Scalar-Tensor Gravity at the First Loop

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3. Effective action
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Motivation

Scalar-tensor gravity:

- Rich phenomenology
- Inflation and early Universe
- Black holes with scalar hairs

Quantum gravity:

- Non-minimal interactions
- Effective action and effective potential
- Quantum corrections to black holes

Horndeski Gravity

$$\mathcal{S} = \int d^4x\sqrt{-g}\,[L_2 + L_3 + L_4 + L_5]$$

$$\begin{aligned}
L_2 =& G_2 \\
L_3 =& G_3\,\Box X \\
L_4 =& G_4\,R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_{\mu\nu}\phi)^2\right] \\
L_5 =& G_5\,G_{\mu\nu}\nabla^{\mu\nu}\phi \\
&- \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\,\Box\phi\,(\nabla_{\mu\nu}\phi)^2 + 2\,(\nabla_{\mu\nu}\phi)^3\right]
\end{aligned}$$

$$G_i = G_i(\phi, X)\,,\,X = (\nabla\phi)^2$$

Horndeski (1974); Kobayashi et al (2011)

Wide phenomenology

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G_N}(R-2\Lambda)+\frac{1}{2}\left(g^{\mu\nu}+\beta G^{\mu\nu}\right)\nabla_\mu\phi\nabla_\nu\phi\right]$$

- Late time accelerated expansion
- Cosmological constant screening
- Stability in the future
- Instability in the past

Starobinsky et al (2016)

Black holes with scalar hair

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G_N} R + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + f(\phi) \mathcal{G} \right]$$

- No-hair theorem is evaded
- Numerical solutions
- Regularity condition

$$\left| \frac{df}{dr} \right| \Bigg|_{r=r_\mathrm{h}} < \frac{r_\mathrm{h}^2}{8\sqrt{3}\,\kappa}$$

Kanti et al (1995); Papageorgiou et al (2022)

Perturbative Quantum Gravity

Perturbative quantum gravity is
quantum theory of small metric perturbations

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$$

$$\kappa^2 = 32\pi G_N$$

- Background dependence
- Mass scale $\kappa^{-1} = M_{\text{Pl}}/32\pi$
- "metric perturbations" = "gravitons"

Perturbative Quantum Gravity

$$\mathcal{Z} = \int \mathcal{D}[g_{\mu\nu}] \exp\left[i\,\mathcal{A}[g_{\mu\nu}]\right]$$

$$= \int \mathcal{D}[\overline{g}_{\mu\nu} + \kappa h_{\mu\nu}] \exp\left[i\,\mathcal{A}[\overline{g}_{\mu\nu} + \kappa h_{\mu\nu}]\right]$$

$$= \int \mathcal{D}[h_{\mu\nu}] \exp\left[i\,\mathcal{A}[\overline{g}_{\mu\nu}] + i\,\frac{\delta\mathcal{A}}{\delta g_{\mu\nu}}\kappa h_{\mu\nu} + i\,\frac{\delta^2\mathcal{A}}{\delta g_{\mu\nu}\delta g_{\alpha\beta}}\kappa^2 h_{\mu\nu}\, h_{\alpha\beta} + \cdots\right]$$

$$= \int \mathcal{D}[h_{\mu\nu}] \exp\left[\frac{i}{2} h^{\mu\nu}\widehat{\mathcal{D}}_{\mu\nu\alpha\beta} h^{\alpha\beta} + i\,\kappa\,\widehat{\mathfrak{V}}^{(3)}_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h^{\mu_1\nu_1} h^{\mu_2\nu_2} h^{\mu_3\nu_3} + \cdots\right]$$

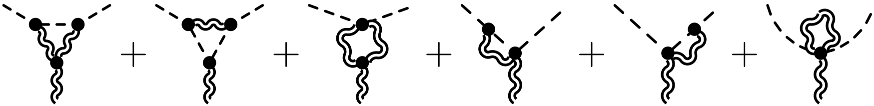
Perturbative theory with infinite number of terms

Latosh (2022), (2023)

Why perturbative quantum gravity?

- Scattering amplitudes
  - † Infrared limit
  - † New operators
- Effective action
  - † Effective potential
  - † Non-minimal coupling

Infrared Limit



$$\int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} \frac{1}{k} = \frac{1}{2\,\pi^2\,r^2}$$

$$\int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} \ln k^2 = -\frac{1}{2\,\pi^2\,r^3}$$

$$V(r) = -\frac{G_N\,m_1\,m_2}{r}\left[1 - \frac{G_N\,(m_1+m_2)}{r\,c^2} - \frac{127}{30\,\pi^2}\frac{G_N\,\hbar}{r^2\,c^3}\right]$$

Donoghue (1994)

Effective Action

Functional $\mathcal{Z} \leftrightarrow$ Green function G

$$\mathcal{Z}[J] = \int \mathcal{D}[\phi] e^{i\mathcal{A}[\phi] + iJ\phi} \leftrightarrow G_n = \frac{\delta^n \mathcal{Z}}{\delta(iJ)^n}$$

Functional $W \leftrightarrow$ Connected Green function G^c

$$\mathcal{Z}[J] = e^{i\,W[J]} \leftrightarrow G_n^c = \frac{\delta^n W}{\delta J^n}$$

Effective action

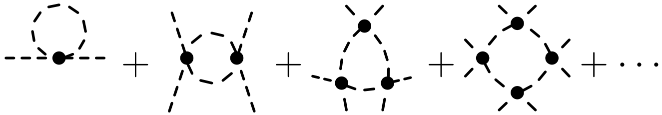
$$\Gamma[\varphi] = W[J] - \varphi \cdot J$$

Buchbinder et al (1992)

Effective Action

$$\Gamma[\varphi] = W[J] - \varphi \cdot J$$

- $\varphi = \langle 0|\widehat{\phi}|0\rangle$
- $\delta\Gamma/\delta\varphi\Big|_{J=0} = 0$
- Γ is a series of one-loop connected one-particle irreducible diagrams



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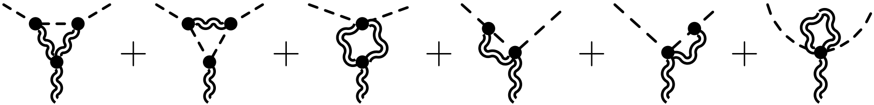
One-loop Scalar-Tensor Gravity

Minimal model

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right]$$

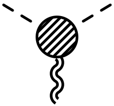


$$\to -\frac{\kappa^3}{2} k^4 \, C_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 (l+p_1)^2 (l-p_2)^2}$$

Latosh (2020)

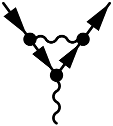
New operators

Non-minimal kinetic coupling



$$\to \kappa^2 G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

Anomalous magnetic moment



$$\to \frac{e}{2\,m} \overline{\psi} [\gamma^\mu, \gamma^\nu] \psi \, \partial_\mu A_\nu$$

Constraints

$$\Gamma = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{1}{2} g^{\mu\nu} \nabla_\mu\phi\nabla_\nu\phi + \beta\,\kappa^2\, G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \right]$$

LIGO data constraint

$$c_t^2 = \frac{2\,G_4 - \left(\dot{\phi}\right)^2 G_{5,\phi} - \left(\dot{\phi}\right)^2 \ddot{\phi}\, G_{5,X}}{2\,G_4 - 2\left(\dot{\phi}\right)^2 G_{4,X} + \left(\dot{\phi}\right)^2 G_{5,\phi} - H\left(\dot{\phi}\right)^3 G_{5,X}}$$

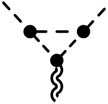
$$= 1 + 2\,\beta\left(16\,\pi\,G_N\,\dot{\phi}\right)^2 + \mathcal{O}\left(G_N^4\right)$$

$$\boxed{-3\times 10^{-15} \leq \beta\left(16\,\pi\,G_N\,\dot{\phi}\right)^2 \leq +7\times 10^{-16}}$$

New operators

Non-minimal model

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \frac{\lambda}{3!} \phi^3 \right]$$



$$\to \lambda^2 R \, \phi^2$$

Constraints

$$\Gamma = \int d^4x\sqrt{-g}\Bigg[-\left(\frac{2}{\kappa^2}+\alpha\,\phi^2\right)R+\frac{1}{2}\,g^{\mu\nu}\,\nabla_\mu\phi\nabla_\nu\phi$$
$$+\beta\,\kappa^2\,G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi+\frac{\lambda}{3!}\,\phi^3\Bigg]$$

Newton constant time variation

$$\left|\frac{\dot{G}_{\mathrm{eff}}}{G_{\mathrm{eff}}}\right| = \left|32\pi\,G_N\,\alpha\,\phi\,\dot{\phi}\right| < (7.1\pm 7.6)\times 10^{-14}\ \mathrm{yr}^{-1}$$

New operators

- Non-minimal kinetic coupling $G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ is generated universally
- Non-minimal Brans-Dicke-like coupling $R\,\phi^2$ is generated is a wide class of models
- New interactions appears to be consistent with the observations

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Effective potential

Minimal model

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{m^2}{2}\phi^2\right]$$

$$\begin{aligned} V_{\text{eff}} = &-\frac{m^4\ln 2}{64\,\pi^2} + \frac{m^2}{2}\left[1 + \frac{m^2\kappa^2}{32\pi^2}(1+\ln 4)\right]\varphi^2 \\ &+ \frac{m^4}{128\pi^2}\Bigg[\left(1 - \sqrt{1-4\,\kappa^2\varphi^2} - 2\,\kappa^2\varphi^2\right)\ln\left[1-\sqrt{1-4\,\kappa^2\varphi^2}\right] \\ &\qquad + \left(1 + \sqrt{1-4\,\kappa^2\varphi^2} - 2\,\kappa^2\varphi^2\right)\ln\left[1+\sqrt{1-4\,\kappa^2\varphi^2}\right]\Bigg] \end{aligned}$$

Arbuzov, Latosh (2021)

Effective potential

Non-minimal model

$$\mathcal{A} = \int d^4x \sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\left(g^{\mu\nu} + \beta G^{\mu\nu}\right)\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}\left(m^2 + \lambda R\right)\phi^2\right]$$

$$V_{\text{eff}}\Big|_{m=0} = \frac{1}{2}\ln\left[1 + \frac{3}{2}\lambda^2\kappa^2\varphi^2\right]\left(\frac{2\,m_{\text{obs}}^2}{3\,\kappa^2\,\lambda^2}\right)$$

Effective potential

Generalization of Coleman-Weinberg model

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right]$$

$$V_{\text{eff}} = \ln \left[1 + \frac{\frac{\lambda}{2} \varphi^2}{m^2} \right] \left[\frac{m^4}{64\pi^2} + \frac{m^2}{2} \varphi^2 \, \frac{\lambda - 2m^2\kappa^2}{32\pi^2} + \frac{\lambda}{4!} \varphi^4 \, \frac{3\lambda - 8m^2\kappa^2}{32\pi^2} - \frac{\kappa^2}{6!} \varphi^6 \, \frac{5\lambda^2}{8\pi^2} \right]$$
$$+ \frac{m^2}{2} \left(1 - \frac{\lambda}{64\pi^2} \right) \varphi^2 + \frac{\lambda}{4!} \left(1 - \frac{3(3\,\lambda - 8\,\kappa^2 m^2)}{64\pi^2} \right) \varphi^4$$
$$+ \frac{g}{6!} \varphi^6 \left(1 - \frac{15\,\lambda^2}{32\pi^2} \, \frac{\lambda - 2\,m^2\,\kappa^2}{g\,m^2} \right)$$

Arbuzov, Latosh, Nikitenko (2022)

Effective potential

Minimal model

- New global maximum
- Controllable UV behavior
- Applicable below the local maximum point

Non-minimal model

- No new minima
- Poorly controllable UV behavior
- No explicit applicability constraints

ϕ^4 model

- New global maximum
- Weakly controllable UV behavior
- Applicable below the local maximum point

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Non-relativistic potential

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + f(\phi) \mathcal{G} \right]$$

$$f(\phi) = \lambda \phi, \qquad \mathcal{E}_\lambda = 1/\sqrt[3]{\kappa^2 \lambda}$$

$$V = \frac{G_N m_1 m_2}{r} \left[1 - \frac{\pi^2}{96} \frac{m_1^4 + m_2^4}{\mathcal{E}_\lambda^6} \frac{1}{r^2} + \cdots \right]$$

$$f(\phi) = \alpha \phi^2, \qquad \mathcal{E}_\alpha = 1/\sqrt{\kappa \sqrt{\alpha}}$$

$$V = \frac{G_N m_1 m_2}{r} \left[1 + 65 \pi^2 \frac{1}{\mathcal{E}_\alpha^4} \frac{m_1^2 + m_2^2}{m_1^2 m_2^2} \frac{1}{r^6} + 2730 \pi^2 \frac{1}{\mathcal{E}_\alpha^4} \frac{1}{m_1^2 m_2^2} \frac{1}{r^8} + \cdots \right]$$

Latosh, Park, to appear on arXiv

Metric corrections

$$f(\phi) = \lambda\,\phi, \qquad \mathcal{E}_\lambda = 1/\sqrt[3]{\kappa^2\lambda}$$

$$\delta g_{00} = -\frac{\pi}{12288}\,\frac{G_N\,m^5}{\mathcal{E}_\lambda^6}\,\frac{1}{r^3}$$

$$\delta g_{0i} = 0$$

$$\delta g_{ij} = \delta_{ij}\left[-\frac{\pi}{12288}\,\frac{G_N\,m^5}{\mathcal{E}_\lambda^6}\,\frac{1}{r^3}\right]$$

$$f(\phi) = \alpha\,\phi^2, \qquad \mathcal{E}_\alpha = 1/\sqrt{\kappa\,\sqrt{\alpha}}$$

$$\delta g_{00} = \frac{165\,\pi}{64}\,\frac{G_N}{\mathcal{E}_\alpha^4\,m}\,\frac{1}{r^7} + \frac{3675\,\pi}{16}\,\frac{G_N}{\mathcal{E}_\alpha^4\,m^4}\,\frac{1}{r^9}$$

$$\delta g_{0i} = 0$$

$$\delta g_{ij} = \delta_{ij}\left[\frac{165\,\pi}{64}\,\frac{G_N}{\mathcal{E}_\alpha^4\,m}\,\frac{1}{r^7} - \frac{2625\,\pi}{16}\,\frac{G_N}{\mathcal{E}_\alpha^4\,m^4}\,\frac{1}{r^9}\right]$$

Massless particles scattering

$$f(\phi) = \lambda\,\phi, \qquad \mathcal{E}_\lambda = 1/\sqrt[3]{\kappa^2\lambda}$$

$$\frac{d\sigma}{d\Omega} = (4G_N M)^2 \left[\left\{ 1 - \frac{3\pi^2}{32}\left(\frac{M}{\mathcal{E}_\lambda}\right)^6 \ln\frac{M^2}{\mu^2} \right\} \frac{1}{\chi^4} + \left\{ 1 - \frac{3\pi^2}{64}\left(\frac{M}{\mathcal{E}_\lambda}\right)^6 \ln\frac{M^2}{\mu^2} \right\} \frac{1}{3!}\frac{1}{\chi^2} \right.$$
$$\left. + \left\{ 1 - \frac{9\pi^2}{176}\left(\frac{M}{\mathcal{E}_\lambda}\right)^6 \ln\frac{M^2}{\mu^2} \right\} \frac{11}{6!} + \cdots \right]$$

$$f(\phi) = \alpha\,\phi^2, \qquad \mathcal{E}_\alpha = 1/\sqrt{\kappa\sqrt{\alpha}}$$

$$\frac{d\sigma}{d\Omega} = (4G_N M)^2 \left[\frac{1}{\chi^4} + \frac{1}{3!}\frac{1}{\chi^2} + \frac{11}{6!} + \frac{31}{6\times 7!}\chi^2 \right.$$
$$\left. + \left\{ \frac{41}{2\times 91} - \frac{35\pi^2}{8\times 4!}\frac{p^6}{\mathcal{E}_\alpha^4 M^2} \ln\left(\frac{p^2}{\mu^2}\sin^2\frac{\chi}{2}\right) \right\} \chi^4 + \cdots \right]$$

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Conclusions

- Non-minimal kinetic coupling is generated at the one-loop level universally
- Brans-Dicke-like coupling is generated at the one-loop level
- Non-trivial one-loop effective potential may serve as a source of inflation
- Non-relativistic potential and metric receive quantum corrections
- Small angle scattering receives quantum corrections relevant for black hole shadow formation

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Thank you for attention!

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