Gravitational Waves from Quasi-stable Strings

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Outline

1 *Introduction*

Standard Model ($SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$)

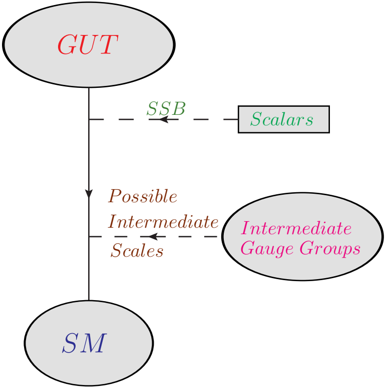
		Fields	Quantum numbers
Spin-$\frac{1}{2}$	Quarks	$Q_g^{i\alpha} = \{ \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L , \begin{pmatrix} c^\alpha \\ s^\alpha \end{pmatrix}_L , \begin{pmatrix} t^\alpha \\ b^\alpha \end{pmatrix}_L \}$	$(2, \frac{1}{6}, 3)$
		$u^C_{\alpha\,L},\ c^C_{\alpha\,L},\ t^C_{\alpha\,L}$	$(1,\text{-}\frac{2}{3}, \overline{3})$
		$d^C_{\alpha L},\ s^C_{\alpha L},\ b^C_{\alpha L}$	$(1, \frac{1}{3}, \overline{3})$
	Leptons	$\ell_g^i = \{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \}$	$(2,\text{-}\frac{1}{2}, 1)$
		$e^C_L,\ \mu^C_L,\ \tau^C_L$	$(1, 1, 1)$
Spin-1	$SU(2)_L$	W^a_μ	$(3, 0, 1)$
	$U(1)_Y$	B_μ	$(1, 0, 1)$
	$SU(3)_C$	G^a_μ	$(1, 0, 8)$
Spin-0	Higgs	$\Phi^i = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$(2, \frac{1}{2}, 1)$

Table: *Fields in the Standard Model.*

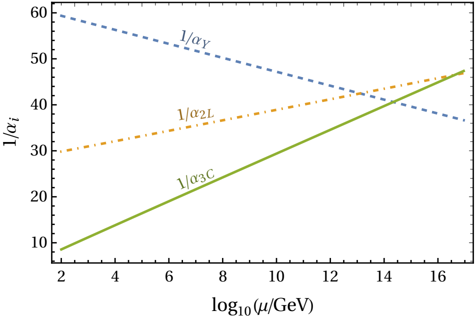
Grand Unification beyond the SM

- The basic idea in a Grand Unified Theory (GUT) is that the SM, $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$, is embedded in a larger simple group, $\mathcal{G}$.

Schematic view



Motivation towards the Grand Unification



- Renormalization Group Evolution of Standard Model gauge couplings gives the hint of the possibility of the Grand Unified Theories.

- Imposition of higher symmetry may constraint some free parameters.
- All fermions including $(\nu^c)_L$ can be put in one representation in GUT.
- One way to understand the charge quantization.

GUT Examples

- $SU(5)$ (rank = 4): $\bar{5} + 10 \Rightarrow$ SM fermions.

  Georgi, Glashow, PRL **32**, 438 (1974)

- $SO(10)$ (rank = 5): $16 \Rightarrow$ SM fermions $\oplus\ \nu_L^C$.

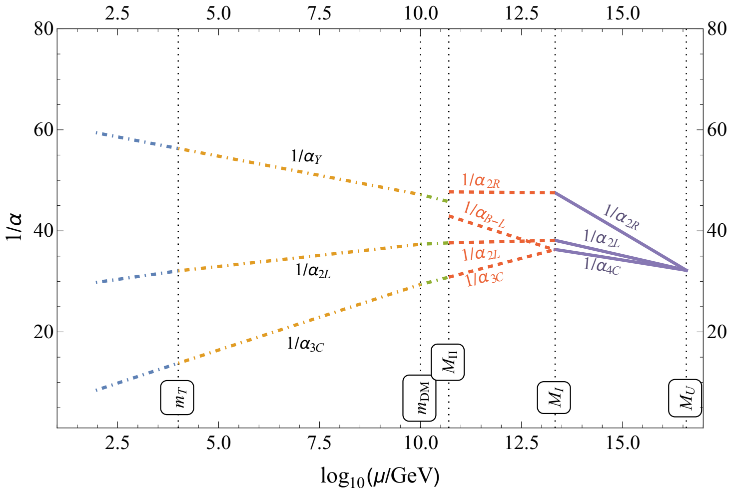
  Fritzsch, Minkowski, Ann. Phys. **93**, 93-266 (1975)

- $E(6)$ (rank = 6): $27 \Rightarrow$ SM fermions $\oplus\ \nu_L^C \oplus$

$$\underbrace{(2, \pm\frac{1}{2}, 1) + (1, -\frac{1}{3}, 3) + (1, \frac{1}{3}, \bar{3}) + (1, 0, 1)}_{\text{Exotic fermions}}.$$

  Shafi, PLB **79** (1978) 301

GUT breaking chain: Example



Lazarides, RM, Roshan, Shafi, Phys. Rev. D **106** (2022) 055009

2 *Topological Defects in GUTs*

Predictions of GUTs: Topological Defects

- Topological defects may appear during the SSB of a group $\mathcal{G}$ down to its subgroup $\mathcal{H}$.
- Non-trivial homotopy group $\Pi_k(\mathcal{M})$ of the vacuum manifold ($\mathcal{M} = \mathcal{G}/\mathcal{H}$) implies formation of topological defects.
- Various types of topological defects which can be formed are : domain walls (k = 0), cosmic strings (k = 1), monopoles (k = 2) etc

Observational Constraints on GUTs and Inflation

- Stable domain walls contradict standard cosmology.

  Y. B. Zeldovich, I. Y. Kobzarev, L. B. Okun, Zh. Eksp. Teor. Fiz. **67**, 3-11 (1974)

- Upper bound on comoving monopole number density from MACRO: $\boxed{Y_M = n_M/s \gtrsim 10^{-27}}$.

  M. Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- The Parkes Pulsar Timing Arrays (PPTA) put a contraint on the tension of the "undiluted" cosmic strings : $\boxed{G\mu \lesssim 10^{-11}}$.

  J.J. Blanco-Pillado, K.D. Olum, X. Siemens, PLB 778, 392 (2018)

- Way out $\Rightarrow$ Inflation: $\frac{a(t_f)}{a(t_i)} = \exp(N_e) \sim \exp[H(t_f - t_i)]$, $H \simeq \sqrt{V/(3m_{\rm Pl}^2)}$.

3. *Quasi-stable Cosmic Strings*

Topologically Unstable Cosmic Strings

- Consider $G \xrightarrow{M_I} H \otimes U(1) \xrightarrow{M_{II}} H$
  with G being simply connected and $\Pi_1(G/H) \cong \Pi_0(H) = I$.
- Strings formed at M_{II} connect monopole-antimonopole ($M\bar{M}$) pairs formed at M_I.
- Strings are topologically unstable: $\Gamma_d = \frac{\mu}{2\pi} \exp\left(-\pi m_M^2/\mu\right)$ with $\mu \sim \pi M_{II}^2$ and $m_M \sim 10 M_I$.
- However, strings are practically stable unless two breaking scales are very close ($(m_M^2/\mu)^{1/2} \lesssim 8.7$).

Preskill, Vilenkin, Phys. Rev. D **47** (1993)

Formation of Quasi-stable Strings

- Intermediate scale magnetic monopoles, created prior to the cosmic strings, experience partial inflation.
- The lifetime of decay of the strings via quantum mechanical tunneling is larger than the age of Universe.
- The strings make random walks with step of the order of the horizon, and form a network of stable strings before the horizon reentry of the monopoles.

Lazarides, RM, Shafi, JCAP 08 (2022) 042

Formation of Quasi-stable Strings

- The strings inter-commute and form loops which decay into gravitational waves.
- As monopoles reenter the horizon we obtain monopole-antimonopole pairs connected by string segments which also decay into gravitational waves.
- We call these quasi-stable strings as they form a stable network until the horizon reentry of monopoles.

Lazarides, RM, Shafi, JCAP 08 (2022) 042

Example in GUT

$$\begin{aligned} SO(10) &\xrightarrow{M_{\rm GUT}} SU(4)_c \times SU(2)_L \times SU(2)_R \\ &\xrightarrow{M_I} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \\ &\xrightarrow{M_{II}} SU(3)_c \times SU(2)_L \times U(1)_Y. \end{aligned}$$

- Symmetry breakings $SU(4)_C \to SU(3)_C \times U(1)_{B-L}$ and $SU(2)_R \to U(1)_R$ produce monopoles; label them 'red' and 'blue' monopoles respectively.

  Lazarides, Shafi, JHEP 10 (2019) 193

- $U(1)_R \times U(1)_{B-L} \to U(1)_Y$ generates cosmic strings which are not topologically stable. These strings connects
  1. a blue monopole to a red monopole,
  2. a monopole to its anti-monopole for both red and blue monopoles.

- Red and blue monopoles combined to form stable Schwinger monopoles $\Rightarrow$ Need to suffer some e-foldings.

4 *Gravitational Waves from Quasi-stable Strings*

Gravitational waves from Quasi-stable Strings

- The stochastic gravitational wave background receives contributions from the oscillating string loops before t_M:

$$\boxed{n_r(l, t < t_M) = \frac{0.18\,\Theta(0.1t - l)}{t^{3/2}(l + \Gamma G\mu t)^{5/2}}}.$$

Blanco-Pillado, Olum, Shlaer, Phys. Rev. D **89** (2014) 023512

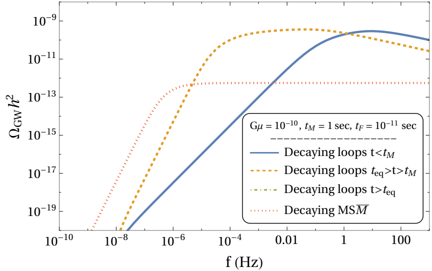
- After t_M, the contributions mainly come
  1. from the decaying string loops formed before t_M:

$$\boxed{n_r(l, t > t_M) = \frac{0.18\,\Theta(0.1t_M - l - \Gamma G\mu(t - t_M))}{t^{3/2}(l + \Gamma G\mu t)^{5/2}}},$$

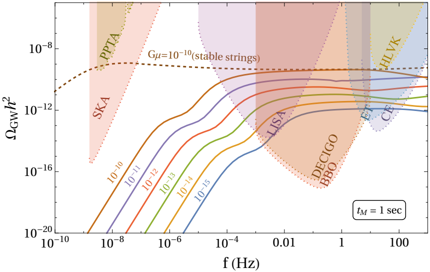
  2. from the oscillating $MS\bar{M}$ structures: $\boxed{\tilde{n}(z) = (2t_M)^{-3}\left(\frac{1+z}{1+z_M}\right)^3.}$

Gravitational Waves from Quasi-stable Strings

- Large string loops and segments ($> 2t_M$) are absent.
- Gravitational wave spectrum in the low frequency region is reduced.



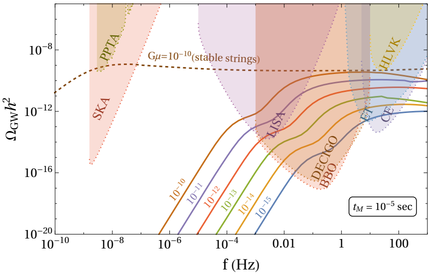
Different components.

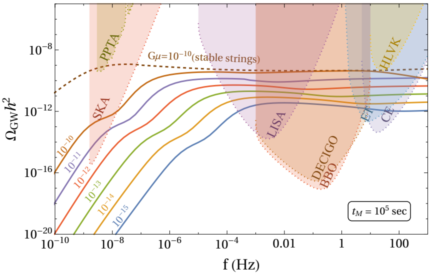


GWs spectra.

Lazarides, Maji, Shafi, JCAP 08 (2022) 042

Gravitational wave background





Lazarides, Maji, Shafi, JCAP 08 (2022) 042

5 *Summary*

Summary

- Topological defects e.g., domain walls, monopoles conflict cosmological observations unless they are inflated away.
- GUTs that predict the presence of these quasi-stable strings accompanied by the somewhat heavier monopoles.
- Quasi-stable strings can produce observable GWs and satisfy the PPTA bound.

Thank You

Back up slides

Inflation with GUT-singlet ϕ

- Inflation driven by Coleman-Weinberg potential of GUT-singlet ϕ

$$V(\phi) = A\phi^4 \left[\log\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + V_0.$$

Here $V_0 = AM^4/4$, M is the vacuum expectation value (VEV) of ϕ, and $A = \beta_D^4 D/(16\pi^2)$, where D is the dimensionality of the representation to which the GUT gauge symmetry breaking real scalar field χ_D belongs, and β determines the coupling $-\beta_D^2\phi^2\chi_D^2/2$ between ϕ and χ_D.

Shafi, Vilenkin, PRL 52, 691 (1984)

- The slow-roll parameters can be written in terms of the potential and its derivatives as follows.

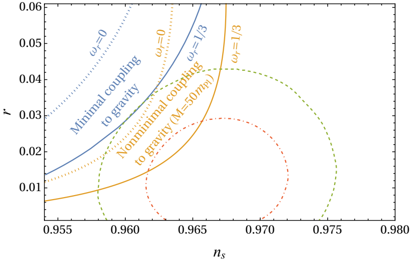
$$\epsilon = \frac{m_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = m_{\rm Pl}^2\frac{V''}{V}, \quad \xi^2 = m_{\rm Pl}^4\frac{V'V'''}{V^2}.$$

Inflation with GUT-singlet ϕ

- Inflationary predictions:

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon, \quad \Delta_R^2 = \frac{V}{24\pi^2\epsilon}.$$

Parameter	68% limits
$\boldsymbol{\Delta_R^2}$	$(2.1 \pm 0.1) \times 10^{-9}$
$\boldsymbol{n_s}$	0.9669 ± 0.0037
$\boldsymbol{r}$	$0.0163^{+0.0061}_{-0.013}$



- The BK18 + BAO + Planck 18 data rules out the CW inflation by 2σ. BICEP, Keck, PRL 127 (2021) 151301

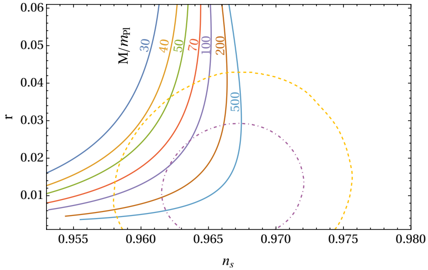
- A non-minimal coupling to gravity: $(1 + \xi(\phi^2 - M^2))R$ helps to satisfy the current data.

Maji, Shafi arXiv:2208.08137

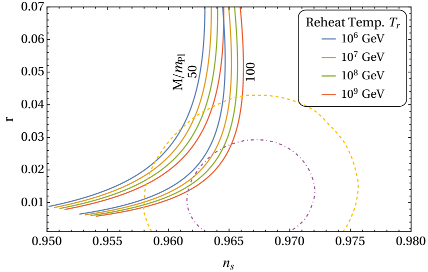
Lazarides, Maji, Roshan, Shafi, 2210.03710

Inflaton with Non-minimal Coupling to Gravity

- $|\xi| \sim 10^{-3}$ brings about inflation below the VEV with $A \sim 10^{-14}$.



$\omega_r = 0, T_r = 10^7$ GeV.



$\omega_r = 0.$

Maji, Shafi arXiv:2208.08137

Phase Transitions and Ginzburg criterion

- The potential $-\frac{1}{2}\beta_D^2\phi^2\chi_D^2 + \frac{\lambda_D}{4}\chi_D^4$ give rise to the spontaneous symmetry breakings via the VEV $\langle\chi_D\rangle = \beta_D/\sqrt{\lambda_D}\phi$.
- Ginzburg criterion for phase transition is

$$\boxed{\xi^3\Delta V \gtrsim T_H.}$$

Ginzburg, Soviet Phys. Solid State 2, 1824 (1961)

- $\Delta V = \frac{m_{\rm eff}^4}{16\lambda_D}$ is the potential difference between the local maximum at $\chi = 0$ and the minima.
- $\xi = \min\left(H^{-1}, m_{\rm eff}^{-1}\right)$ is the correlation length.
- $m_{\rm eff}^2 = 2\left(\beta_D^2\phi^2 - \sigma_{\chi_D}T_H^2\right)$ be the squared effective mass of $\chi_{\rm str}$.
- $T_H = H/2\pi$ is the Hawking temperature.

Evolution of Intermediate-mass Monopoles

Number density at production, $\xi = \min\left(H^{-1}, m_{\rm eff}^{-1}\right)$

Dilution during Inflation

Dilution from Inflaton oscillation

Monopole yield after reheating :

$$Y_M \simeq \frac{\frac{\xi^{-3}}{10}\; \exp(-3N_M)\; \left(\frac{\tau}{t_r}\right)^2}{\frac{2\pi^2}{45} g_* T_r^3}$$

Entropy density after reheating

- MACRO bound: $Y_M \lesssim 10^{-27}$.

  Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- Adopted threshold for observability: $Y_M \gtrsim 10^{-35}$.

Intermediate Mass Monopoles and MACRO

$\frac{M}{m_{\rm Pl}}$	$\log_{10}\left(\frac{V_0^{1/4}}{\rm GeV}\right)$	$\phi_+/m_{\rm Pl}$	$\phi_-/m_{\rm Pl}$	H_+	H_-	M_{I+}	M_{I-}	N_+	N_-
				(10^{13} GeV)					
50	16.45	43.62	42.16	3.74	4.15	5.58	6.40	10.9	17.1
70	16.54	63.57	62.07	3.67	4.04	5.27	5.93	10.9	17.1
100	16.64	93.57	92.04	3.51	3.81	4.89	5.39	10.8	17.1
200	16.86	193.71	192.16	3.03	3.21	4.08	4.35	10.8	17.0
500	17.16	494.33	492.79	2.33	2.40	3.07	3.18	10.7	16.9

Table: Values of the various parameters (indicated by a subscript $+$) corresponding to the MACRO bound ($Y_M < 10^{-27}$) on the flux of monopoles formed at the scale M_I and their values (indicated by a subscript $-$) corresponding to the adopted observability threshold ($Y_M > 10^{-35}$) for the monopole flux for a typical choice of nonminimal coupling, $\xi = -0.001$.

Maji, Shafi arXiv:2208.08137

Evolution of Strings in Inflationary Cosmology

- The mean inter-string distance at cosmic time t (temp $= T$):

$$d_{\rm str} = p\,\xi(\phi_I)\ \exp(N_{\rm str})\ \left(\frac{t_r}{\tau}\right)^{\frac{2}{3}}\ \frac{T_r}{T}$$

Inter-string sepera-tion at production $\xi = \min\left(H^{-1}, m_{\rm eff}^{-1}\right)$

Expansion during Infla-tion

Expansion dur-ing Inflaton oscillation

Expansion after reheat-ing

- The string network re-enters the post-inflationary horizon at cosmic time t_F if

$$d_{\rm str}(t_F) = d_{\rm hor}(t_F)$$

$$\text{with}\ \ d_{\rm hor}(t_F) = \begin{cases} 2t_F & \text{(radiation dominance)} \\ 3t_F & \text{(matter domination).} \end{cases}$$

Chakrabortty, Lazarides, Maji, Shafi JHEP 02 (2021) 114

String Loops and Gravitational Waves

- After horizon re-entry, the strings inter-commute and form loops at any subsequent time t_i.

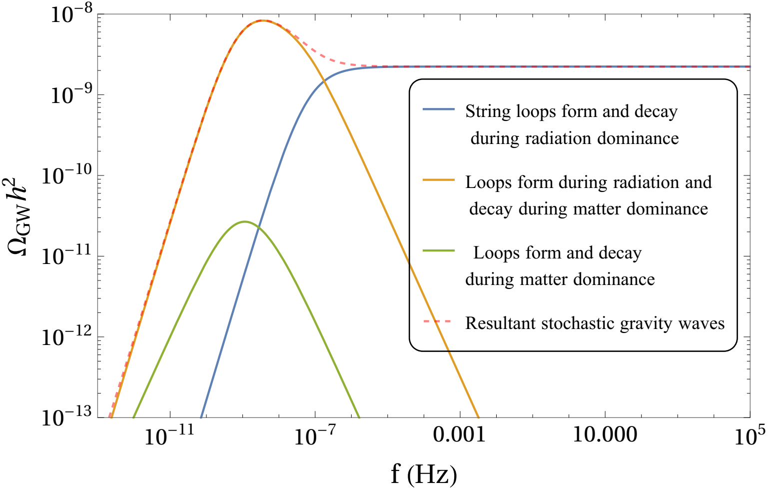
- Loops of initial length $l_i = \alpha t_i$ decay via emission of gravity waves.

$$\frac{dE_{\text{GW}}^{(k)}}{dt} = \Gamma_k G\mu^2 \quad \text{with} \;\; k = 1, 2, 3, ...$$

- The redshifted frequency of a normal mode k, emitted at time $\tilde{t}$, as observed today, is given by

$$f = \frac{a(\tilde{t})}{a(t_0)} \frac{2k}{\alpha t_i - \Gamma G\mu(\tilde{t} - t_i)}, \;\; \text{with} \;\; \Gamma = \sum \Gamma_k$$

Stochastic Gravitational Wave Background



Sousa, Avelino, Guedes, PRD 101 (2020) 10, 103508

Loop distribution

$$n_{rm}(l, t > t_{eq} > t_M) = \frac{0.18 t_{eq}^{1/2} \Theta(0.1 t_M - l - \Gamma G\mu(t - t_M))}{t^2 (l + \Gamma G\mu t)^{5/2}} \quad (1)$$