Detection of gravitational waves by electromagnetic cavity: a review

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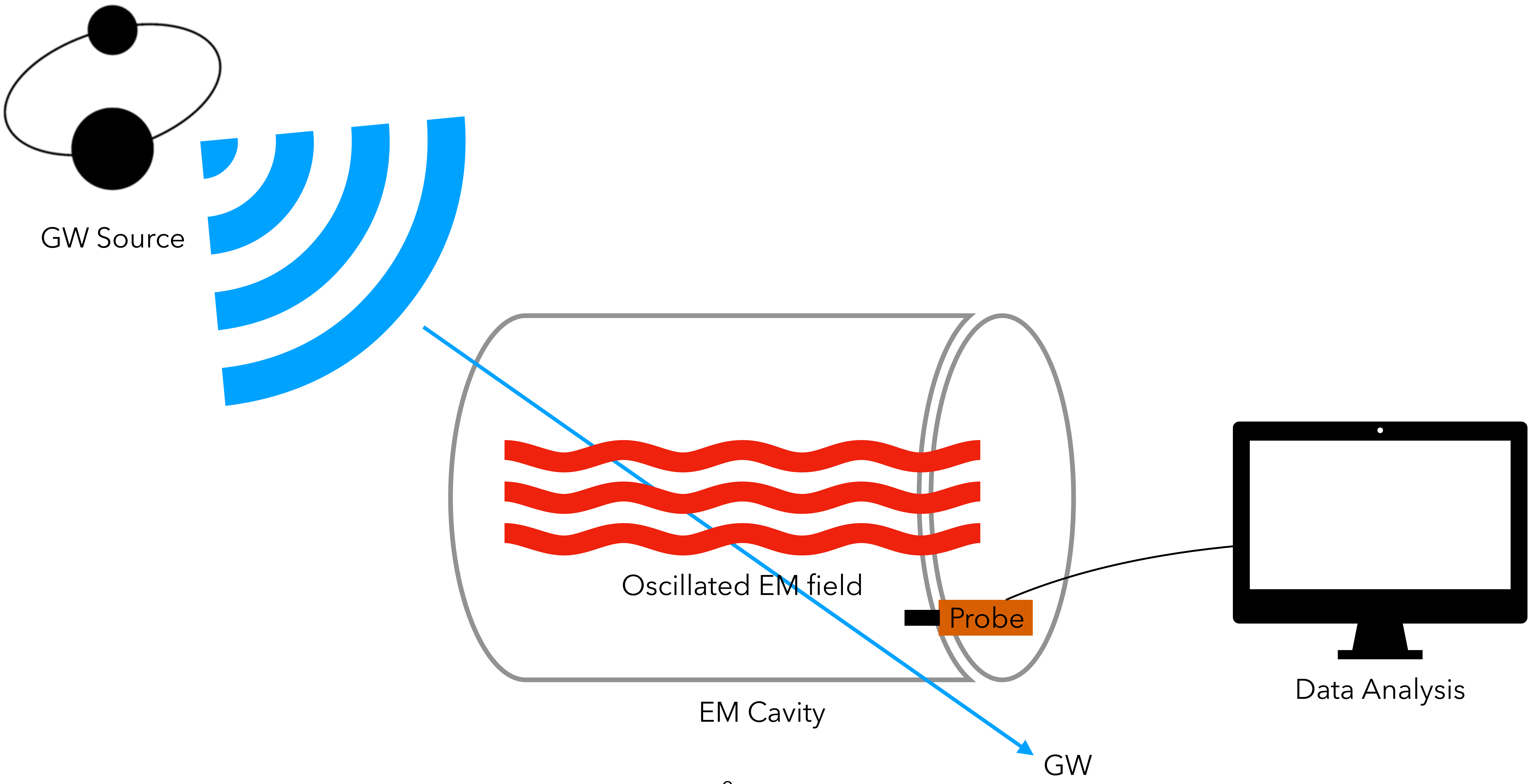
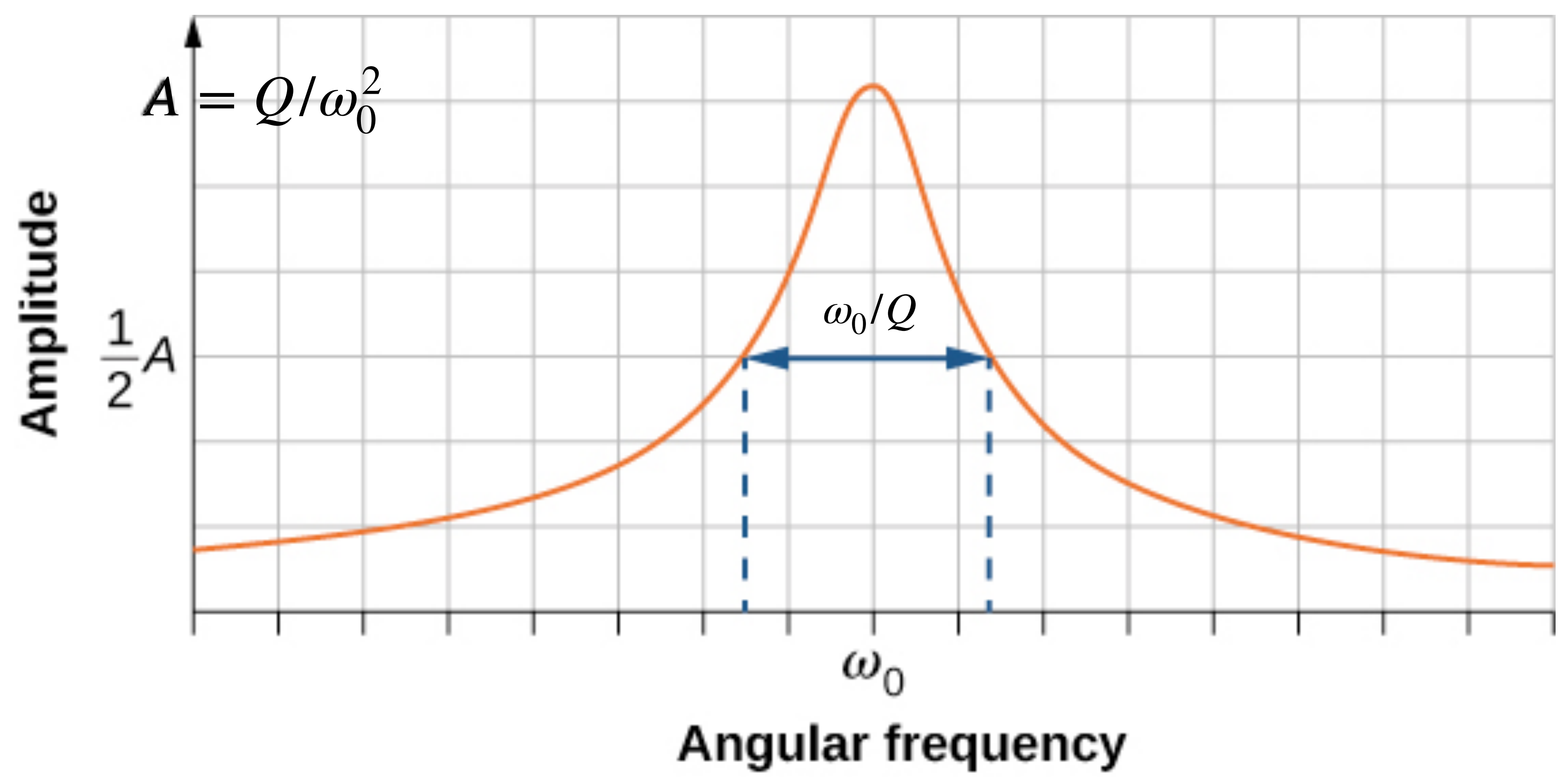
Overview

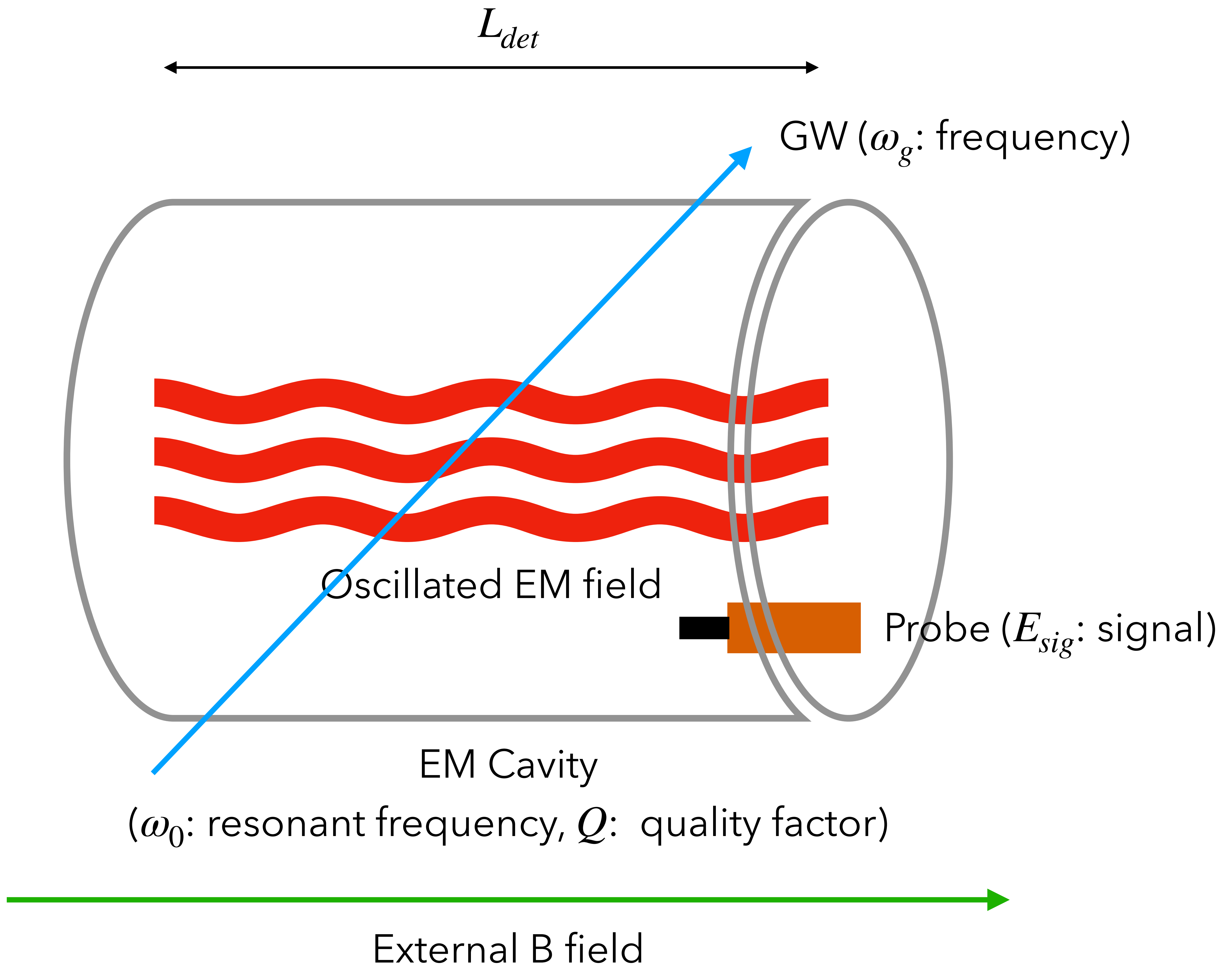

GW Source
Oscillated EM field
Probe
EM Cavity
GW
Data Analysis

Brief Working Principle of Detection

- GW induces a forced oscillation of EM field. When $\omega_g \to \omega_0$, the cavity is resonantly excited.
- Detectable GW frequency:
$$\omega_g \sim \omega_0 \sim \frac{1}{L_{det}} \sim 1\ \mathrm{GHz}$$
- Frequency Band: $\Delta\omega_g \sim \Delta\omega_0 \sim \frac{\omega_0}{Q} \sim 10\ \mathrm{kHz}$



Amplitude of forced oscillation



Reference Papers

- Detecting high-frequency gravitational waves with microwave cavities / A. Berline+ / 2022 PRD
- Novel Search for High-Frequency Gravitational Waves with Low-Mass Axion Haloscopes / V. Domcke / 2022 PRL

Detecting high-frequency gravitational waves with microwave cavities

Asher Berlin,[1,2,3] Diego Blas,[4,5] Raffaele Tito D'Agnolo,[6] Sebastian A. R. Ellis,[7,6] Roni Harnik,[2,3] Yonatan Kahn,[8,9,3] and Jan Schütte-Engel[8,9,3]

We give a detailed treatment of electromagnetic signals generated by gravitational waves (GWs) in resonant cavity experiments. Our investigation corrects and builds upon previous studies by carefully accounting for the gauge dependence of relevant quantities. We work in a preferred frame for the laboratory, the proper detector frame, and show how to resum short-wavelength effects to provide analytic results that are exact for GWs of arbitrary wavelength. This formalism allows us to firmly establish that, contrary to previous claims, cavity experiments designed for the detection of axion dark matter only need to reanalyze existing data to search for high-frequency GWs with strains as small as $h \sim 10^{-22}$–10^{-21}. We also argue that directional detection is possible in principle using readout of multiple cavity modes. Further improvements in sensitivity are expected with cutting-edge advances in superconducting cavity technology.

Novel Search for High-Frequency Gravitational Waves with Low-Mass Axion Haloscopes

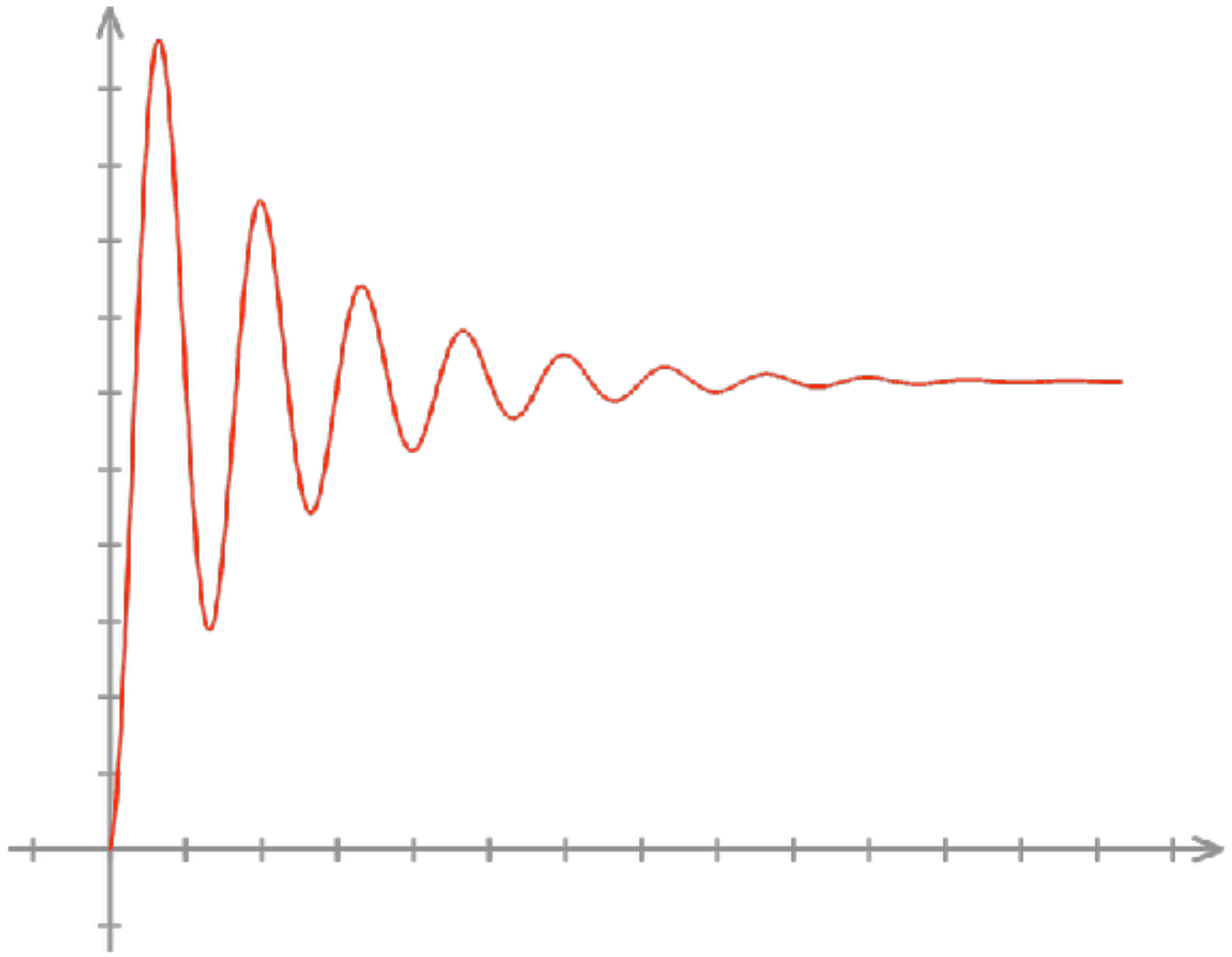
Valerie Domcke,[1,2] Camilo Garcia-Cely,[3] and Nicholas L. Rodd[1]

Gravitational waves (GWs) generate oscillating electromagnetic effects in the vicinity of external electric and magnetic fields. We discuss this phenomenon with a particular focus on reinterpreting the results of axion haloscopes based on lumped-element detectors, which probe GWs in the 100 kHz–100 MHz range. Measurements from ABRACADABRA and SHAFT already place bounds on GWs, although the present strain sensitivity is weak. However, we demonstrate that the sensitivity scaling with the volume of such instruments is significant—faster than for axions—and so rapid progress will be made in the future. With no modifications, DMRadio-m^3 will have a GW strain sensitivity of $h \sim 10^{-20}$ at 200 MHz. A simple modification of the pickup loop used to readout the induced magnetic flux can parametrically enhance the GW sensitivity, particularly at lower frequencies.

Sources of GWs

Source of Gravitational Waves (GWs)

- It should be violent event to emit GWs as strong as enough to detect.
- Source Type
  - Transient Source
  - Stochastic Source

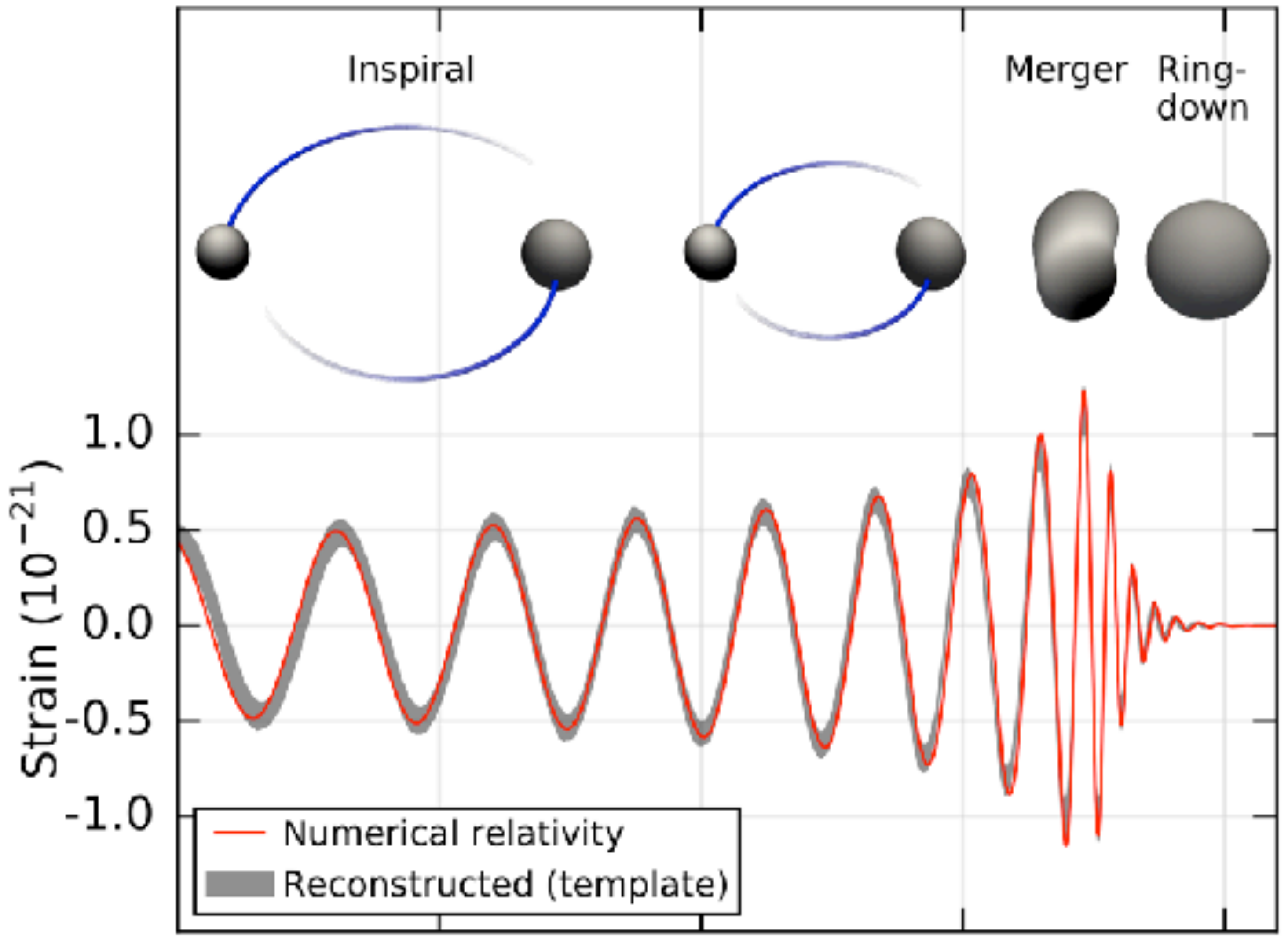


Transient Signal

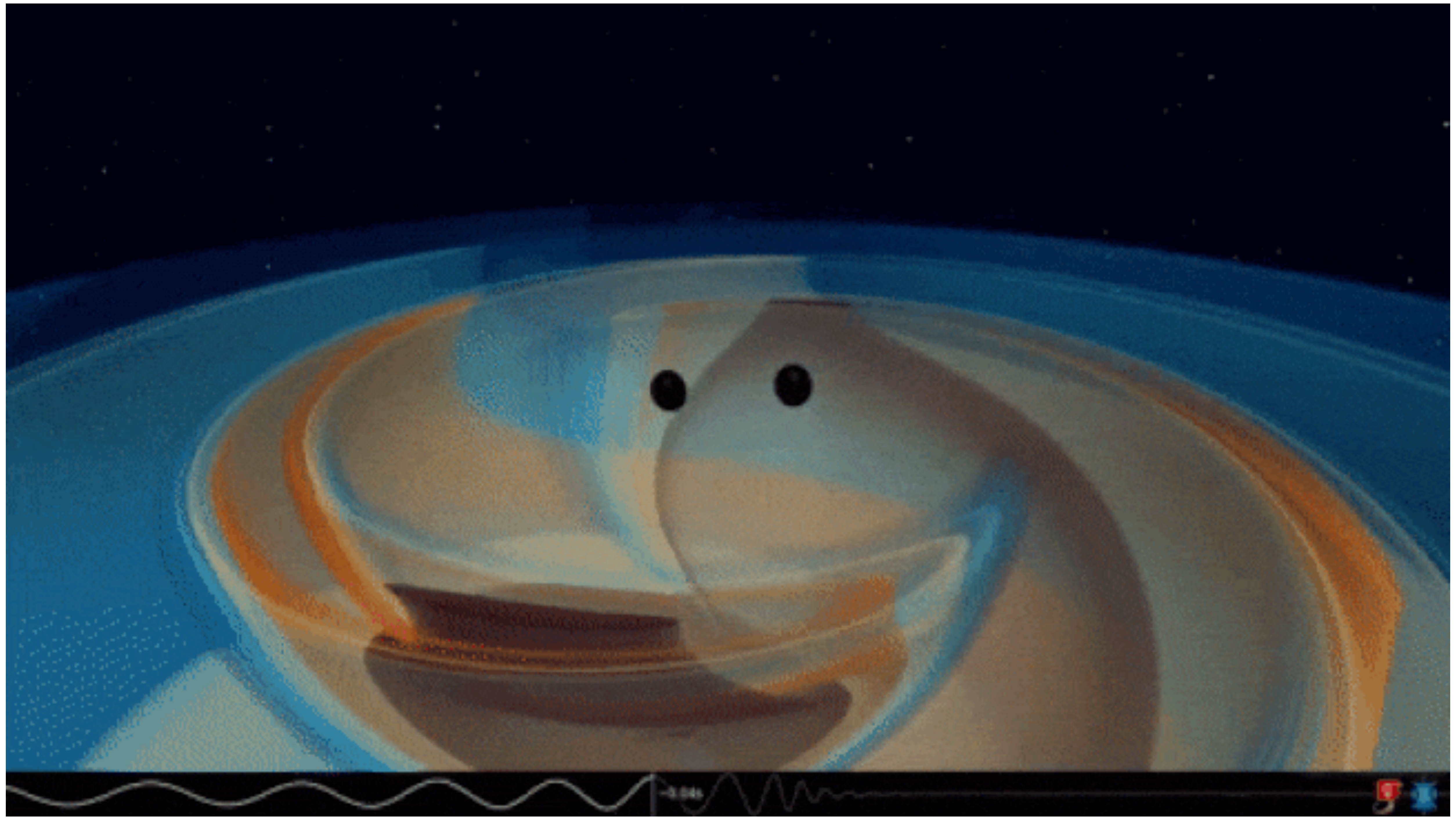
Stochastic Water Waves from Distribution of Rain Drops

Sources of GWs: Transient GWs

- Merger of Binary Compact Stars
  - Binary Black Holes (BH) Merger
  - Binary Neutron Star (NS) Merger
  - BH-NS Merger



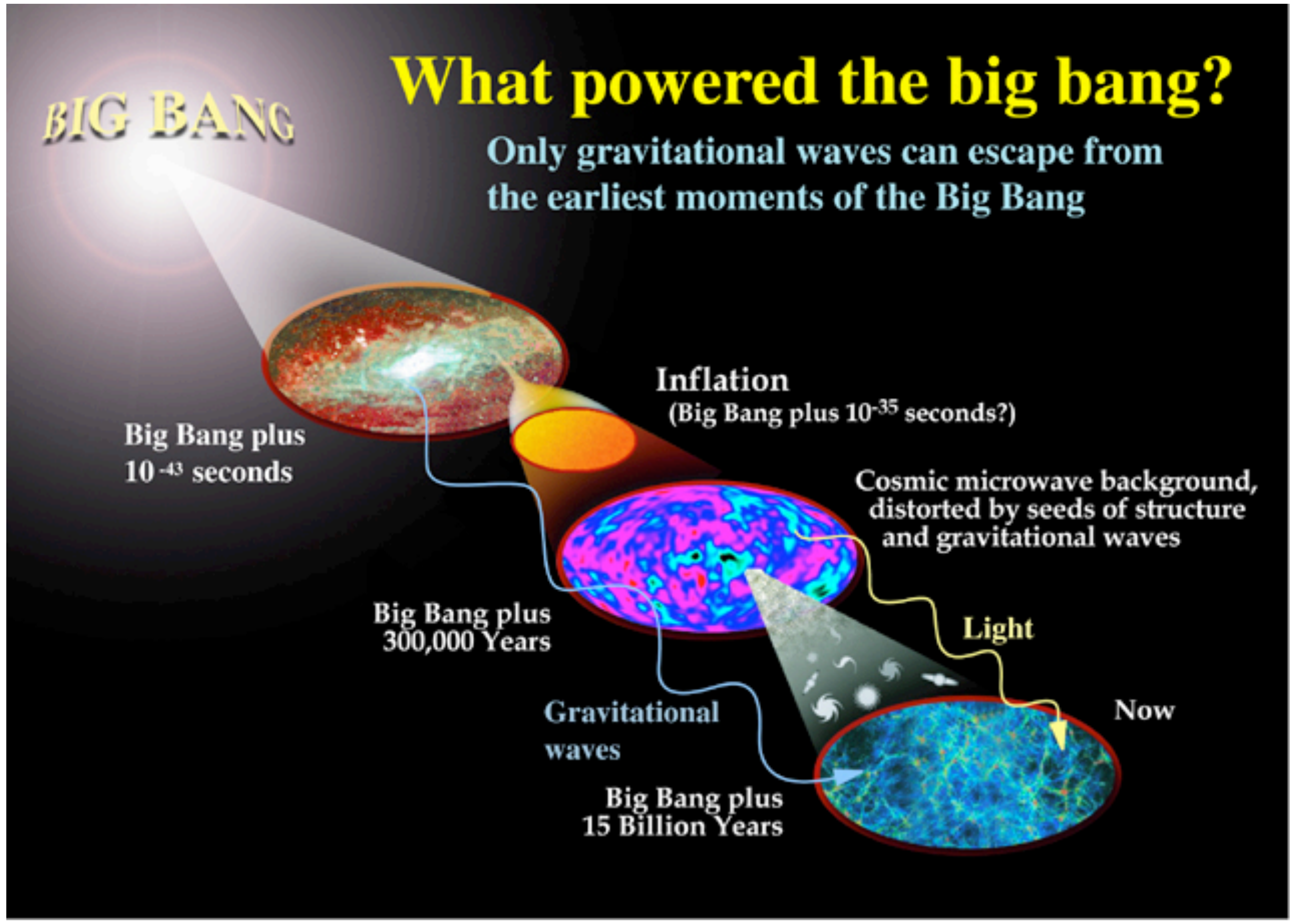
GW Waveform of a Transient Source / PRL 116, 061102 (2016)



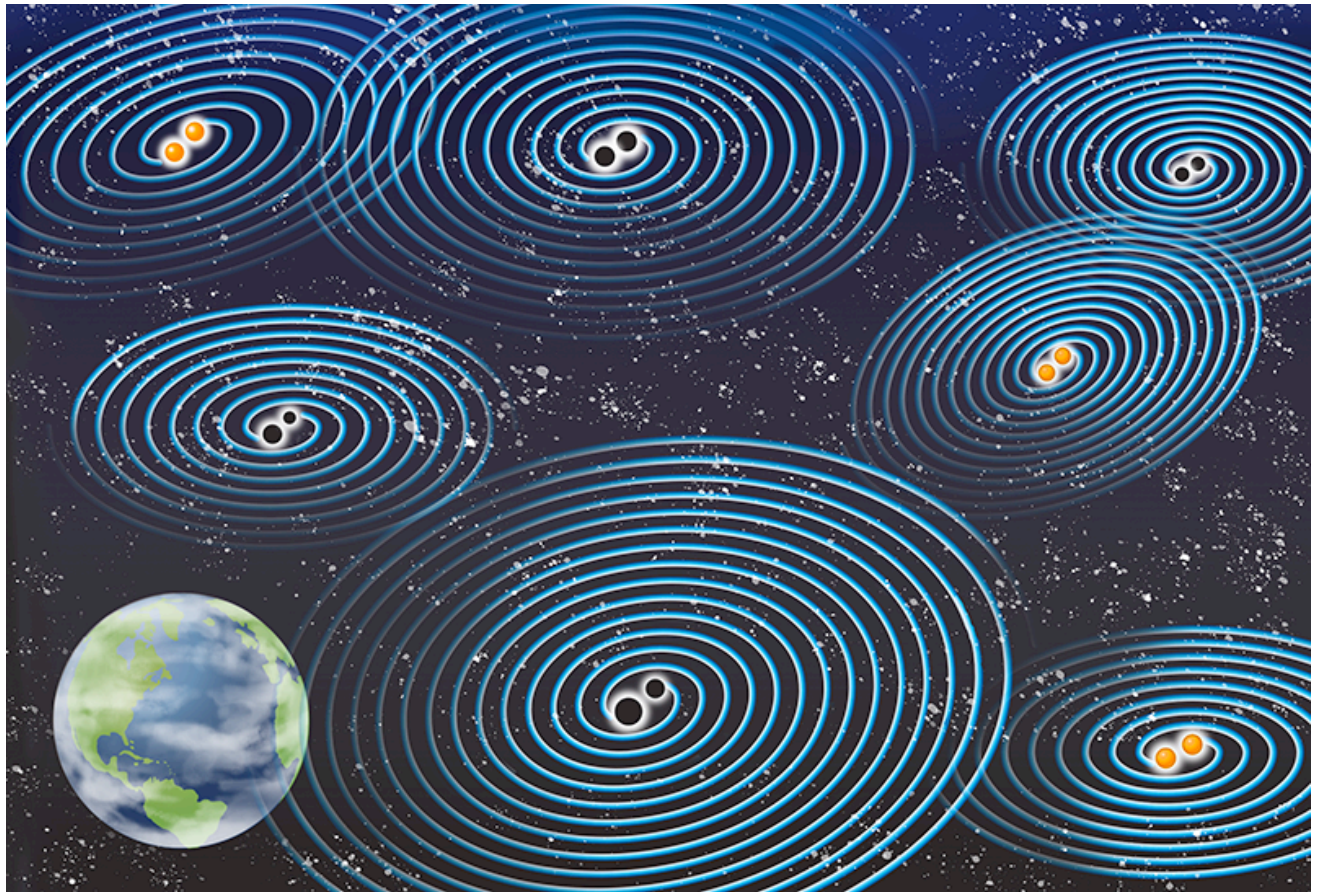
Merger of Binary Black Holes and Its Gravitational Waves / Credit: Simulating Extreme Spacetimes

Sources of GWs: Stochastic GWs

- Cosmological origin: Quantum state in early universe
- Astrophysical origin: Distribution of compact binaries



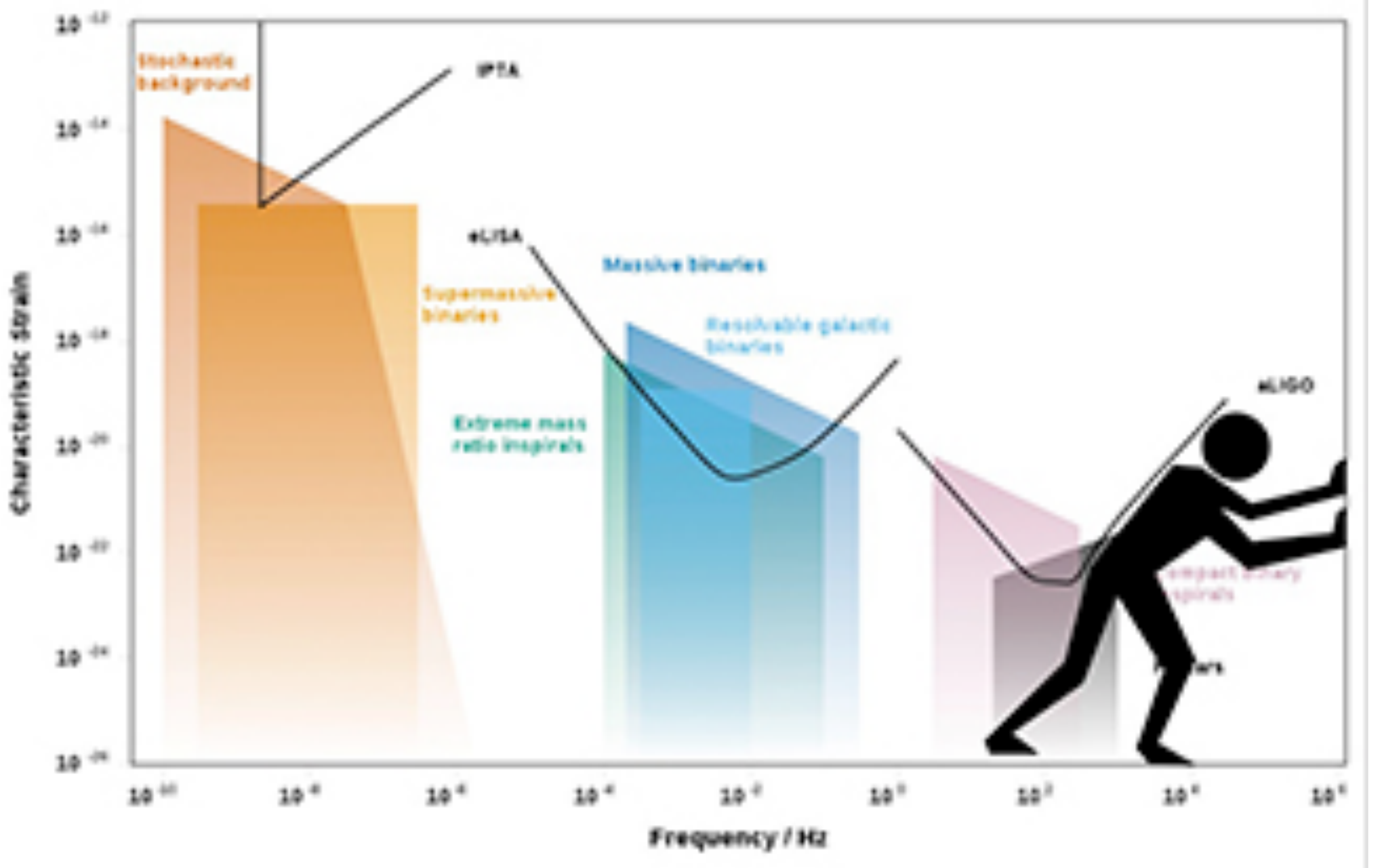
Stochastic GW from Big Bang / Credit: NASA



Distribution of Compact Binaries / Credit: APS

Ultra-High-Frequency (~GHz) GWs

- Mergers of compact objects with subsolar masses
  - Primordial black holes $\sim 10^{-6} M_{\odot}$
  - Exotic compact objects: boson stars, fermion stars, gravitino stars, gravistars, dark matter blobs
- Cosmological stochastic GWs
  - first-order phase transition, cosmic string, inflation, preheating
- We are welcome to your novel scenario!



Credit: UHF-GW initiative

EM Cavity with GWs

EM Field induced by GW

- Vacuum Maxwell Equation
  - $\nabla^b F_{ab} = J_a$
  - $\nabla_{[a} F_{bc]} = 0$
- Wave Equation
  - $\nabla^c \nabla_c F_{ab} = -2\nabla_{[a} J_{b]} - F_{cd} R^{cd}{}_{ab} - 2F_{d[a} R^d{}_{b]}$
- in Background Spacetime
  - $R^a{}_{bcd} = 0$
  - $F_{ab} = \epsilon^c{}_{ab} B_c = \mathrm{const}$
  - $J_a = 0$
- GW in Transverse-Traceless Gauge
  - $\nabla^b h_{ab} = 0 \qquad h^a{}_a = 0 \quad h_{ab} u^b = 0$
- Perturbation
  - ${}^\epsilon F_{ab} = \epsilon \left\{ F_{ab} + \epsilon \, (\delta F)_{ab} + O\left(\epsilon^2\right) \right\}$
  - ${}^\epsilon J_a = \epsilon \left\{ \cancel{J_a} + \epsilon \, (\delta J)_a + O\left(\epsilon^2\right) \right\}$
  - $\nabla^b (\delta F)_{ab} = (\delta J)_a - F_{bc} \nabla^b h^c{}_a$
  - $\nabla_{[a} (\delta F)_{bc]} = 0$
  - $\nabla^c \nabla_c (\delta F)_{ab} = -2\nabla_{[a} (\delta J)_{b]} - F_{cd} (\delta R)^{cd}{}_{ab}$

Perturbation of Electric Field

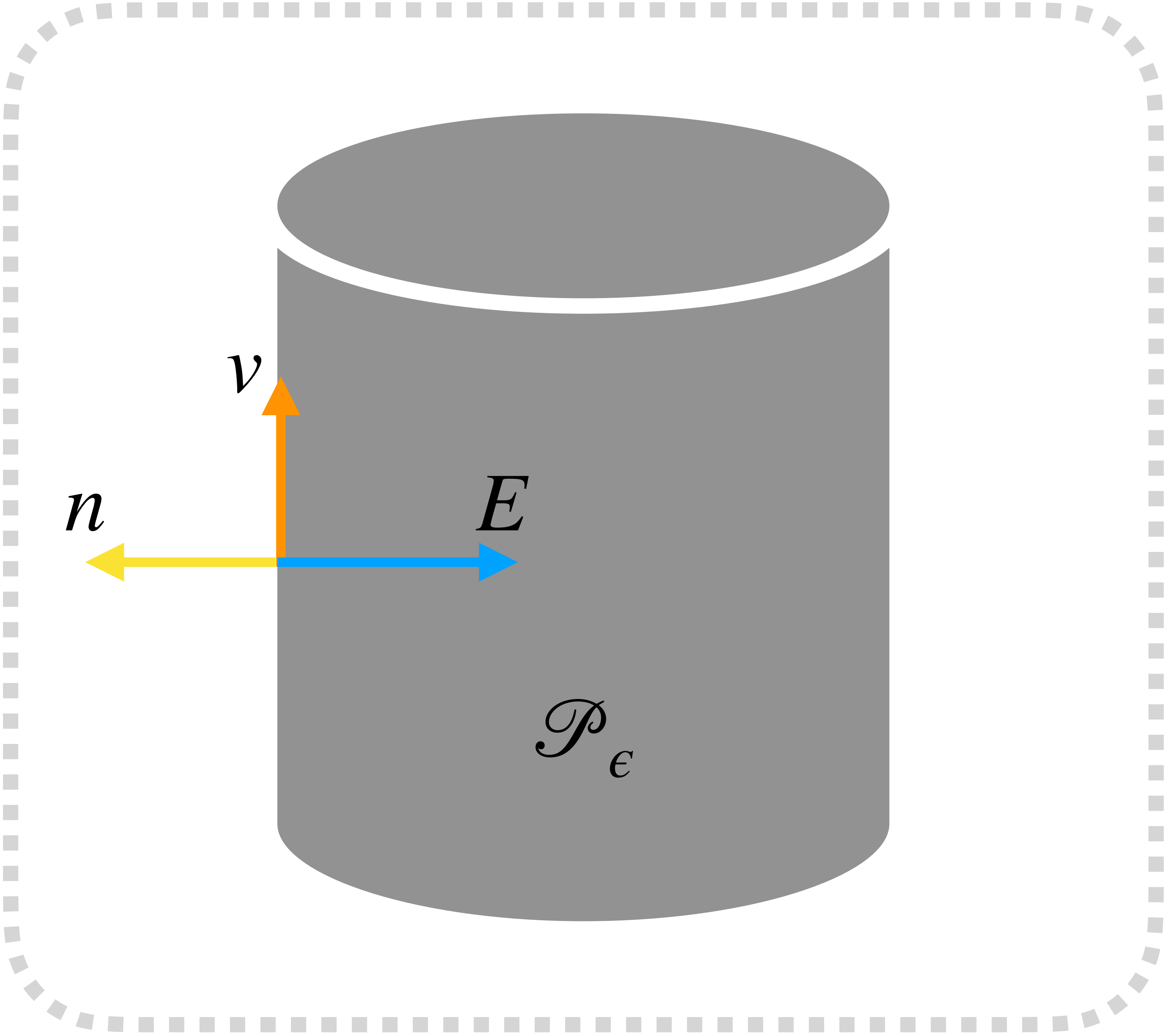
- 4-velocity for inertial observers
  - ${}^{\epsilon}u^a = u^a + \epsilon\cancel{(\delta u)^a} + O\left(\epsilon^2\right)$
- Perturbation of electric field
  - ${}^{\epsilon}E_a = \epsilon\left\{\cancel{E_a} + \epsilon\,(\delta E)_a + O\left(\epsilon^2\right)\right\}$
  - $(\delta E)_a = (\delta F)_{ab}\,u^b + \cancel{F_{ab}\,(\delta u)^b}$
  - Note that δE is gauge-invariant
- Maxwell equation
  - $\nabla^a(\delta E)_a = -\,u^a\,(\delta J)_a$
- Ohm's Law
  - $\perp\,\delta J^a = \sigma\,(\delta E)^a + \cancel{\sigma h^{ab}E_b} + \cancel{(\delta\sigma)\,E_a}$
  - σ: conductivity

We only consider this mode!

- Wave equation for solenoidal mode $\partial^i\,(\delta E)_i = 0$
  - $\left(-\partial_t^2 - \sigma\partial_t + \partial^i\partial_i\right)(\delta E)_a = -\,F_{cd}(\delta R)^{cd}{}_{ab}u^b$
- Wave equation for irrotational mode $\epsilon^{abc}D_b\,(\delta E)_c = 0$
  - $\left(-\partial_t^2 - \sigma\partial_t\right)(\delta E)_a = -\,F_{cd}(\delta R)^{cd}{}_{ab}u^b$

Boundary Condition For δE

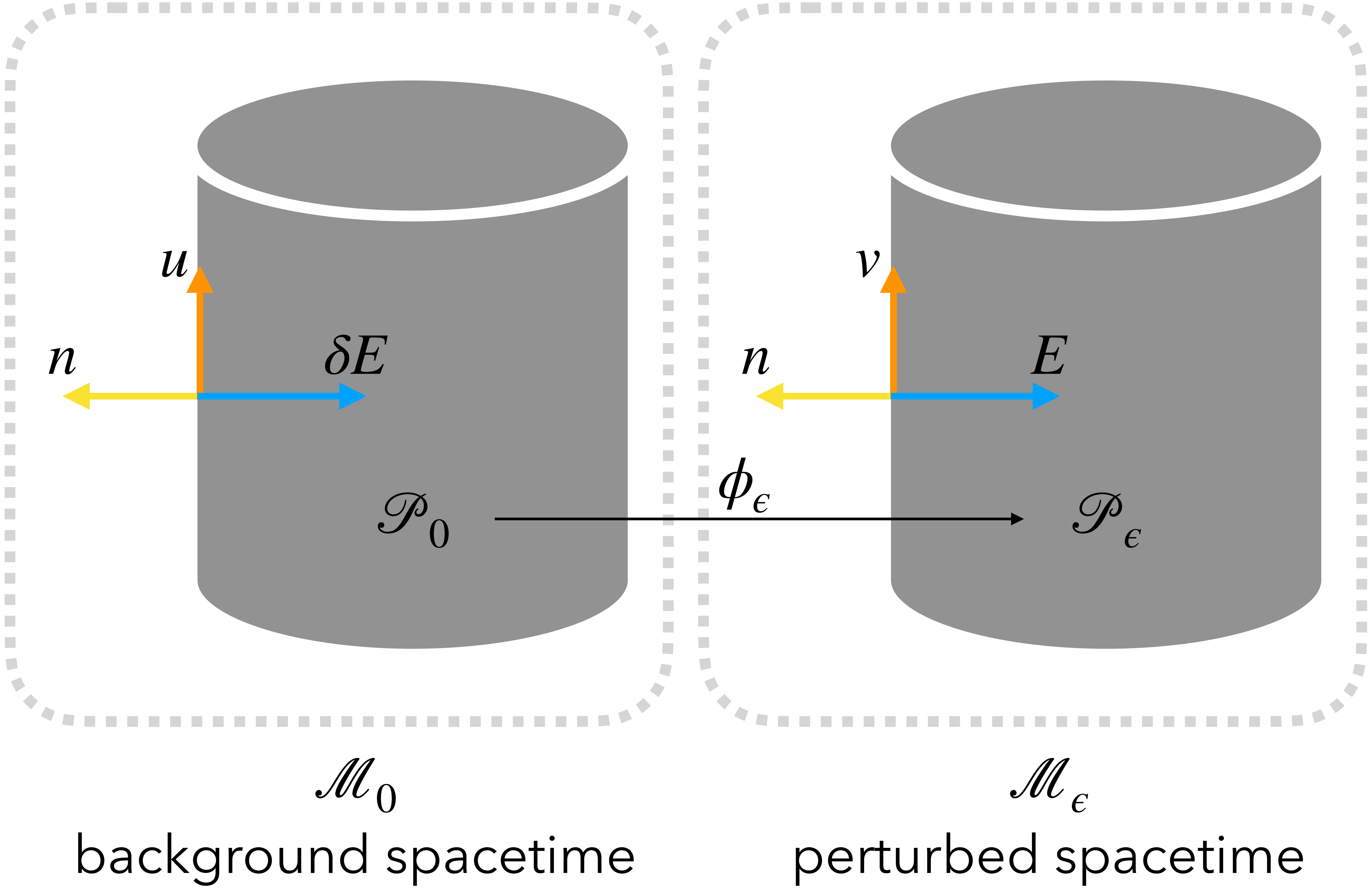
- Boundary condition on $\mathscr{P}_\epsilon$ for perfect conductor
- $\epsilon_{abc}{}^{d} v^b n^c E_d = 0$
- where
- $\mathscr{P}_\epsilon$: 3-dimensional timelike hypersurface for surface conductor in $\mathscr{M}_\epsilon$.
- n: vector field normal to $\mathscr{P}_\epsilon$
- v: vector field of 4-velocity for the conductor



$\mathscr{M}_\epsilon$
perturbed spacetime

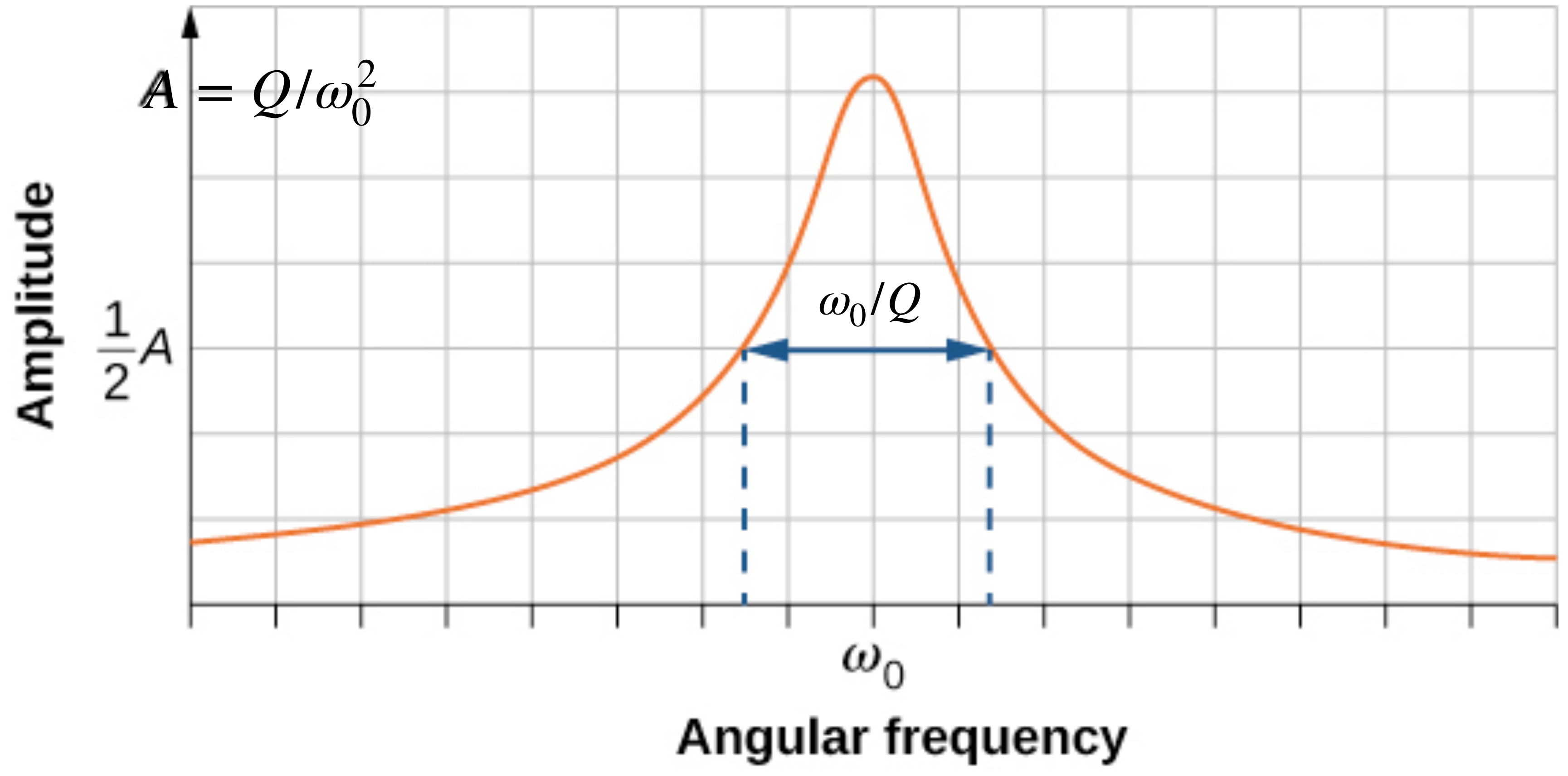
Boundary Condition For δE

- $\phi_\epsilon : \mathscr{M}_0 \to \mathscr{M}_\epsilon$: a diffeomorphism s.t. $\phi_\epsilon\left[\mathscr{P}_0\right] = \mathscr{P}_\epsilon$
- $\epsilon_{abc}{}^{d} v^b n^c (\delta E)_d = 0$ on $\mathscr{P}_0$ in $\mathscr{M}_0$ by the perturbation using ϕ_ϵ.
- Because δE is gauge-invariant, in our TT gauges we also have
- $\epsilon_{abc}{}^{d} u^b n^c (\delta E)_d = 0$ on $\mathscr{P}_0$



Mode Expansion

- Mode Expansion (depends on shape of cavity)
  - $$(\delta E)_a(t,\vec{x}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_n \tilde{e}_n(\omega)\, \tilde{E}^n_a(\vec{x})\, e^{-i\omega t}$$
- where $\tilde{E}$ satisfies
  - $\partial^i \tilde{E}^n_i = 0$ (solenoidal mode)
  - $\partial^i \partial_i \tilde{E}^n_a = -\omega_n^2 \tilde{E}^n_a$ (dispersion)
  - $\int_{\mathscr{V}} d\mathscr{V}\, \tilde{E}^n \cdot \tilde{E}^{m*} = \delta_{nm} \int_{\mathscr{V}} d\mathscr{V}\, \left|\tilde{E}^n\right|^2$ (orthogonality)
  - $\left|\tilde{E}^n\right| \propto B$ (to make $\tilde{e}$ dimensionless)
- Amplitude of forced oscillation given by the wave equation
  - $$\tilde{e}(\omega) = \frac{\omega^2}{\left(-\omega^2 - i\frac{\omega_n}{Q_n}\omega + \omega_n^2\right)} \int d^2\kappa\, (B\times\kappa)^a\, \tilde{h}^b{}_a(\omega,\kappa) \int_{\mathscr{V}} d\mathscr{V}\, \tilde{E}^{n*}_b(\vec{x})\, e^{i\omega\kappa\cdot\vec{x}} \left[\int_{\mathscr{V}} d\mathscr{V}\, \left|\tilde{E}^n\right|^2\right]^{-1}$$



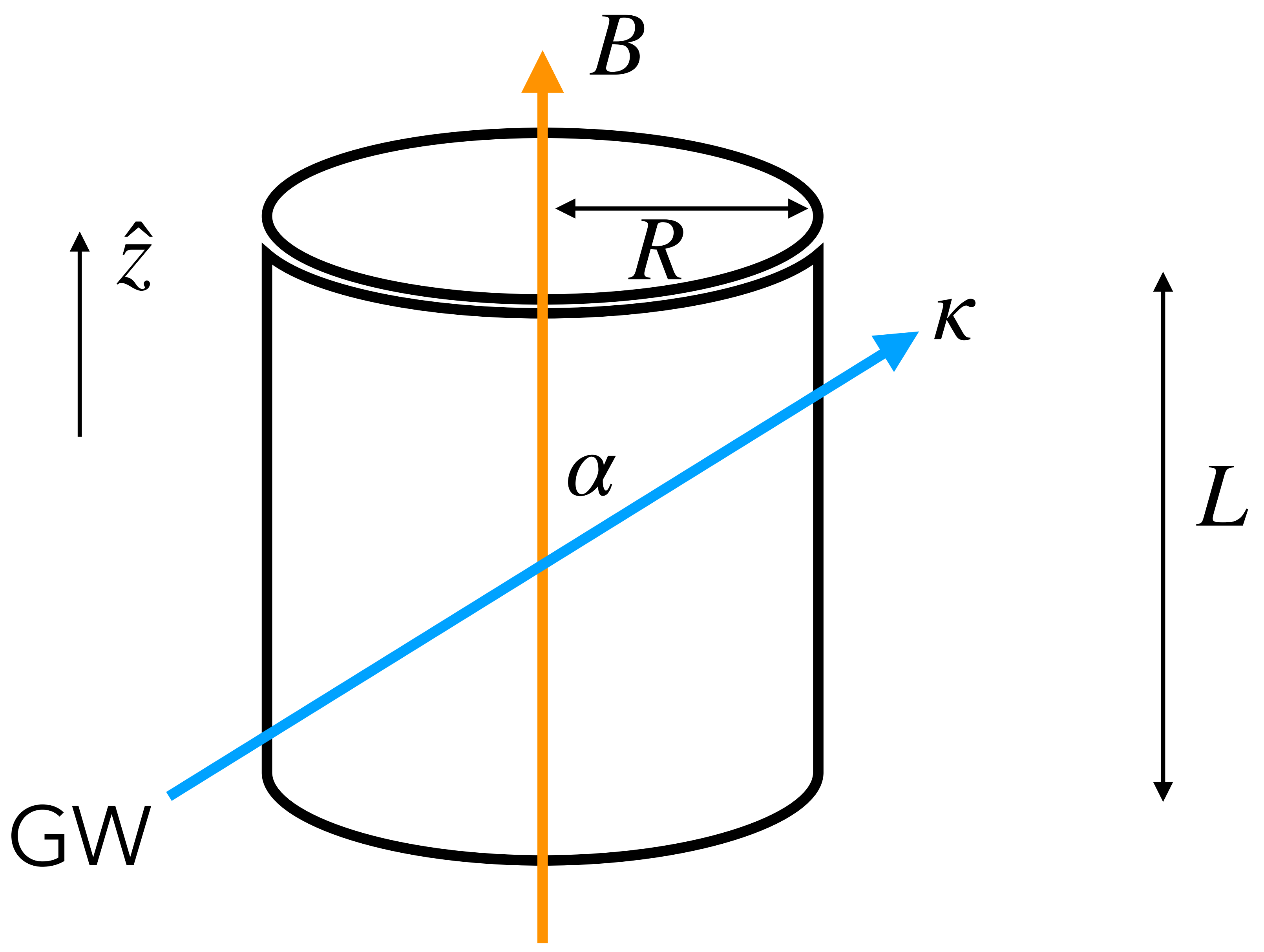
Amplitude of forced oscillation

GW Signal

- We choose a specific mode with n and frequency band $\omega_1 < \omega_n < \omega_2$
- $(\delta E)_a \left(t, \vec{x}\right) \simeq \int_{|\omega|=\omega_1}^{\omega_2} \frac{d\omega}{2\pi} \tilde{e}_n(\omega)\, \tilde{E}_a^n \left(\vec{x}\right) e^{-i\omega t}$
- By measuring δE using the probe at $\vec{x}_p$ directed to $\hat{z}$, we can extract $\tilde{e}(\omega)$ because $\tilde{E}^n \left(\vec{x}_p\right) \cdot \hat{z}$ can be given in advance.
- We define GW signal as
- $h(t) = \int_{|\omega|=\omega_1}^{\omega_2} \frac{d\omega}{2\pi} \tilde{e}_n(\omega)\, e^{-i\omega t}$
- Output of the detector will be $s(t) = h(t) + n(t)$

Example: TM010 mode of Cylindrical Cavity

- $\tilde{E}_{010}\left(\rho, \phi, z\right) = BJ_0\left(\omega_{010}\rho\right)\hat{z}$
- B: strength of external magnetic field
- $\omega_{010} = j_{0,1}/R$: resonance frequency
- $j_{0,1}$: first zero of Bessel function J_0
- R: radius of cylinder



- $\tilde{e}\left(\omega\right) = \frac{\omega^2}{\left(-\omega^2 - i\frac{\omega_{010}}{Q_{010}}\omega + \omega_{010}^2\right)} \frac{\left(2\pi\right)^2 j_{0,1}}{\pi L R^2 J_1\left(j_{0,1}\right)} \int d^2\kappa\, \left(\hat{z}\times\kappa\right)^a \tilde{h}_{ab}\left(\omega,\kappa\right)\hat{z}^b \frac{\sin\left(\frac{1}{2}\omega L\cos\alpha\right)}{\frac{1}{2}\omega\cos\alpha} \frac{J_0\left(R\omega\sin\alpha\right)}{\omega_{010}^2 - \left(\omega\sin\alpha\right)^2}$

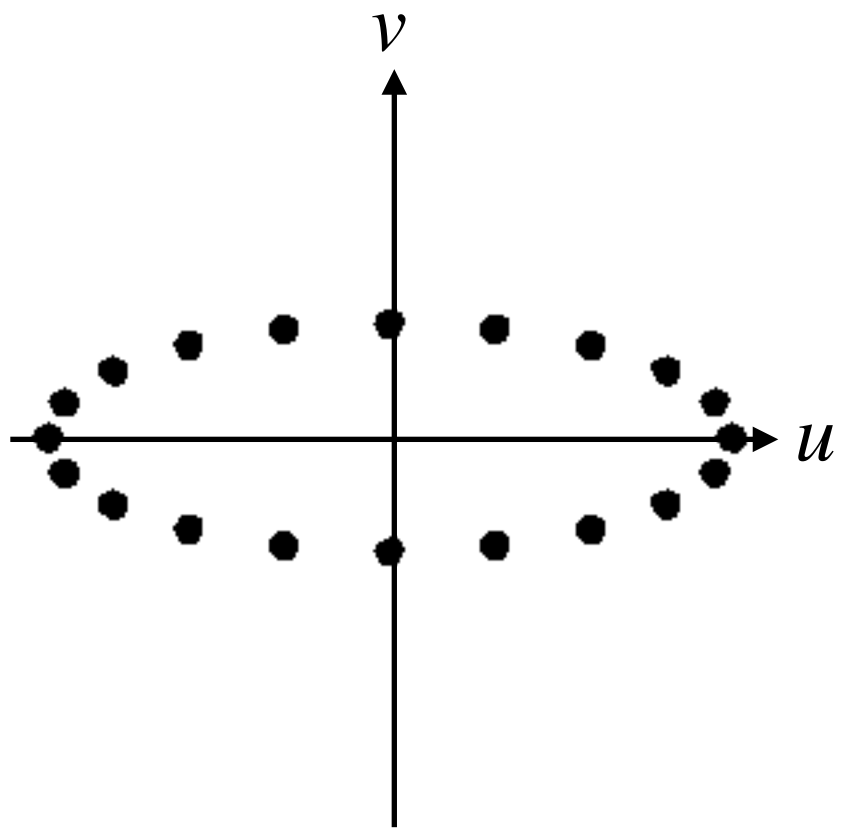
Data Analysis

Statistical Assumptions on SGWB

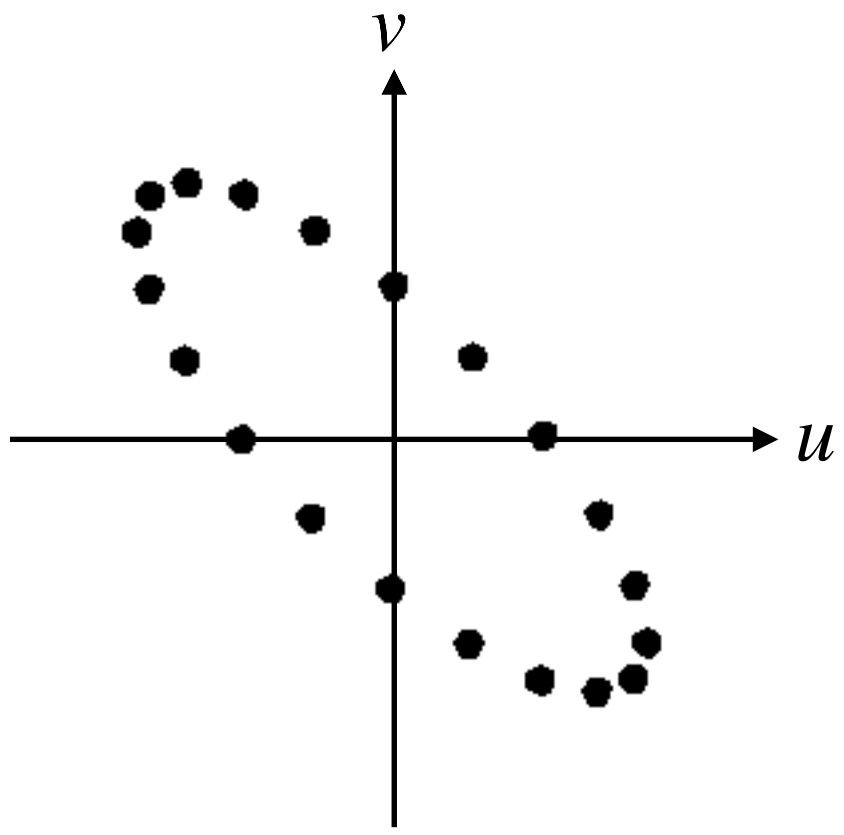
- GWs
  - $h_{ab}\left(t,\vec{x}\right) = \int d^2\kappa \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{h}_{ab}(\omega,\kappa)\, e^{iP\left(t,\vec{x};\,\omega,\kappa\right)}$
- Gaussian and stationary assumptions
  - $\left\langle \tilde{h}_{ab}(\omega,\kappa) \right\rangle = 0$
  - $\left\langle \tilde{h}_{ab}(\omega,\kappa)\, \tilde{h}^{*}_{cd}(\omega',\kappa') \right\rangle \propto S_h(\omega)\, \delta(\omega-\omega')$
  - $S_h(\omega)$: power spectral density (real and even)
- Isotropic assumption
  - $\left\langle \tilde{h}_{ab}(\omega,\kappa)\, \tilde{h}^{*}_{cd}(\omega',\kappa') \right\rangle \propto \delta^2(\kappa-\kappa')$
  - $\delta^2(\kappa-\kappa') = \delta(\cos\theta - \cos\theta')\, \delta\left(\phi-\phi'\right)$
  - $\kappa = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$

Statistical Assumptions on SGWB

- No prefer polarization assumption
  - $\left\langle \tilde{h}_{ab}(\omega,\kappa)\,\tilde{h}^{*}_{cd}(\omega',\kappa') \right\rangle \propto \Lambda_{abcd}(\kappa)$
  - $\Lambda^{ab}{}_{cd} = e^{ab}_{A} e^{A}_{cd} = P^{a}{}_{(c} P^{b}{}_{d)} - \frac{1}{2} P^{ab} P_{cd}$: projection operator for symmetric traceless rank-2 tensors in the tidal plane
  - $P^{a}{}_{b} = \delta^{a}{}_{b} + n^{a} n_{b} - \kappa^{a} \kappa_{b}$: projection operator for vector to the tidal plane



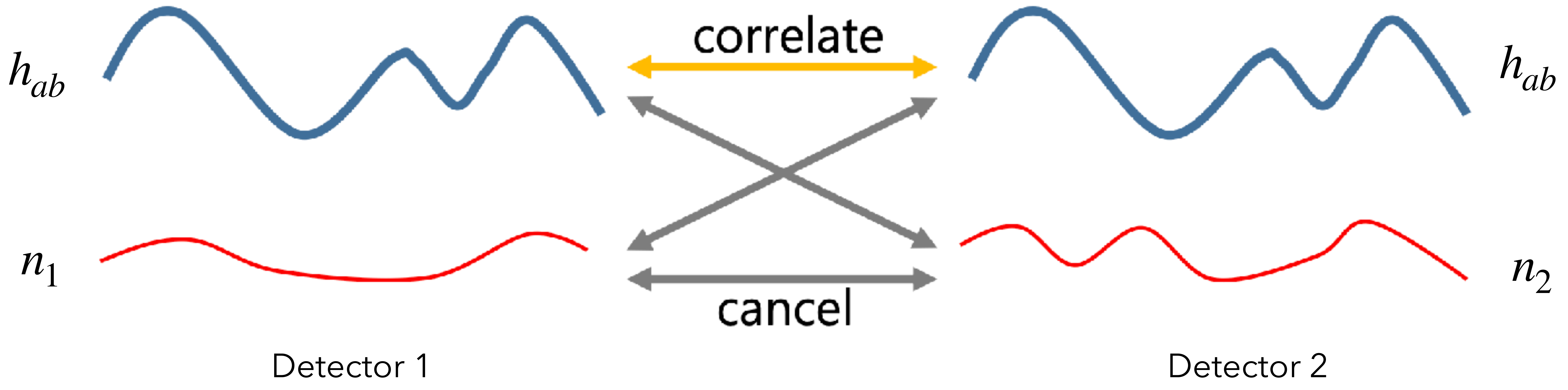
+ polarization



× polarization

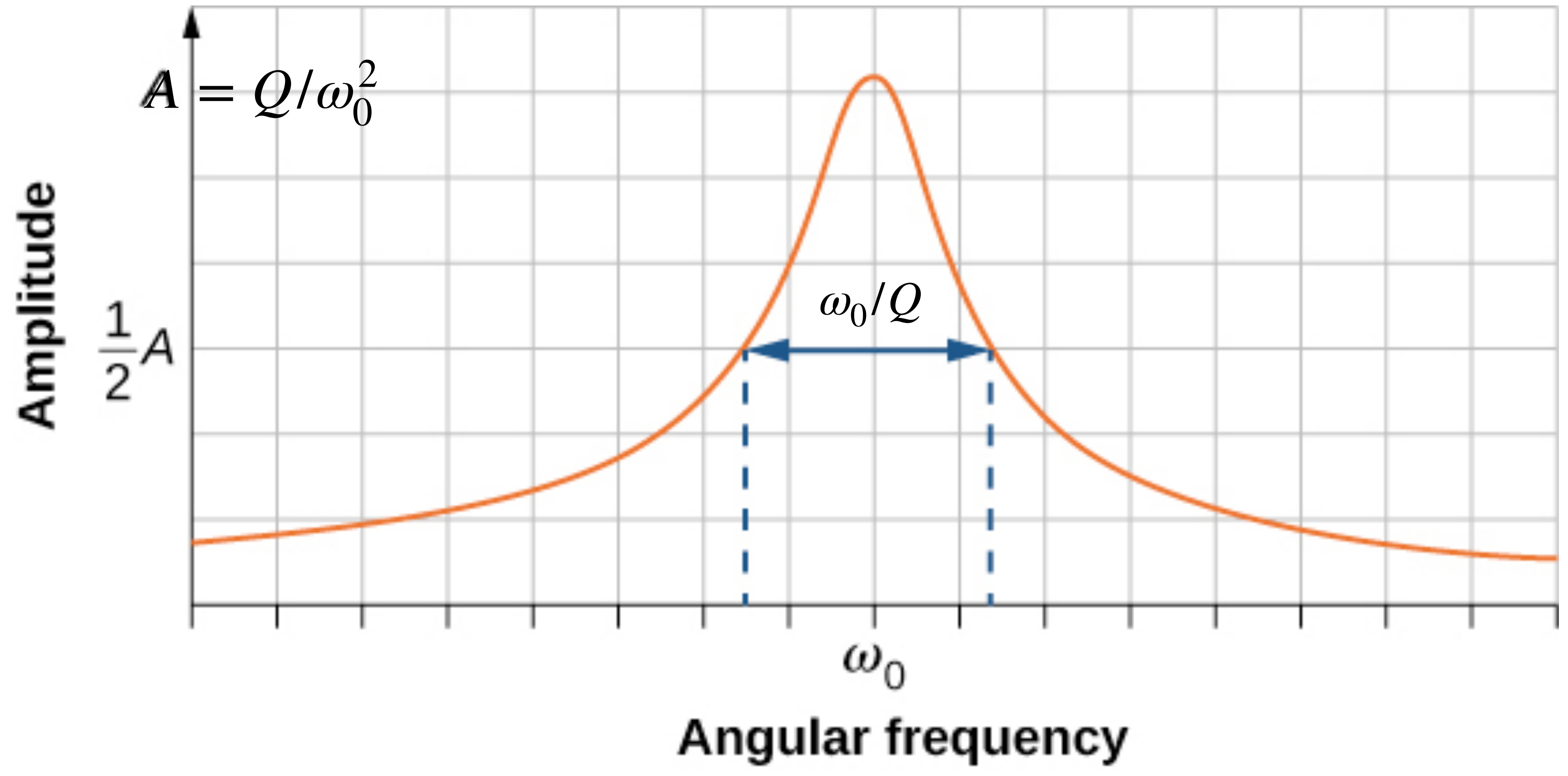
Correlation Method

- Output for two-detectors
  - $s_1(t) = h_1(t) + n_1(t) = \int d^2\kappa \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{D}_1^{ab}(\omega, \kappa)\, \tilde{h}_{ab}(\omega, \kappa)\, e^{i(-t + \kappa \cdot \vec{x}_1)} + n_1(t)$
  - $s_2(t) = h_2(t) + n_2(t) = \int d^2\kappa \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{D}_2^{ab}(\omega, \kappa)\, \tilde{h}_{ab}(\omega, \kappa)\, e^{i(-t + \kappa \cdot \vec{x}_2)} + n_2(t)$



Signal to Noise Ratio

- Correlation Measure
  - $Y = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \, s_1(t) \, s_2(t') \, Q(t - t')$
  - $Q(t)$: real filter function
- Maximal SNR
  - $\frac{S}{N} = \sqrt{T} \left[\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left| \tilde{\Gamma}(\omega) \right|^2 \frac{S_h^2(\omega)}{S_{n,1}(\omega) \, S_{n,2}(\omega)} \right]^{1/2}$
  - T: observation time
  - $S_{n,i}(\omega)$: noise spectral density for i-th detector



Amplitude of forced oscillation

- $\tilde{\Gamma}(\omega) = \frac{\omega^4}{\left(\omega_{010}^2 - \omega^2\right)^2 + \left(\frac{\omega_{010}}{Q_{010}}\omega\right)^2} \frac{32\pi^3 j_{0,1}^2}{J_1^2\left(j_{0,1}\right)} \int_0^{\pi} d\alpha \, \sin^5\alpha \cos\left(\omega d \cos\alpha\right) \left[\frac{\sin\left(\frac{1}{2}L\omega\cos\alpha\right)}{\frac{1}{2}L\omega\cos\alpha} \frac{J_0\left(R\omega\sin\alpha\right)}{j_{0,1}^2 - \left(R\omega\sin\alpha\right)^2} \right]^2$

Summary

- There are many candidate scenarios for existence of UHF-GW.
- EM cavity can be utilize for detection of UHF-GW.
- By two-detector correlation method, we can extract stochastic gravitational wave background. And we have almost finished theoretical expectation.
- Stay tuned!
- Thank you for listening 😄