

New physics searches at future muon colliders



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Based on work in collaboration with S. Nasri, T. Ahmed and S. Saad

arXiv: 2301.12524 and 2306.01255

A. Introduction

Introduction

- Lepton colliders are suitable for precise measurements due to their clean environment.
- Hadron colliders on the other hand can reach higher energies and are the best places to make discoveries.

Can we have a collider which has the best of these two?

In principle a future muon collider can both run at a very high energy and possess a clean environment.

There is only one challenge: a muon is unstable and decays weakly into an electron and neutrinos.

=> Can be solved by either the Muon Accelerator Program (R. B. Palmer; 2014) or LEMMA (M. Antonelli et al., 2015).



Muons vs Protons

What is the center-of-mass energy that is necessary to achieve the same beam-level cross section for protons?

Example: $2 \rightarrow 1$ annihilation

$$\sigma_p(2 \rightarrow 1) = \sum_{i,j} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}_{ij}}{d\tau} [\hat{\sigma}_{ij}]_p \delta\left(\tau - \frac{M^2}{s_p}\right)$$

$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{dx}{x} [f_{i/p}(x, \mu_F) f_{j/p}(x, \mu_F) + (i \leftrightarrow j)]$$

Let us have the following assumptions:

$$\mu_F = \sqrt{\hat{s}}/2; s_{\mu} = \hat{s} = M^2 \quad \text{and} \quad \sigma_{\mu} = [\hat{\sigma}]_{\mu}$$

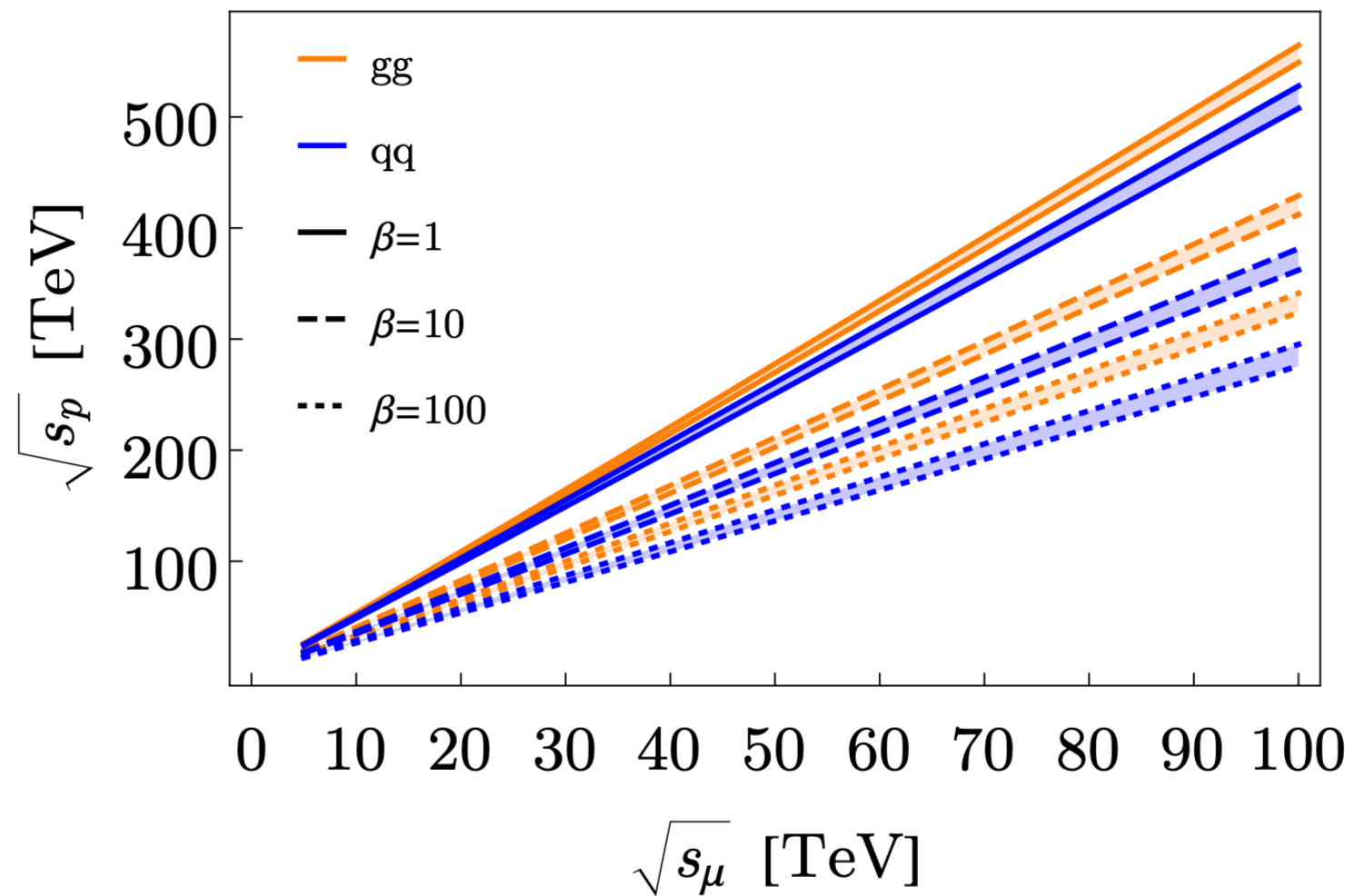
Therefore

$$\sigma_p = \sigma_{\mu} \implies \frac{[\hat{\sigma}]_p}{[\hat{\sigma}]_{\mu}} \sigma_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \left(\frac{s_{\mu}}{s_p}, \frac{\sqrt{s_{\mu}}}{2} \right) = 1$$

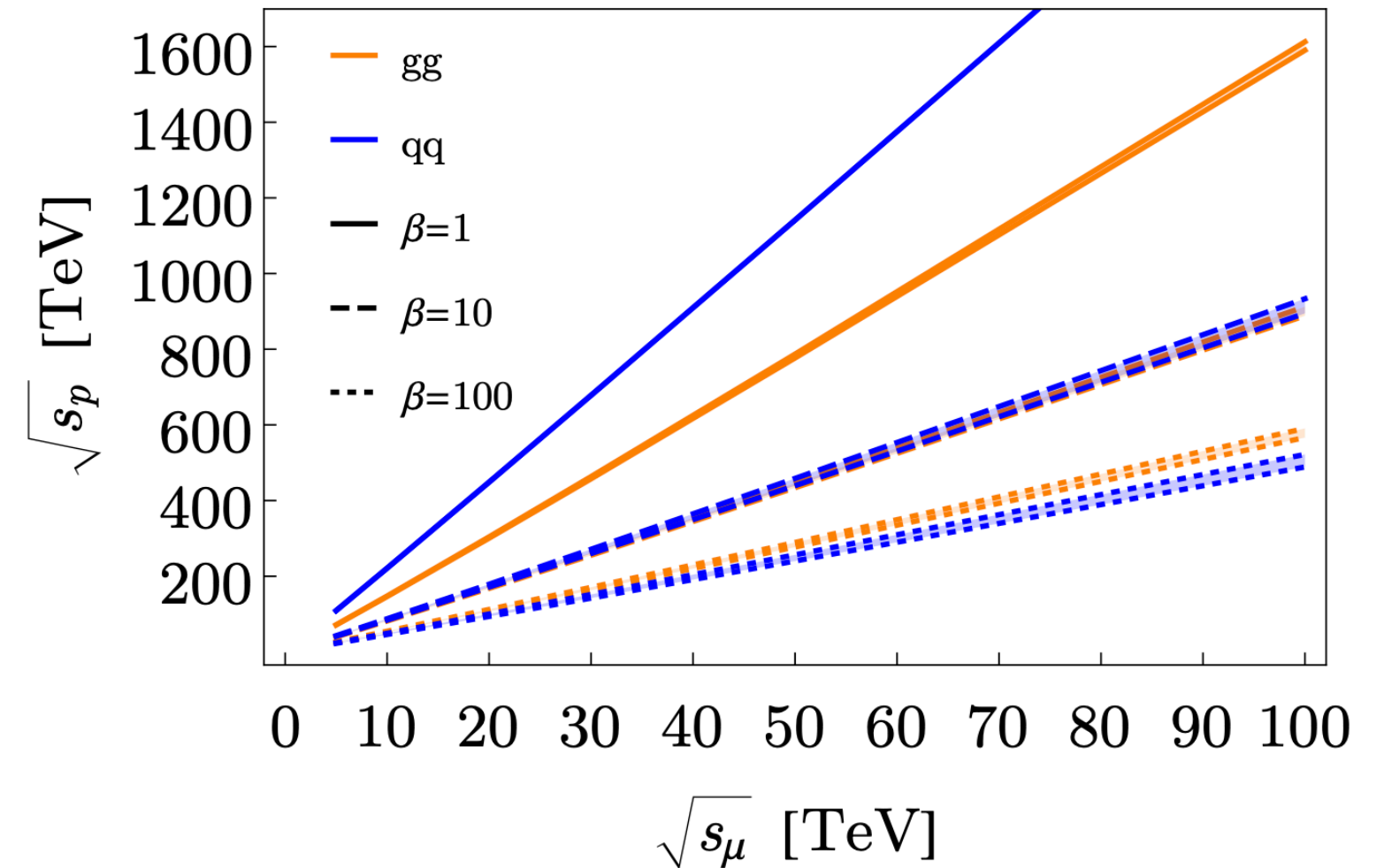
We can solve this numerically for different values of the ratio $\beta \equiv [\hat{\sigma}]_p / [\hat{\sigma}]_{\mu}$

Muons vs Protons

$2 \rightarrow 1$



$2 \rightarrow 2$

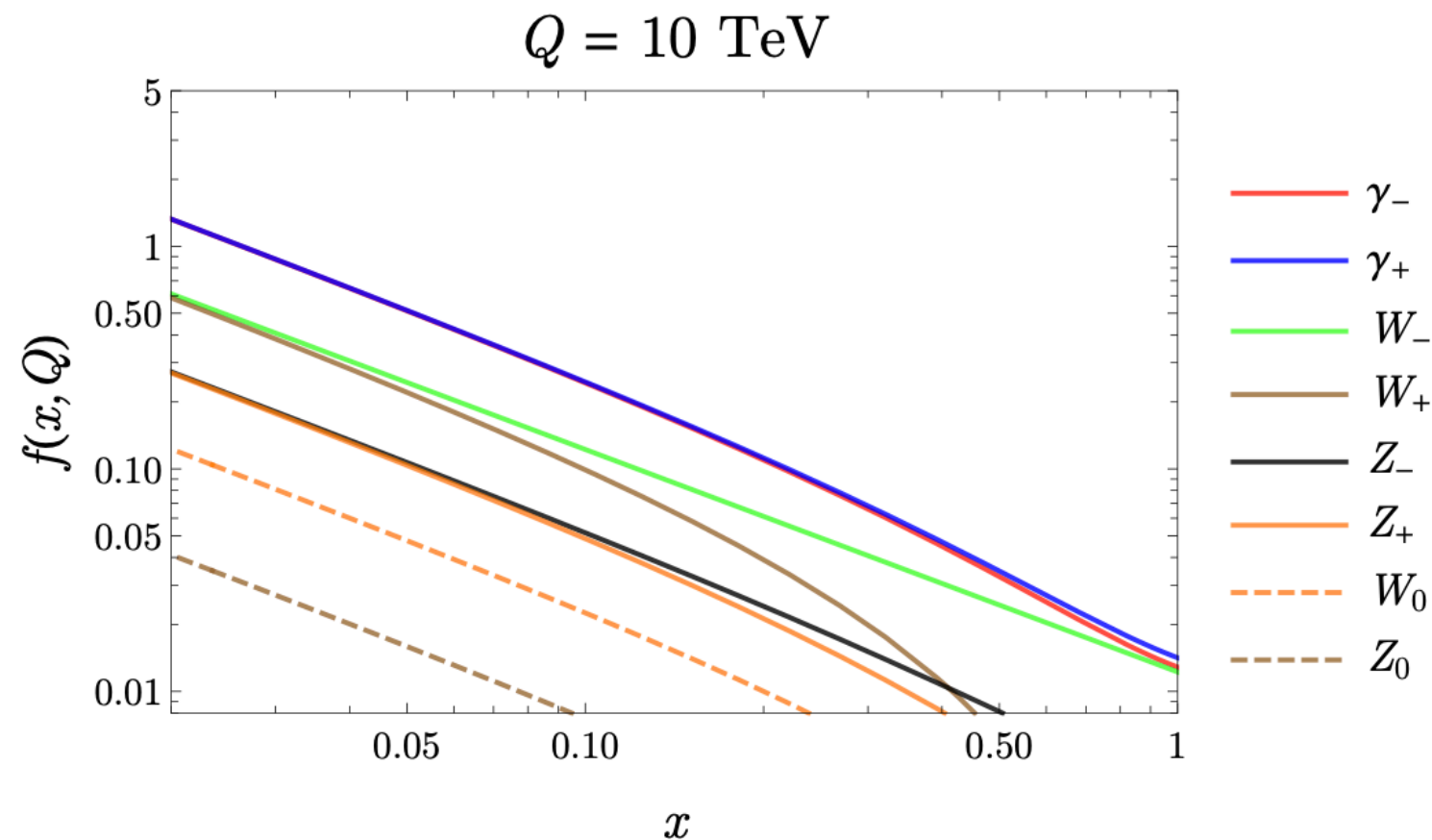


Taken from the Muon's Smasher Guide; 2103.14043

Vector-boson fusion

Well above the production threshold of a state X , the production cross section gets important contribution from vector-boson fusion (VBF) channels or equivalently the virtual gauge boson contents of a muon become very relevant

$$\sigma(\mu\mu \rightarrow F) = \sum_{i,j} \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 f_{i/\mu^+}(x_1, \mu_F) f_{j/\mu^-}(x_2, \mu_F) \hat{\sigma}(ij \rightarrow F)$$



$Q \equiv \mu_F$ is factorisation scale

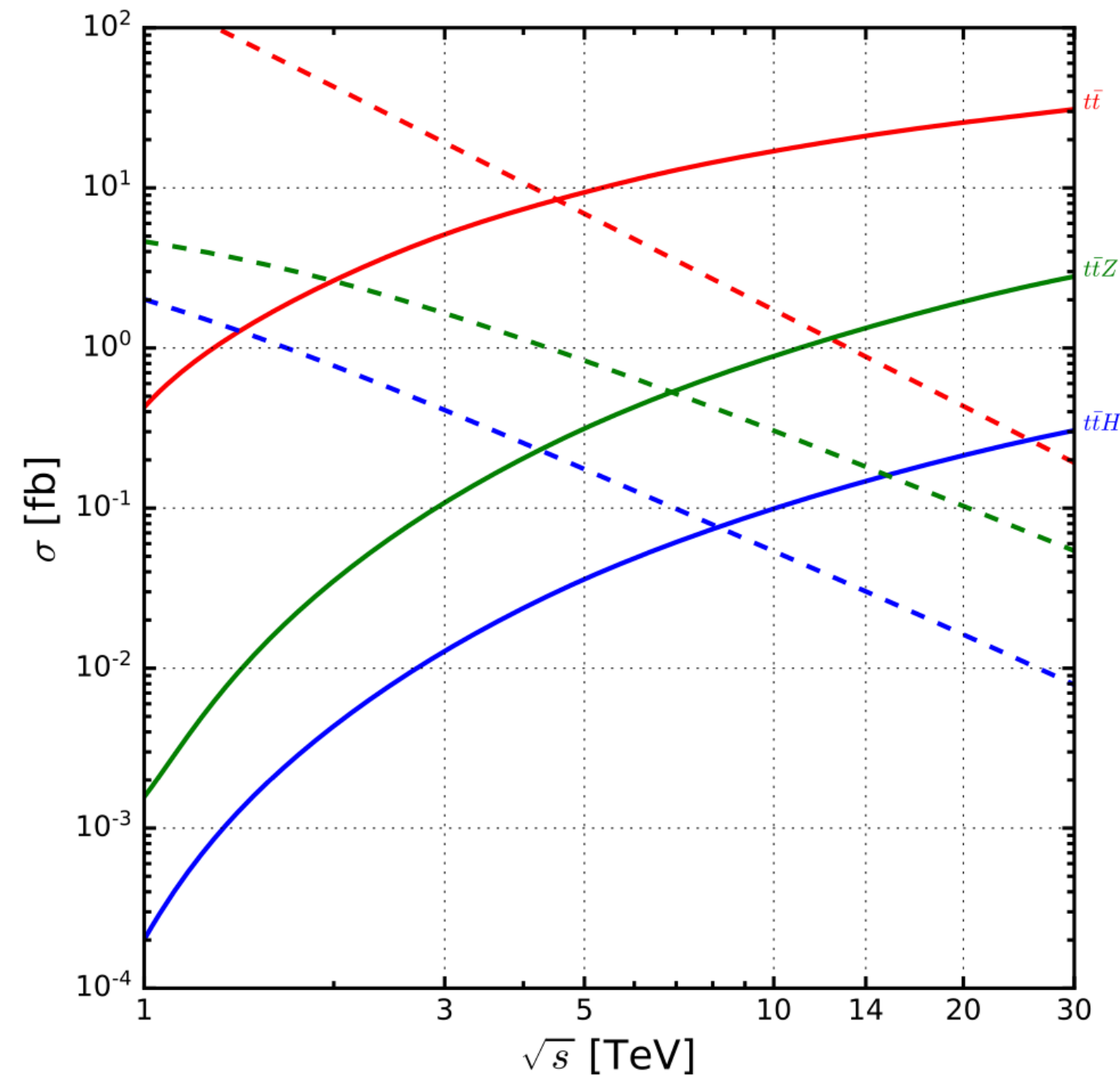
Vector-boson fusion

There is a crossover between VBF contribution and annihilation channel at around a few TeVs
(Constantini et al., 2005.10289)

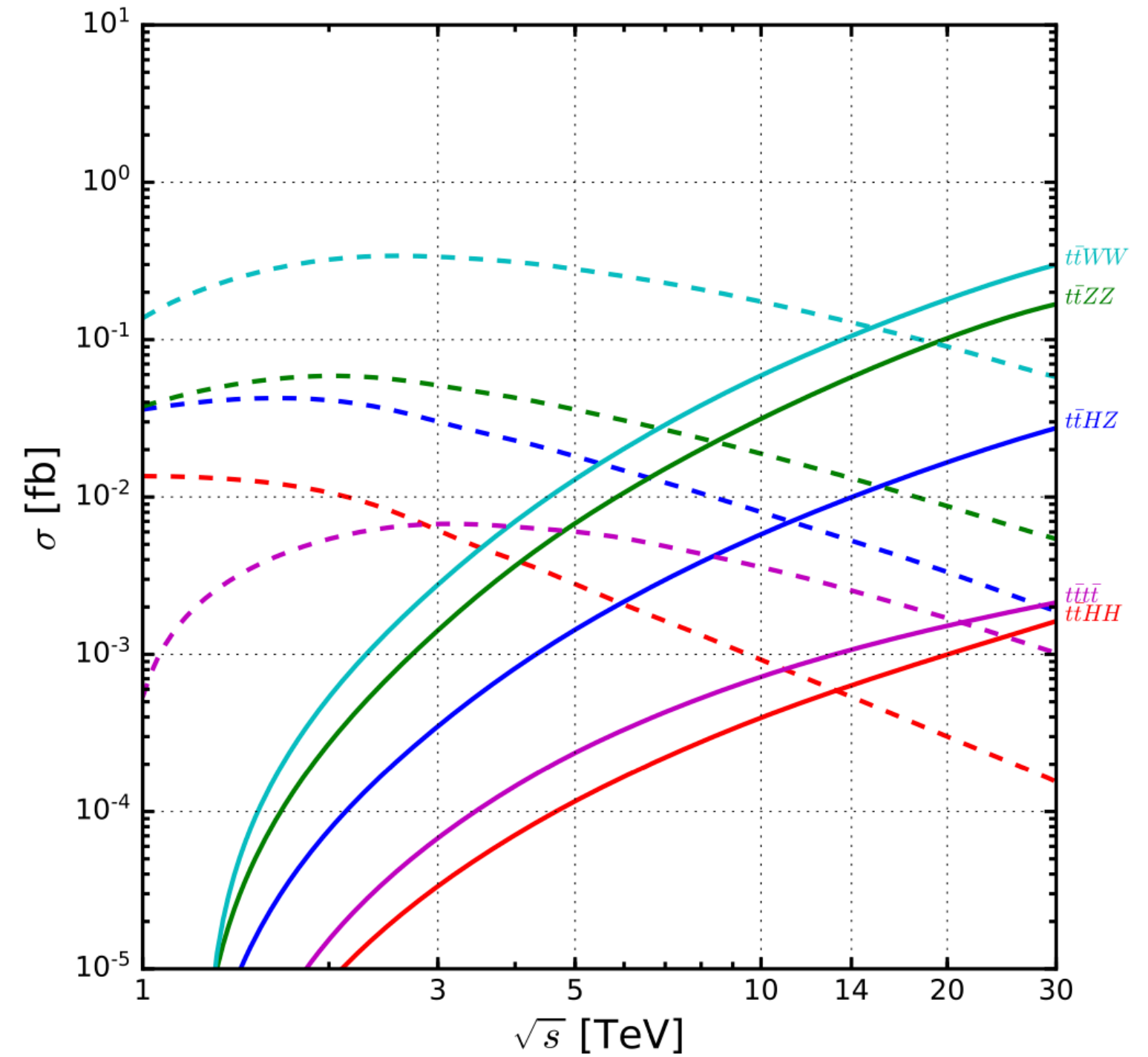
$$\frac{\sigma_{\text{VBF}}}{\sigma_{\text{ann}}} \propto \begin{cases} \alpha_W^2 \frac{s}{M_V^2} \log^3 \frac{s}{M_V^2} & \text{for SM} \\ \alpha_W^2 \frac{s}{M_X^2} \log^2 \frac{s}{M_V^2} \log \frac{s}{M_X^2} & \text{for BSM} \end{cases} \quad \text{where we assume } M_X^2 \ll s$$

The position of the crossover depends on the number of particles in the final state and their masses.

Vector-boson fusion (SM)



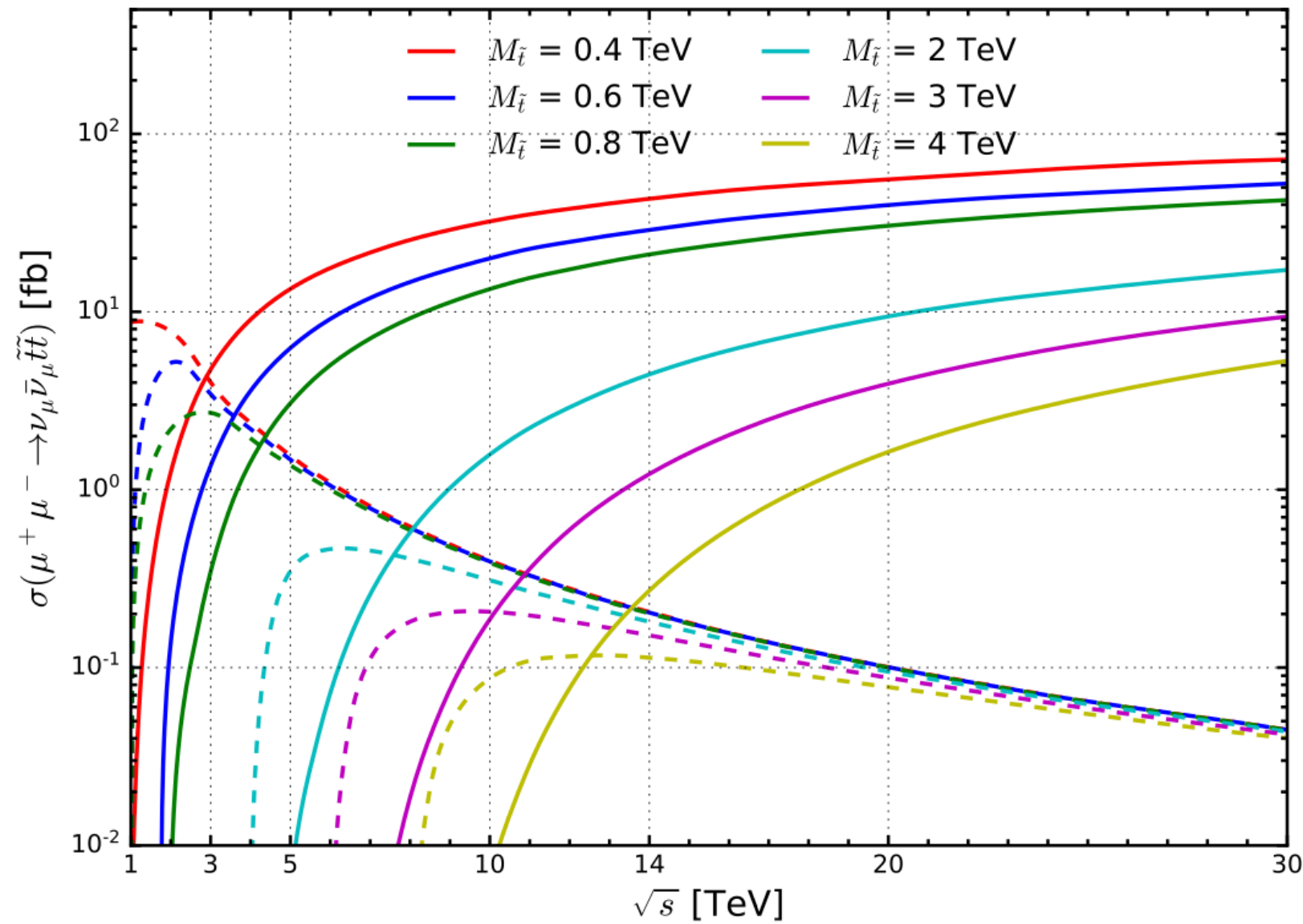
(a)



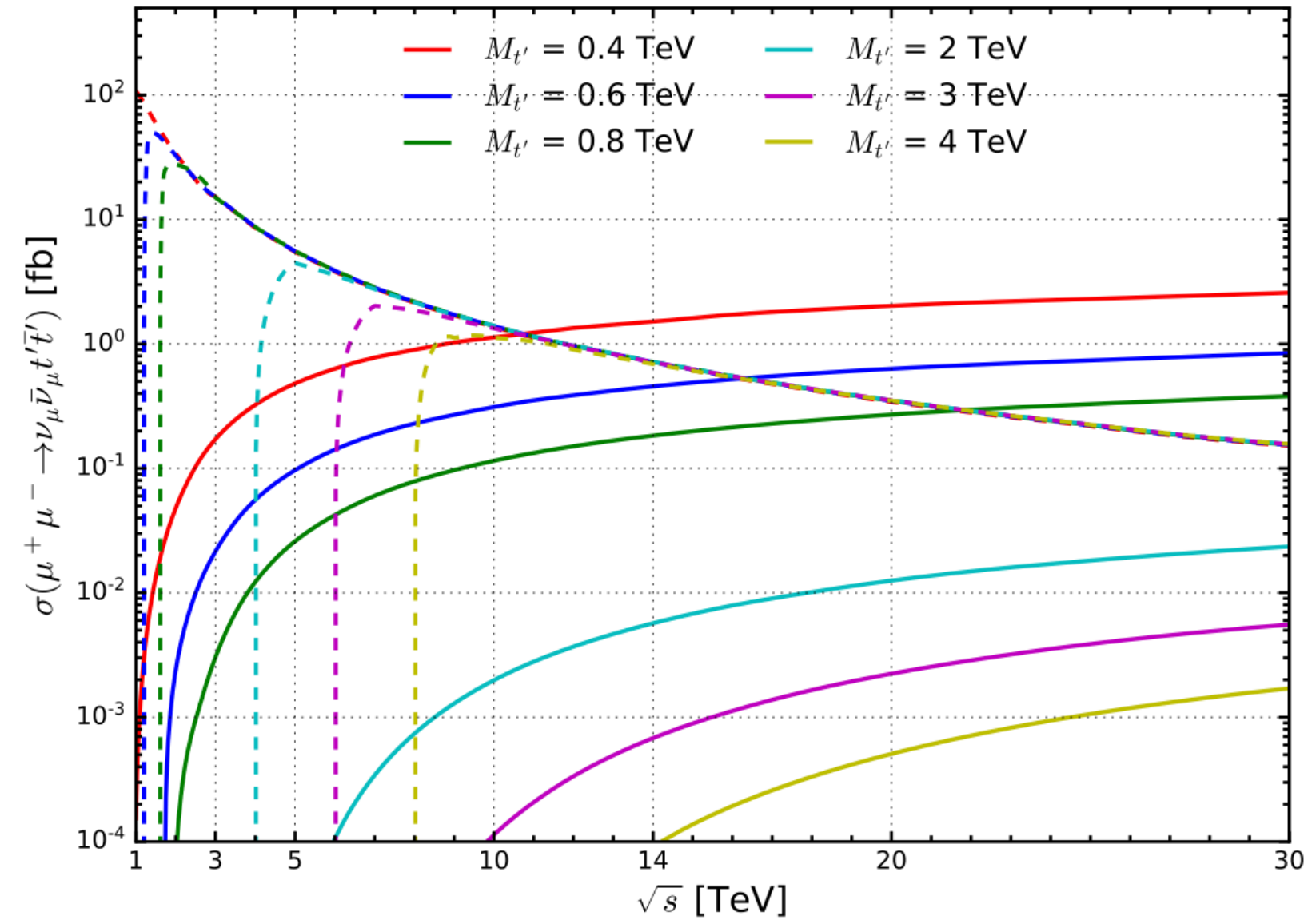
(b)

Vector-boson fusion (BSM)

$$\mu^+ \mu^- \rightarrow \tilde{t} \tilde{t}$$

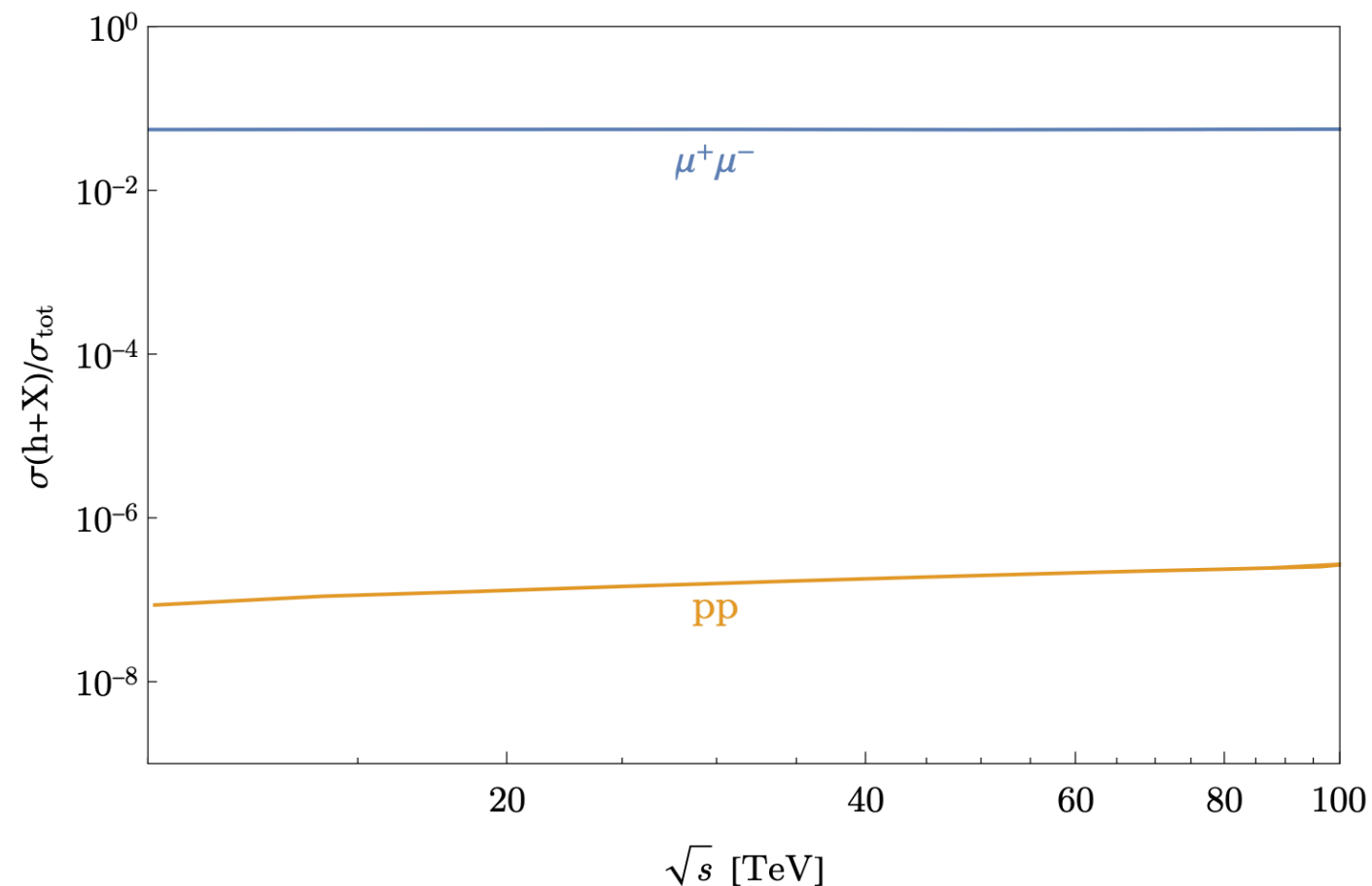


$$\mu^+ \mu^- \rightarrow T \bar{T}$$



Signal vs backgrounds

One of the advantages of the muon colliders is that it has larger signal-to-background ratios than in proton colliders.



$$\sigma(\mu\mu \rightarrow h + X) = \sigma(\mu\mu \rightarrow h\nu\nu)$$

$$\sigma(\mu\mu)_{\text{total}} = \sum_{i=h,\gamma,Z,W} \sigma(\mu\mu \rightarrow i + X)^{\text{VBF}}$$

$$\sigma(pp \rightarrow h + X) = \sigma(gg \rightarrow h)^{\text{N3LO}}$$

$$\sigma(pp)_{\text{total}} = \sigma(pp \rightarrow b\bar{b})^{\text{NNLO}}$$

Even the cross section for the SM Higgs Boson at the LHC is about 50 times larger than at muon colliders!!! (at 14 TeV).

B. Minimal Lepton Portal DM

AJ, S. Nasri: 2301.12524

A Minimal dark-matter model

We suggest a new minimal model where extend the Standard Model with two gauge-singlets; a charged scalar S and a right-handed singlet Majorana fermion N_R .

They transform under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as

$$S : (\mathbf{1}, \mathbf{1})_{+2} \text{ and } N_R : (\mathbf{1}, \mathbf{1})_0$$

These extra states are odd under an extra Z_2 symmetry (called matter parity) while all the SM particles are even, i.e. $\{S, N_R\} \rightarrow \{-S, -N_R\}$ and $\{V^\mu, f, \Phi\} \rightarrow \{V^\mu, f, \Phi\}$

The most general interaction Lagrangian can be written as

$$\mathcal{L}_{\text{int}} \supset \sum_{\ell=e,\mu,\tau} y_\ell \bar{\ell}_R^c S N_R + \lambda_2 |S^\dagger S|^2 + \lambda_3 |\Phi^\dagger \Phi| |S^\dagger S|$$

The scalar singlet (S) is electrically-charged and plays the role of a mediator between dark matter and the SM sectors:

$$\mathcal{L}_{\text{gauge}} \supset -i \left(e A^\mu - e \tan \theta_W Z^\mu \right) S^\dagger \partial_\mu S$$



UV realizations

- If you would like neutrinos to be massive, add two extra right handed neutrinos (N_2, N_3) and an inert scalar isodoublet Φ_2 .
 $\iff$ decouple the two other right-neutrinos and make the other couplings small and you will get Leptogenesis as a bonus.
- We can embed this into e.g. a $SU(5)$ theory: the matter fields in the $\mathbf{10}_F$ and $\bar{\mathbf{5}}_F$ representations and the charged singlet belongs to the $\mathbf{10}_H$ representation, while N_R belongs to the singlet representation $\mathbf{1}_N$

$$\mathcal{L} = g_{\alpha\beta} \bar{\mathbf{10}}_{F_\alpha} \otimes \mathbf{10}_H \otimes \mathbf{1}_{N_\beta} \supset g_{\alpha\beta} \ell_{R\alpha}^T C N_\beta S^+$$

- You can also have it in a flipped- $SU(5) \otimes U(1)_X$ grand-unified theory: The lepton field is a singlet of $SU(5)$, and N_R is a member of the $\mathbf{10}_F$ representation

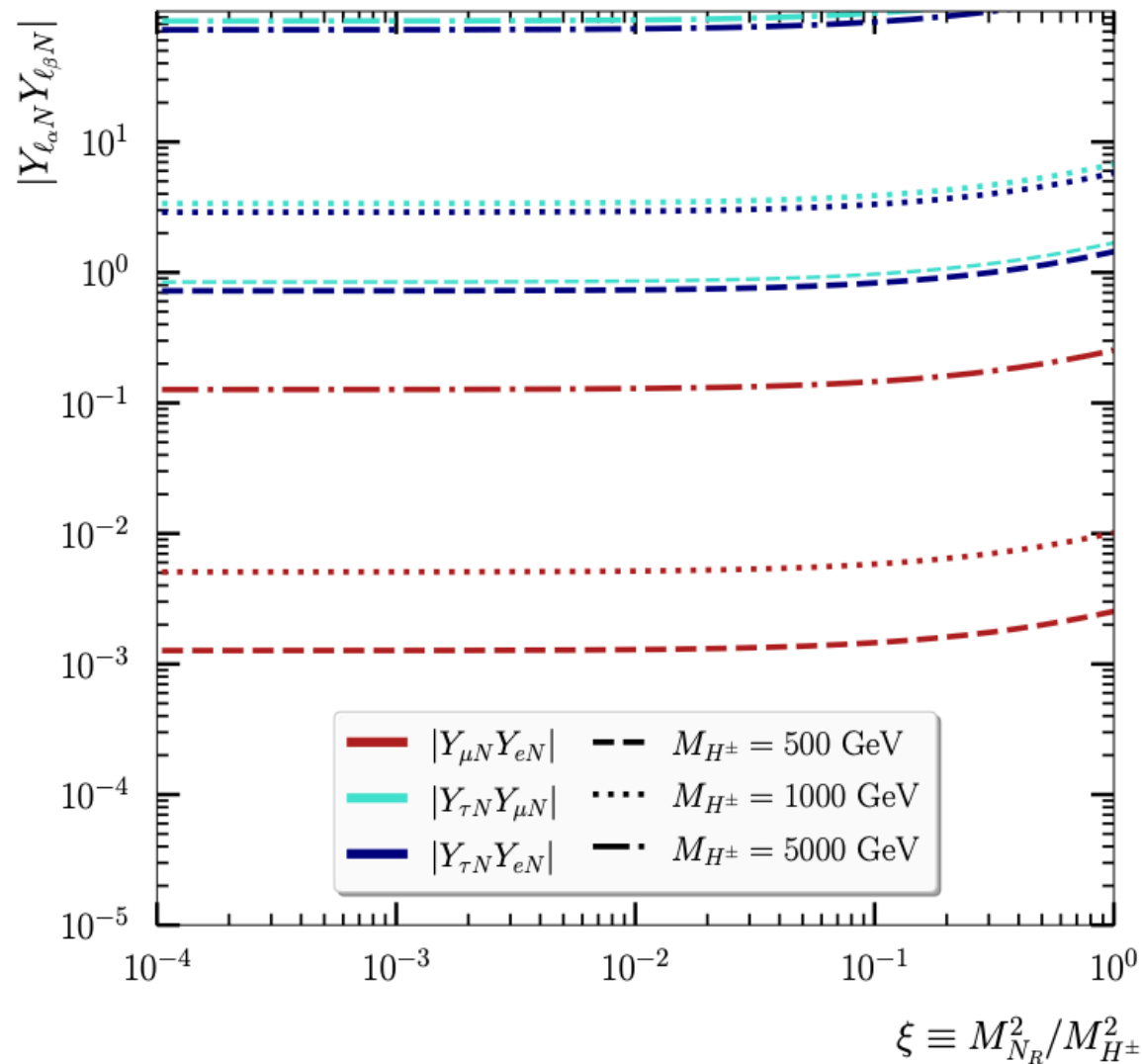
$$\mathcal{L} = \frac{h_{\alpha\beta}}{\Lambda} \bar{\mathbf{10}}_{F_\alpha} \otimes \bar{\mathbf{1}}_{F_\beta} \otimes \mathbf{10}_H \otimes \mathbf{1}_S \supset \frac{h_{\alpha\beta} \langle \mathbf{10}_H \rangle}{\Lambda} N^T C \ell_R S^-$$



Charged lepton flavor violation

Bounds on $\text{BR}(\ell_\alpha \rightarrow \ell_\beta \gamma)$ from MEG and BABAR

$$\left\{ |Y_{eN}Y_{\mu N}|, |Y_{\mu N}Y_{\tau N}|, |Y_{\tau N}Y_{eN}| \right\} < \left\{ \left(\frac{2.85 \times 10^{-5}}{\text{GeV}} \right)^2, \left(\frac{3.07 \times 10^{-4}}{\text{GeV}} \right)^2, \left(\frac{2.87 \times 10^{-4}}{\text{GeV}} \right)^2 \right\} \times \frac{M_{H^\pm}^2}{|F(\xi)|}$$



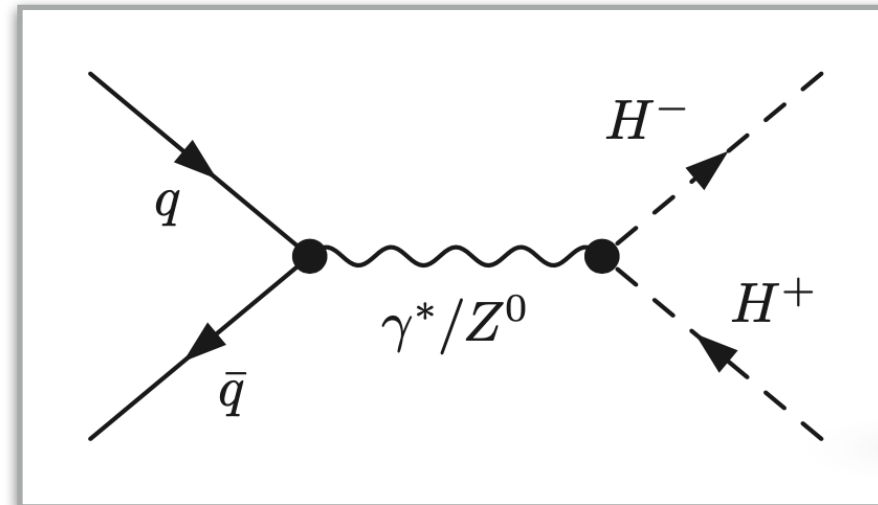
The constraints on $|Y_{iN}Y_{jN}|$ are very severe if $H^\pm$ is light and become weak for $m_{H^\pm} \simeq \mathcal{O}(10^3)$ GeV.



- $Y_{eN} \simeq \mathcal{O}(1) \gg Y_{\tau N} \geq Y_{\mu N}$: Interesting for e^+e^- colliders.
- $Y_{eN} \simeq Y_{\mu N} \simeq Y_{\tau N} \simeq \mathcal{O}(10^{-2})$: can be tested in hadronic collisions (very hard to achieve the correct relic density).
- $Y_{\mu N} \simeq \mathcal{O}(1) \gg (\geq) Y_{\tau N} \gg Y_{eN}$: Interesting for $\mu^+\mu^-$ colliders.

Status at the Large Hadron Collider

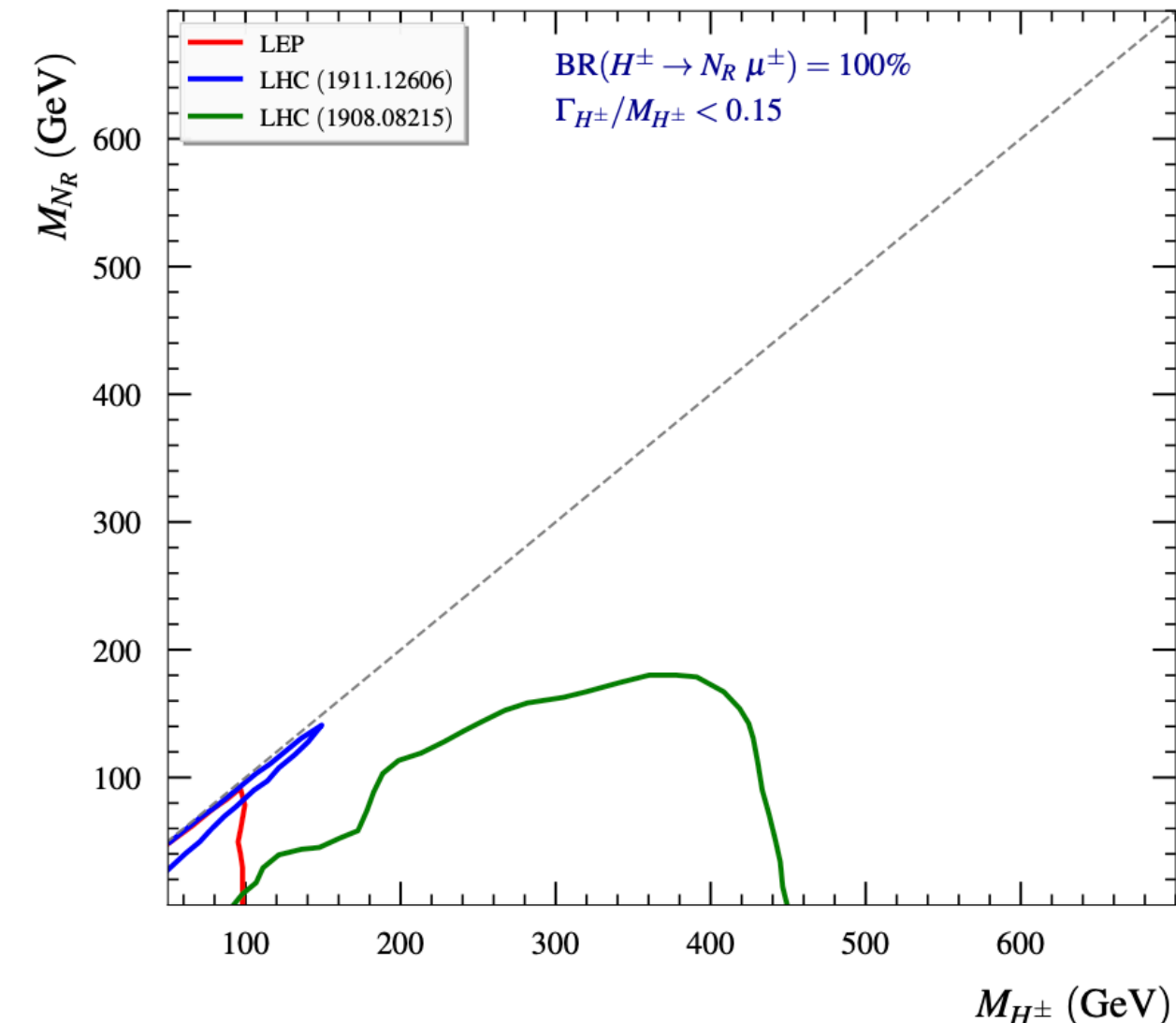
- The model can be constrained from re-interpretation of the results of sleptons/charginos (using MadAnalysis 5).
- In our model, we can pair produce the charged Higgs boson through $q\bar{q}$ annihilation and then decay them to charged leptons plus large MET.
- ATLAS has searched for sleptons/charginos defining eight signal regions – depend on the jet multiplicity $n_{\text{jet}} = 0, 1$ and the bins for the transverse mass M_{T2} –.
- Masses of the charged Higgs boson up to 400 GeV can be excluded.
- No sensitivity at all for small mass splitting ($m_{H^\pm} - m_N$).



+ ≤ 2 partons

+ Merging

Destructive interference
between γ^* and Z^0



Phenomenology at muon colliders: BPs

For the case of muon colliders, we need to choose the following scenario

$$Y_{\mu N} \geq (\approx) Y_{\tau N} \gg Y_{eN}$$

Benchmark scenario	BP1	BP2	BP3	BP4
<i>Parameters</i>				
M_{N_R} (GeV)	50	200	598	1000
$M_{H^\pm}$ (GeV)	500	500	600	1500
Y_{Ne}	10^{-4}	5×10^{-4}	10^{-3}	5×10^{-3}
$Y_{N\mu}$	2.8	1.6	1	2
$Y_{N\tau}$	5×10^{-2}	5×10^{-1}	5×10^{-1}	2
λ_3	4	5	5	6

DM production at muon colliders

Work is ongoing for $N_R N_R + Z(\rightarrow \ell \ell)/H(\rightarrow b \bar{b})$ for the following center-of-mass energies

$$\sqrt{s_{\mu\mu}} = 3, 10, \text{ and } 30 \text{ TeV}$$

$$\int \mathcal{L} = 1, 10, \text{ and } 90 \text{ ab}^{-1}$$



Decent statistics for signal events!

DM production at muon colliders

(i) DM production plus X ($N_R N_R + X$)

- $N_R N_R + \gamma \implies$ High-energetic photon plus MET.
- $N_R N_R + Z \implies$ 2 leptons or two jets plus MET.
- $N_R N_R + H_{\text{SM}} \implies b\bar{b} + \text{MET}; gg + \text{MET}; \dots$

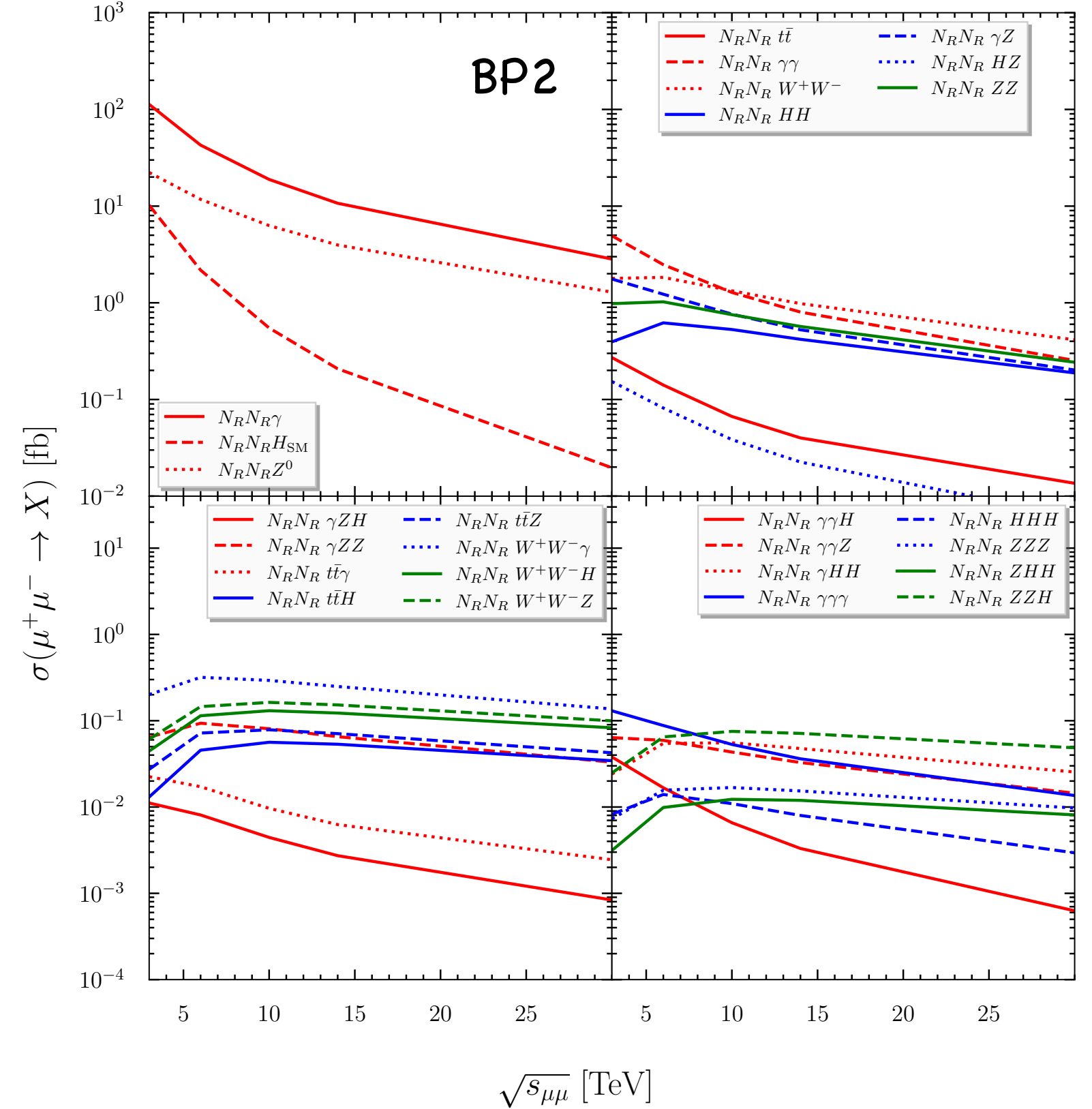
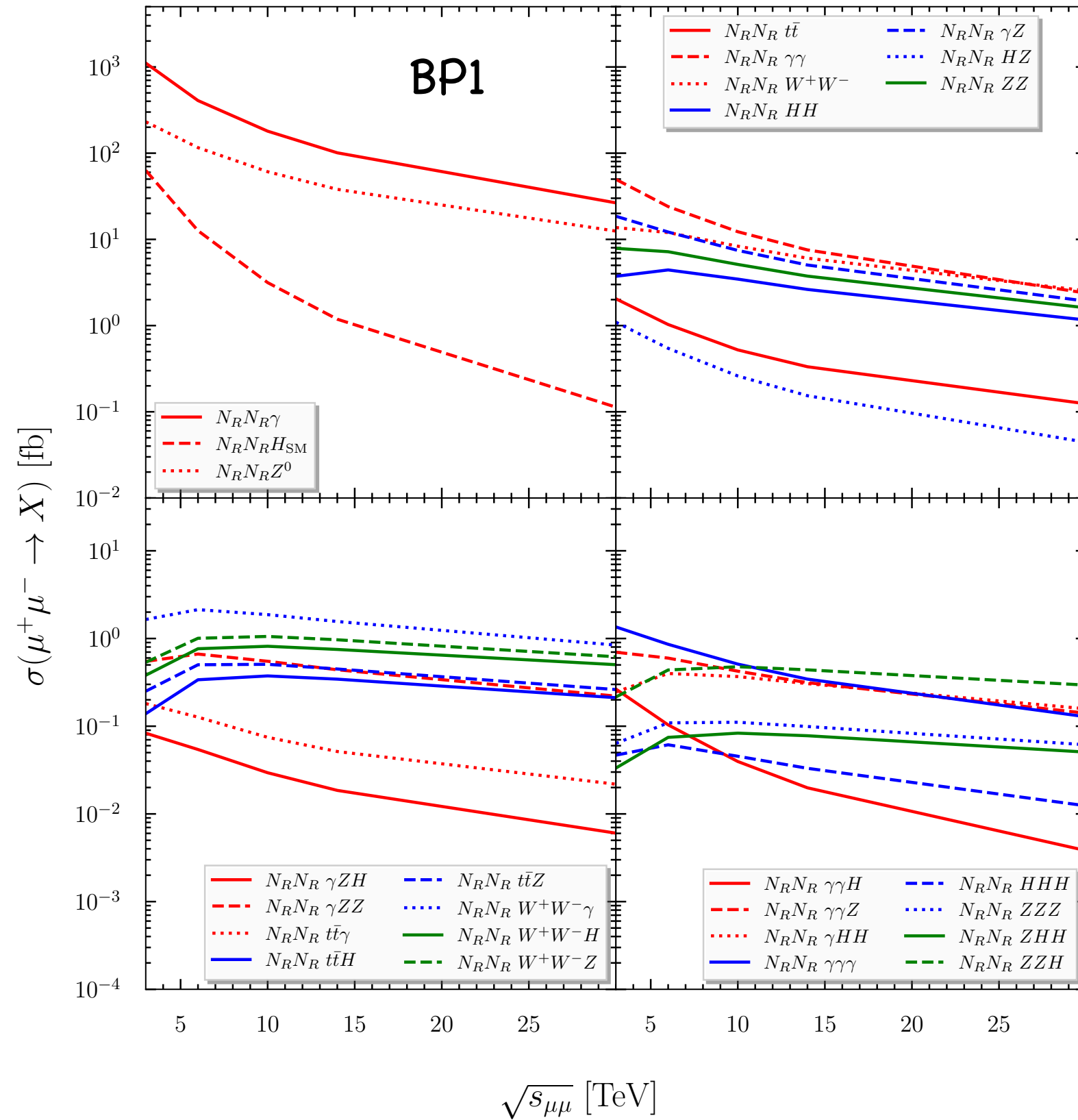
(ii) DM production plus XY ($N_R N_R + XY$)

- $N_R N_R + \gamma\gamma \implies$ 2 photons plus MET.
- $N_R N_R + \gamma Z \implies$ one photon + 2 leptons or two jets plus MET.
- $N_R N_R + ZZ/HZ/W^+W^-/HH/t\bar{t} \implies$ variety of final-state particles depending on the decay products of the heavy resonances.

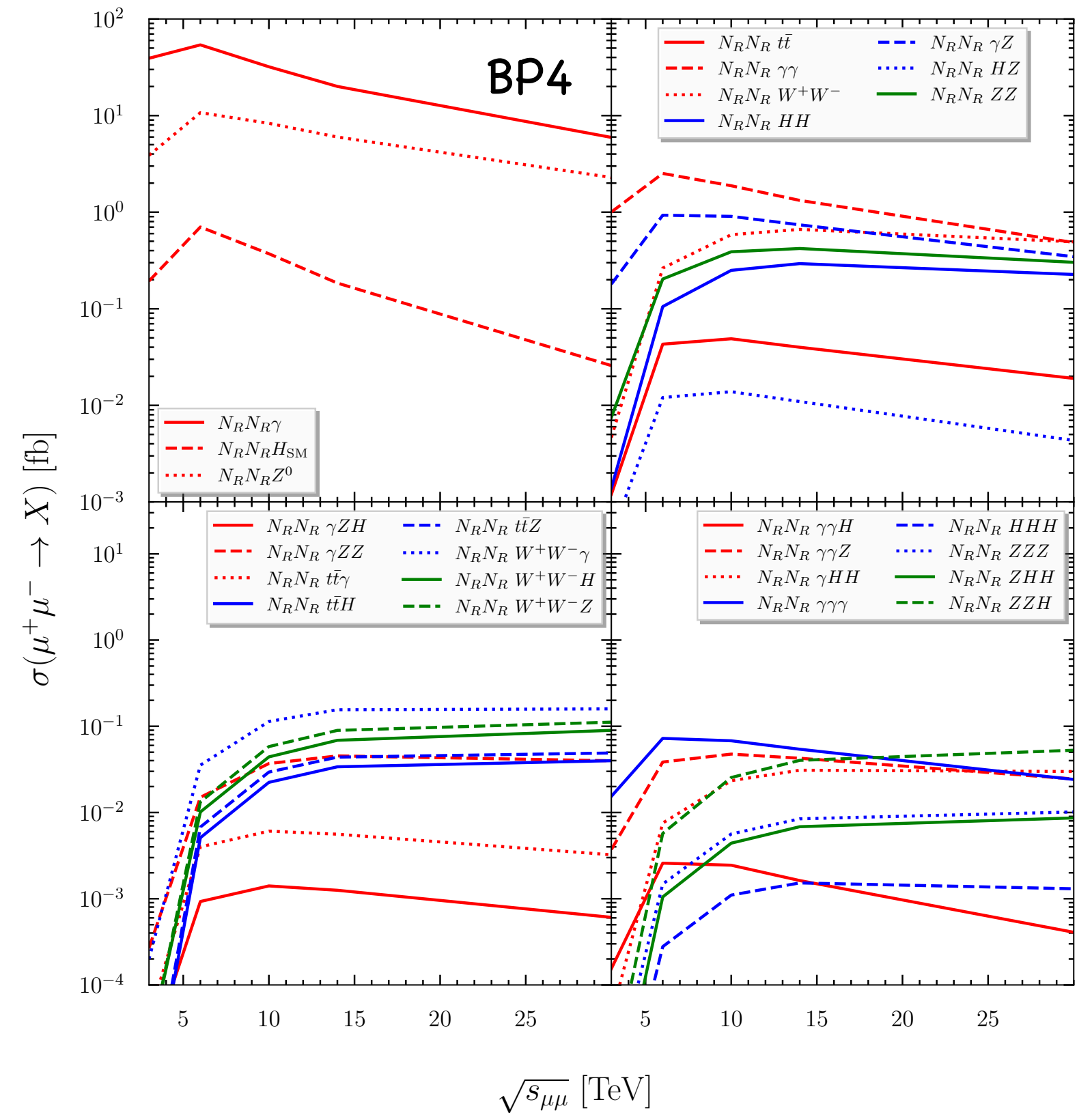
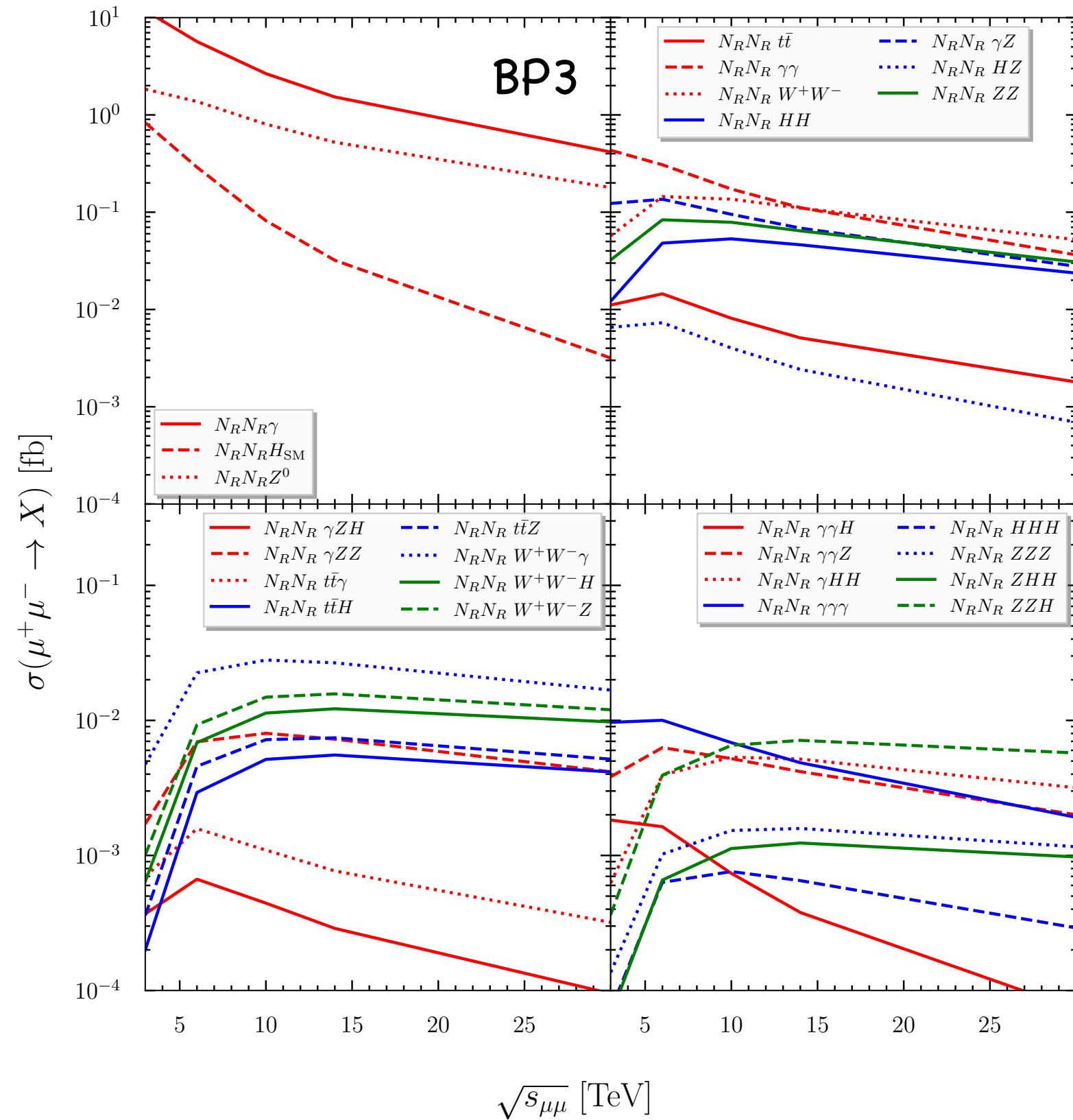
(iii) DM production plus XYZ ($N_R N_R + XYZ$)

- About 16 different production channels

DM production at muon colliders: results



DM production at muon colliders: results



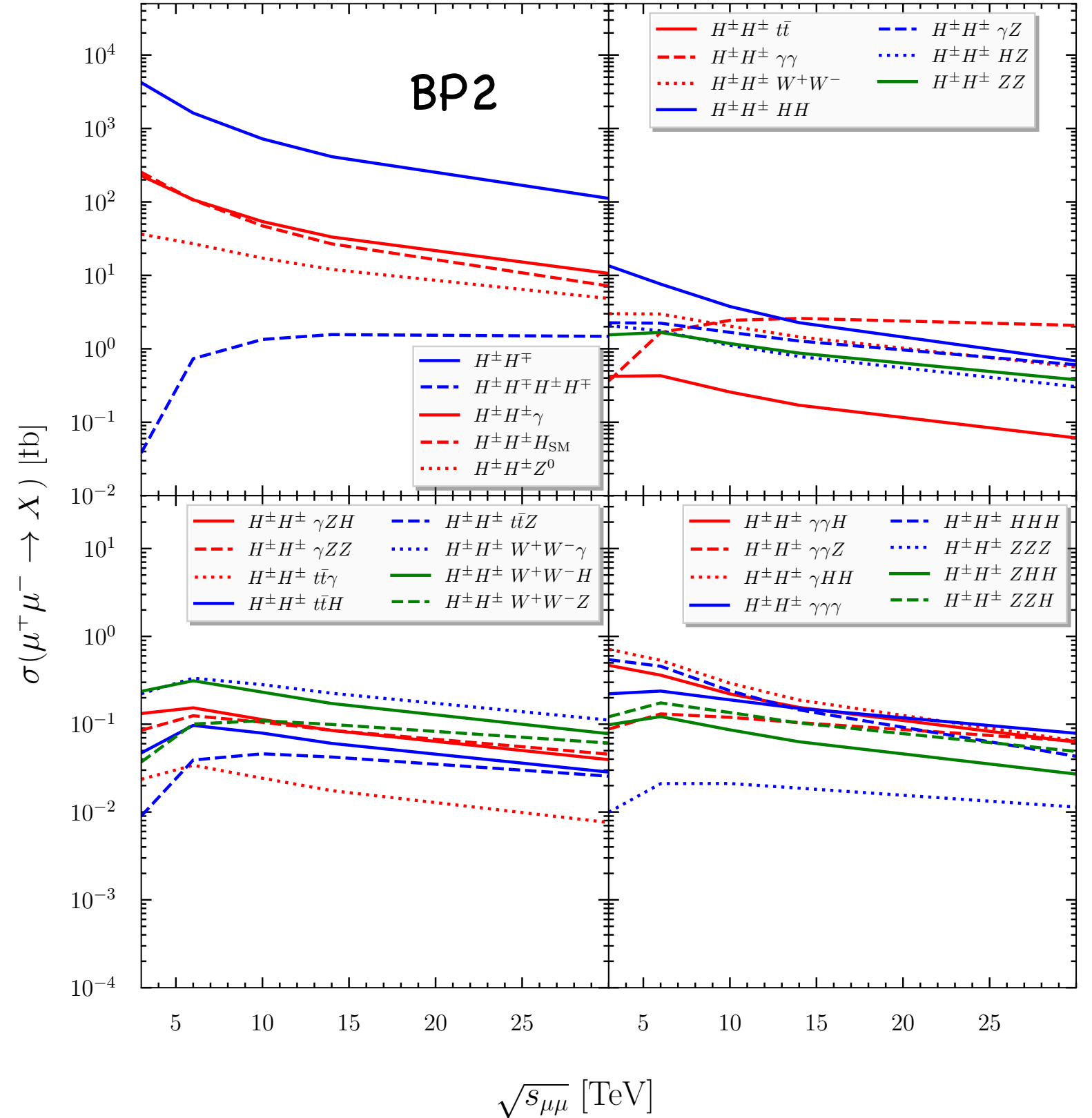
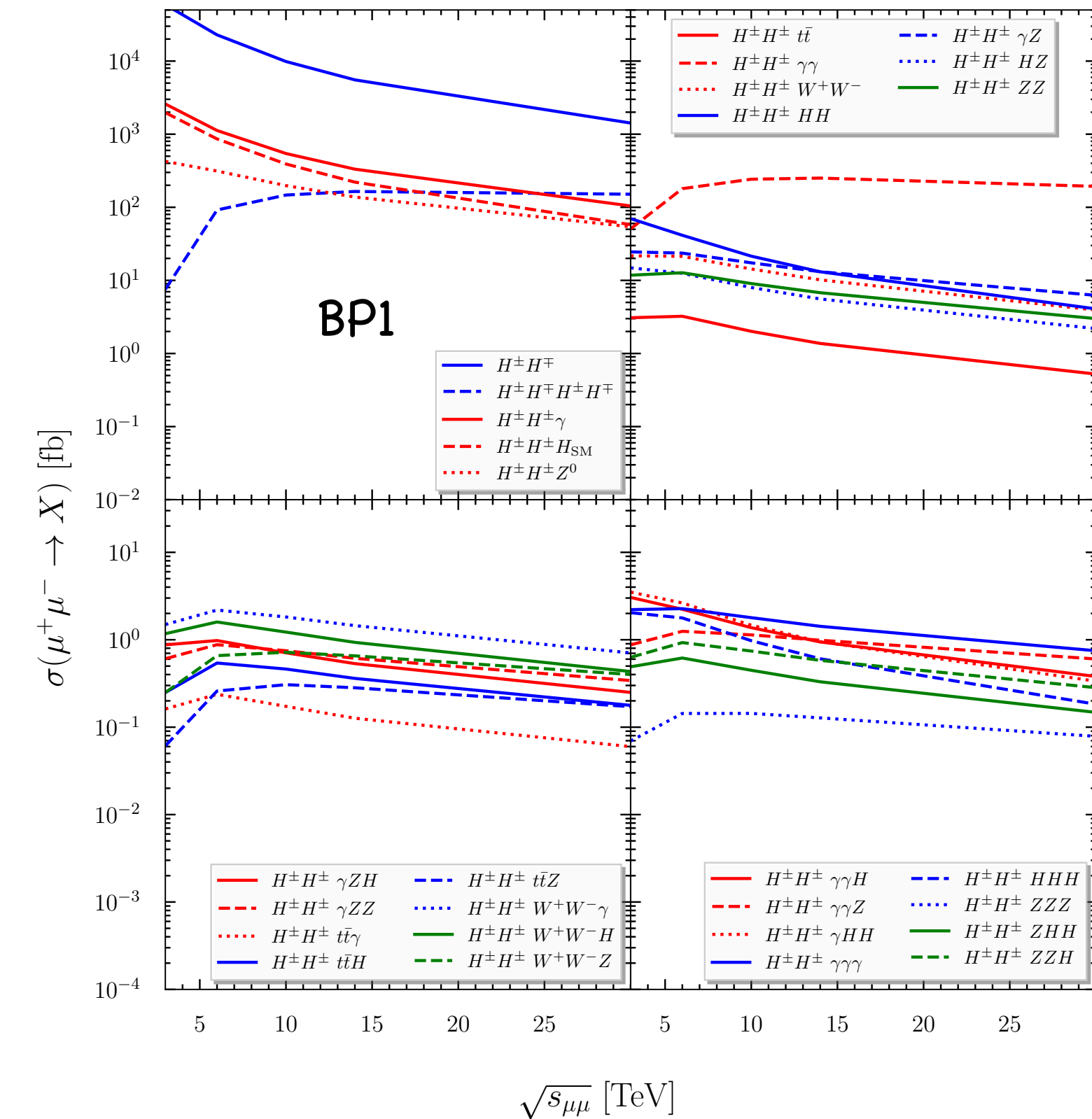
DM production at muon colliders: signal vs backgrounds

		$\sigma \times \text{BR} [\text{fb}]$ (number of events)			
		3 TeV	10 TeV	30 TeV	
$N_R N_R + \gamma$	BP1	1.11×10^3 (1.11×10^6)	1.80×10^2 (1.80×10^6)	2.38×10^1 (2.65×10^6)	
	BP2	1.13×10^2 (1.13×10^5)	1.88×10^1 (1.88×10^5)	2.83×10^0 (2.55×10^5)	
	BP3	1.18×10^1 (1.18×10^3)	2.65×10^0 (2.65×10^4)	0.41×10^0 (3.69×10^4)	
	BP4	3.92×10^1 (3.95×10^4)	3.20×10^1 (3.20×10^5)	5.94×10^0 (5.35×10^5)	
	bkg	3.02×10^3 (3.02×10^6)	3.29×10^3 (3.29×10^7)	3.36×10^3 (3.02×10^8)	
$N_R N_R + Z(\rightarrow \ell\bar{\ell})$	BP1	1.68×10^1 (1.68×10^4)	4.44×10^0 (4.44×10^4)	0.91×10^0 (8.19×10^4)	
	BP2	1.62×10^0 (1.62×10^3)	0.46×10^0 (4.58×10^3)	9.39×10^{-2} (8.45×10^3)	
	BP3	0.13×10^0 (0.13×10^3)	0.58×10^{-1} (0.58×10^3)	1.30×10^{-2} (1.17×10^3)	
	BP4	0.28×10^0 (0.28×10^3)	0.61×10^0 (0.61×10^4)	0.17×10^0 (1.53×10^4)	
	bkg	2.75×10^1 (2.75×10^4)	2.57×10^1 (2.57×10^5)	4.69×10^1 (4.22×10^6)	
$N_R N_R + Z(\rightarrow q\bar{q})$	BP1	1.59×10^2 (1.59×10^5)	4.20×10^1 (4.20×10^5)	8.61×10^0 (7.75×10^5)	
	BP2	1.53×10^1 (1.53×10^4)	4.33×10^0 (4.33×10^4)	0.89×10^0 (8.00×10^4)	
	BP3	1.26×10^0 (1.26×10^3)	0.55×10^0 (5.54×10^3)	0.12×10^0 (1.11×10^4)	
	BP4	2.67×10^0 (2.67×10^3)	5.73×10^0 (5.73×10^4)	1.57×10^0 (1.41×10^5)	
	bkg	4.76×10^2 (4.76×10^5)	6.71×10^2 (6.71×10^6)	1.01×10^3 (0.91×10^8)	
$N_R N_R + H_{\text{SM}}(\rightarrow b\bar{b})$	BP1	2.05×10^1 (2.05×10^4)	1.02×10^0 (1.02×10^4)	3.67×10^{-2} (3.30×10^3)	
	BP2	5.83×10^0 (5.83×10^3)	0.31×10^0 (0.31×10^4)	1.12×10^{-2} (1.01×10^3)	
	BP3	0.47×10^0 (0.47×10^3)	0.47×10^{-1} (0.47×10^3)	1.81×10^{-3} (1.63×10^2)	
	BP4	0.11×10^0 (0.11×10^3)	0.21×10^0 (0.21×10^4)	1.47×10^{-2} (1.32×10^3)	
	bkg	4.76×10^2 (4.76×10^5)	6.71×10^2 (6.71×10^6)	1.01×10^3 (0.91×10^8)	

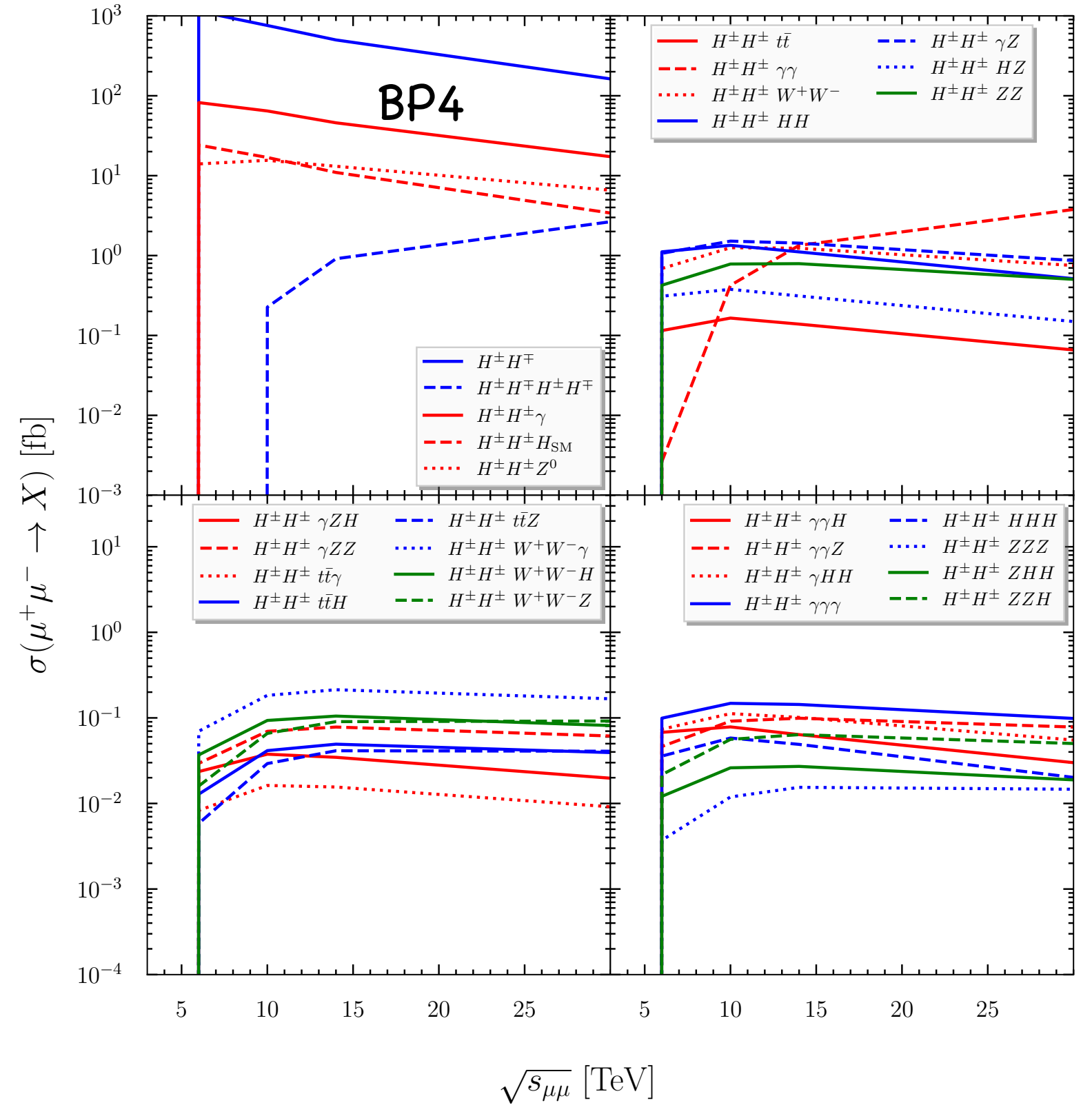
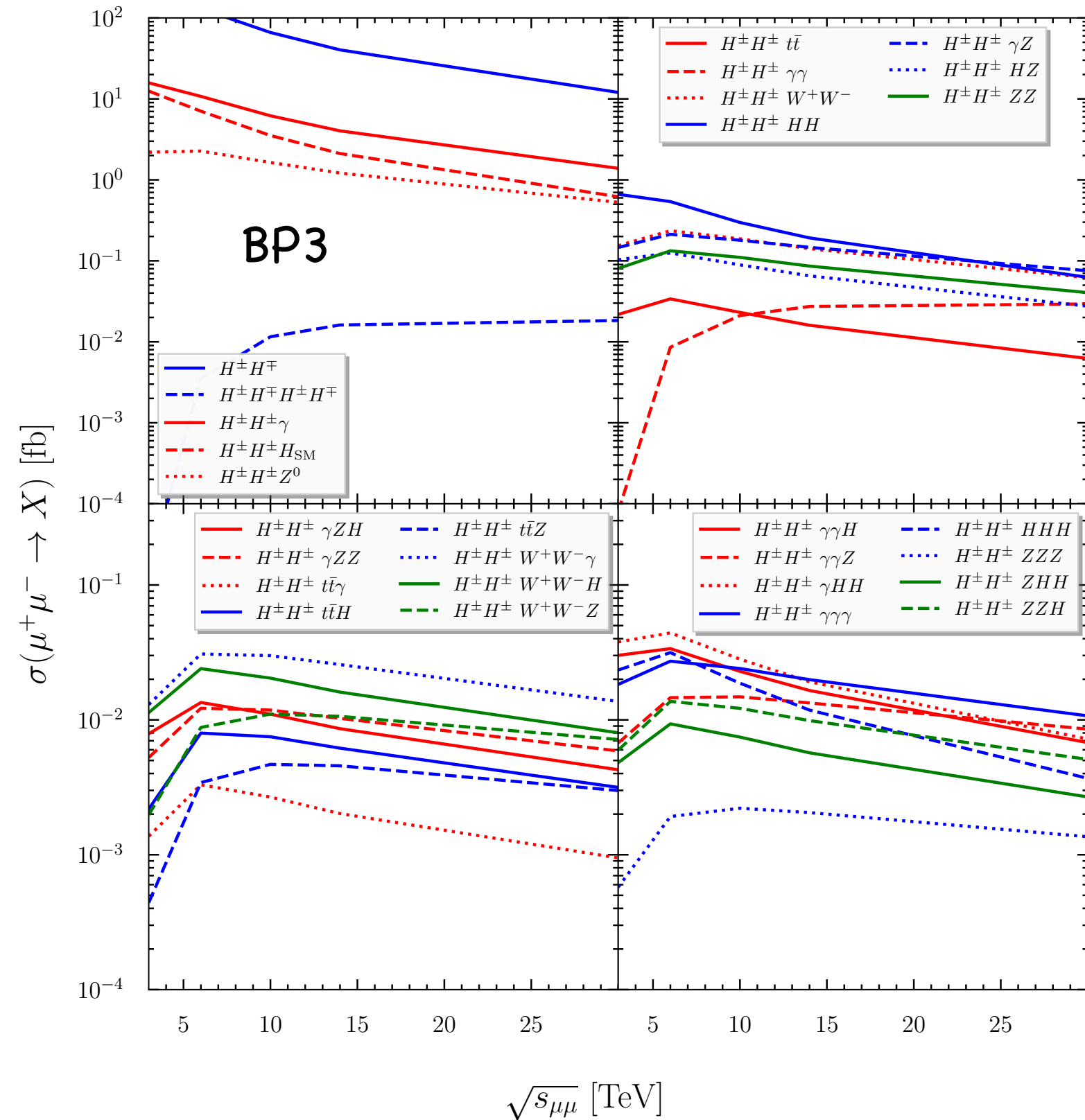
DM production at muon colliders: signal vs backgrounds

		$\sigma \times \text{BR} [\text{fb}]$ (number of events)			
		3 TeV	10 TeV	30 TeV	
$N_R N_R + \gamma\gamma$	BP1	4.97×10^1 (4.97×10^4)	1.23×10^1 (1.23×10^5)	2.38×10^0 (2.38×10^5)	
	BP2	4.93×10^1 (4.93×10^3)	1.28×10^0 (1.28×10^4)	0.25×10^0 (2.53×10^4)	
	BP3	0.43×10^0 (0.43×10^3)	0.17×10^0 (1.73×10^3)	0.36×10^{-1} (3.64×10^3)	
	BP4	1.00×10^0 (1.00×10^3)	1.87×10^0 (1.87×10^4)	0.48×10^0 (4.85×10^4)	
	bkg	8.73×10^1 (8.73×10^4)	1.04×10^2 (1.04×10^6)	1.12×10^2 (1.12×10^7)	
$N_R N_R + \gamma Z(\rightarrow \ell\ell)$	BP1	1.24×10^0 (1.24×10^3)	0.49×10^0 (4.98×10^3)	1.29×10^{-1} (1.29×10^4)	
	BP2	1.76×10^0 (1.76×10^3)	0.76×10^0 (7.64×10^3)	0.20×10^0 (2.02×10^4)	
	BP3	0.12×10^0 (1.23×10^2)	9.50×10^{-2} (9.50×10^2)	2.80×10^{-2} (2.80×10^3)	
	BP4	0.18×10^0 (1.79×10^2)	9.05×10^{-1} (9.05×10^3)	3.46×10^{-1} (3.46×10^4)	
	bkg	1.57×10^0 (1.57×10^3)	1.59×10^0 (1.59×10^4)	2.97×10^0 (2.67×10^5)	
$N_R N_R + Z(\rightarrow \ell\ell)Z(\rightarrow \ell\ell)$	BP1	3.53×10^{-2} (3.53×10^1)	2.29×10^{-2} (2.29×10^2)	7.21×10^{-3} (7.21×10^2)	
	BP2	0.98×10^0 (9.80×10^2)	0.75×10^0 (7.54×10^3)	0.24×10^0 (2.42×10^4)	
	BP3	3.23×10^{-2} (3.23×10^1)	7.87×10^{-2} (7.87×10^2)	3.08×10^{-2} (3.08×10^3)	
	BP4	7.50×10^{-3} (7.50×10^0)	0.39×10^0 (3.89×10^3)	0.30×10^0 (3.02×10^4)	
	bkg	1.08×10^{-1} (1.08×10^2)	1.39×10^{-1} (1.39×10^3)	3.74×10^{-1} (3.36×10^4)	
$N_R N_R + V(\rightarrow q\bar{q})V(\rightarrow q\bar{q})$	BP1	1.05×10^1 (1.05×10^4)	6.57×10^0 (6.57×10^4)	2.02×10^0 (2.02×10^5)	
	BP2	2.76×10^0 (2.76×10^3)	2.08×10^0 (2.08×10^4)	0.65×10^0 (6.57×10^4)	
	BP3	8.90×10^{-2} (8.90×10^1)	2.15×10^{-1} (2.15×10^3)	8.30×10^{-2} (8.30×10^3)	
	BP4	1.30×10^{-2} (1.30×10^1)	9.74×10^{-1} (9.74×10^3)	7.96×10^{-1} (7.96×10^4)	
	bkg	6.63×10^1 (6.63×10^4)	1.71×10^2 (1.71×10^6)	3.34×10^2 (3.01×10^7)	
$N_R N_R + H_{\text{SM}}(\rightarrow b\bar{b})H_{\text{SM}}(\rightarrow b\bar{b})$	BP1	1.21×10^0 (1.21×10^3)	1.12×10^0 (1.12×10^4)	3.77×10^{-1} (3.77×10^4)	
	BP2	3.95×10^{-1} (3.95×10^2)	5.29×10^{-1} (5.29×10^3)	1.88×10^{-1} (1.88×10^4)	
	BP3	1.22×10^{-2} (1.22×10^1)	5.32×10^{-2} (5.32×10^2)	2.36×10^{-2} (2.36×10^3)	
	BP4	1.40×10^{-3} (1.40×10^0)	2.49×10^{-1} (2.49×10^3)	2.27×10^{-1} (2.27×10^4)	
	bkg	6.63×10^1 (6.63×10^4)	1.71×10^2 (1.71×10^6)	3.34×10^2 (3.01×10^7)	

Production of charged scalars at muon colliders



Production of charged scalars at muon colliders



C. Zee-Babu model

T. Ahmed, AJ, S. Nasri and S. Saad: 2306.01255

The model

The Standard Model is extended with two $SU(2)$ gauge-singlets both of them are electrically charged ($\phi^\pm$ and $\kappa^{\pm\pm}$)

They transform under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as

$$\phi : (1, 1)_{+2} \text{ and } \kappa : (1, 1)_{+4}$$

These extra states have lepton number ($L_\phi = 2; L_\kappa = 2$).

- Yukawa Lagrangian

$$\mathcal{L}_Y \supset f_{ij} L_i^{aT} C L_j^b \epsilon_{ab} \phi^+ + g_{ij} \ell_i^T C \ell_j \kappa^{++} + \text{h.c.}$$

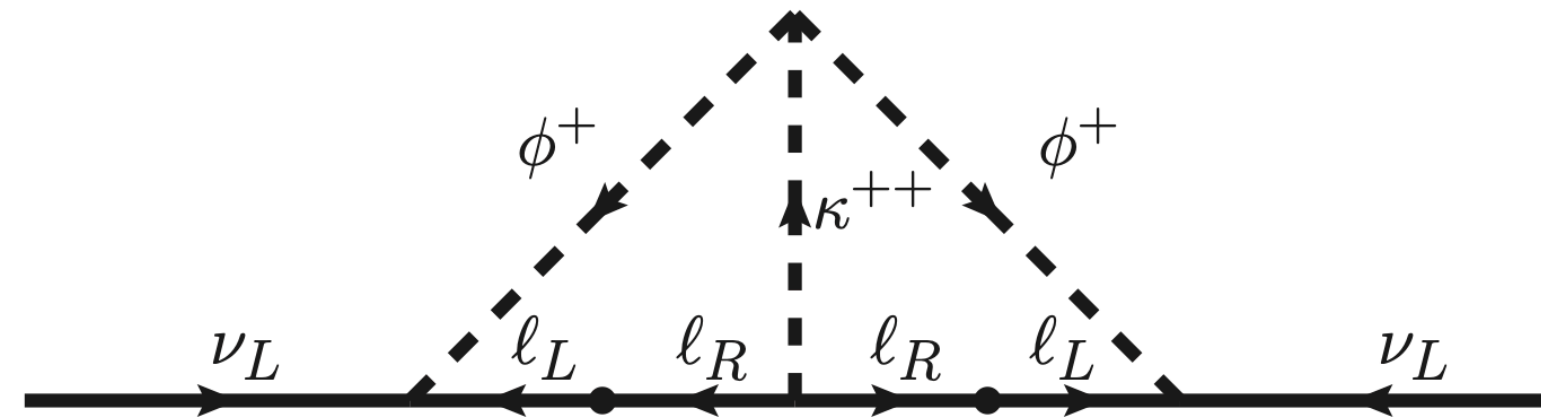
- Scalar potential

$$V_{\text{NP}} = \text{mass terms} + \lambda_\phi |\phi|^4 + \lambda_\kappa |\kappa|^4 + \lambda_{\phi\kappa} |\phi|^2 |\kappa|^2 + \lambda_{H\phi} |H|^2 |\phi|^2 + \lambda_{H\kappa} |H|^2 |\kappa|^2 + \underbrace{(\mu \phi^+ \phi \kappa^{--} + \text{h.c.})}$$

Break lepton number by two units

Neutrino mass

Since the trilinear $\phi\phi\kappa$ term in the potential breaks lepton number by two units we can generate tiny neutrino mass at the two-loop order



Neutrino mass

$$\mathcal{M}_{ij}^\nu = 16\mu f_{ik} m_k \hat{g}_{kl} m_l f_{lj} I_{kl}$$

LNv term

Lepton masses

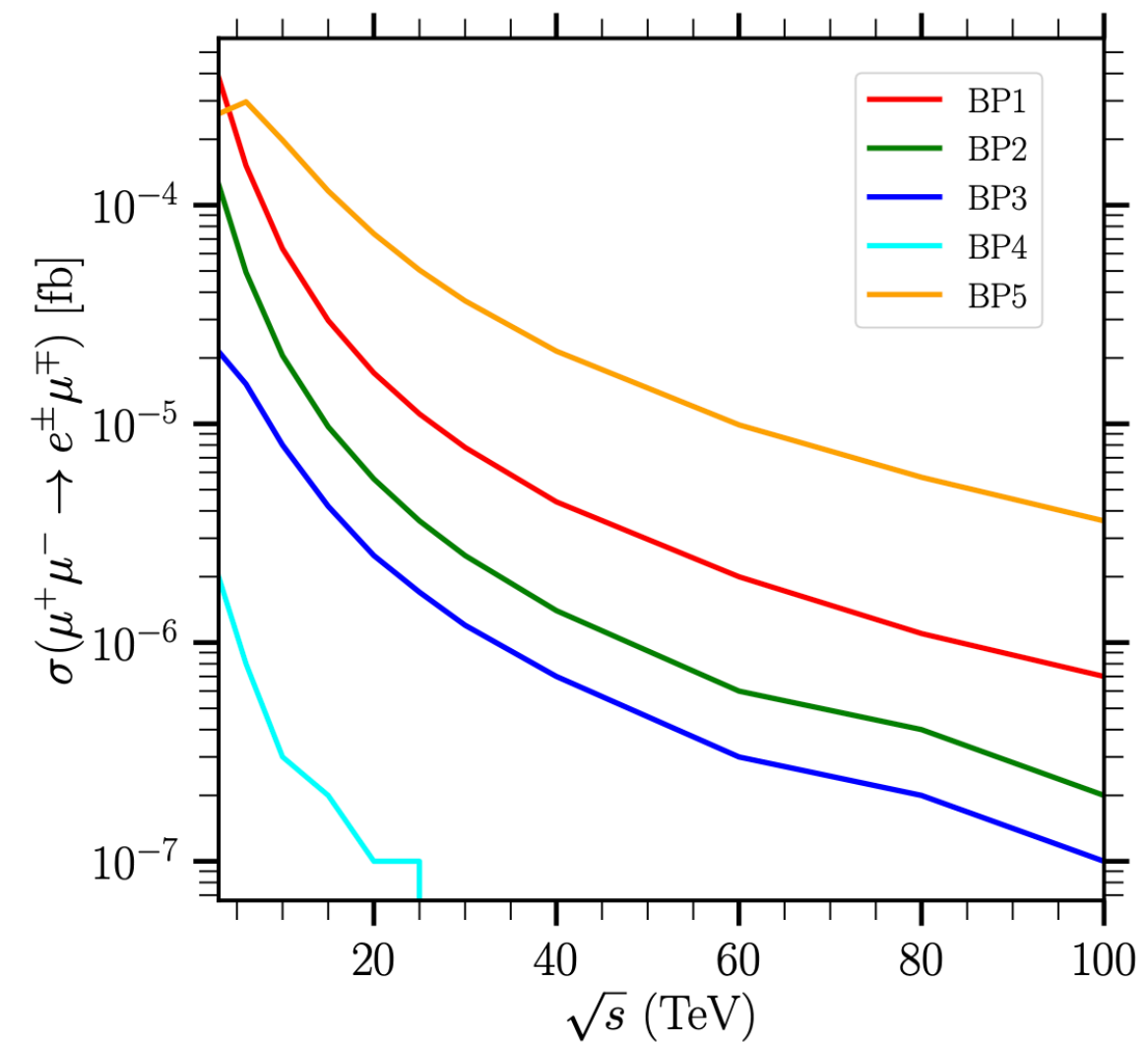
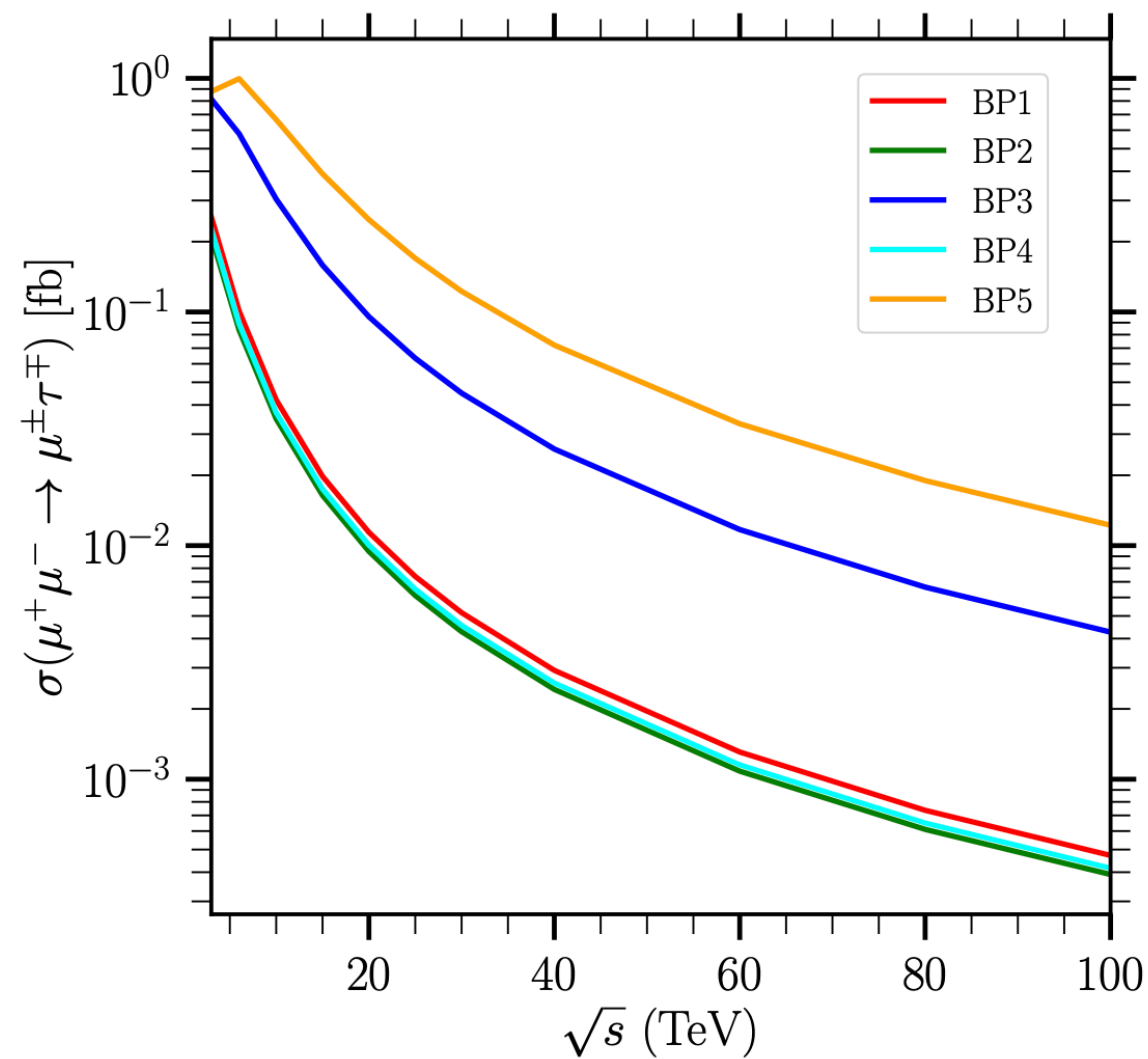
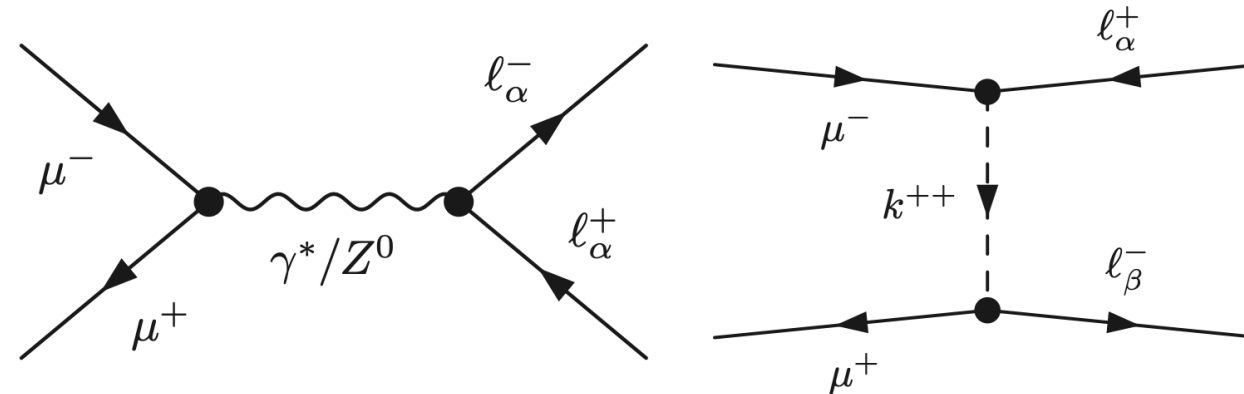
Two-loop integral



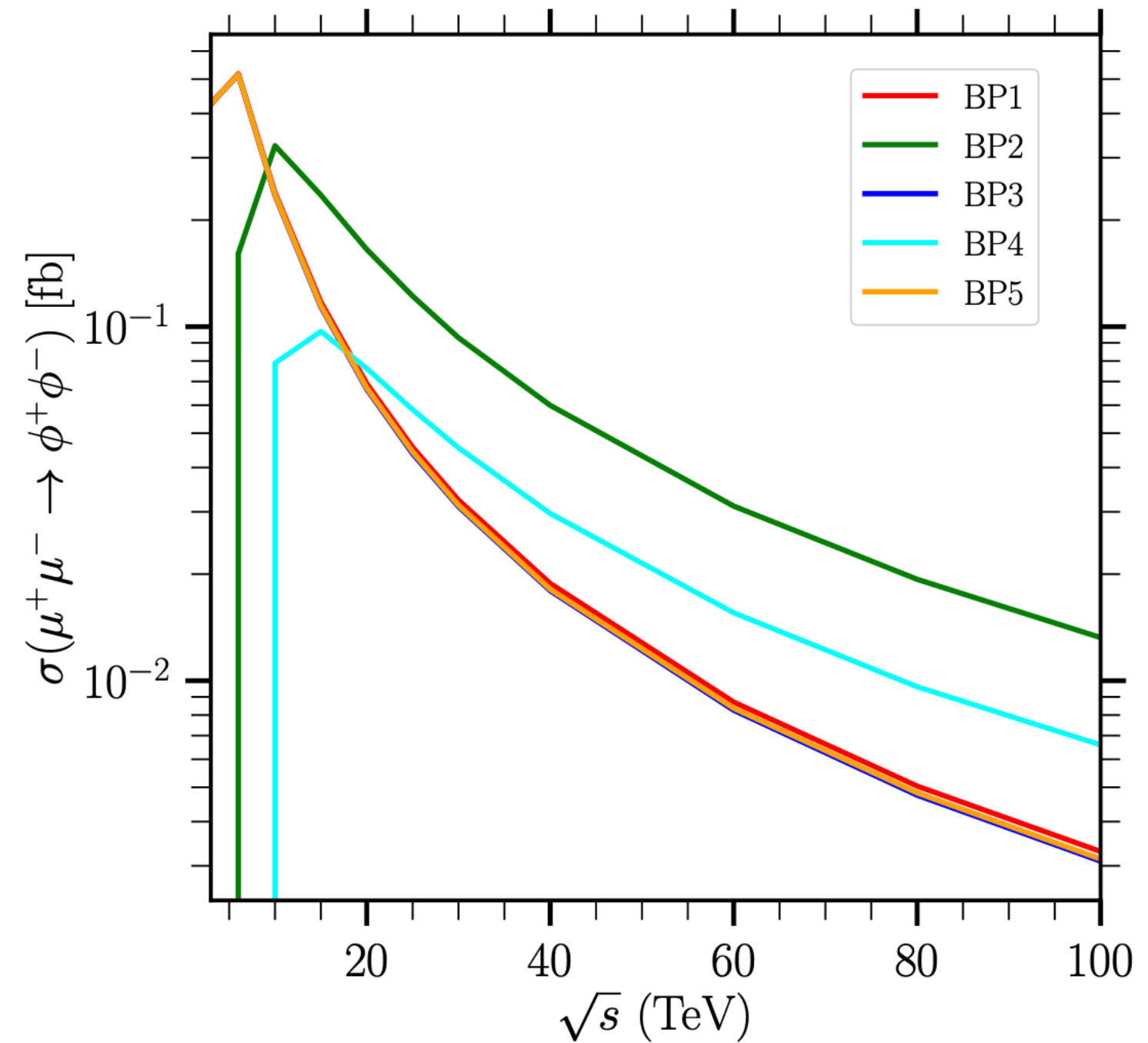
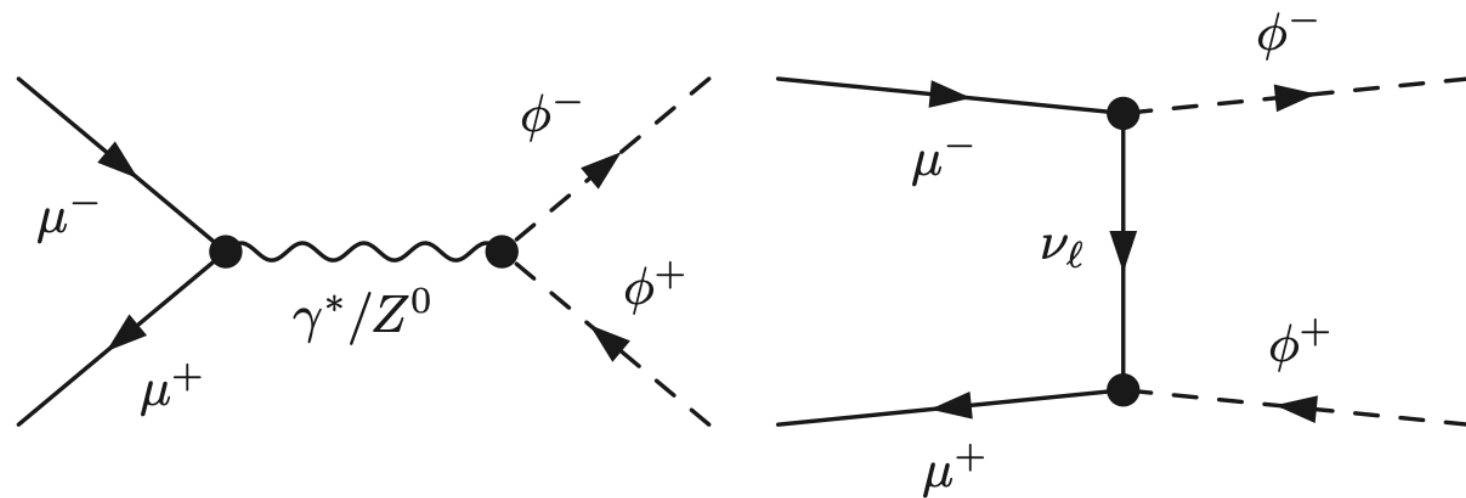
Benchmark points

Benchmark point	BP1	BP2	BP3	BP4	BP5
<i>Parameters</i>					
m_κ (GeV)	1250	1250	2500	1250	3750
m_ϕ (GeV)	1250	2500	1250	3750	1250
μ (GeV)	3467.38	3264.21	3336.07	9544.89	3308.84
$f_{e\mu}$	-0.03616	-0.0688	-0.0339	-0.05738	-0.03448
$f_{e\tau}$	0.0193	0.0368	0.01813	0.03069	0.01835
$f_{\mu\tau}$	0.0598	0.1138	0.05616	0.09489	0.05704
g_{ee}	-0.04436	0.007757	0.1437	0.0297	-0.00628
$g_{e\mu}$	-0.00026	-0.000353	-0.000136	-0.000057	0.00078
$g_{e\tau}$	0.00236	0.0117	0.00847	0.0127	-0.0092
$g_{\mu\mu}$	0.4426	0.55	1.0	0.43	1.0
$g_{\mu\tau}$	0.01987	0.01455	0.02649	0.0192	0.04503
$g_{\tau\tau}$	0.002157	0.00254	0.00464	0.00208	0.0049

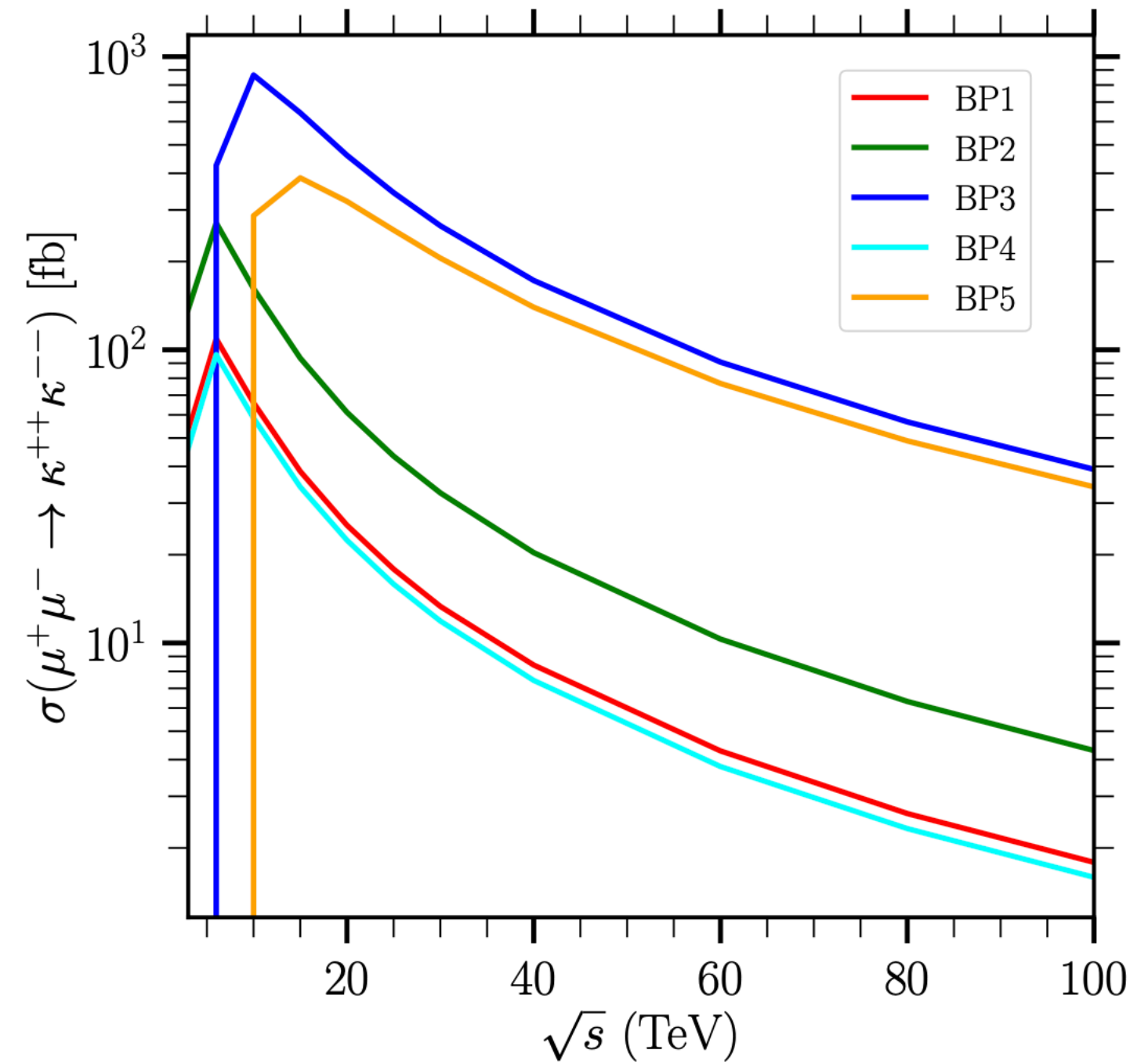
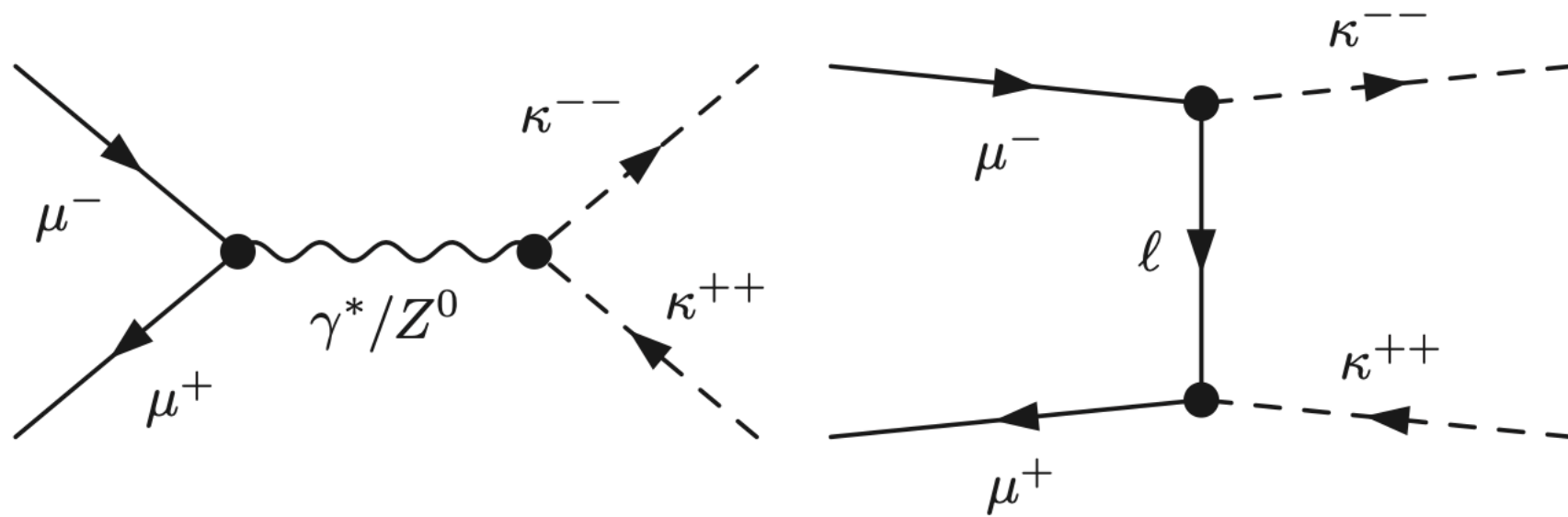
Production cross sections: $\mu^+\mu^- \rightarrow \ell_\alpha^+\ell_\beta^-$



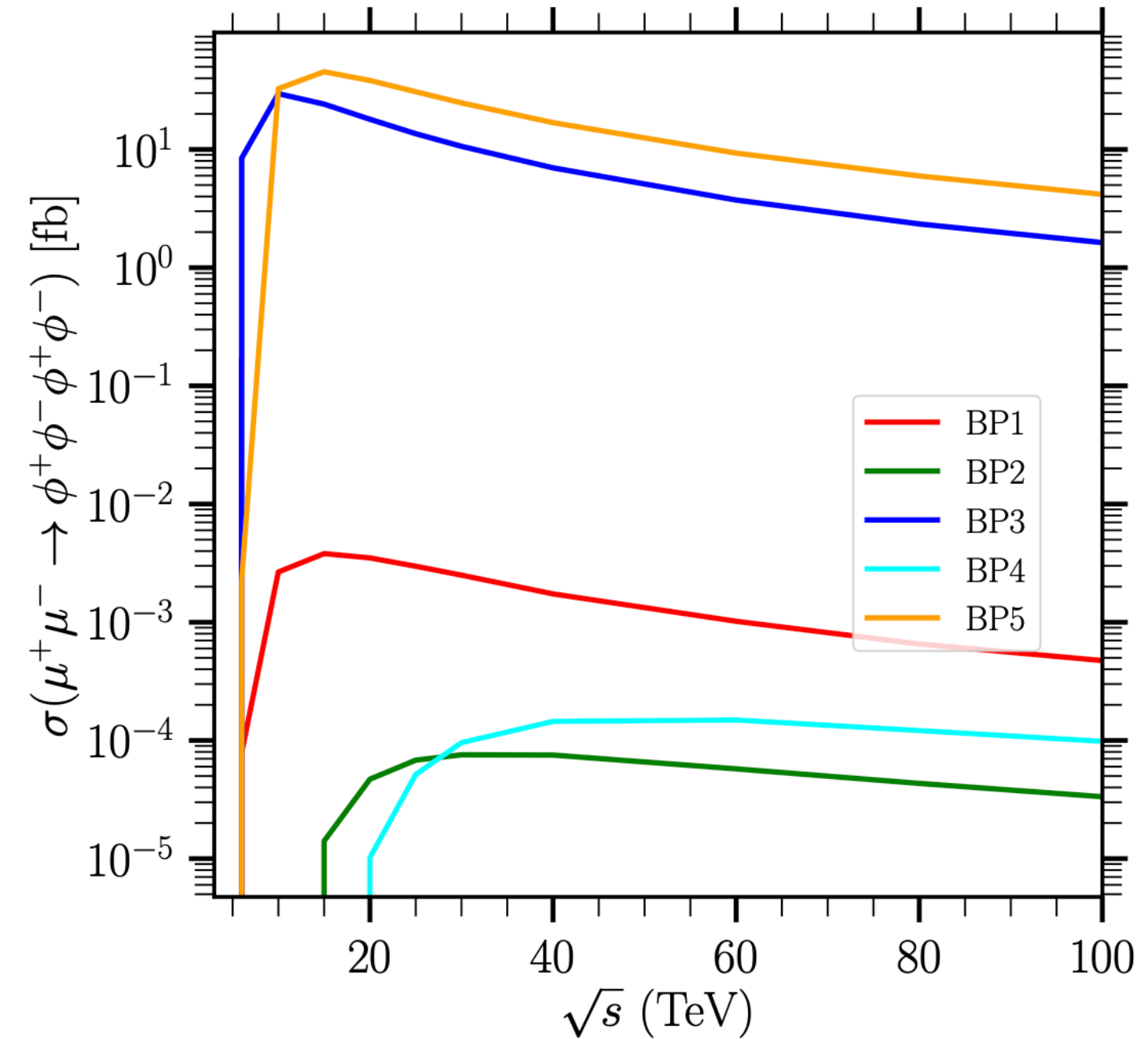
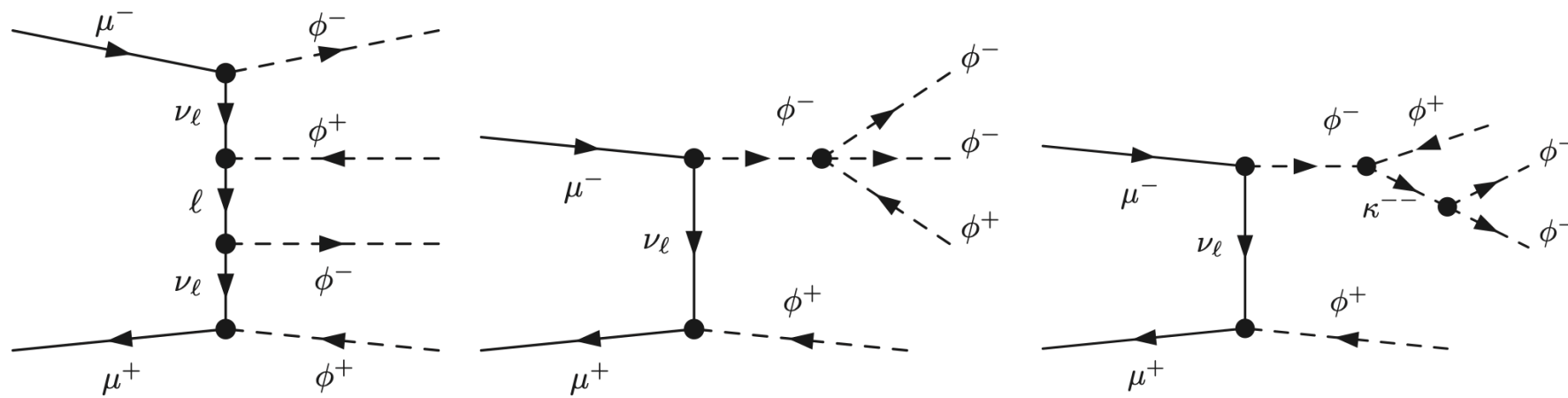
Production cross sections: $\mu^+\mu^- \rightarrow \phi^+\phi^-$



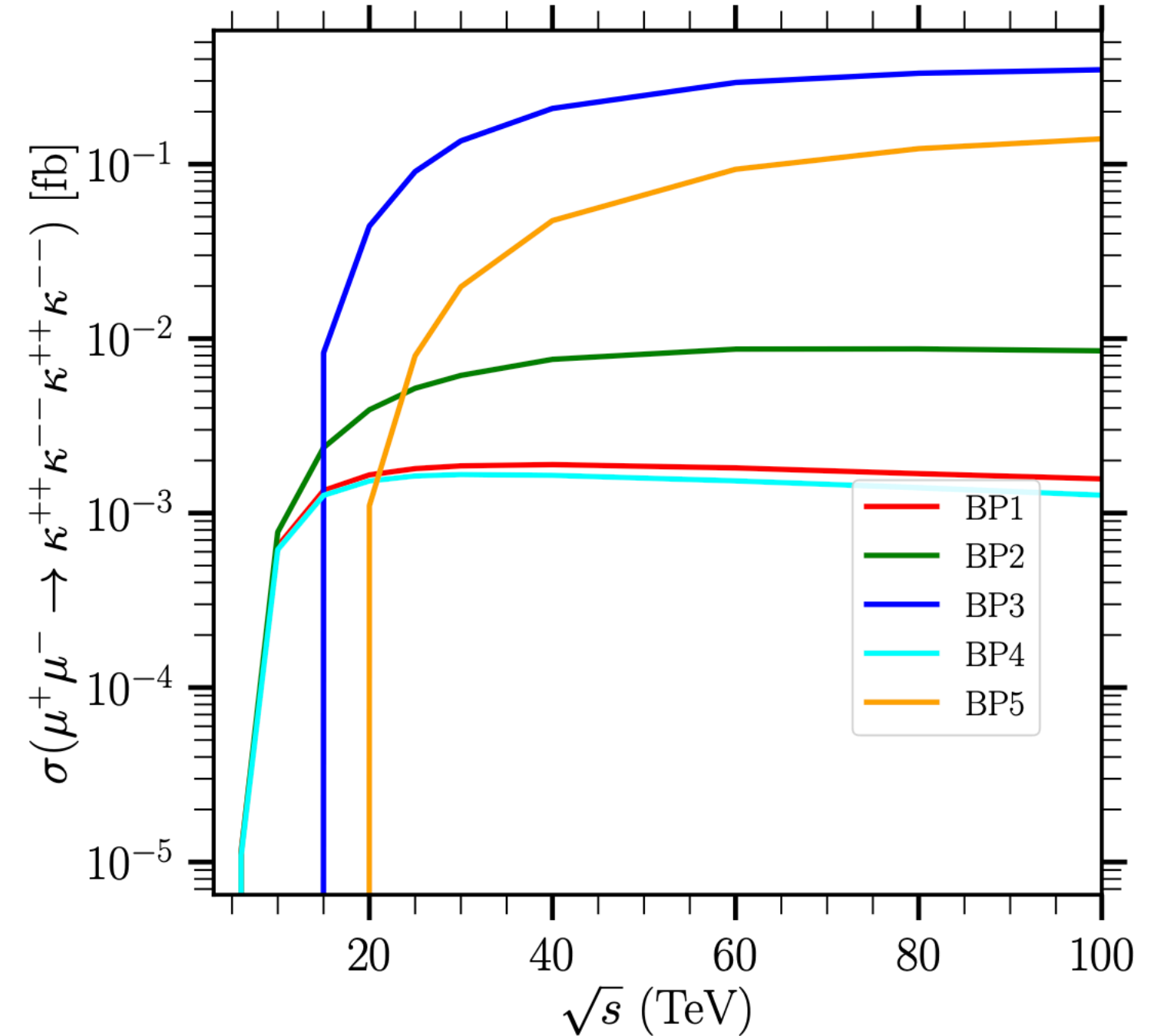
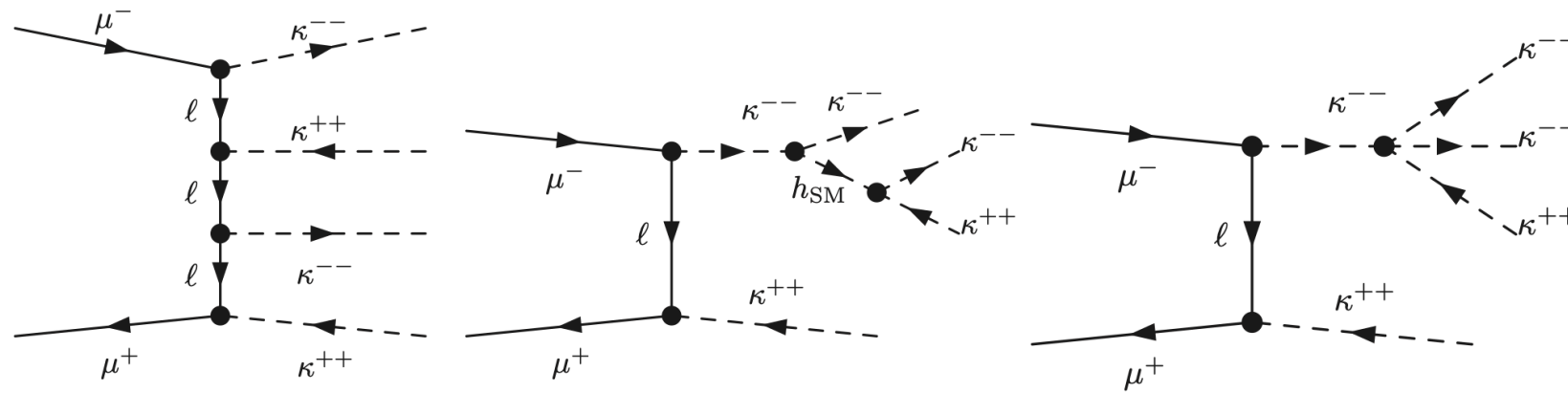
Production cross sections: $\mu^+\mu^- \rightarrow \kappa^{++}\kappa^{--}$



Production cross sections: $\mu^+\mu^- \rightarrow \phi^+\phi^-\phi^+\phi^-$



Production cross sections: $\mu^+\mu^- \rightarrow \kappa^{++}\kappa^{--}\kappa^{++}\kappa^{--}$

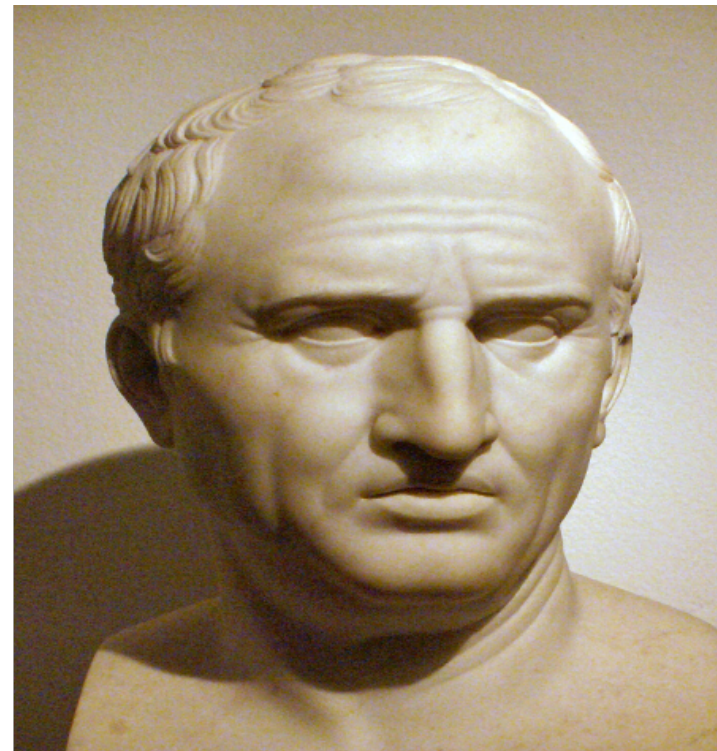


Conclusions and Outlook

- The first results are promising ... but ...
- **MORE WORK IS NEEDED**

Guiding principle: History

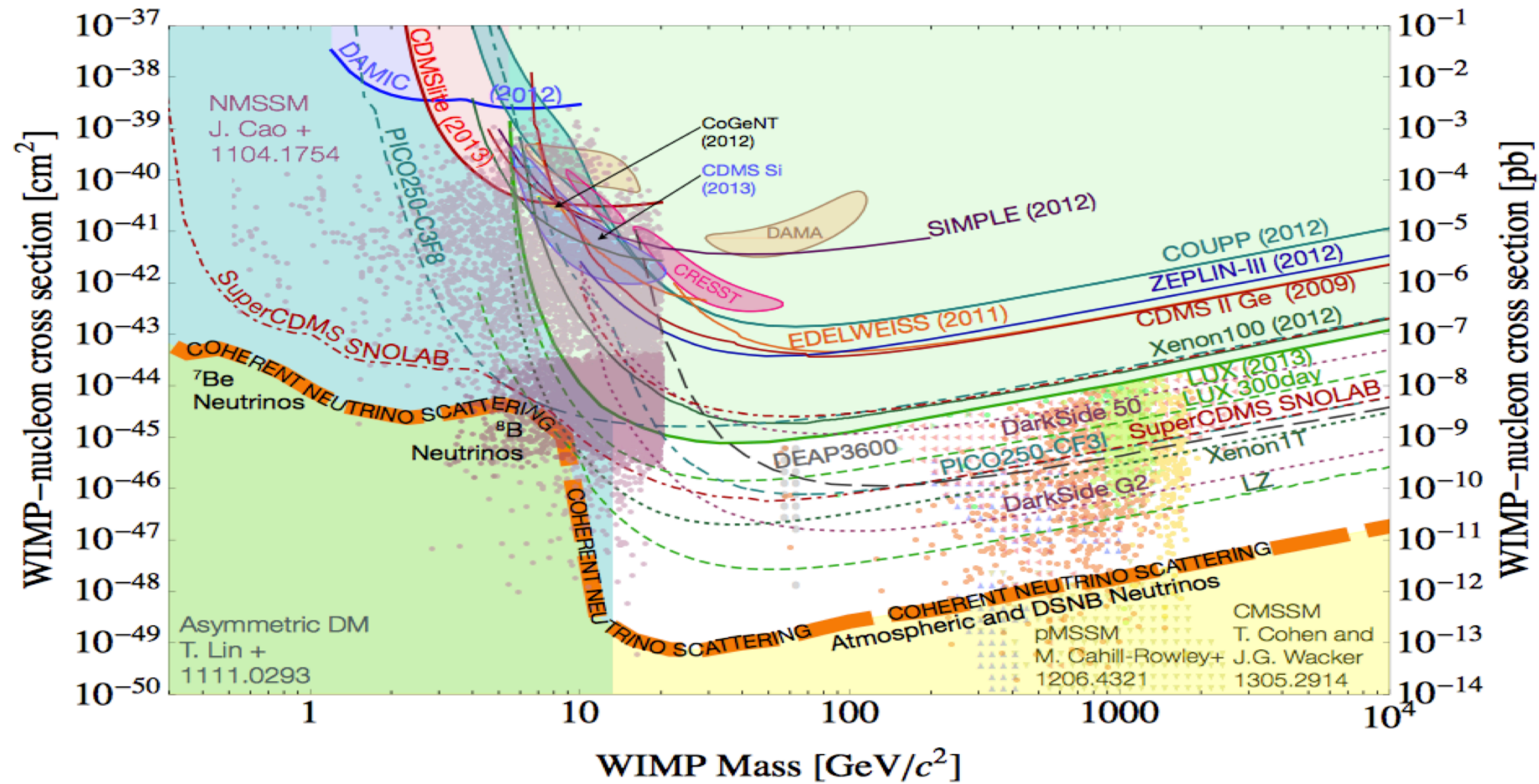
By what other voice, too, than that of the orator, is history, the witness of time, the light of truth, the life of memory, the directress of life, the herald of antiquity, committed to immortality?



Cicero, De Oratore II, 36

Back-up slides

Direct detection is more harsh for WIMP: Close to the neutrino floor



PROBLEM: dark-matter direct searches are strongly correlated with collider searches.

➡ Strong bounds imply expected weak signals at colliders; (excluding simple models)

- The strong bounds from direct-detection experiments tend to exclude the simplest dark-matter model; e.g. the singlet model.

What if the dark matter candidate is a singlet Majorana fermion?

- Usually, these simple dark-matter models lead to s-wave annihilation channels; Models with s-wave annihilations are almost excluded (model-independent analysis by [Leane, Slatyer, Beacom and Ng; 2018](#)).
- Collider searches at the Large Hadron Collider tend to exclude couplings of order $\mathcal{O}(1)$ and light masses (see e.g. the summary plots in [ATL-PHYS-PUB-2020-021](#))
- An alternative solution is to consider (or reconsider) Majorana singlet fermions as dark-matter candidates:
 - i. The elastic scattering of dark-matter off the nucleus is induced at the one-loop order  The corresponding cross-section is always suppressed even for couplings of order $\mathcal{O}(1)$.
 - ii. Hard to produce at hadron colliders for a wide class models  Explain why it is not observed so far?
 - iii. Annihilation cross section occurs through p-wave amplitudes; no signal, no problem.
 - iv. Lepton colliders may play the role of discovery machines for these models.

Important: Minimal is not Simplified

- Take a simplified s -channel model with spin-0 mediator and Dirac dark matter

$$\mathcal{L}^{Y_0} \supset \sum_i \left(\frac{y_i^q}{\sqrt{2}} g_{q_i}^S \bar{q}_i q_i + g_\chi^S \bar{\chi} \chi \right) Y_0 \quad (\text{Assuming flavour-diagonal couplings})$$

Couplings DM Mediator

- In principle, you cannot get this interaction unless: (i) Y_0 is a member of a Higgs multiplet (doublet for example) or (ii) Y_0 is a singlet that mixes with the SM Higgs boson after EWSB $\implies$ models get more complicated with many additional parameters and smaller rates due to constraints from e.g. Higgs boson data.
- Minimal dark matter models, on the other hand, do not rely on any extra assumption except (may be) unification at some higher scales...

Models with Majorana dark matter: directions

- Singlet Majorana fermions can be accommodated in many extensions of the SM; Mainly for neutrino mass generation through loops

Examples: One-loop (E. Ma; 2006), Three-loops (Krauss-Nasri-Trodden; 2003) and Three-loops with multi-Doublets (Aoki-Kanemura-Seto; 2009)

- What if follows a bottom-up approach? Any model of this kind should fulfill these four pillars (taken from Stephen King)

Minimality

It must be simple/elegant to have a chance of being correct

Predictivity

It must be possible to exclude such models by experiments

Robustness

It must be firmly based on some theoretical symmetry and/or dynamics

Unification

It must be capable of being embedded into a grand-unified theory



What about the various constraints (DM)?

After electroweak symmetry breaking; one lefts with two extra states (N_R and $H^\pm$) and seven extra parameters (three are interconnected via lepton-flavor violation and one is irrelevant in phenomenological studies). The parameters are

- General case: $\{M_{H^\pm}, M_{N_R}, \lambda_2, \lambda_3, Y_{eN}, Y_{\mu N}, Y_{\tau N}\}$

- Relevant for DM: $\{M_{H^\pm}, M_{N_R}, \lambda_2, \lambda_3, Y_{\ell N}\}$

Good approximation for massless leptons;
$$Y_{\ell N} = \sqrt{Y_{eN}^2 + Y_{\mu N}^2 + Y_{\tau N}^2}$$

Theoretical constraints

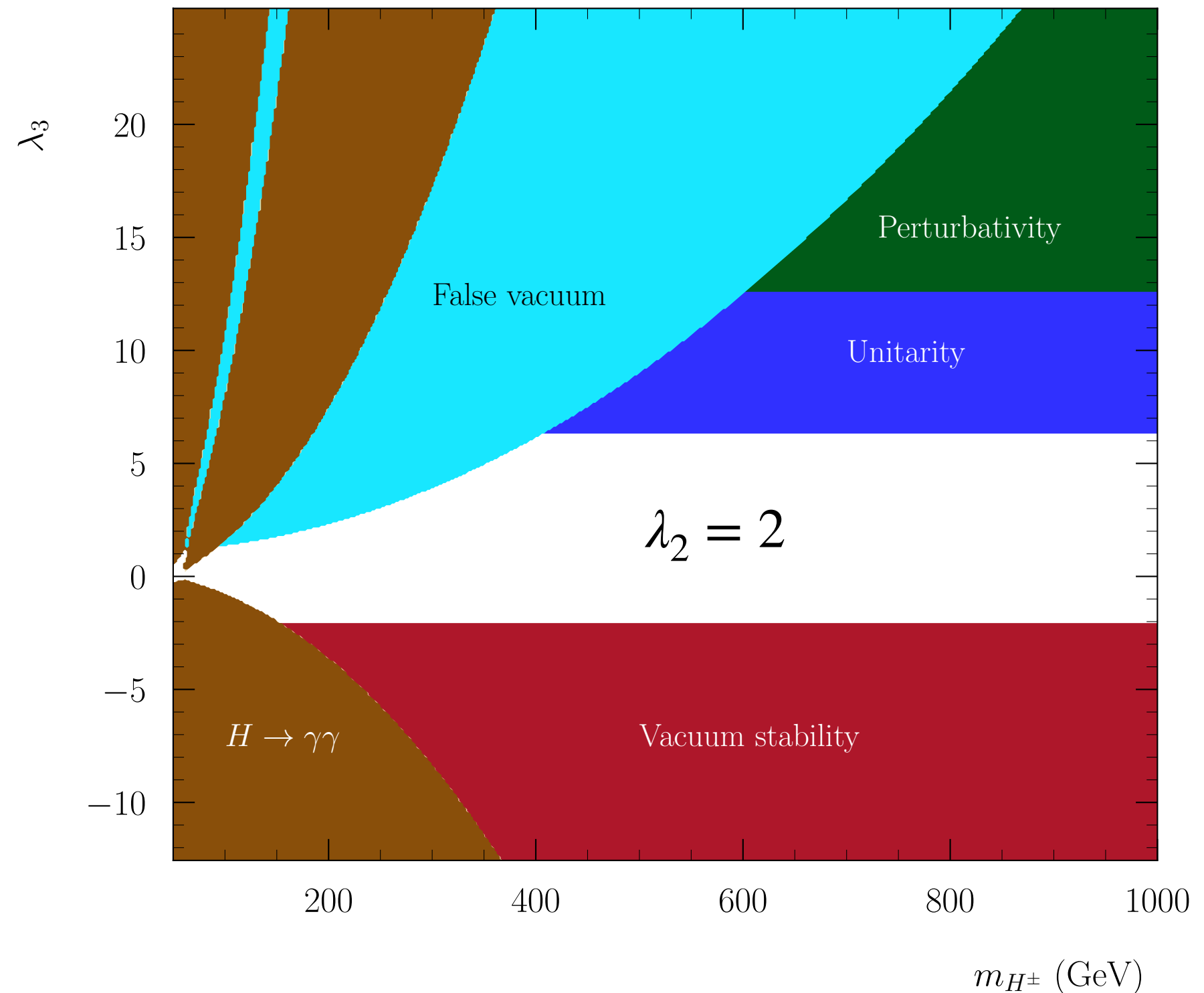
- (i) Vacuum stability: the scalar potential should be bounded from below (Branco et al.; 2012)
- (ii) Perturbativity & Perturbative unitarity
- (iii) False vacuum

Experimental constraints

- (i) $H \rightarrow \gamma\gamma$
- (ii) Higgs invisible decay ($H \rightarrow NN$): relevant for $m_H > 2m_N$
- (iii) Charged lepton flavor violating decays;
 $\ell_\alpha \rightarrow \ell_\beta \gamma$ and $\ell_\alpha \rightarrow \ell_\beta \ell_\gamma \bar{\ell}_\gamma$
- (iv) Searches of charginos at LEP-II.

Summary of theoretical and experimental constraints

- Perturbativity and unitarity constraints exclude large values of λ_3 .
- The bounds on the charged Higgs mass do not depend on λ_3 for $\lambda_3 \approx \mathcal{O}(1)$.
- If λ_3 is large, false vacuum constraints exclude light charged scalar masses; i.e. one has $m_{H^\pm} \geq 350$ GeV for $\lambda_3 = 5$.
- For $\lambda_3 > 0$, there is a region where the constraints from $H \rightarrow \gamma\gamma$ completely vanish.



Higgs invisible decay

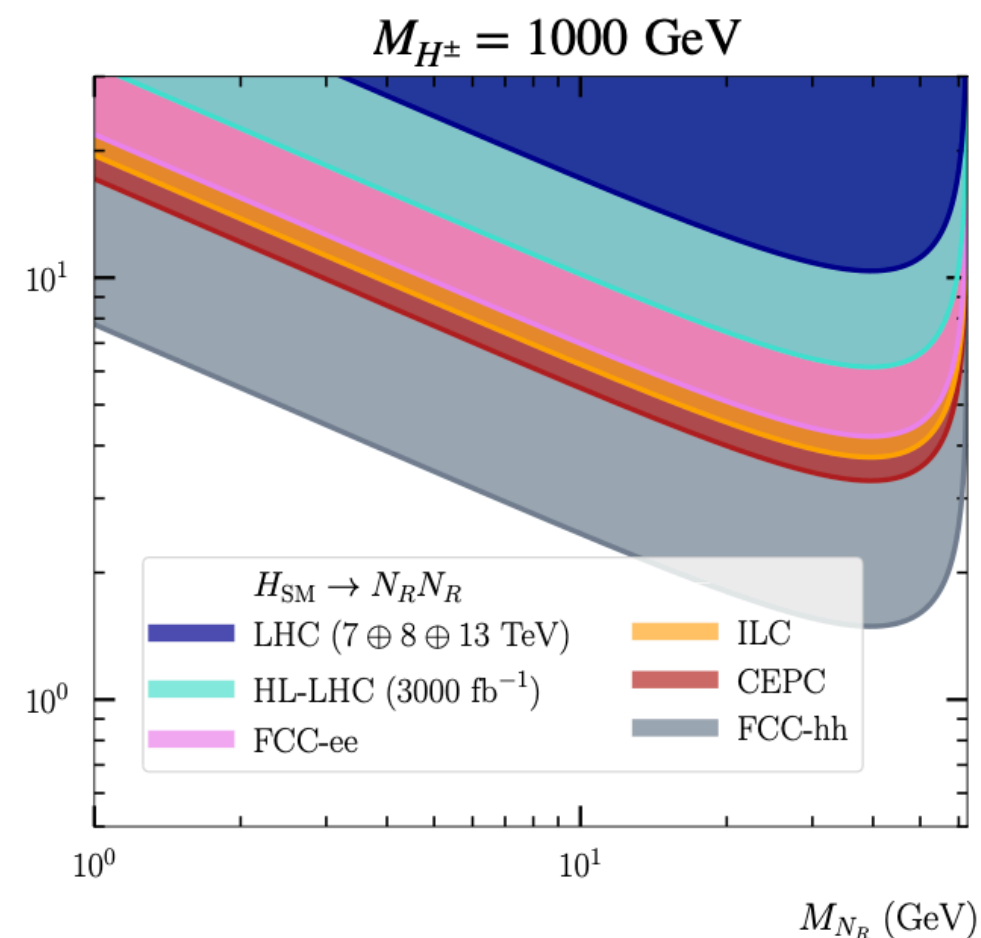
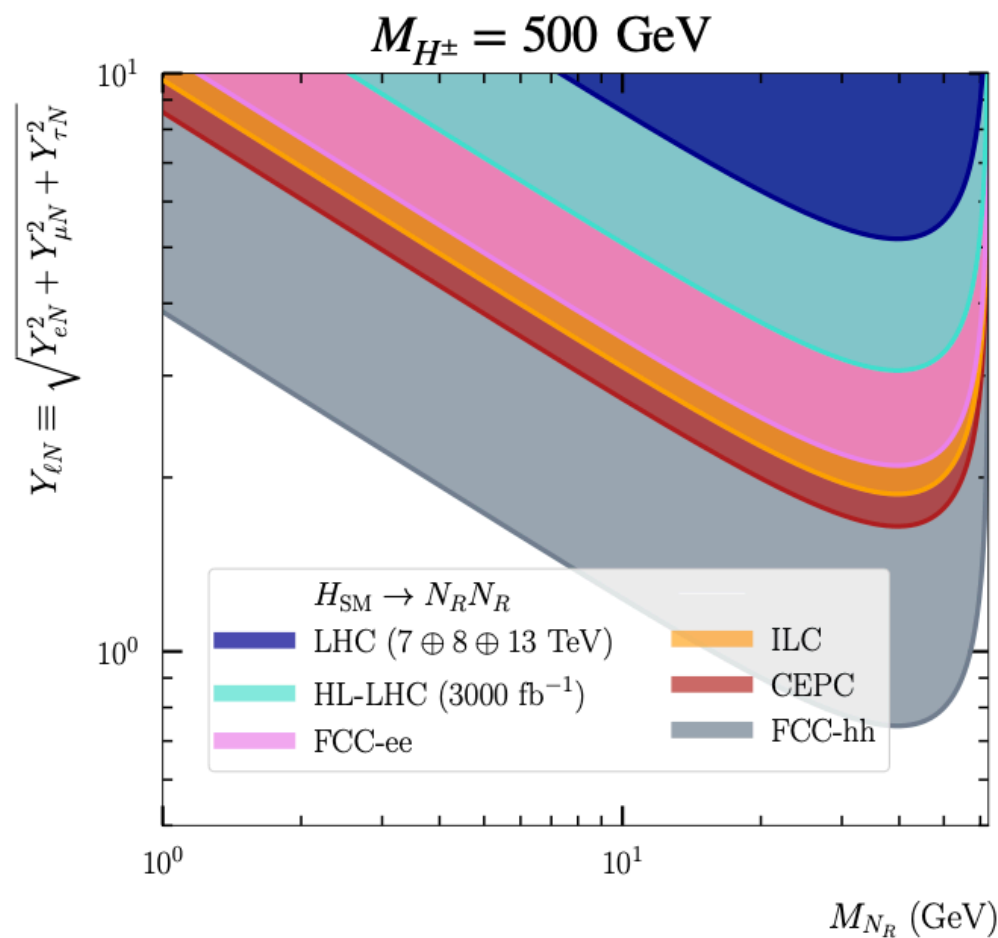
$$Y_{\ell N} < \left(\frac{2^{11} \pi^5 \Gamma_H^{\text{SM}}}{\beta_N^3 m_H \lambda_3^2 v^2 m_N^2 |C_0 + C_2|^2 \mathcal{R}_{\text{exp}}} \right)^{1/4}$$

$$\mathcal{R}_{\text{exp}} = \frac{1}{B_{H \rightarrow \text{invisible}}^{\text{up.bound}}} - 1$$

$$C_{0,2} \equiv C_{0,2}(m_N^2, m_H^2, m_N^2, m_\ell^2, m_{H^\pm}^2, m_{H^\pm}^2)$$

Passarino-Veltman functions

- The future constraints on $Y_{\ell N}$ are expected to be very important for light charged Higgs boson.
- Still some room for future studies to be focused on light dark-matter masses.
- Note that it's very hard to produce the correct relic density for $M_{N_R} < 10$ GeV if we assume the perturbativity of the couplings.



Dark matter relic abundance

The relic abundance of N_R gets contributions that can be categorized into sets (assuming freeze-out mechanism):

(i) Annihilation into SM particles: important for $Y_{\ell N} = \sqrt{Y_{eN}^2 + Y_{\mu N}^2 + Y_{\tau N}^2} \approx \mathcal{O}(1)$

$$N_R N_R \rightarrow \ell_{\alpha}^{\pm} \ell_{\beta}^{\mp}$$

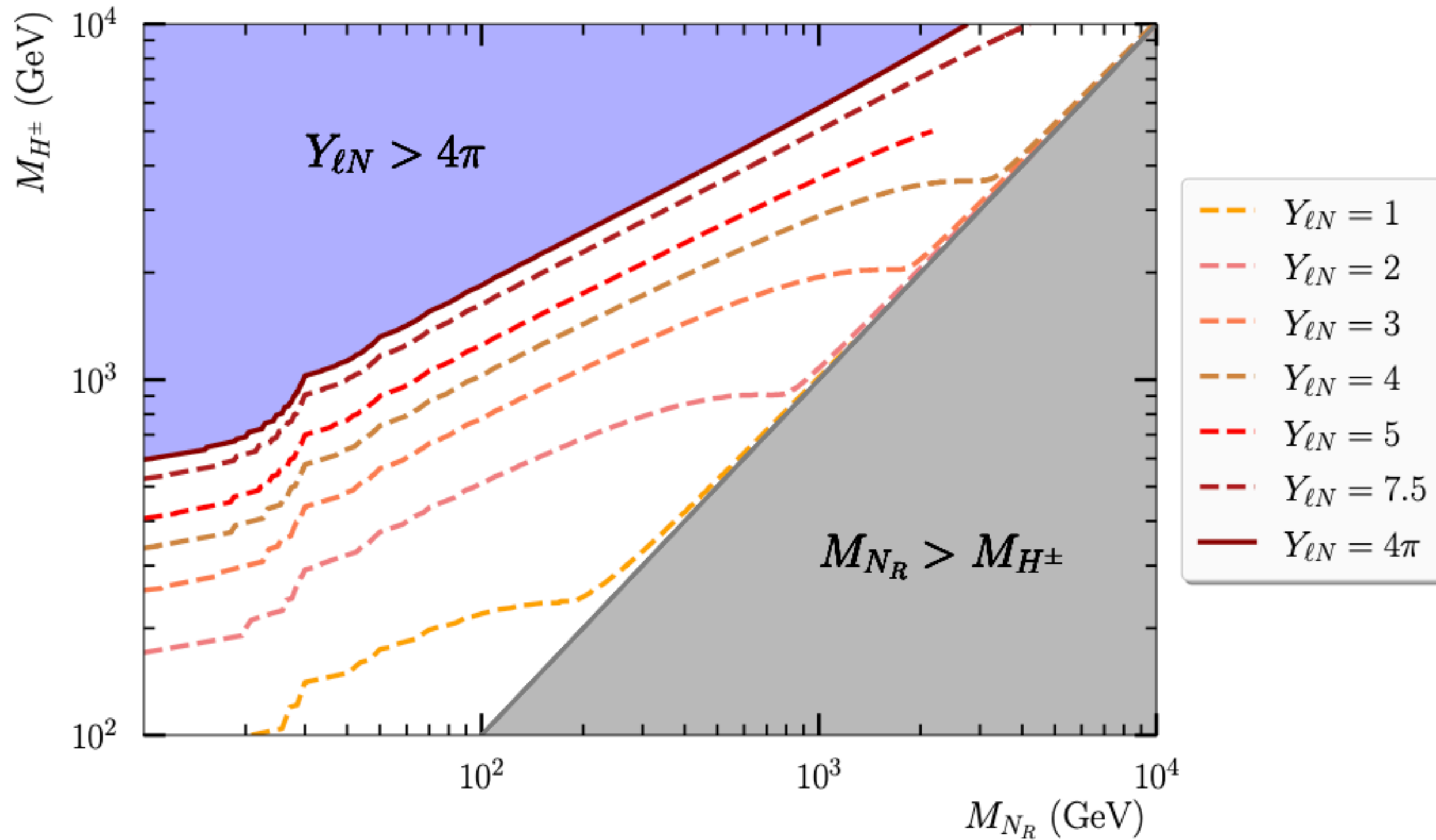
$$N_R N_R \rightarrow H^* \rightarrow \tau\tau, b\bar{b}, t\bar{t}, Z^0 Z^0, W^+ W^-, HH$$

(ii) Co-annihilation channels: dominates for tiny mass-splitting ($\Delta < m_N/10$)

$$N_R H^{\pm} \rightarrow \ell^{\pm} H, W^{\pm} \nu_{\ell}, \ell^{\pm} Z, \ell^{\pm} \gamma$$

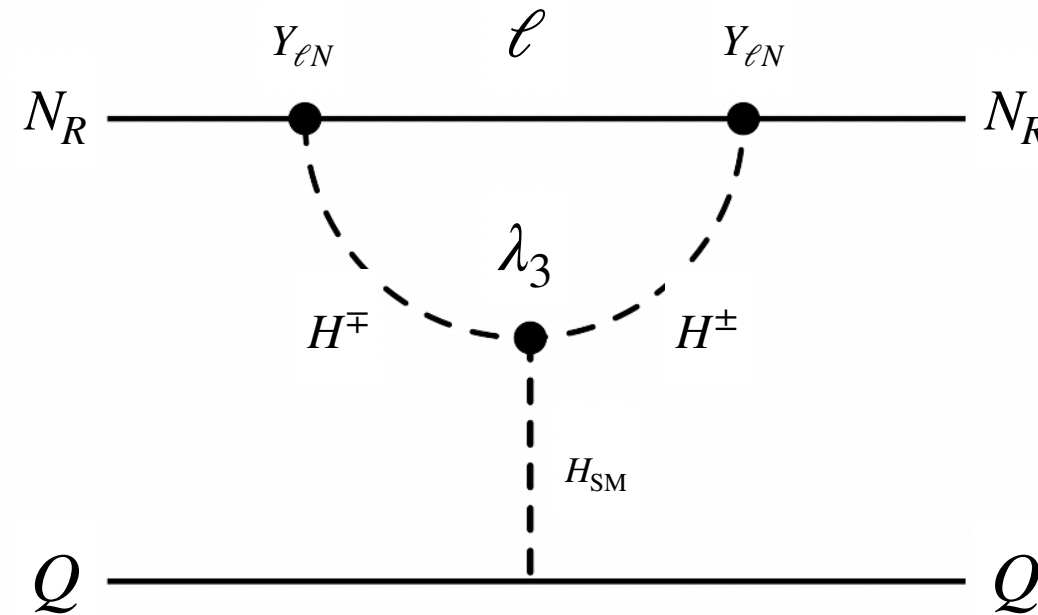
$$H^{\pm} H^{\mp} \rightarrow \ell^{\pm} \ell^{\mp}, q\bar{q}, HH, ZZ, W^{\pm} W^{\mp}, ZH, t\bar{t}$$

Dark matter relic abundance



Direct detection constraints

The spin-independent nucleus- N_R elastic cross section occurs at the one(two)-loop order



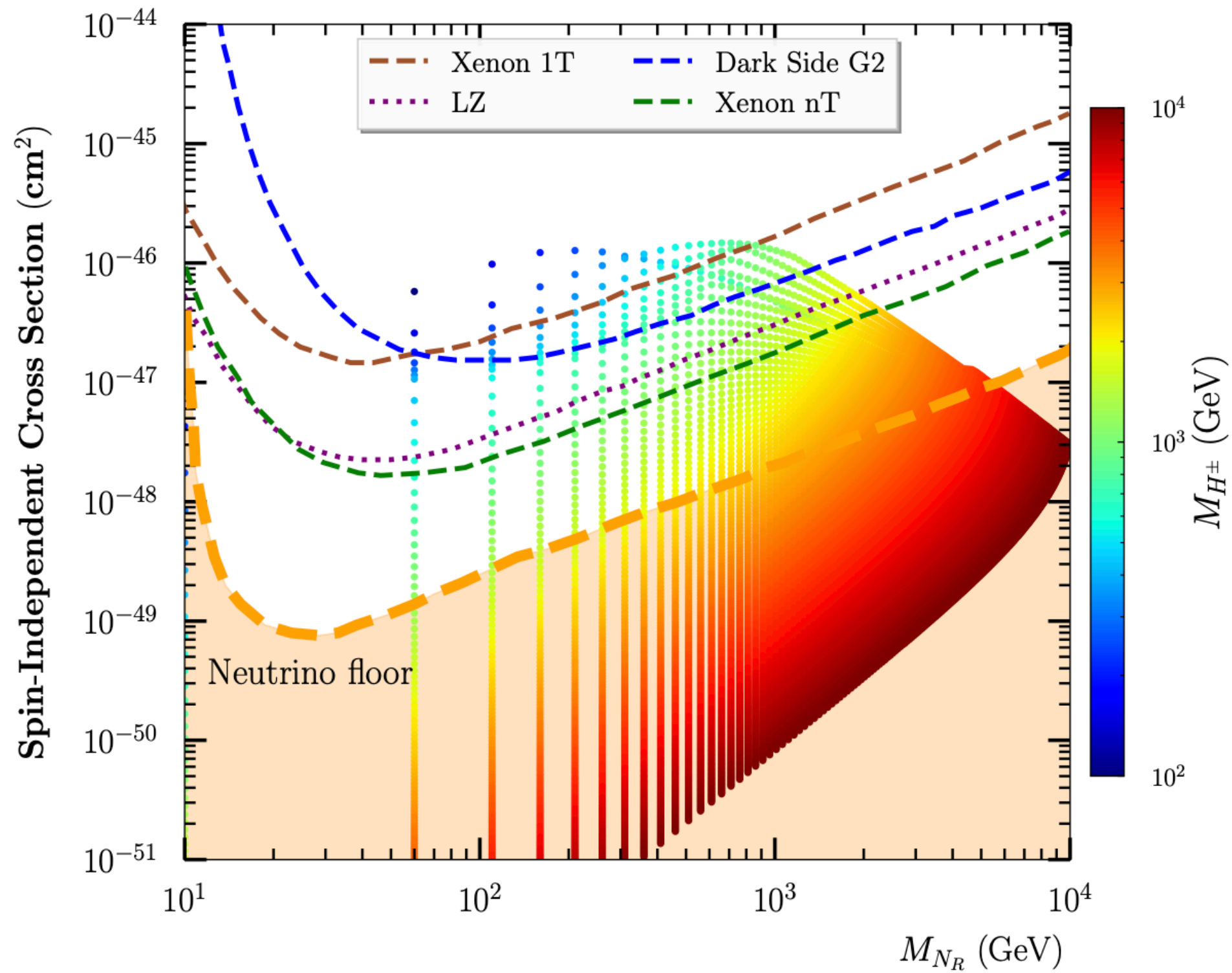
We get something like

$$\sigma_{\text{SI}} \propto \left(\text{Nuclear matrix elements} \right)^2 \times \left| \frac{\tilde{y}(Q^2 \approx 0)}{\downarrow M_{H^\pm}^2} \right|^2 \times \text{phase space}$$

Effective Higgs- N_R coupling

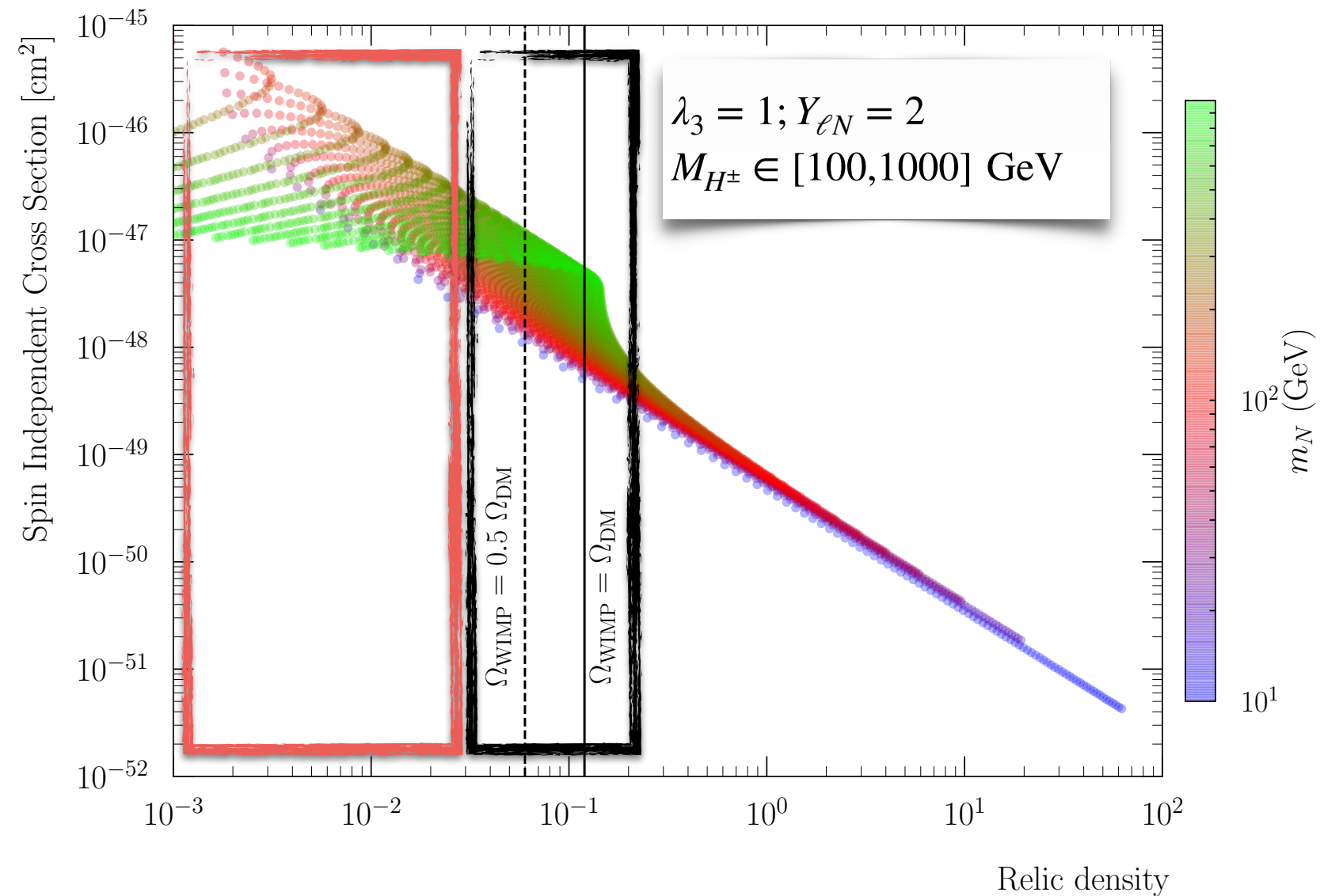
$$\tilde{y}(Q^2 \approx 0) = -\frac{\lambda_3 v |Y_{\ell N}^2|}{16\pi M_{H^\pm}} q_N \times \left[1 - (1 - q_N^{-2}) \log(1 - q_N^2) \right] \quad (q_N = M_{N_R}/M_{H^\pm})$$

Direct detection constraints

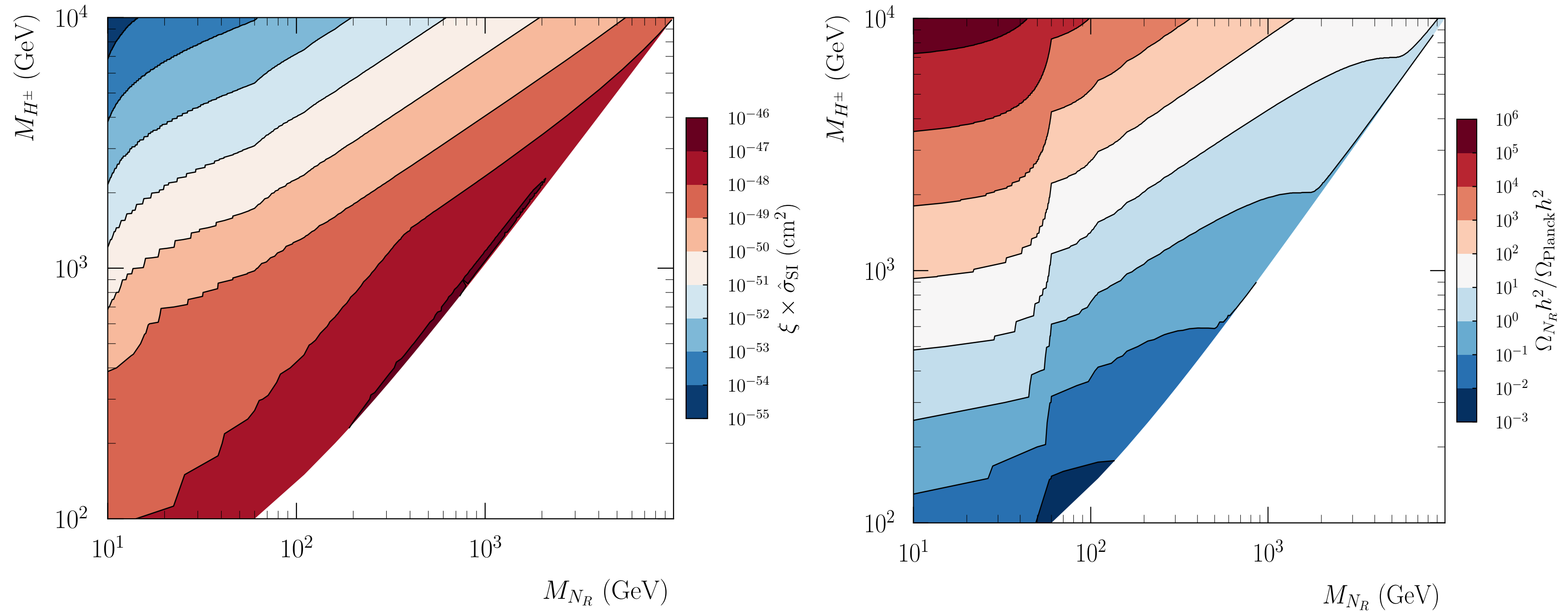


Correlations: Ωh^2 vs σ_{SI}

- Strong anti-correlation is observed between the spin-independent cross section (σ_{SI}) and the relic abundance of N_R .
- Regions where the predicted σ_{SI} is enhanced are hard to exclude as they correspond to $\xi \equiv \Omega_N h^2 / \Omega_{\text{Planck}} h^2 \ll 1$



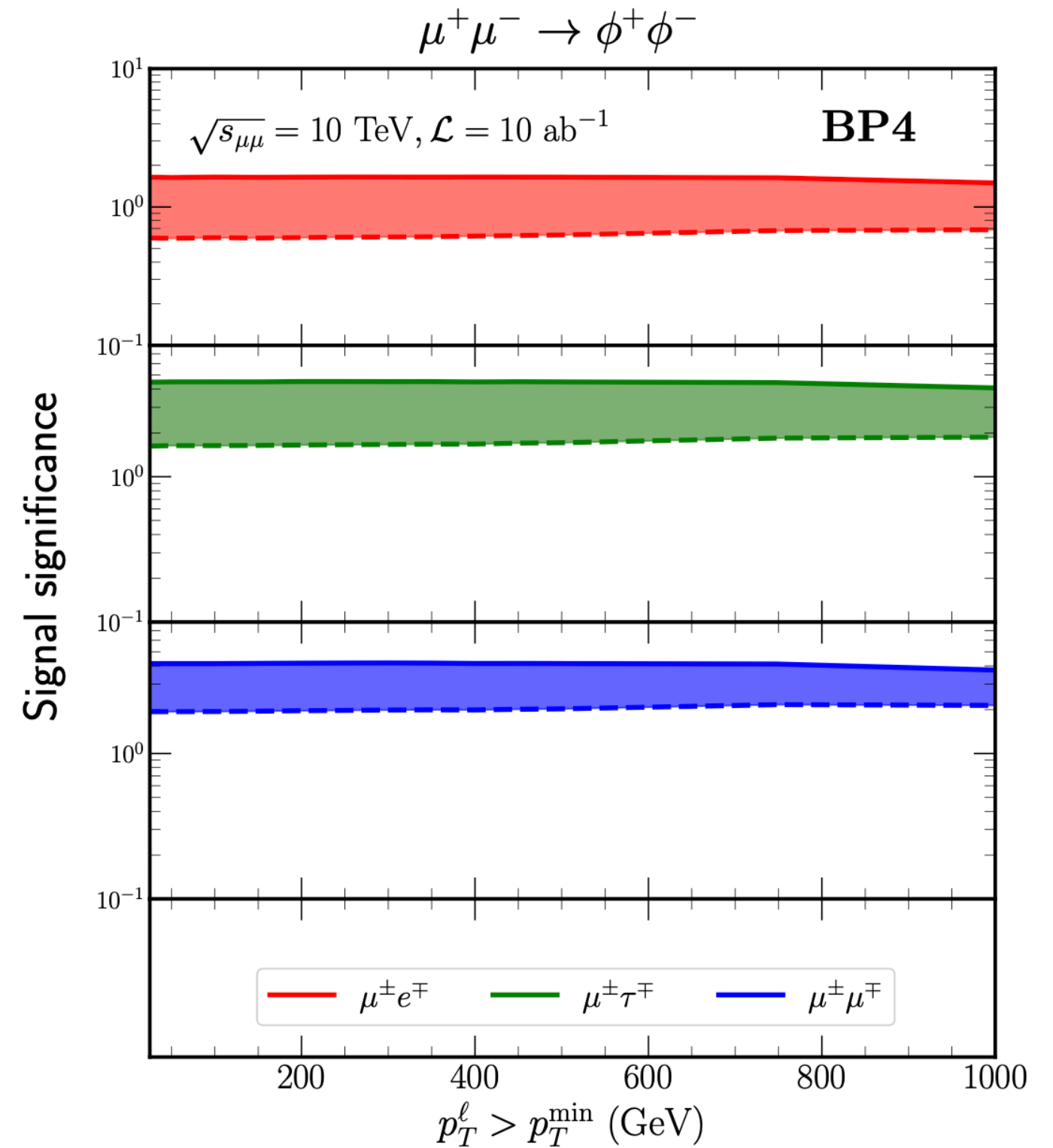
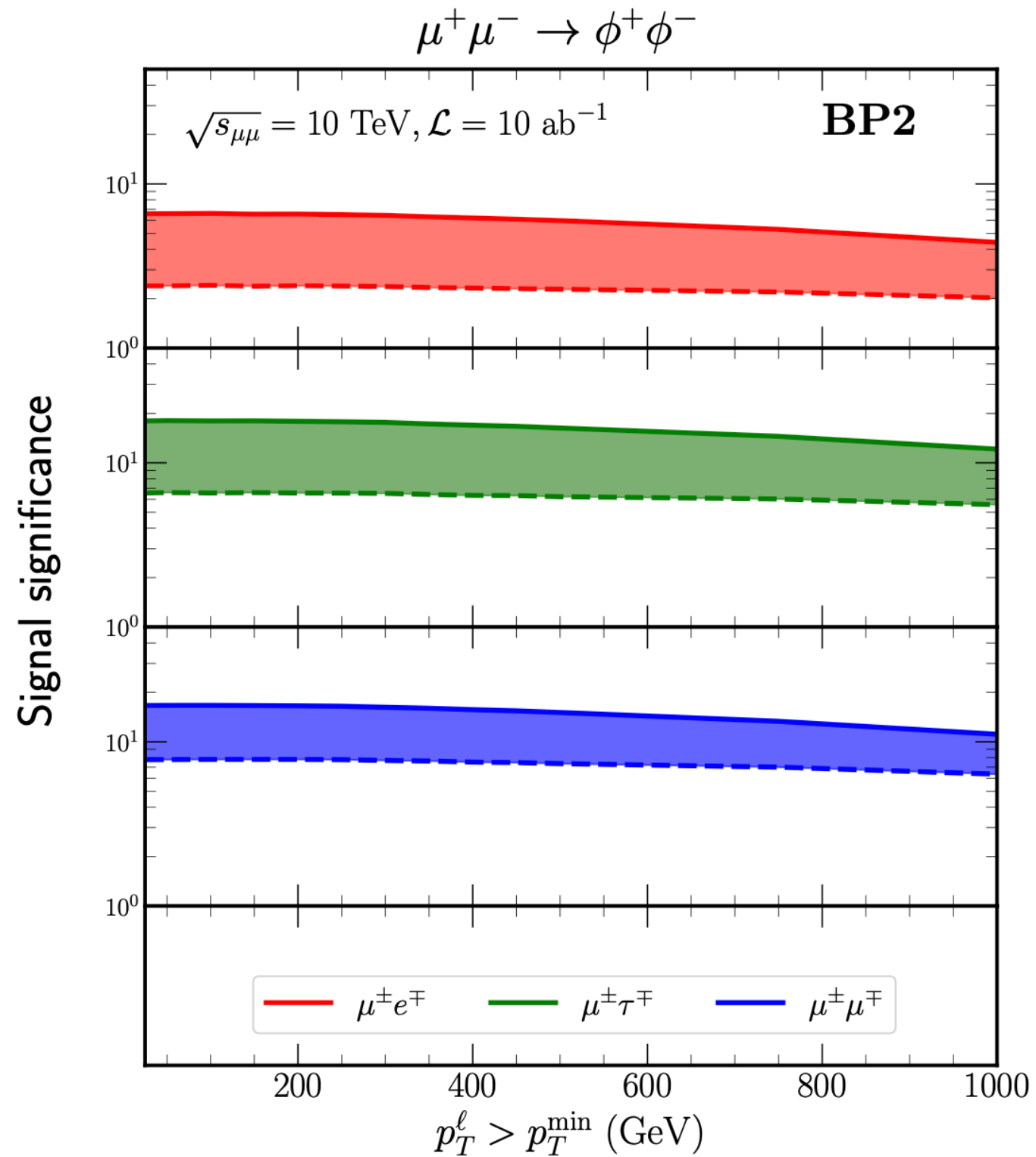
Correlations: Ωh^2 vs σ_{SI}



$$Y_{\ell N} \equiv \sqrt{Y_{eN}^2 + Y_{\mu N}^2 + Y_{\tau N}^2} \approx Y_{\mu N} = 2$$



Sensitivity reach: $\mu^+\mu^- \rightarrow \phi^+\phi^-$



Sensitivity reach: $\mu^+\mu^- \rightarrow \kappa^{++}\kappa^{--}$

