



**I FOUND THE HUGS BISON.**

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# BSM Higgs Sectors, triple Higgs couplings and Gravitational Waves

*Sven Heinemeyer, IFT (CSIC, Madrid)*

Daejeon, 06/2023

1. Introduction
2. FOEWPT and GWs in the 2HDM
3. THCs in the 2HDM at the HL-LHC and the ILC
4. Bonus track: Discovery of the second Higgs boson
5. Conclusions

# 1. Introduction

## 1. Introduction

**HIGGS 2013**



**Gravitational  
Waves 2017**



## 1. Introduction

**HIGGS 2013**

**Gravitational  
Waves 2017**



⇒ Why is there more matter than antimatter? ⇒ (EW) baryogenesis

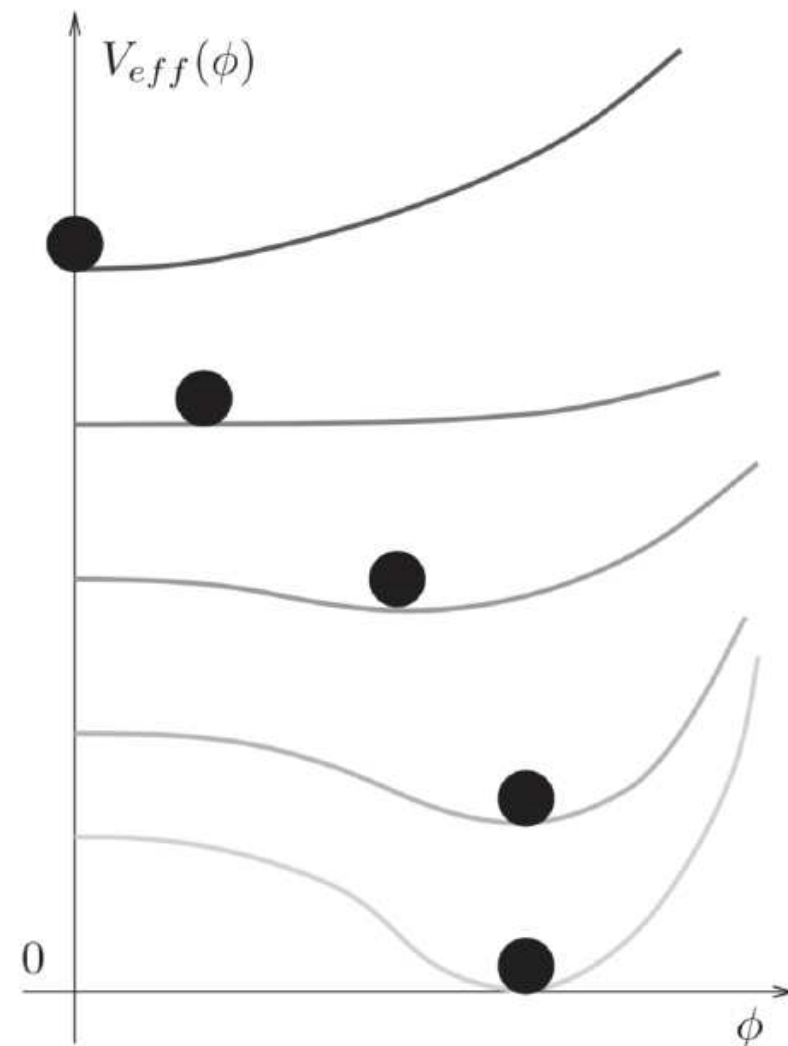
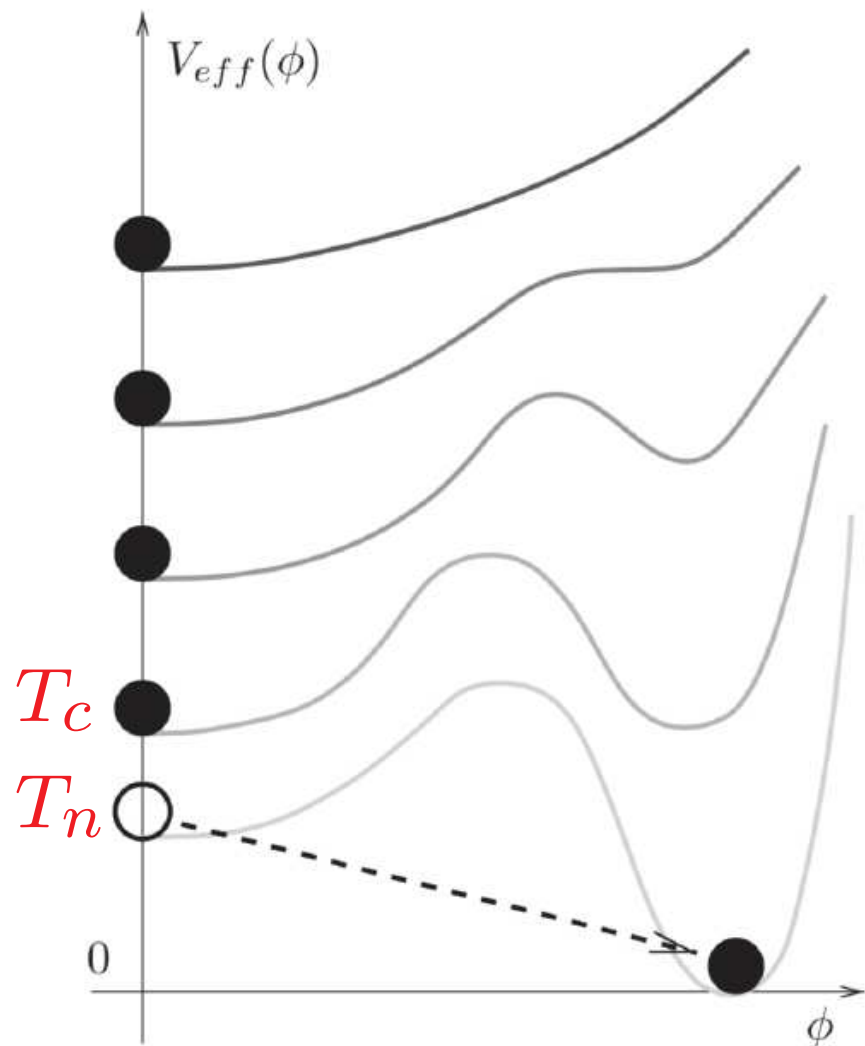
⇒ requires First Order EW Phase Transition (FOEWPT)

FOEWPT not possible in the SM ⇒ BSM THC's required

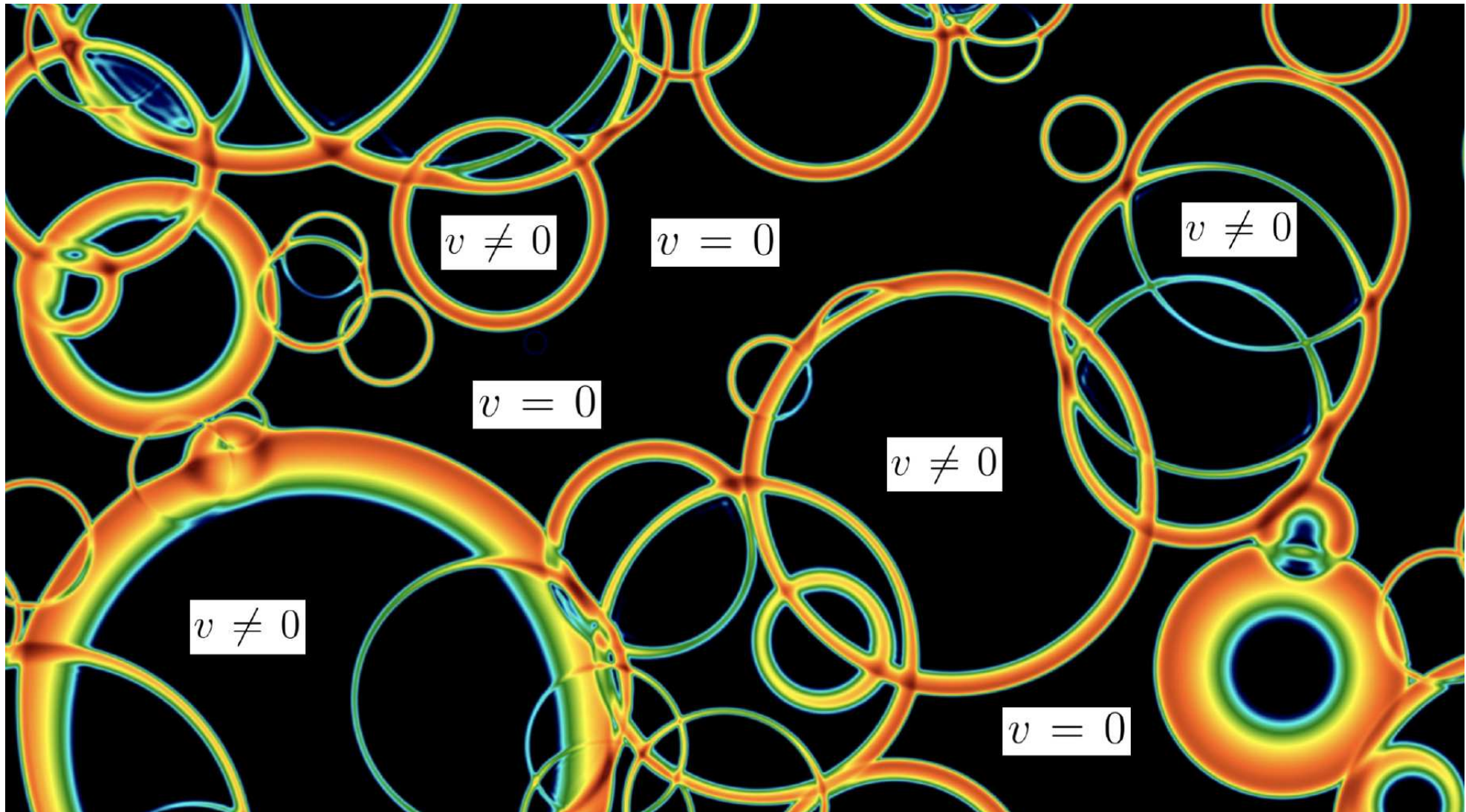
FOEWPT can cause Gravitational Waves (GW), detectable with LISA, ...

## Phase transition: BSM vs. SM

[taken from V. A. Rubakov and D. S. Gorbunov]



⇒ BSM Higgs sector required to realized FOEWPT



⇒ Can this happen in the 2HDM? Implications for THC's?

## 2. FOEWPT and GWs in the 2HDM

Two Higgs Doublet Model (2HDM):

Fields:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix}$$

Potential:

$$\begin{aligned} V = & m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c.) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + h.c.] \end{aligned}$$

Physical states:  $h$ ,  $H$ , ( $\mathcal{CP}$ -even),  $A$  ( $\mathcal{CP}$ -odd),  $H^\pm$  (charged)

“Physical” input parameters:

$$c_{\beta-\alpha}, \quad \tan \beta, \quad v, \quad M_h, \quad M_H, \quad M_A, \quad M_{H^\pm}, \quad m_{12}^2$$

Alignment limit:  $c_{\beta-\alpha} \rightarrow 0$  (for  $M_h \sim 125$  GeV)

Many triple Higgs couplings:  $\lambda_{hhh}, \lambda_{hhH}, \lambda_{hHH}, \lambda_{hH+H-}, \lambda_{HAA}, \dots$



Assumption:  $h \sim h_{125}$

$Z_2$  symmetry to avoid FCNC:

$$\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2$$

Extension of the  $Z_2$  symmetry to fermions determines four types:

|                            | $u$ -type | $d$ -type | leptons  |                         |
|----------------------------|-----------|-----------|----------|-------------------------|
| type I                     | $\Phi_2$  | $\Phi_2$  | $\Phi_2$ |                         |
| type II                    | $\Phi_2$  | $\Phi_1$  | $\Phi_1$ | $\rightarrow$ SUSY type |
| type III (lepton-specific) | $\Phi_2$  | $\Phi_2$  | $\Phi_1$ |                         |
| type IV (flipped)          | $\Phi_2$  | $\Phi_1$  | $\Phi_2$ |                         |

Sum rule (with  $h$  SM-like):  $\sin(\beta - \alpha) \approx 1, \cos(\beta - \alpha) \approx 0$

Unitarity/perturbativity and EWPO :  $\Rightarrow M_A \sim M_H \sim M_{H^\pm}$

$\Rightarrow$  Parameter scan in the 2HDM type II  $\Rightarrow$  ScannerS

$$\tan \beta = 3, c_{\beta-\alpha} = 0, m_{12}^2 = m_H^2 s_\beta c_\beta$$

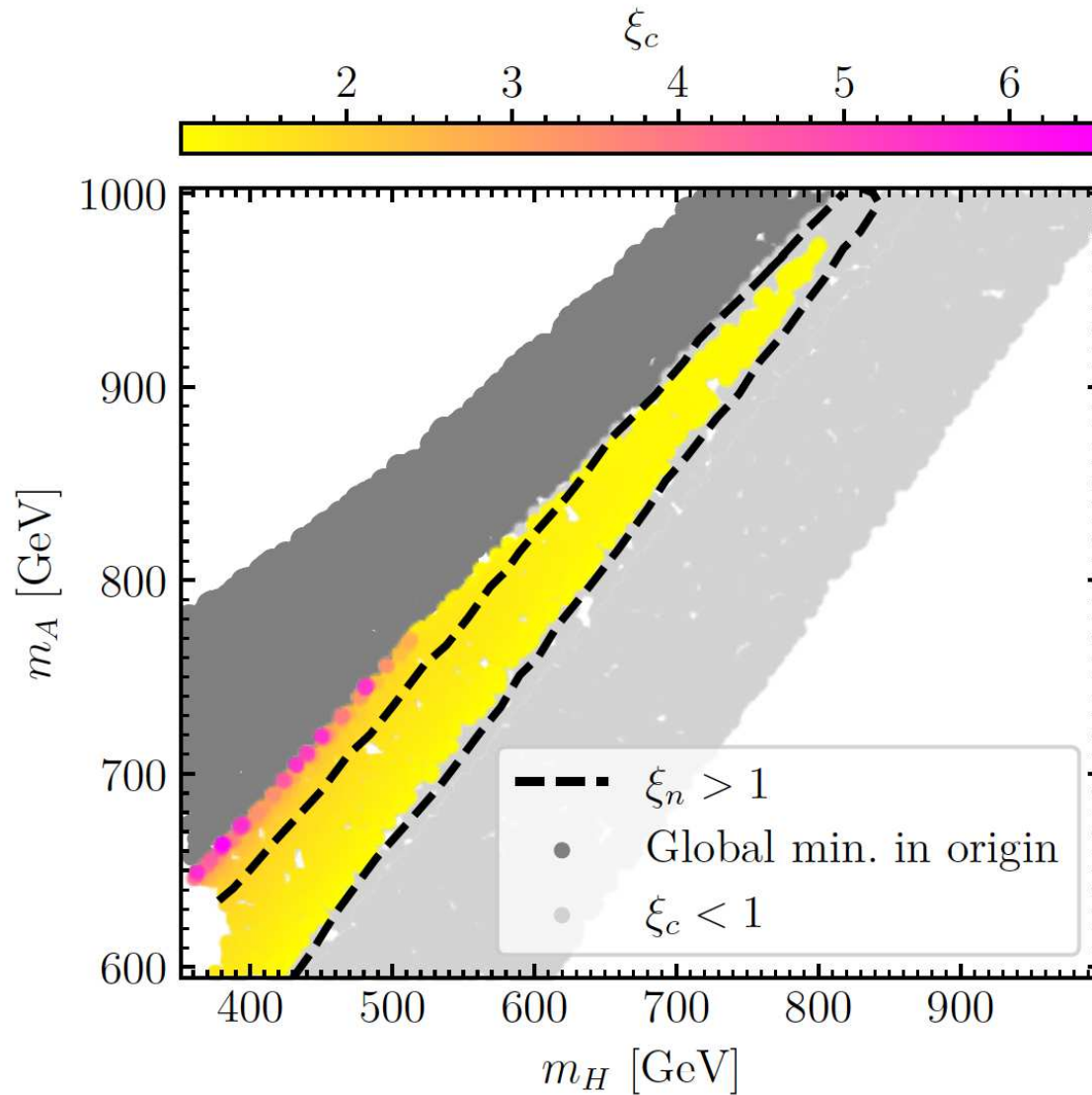
$$0.2 \text{ TeV} \leq m_H \leq 1 \text{ TeV}, 0.6 \text{ TeV} \leq m_A = m_{H^\pm} \leq 1.2 \text{ TeV}$$

Constraints:

- Tree-level perturbativity  $\Rightarrow$  ScannerS
- Minimum of potential is global minimum  $\Rightarrow$  ScannerS  
... or sufficiently long-lived  $\Rightarrow$  Evade
- Higgs searches at LEP, Tevatron, LHC  $\Rightarrow$  HiggsBounds
- SM-like Higgs properties  $\Rightarrow$  HiggsSignals (2HDECAY, SusHi)  
 $\Rightarrow \chi_{125}^2$  (with  $\chi_{\text{SM},125}^2 = 84.4$ )
- Flavor physics (mainly  $\text{BR}(B_s \rightarrow X_s \gamma)$ ,  $\Delta M_{B_s}$ )  $\Rightarrow$  SuperIso bounds
- Electroweak precision data ( $T$  and  $S$ )  $\Rightarrow$  ScannerS

## GWs in the 2HDM: $\xi_c := v_c/T_c \gtrsim 1$

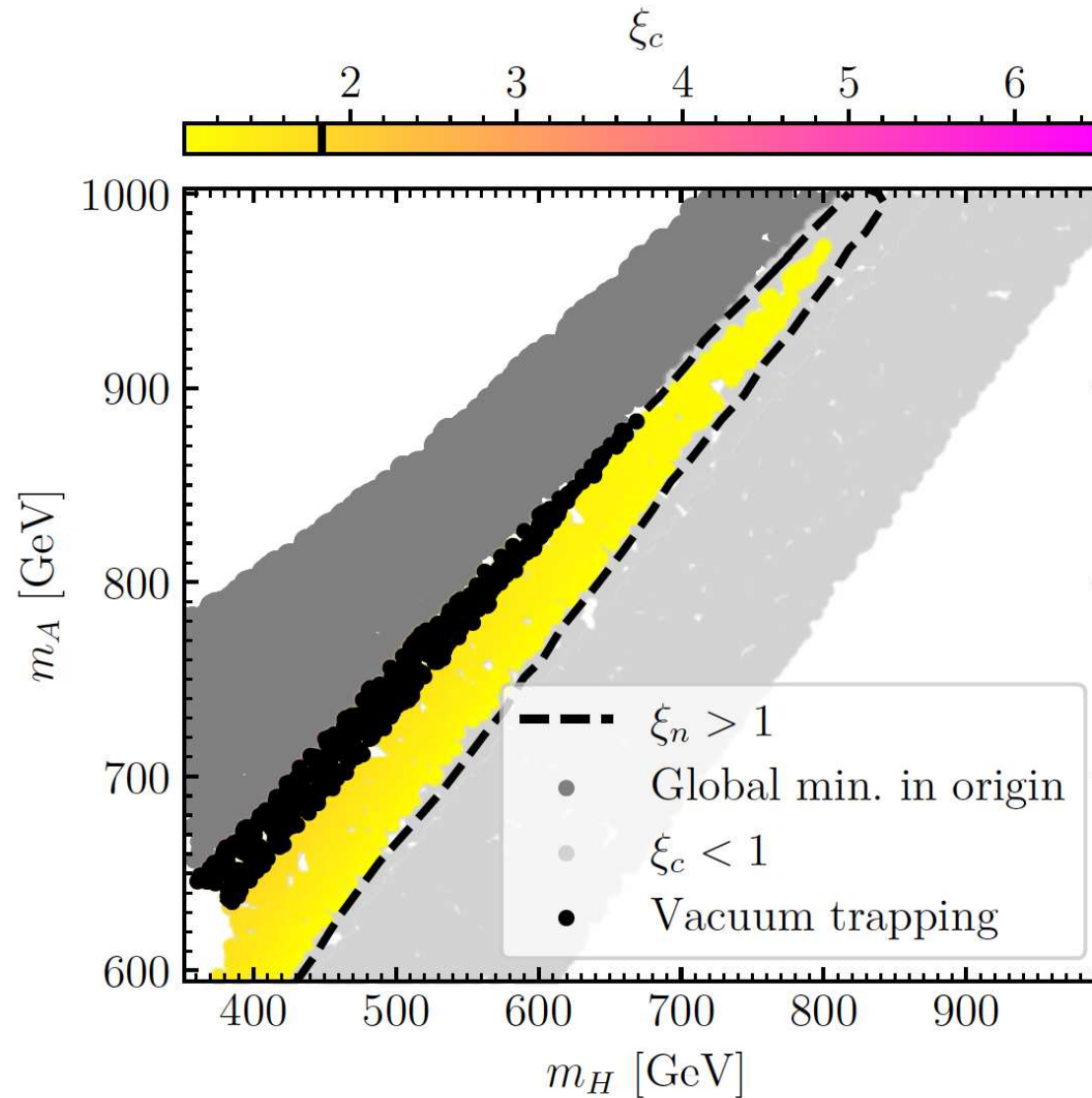
[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]



$\Rightarrow$  large  $\xi_c$  found in the 2HDM  $\Rightarrow$  strong GW signal?

## GWs in the 2HDM: $\xi_n := v_n/T_n \gtrsim 1$

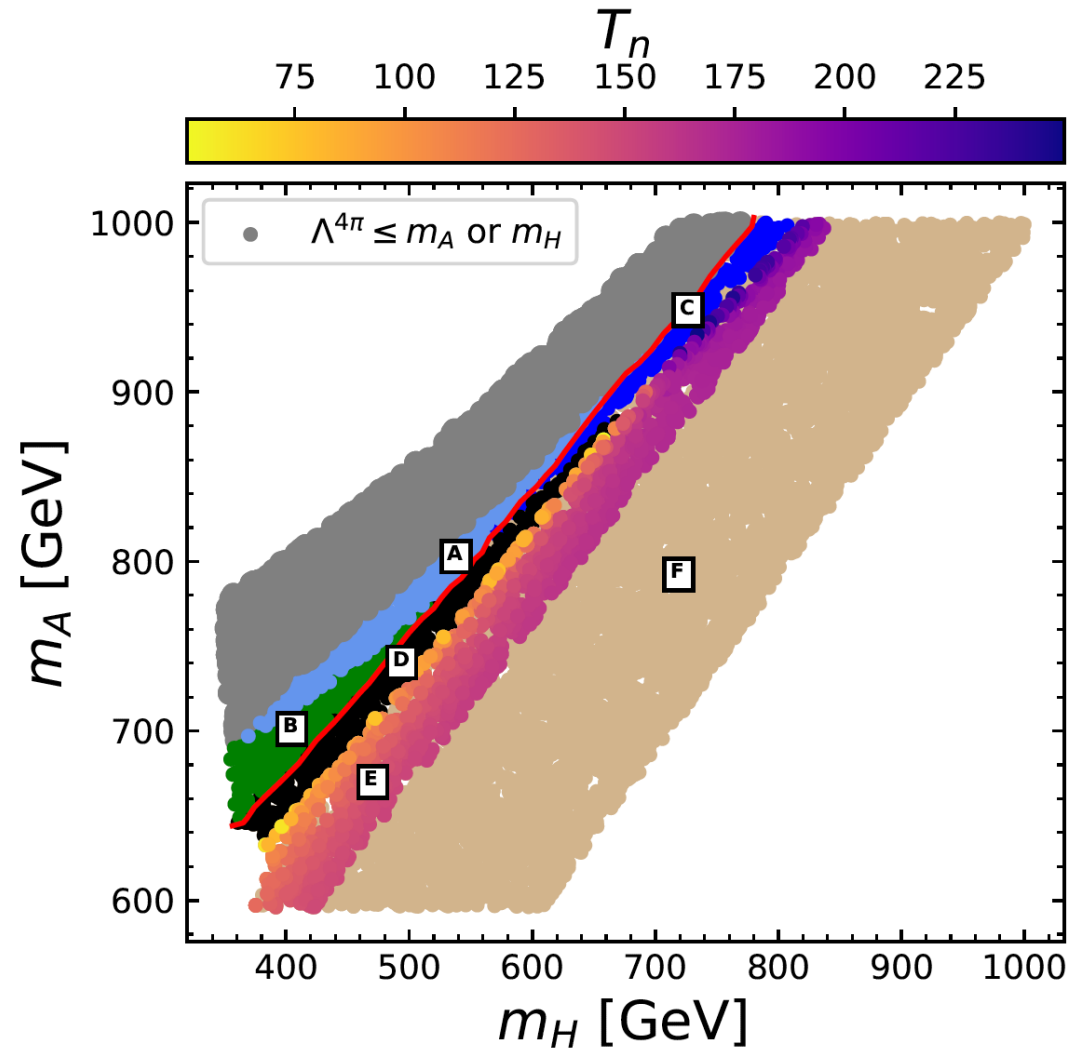
[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]



⇒ potentially strongest GW signal: forbidden by vacuum trapping

## Six thermal histories in the 2HDM:

[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]

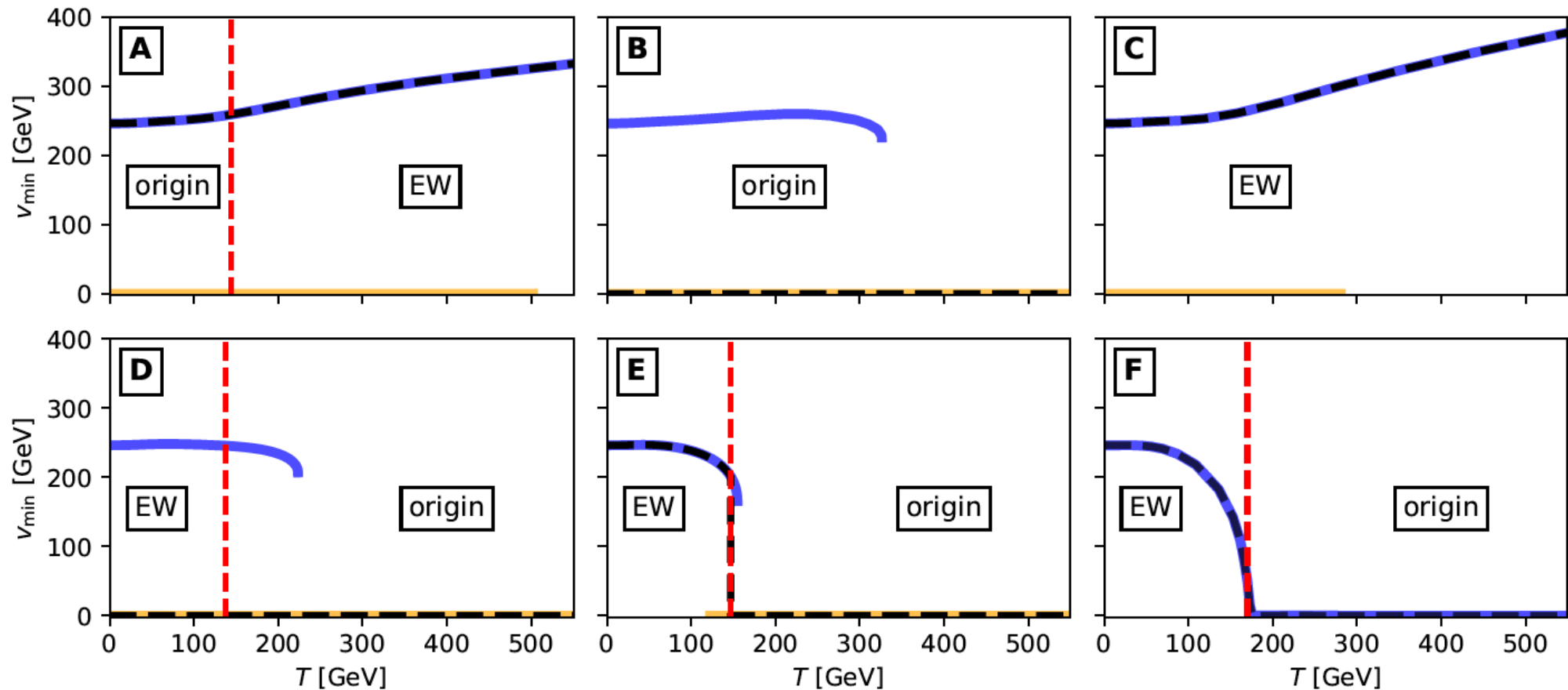


**E:** viable for **FOEWPT**, **GWs** are induced (detectable?)

**F:** no FOEWPT, no GWs are induced

## Six thermal histories in the 2HDM:

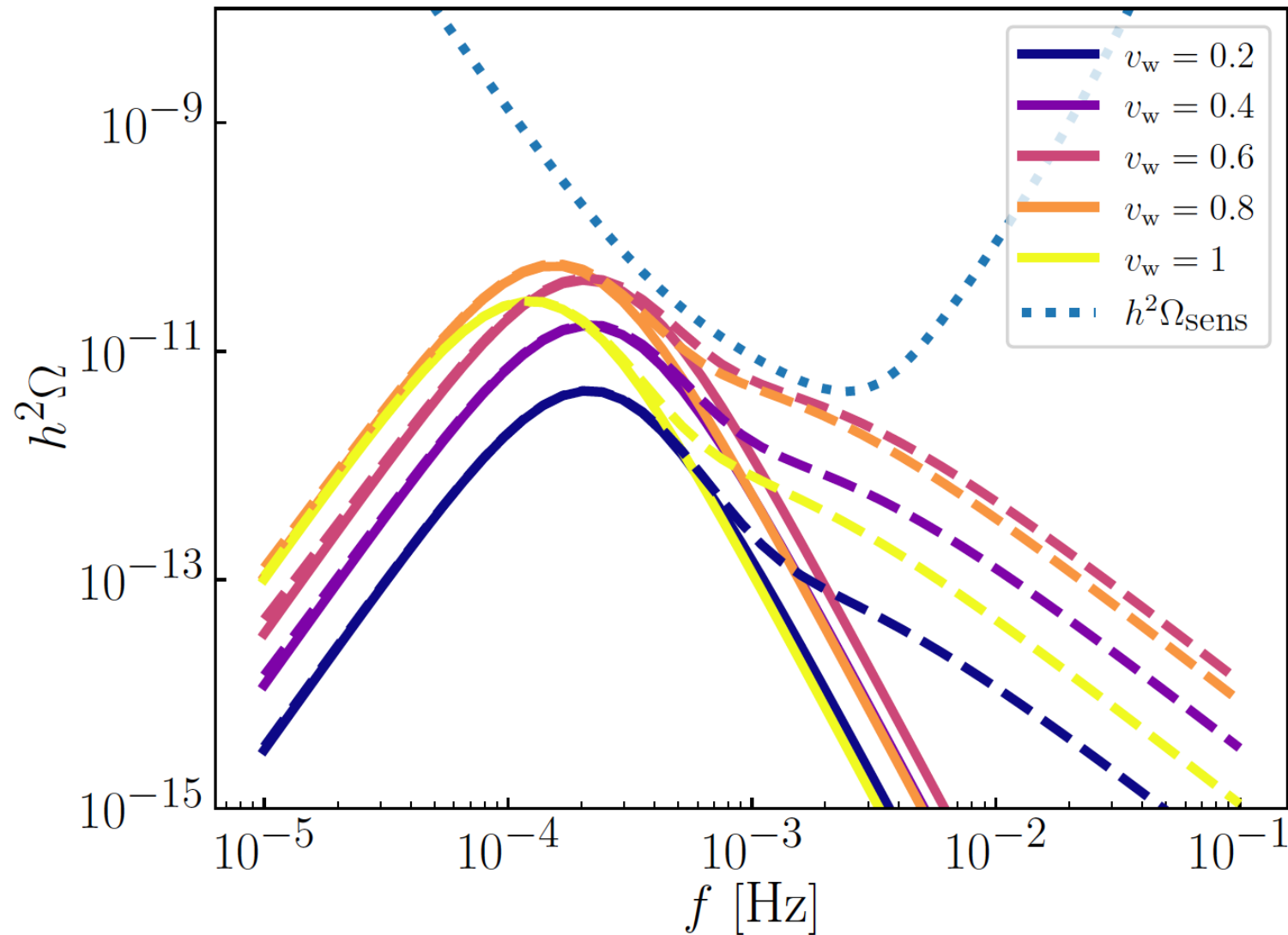
[*T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22*]



⇒ Zone E preferred by phenomenology/FOEWPT

## GWs vs. LISA: ( $m_H = 419$ GeV, $m_A = m_{H^\pm} = 663$ GeV)

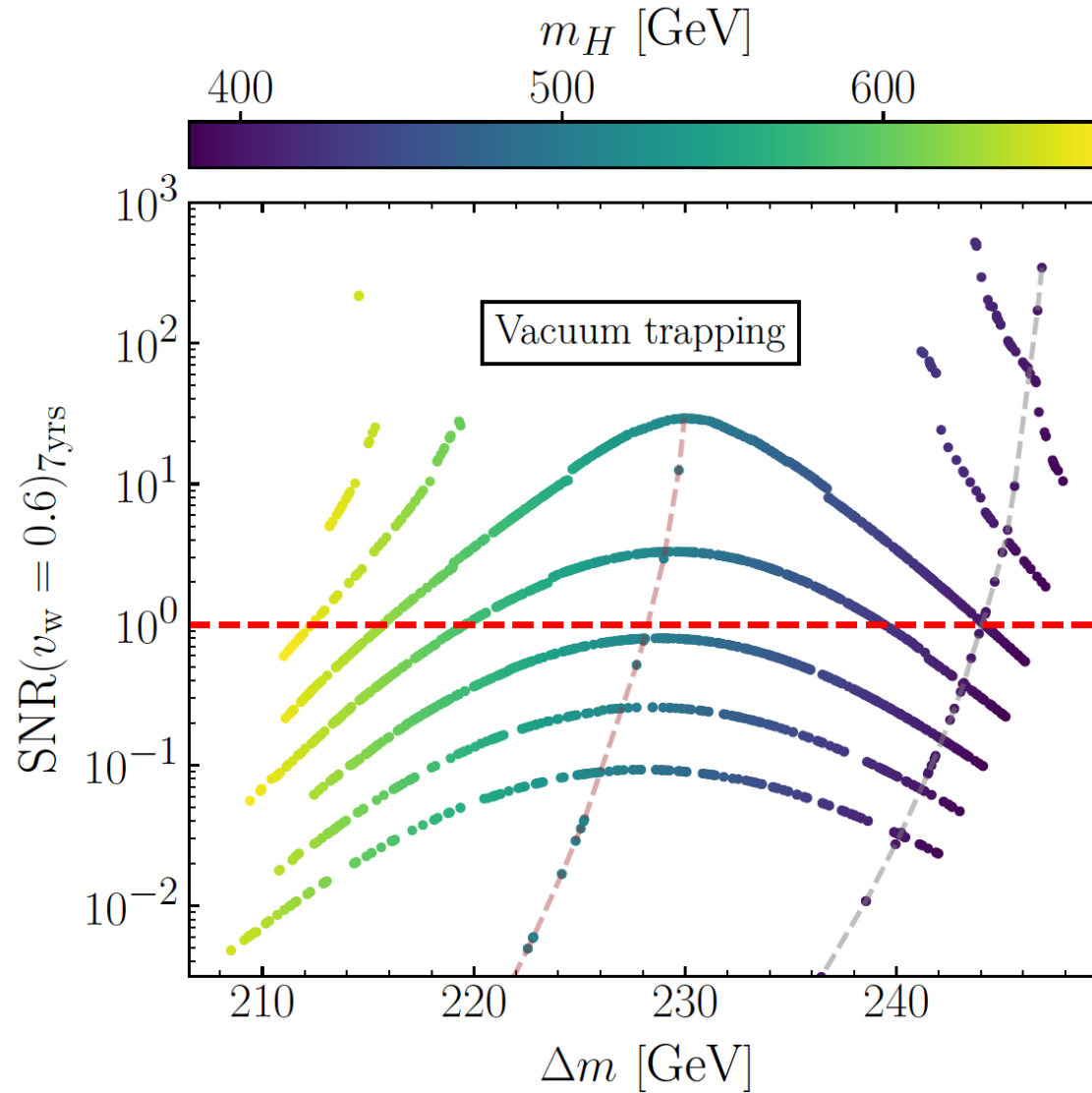
[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]



⇒ bubble wall velocity and turbulence important

## GWs vs. LISA: ( $v_w = 0.6$ , 7 years of LISA data)

[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]

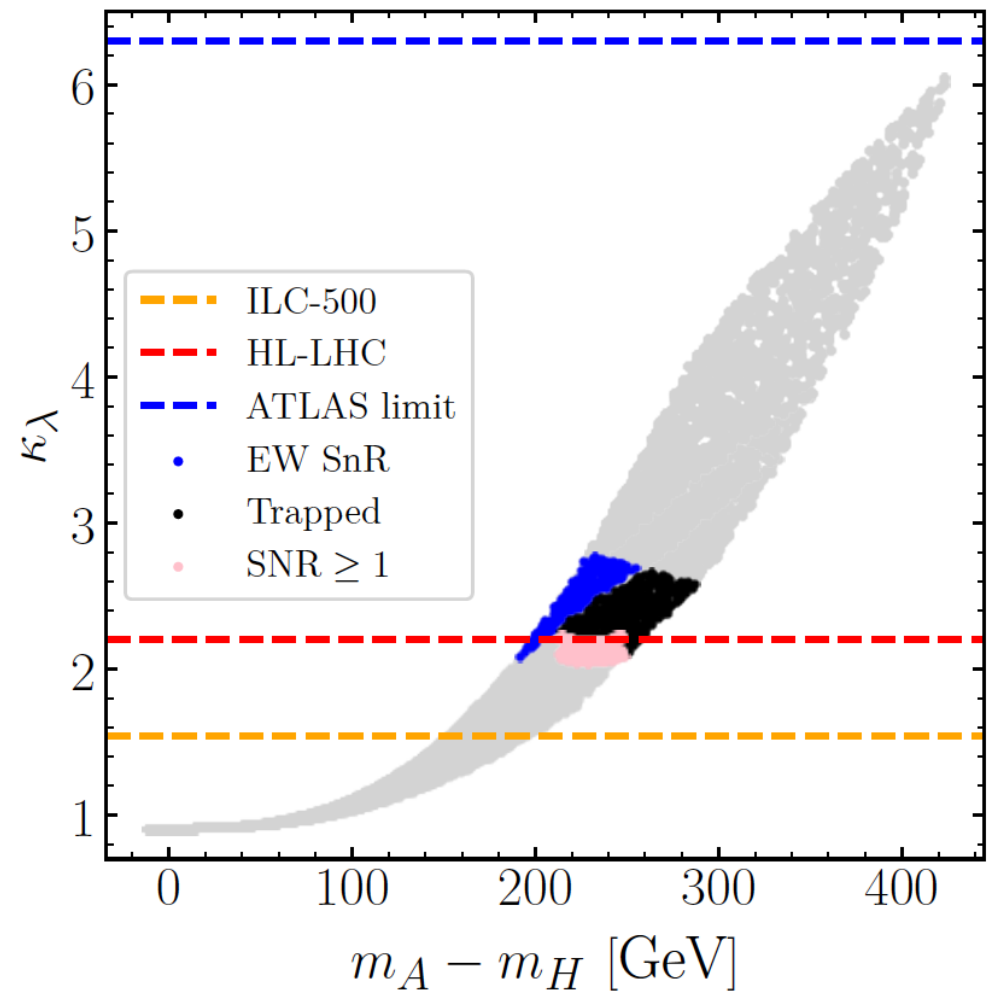
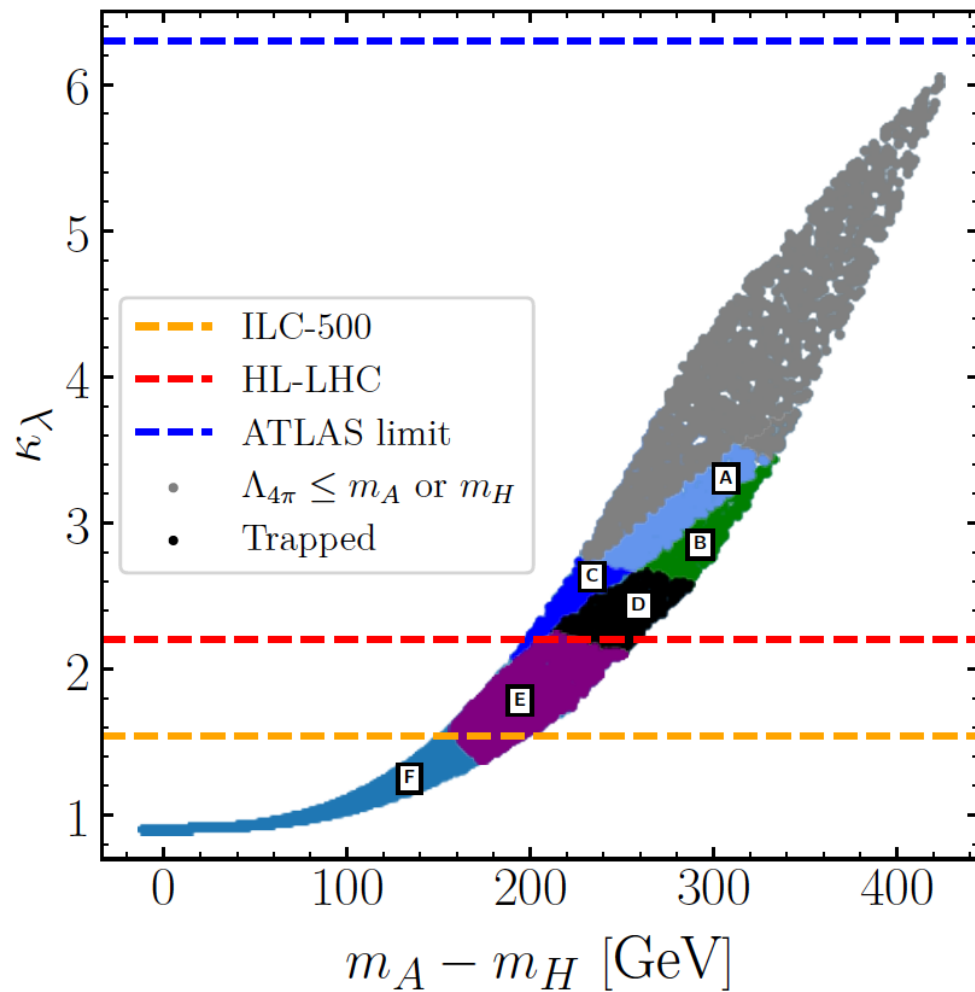


⇒ detectable GWs only in a very small zone close to VT



## 2HDM parameter scan to yield FOEWPT:

[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]

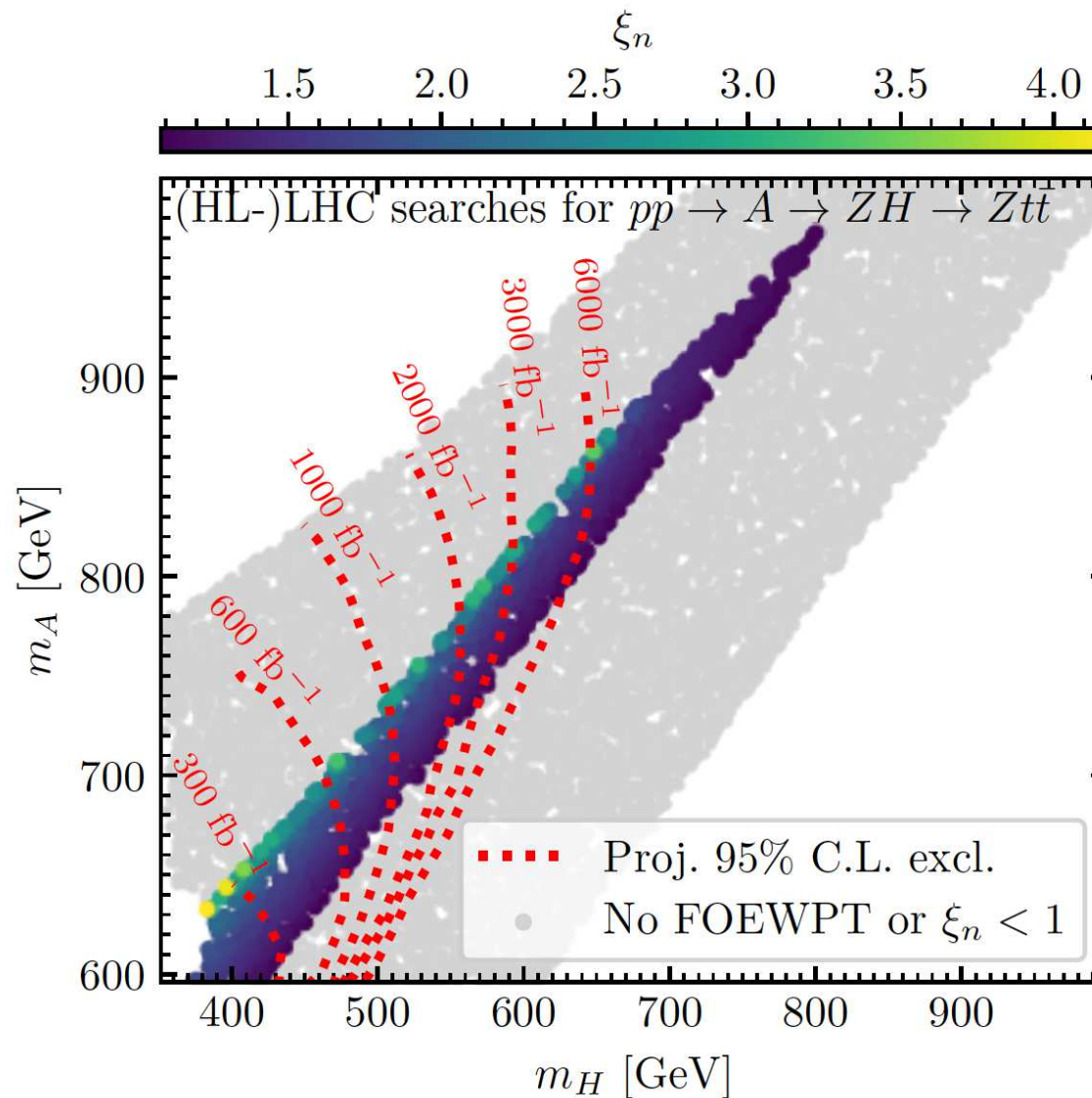


$\Rightarrow$  FOEWPT requires  $\kappa_\lambda \lesssim 2$

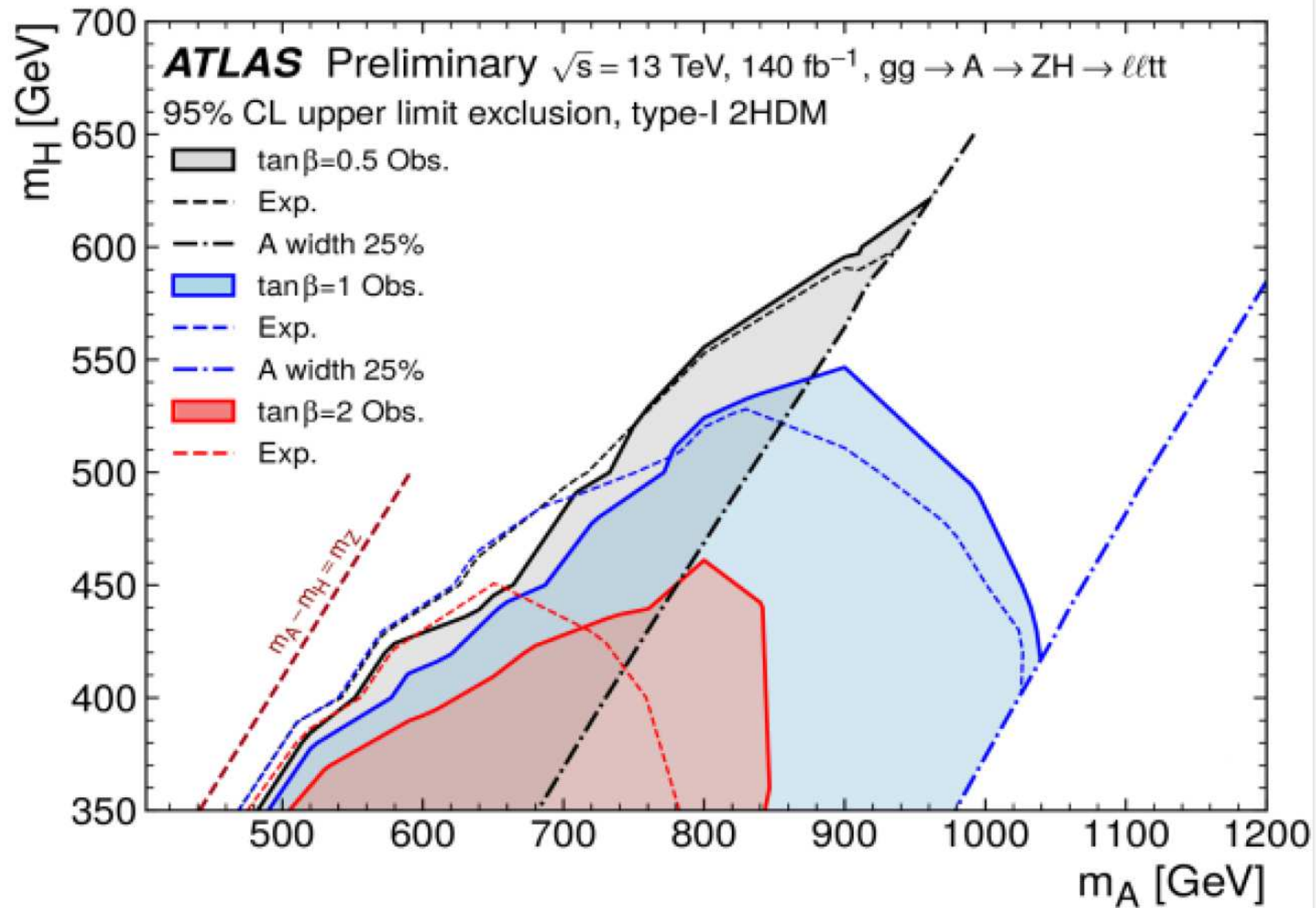
$\Rightarrow$  GW signal requires  $\kappa_\lambda \sim 2$

## Smoking gun signature: gap between $m_A$ and $m_H$

[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]



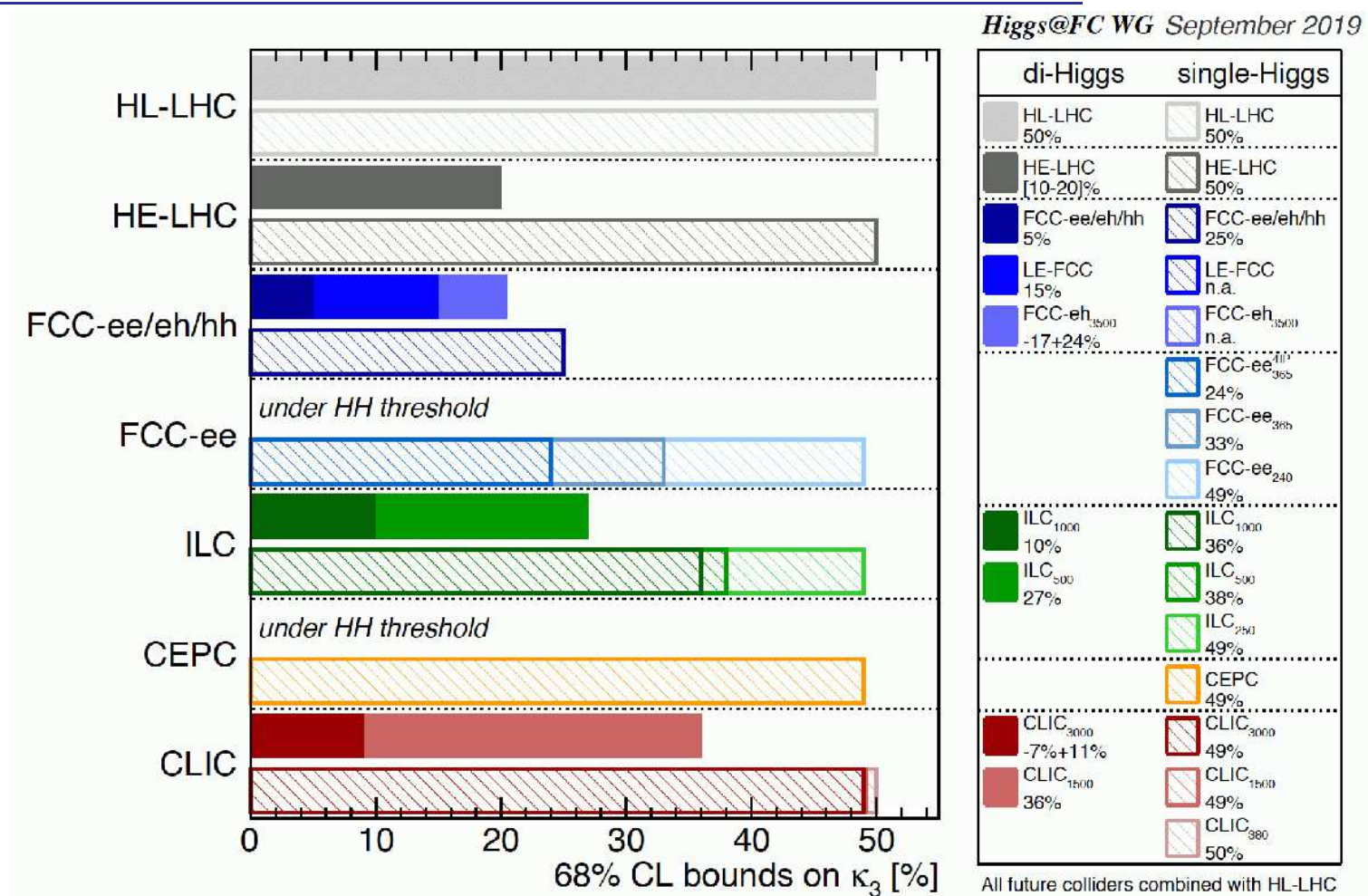
⇒ GW zone can be covered at the HL-LHC



$\Rightarrow$  interesting excess in the “right spot” :- ) ( $m_H = 420 \text{ GeV}$ ,  $m_A = 650 \text{ GeV}$ )

### 3. THCs in the 2HDM at the HL-LHC and the ILC

SM triple Higgs coupling: comparison of all colliders:

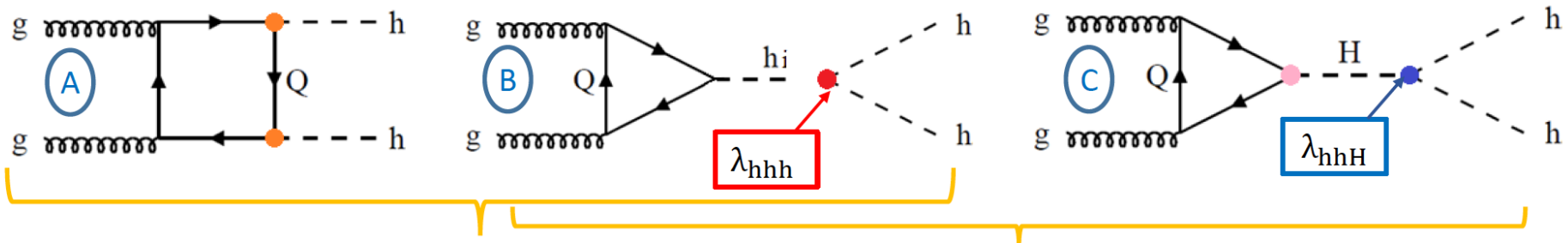


⇒ focus on “SM triple Higgs coupling”,  $\kappa_\lambda := \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$

BSM case 1:  $\kappa_\lambda \neq 1$

BSM case 2: THC that involves BSM Higgses:  $\lambda_{hhH}, \dots$

## Di-Higgs production at the LHC:

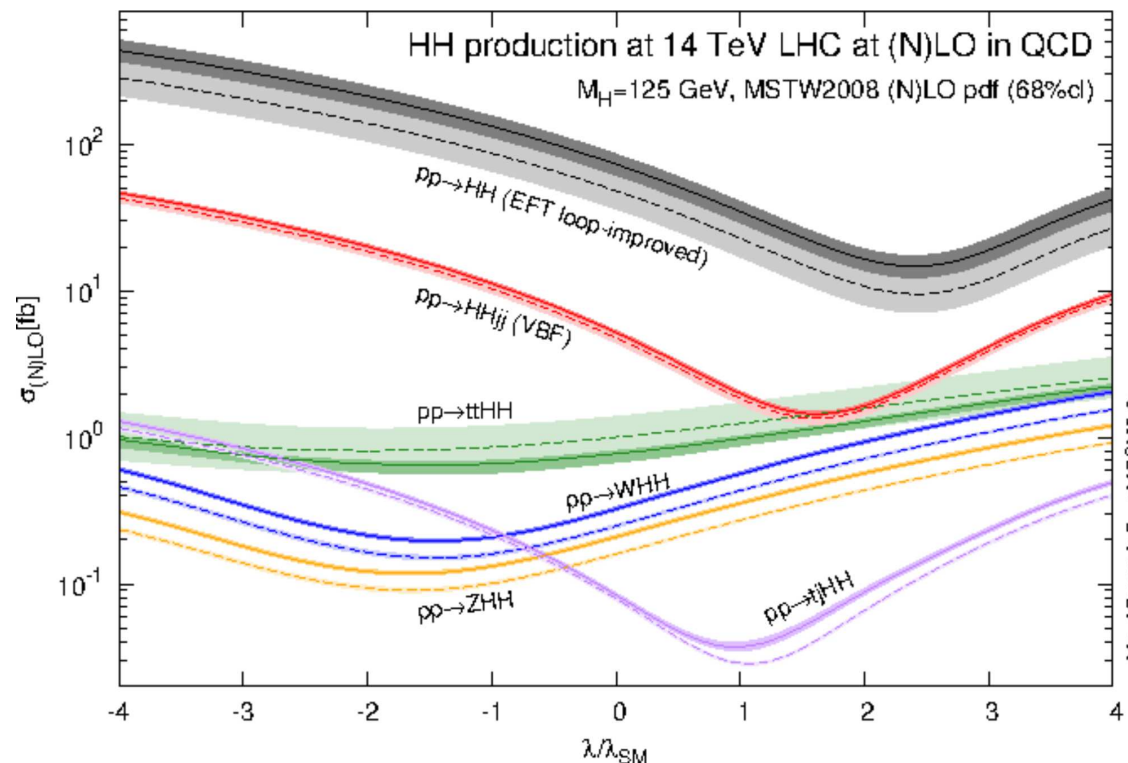


$\sigma_{\text{SM}} \sim 38 \text{ fb at NLO}$

Diagrams that exist in the SM:  
They have a negative interference

Diagrams that are sensitive to triple Higgs couplings

⇒ strong interference of “box” and “SM-like Higgs”

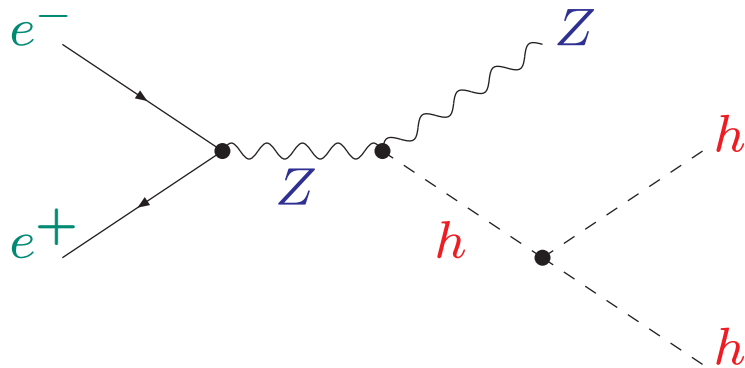




## Di-Higgs production at ILC/CLIC:

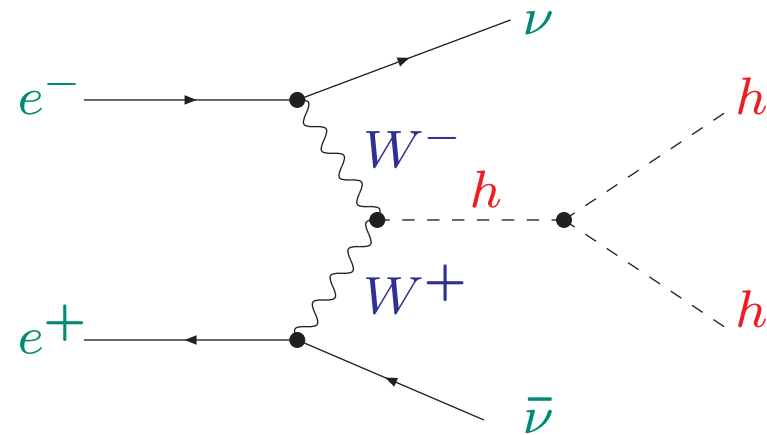
Higgs-strahlung:

$$e^+e^- \rightarrow Z^* \rightarrow Zh h$$

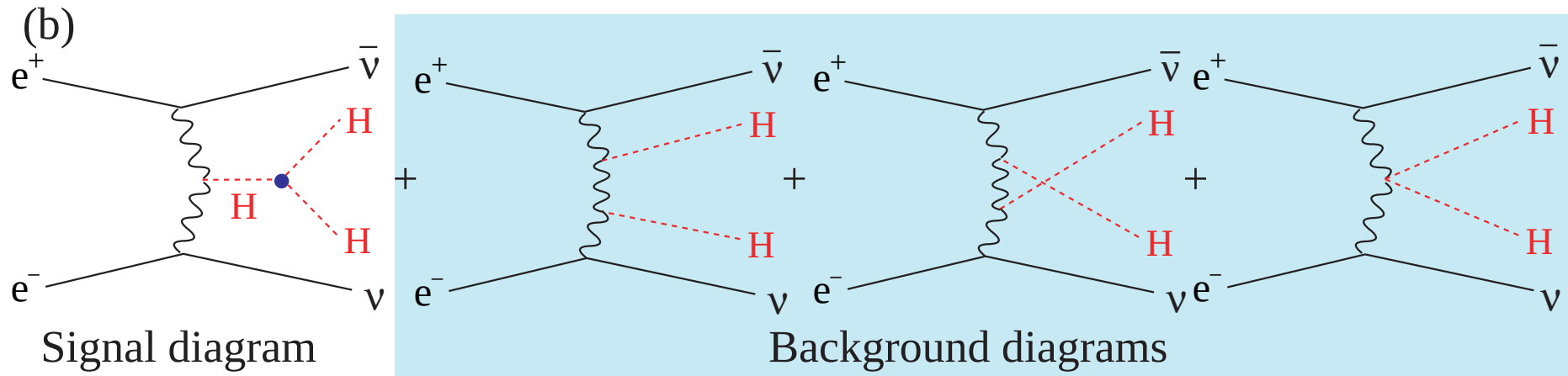


weak boson fusion (WBF):

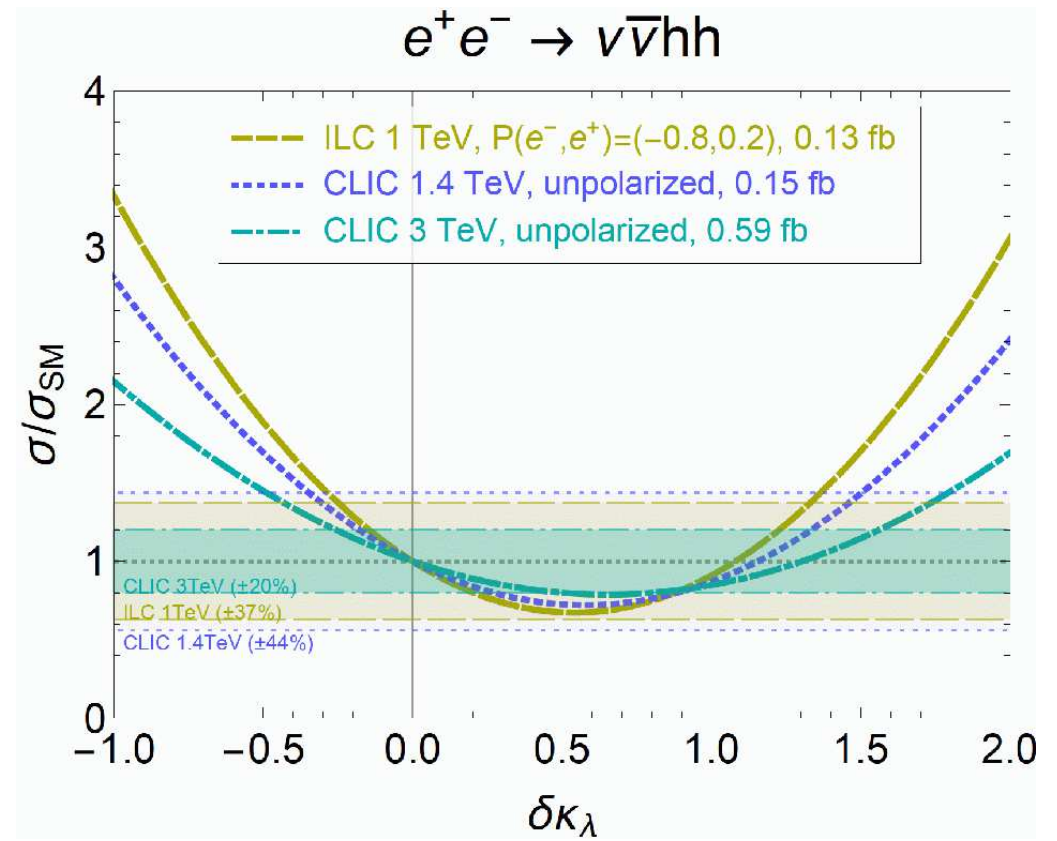
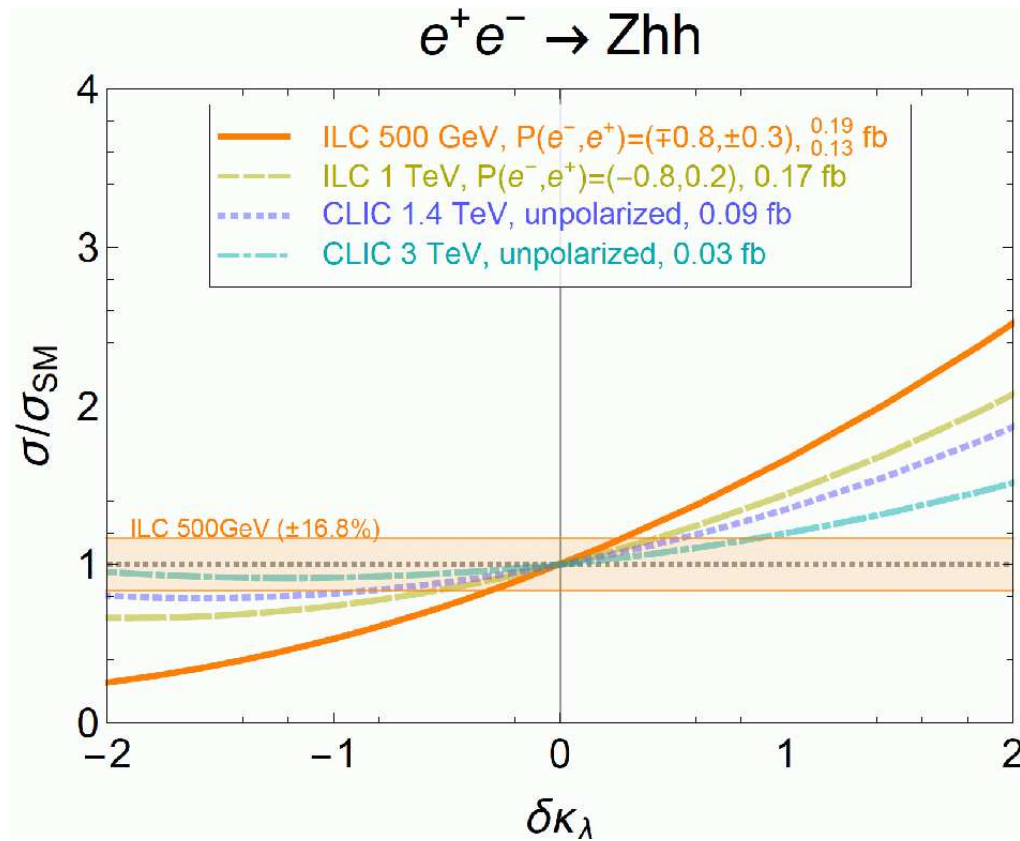
$$e^+e^- \rightarrow \nu\bar{\nu}hh$$



Signal and background interference:



## Di-Higgs production at ILC/CLIC:

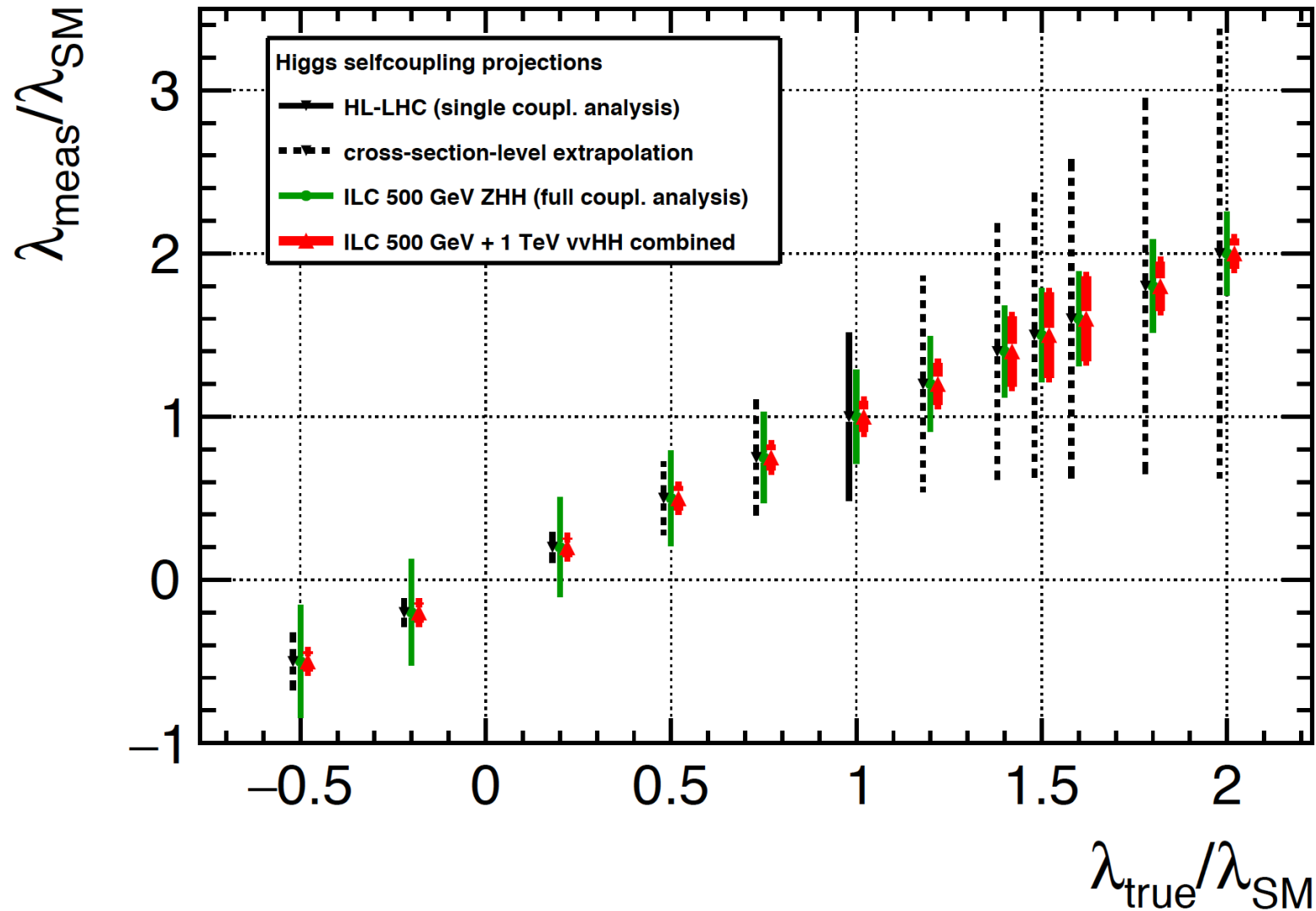


$$\kappa_\lambda := 1 + \delta\kappa_\lambda$$

⇒ strong and different dependence on  $\kappa_\lambda$

# Measurement of $\kappa_\lambda$ selfcoupling at HL-LHC/ILC:

[J. List et al. – PRELIMINARY]

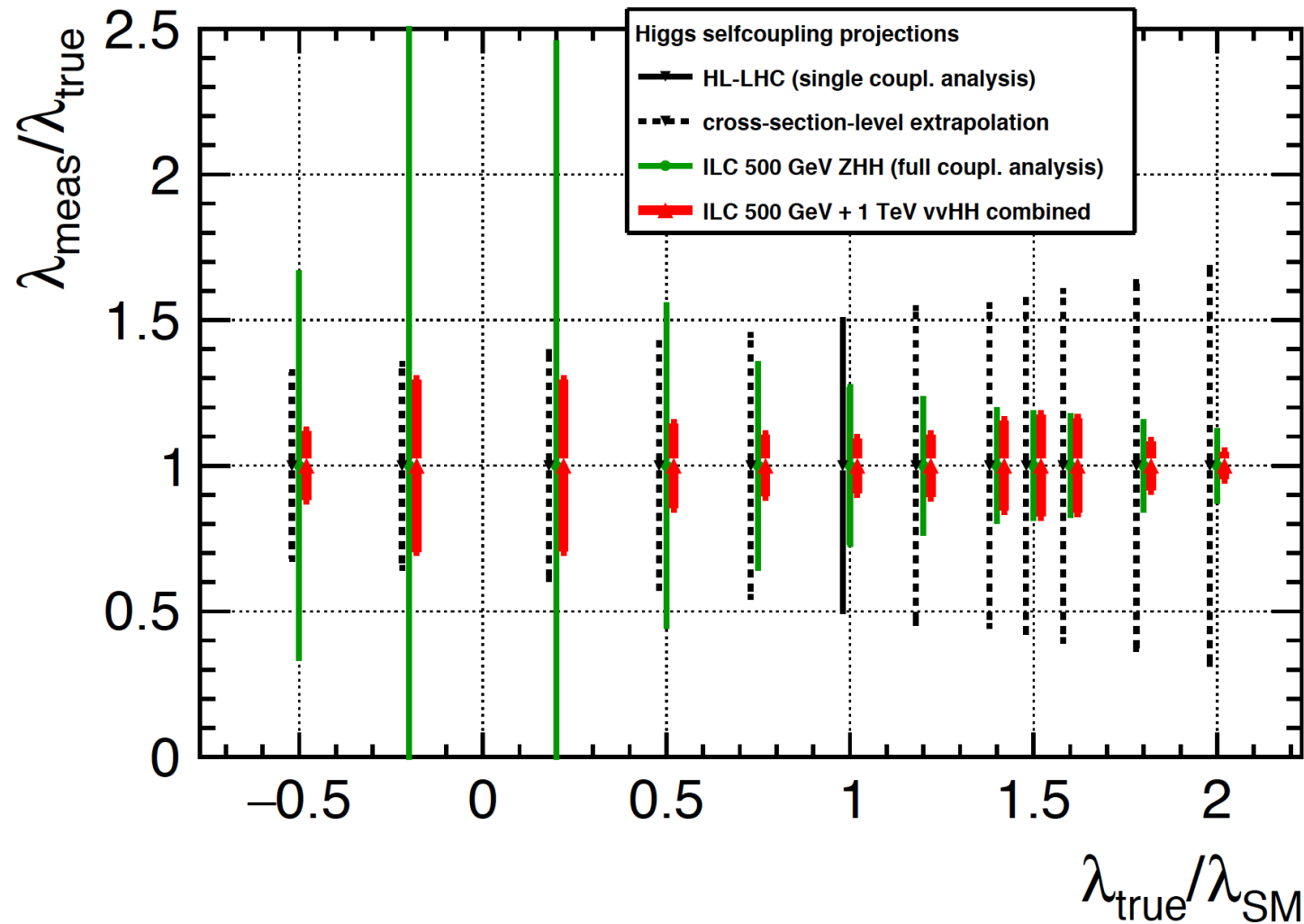


FOEWPT/GW:  $\lambda_{hhh} \lesssim 2 \Rightarrow$  bad for HL-LHC, good for ILC



## Measurement of $\kappa_\lambda$ selfcoupling at HL-LHC/ILC:

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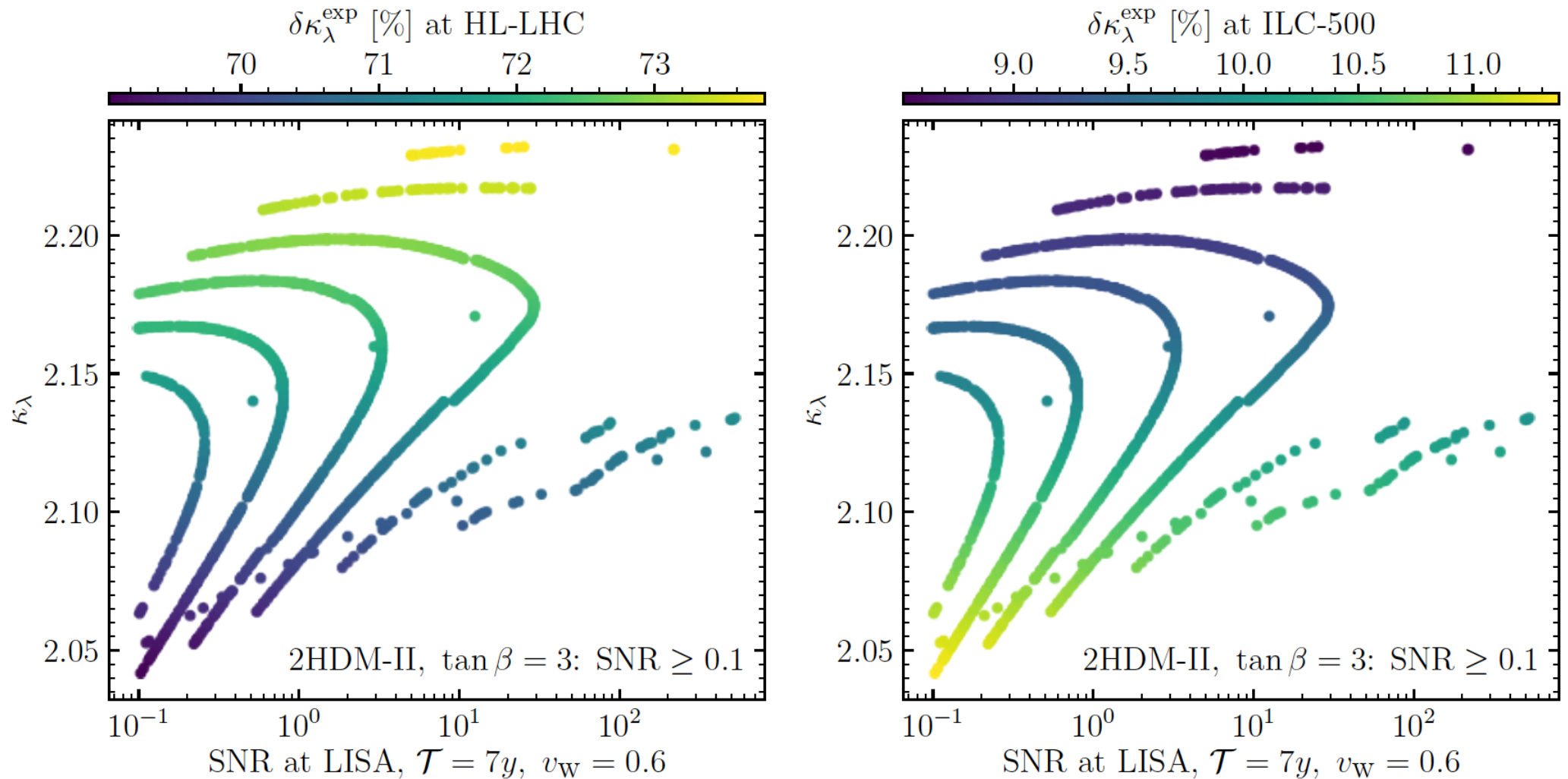


⇒ over most of the parameter space ILC is clearly superior to HL-LHC

## Example: 2HDM $\Rightarrow$ FOEWPT $\Rightarrow$ GW's

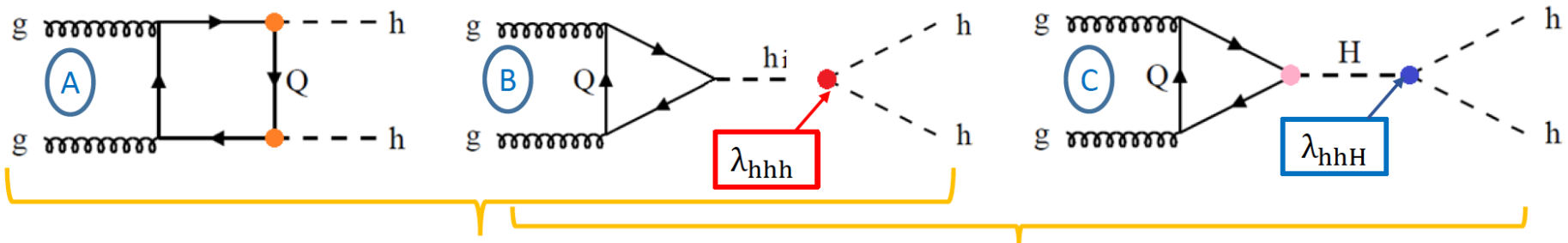
[T. Biekötter, S.H., J. No, O. Olea, G. Weiglein '22]

$\Rightarrow$  Synergies: collider:  $\lambda_{hhh}$   $\Leftrightarrow$  LISA: GW signals



$\Rightarrow$  FOEWPT requires large  $\lambda_{hhh}$  and can induce GW signals

## BSM THCs at the HL-LHC and ILC



$\sigma_{\text{SM}} \sim 38 \text{ fb at NLO}$

Diagrams that exist in the SM:  
They have a negative interference

Diagrams that are sensitive  
to triple Higgs couplings

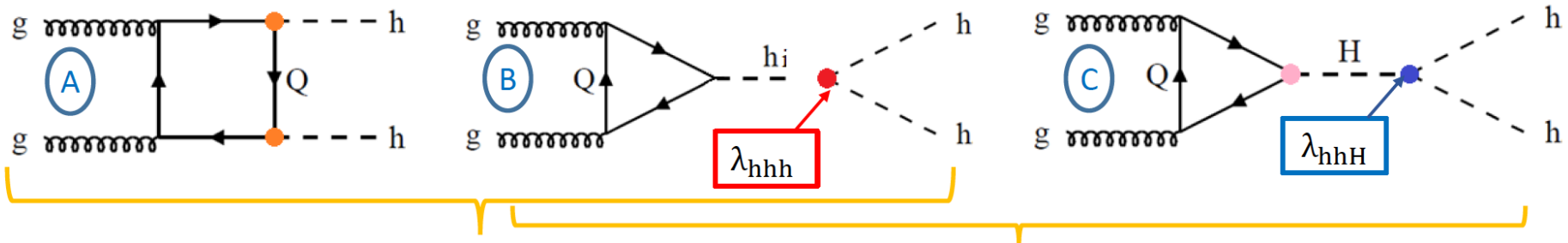
⇒ possible strong resonance with BSM Higgs

Sideremark: experimental limits are obtained for

- non-resonant production
- purely resonant production

⇒ no limits available for mixed scenarios :-)

# BSM THCs at the HL-LHC and ILC



$\sigma_{\text{SM}} \sim 38 \text{ fb at NLO}$

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They have a negative interference

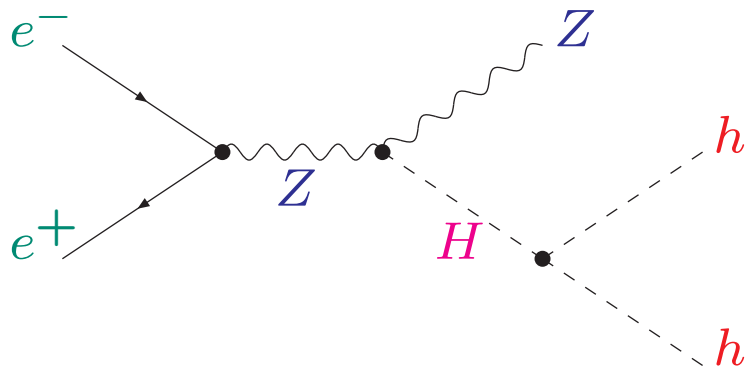
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⇒ possible strong resonance with BSM Higgs

## ILC/CLIC:

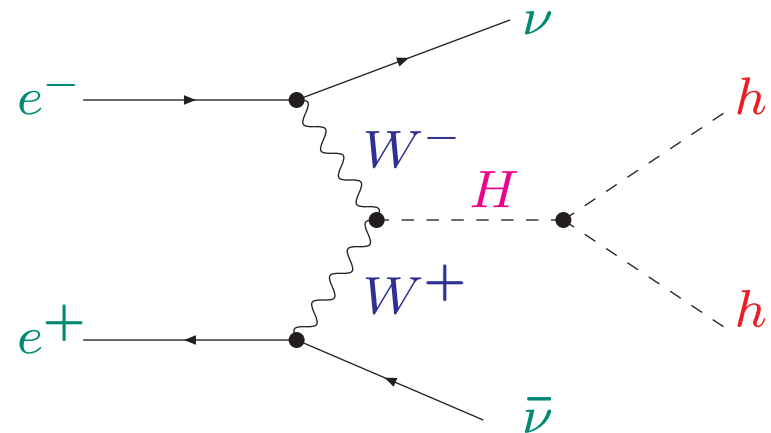
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weak boson fusion (WBF):

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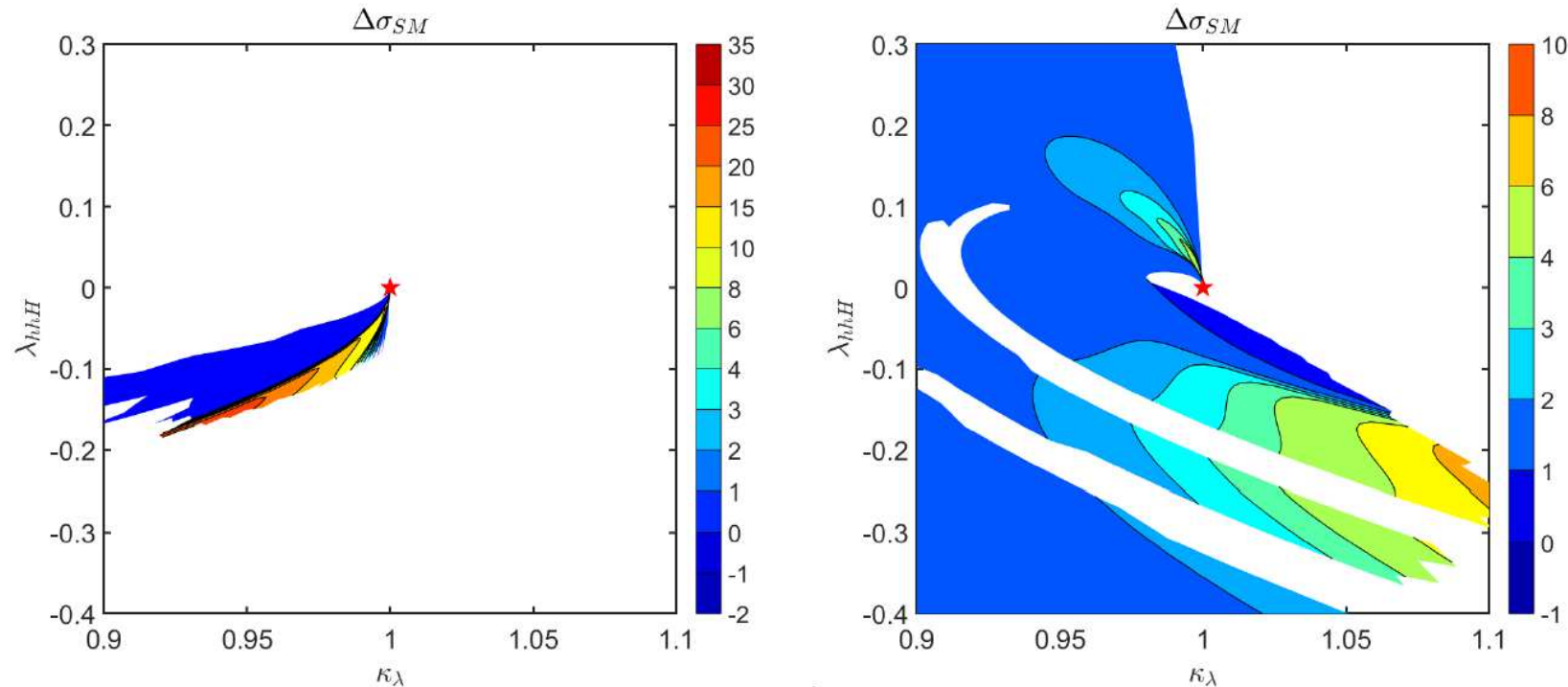


Di-Higgs production at the HL-LHC: [F. Arco, S.H., M. Mühlleitner, K. Radchenko '22]

$gg \rightarrow hh$  in the 2HDM with **HPAIR**

Benchmark plane: 2HDM type I,  $m_{12}^2 = (m_H^2 \cos^2 \alpha) / \tan \beta$ ,  $\tan \beta = 10$

$\Delta\sigma_{SM}$ : precision in  $\sigma$  (based on  $\kappa_\lambda = 1$ )



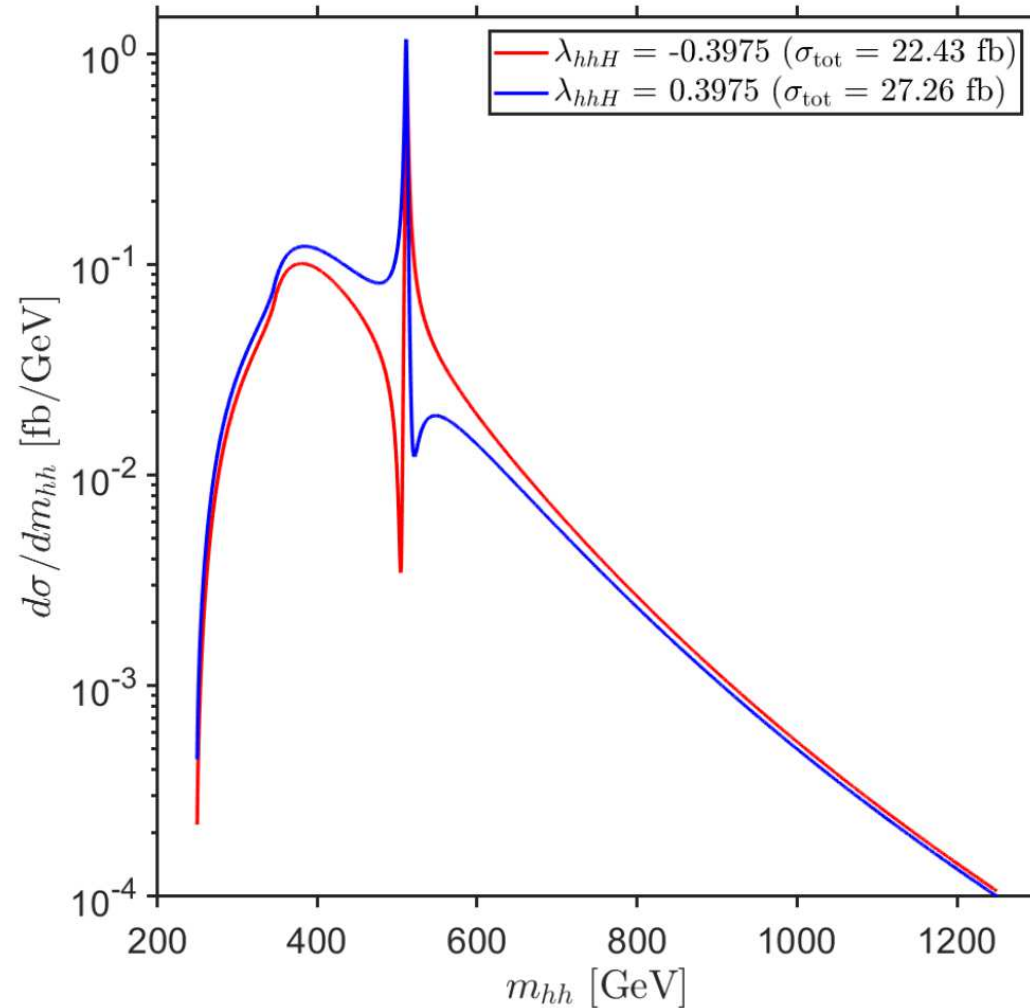
$\Rightarrow \kappa_\lambda \sim 1$ , no effect on di-Higgs cross section

$\Rightarrow$  strong variation with  $\lambda_{hhH}$  from resonant  $H$ -exchange

Di-Higgs production at the HL-LHC: [F. Arco, S.H., M. Mühlleitner, K. Radchenko '22]

Benchmark point: 2HDM type I,

$$m_{A,H^\pm} = 545 \text{ GeV}, m_H = 515 \text{ GeV}, t_\beta = 10, c_{\beta-\alpha} = 0.2, m_{12}^2 = m_H^2 c_\alpha^2 / t_\beta$$

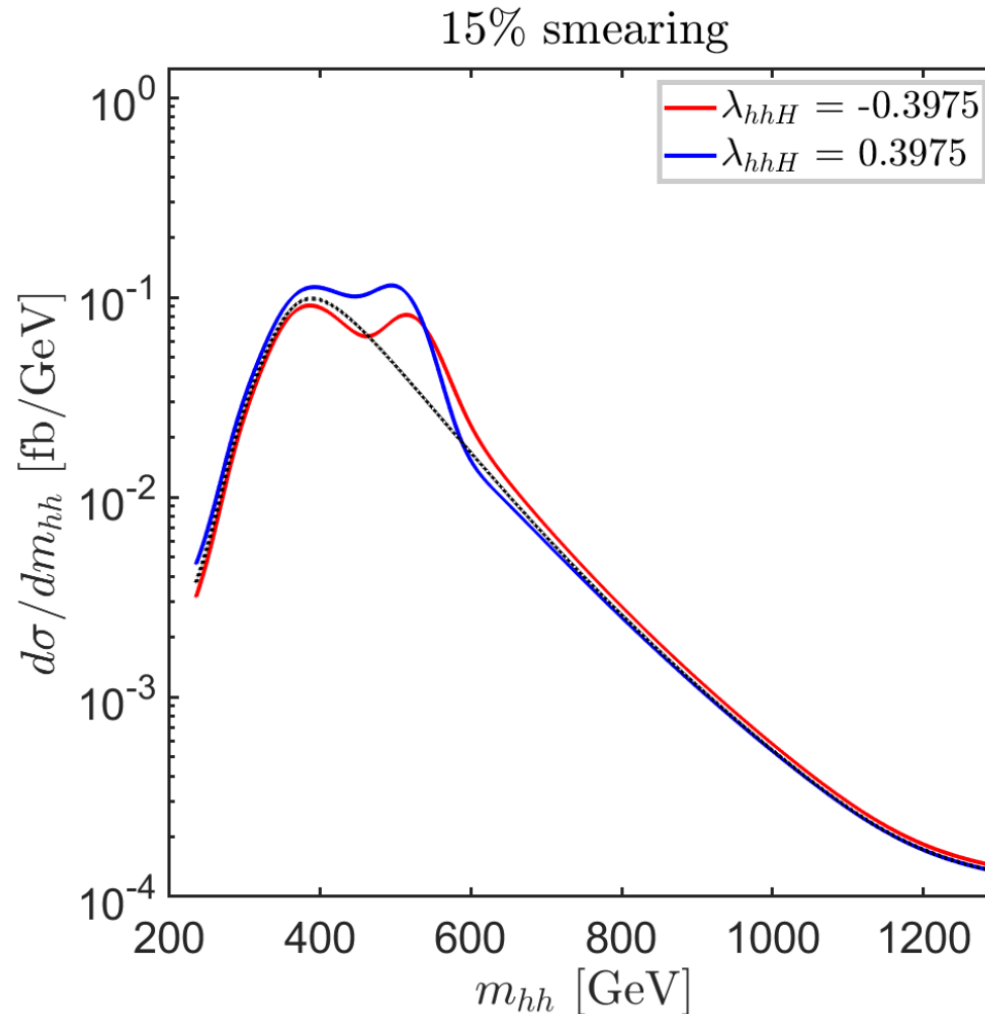


$\Rightarrow$  dip-peak / peak-dip from resonant  $H$ -exchange  $\Rightarrow$  access to  $\lambda_{hhH}$ ?

## Di-Higgs production at the HL-LHC: [F. Arco, S.H., M. Mühlleitner, K. Radchenko '22]

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$\Rightarrow$  smearing of 15% applied (optimistic?)  $\Rightarrow$  access to  $\lambda_{hhH}$ ?

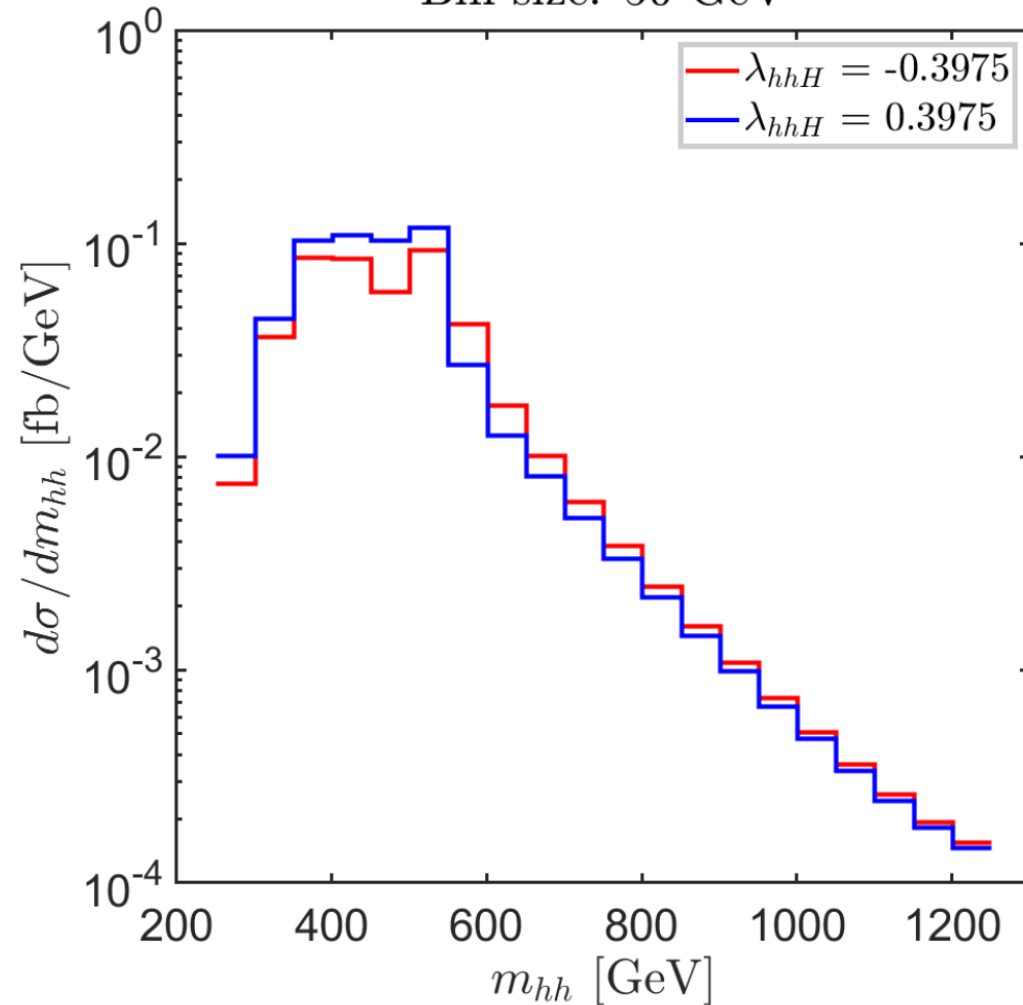


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Bin size: 50 GeV



⇒ binning of 50 GeV applied (realistic?) ⇒ access to  $\lambda_{hhH}$ ?

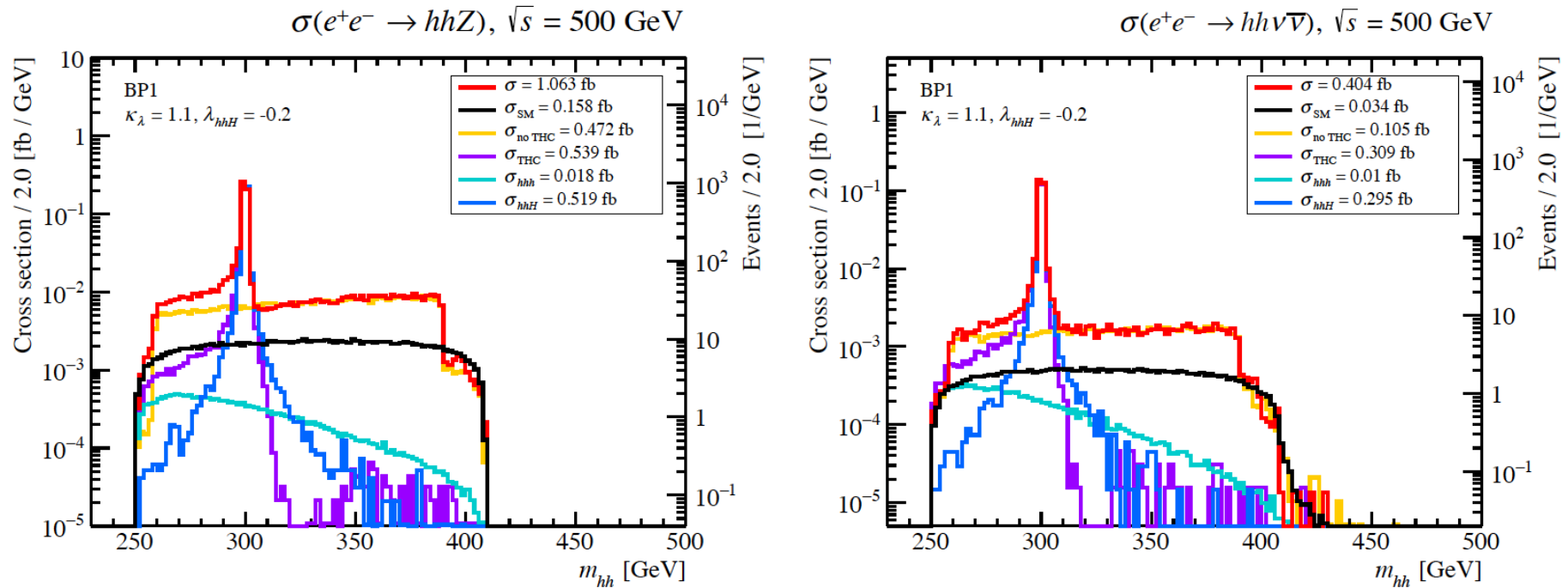
## Di-Higgs production at the ILC:

[F. Arco, S.H., M. Herrero '21]

Example: 2HDM type I

$e^+e^- \rightarrow Zhh/\nu\bar{\nu}hh$  with MadGraph

$m_{H,A,H^\pm} = 300 \text{ GeV}, t_\beta = 10, c_{\beta-\alpha} = 0.25, m_{12}^2 = m_H^2 c_\alpha^2 / t_\beta \Rightarrow \kappa_\lambda = 1.1, \lambda_{hhH} = -0.2$



theory analysis:  $R := (\bar{N}^R - \bar{N}^C) / \sqrt{\bar{N}^C}$

R = “resonance”, C = “continuum”,  $\bar{N}$  incl. cuts and  $b$ -tagging efficiencies

$$\sqrt{s} = 500 (1000) \text{ GeV} \Rightarrow R = 58 (205)$$

experimental analysis: crucially needed!

#### 4. Bonus track: Discovery of the second Higgs boson



## Measurement of Higgs boson production and search for new resonances in final states with photons and Z bosons

by Chiara Arcangeletti (INFN e Laboratori Nazionali di Frascati (IT))

 Tuesday 6 Jun 2023, 11:00 → 12:00 Europe/Zurich

 500/1-001 - Main Auditorium (CERN)

New ATLAS result on the low-mass Higgs search in  $pp \rightarrow \phi \rightarrow \gamma\gamma$

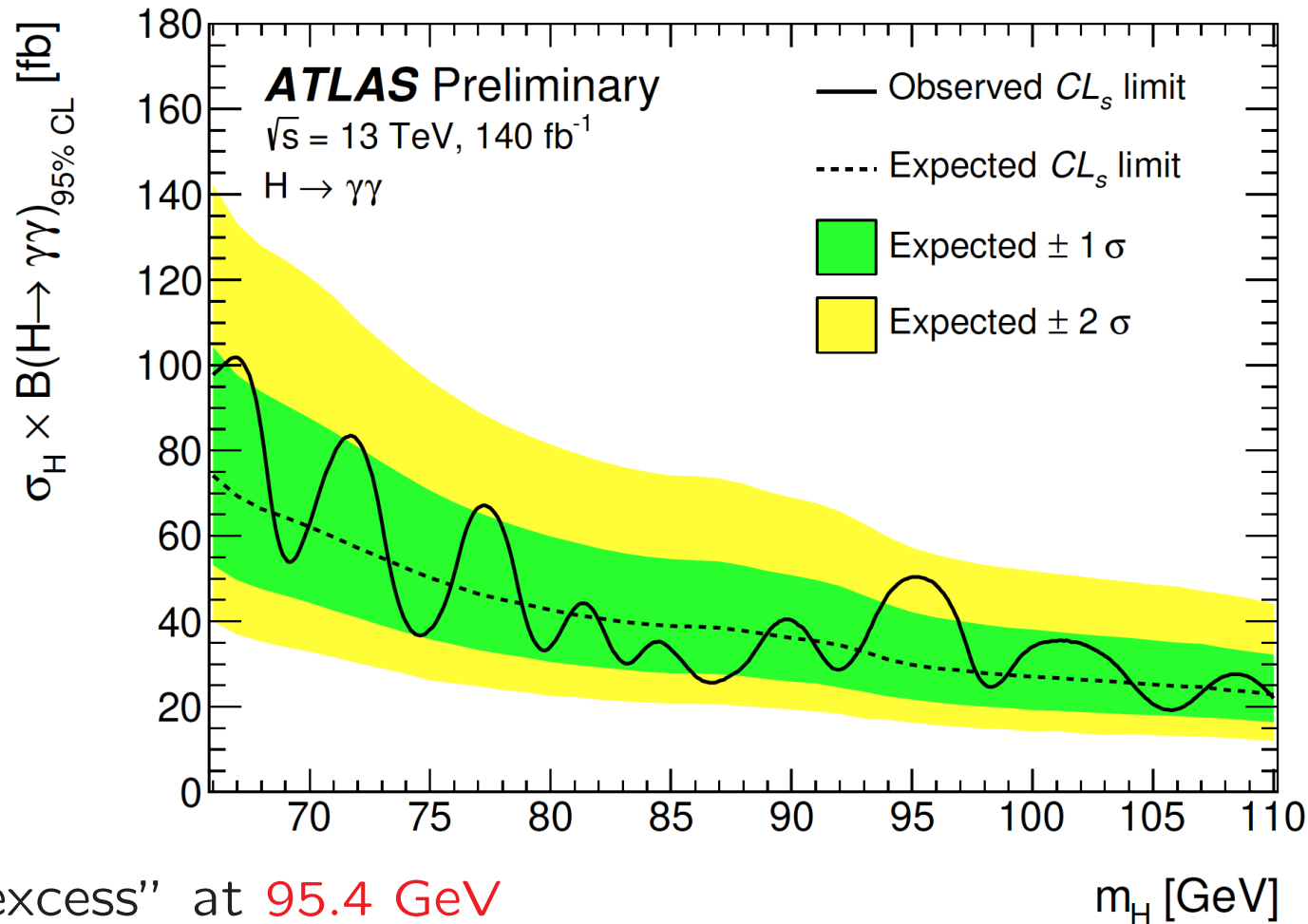
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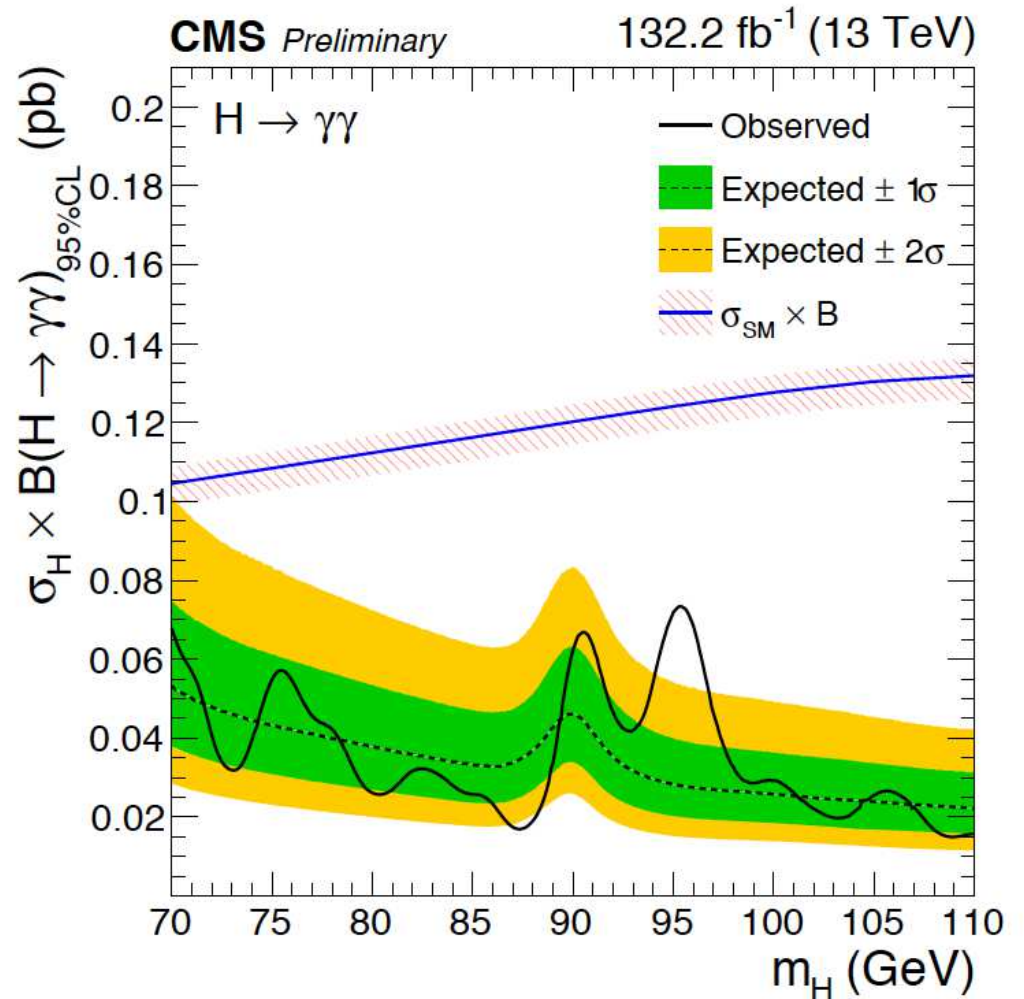
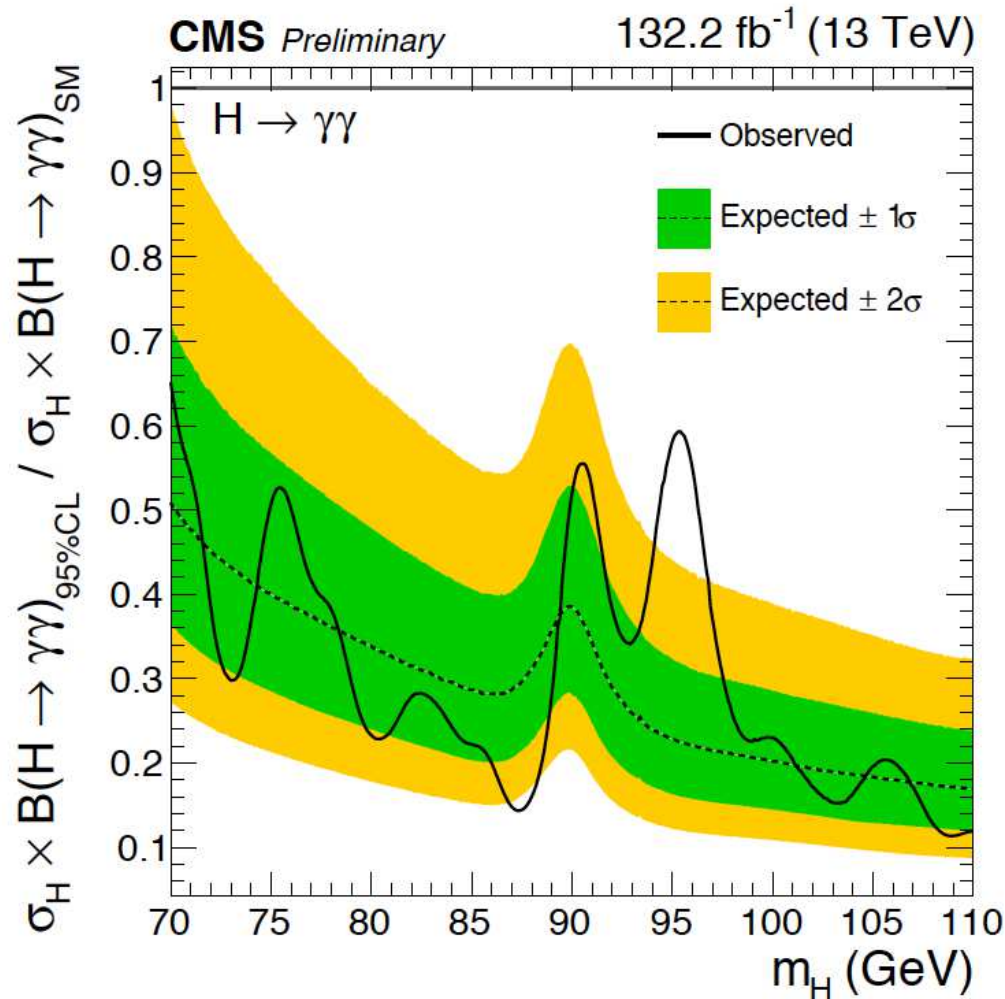
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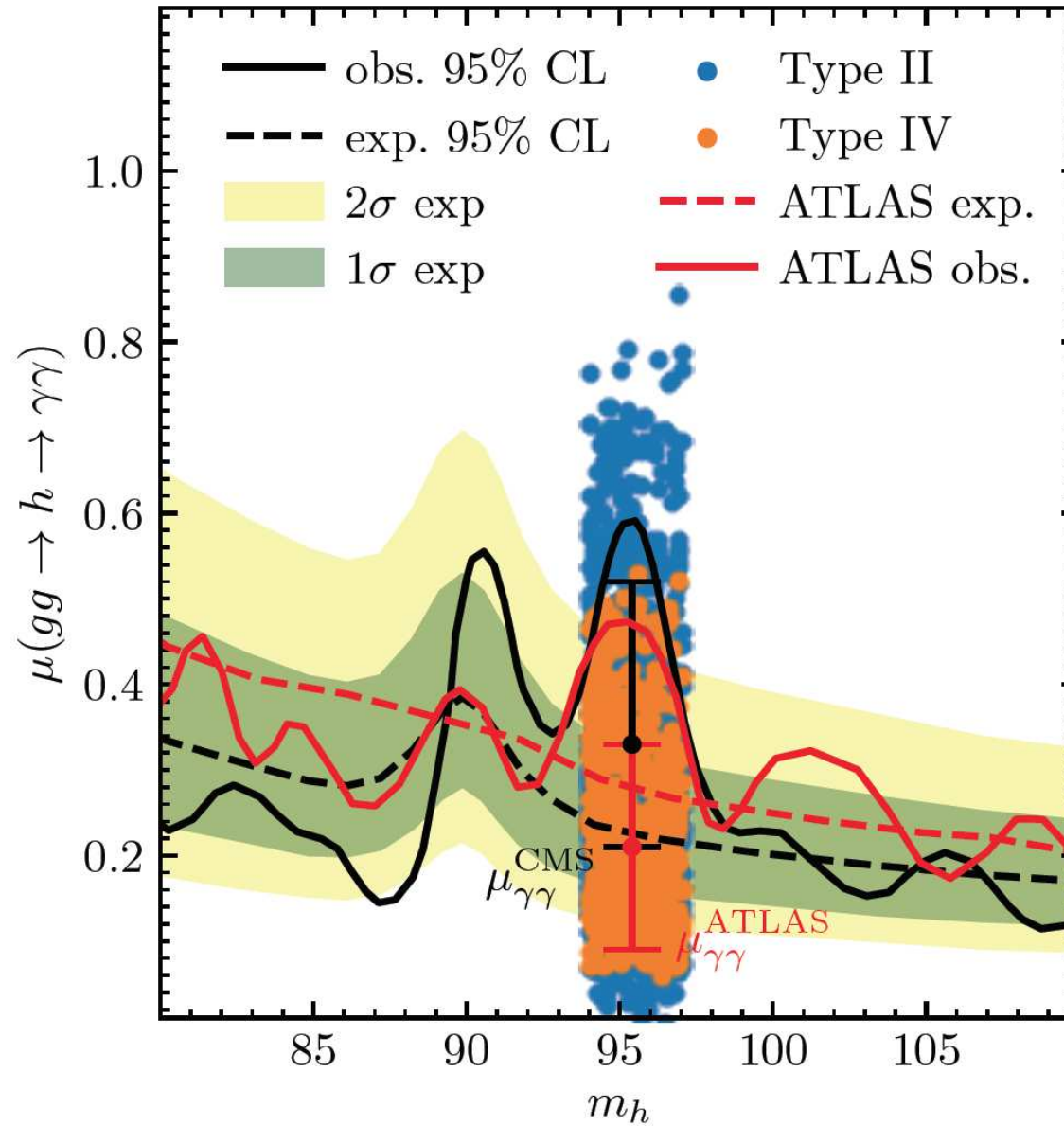




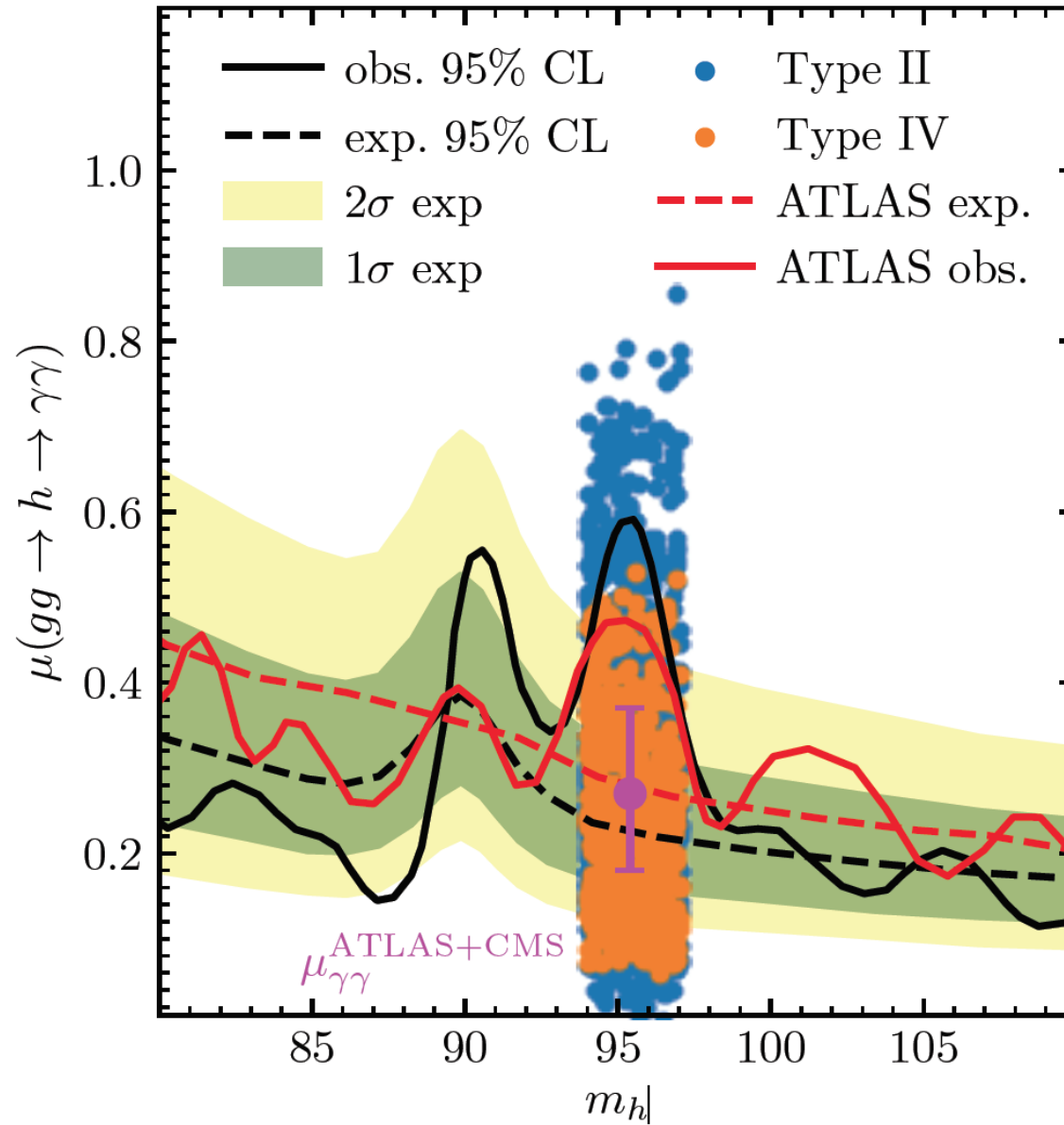
$$\mu_{\gamma\gamma}^{\text{CMS}} = [\sigma(gg \rightarrow h_{95}) \times \text{BR}(h_{95} \rightarrow \gamma\gamma)]_{\text{exp/SM}} = 0.33_{-0.12}^{+0.19} \quad (2.9 \sigma)$$

$$\mu_{\gamma\gamma}^{\text{ATLAS}} = 0.21 \pm 0.12 \quad (1.7 \sigma)$$





⇒ agreement between ATLAS and CMS!

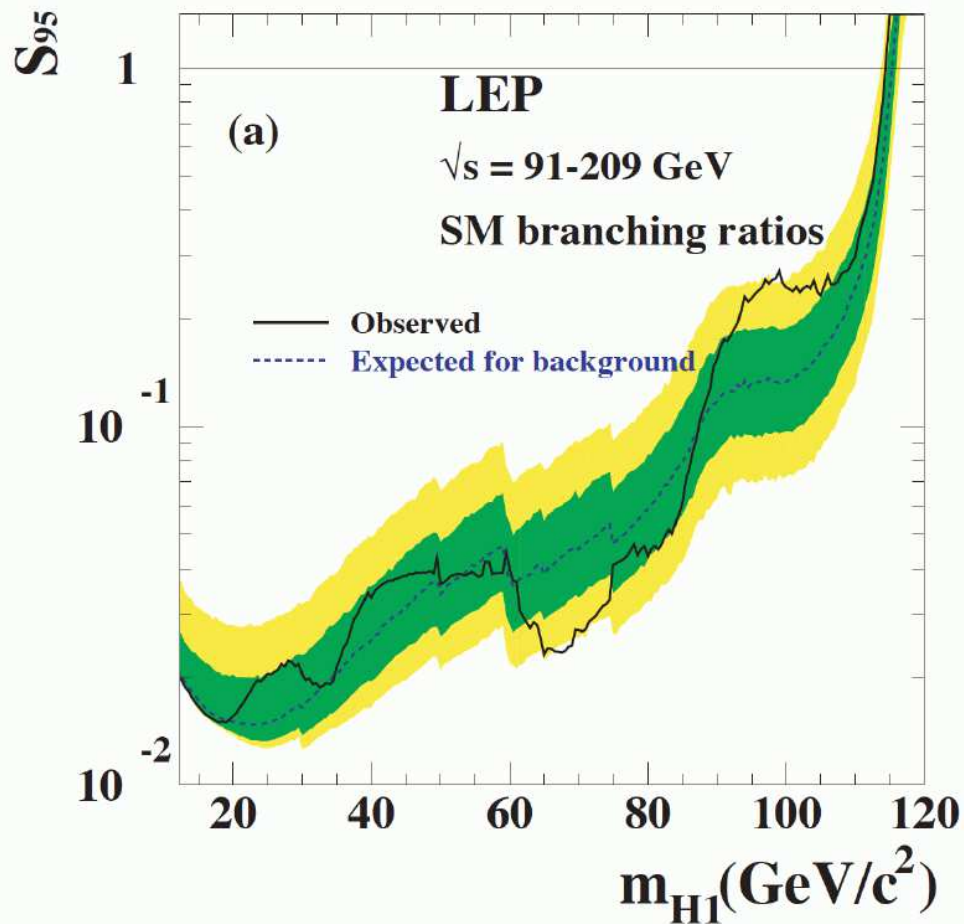


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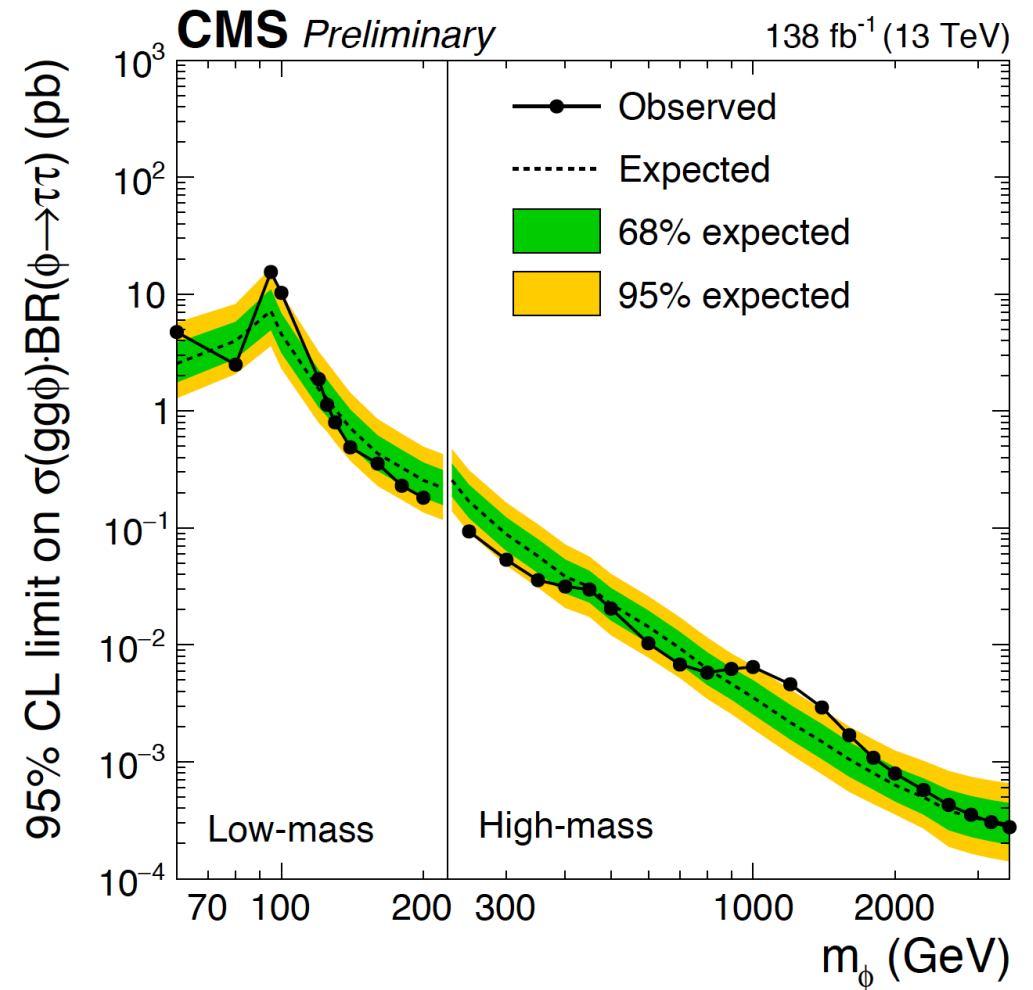
$$\mu_{\gamma\gamma} = 0.27^{+0.10}_{-0.09} \quad (3.2 \sigma)$$



LEP:  $e^+e^- \rightarrow Z\phi \rightarrow Zb\bar{b}$  ( $2\sigma$ )



CMS:  $pp \rightarrow \phi \rightarrow \tau^+\tau^-$  ( $2.4\sigma$ )



⇒ no LEE (as theorist I am allowed to add naively)

⇒  $\sim 4.6\sigma$

S2HDM: 2 Higgs doublets (complex) + 1 Higgs singlet (complex)

Three neutral  $\mathcal{CP}$ -even Higgses:

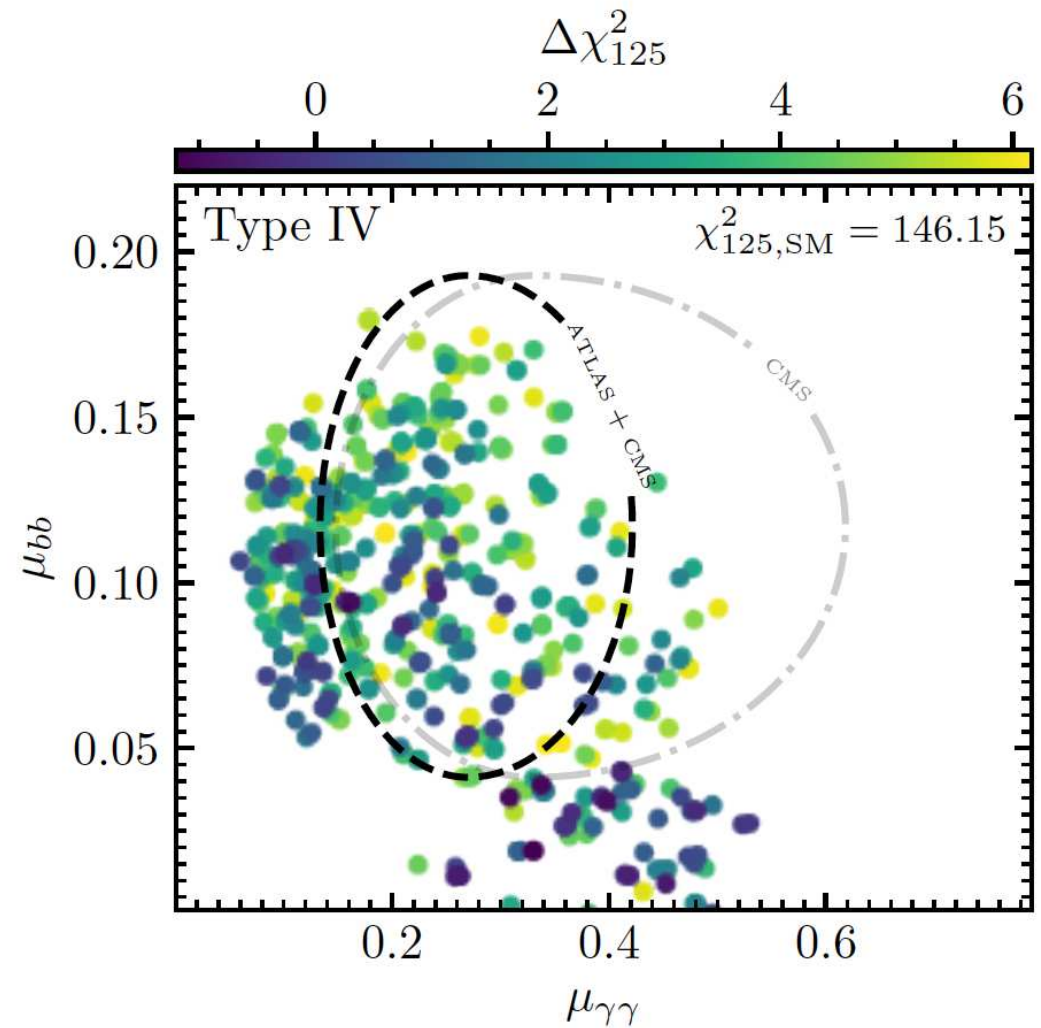
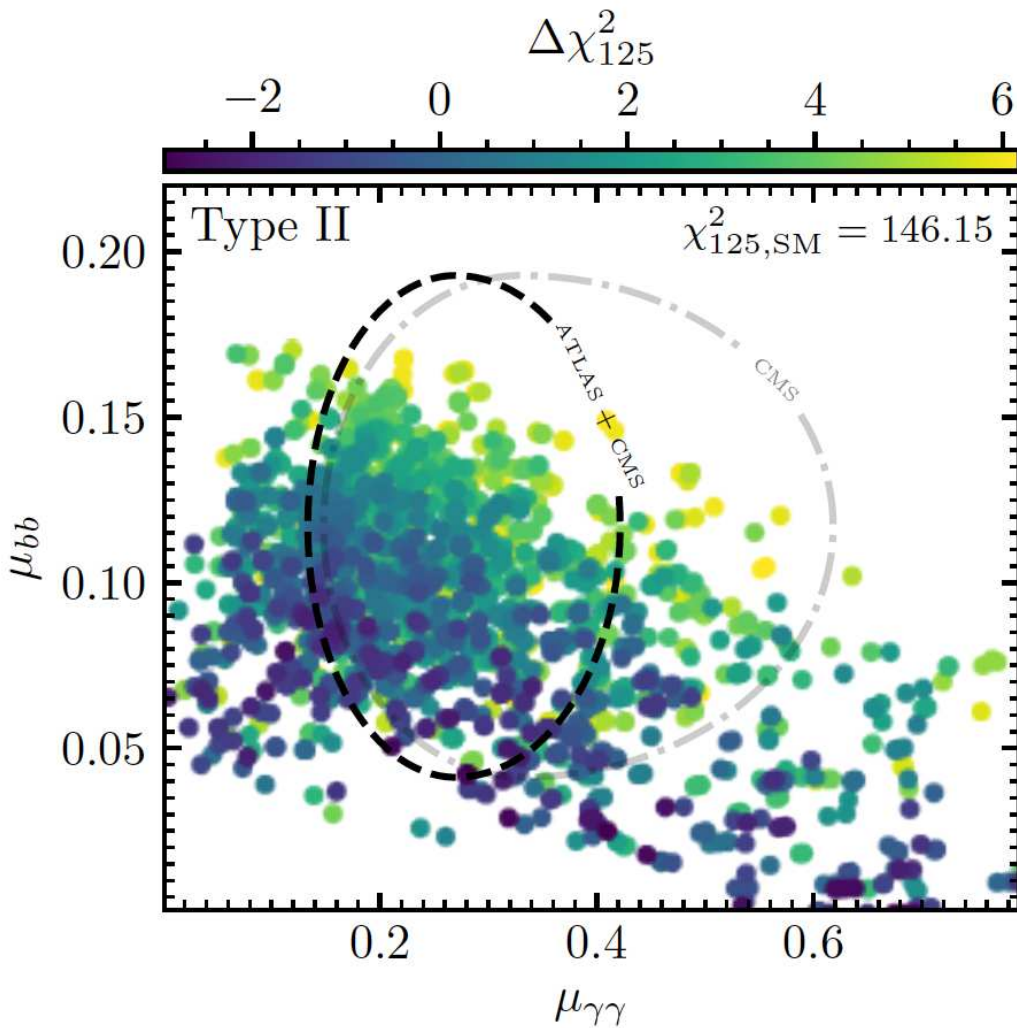
$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_S \end{pmatrix}, \quad R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -(c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & -(c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}$$

Coupling to fermions: (same pattern as in 2HDM)

|                            | $u$ -type ( $c_{h_i tt}$ ) | $d$ -type ( $c_{h_i bb}$ ) | leptons ( $c_{h_i \tau\tau}$ ) |
|----------------------------|----------------------------|----------------------------|--------------------------------|
| type I                     | $\frac{R_{i2}}{s_\beta}$   | $\frac{R_{i2}}{s_\beta}$   | $\frac{R_{i2}}{s_\beta}$       |
| type II                    | $\frac{R_{i2}}{s_\beta}$   | $\frac{R_{i1}}{c_\beta}$   | $\frac{R_{i1}}{c_\beta}$       |
| type III (lepton-specific) | $\frac{R_{i2}}{s_\beta}$   | $\frac{R_{i2}}{s_\beta}$   | $\frac{R_{i1}}{c_\beta}$       |
| type IV (flipped)          | $\frac{R_{i2}}{s_\beta}$   | $\frac{R_{i1}}{c_\beta}$   | $\frac{R_{i2}}{s_\beta}$       |

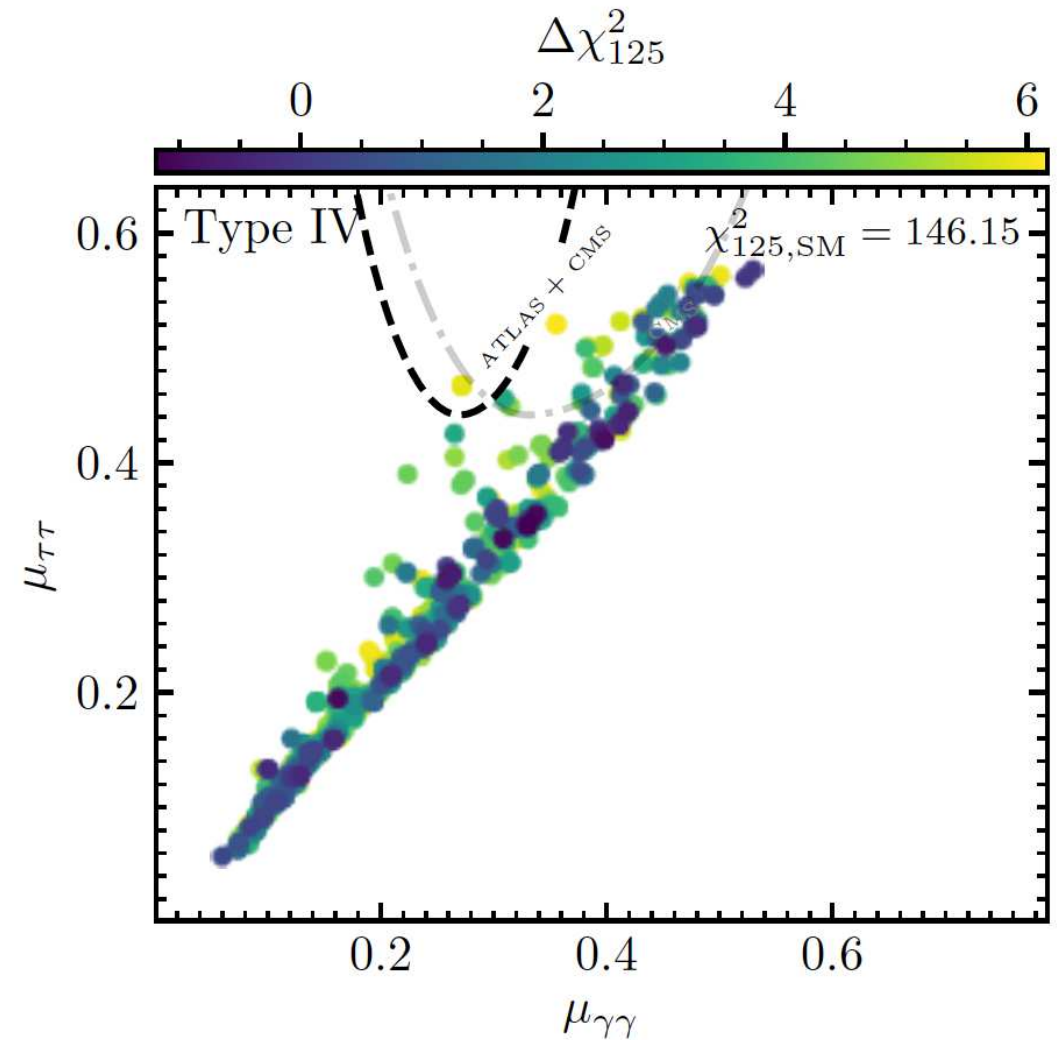
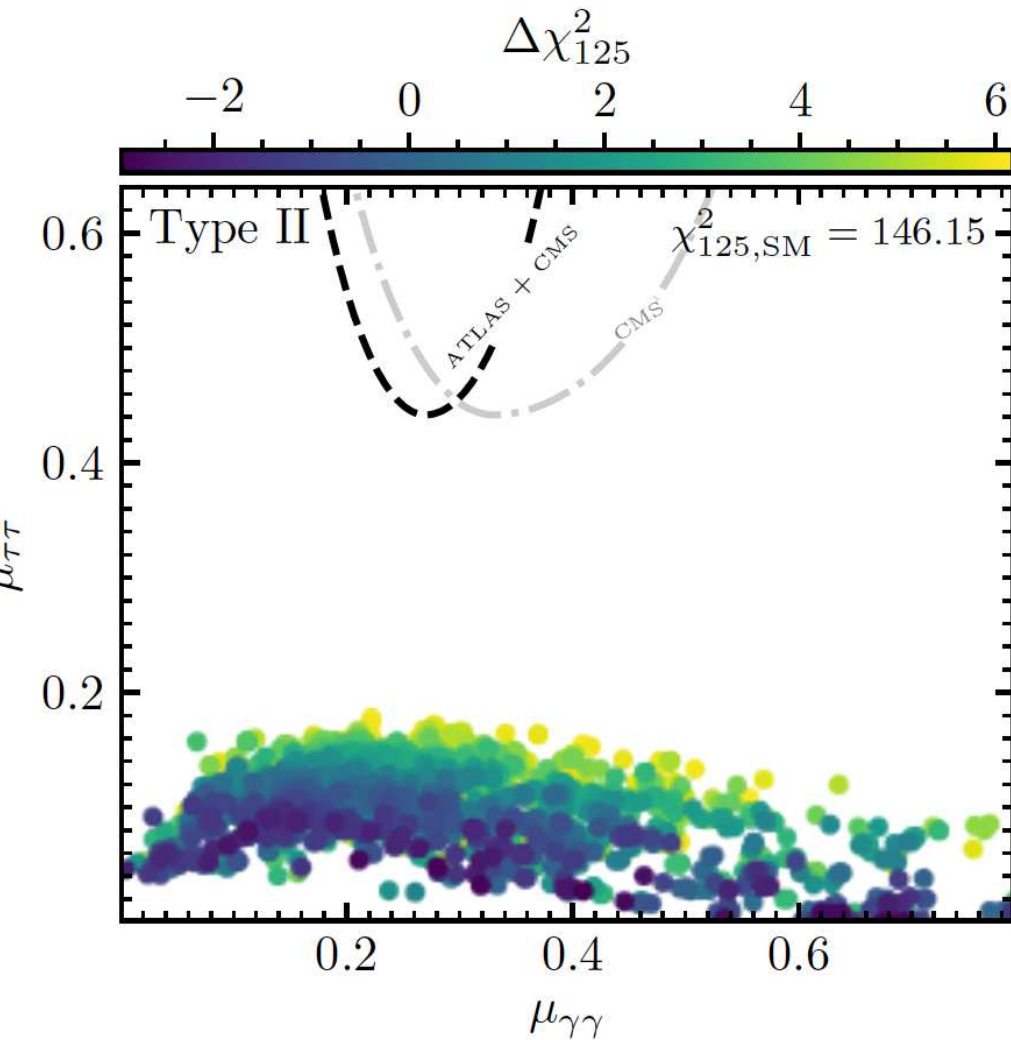
“Physical” input parameters:

$$\alpha_{1,2,3}, \quad \tan \beta, \quad v, \quad v_S, \quad m_{h_{1,2,3}}, \quad m_A, \quad M_{H^\pm}, \quad m_\chi, \quad m_{12}^2$$



Color coding:  $\chi^2_{125}$  from HiggsSignals

⇒ both type II and IV can fit the  $\gamma\gamma$  and  $bb$  excesses



Color coding:  $\chi^2_{125}$  from HiggsSignals

⇒ only type IV can fit marginally the  $\gamma\gamma$  and  $\tau\tau$  excesses

## 5. Conclusions

- $\Rightarrow$  Why is there more matter than antimatter?  $\Rightarrow$  (EW) baryogenesis  
 $\Rightarrow$  requires First Order EW Phase Transition (FOEWPT)  
FOEWPT not possible in the SM  $\Rightarrow$  BSM THC's required  
FOEWPT can cause Gravitational Waves (GW), detectable with LISA  
**Q:** Can this happen in the 2HDM? Implications for THCs?
- 2HDM:  $\Rightarrow$  FOEWPT requires  $\kappa_\lambda \lesssim 2 \Rightarrow$  GW signal requires  $\kappa_\lambda \sim 2$   
 $\Rightarrow$  bad for HL-LHC ( $\delta\lambda_{hhh} \sim 70\%$ ), good for ILC ( $\delta\lambda_{hhh} \sim 10\%$ )  
 $\Rightarrow$  FOEWPT favors  $A$ - $H$  mass gap  
 $\Rightarrow$  Smoking gun signature:  $A \rightarrow ZH \rightarrow Zt\bar{t}$
- Triple Higgs couplings are in the focus of current and future colliders  
 $\Rightarrow$  focus on “SM triple Higgs coupling”,  $\kappa_\lambda := \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$   
BSM case 1:  $\kappa_\lambda \neq 1$   
BSM case 2: THC that involves BSM Higgses:  $\lambda_{hhH}, \dots$
- Access to BSM THC's:  $\lambda_{hhH}$   
 $\Rightarrow$  benchmark point in the 2HDM type I with  $m_H = 300$  GeV  
HL-LHC:  $\Rightarrow$  dip-peak / peak-dip from resonant  $H$ -exchange  
smearing/binning crucial  $\Rightarrow$  full exp. study needed!  
ILC: theory analysis looks very good, exp. analysis: crucially needed



# Higgs Days at Santander 2023

## Theory meets Experiment

4 - 8 September

Contact: [Sven.Heinemeyer@cern.ch](mailto:Sven.Heinemeyer@cern.ch)  
Local: [Alicia.Calderon@cern.ch](mailto:Alicia.Calderon@cern.ch)  
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<http://hdays.csic.es>





Further Questions?



Needed to fit the two excesses:  $m_{h_1} \sim 95 \text{ GeV}$ ,  $m_{h_2} \sim 125 \text{ GeV}$

- $c_{h_1 VV}^2$  strongly reduced for  $\mu_{\text{LEP}}$
- $c_{h_1 bb}$  reduced to enhance  $\text{BR}(h_1 \rightarrow \gamma\gamma)$
- $c_{h_1 tt}$  not reduced for  $\mu_{\text{CMS}}$
- $c_{h_1 \tau\tau}$  possibly reduced to enhance  $\text{BR}(h_1 \rightarrow \gamma\gamma)$

|          | Decrease $c_{h_1 b\bar{b}}$    | No decrease $c_{h_1 t\bar{t}}$            | No enhancement $c_{h_1 \tau\bar{\tau}}$ |
|----------|--------------------------------|-------------------------------------------|-----------------------------------------|
| type I   | $(\frac{R_{12}}{s_\beta}) :-)$ | $(\frac{R_{12}}{s_\beta}) :-(\text{red})$ | $(\frac{R_{12}}{s_\beta})$              |
| type II  | $(\frac{R_{11}}{c_\beta}) :-)$ | $(\frac{R_{12}}{s_\beta}) :-)$            | $(\frac{R_{11}}{c_\beta})$              |
| type III | $(\frac{R_{12}}{s_\beta}) :-)$ | $(\frac{R_{12}}{s_\beta}) :-(\text{red})$ | $(\frac{R_{11}}{c_\beta})$              |
| type IV  | $(\frac{R_{11}}{c_\beta}) :-)$ | $(\frac{R_{12}}{s_\beta}) :-)$            | $(\frac{R_{12}}{s_\beta})$              |

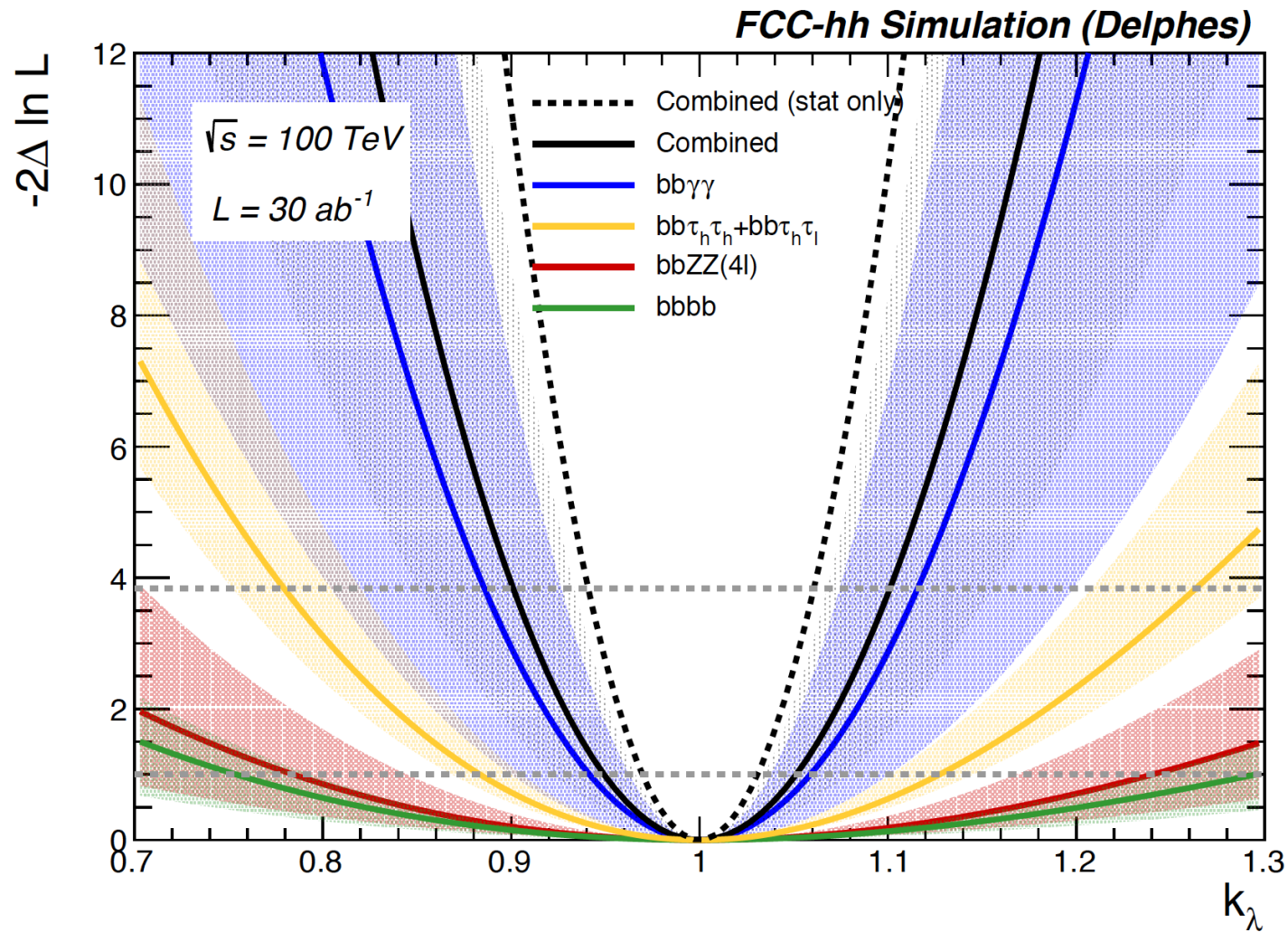
Type II and IV:  $c_{h_1 bb}$  and  $c_{h_1 tt}$  independent

Type II vs. IV:  $c_{h_1 \tau\tau}$  can be suppressed or enhanced

$\Rightarrow$  only type II and IV can fit CMS and LEP excesses

## Measurement of $\kappa_\lambda$ at the FCC-hh:

[Mangano, Ortona, Selvaggi '20]

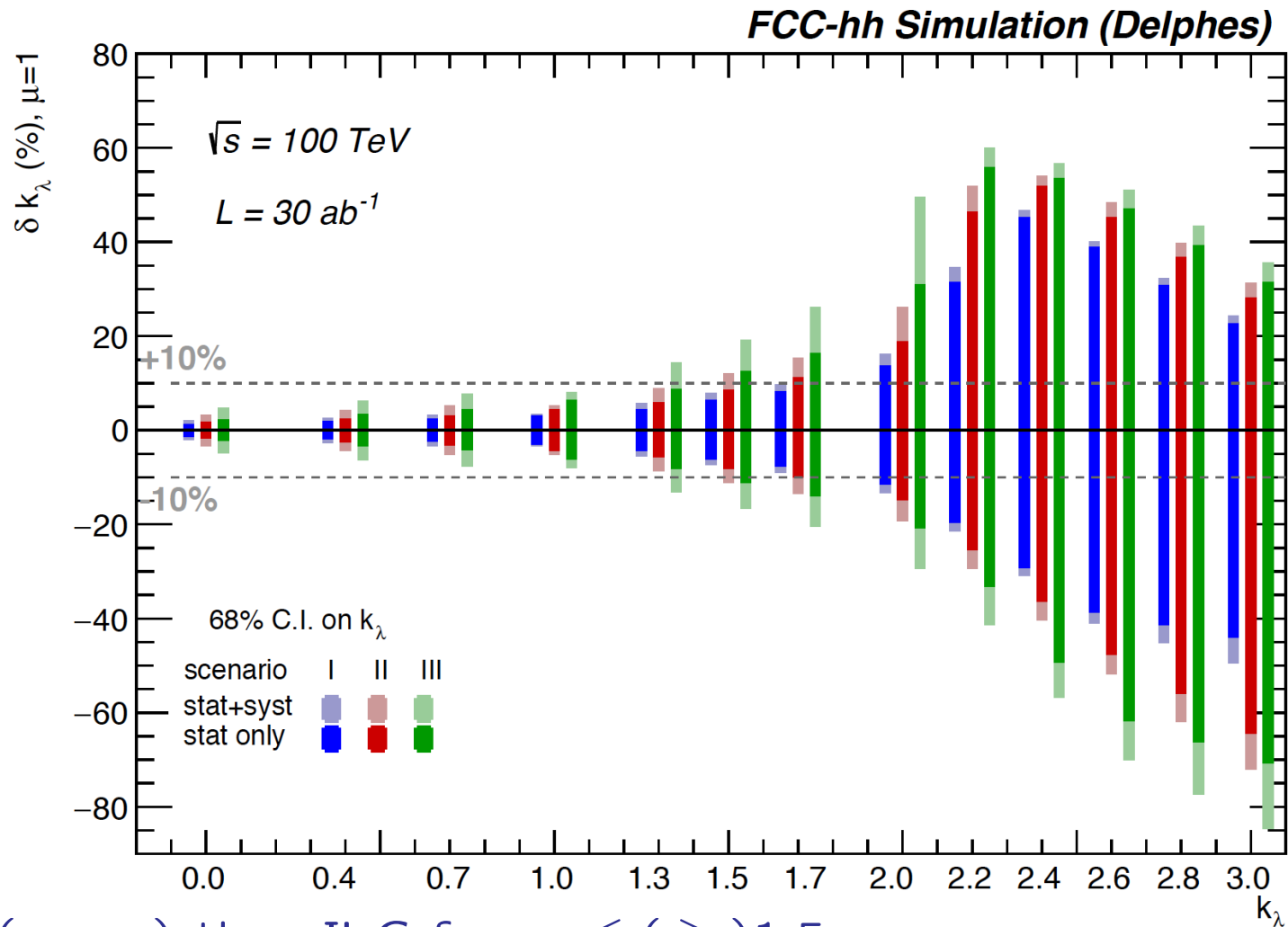


⇒ result only for  $\kappa_\lambda = 1$

⇒ pile-up neglected ...

# Measurement of $\kappa_\lambda$ at the FCC-hh:

[Mangano, Ortona, Selvaggi '20]



⇒ better (worse) than ILC for  $\kappa_\lambda \lesssim (\gtrsim) 1.5$

⇒ no results for  $\kappa_\lambda \leq 0$

⇒ pile-up neglected ...