

$$\overset{0}{}\quad\overset{0}{}\quad\overset{0}{}$$

$$d^c = 1 \oplus 1 \oplus 1$$

RH down quark

(2,3,1)

$$Q = 3$$

LH doublet quark

small angles and mass hierarchy mainly from Q

- Lepton sector

$\mathbb{Z}_4^T$-charge

$$\overset{0}{}\quad\overset{0}{}\quad\overset{0}{}$$

$$L = 1 \oplus 1 \oplus 1$$

LH doublet lepton

(2,3,1)

$$e^c = 3$$

RH charged lepton

large angles from L, while mass hierarchy from e^c

Quark and lepton hierarchies

- masses and CKM /PMNS matrix

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t$$

$$(m_d, m_s, m_b) \sim \left(m_e, m_\mu, m_\tau\right) \sim (\epsilon^3, \epsilon^2, \epsilon)\, m_t/t_\beta$$

$$V^{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \qquad V^{PMNS} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$N = 4$ is minimal to have ϵ^3

*see also models based on A_4 (2212. 13336), Γ_6 (2301.03737), A_4^3 (2302.03326) for Q

- Additional factors

  - powers of $(2\mathrm{Im}\tau\,)^k$ from canonical normalization
  - numerical factors from modular forms

Yukawa couplings 2302.11183

$$W_u = H_u \left\{ \sum_{a=1}^{2} \alpha_a \left(Q Y_3^{(k_{u_a}+k_Q)} \right)_1 u_i^c + \alpha_3 \left(Q Y_{\hat{3}}^{(k_{u_3}+k_Q)} u_3^c \right)_1 \right\}$$

$$W_d = H_d \left\{ \sum_{i=13} \beta_i \left(Q Y_3^{(k_{d_i}+k_Q)} \right)_1 d_i^c + \gamma_i \left(L_i Y_3^{(k_e+k_{L_i})} e^c \right)_1 \right\}$$

$$W_\nu = \sum_{i,j=1}^{3} \frac{c_{ij}}{\Lambda} Y_1^{(k_{L_i}+k_{L_j})} L_i H_u L_j H_u$$

$(\cdots)_1$: singlet combination

$\alpha_i, \beta_i, \gamma_i, c_{ij} : \mathcal{O}(1)$ coefficients

For neutrinos, we assume

- Majorana masses from Weinberg operator
- L_i is singlet, s.t. Yukawa is singlet ($L_i = \hat{1} \rightarrow$ Y is $1'$)

Modular weights

2302.11183

$$(k_{u_1}, k_{u_2}, k_{u_3}) = (0, 4, 3), \quad (k_{d_1}, k_{d_2}, k_{d_3}) = (0, 2, 4), \; k_Q = 4$$

$$(L_1, k_{L_2}, k_{L_3}) = (0, 2, 4), \qquad k_e = 4$$

→ Modular weights of Yukawa couplings

$$k_{Y_u} = (4,8,7), \qquad k_{Y_d} = k_{Y_e} = (4,6,8), \qquad k_{c_{ij}} = 2i + 2j - 4$$

- minimal combinations for rank-3 Yukawa matrices up to $\mathcal{O}(\epsilon^3)$
- neutrino mass hierarchy from $2\mathrm{Im}\,\tau \sim 5$

➢ Modular forms:

ex) $Y_{\hat{3}}^{(7)} \sim \epsilon\,(1, \sqrt{2}\epsilon, 7\sqrt{2}\epsilon^2)$

→ Cabbibo angle is enhanced as $s_C \sim 7\sqrt{2}\epsilon \sim 0.2$

Fit results

$\tan\beta = 3.7, \mathrm{Im}\,\tau = 2.8, |\alpha_3^1| = 0.0012$

$$\frac{1}{|\alpha_3^1|}\begin{pmatrix}\alpha_1\\ \alpha_2^1\\ \alpha_2^2\\ \alpha_3^1\\ \alpha_3^2\end{pmatrix} = \begin{pmatrix}-0.69\\ -1.8\\ 0.84\\ e^{-1.6i}\\ -1.1\end{pmatrix} \qquad \frac{1}{|\alpha_3^1|}\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3^1\\ \beta_3^2\end{pmatrix} = \begin{pmatrix}1.6\\ -1.8\\ 0.6\\ -0.62\end{pmatrix} \qquad \frac{1}{|\alpha_3^1|}\begin{pmatrix}\gamma_1\\ \gamma_2\\ \gamma_3^1\\ \gamma_3^2\end{pmatrix} = \begin{pmatrix}-1.7\\ -2.8\\ 0.80\, e^{0.93i}\\ 2.75\end{pmatrix}$$

$$\frac{1}{|\alpha_3^1|}\begin{pmatrix}c_{11}\\ c_{22}\\ c_{33}\\ c_{13}\\ c_{23}\end{pmatrix} = \begin{pmatrix}1.7\\ -1.9\\ -0.67\\ -5.4\\ -2.0\end{pmatrix}$$

- coefficients are in [0.62,5.4]
- two phases are introduced

→

obs.	value	center	error
$y_u/10^{-6}$	4.44	2.85	0.88
$y_c/10^{-3}$	1.481	1.479	0.052
y_t	0.5322	0.5320	0.0053
$y_d/10^{-5}$	1.94	1.93	0.21
$y_s/10^{-4}$	3.88	3.82	0.21
$y_b/10^{-2}$	2.097	2.100	0.021
$y_e/10^{-6}$	7.816	7.816	0.047
$y_\mu/10^{-3}$	1.6496	1.6500	0.0099
$y_\tau/10^{-2}$	2.808	2.805	0.028

s_{12}	0.22520	0.22541	0.00072
$s_{23}/10^{-2}$	4.007	3.998	0.064
$s_{13}/10^{-3}$	3.43	3.48	0.13
δ_{CKM}	1.2395	1.2080	0.0540
$R_{32}^{21}/10^{-2}$	3.053	3.070	0.084
s_{12}^2	0.302	0.307	0.013
s_{23}^2	0.547	0.546	0.021
$s_{13}^2/10^{-2}$	2.203	2.200	0.070
δ_{PMNS}	-0.85	-2.01	0.63

Well-fit ($< 1.8\sigma$) to the data by $\mathcal{O}(1)$ coefficients

Summary

- Modular flavor symmetry realizes
  - generalized non-Abelian discrete symmetry
  - hierarchical Yukawa matrix via Froggatt-Nielsen mechanisms
- S_4' model
  - maybe minimal possibility to realize the Q and L hierarchy
  - successfully explains the quark and lepton hierarchies
  - can not be embedded into e.g. SU(5) GUT   ongoing

Thank you

backups

S_3 symmetry

the coefficients have the "hierarchy" structure

$$\alpha_1 \ll \alpha_2, \alpha_3 \qquad\qquad \beta_{11} \gg \beta_{21}, \beta_{22}, \beta_{23}$$

first low is smaller (larger) than others in up (down) quarks

- S_3 model with another modulus τ_2

d^c, q_1: singlet 1 $\qquad$ u^c, q_2: non-trivial singlet $1'$

$$\rightarrow \quad Y_u \propto \begin{pmatrix} \epsilon_2 & \epsilon_2 & \epsilon_2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \alpha_1 \\ \\ \alpha_2, \alpha_3 \end{matrix} \qquad Y_d \propto \begin{pmatrix} 1 & 1 & 1 \\ \epsilon_2 & \epsilon_2 & \epsilon_2 \\ \epsilon_2 & \epsilon_2 & \epsilon_2 \end{pmatrix} \begin{matrix} \beta_{11} \\ \\ \beta_{21}, \beta_{22}, \beta_{23} \end{matrix}$$

the hierarchy is explained by $\epsilon_2(\tau_2) \sim 0.1$

Canonical normalization

- modular invariant kinetic term

kinetic term $\frac{\bar{Q}Q}{(-i\tau + i\bar{\tau})^{k_q}}$ → $\bar{Q}Q$

canonical basis

Yukawa coup. $Y^{(k_Y)}(\tau)$ → $(2\mathrm{Im}\tau)^{k_Y/2}\, Y^{(k_Y)}$

- When $\epsilon(\tau) \ll 1$

$\epsilon(\tau) \sim 0.05$ → $t \coloneqq 2\mathrm{Im}\,\tau \sim 5$ gives additional structure

another FN-like mechanism controlled by modular weights

Spontaneous CP violation

* Thank to Tanimoto-san for comments

- from S_4'

$\epsilon(\tau) \sim 2\exp\left(\frac{2\pi\tau i}{4}\right)$ is a complex parameter

However, it induces only unphysical phases in CKM matrix up to ϵ^3

→ spontaneous CP violation is too small $\sim \epsilon^4$

- from S_3

CPV from $\epsilon_2 \sim 0.1$ does not physical phase up to ϵ_2

However, $\epsilon_2^2 \sim 0.01$ ($\mathbb{Z}_2^T$ neutral) is enough for CKM phase

→ moderate CP violation from S_3

Quark hierarchies

see also models based on A_4 (2212. 13336), Γ_6 (2301.03737), $A_4 \times A_4 \times A_4$ (2302.03326)

- masses and CKM matrix

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t$$

$$(m_d, m_s, m_b) \sim \epsilon^p (\epsilon^2, \epsilon^2, 1)\, m_t / t_\beta$$

$$V^{CKM} \sim \begin{pmatrix} 1 & 1 & \epsilon^2 \\ 1 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$$

$\epsilon \sim 0.05$

$p = 0{,}1$

$t_\beta = v_u / v_d$

- $N = 4$ is the minimal number for the hierarchy with ϵ^3
- $\mathcal{O}(0.1)$ deviations could be explained by modular forms

- representations of quarks for $p = 1$

c.f. Novichkov, Penedo, Petkov, 21'

there is only one combination of reps. for the quark hierarchy *

* assume no coexistence of $\mathbb{Z}_2^R$-even and –odd states <u>in</u> same quark

$\mathbb{Z}_4^T$-charge :

(2,3,1)	2 2 2	0 (0, 2)
$u^c = 3$	$d^c = 1' \oplus 1' \oplus 1'$	$Q = 1 \oplus 2$
RH up quark	RH down quark	LH doublet quark

Representation matrices in S_4'

$$\rho_S(2) = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}, \quad \rho_T(2) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and

$$\rho_S(3) = -\frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix}, \quad \rho_T(3) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix}.$$

The primed and hatted representations are related as

$$\begin{aligned} \rho_S(r) &= -\rho_S(r') = -i\rho_S(\hat{r}) = i\rho_S(\hat{r}'), \\ \rho_T(r) &= -\rho_T(r') = i\rho_T(\hat{r}) = -i\rho_T(\hat{r}'), \\ \mathbb{I} = \rho_R(r) &= \rho_R(r') = -\rho_R(\hat{r}) = -\rho_R(\hat{r}'). \end{aligned}$$

$\tau \sim i\infty$ $\rightarrow$

$$Y_1 \sim 1, \quad Y_{1'} \sim \epsilon^2, \quad Y_{\hat{1}} \sim \epsilon^3, \quad Y_{\hat{1}'} \sim \epsilon, \quad Y_2 \sim \begin{pmatrix} 1 \\ \epsilon^2 \end{pmatrix}, \quad Y_{\hat{2}} \sim \begin{pmatrix} \epsilon^3 \\ \epsilon \end{pmatrix},$$

$$Y_3 \sim \begin{pmatrix} \epsilon^2 \\ \epsilon^3 \\ \epsilon \end{pmatrix}, \quad Y_{3'} \sim \begin{pmatrix} 1 \\ \epsilon \\ \epsilon^3 \end{pmatrix}, \quad Y_{\hat{3}} \sim \begin{pmatrix} \epsilon \\ \epsilon^2 \\ 1 \end{pmatrix}, \quad Y_{\hat{3}'} \sim \begin{pmatrix} \epsilon^3 \\ 1 \\ \epsilon^2 \end{pmatrix},$$

Yukawa matrices

- model for $p = 1$

$$Y_u = \begin{pmatrix} \alpha_1 [Y_3^{(4)}]_1 & \alpha_1 [Y_3^{(4)}]_3 & \alpha_1 [Y_3^{(4)}]_2 \\ -2\alpha_2 [Y_3^{(6)}]_1 & \alpha_2 [Y_3^{(6)}]_3 + \sqrt{3}\alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_2 & \alpha_2 [Y_3^{(6)}]_2 + \sqrt{3}\alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_3 \\ -2\alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_1 & \alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_3 - \sqrt{3}\alpha_2 [Y_3^{(6)}]_2 & \alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_2 - \sqrt{3}\alpha_2 [Y_3^{(6)}]_3 \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \beta_{11} Y_{1'}^{(6)} & 0 & 0 \\ -\beta_{21}^{i_Y} [Y_2^{i_Y (8)}]_2 & -\beta_{22} [Y_2^{(6)}]_2 & -\beta_{23} [Y_2^{(4)}]_2 \\ \beta_{21}^{i_Y} [Y_2^{i_Y (8)}]_1 & \beta_{22} [Y_2^{(6)}]_1 & \beta_{23} [Y_2^{(4)}]_1 \end{pmatrix},$$

- model for $p = 0$

$$Y_u = \begin{pmatrix} \alpha_1 [Y_3^{(6)}]_1 & \alpha_1 [Y_3^{(6)}]_3 & \alpha_1 [Y_3^{(6)}]_2 \\ -2\alpha_2 [Y_3^{(6)}]_1 & \alpha_2 [Y_3^{(6)}]_3 + \sqrt{3}\alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_2 & \alpha_2 [Y_3^{(6)}]_2 + \sqrt{3}\alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_3 \\ -2\alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_1 & \alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_3 - \sqrt{3}\alpha_2 [Y_3^{(6)}]_2 & \alpha_3^{i_Y} [Y_{3'}^{i_Y (6)}]_2 - \sqrt{3}\alpha_2 [Y_3^{(6)}]_3 \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \beta_{11} Y_{\hat{1}}^{(9)} & 0 & 0 \\ \beta_{21} [Y_{\hat{2}}^{(9)}]_1 & \beta_{22} [Y_{\hat{2}}^{(7)}]_1 & \beta_{23} [Y_{\hat{2}}^{(5)}]_1 \\ \beta_{21} [Y_{\hat{2}}^{(9)}]_2 & \beta_{22} [Y_{\hat{2}}^{(7)}]_2 & \beta_{23} [Y_{\hat{2}}^{(5)}]_2 \end{pmatrix}.$$

Model for $p = 0$

$u^c = 3 \quad d^c = \hat{1}' \oplus \hat{1}' \oplus \hat{1}' \quad Q = 1 \oplus 2$

$d_1^c \quad d_2^c \quad d_3^c \quad\quad q_1 \quad q_2$

$$k_u = 2, \ \ (k_{d_1}, k_{d_2}, k_{d_3}) = (5\ 3, 1), \quad (k_{q_1}, k_{q_2}) = (4, 4)$$

$$\tan\beta = 1.6, \tau = 1.5 + 2.7i, |\alpha_3| = 0.0013$$

at GUT scale

S.Antusch, V.Maurer 1306.6879

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_3'\end{pmatrix} = \begin{pmatrix}-0.27\\ -1.7\\ e^{-3.1i}\\ -1.4\end{pmatrix}$$

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\beta_{11}\\ \beta_{21}\\ \beta_{22}\\ \beta_{23}\end{pmatrix} = \begin{pmatrix}-6.9\\ 0.13\\ 0.28\\ 0.41\end{pmatrix}$$

→

obs.	value	center	error
$y_u/10^6$	2.9	2.9	1.3
$y_c/10^3$	1.560	1.508	0.095
y_t	0.5464	0.5464	0.0084
$y_d/10^6$	9.00	9.06	0.87
$y_s/10^4$	1.73	1.79	0.14
$y_b/10^2$	1.011	0.994	0.013
s_{12}	0.2274	0.2274	0.0007
$s_{23}/10^2$	3.991	3.989	0.065
$s_{13}/10^3$	3.47	3.47	0.13
$\delta_{\rm CP}$	1.204	1.208	0.054

our model exp. error

- similar to the model for $p = 1$, but with $\tan\beta \sim 1$
- the sizes of coefficients are in $[0.13, 6.9]$, ratio is 50

Rank condition

- e.g. if $k_d = (4,2,2)$, $k_Q = (2,0)$

$$Y_d = \begin{pmatrix} \beta_{11} Y_{1'}^{(6)} & 0 & 0 \\ \beta_{21} Y_2^{(4)} & \beta_{22} Y_2^{(2)} & \beta_{23} Y_2^{(2)} \end{pmatrix}$$ is "rank-2" up to ϵ^3 $\rightarrow$ need larger weights

- # of representations * reps. are hatted for odd weights

weight	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	1	0	1	0	1	1	1	0
$1'$	0	0	1	0	0	1	1	0	1	1	1
2	0	1	0	1	1	1	1	2	1	2	2
3	1	0	1	1	2	1	2	2	3	2	3
$3'$	0	1	1	1	1	2	2	2	2	3	3

there are $2k+1$ independent modular forms at a weight k

of representations under S_4'

weight	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	1	0	1	0	1	1	1	0
$1'$	0	0	1	0	0	1	1	0	1	1	1
2	0	1	0	1	1	1	1	2	1	2	2
3	1	0	1	1	2	1	2	2	3	2	3
$3'$	0	1	1	1	1	2	2	2	2	3	3

* reps. for odd weights are hatted ones

there are $2k+1$ independent modular forms at a weight k

Bad case for CKM hierarchy

- Quark masses

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t \qquad (m_d, m_s, m_b) \sim \epsilon\, (\epsilon^2, \epsilon, 1)\, m_t / t_\beta$$

is also realized if

c.f. Novichkov, Penedo, Petkov, 21'

$$\overset{(2,3,1)}{Q = 3} \qquad u^c = \overset{2}{1'} \oplus \overset{0}{1} \oplus \overset{0}{1} \qquad d^c = \overset{0}{1} \oplus \overset{0}{1} \oplus \overset{0}{1}$$

- Yukawa hierarchies

$$Y_u \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ \epsilon & \epsilon & \epsilon^3 \\ \epsilon^3 & \epsilon^3 & \epsilon \end{pmatrix} \begin{pmatrix} t \\ c \\ u \end{pmatrix}_L \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ \epsilon^3 & \epsilon^3 & \epsilon^3 \end{pmatrix} \begin{pmatrix} s \\ b \\ d \end{pmatrix}_L$$

$$\rightarrow \quad V^{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \mathcal{O}(\epsilon)$$

not identity at LO