
Primordial cosmology

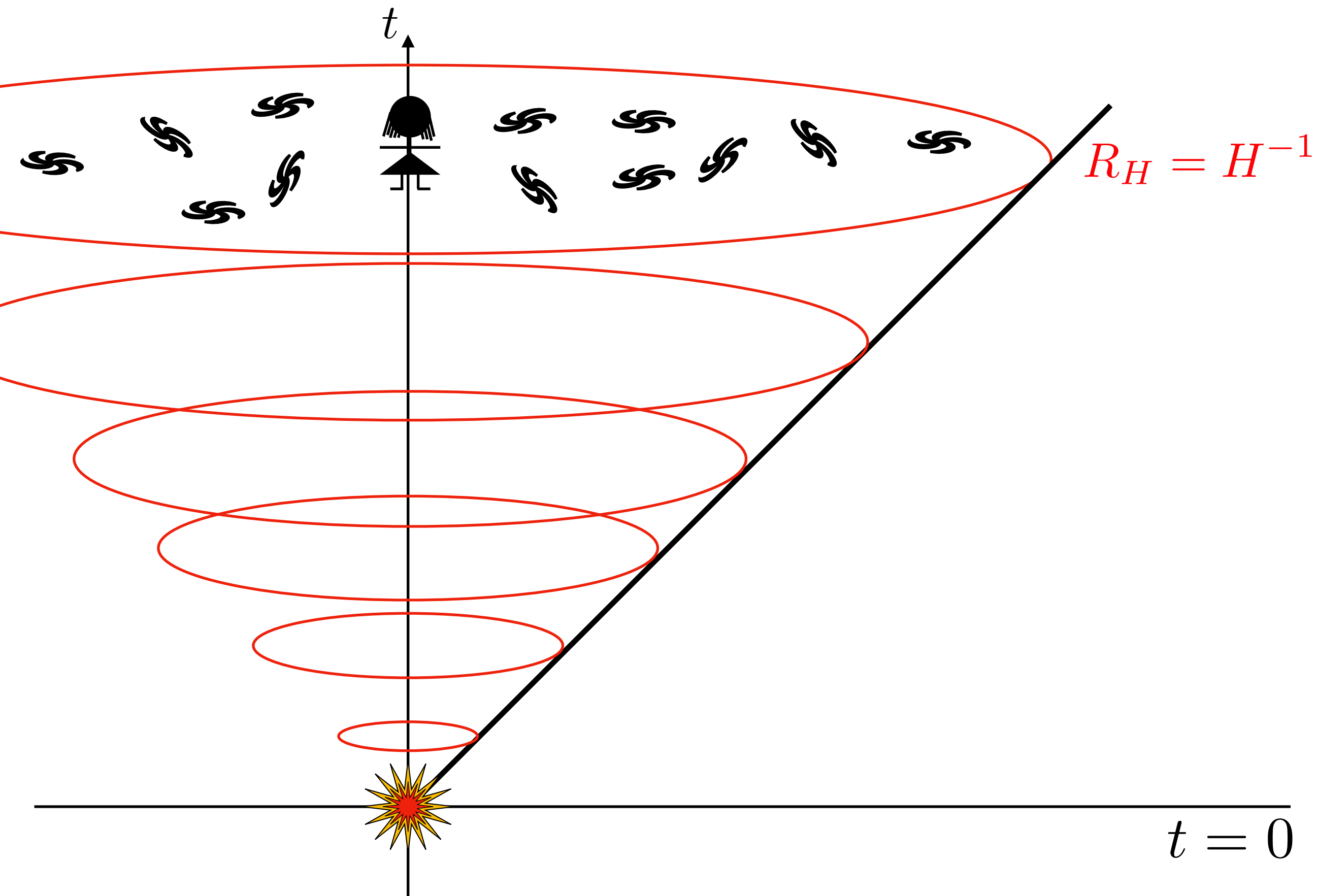
(Inflation in cosmology)

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PPC, Daejeon
June 12, 2023

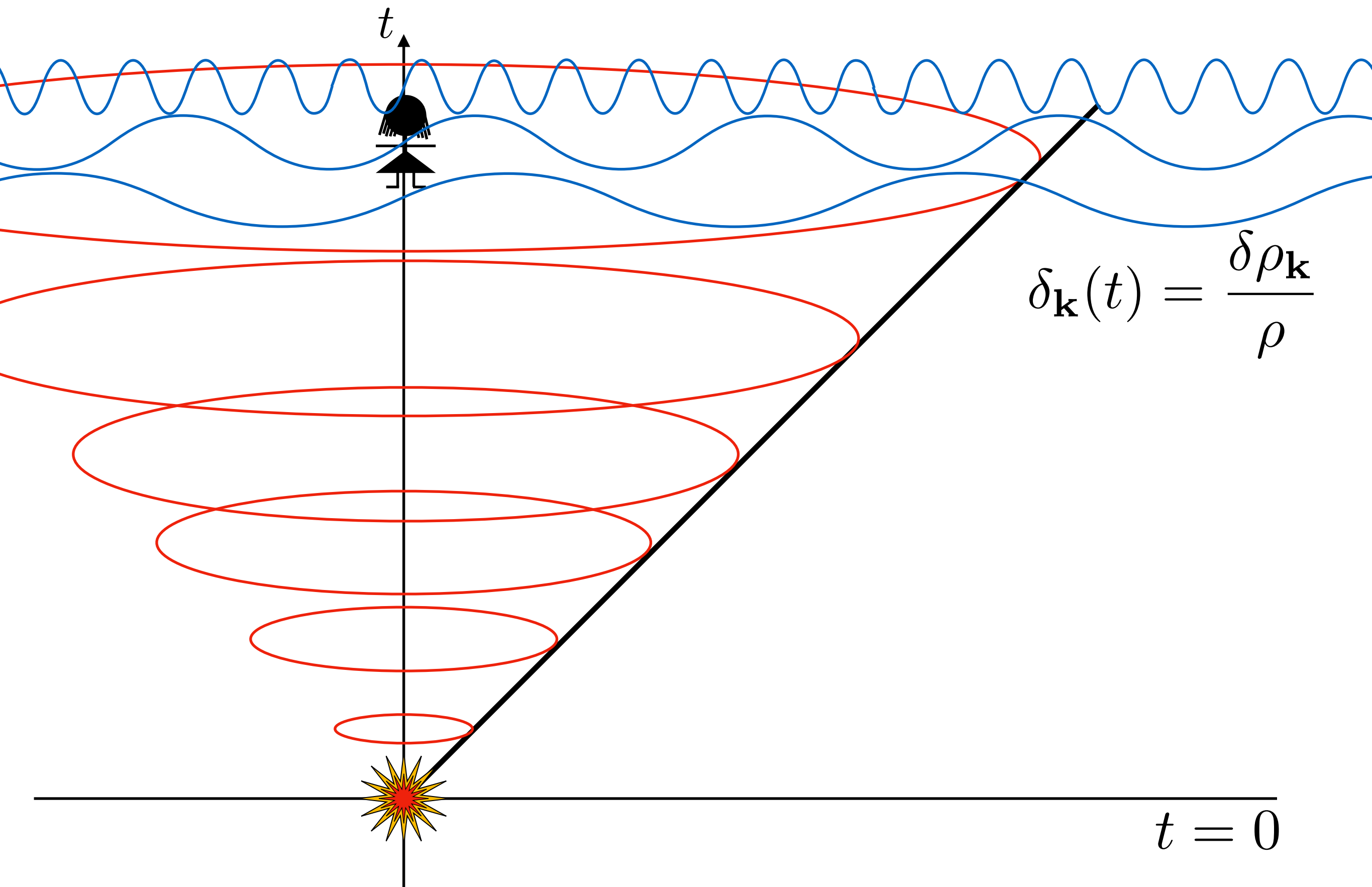
Primordial fluctuations

01



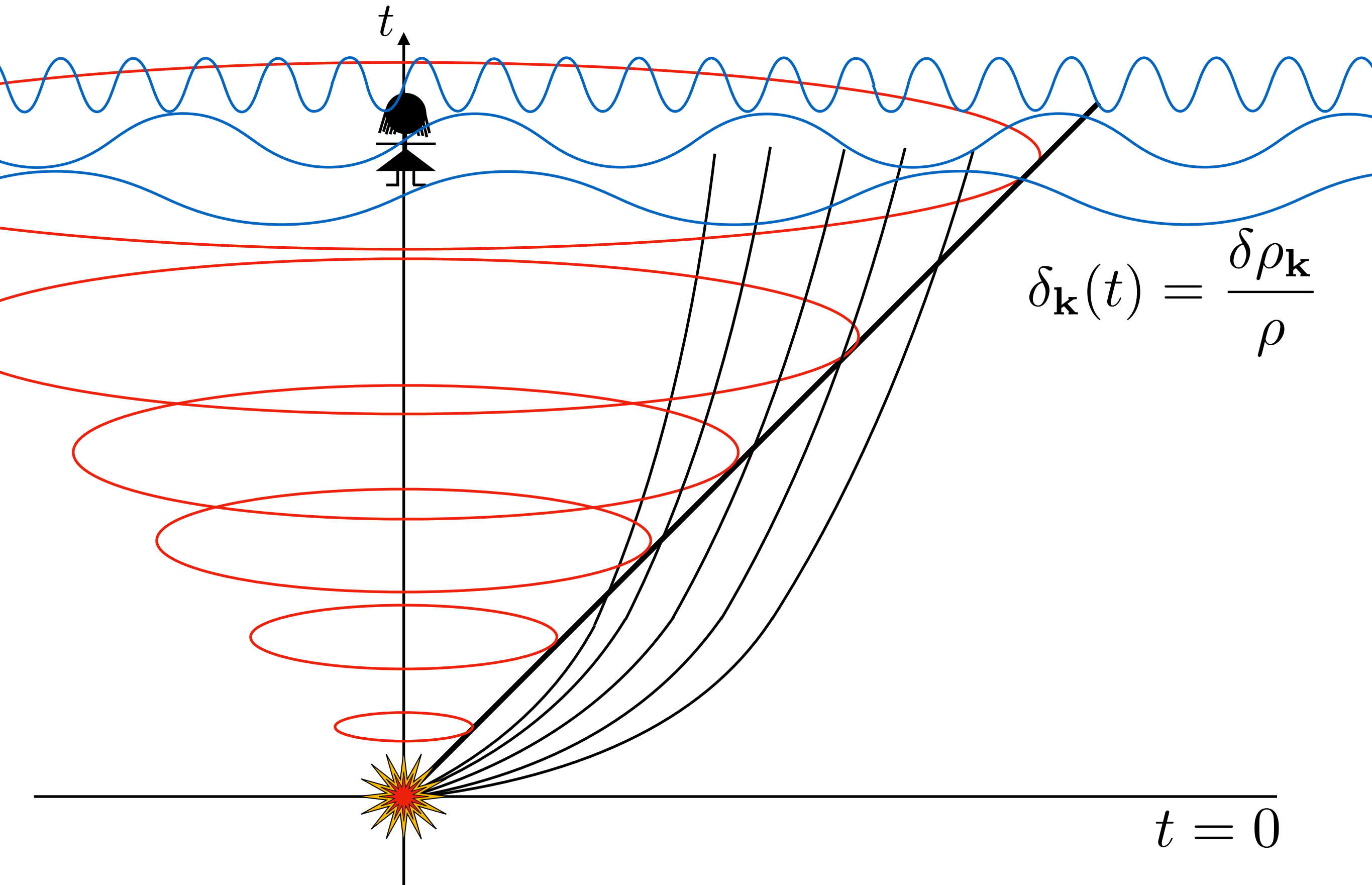
Primordial fluctuations

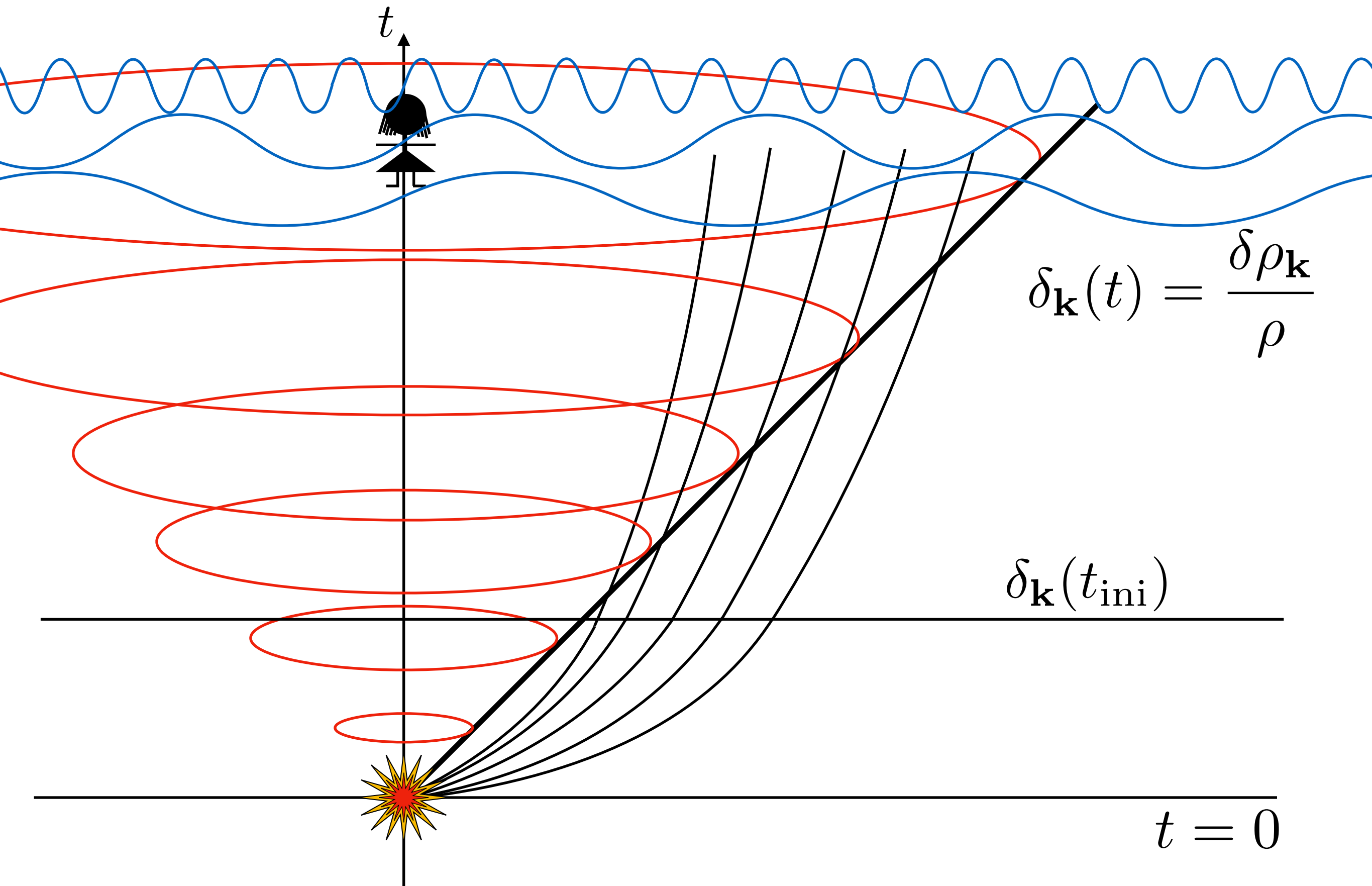
01

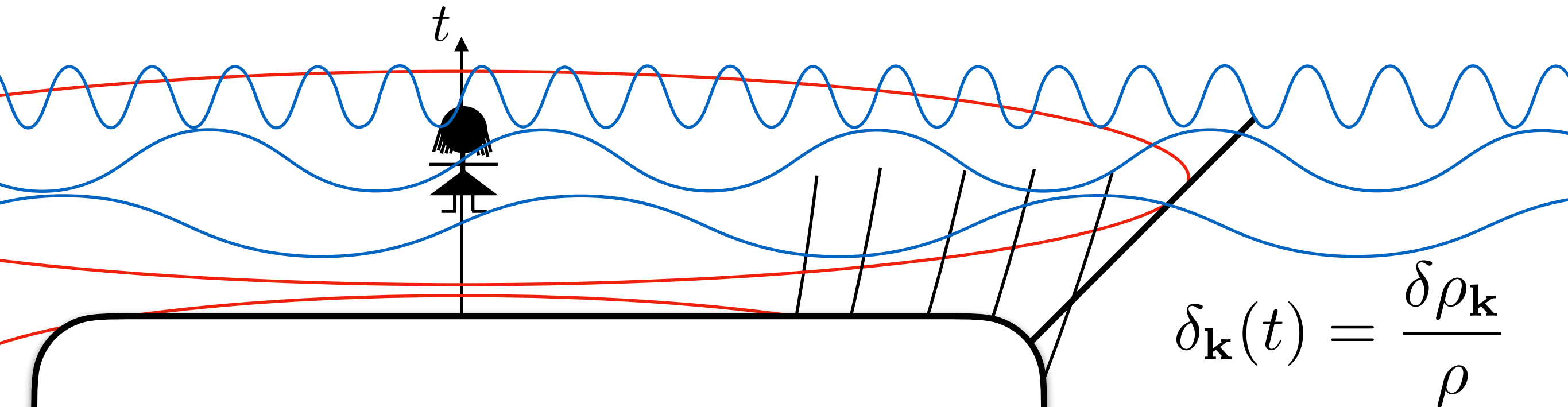


Primordial fluctuations

01







CMB and LSS tell us that the Statistics of $\delta_{\mathbf{k}}(t_{\text{ini}})$ is:

- * Adiabatic
- * Gaussian
- * Almost scale independent

$$\delta_{\mathbf{k}}(t_{\text{ini}})$$

$$t = 0$$

* Adiabaticity

Every inhomogeneity is determined by a single fluctuation

$$\delta_{\mathbf{k}}^{\gamma}(t_{\text{ini}}) \propto \mathcal{R}_{\mathbf{k}}$$

$$\delta_{\mathbf{k}}^{\nu}(t_{\text{ini}}) \propto \mathcal{R}_{\mathbf{k}}$$

$$\delta_{\mathbf{k}}^{\text{Bar}}(t_{\text{ini}}) \propto \mathcal{R}_{\mathbf{k}}$$

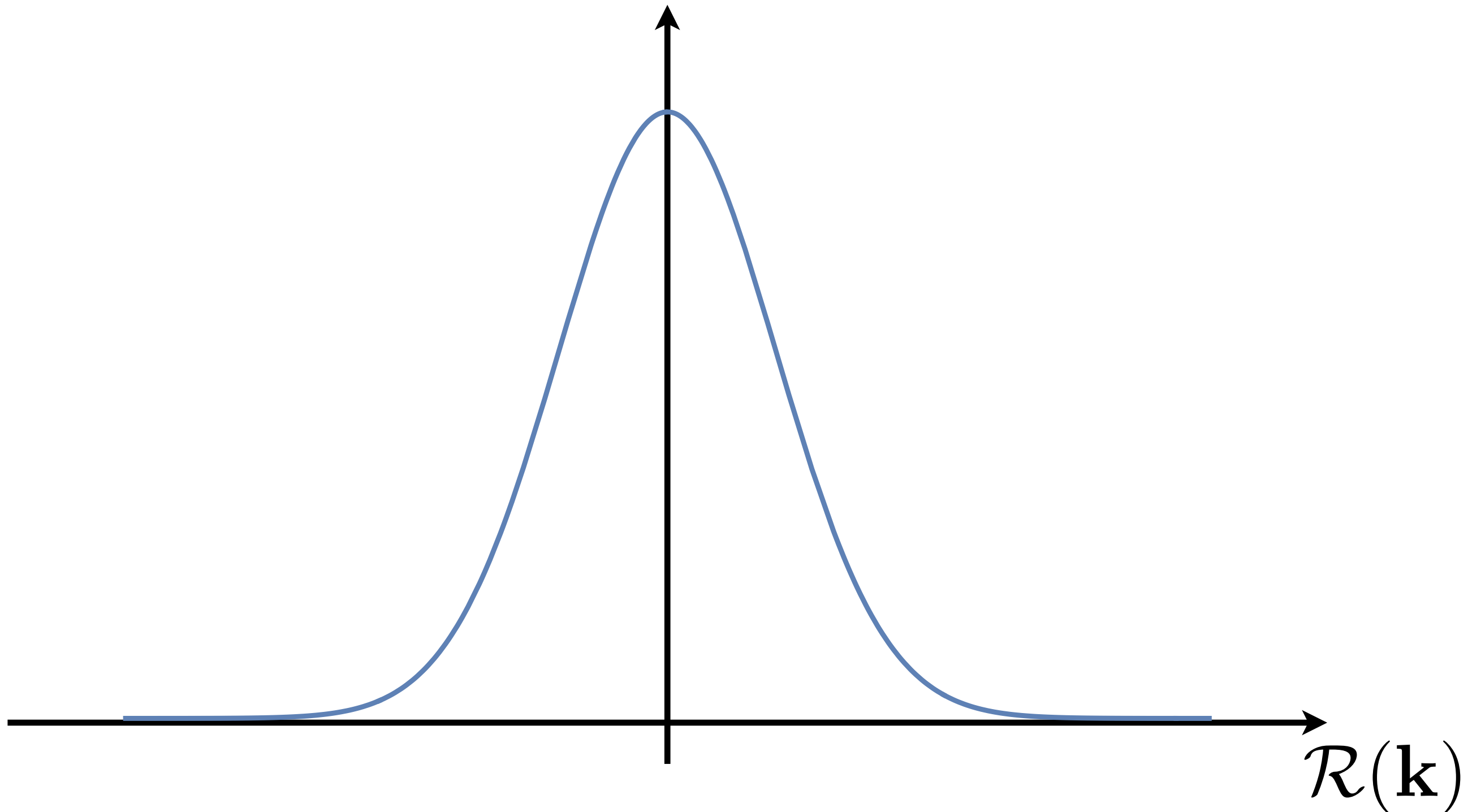
$$\delta_{\mathbf{k}}^{\text{DM}}(t_{\text{ini}}) \propto \mathcal{R}_{\mathbf{k}}$$

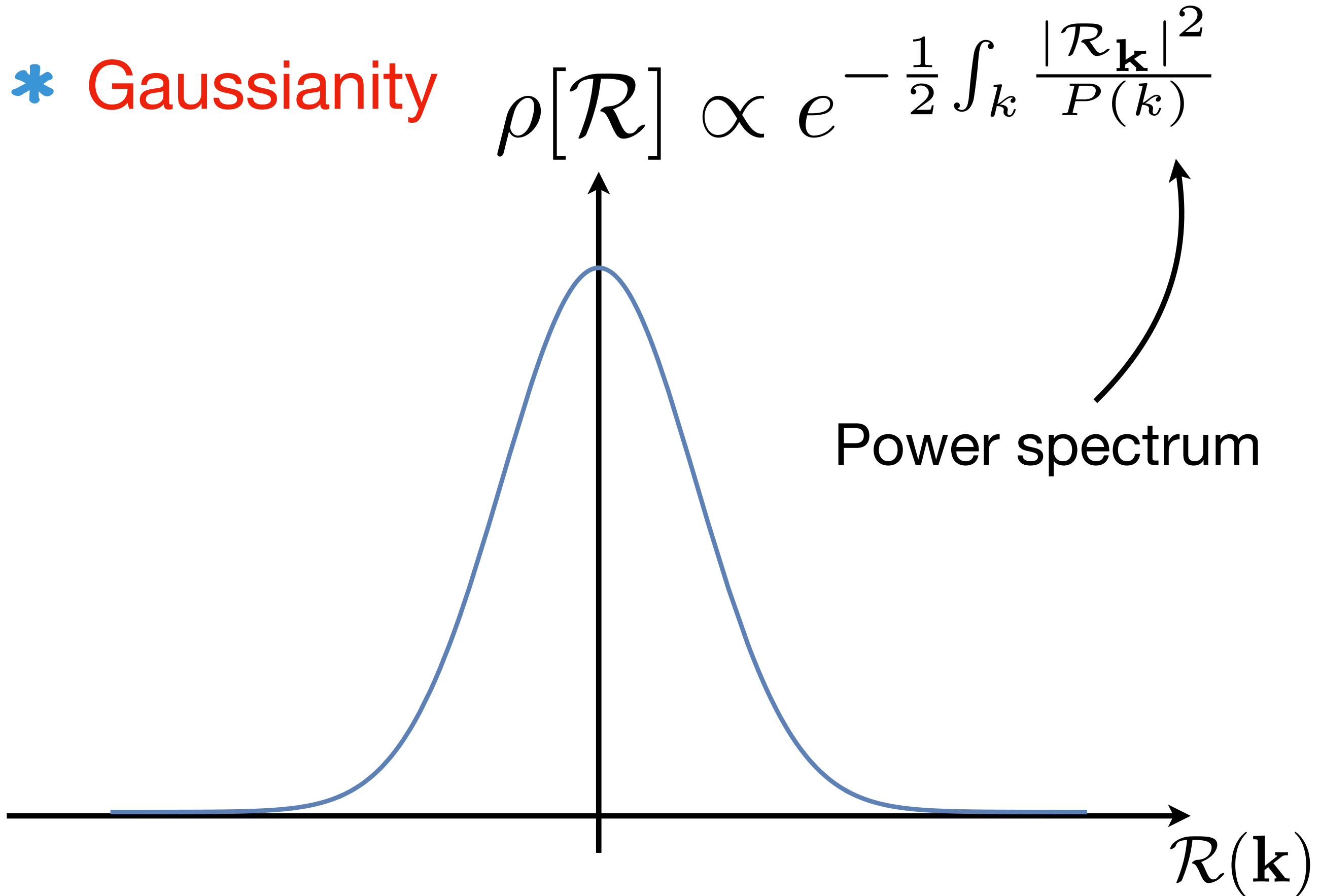
$$ds^2 = -dt^2 + a^2(t)e^{2\mathcal{R}(t,\mathbf{x})}d\mathbf{x}^2$$

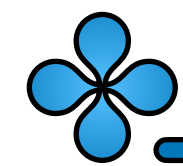
✿ Primordial fluctuations

03

* Gaussianity $\rho[\mathcal{R}] \propto e^{-\frac{1}{2} \int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}}$

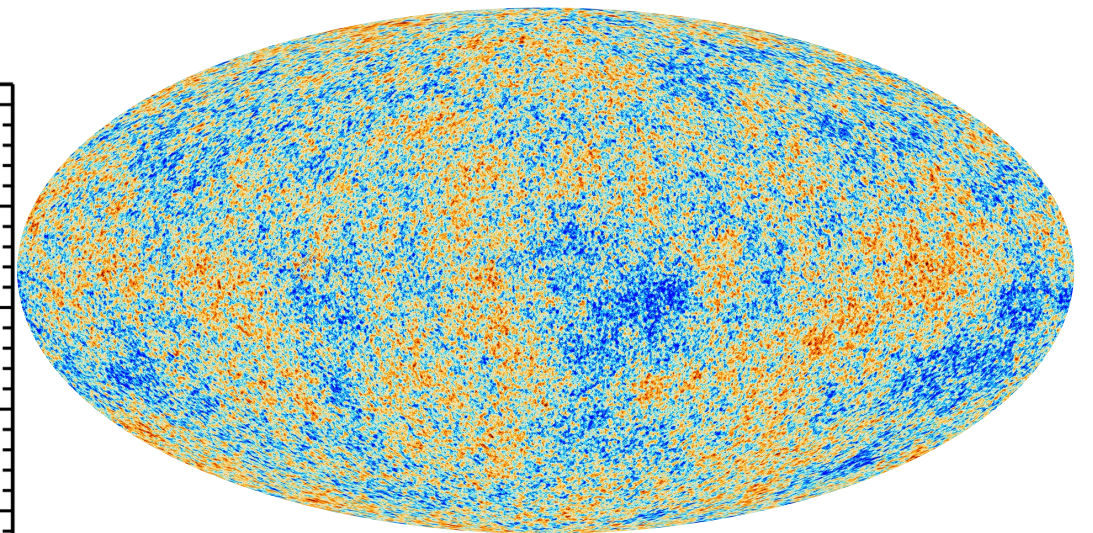
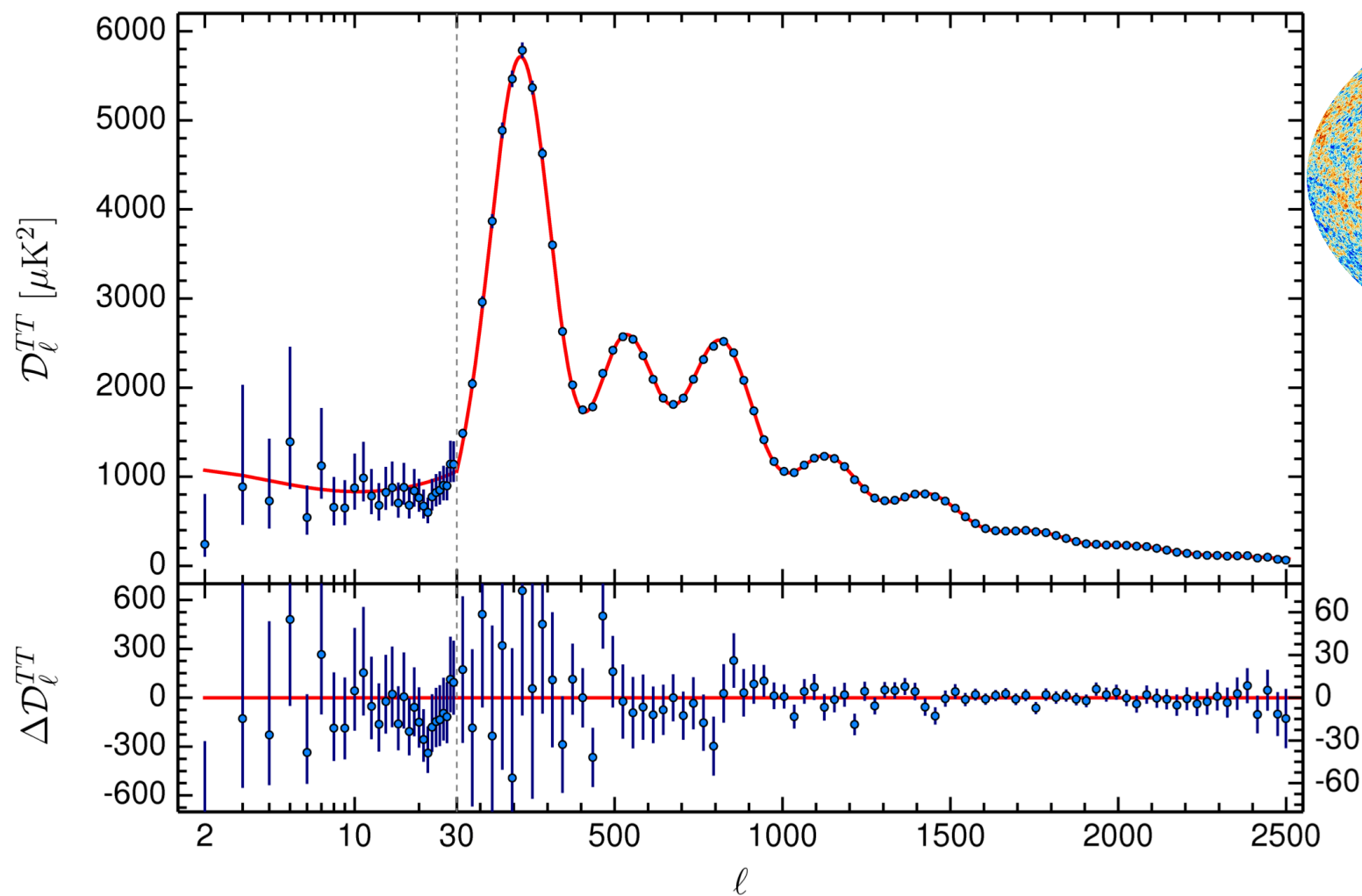






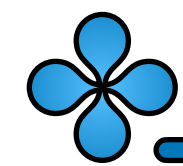
* Almost scale independent

$$P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3} \Delta_{\mathcal{R}}(k) \quad \Delta_{\mathcal{R}}(k) = A \left(\frac{k}{k_*} \right)^{n_s - 1}$$



$$n_s \simeq 0.96$$

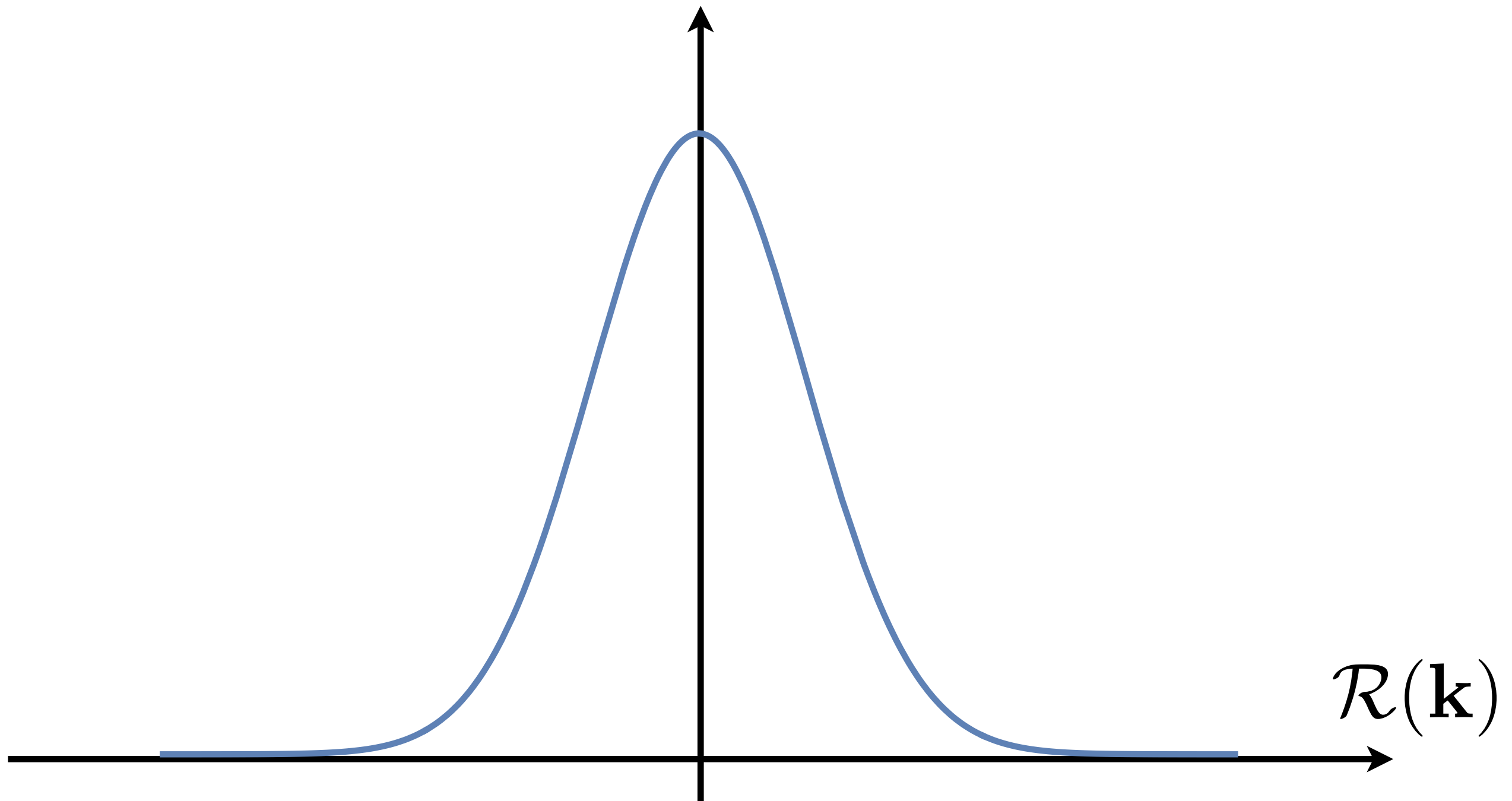
Planck collaboration (2018)

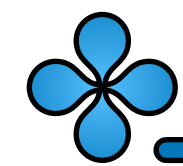


Non-Gaussianity?

05

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2} \int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}}$$

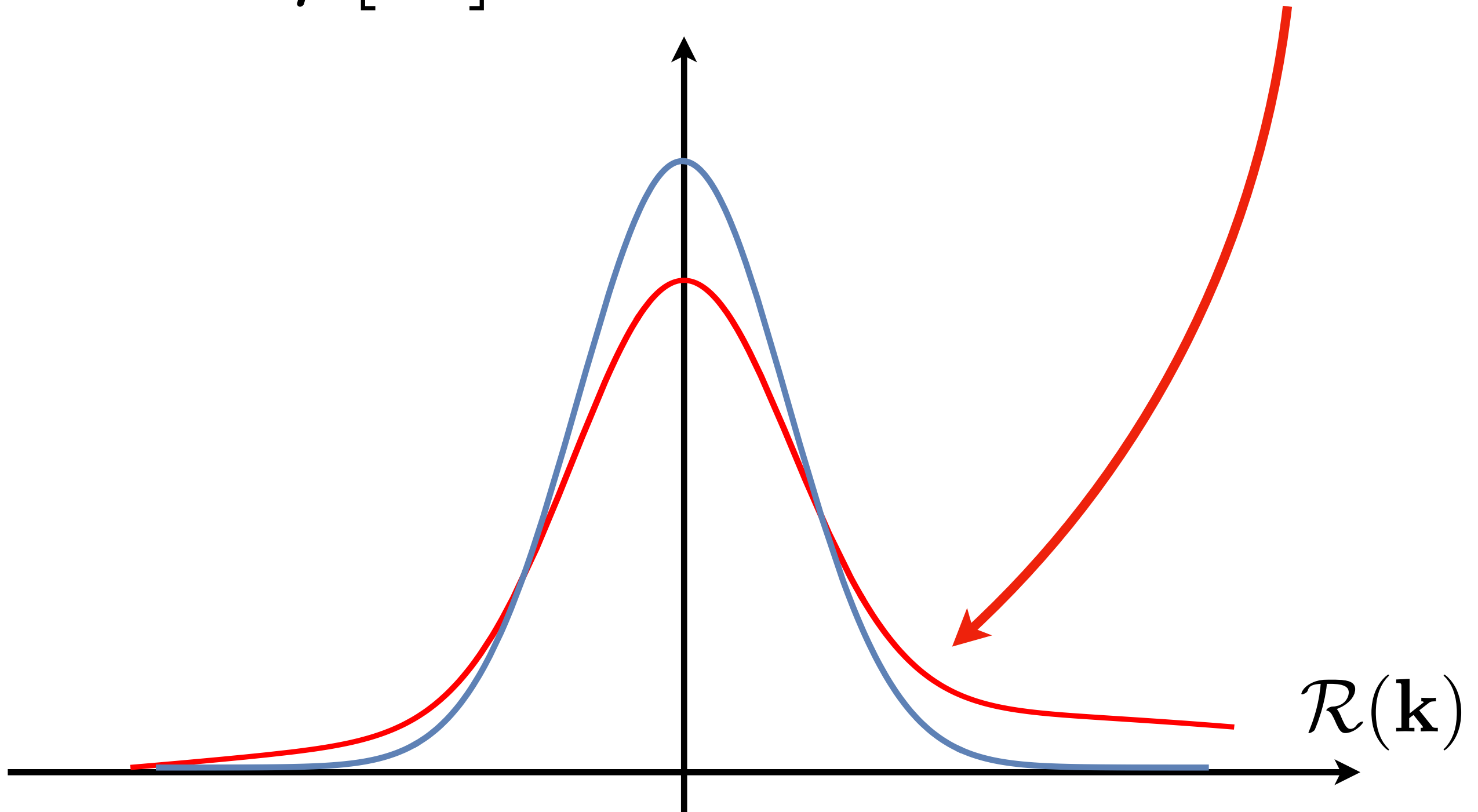




Non-Gaussianity?

05

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2} \int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \dots$$



$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2} \int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \dots$$

$$Z[J] \equiv \int \mathcal{D}\mathcal{R} \rho[\mathcal{R}] e^{-i \int_k J_{\mathbf{k}} \mathcal{R}_{\mathbf{k}}}$$

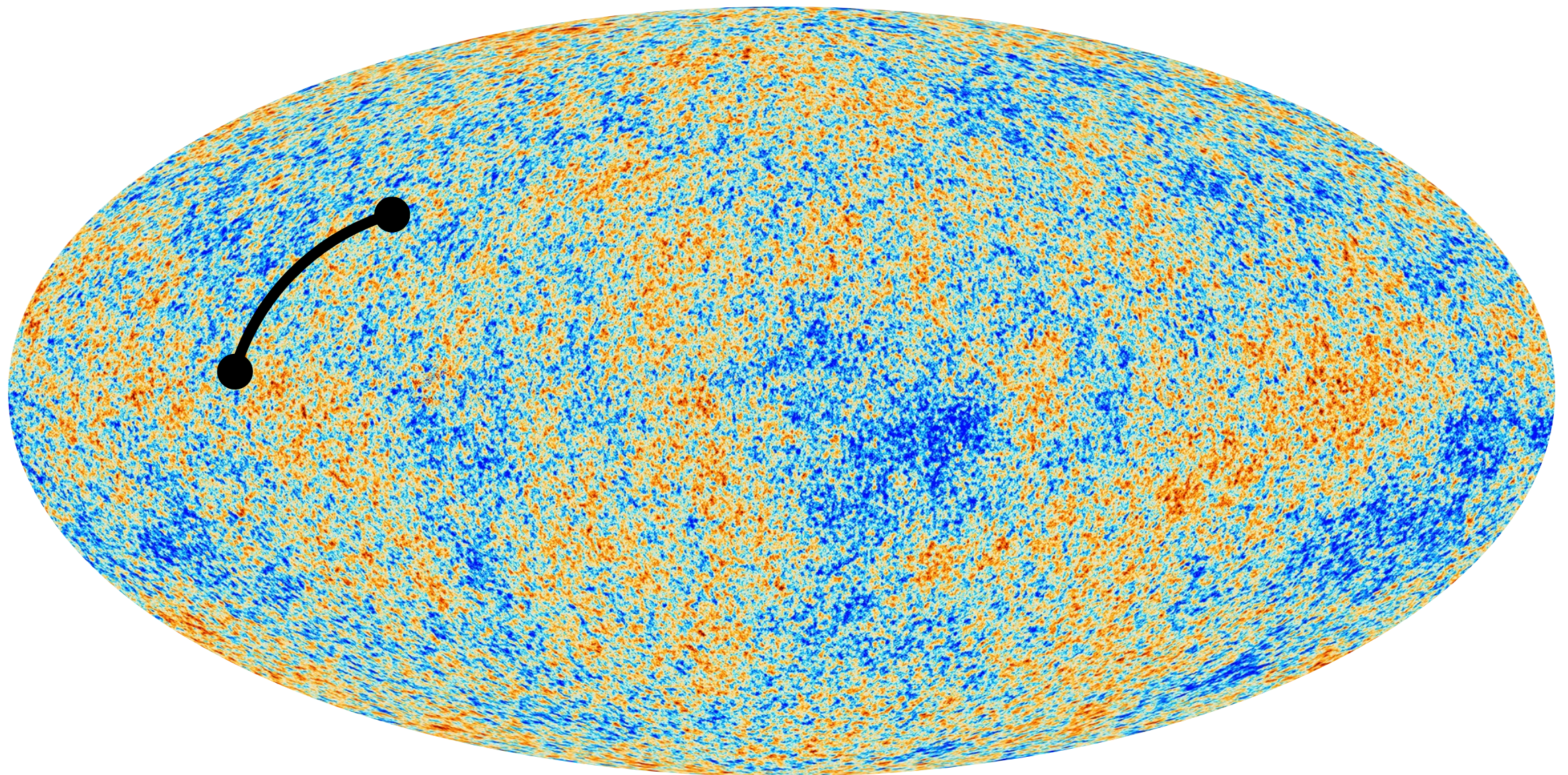
$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle = i^N \frac{\delta}{\delta J_{\mathbf{k}_1}} \cdots \frac{\delta}{\delta J_{\mathbf{k}_N}} Z[J] \Big|_{J=0}$$

$\mathcal{R}(\mathbf{k})$

✿ Non-Gaussianity?

06

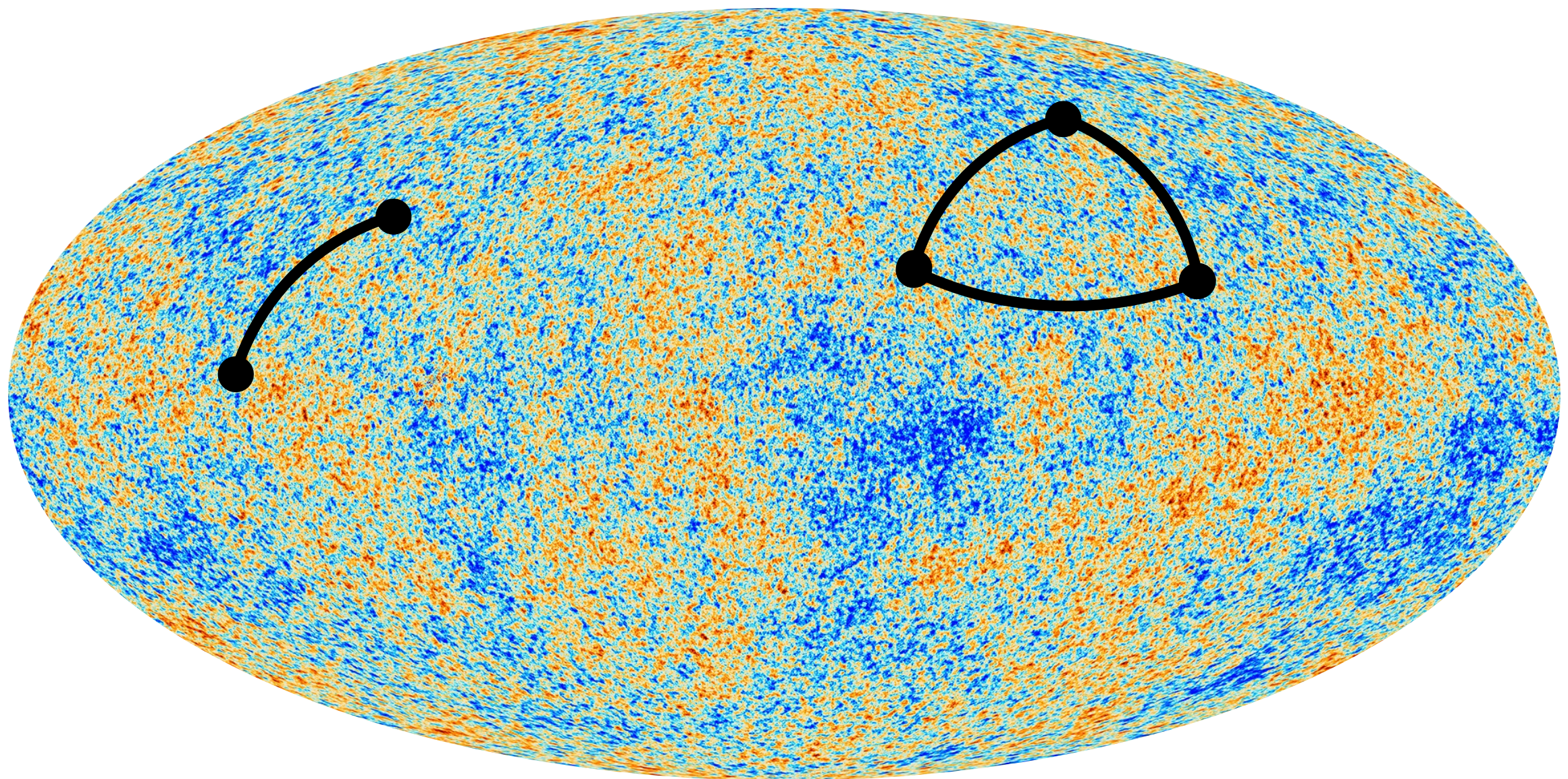
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✿ Non-Gaussianity?

06

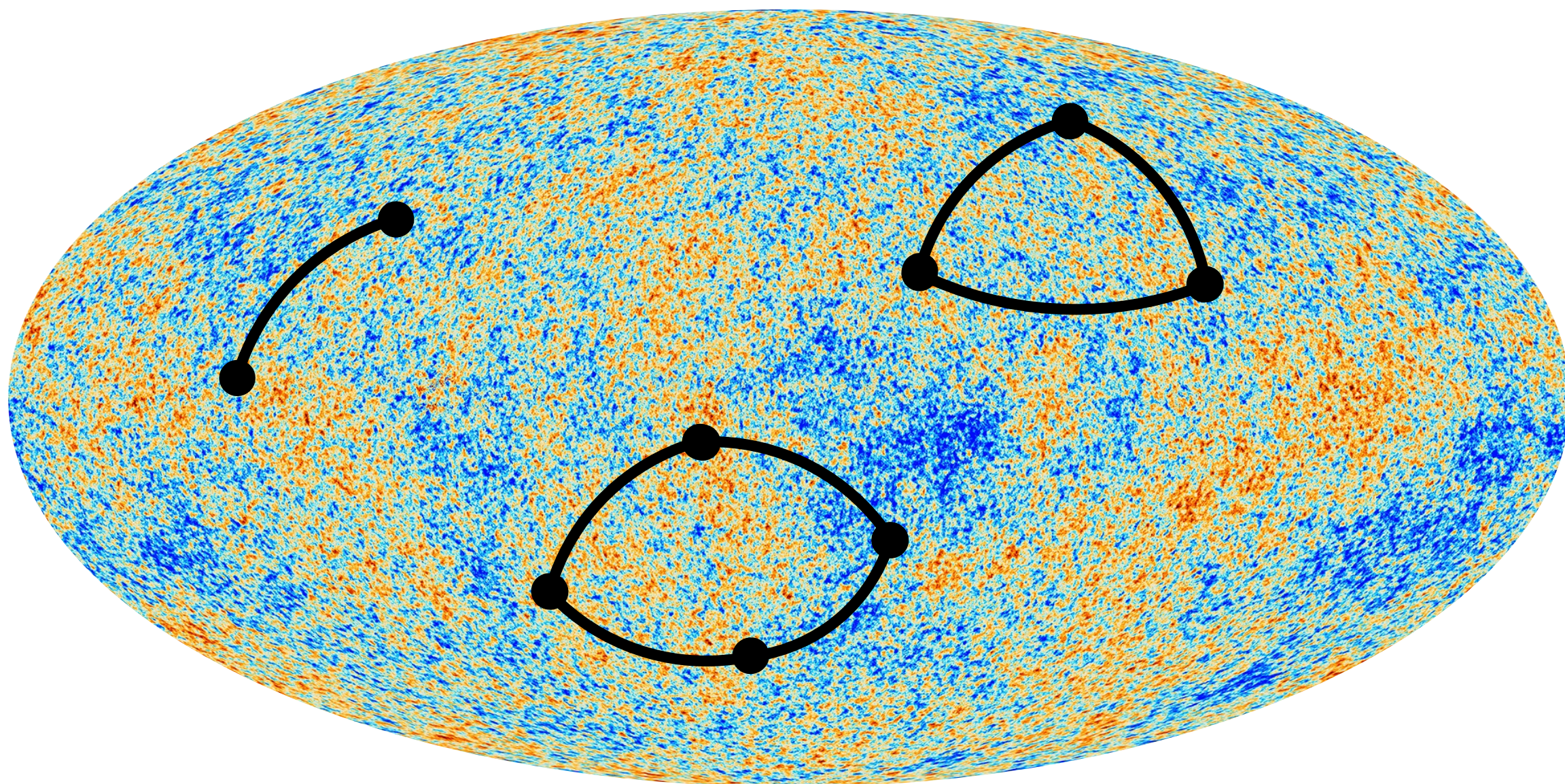
$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle = i^N \frac{\delta}{\delta J_{\mathbf{k}_1}} \cdots \frac{\delta}{\delta J_{\mathbf{k}_N}} Z[J] \Big|_{J=0}$$

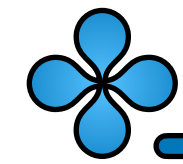


✿ Non-Gaussianity?

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$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle = i^N \frac{\delta}{\delta J_{\mathbf{k}_1}} \cdots \frac{\delta}{\delta J_{\mathbf{k}_N}} Z[J] \Big|_{J=0}$$





Non-Gaussianity?

07

The bispectrum parametrizes the simplest deviation to NG

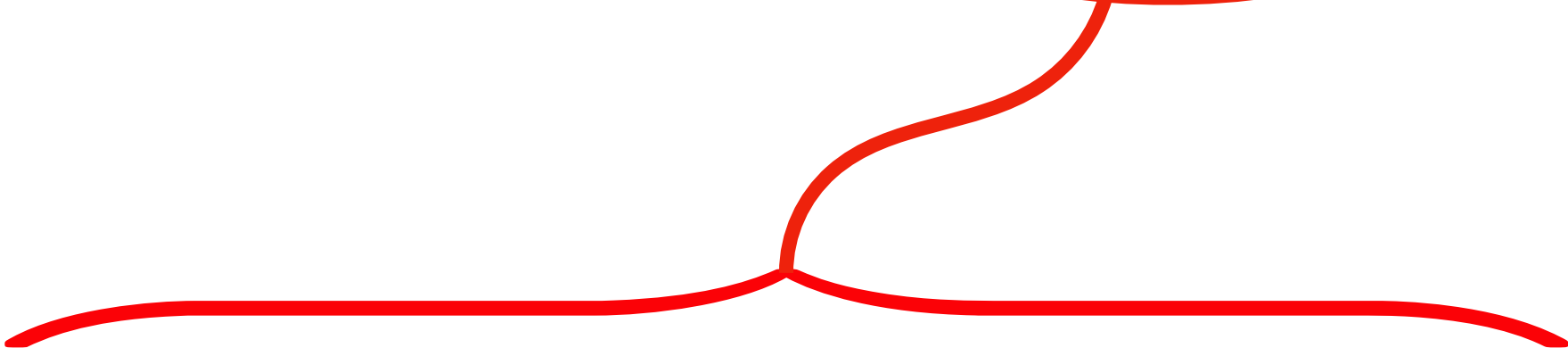
$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2} \int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \dots$$

Non-Gaussianity?

07

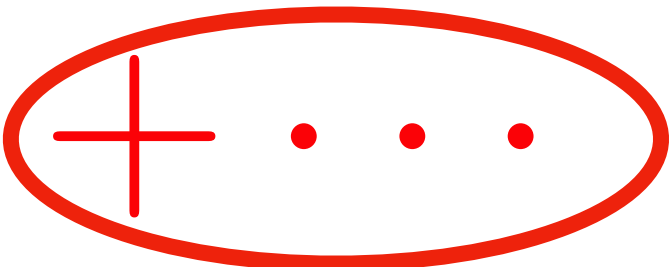
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$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2} \int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} \quad (+ \dots)$$

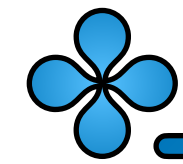


$$+ \dots = \int_{k_1} \int_{k_2} \int_{k_3} B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} + \dots$$

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$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

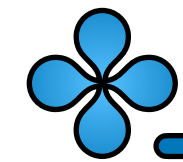


Non-Gaussianity?

08

In the absence of a specific theory, a common parametrization for the bispectrum is the f_{NL} parameter

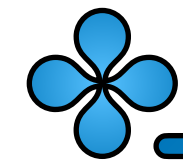
$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_G(\mathbf{x}) + \frac{3}{5} f_{\text{NL}}^{\text{loc}} \mathcal{R}_G^2(\mathbf{x})$$



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$$B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \propto f_{\text{NL}}^{\text{loc}} \times \left(P(\mathbf{k}_1)P(\mathbf{k}_2) + P(\mathbf{k}_2)P(\mathbf{k}_3) + P(\mathbf{k}_3)P(\mathbf{k}_1) \right)$$



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$$B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{18}{5} A^2 \sum_{\text{type}} f_{\text{NL}}^{\text{type}} S_{\text{type}}(k_1, k_2, k_3)$$

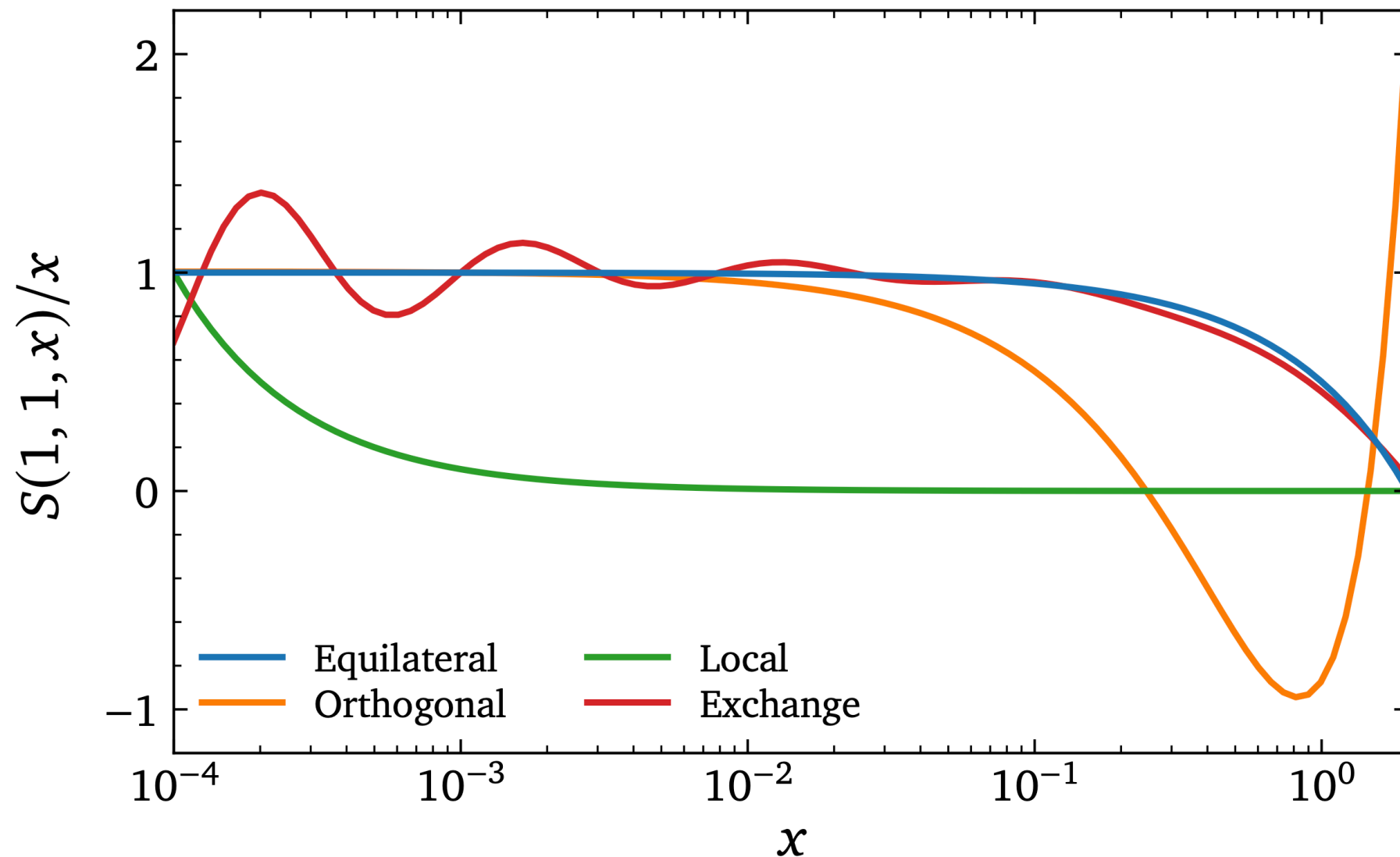
Non-Gaussianity?

08

In the
bispe

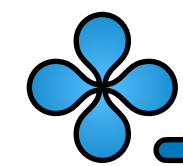
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$B($



Pimentel, Wallisch, Wu, Achúcarro et al. (2022)

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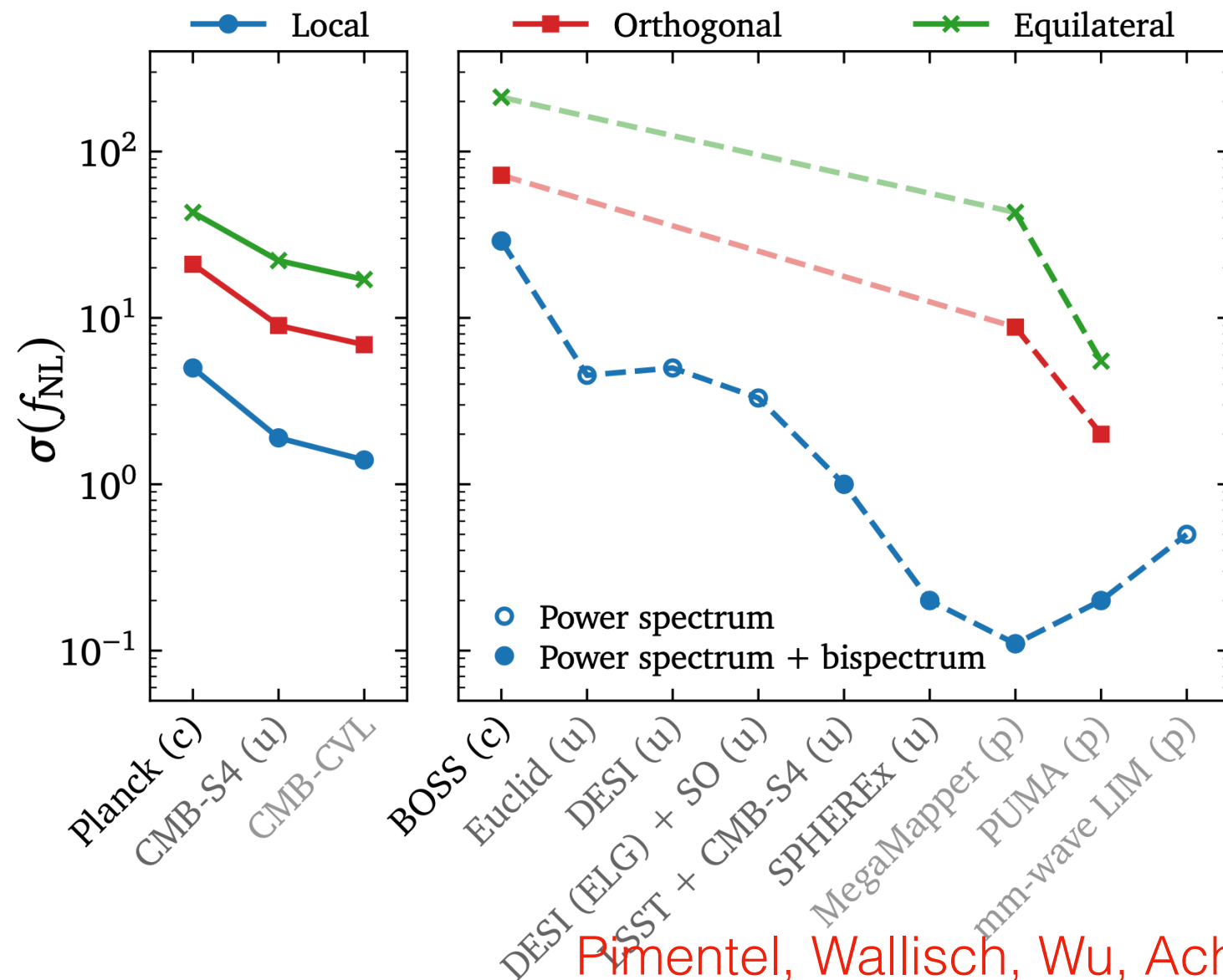
Non-Gaussianity?

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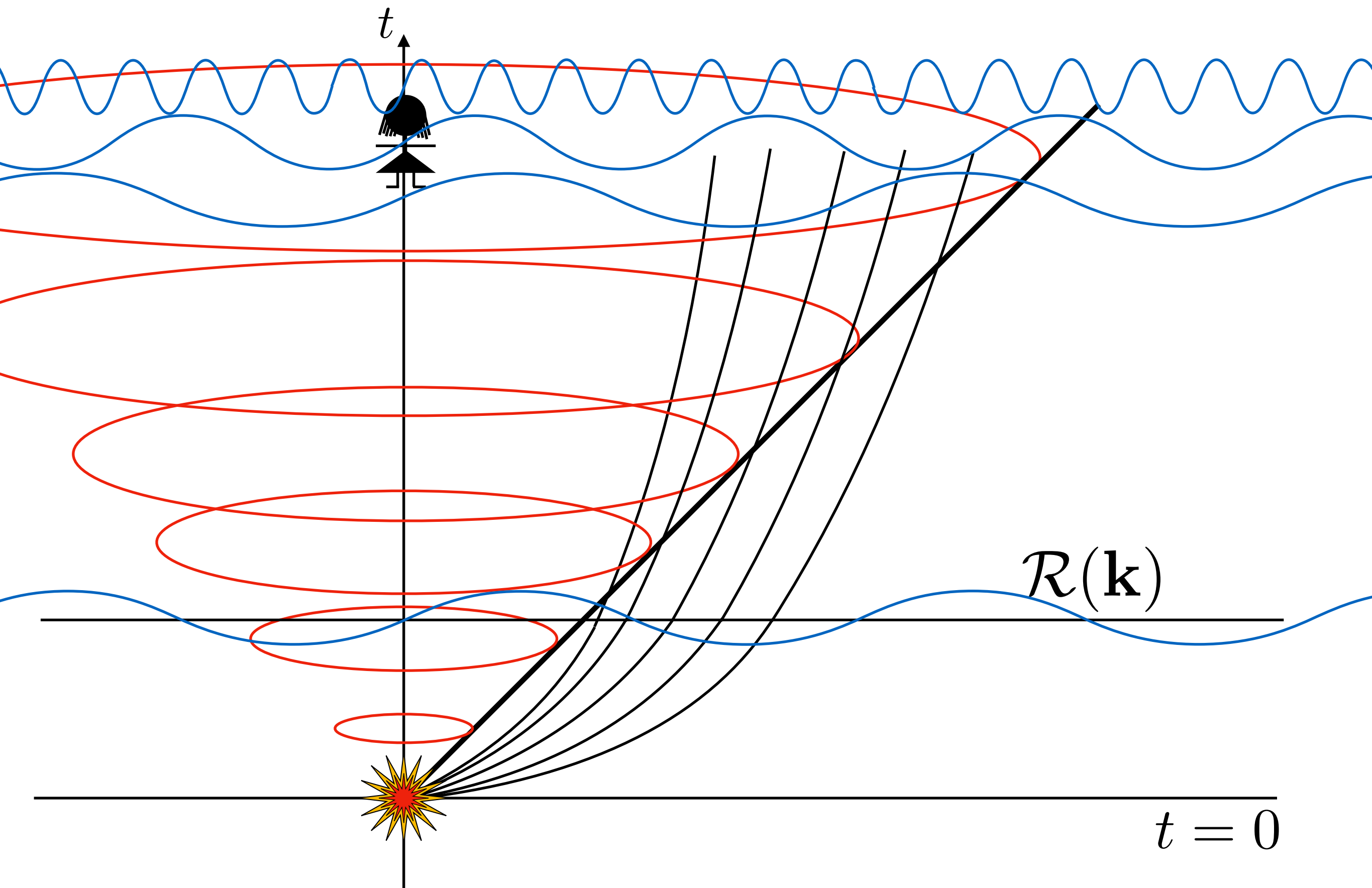
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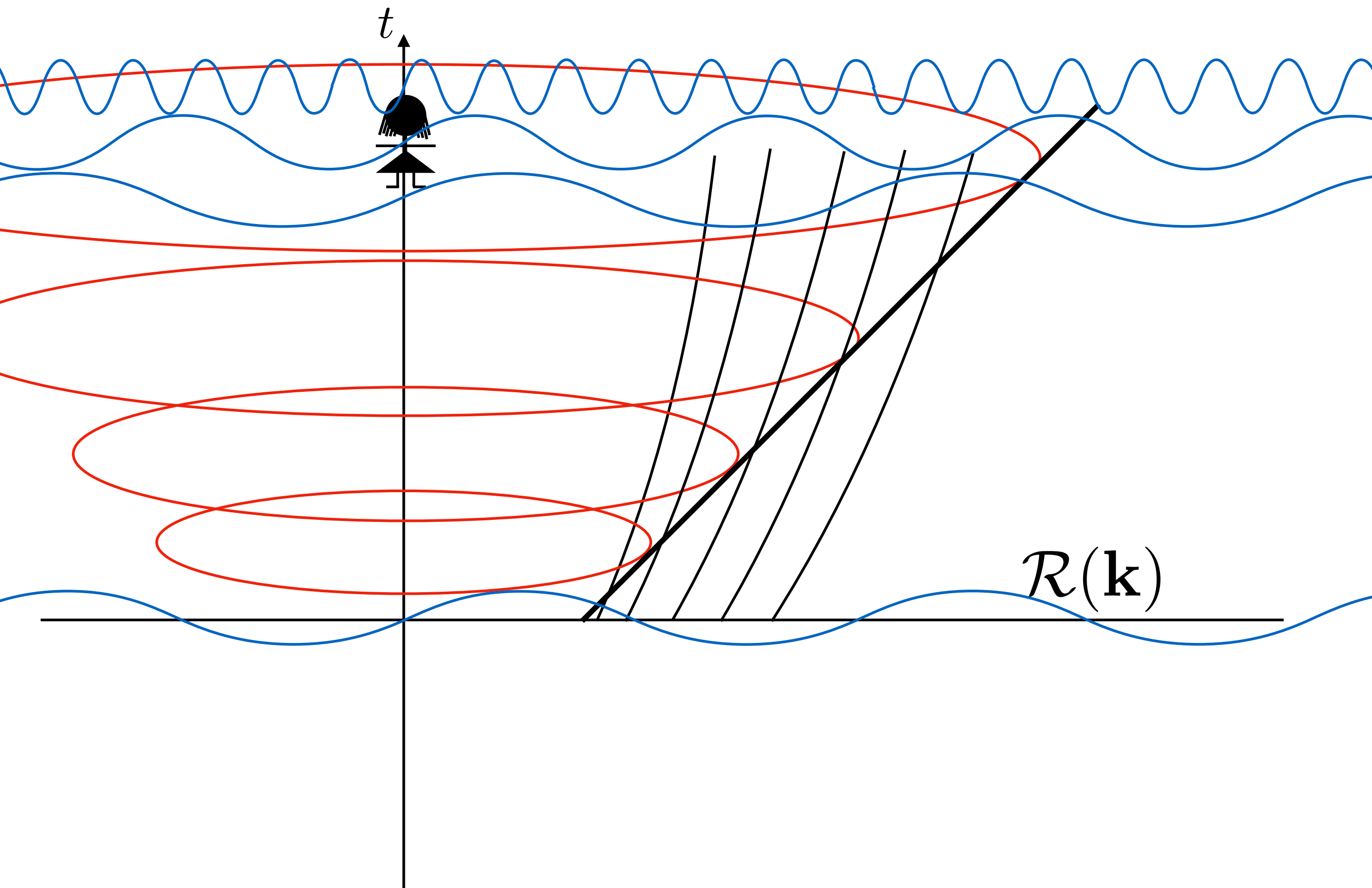
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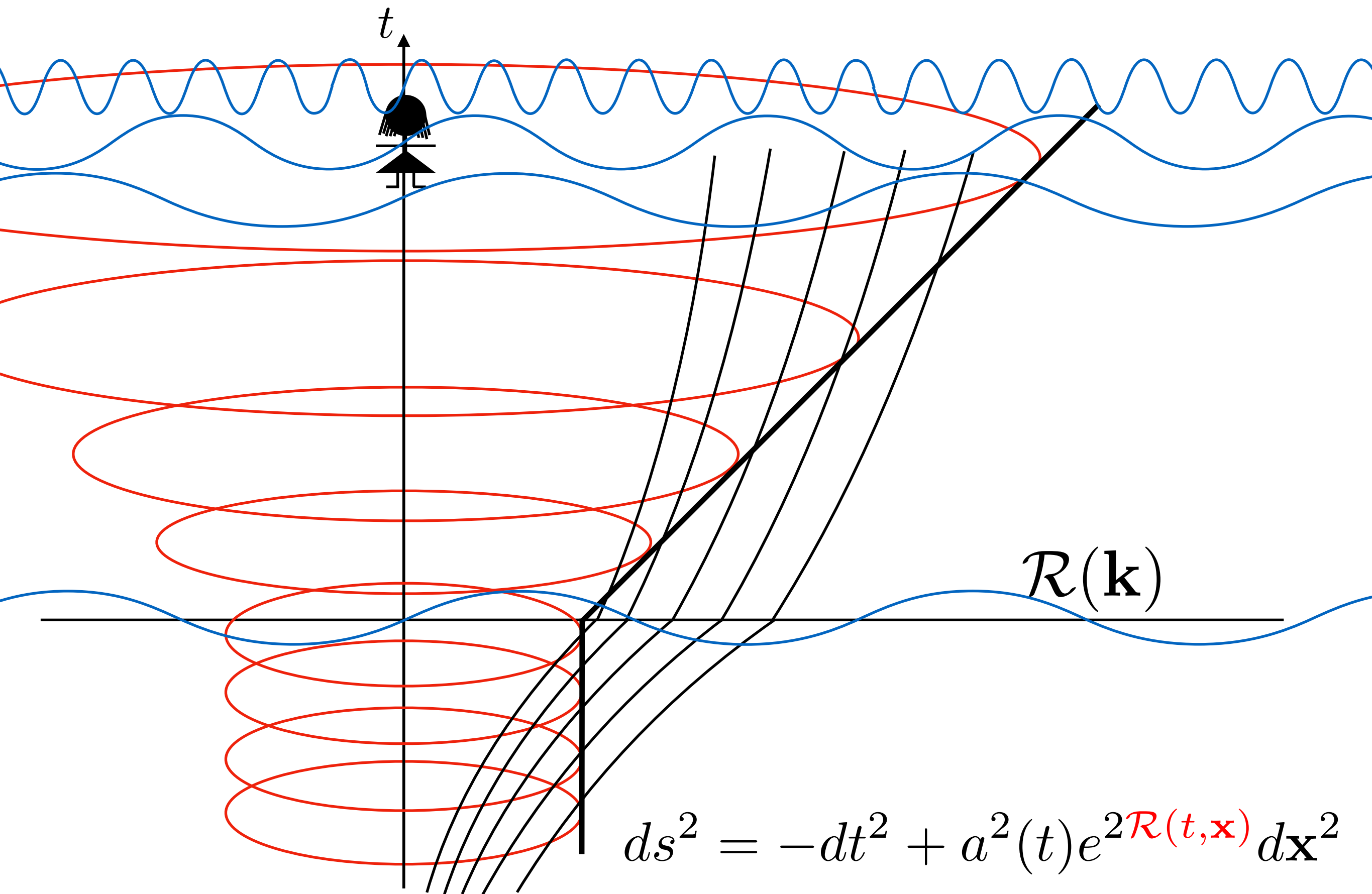


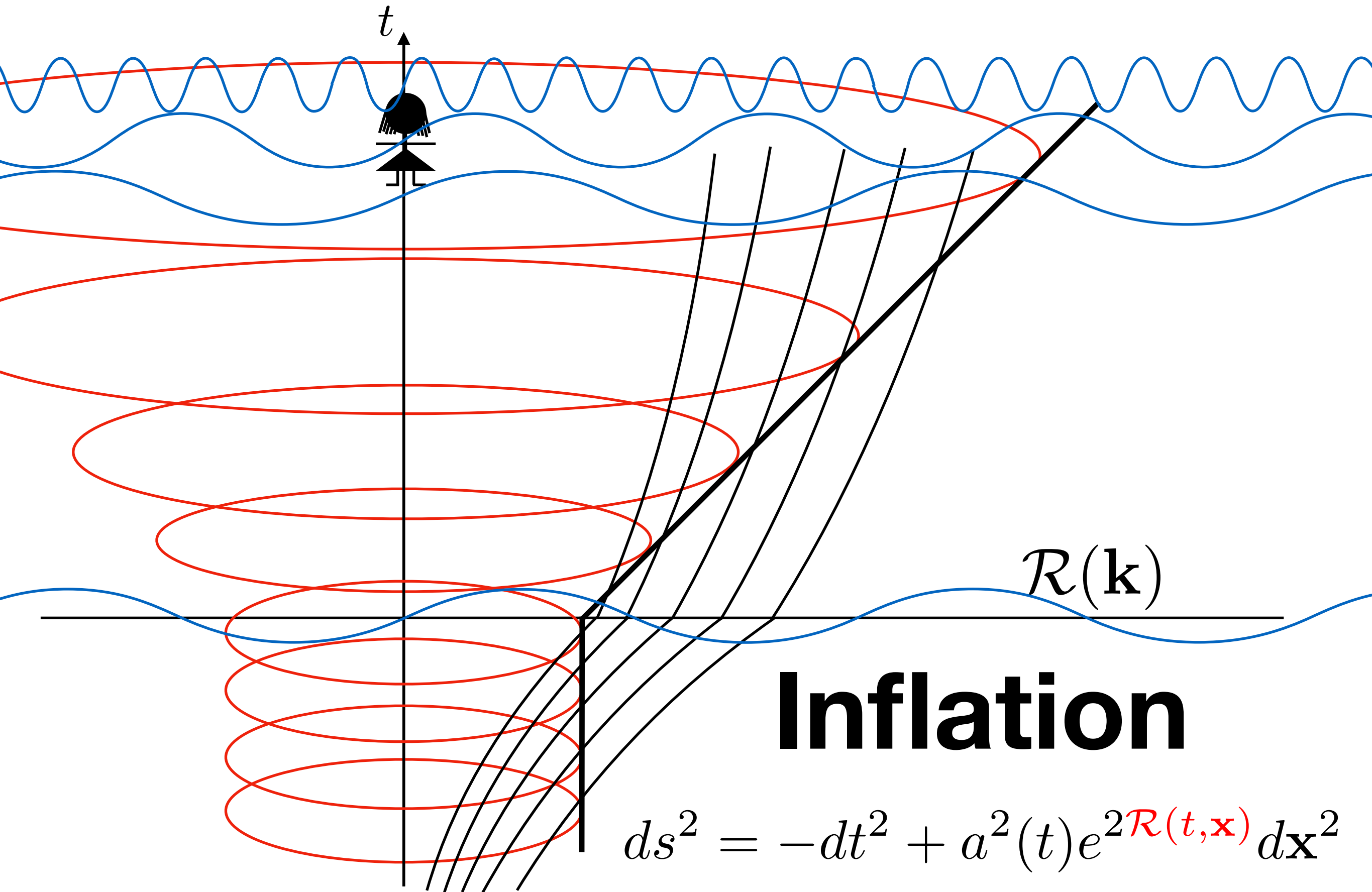
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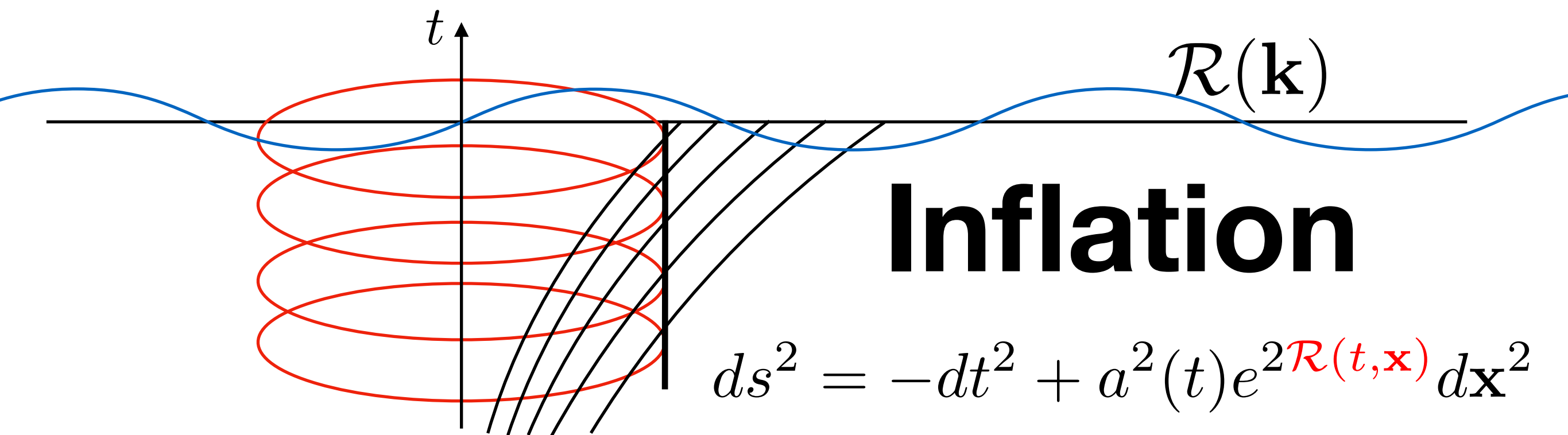








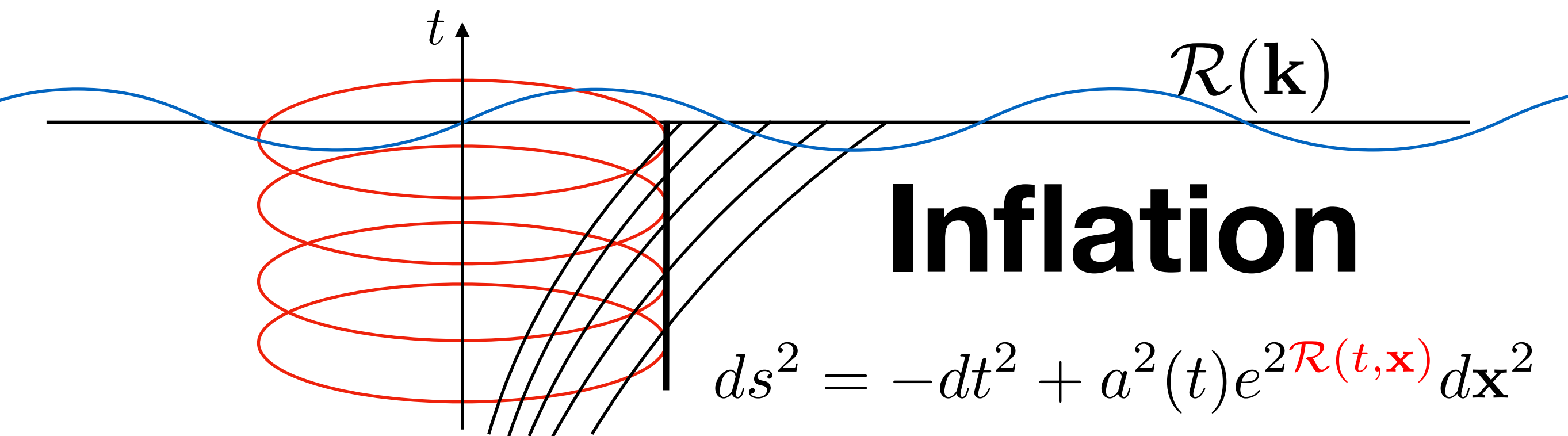
We can now write a theory for $\mathcal{R}(\mathbf{k})$



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$$S = \int d^4x a^3 \epsilon \left[\dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 + \mathcal{O}(\mathcal{R}^3) + \mathcal{O}(\mathcal{R}^4) + \dots \right]$$

There are three favorite ways to compute observables from this theory



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There are three favorite ways to compute observables from this theory

(1) Hamiltonian in-in formalism:

$$H(t) = H^{(2)}(t) + H^{(3)}(t) + \dots$$

$$U(t) = \mathcal{T} \exp \left\{ -i \int_{-\infty}^t dt' H_I(t') \right\}$$

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle = \langle 0 | U^\dagger(t) \mathcal{R}_{\mathbf{k}_1}^I(t) \cdots \mathcal{R}_{\mathbf{k}_N}^I(t) U(t) | 0 \rangle$$

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$$S = \int d^4x a^3 \epsilon \left[\dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 + \mathcal{O}(\mathcal{R}^3) + \mathcal{O}(\mathcal{R}^4) + \dots \right]$$

There are three favorite ways to compute observables from this theory

(2) Wavefunction of the Universe:

$$\Psi[\mathcal{R}(\mathbf{x})] = \int_{\mathcal{R}_{BD}}^{\mathcal{R}(\mathbf{x})} D\mathcal{R} e^{iS[\mathcal{R}]/\hbar} \quad \rho[\mathcal{R}(\mathbf{x})] = \left| \Psi[\mathcal{R}(\mathbf{x})] \right|^2$$

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle = \int D\mathcal{R}(\mathbf{x}) \rho[\mathcal{R}(\mathbf{x})] \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t)$$

We can now write a theory for $\mathcal{R}(\mathbf{k})$

$$S = \int d^4x a^3 \epsilon \left[\dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 + \mathcal{O}(\mathcal{R}^3) + \mathcal{O}(\mathcal{R}^4) + \dots \right]$$

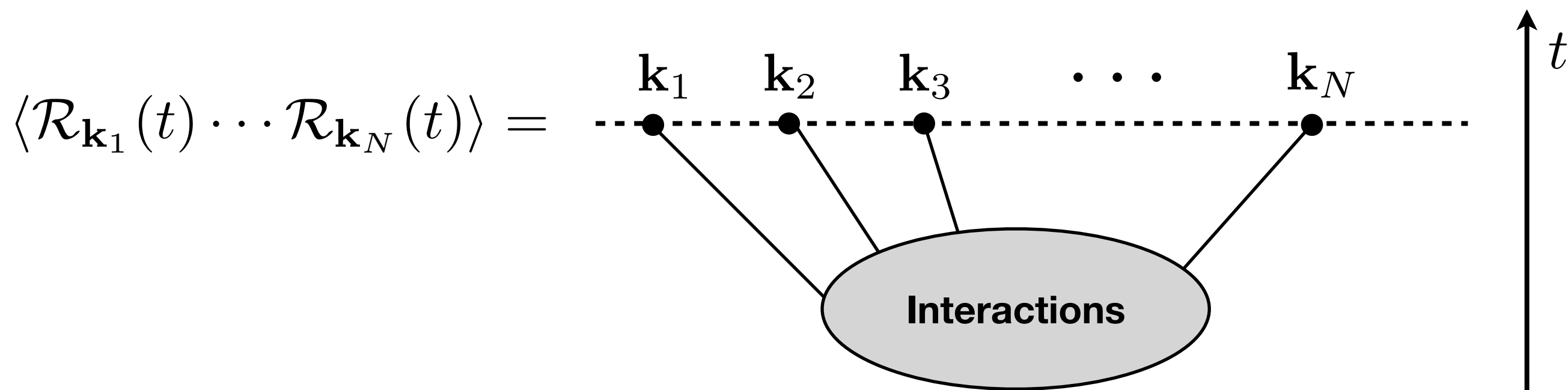
There are three favorite ways to compute observables from this theory

(3) Schwinger-Keldysh in-in formalism:

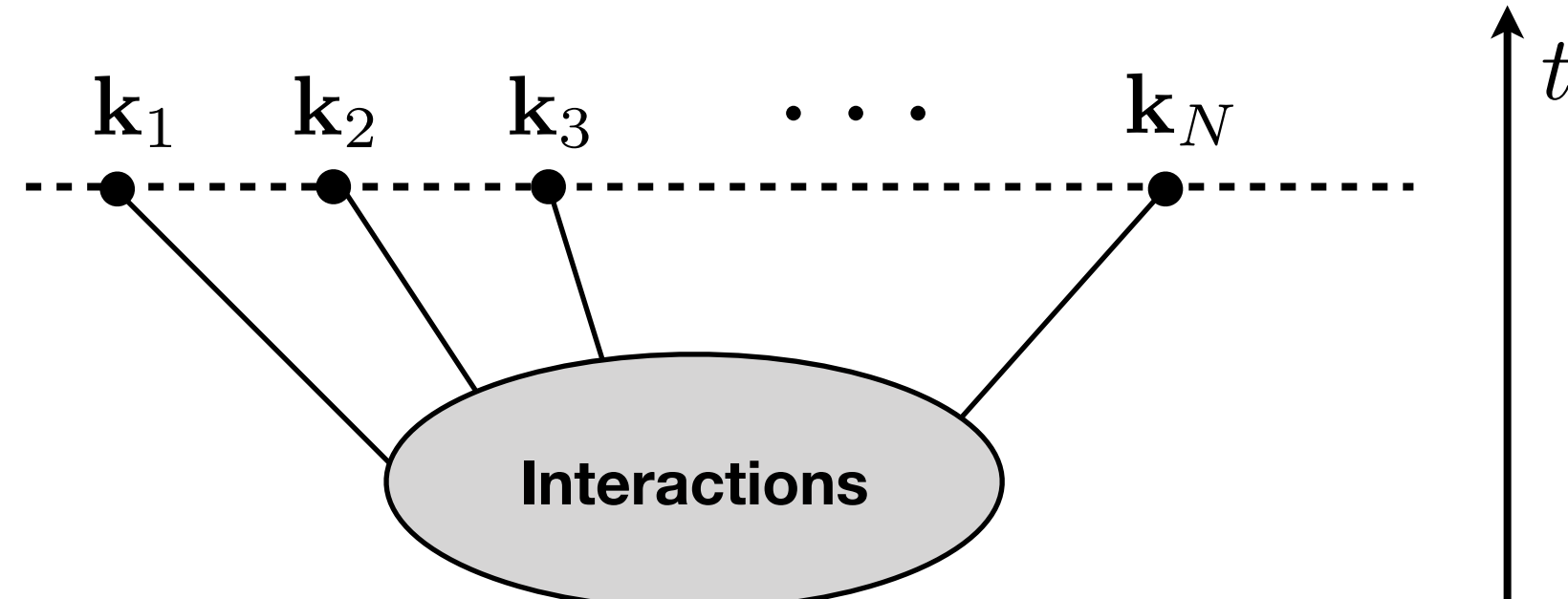
$$Z[J_+, J_-] = \int_{\mathcal{R}_+(t_{\text{end}}) = \mathcal{R}_-(t_{\text{end}})} D\mathcal{R}_+ D\mathcal{R}_- e^{iS[\mathcal{R}_+] - iS[\mathcal{R}_-] + i \int_x \mathcal{R}_+ J_+ - i \int_x \mathcal{R}_- J_-}$$

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle = \frac{\delta}{\delta J_+(\mathbf{k}_1)} \cdots \frac{\delta}{\delta J_+(\mathbf{k}_N)} Z[J_+, J_-]$$

These methods offer a diagrammatic procedure to compute n-points

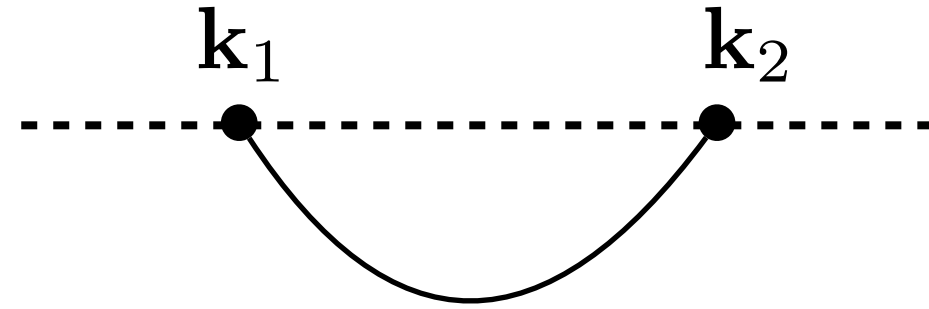


These methods offer a diagrammatic procedure to compute n-points

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle =$$


The diagram shows a horizontal dashed line representing time t . Points on this line are labeled $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \dots, \mathbf{k}_N$. A shaded oval labeled "Interactions" is connected by solid lines to the points $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$, and $\mathbf{k}_N$.

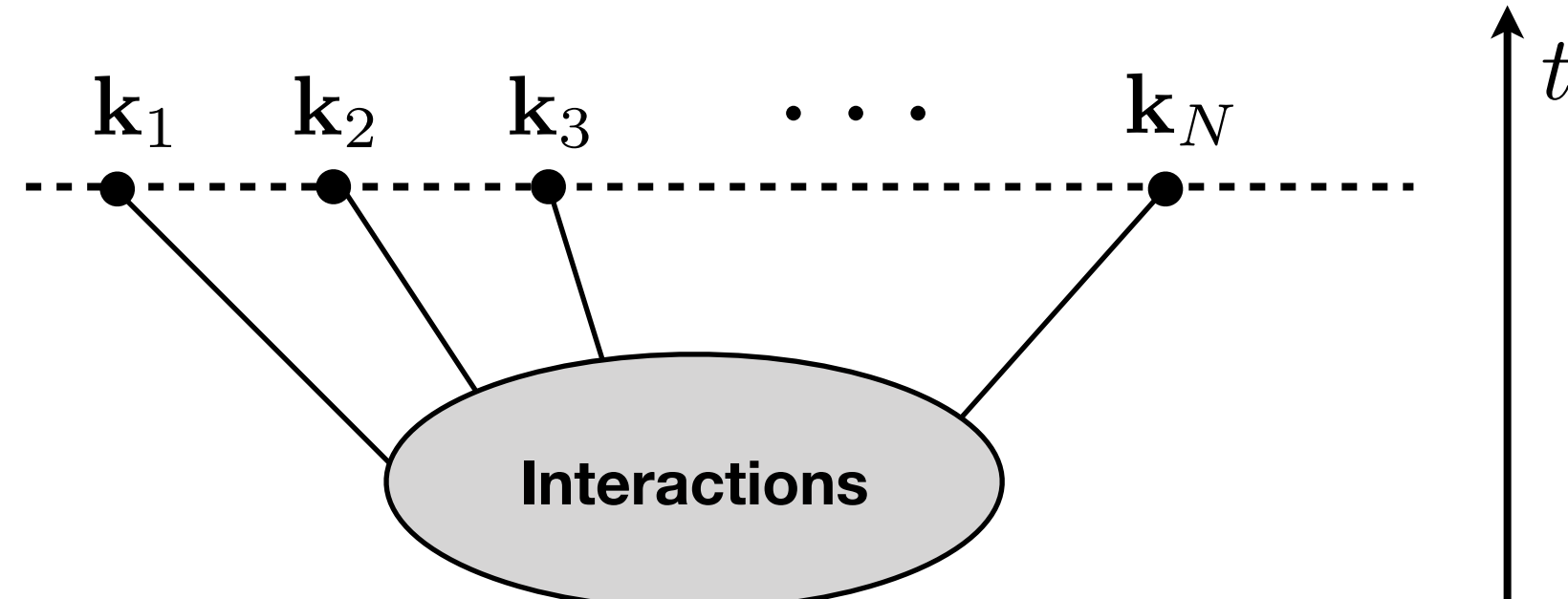
Power spectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \rangle =$$


$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k)$$

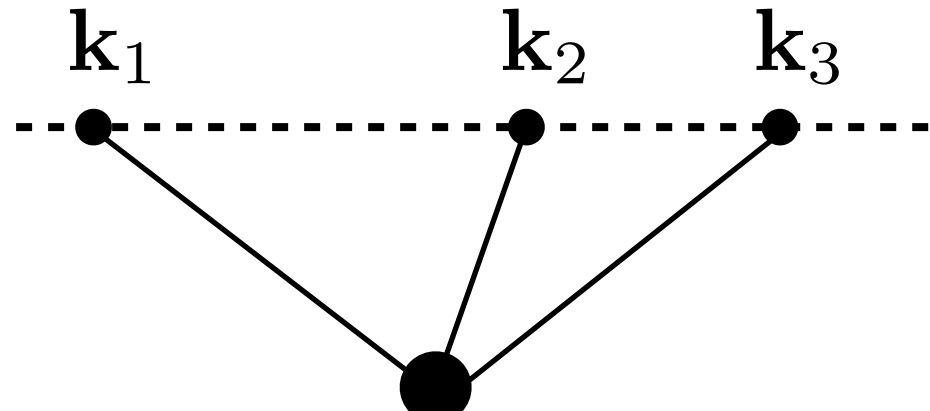
The diagram shows a horizontal dashed line with two points labeled $\mathbf{k}_1$ and $\mathbf{k}_2$. A curved line connects these two points below the dashed line.

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$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle =$$


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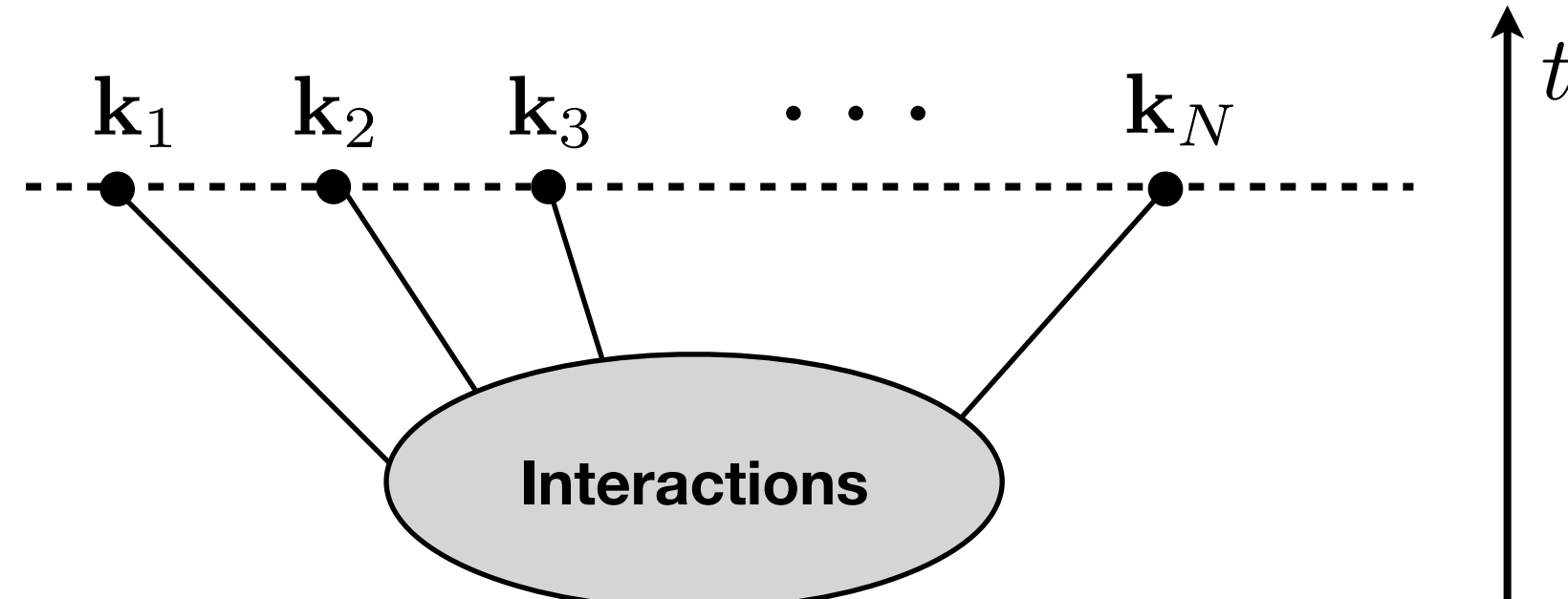
Bi-spectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle =$$


The diagram shows a horizontal dashed line with three points labeled $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$. Solid lines connect these three points to a single vertex below the line.

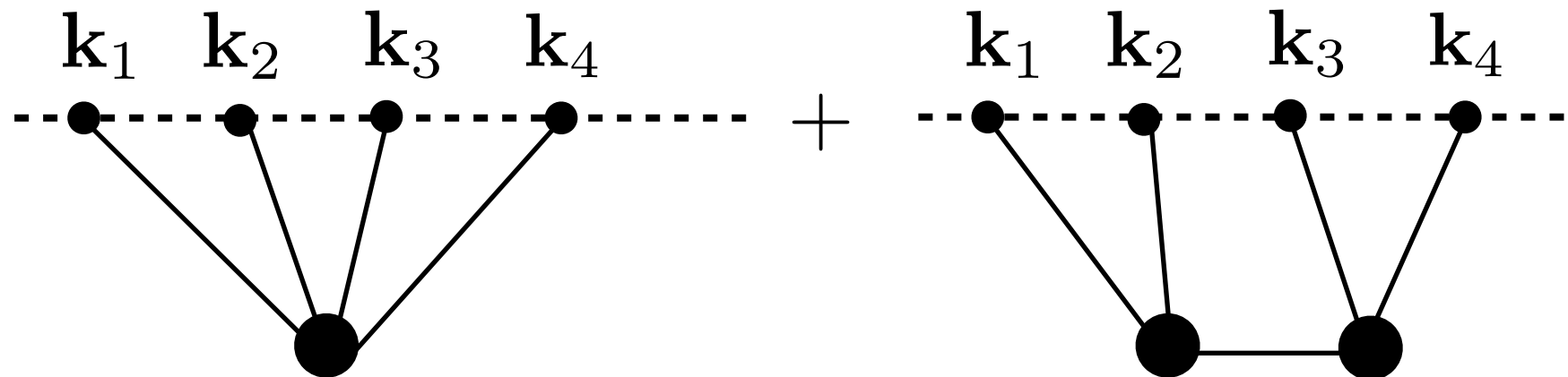
$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

These methods offer a diagrammatic procedure to compute n-points

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle =$$


The diagram shows a horizontal dashed line representing time t . Points on this line are labeled $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \dots, \mathbf{k}_N$. Lines connect $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ and $\mathbf{k}_N$ to a central shaded oval labeled "Interactions".

Tri-spectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle =$$


The diagram shows two terms for the tri-spectrum, separated by a plus sign. Each term has a horizontal dashed line with four points labeled $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4$. In the first term, lines connect $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ and $\mathbf{k}_4$ to a single vertex below. In the second term, lines connect $\mathbf{k}_1, \mathbf{k}_2$ and $\mathbf{k}_3$ to a vertex, and $\mathbf{k}_4$ to another vertex, which are connected by a horizontal line.

$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \tau(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_3)$$

Single-field slow-roll inflation predicts small amounts of NG:

$$f_{\text{NL}}^{\text{type}} \simeq \mathcal{O}(\epsilon, \eta)$$

Maldacena (2002)

$$f_{\text{NL}}^{\text{loc}} = 0$$

Tanaka & Urakawa (2011)

Pajer, Schmidt & Zaldarriaga (2013)

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More general types of single-field inflation can enhance NG:

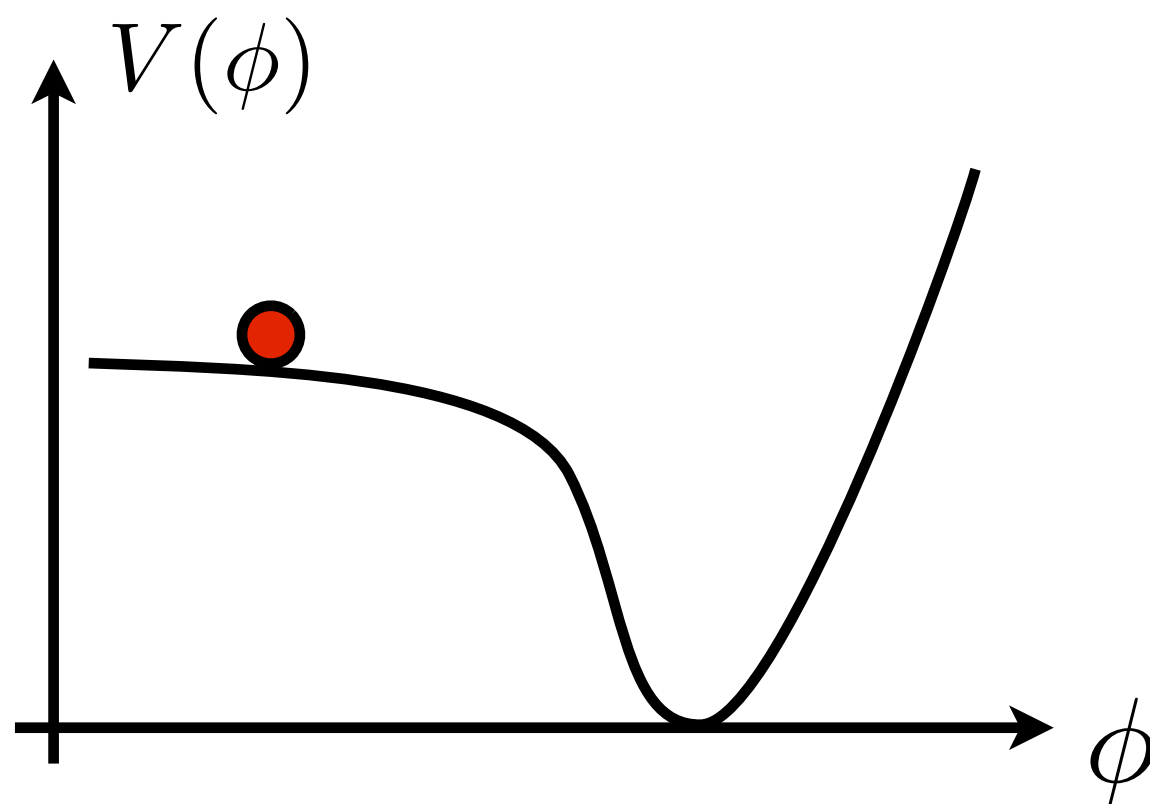
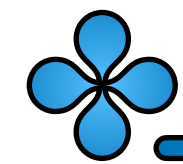
$$\mathcal{L} = \epsilon \left(c_s^2 \dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 \right) + \left(\frac{1}{c_s^2} - 1 \right) \times \mathcal{O}(\mathcal{R}^3) + \dots$$

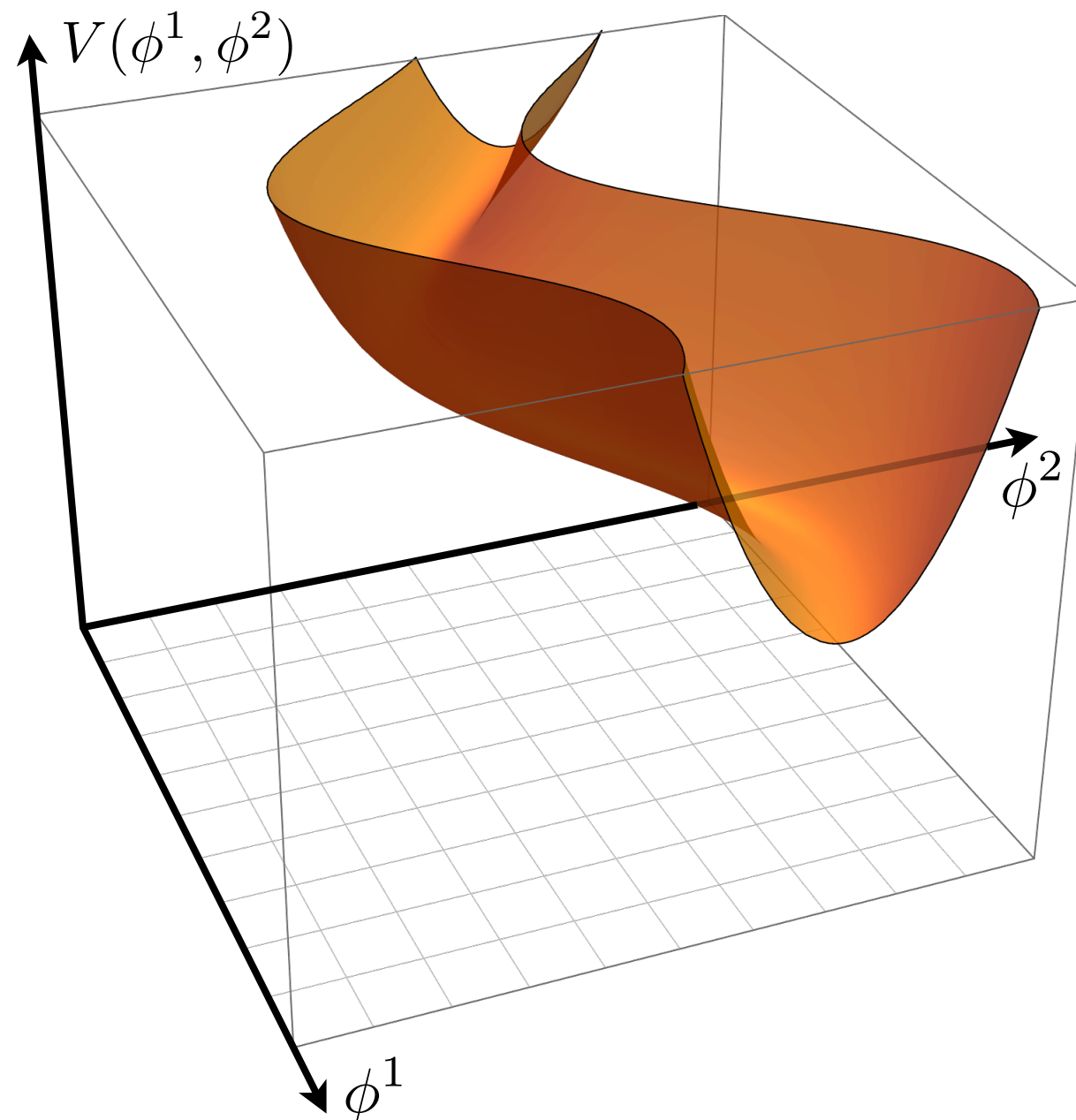
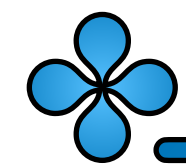
$$f_{\text{NL}}^{\text{equil}} \simeq \mathcal{O} \left(\frac{1}{c_s^2} - 1 \right)$$

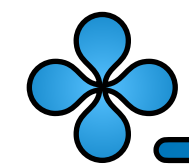
Chen, Huang, Kachru & Shiu (2007)

$$f_{\text{NL}}^{\text{loc}} = 0$$

Creminelli & Zaldarriaga (2004)

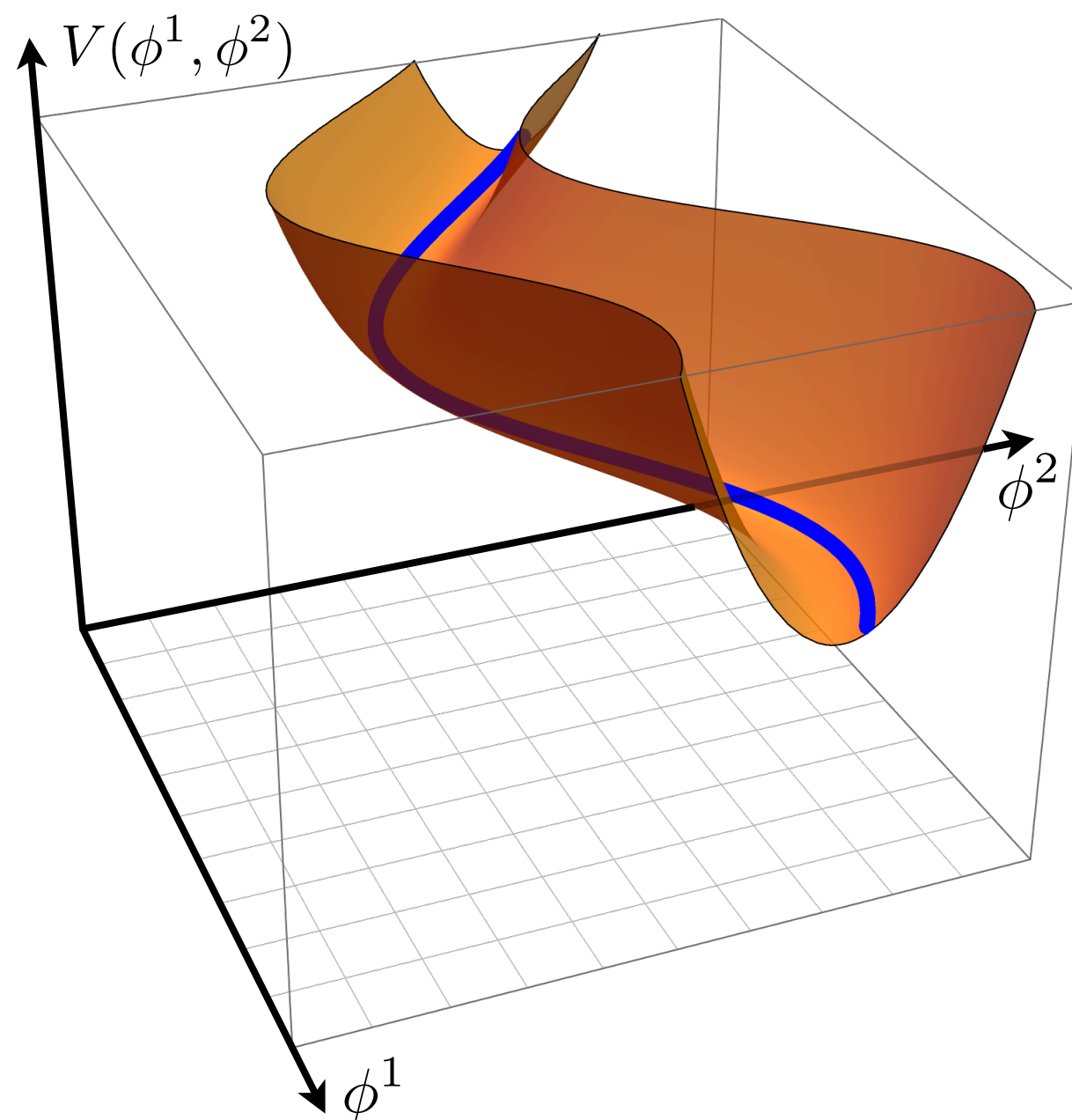


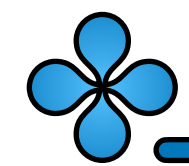




Multi-field inflation

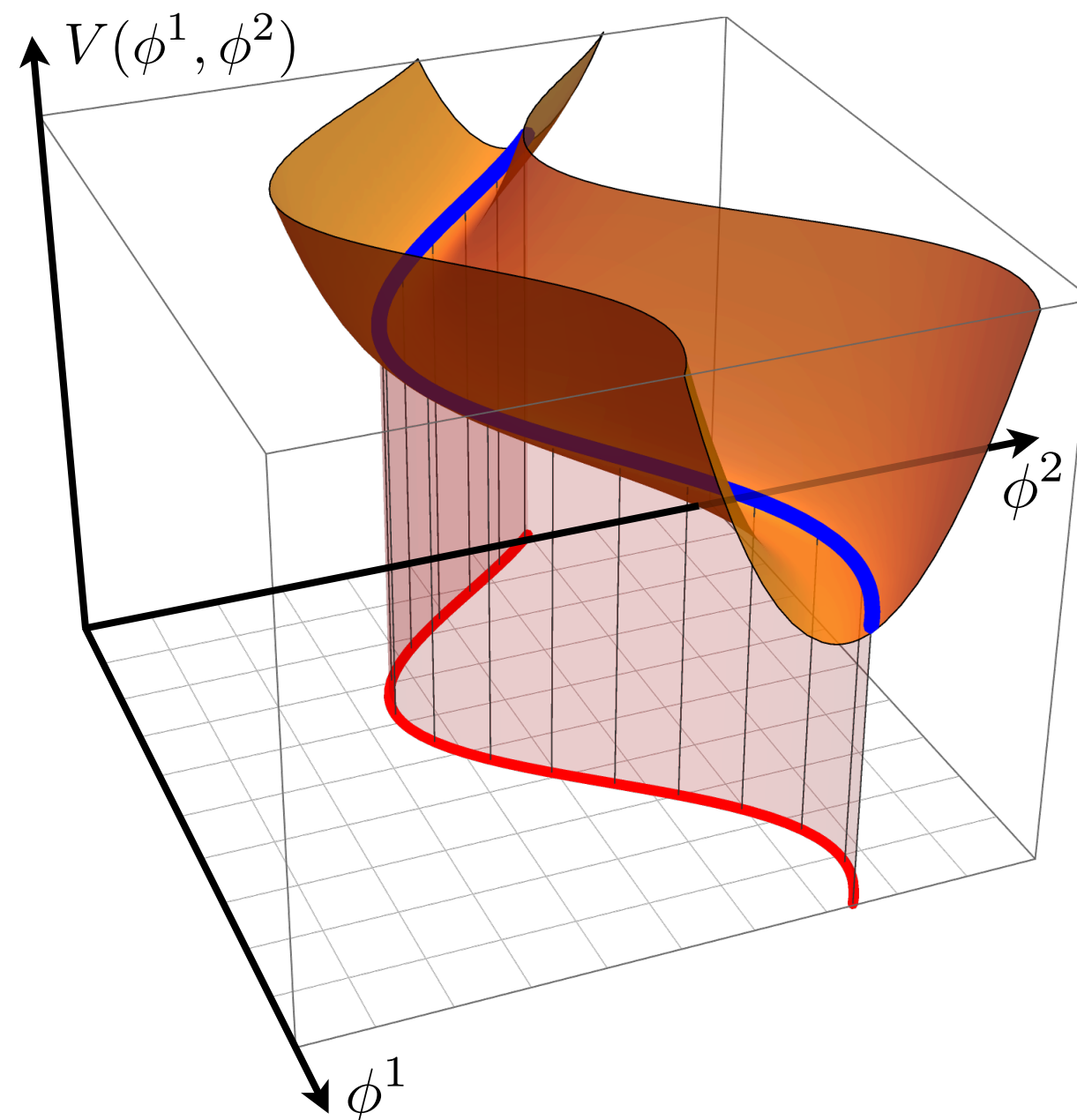
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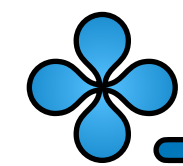




Multi-field inflation

12

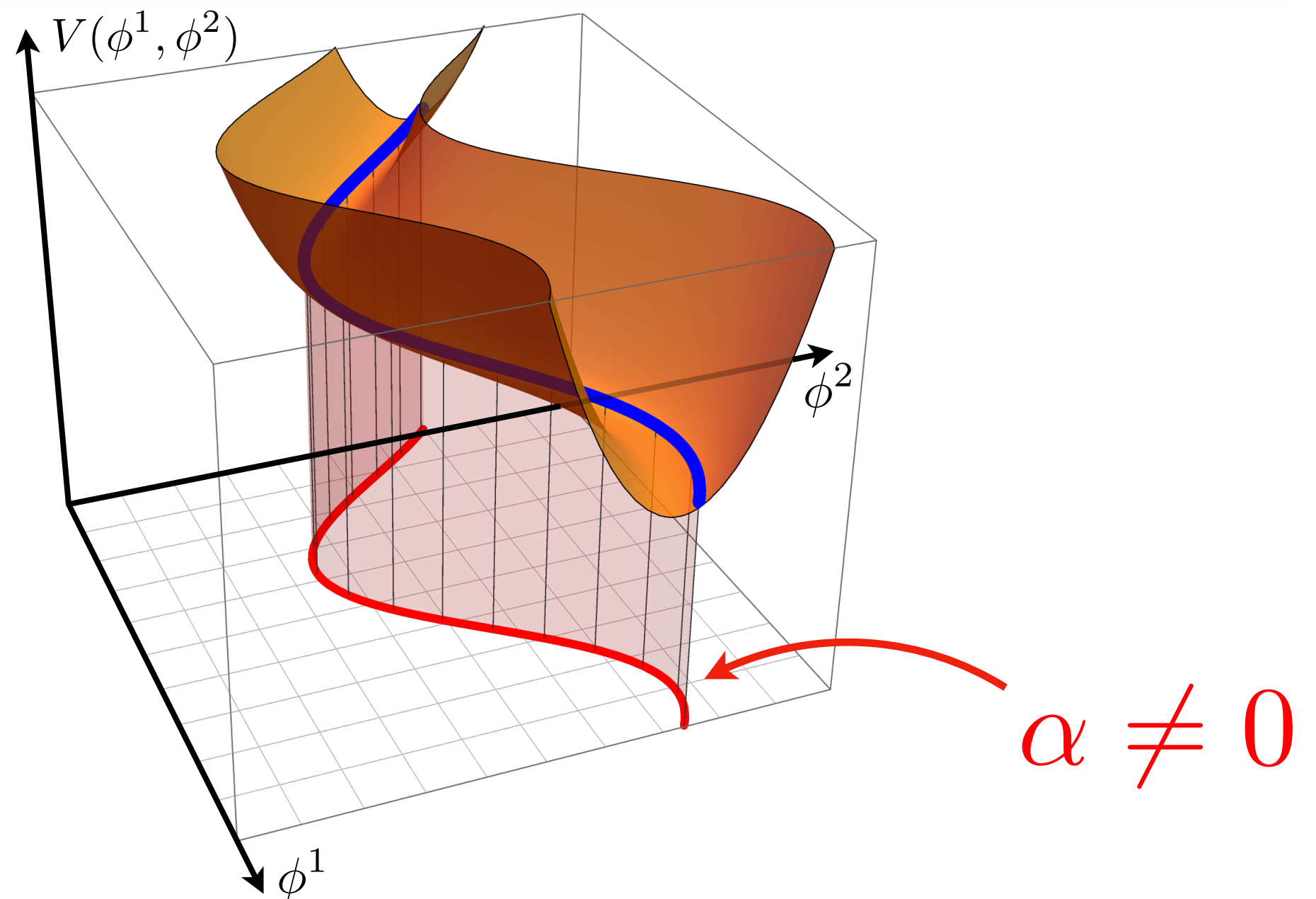


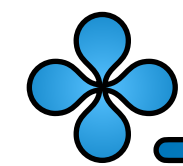


Multi-field inflation

12

$$\mathcal{L} = \epsilon \left(\dot{\mathcal{R}} - \alpha \psi \right)^2 - \frac{\epsilon}{a^2} (\nabla \mathcal{R})^2 + \frac{1}{2} \dot{\psi}^2 - \frac{1}{a^2} (\nabla \psi)^2 \\ + \epsilon \left(\dot{\mathcal{R}} - \alpha \psi \right)^3 - V(\psi) + \dots$$

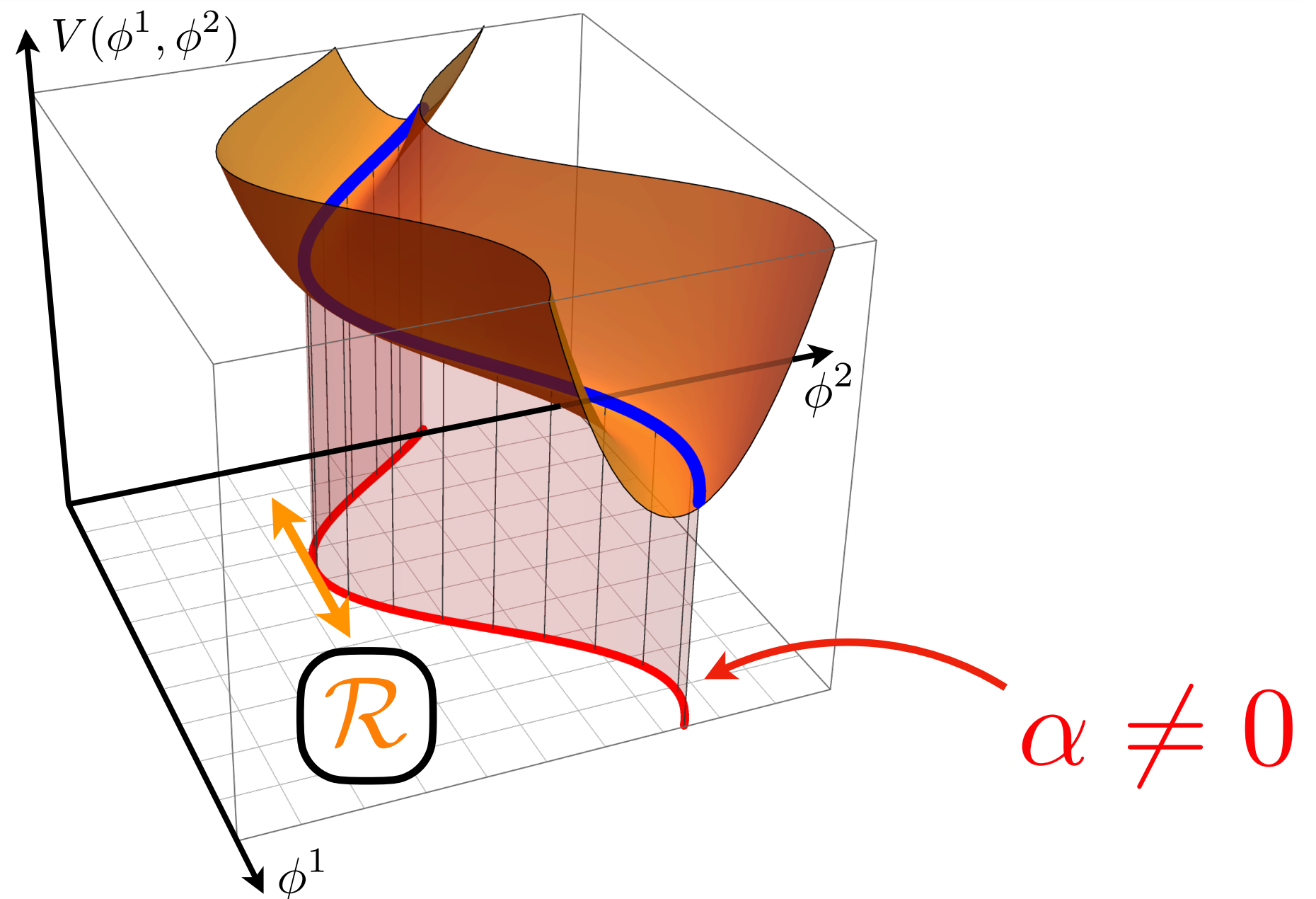




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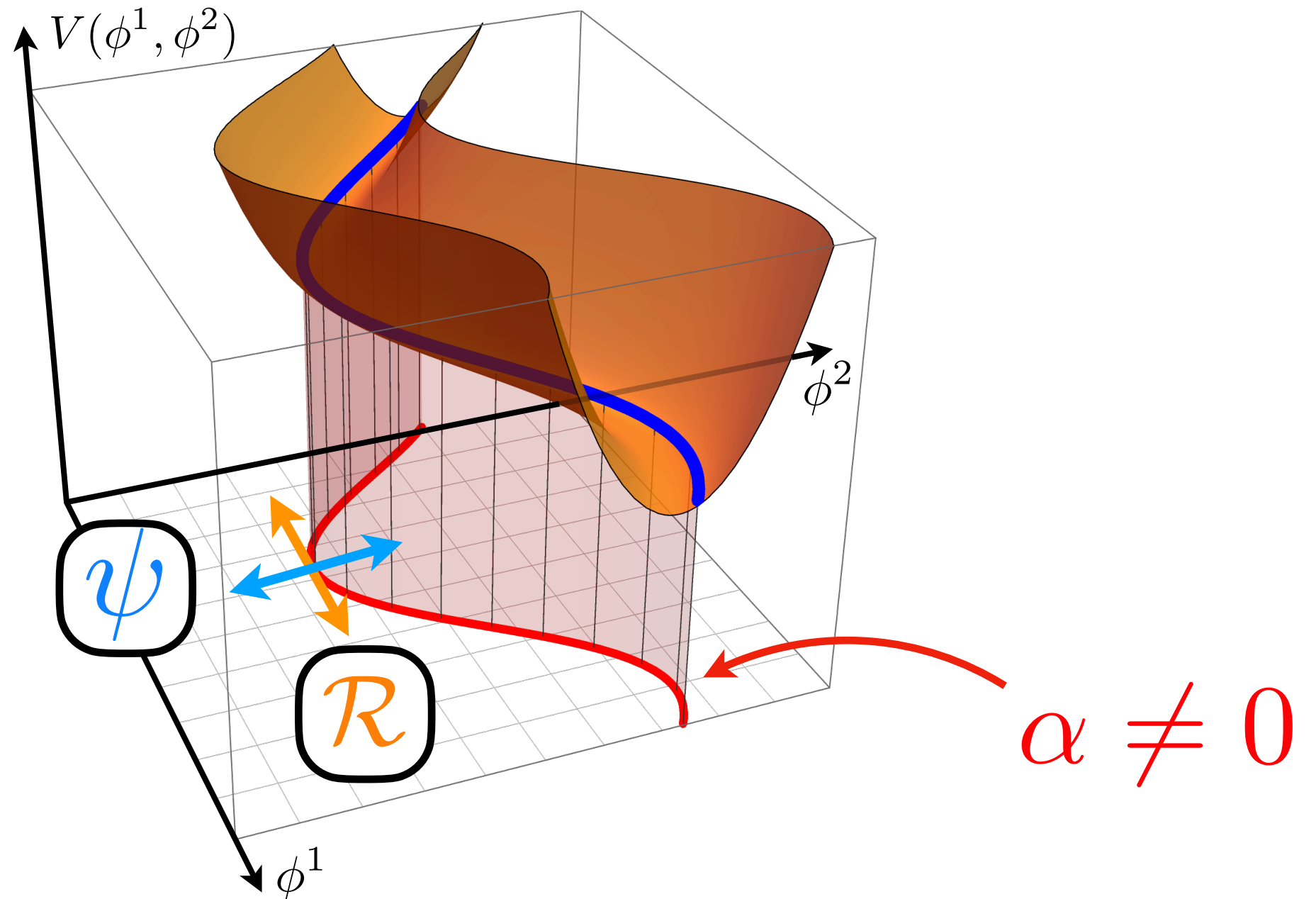
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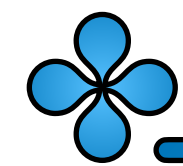
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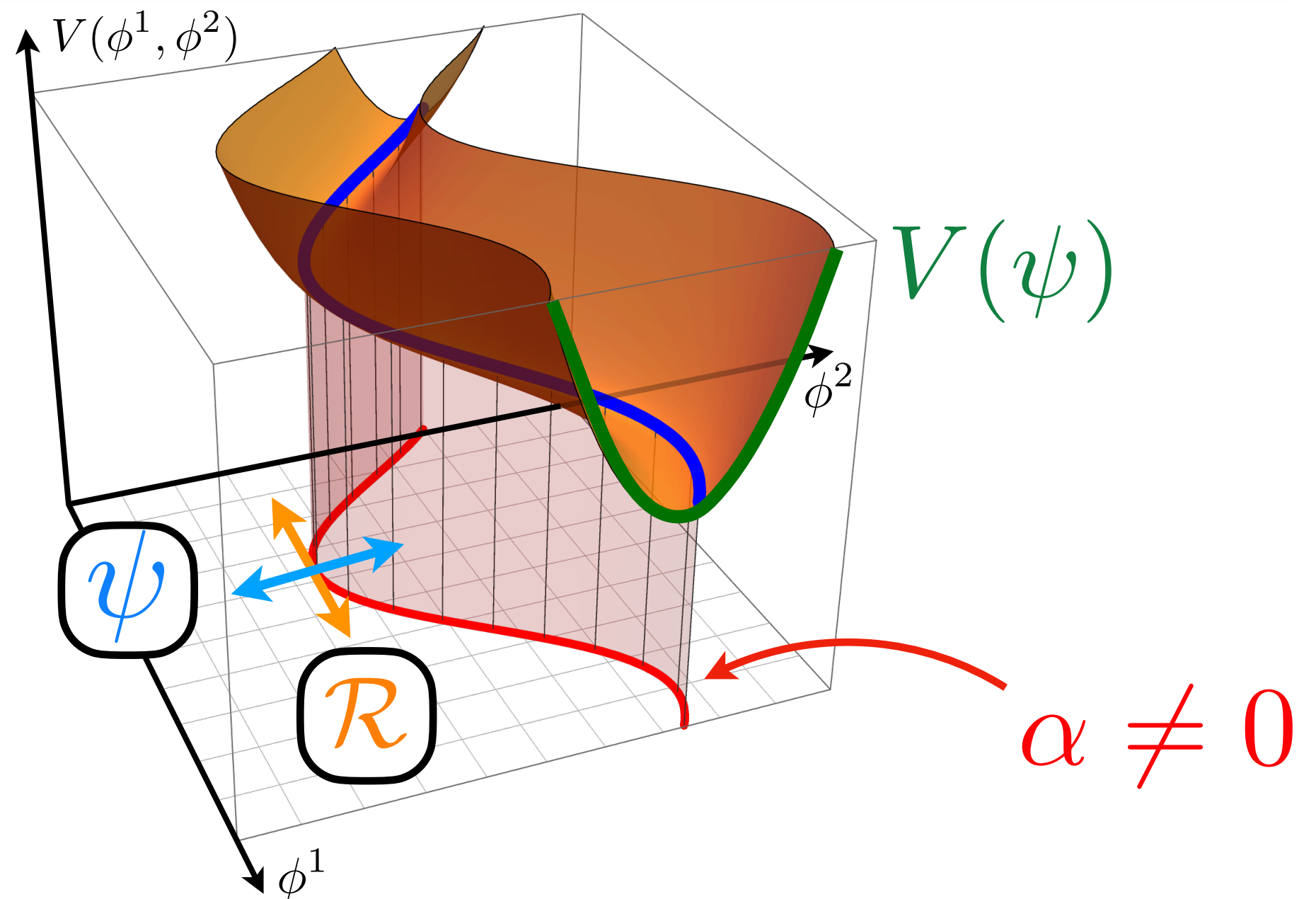


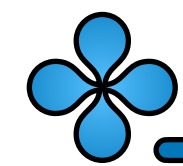


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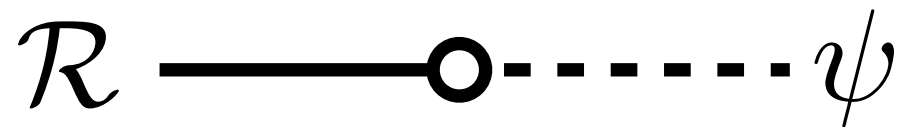


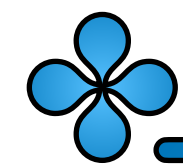


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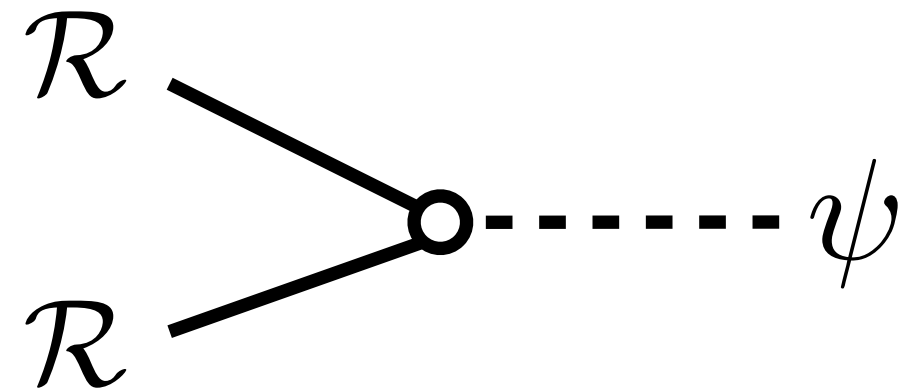
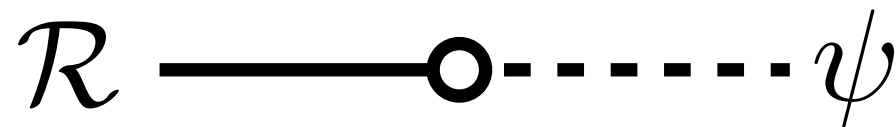


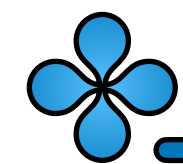


Multi-field inflation

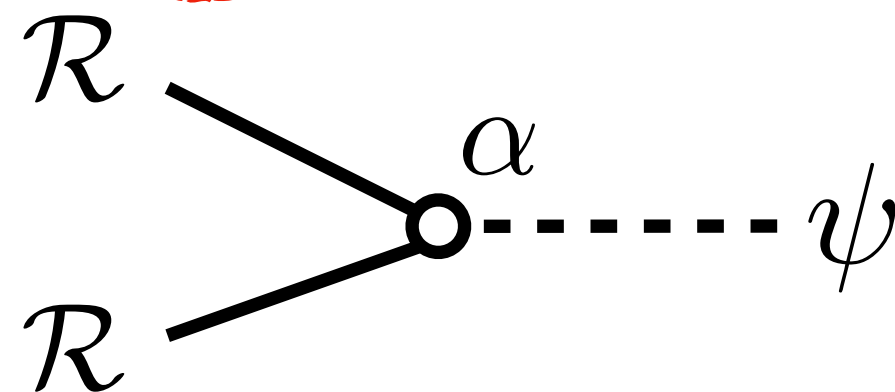
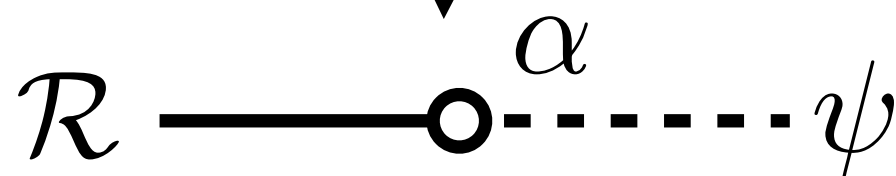
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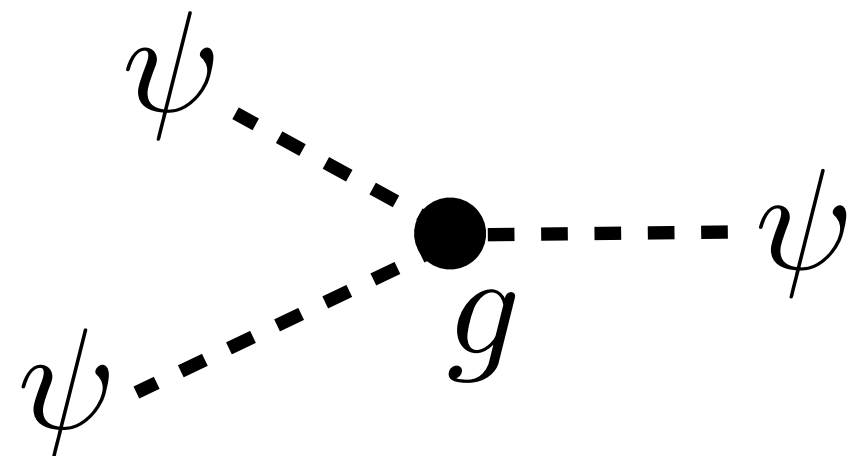


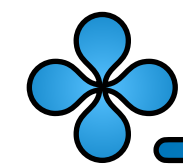


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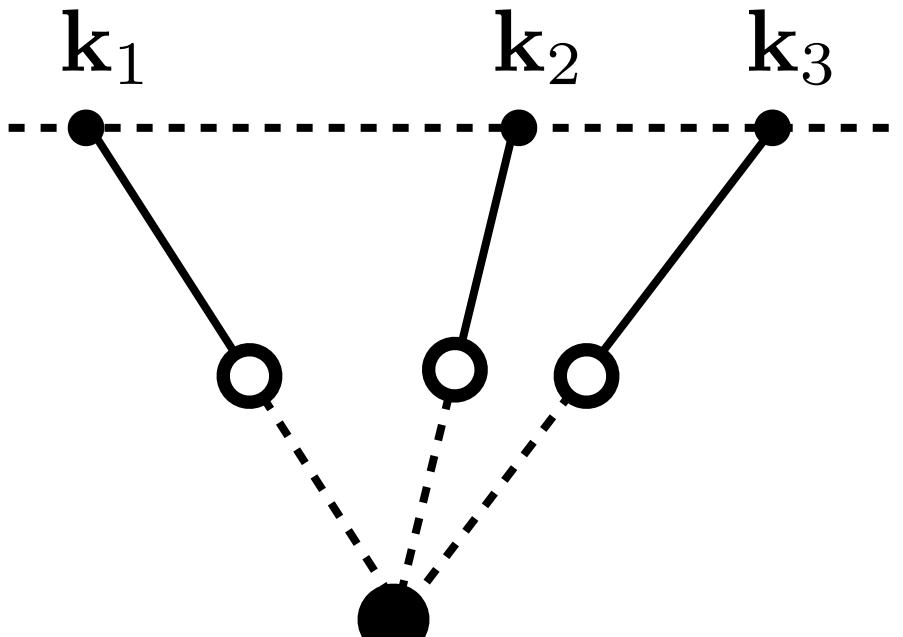


Three-point statistics:

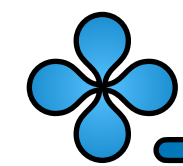
$$\mu \neq 0$$

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle =$$

(Quasi-single field)

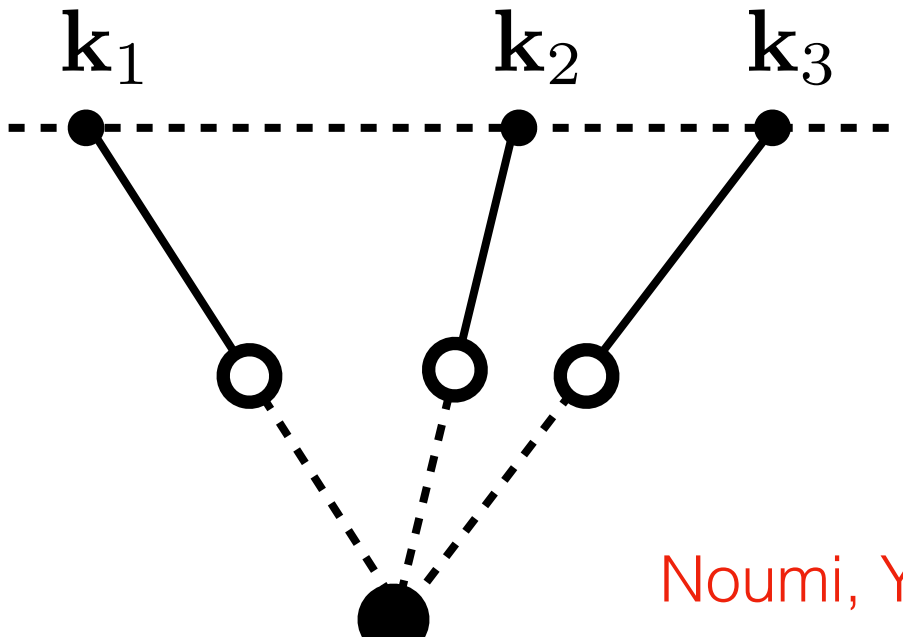


Chen & Wang (2012)
see also Assassi et al. (2013)



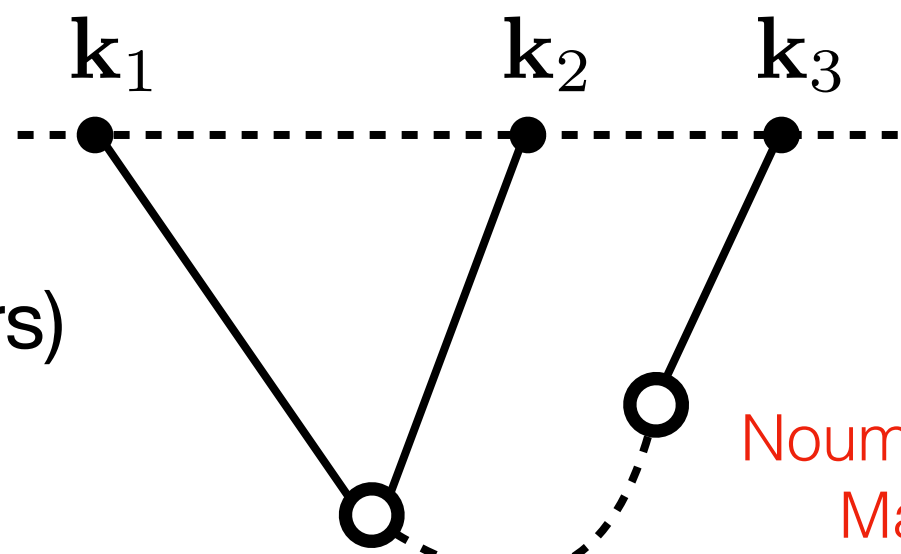
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(Cosmological colliders)

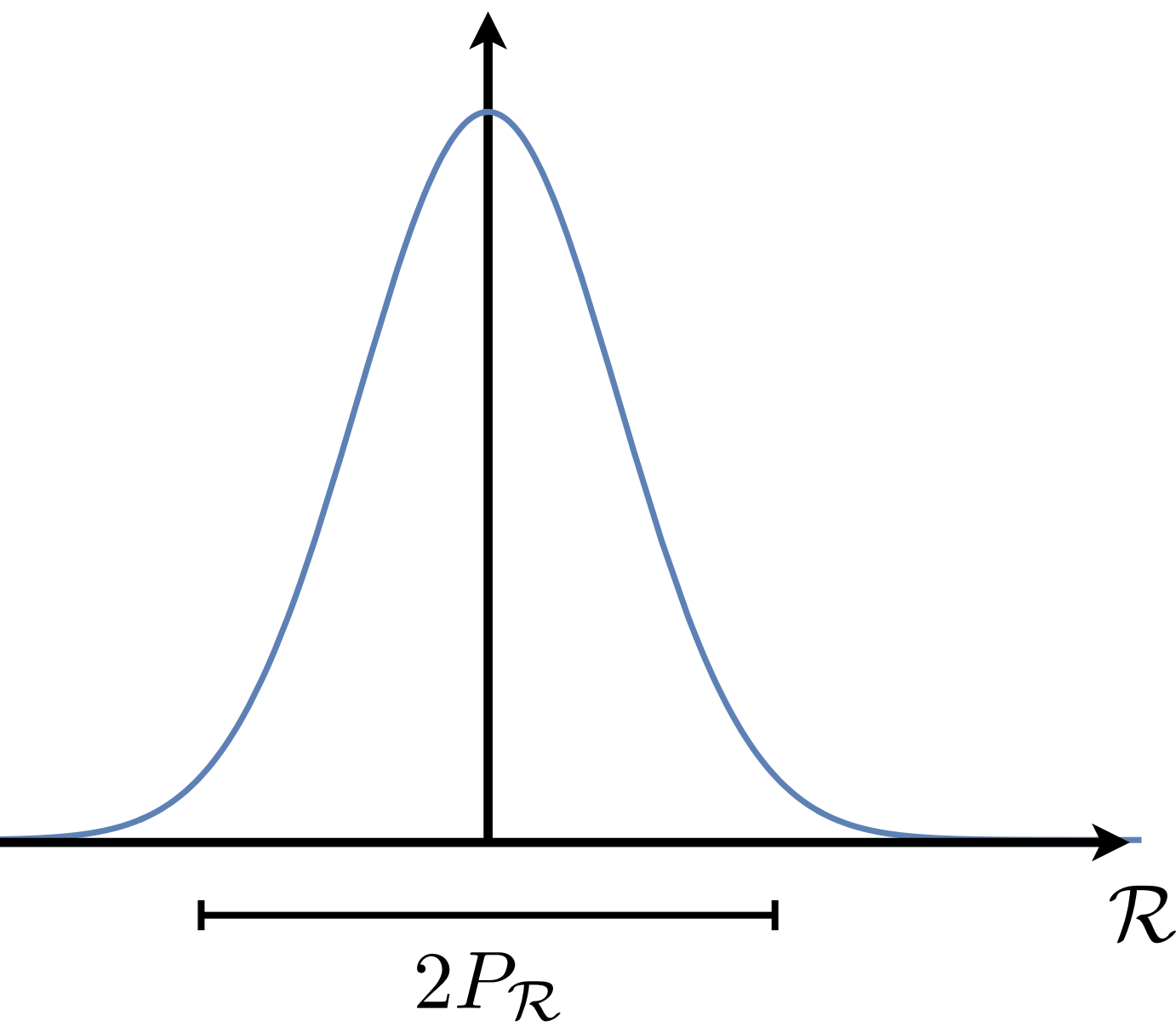
Noumi, Yamaguchi & Yokoyama (2013)
Maldacena & Arkani-Hamed (2016)
Chen & Wang (2016)
Lee, Baumann & Pimentel (2016)

Beyond the bispectrum

14

The perturbative schemes seem to imply a hierarchical reconstruction of the probability distribution function

$$\rho[\mathcal{R}] \sim \exp \left\{ -\frac{\mathcal{R}^2}{2P_{\mathcal{R}}} \left(1 + f_{\text{NL}}\mathcal{R} + g_{\text{NL}}\mathcal{R}^2 + \cdots \right) \right\}$$

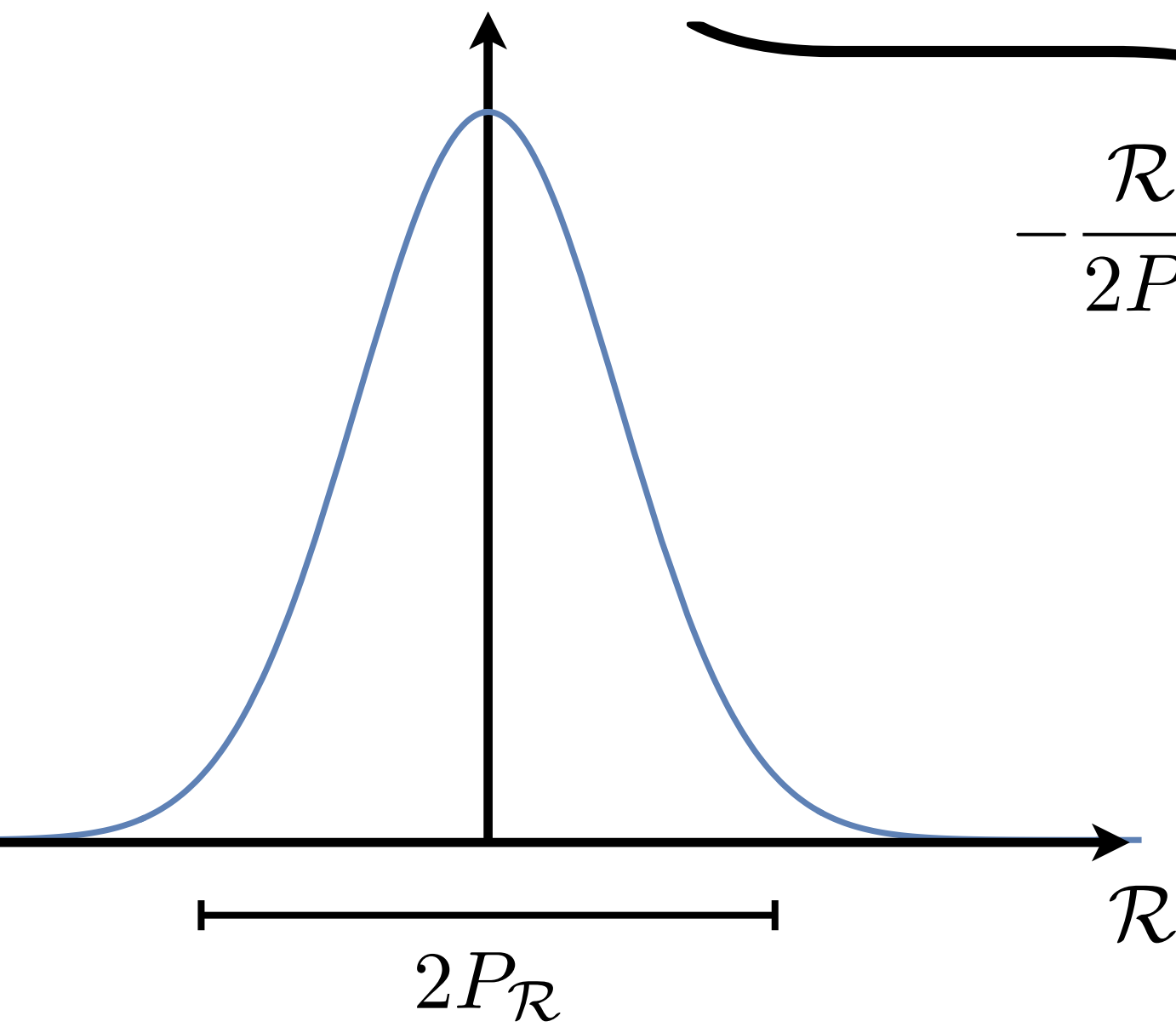


Beyond the bispectrum

14

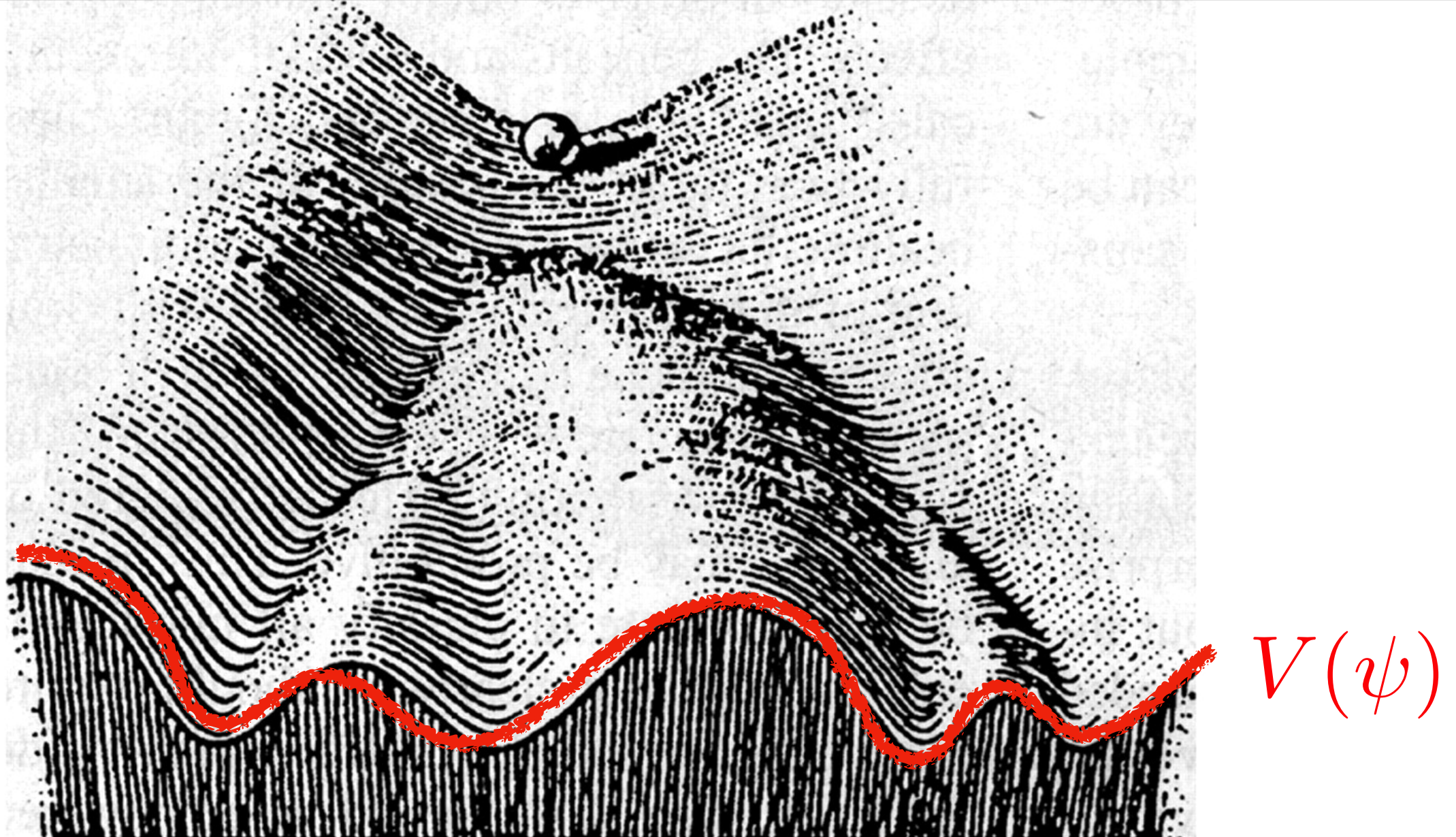
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✿ Beyond the bispectrum

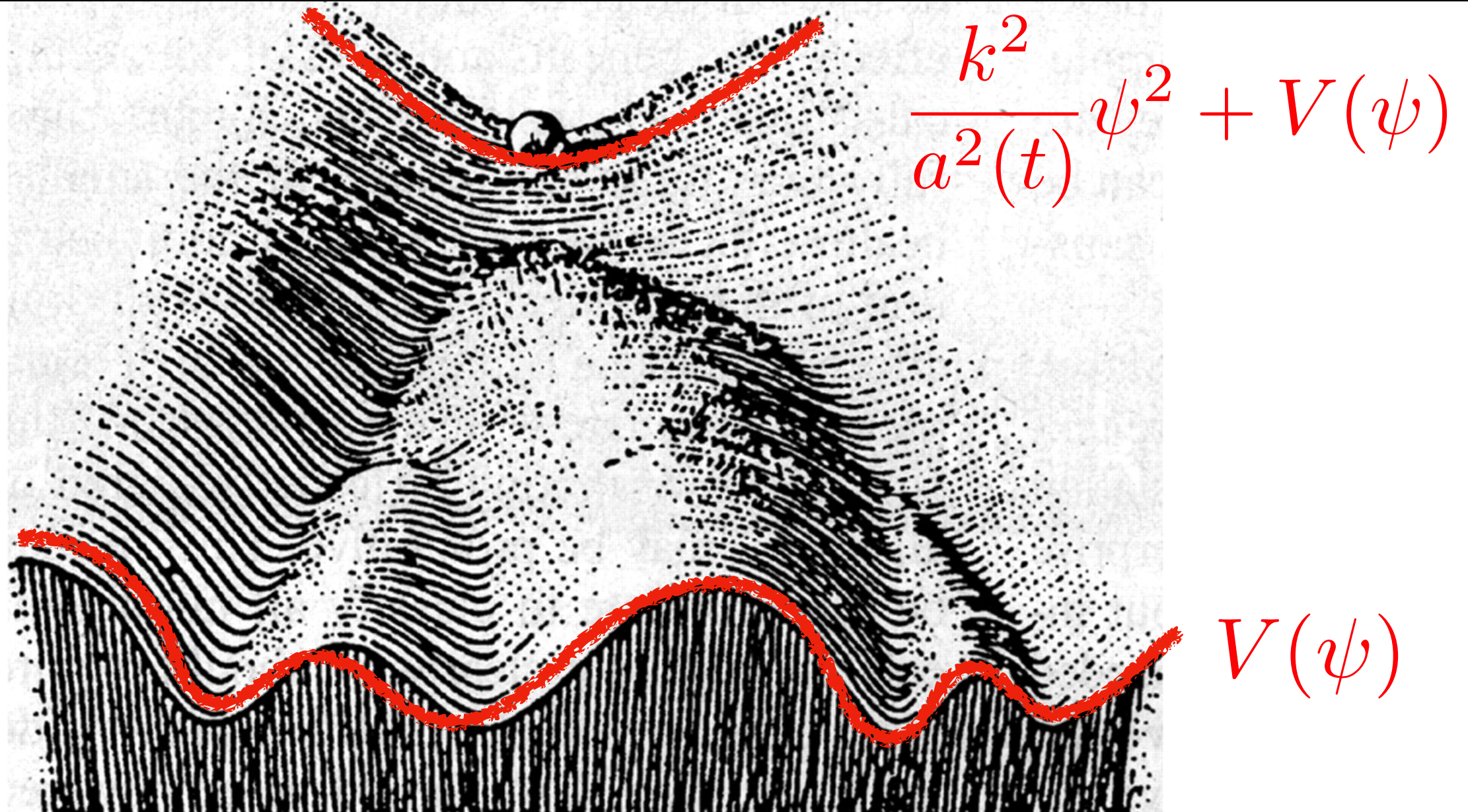
15



Consider a spectator field ψ during inflation with it's own potential $V(\psi)$
(This potential is not driving inflation)

✿ Beyond the bispectrum

15

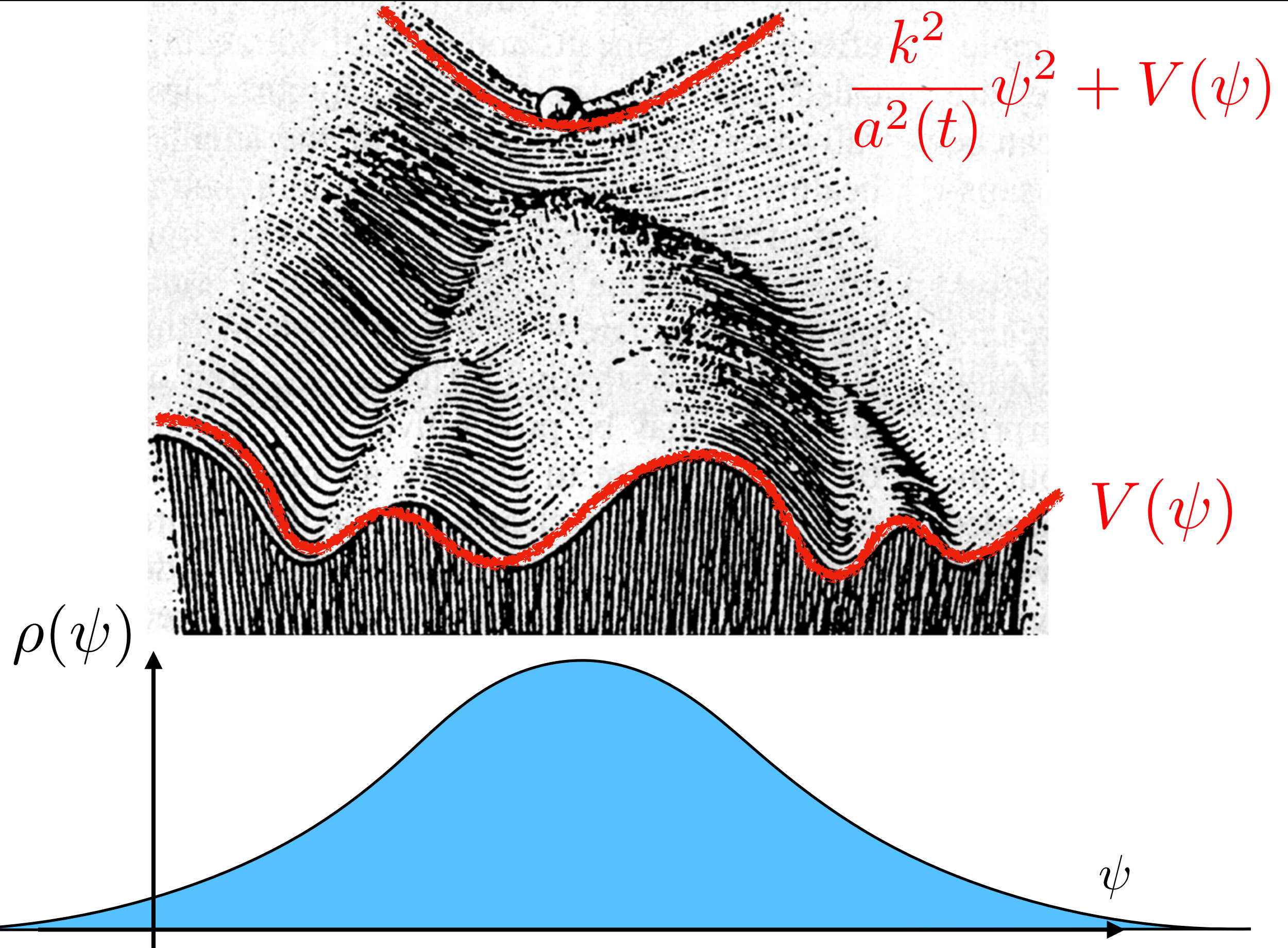


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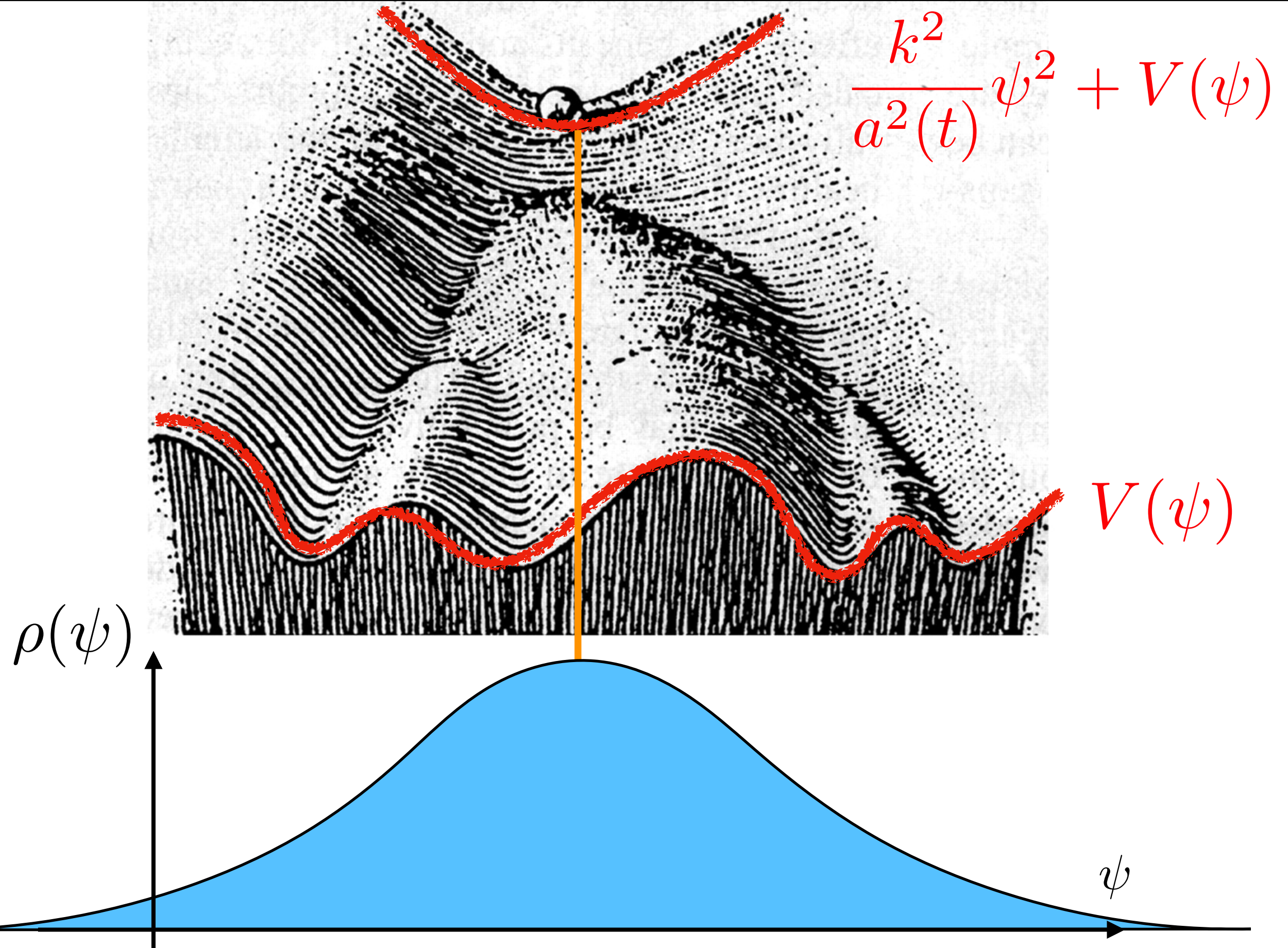
✿ Beyond the bispectrum

15



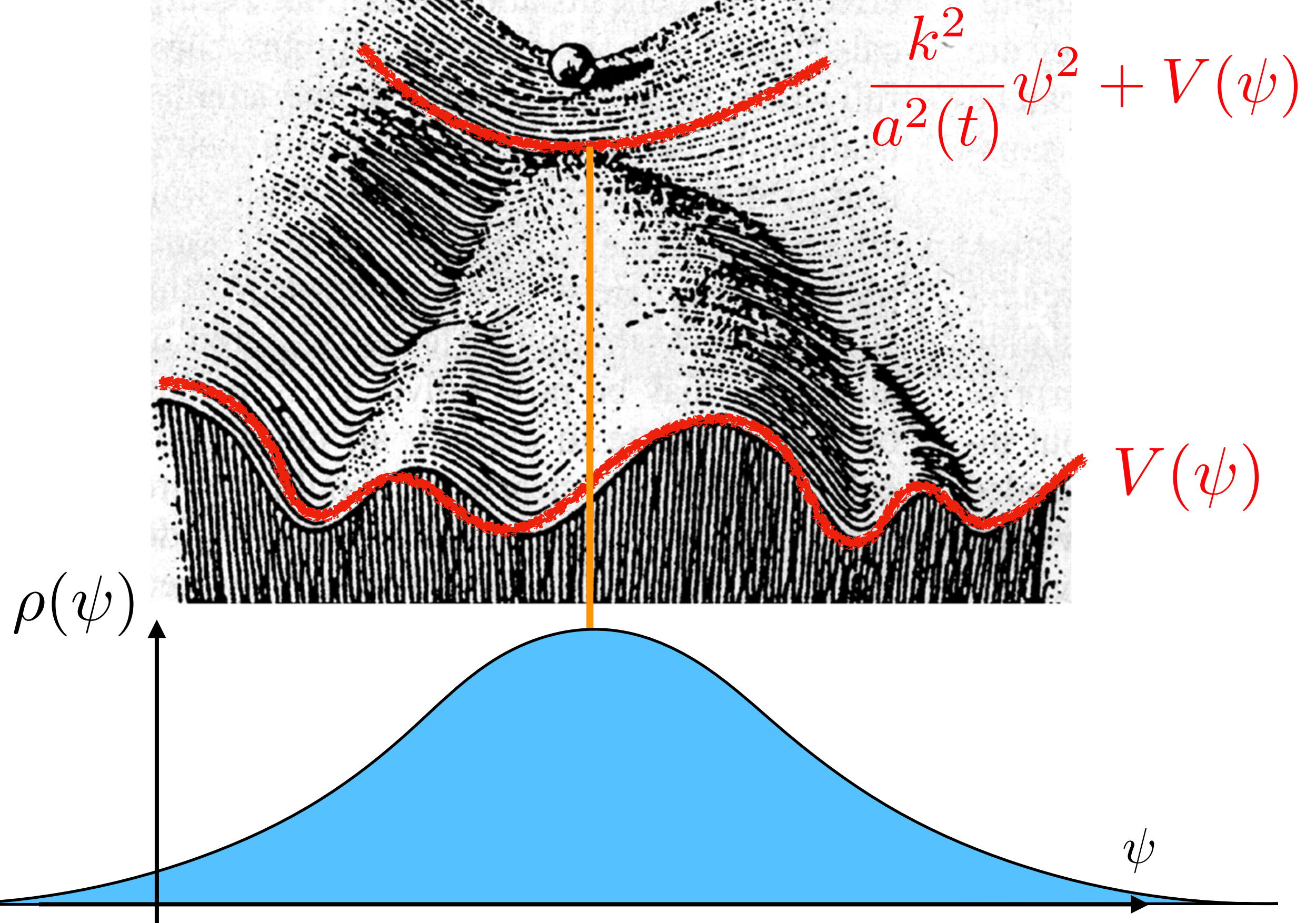
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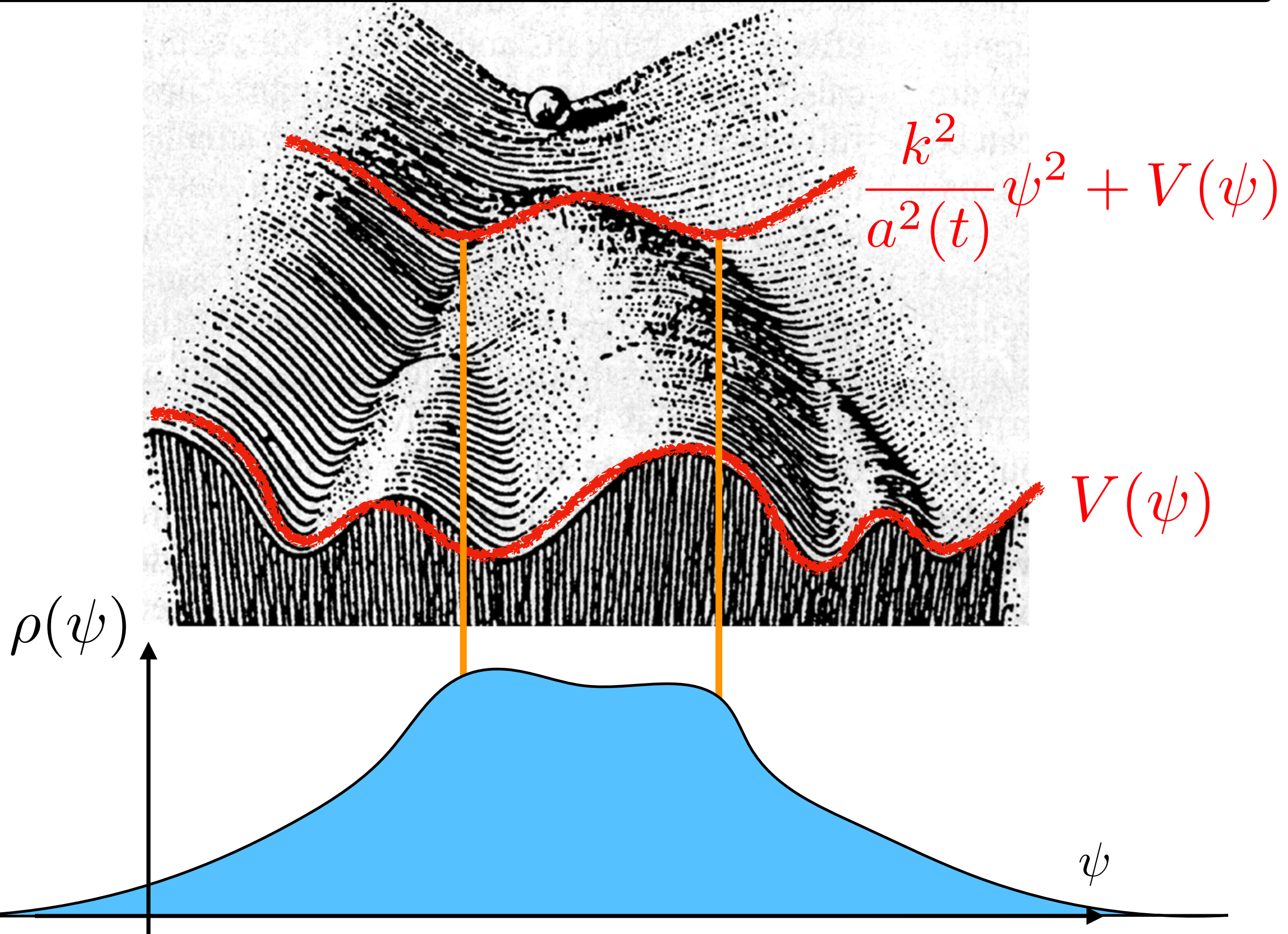
Beyond the bispectrum

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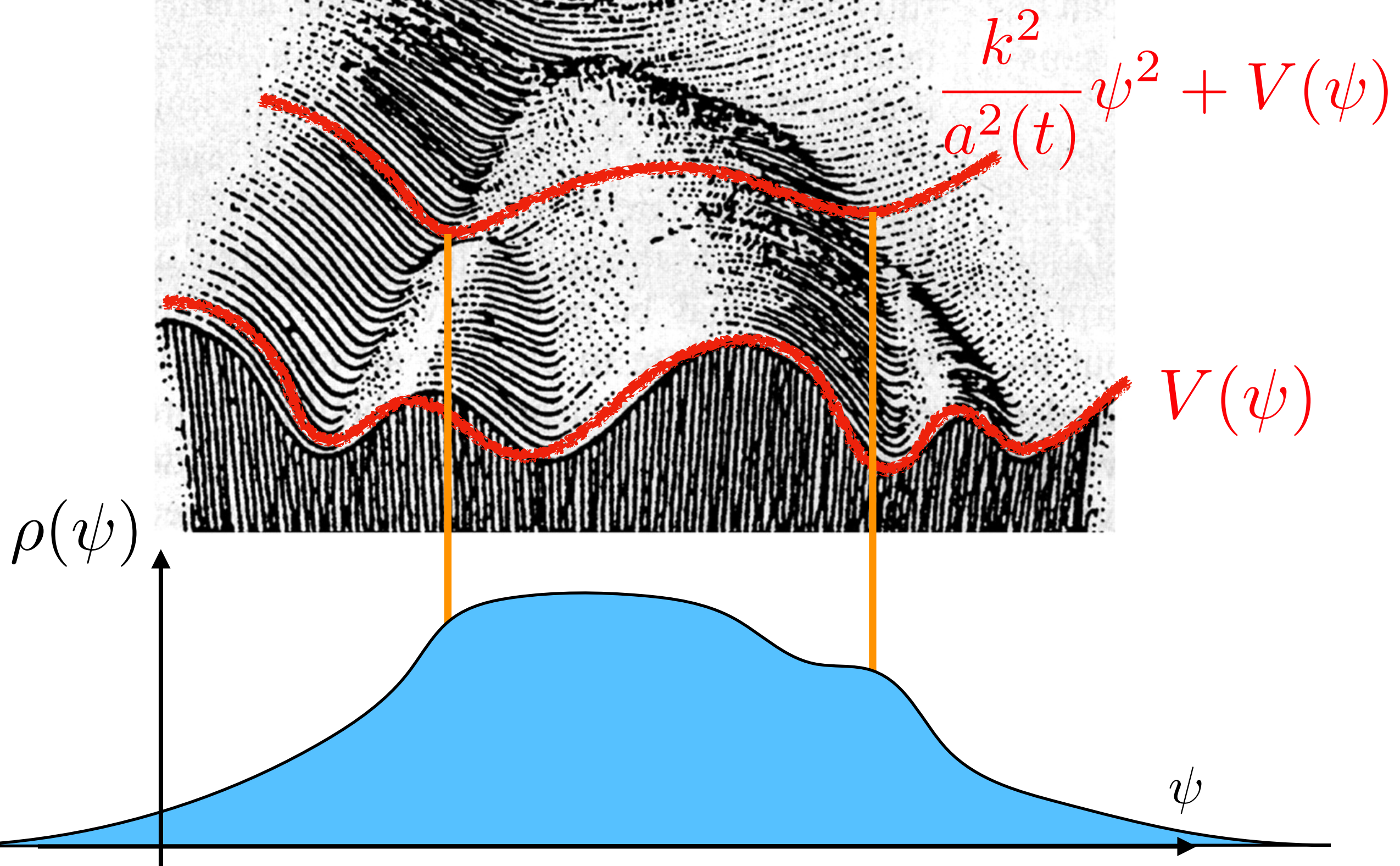
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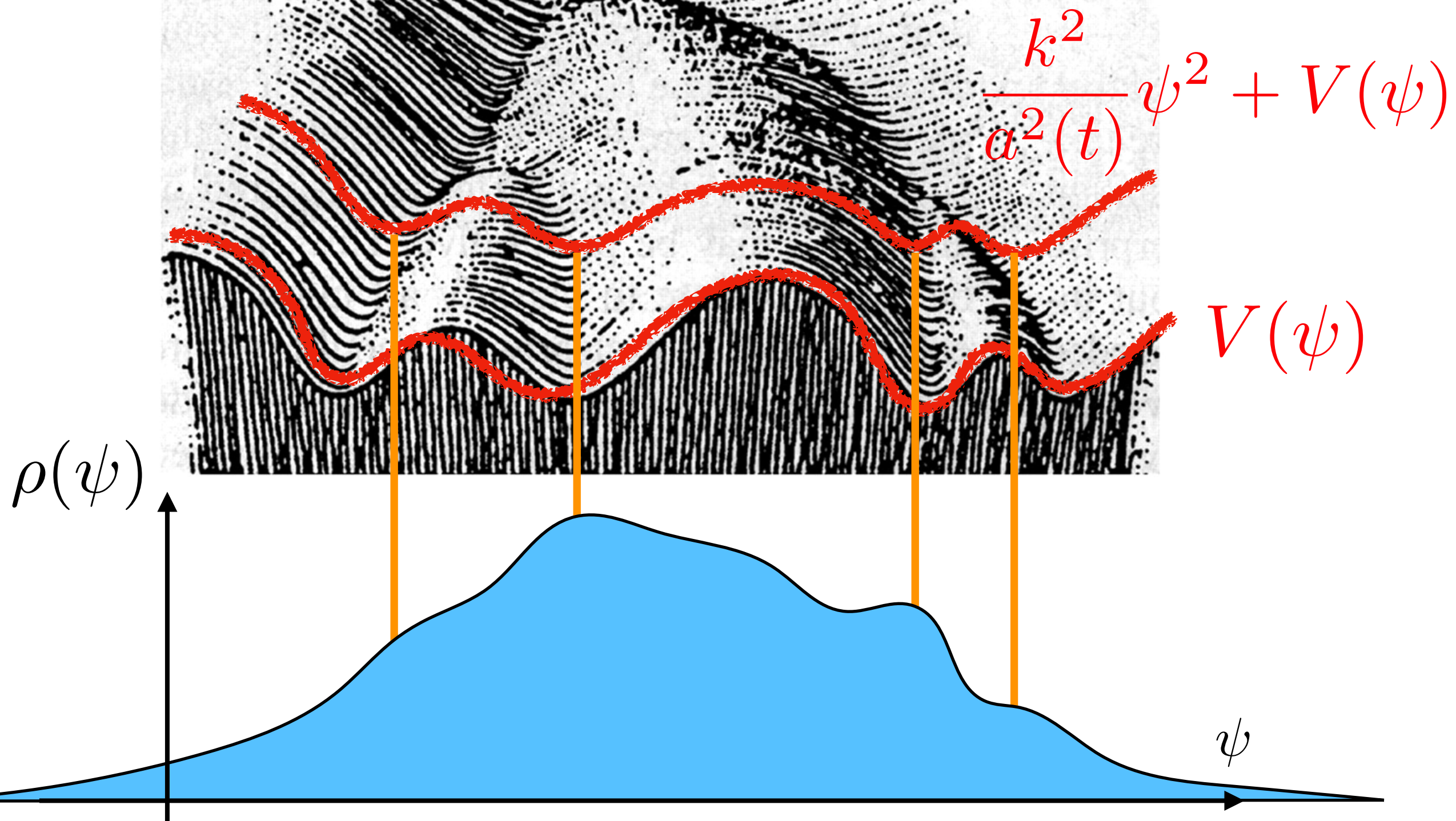
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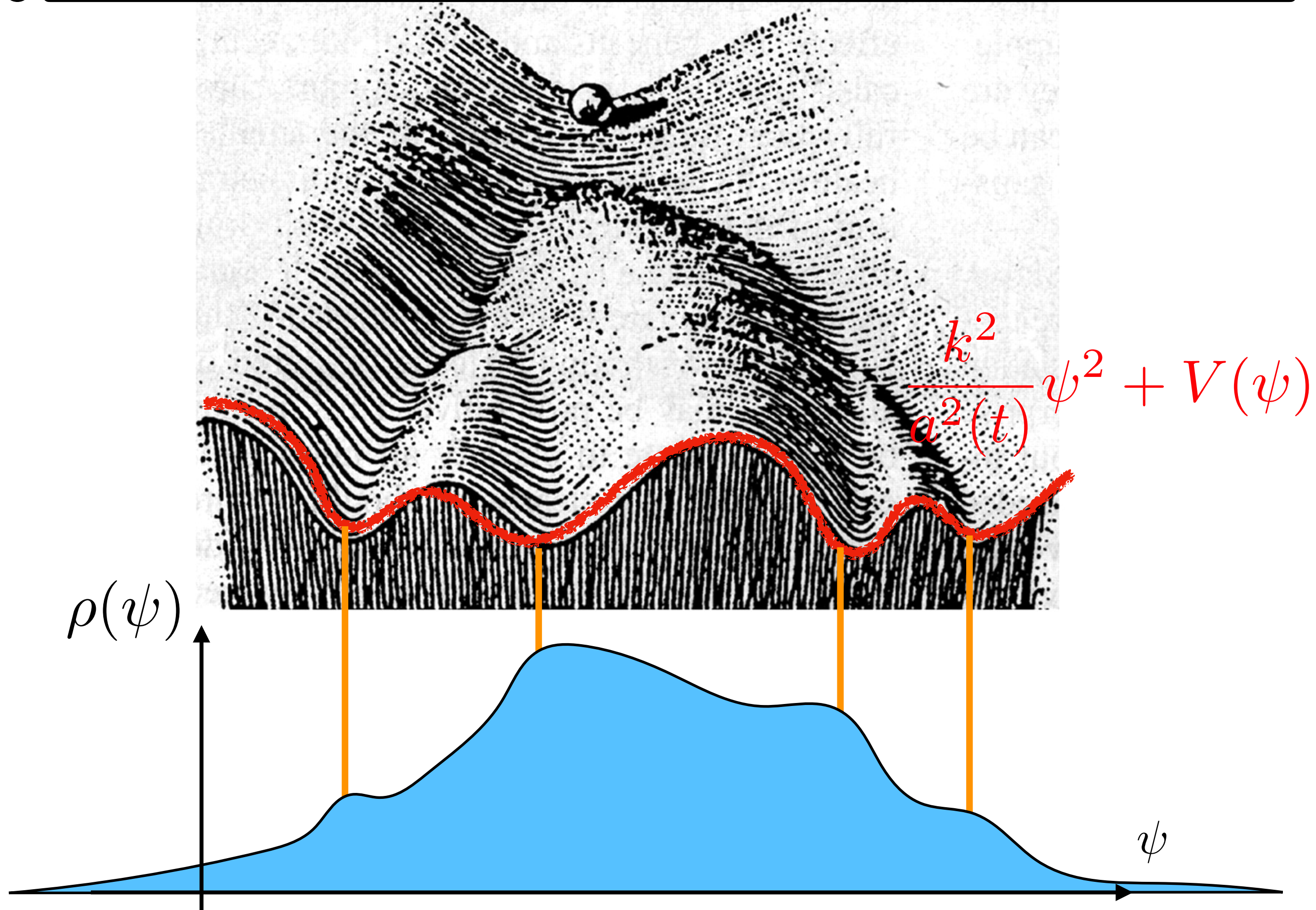
Beyond the bispectrum

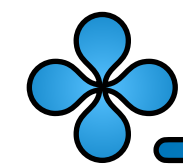
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Beyond the bispectrum

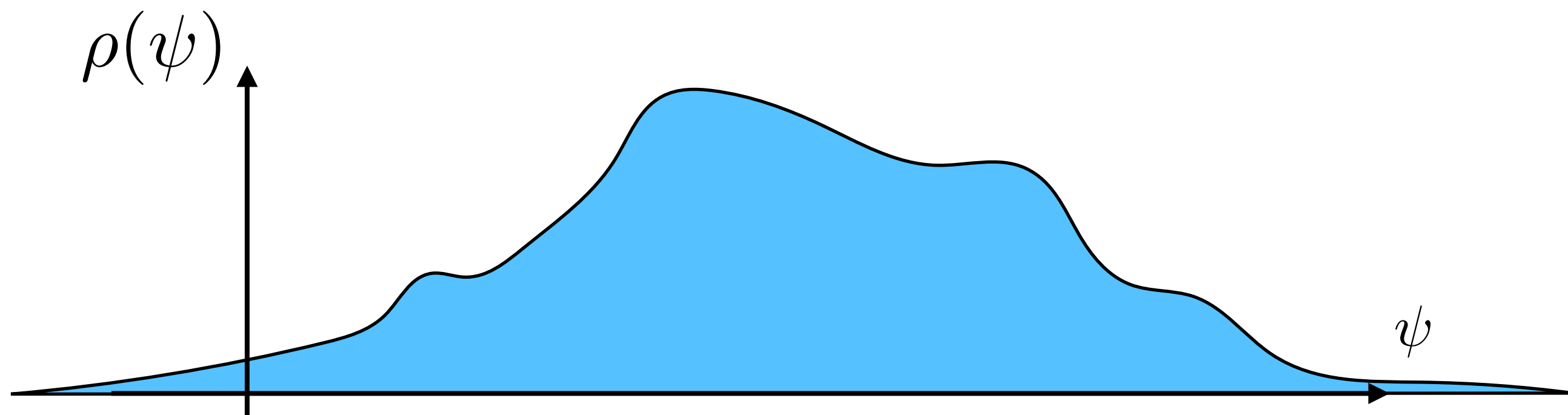
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Beyond the bispectrum

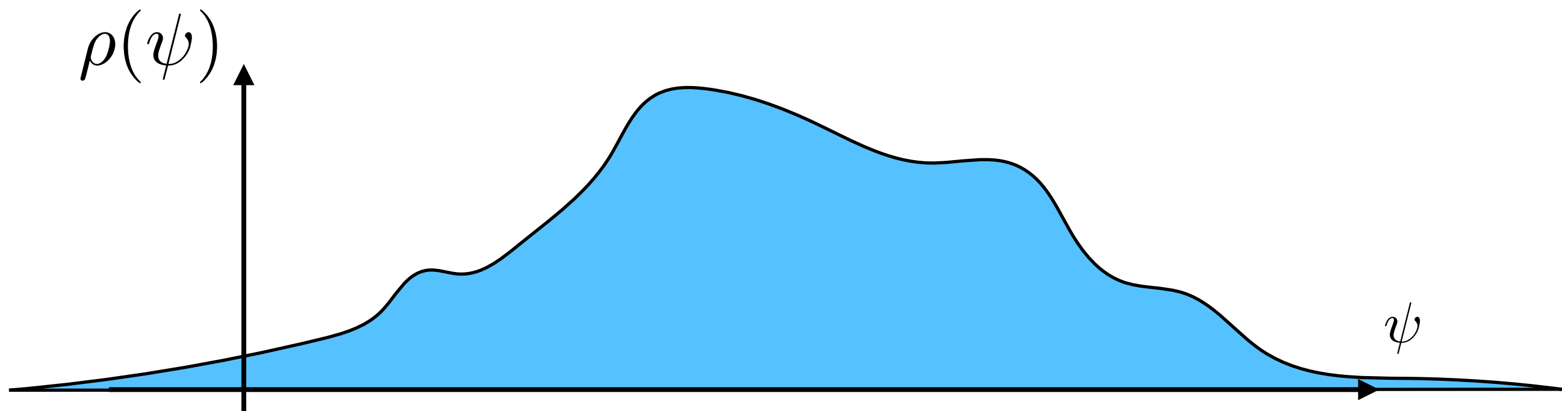
15



✿ Beyond the bispectrum

15

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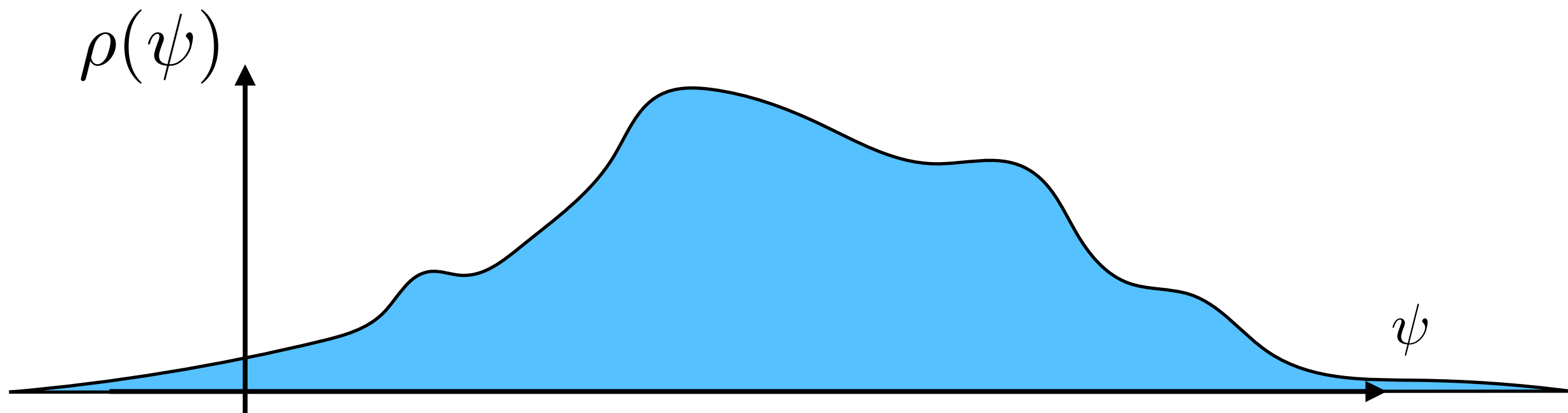


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$\mathcal{R} \text{ --- } \bigcirc \text{ --- } \psi$



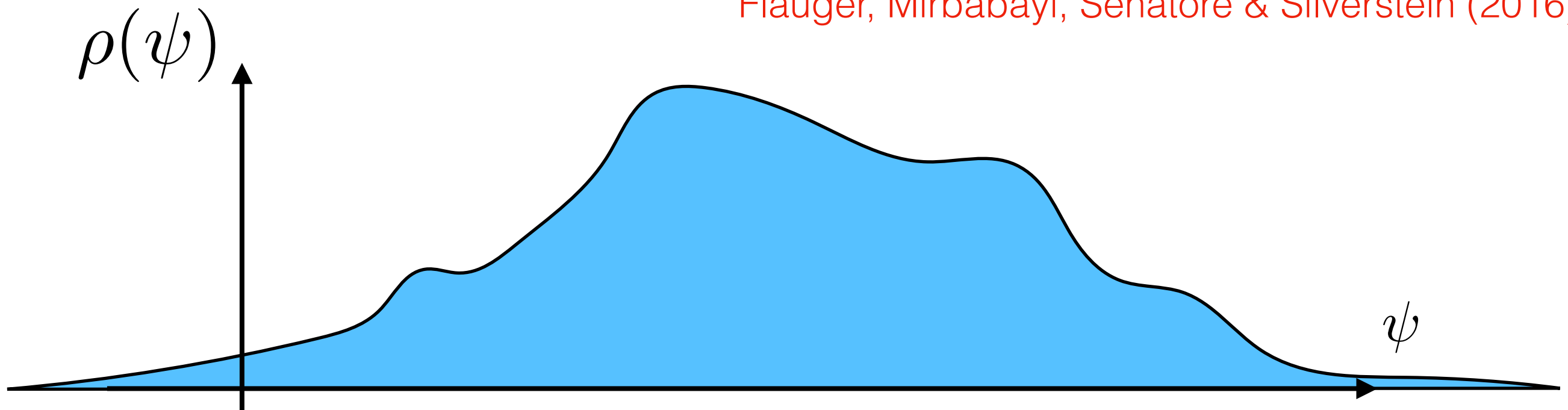
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$\mathcal{R} \text{ --- } \bigcirc \text{ --- } \psi \implies \rho(\mathcal{R}) \neq \exp \left[-\frac{\mathcal{R}^2}{2\sigma_{\mathcal{R}}^2} \right]$

Flauger, Mirbabayi, Senatore & Silverstein (2016)



Beyond the bispectrum

The previous idea can be examined non-perturbatively. On long wavelengths a spectator field satisfies the Fokker-Planck equation:

$$\frac{\partial \rho}{\partial t} = \frac{1}{3H} \frac{\partial}{\partial \psi} \left(V'(\psi) \rho \right) + \frac{H^3}{8\pi^2} \frac{\partial^2 \rho}{\partial \psi^2}$$

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Starobinsky & Yokoyama (1994)

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Cohen, Green & Premkumar (2022)

Beyond the bispectrum

17

Another approach consists of directly reconstructing $\rho(\mathcal{R})$ out of n-point functions

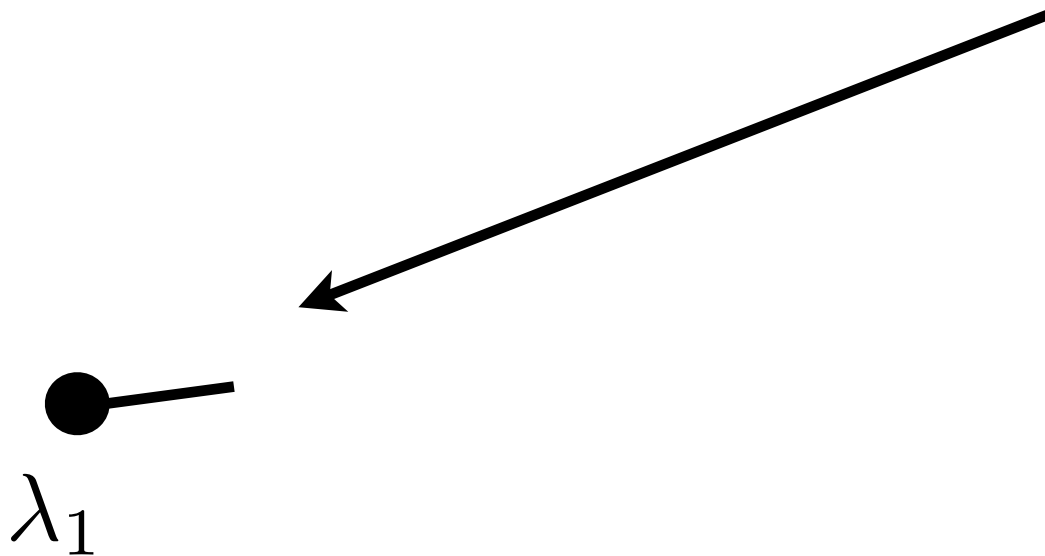
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17

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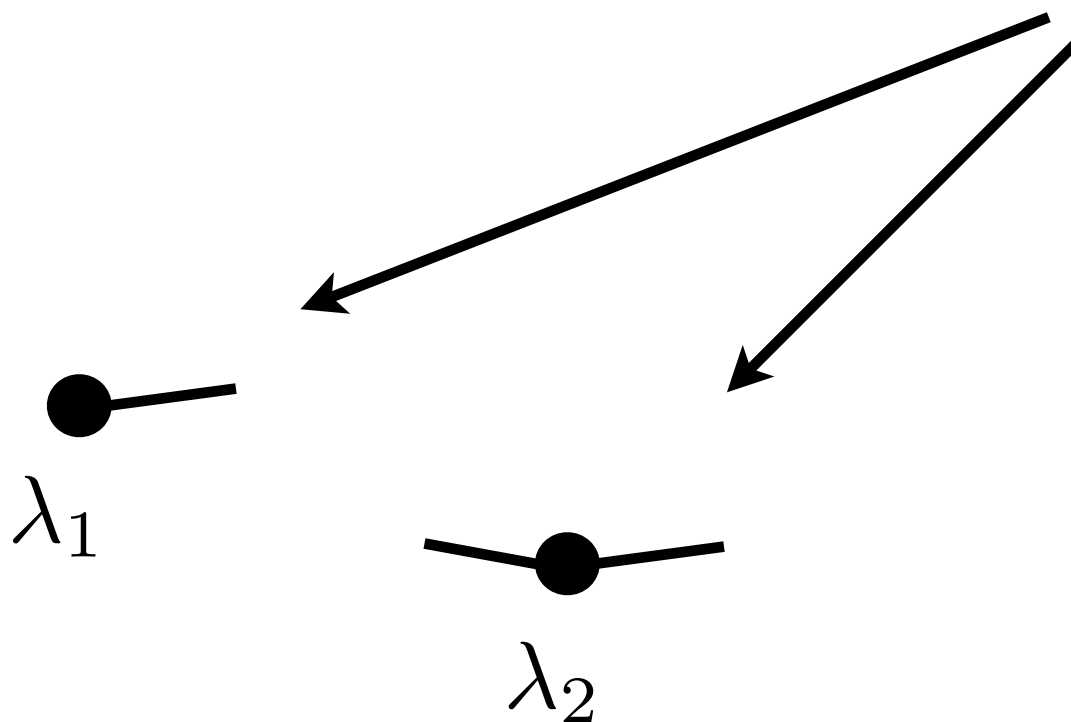


Beyond the bispectrum

17

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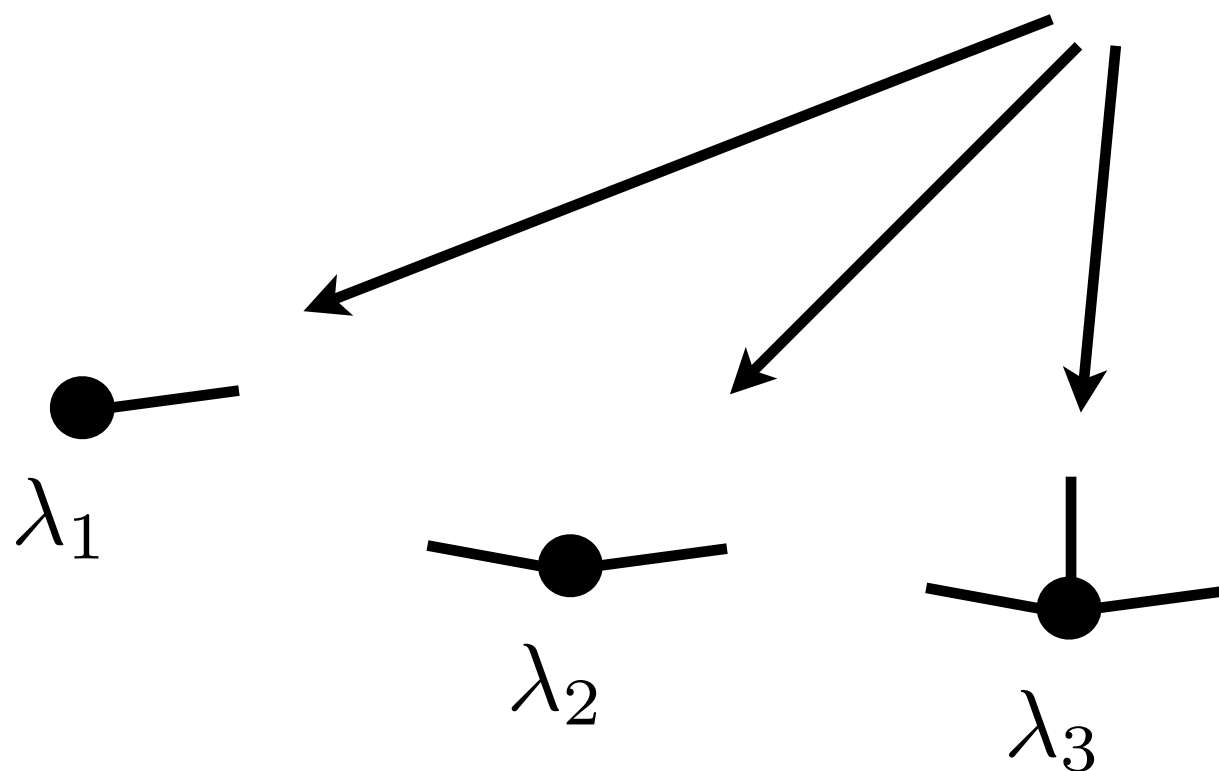


Beyond the bispectrum

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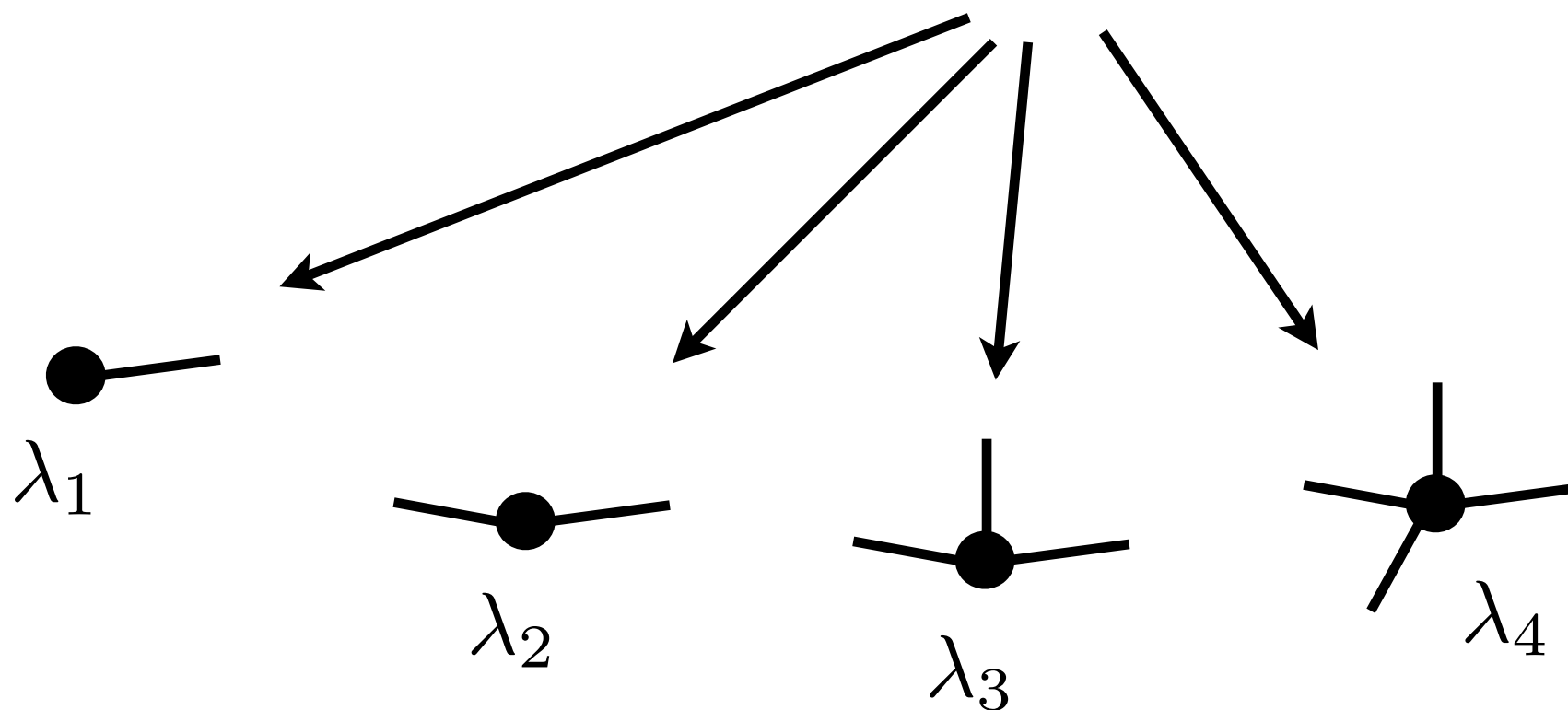


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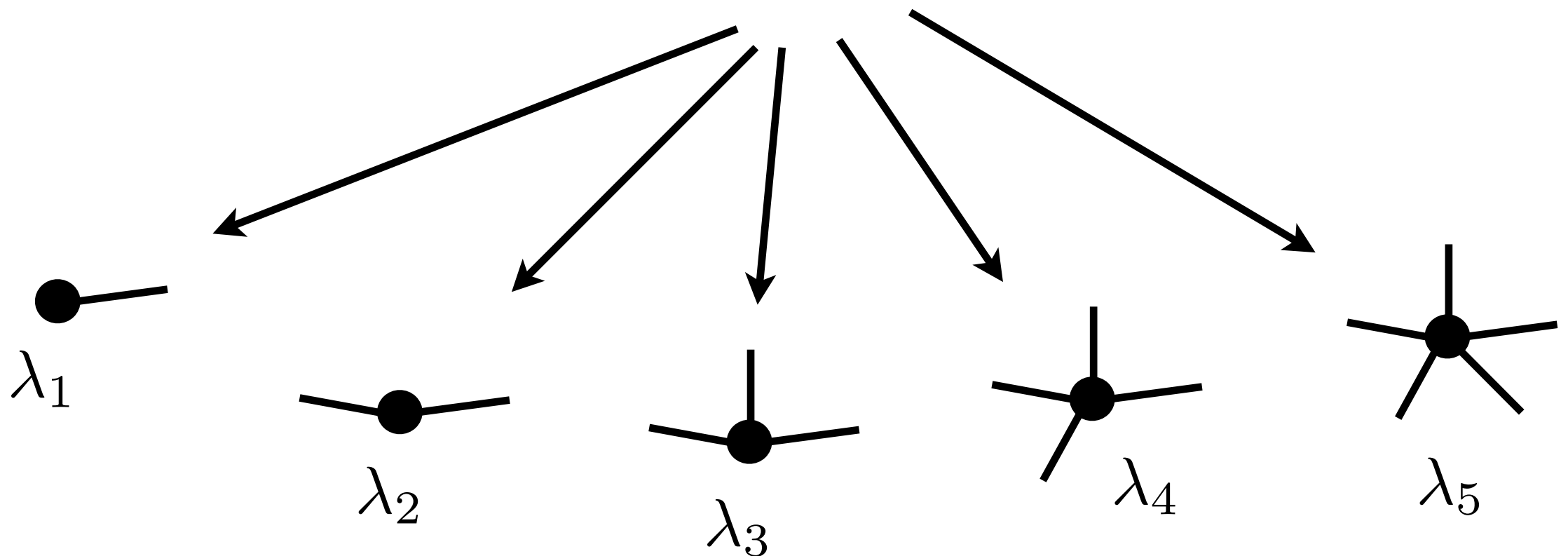


Beyond the bispectrum

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Beyond the bispectrum

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One can prove in general:

$$\begin{aligned} \langle \psi(\mathbf{k}_1) \cdots \psi(\mathbf{k}_n) \rangle'_{(n)} &\sim [V(\psi) \log a]^n \\ &\quad + [V(\psi)]^n [\log a]^{n-1} + \dots \end{aligned}$$

Beyond the bispectrum

17

Another approach consists of directly reconstructing $\rho(\mathcal{R})$ out of n-point functions

$$V(\psi) = \sum_n \frac{\lambda_n}{n!} \psi^n$$

One can prove in general:

$$\langle \psi^n \rangle_c \propto \log a \int d\psi \frac{e^{-\frac{\psi^2}{2\sigma^2}}}{\sqrt{2\psi}\sigma} \text{He}_n(\psi/\sigma) \left(\sigma^2 \partial_\psi - \psi \right) u'(\psi)$$

$$u(\psi) = e^{-\frac{\sigma^2}{2}} \partial_\psi^2 V(\psi)$$

Using:

$$\rho(\psi) = \frac{e^{-\frac{1}{2} \frac{\psi^2}{\sigma^2}}}{\sqrt{2\pi}\sigma} \sum_{N=0}^{\infty} \frac{1}{N!} \sum_{n_1=0}^{\infty} \cdots \sum_{n_N=0}^{\infty} \frac{1}{n_1!} \cdots \frac{1}{n_N!} \frac{\langle \psi^{n_1} \rangle_c}{\sigma^{n_1}} \cdots \frac{\langle \psi^{n_N} \rangle_c}{\sigma^{n_N}} \text{He}_{n_1+\cdots+n_N}(\psi/\sigma)$$

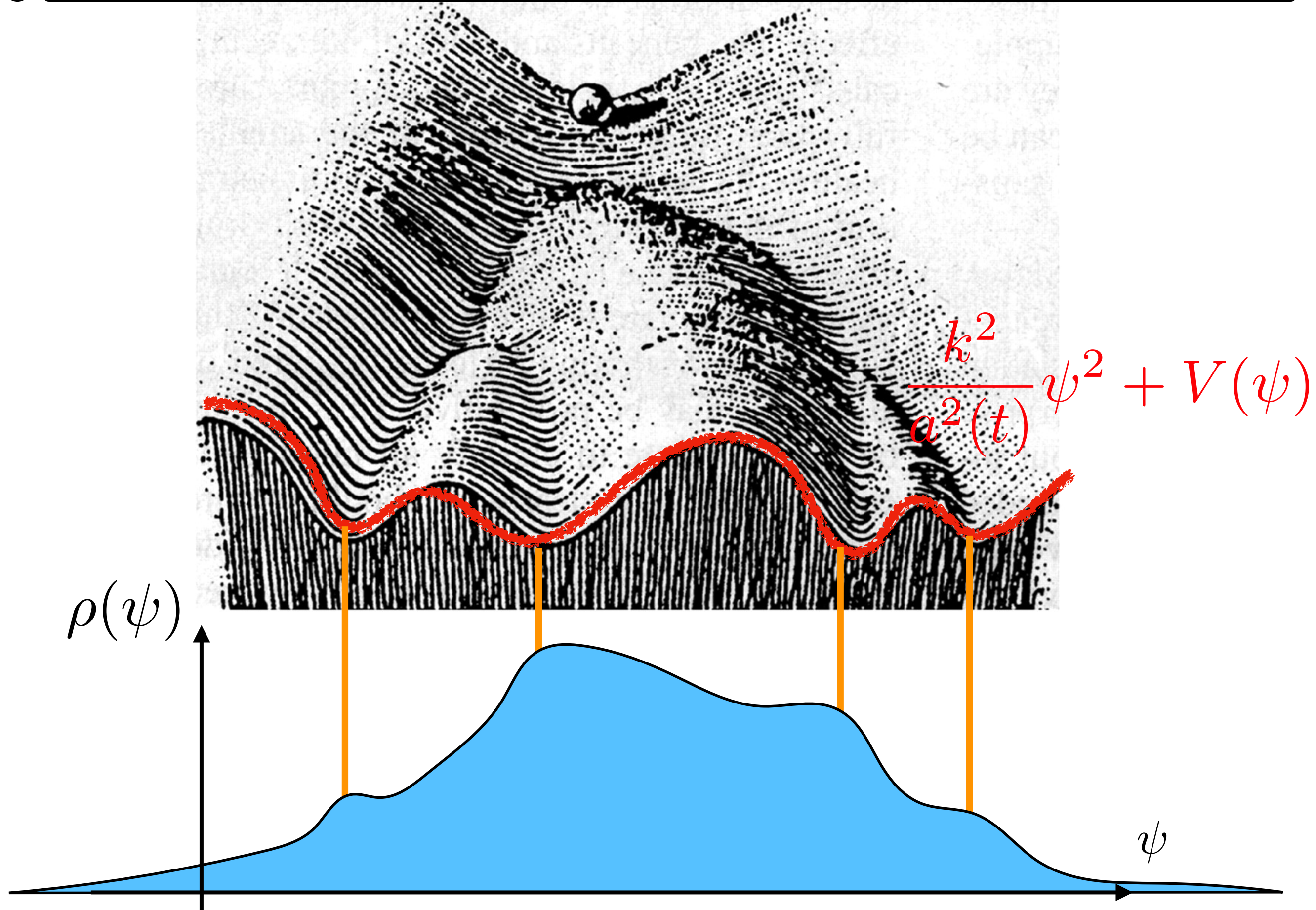
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$$\begin{aligned} \rho(\psi) = & \frac{e^{-\frac{1}{2} \frac{\psi^2}{\sigma^2}}}{\sqrt{2\pi}\sigma} \left[1 + \log a \left(\sigma^2 u'' - \psi u' \right) \right. \\ & + (\log a)^2 \left(\left(\sigma^2 u'' - \psi u' \right)^2 + \mathcal{O}(u^2) \right) \\ & \left. + \cdots \right] \end{aligned}$$

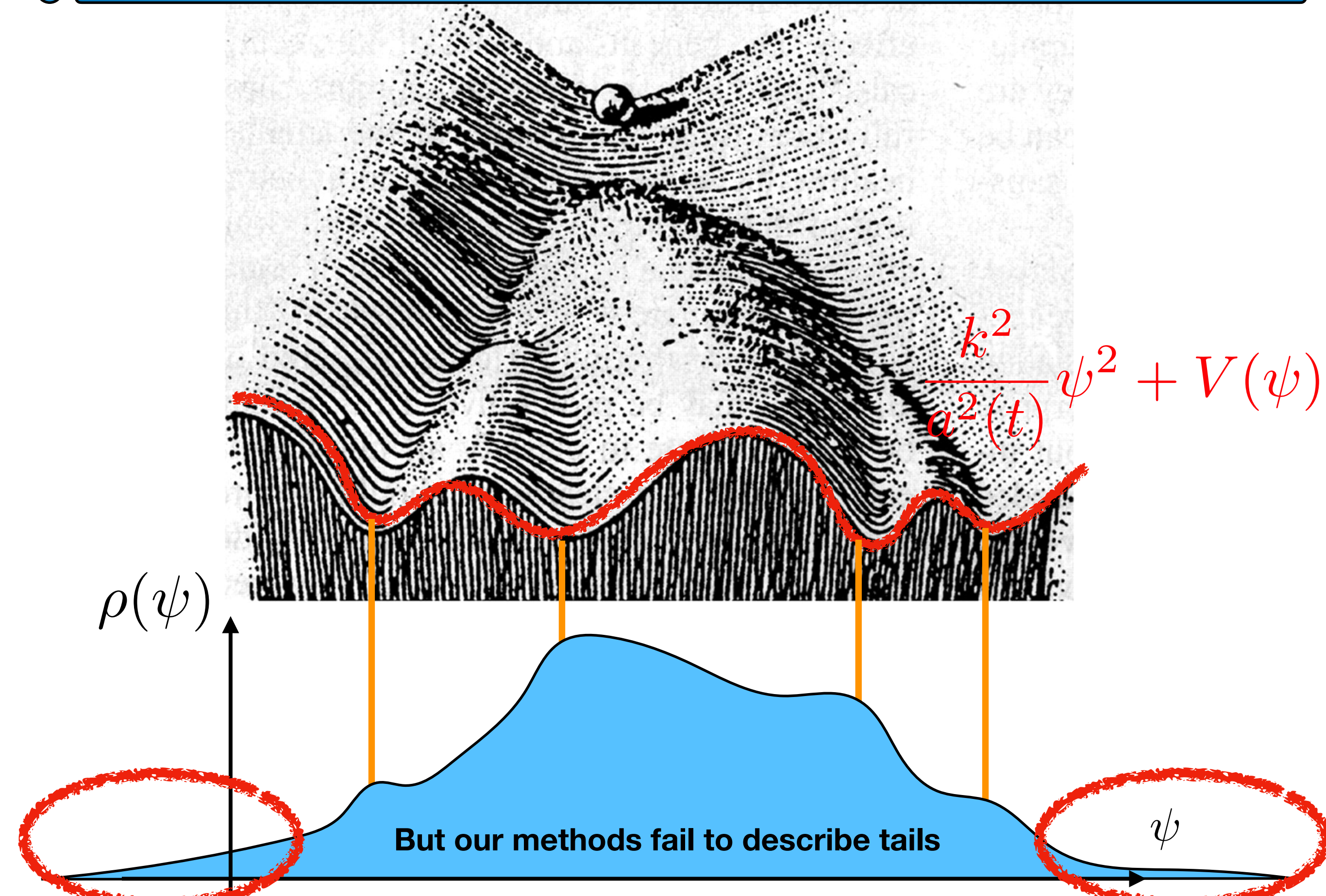
Beyond the bispectrum

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Beyond the bispectrum

19

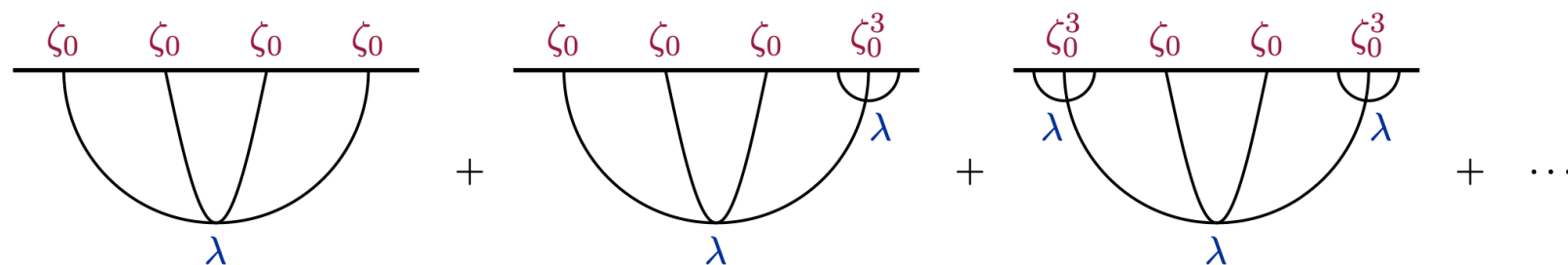


Beyond the bispectrum

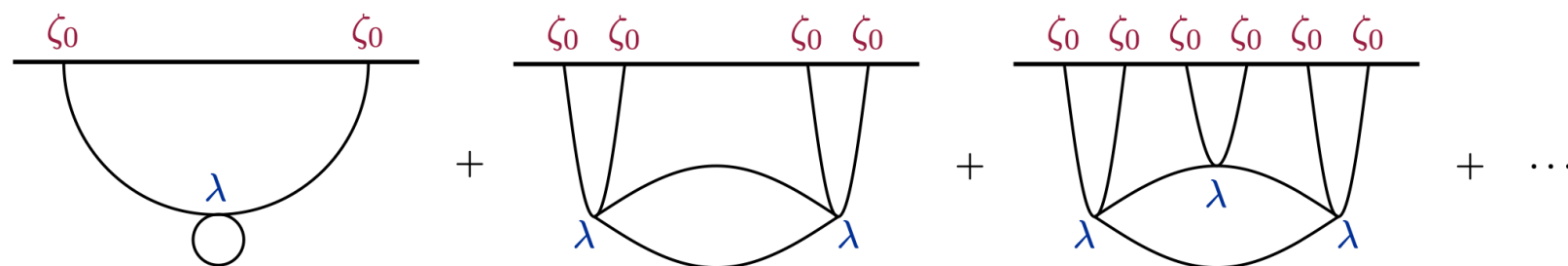
20

NG tails might be possible in single field inflation:

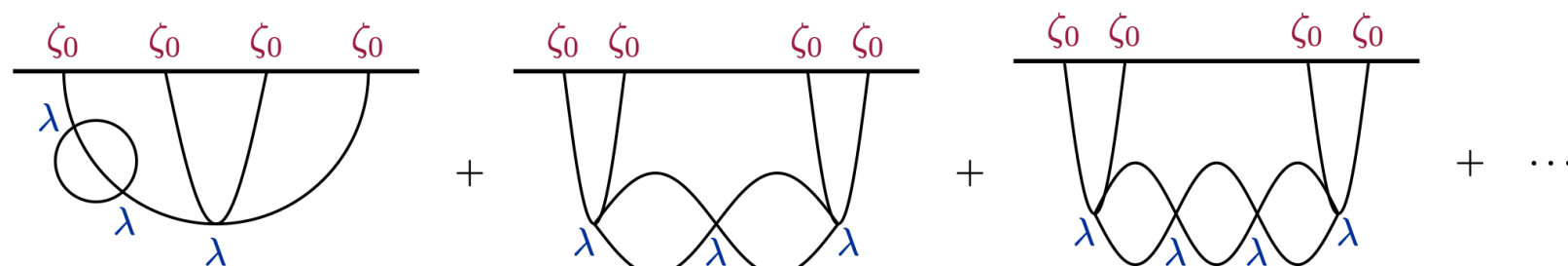
$$\mathcal{L} = \epsilon \left(\dot{\mathcal{R}}^2 - (\nabla \mathcal{R})^2 + \frac{\lambda}{4! H^2} \dot{\mathcal{R}}^4 \right)$$



(a)



(b)



(c)

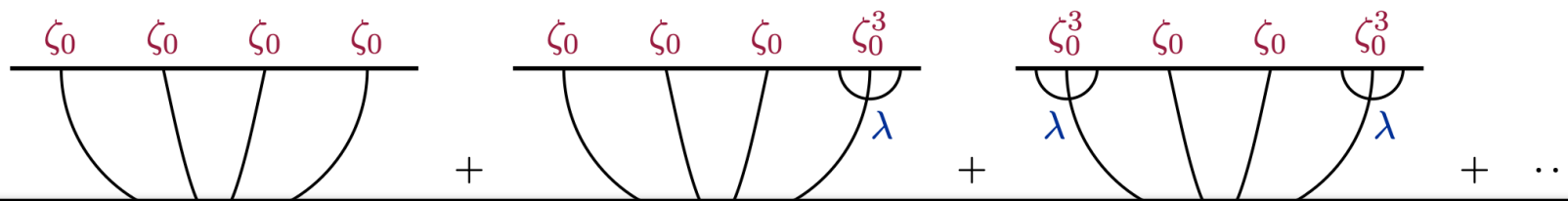
Celoria, Creminelli, Tambalo & Yingcharoenrat (2021)

✿ Beyond the bispectrum

20

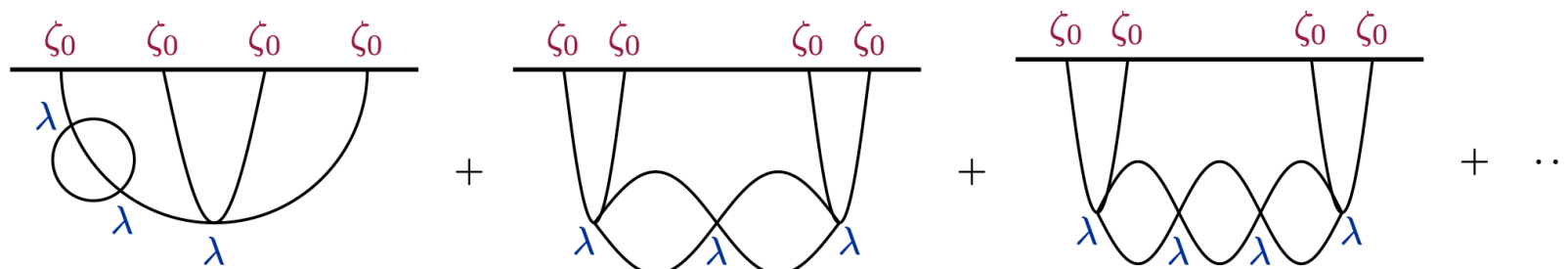
NG tails might be possible in single field inflation:

$$\mathcal{L} = \epsilon \left(\dot{\mathcal{R}}^2 - (\nabla \mathcal{R})^2 + \frac{\lambda}{4! H^2} \dot{\mathcal{R}}^4 \right)$$



$$\rho(\mathcal{R}) \sim \exp \left[- \frac{\mathcal{R}^{3/2}}{\lambda^{1/4}} \right]$$

(b)

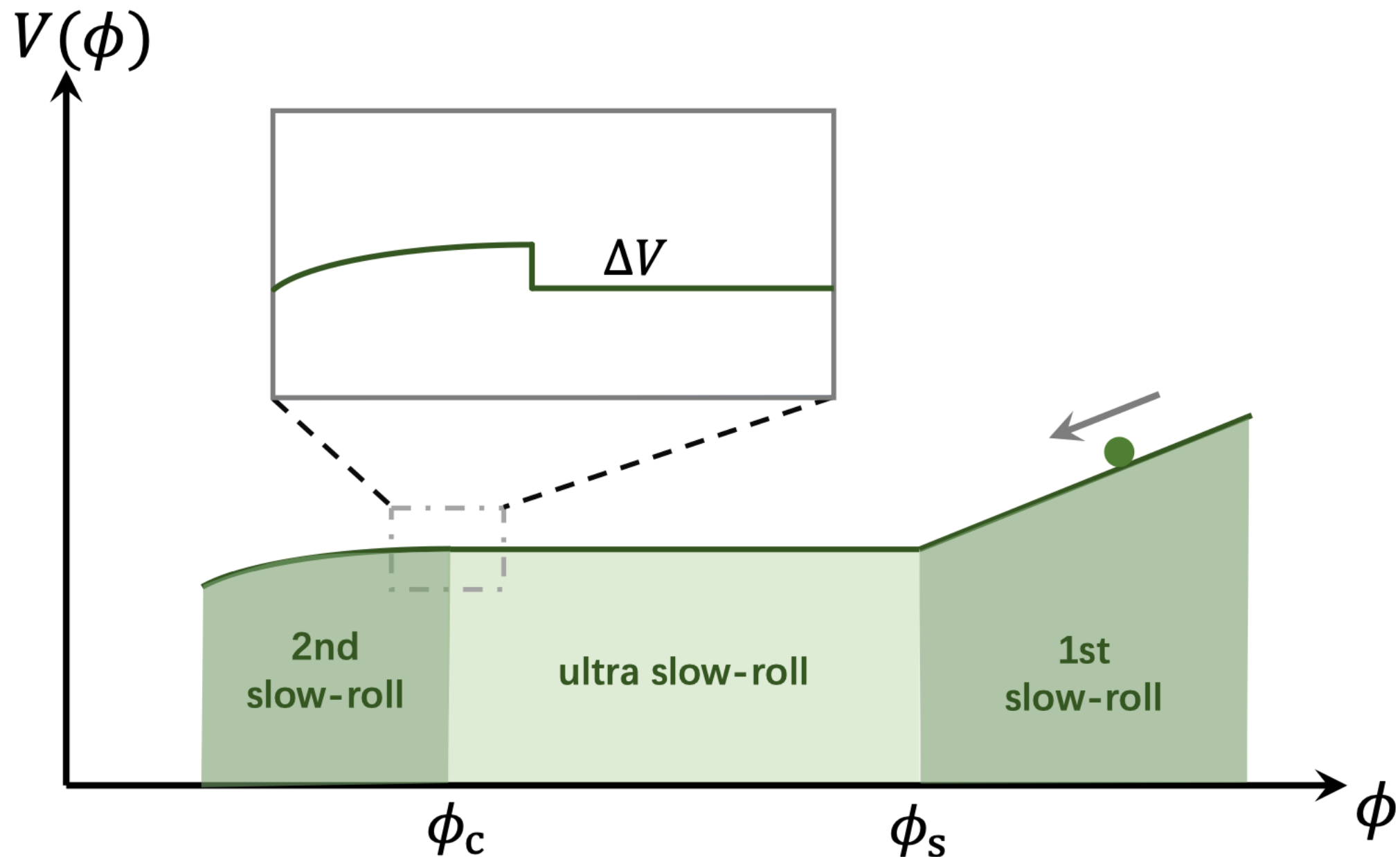


(c)

Celoria, Creminelli, Tambalo & Yingcharoenrat (2021)

✿ Beyond the bispectrum

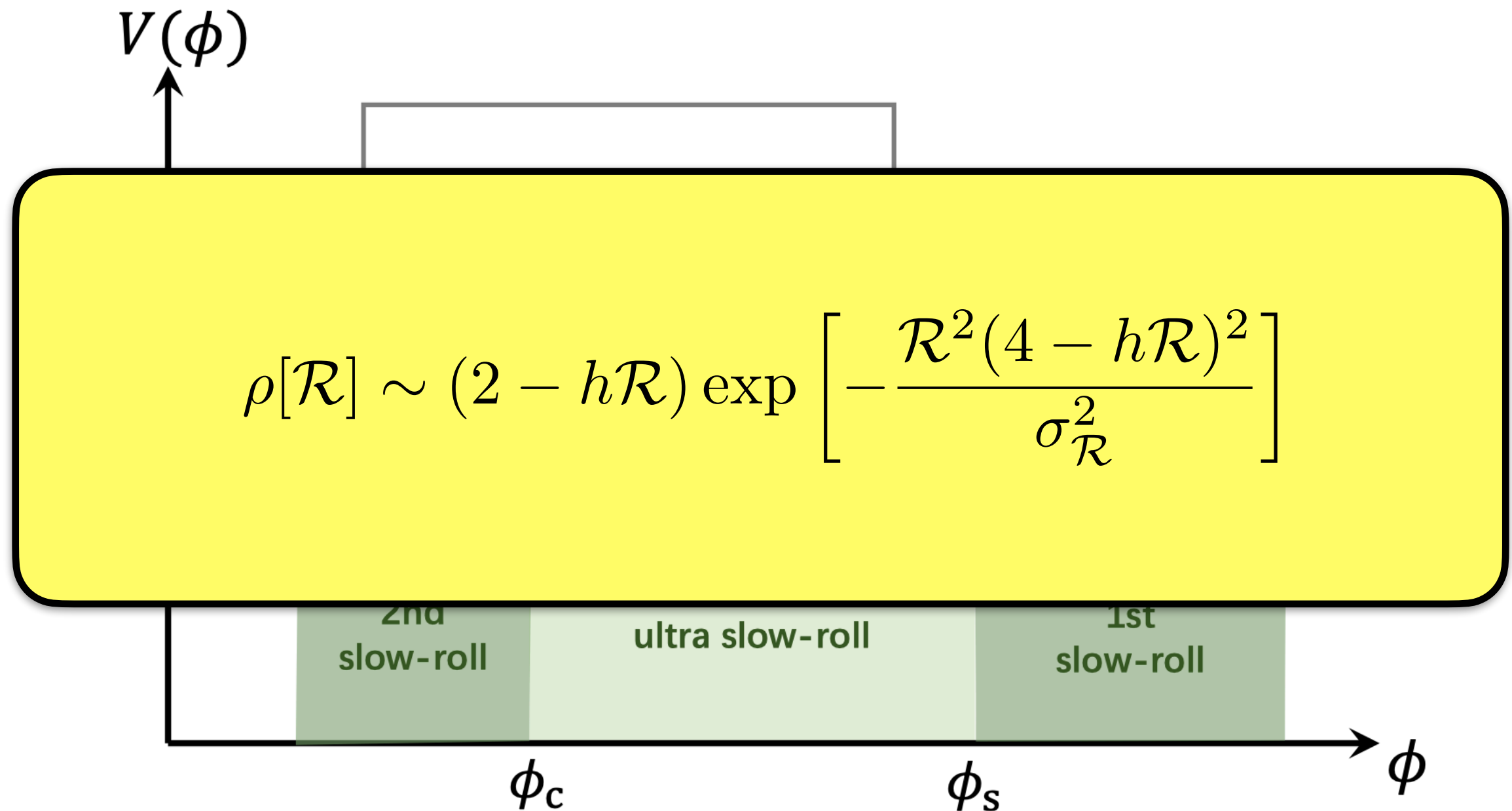
Other recent results on highly non-Gaussian tails:



✿ Beyond the bispectrum

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Other recent results on highly non-Gaussian tales:



- The primordial statistics may deviate significantly from Gaussianity in a way not parametrized by the bispectrum
- These effects could escape conventional data analysis
- New perturbative and nonperturbative techniques are necessary to uncover this type of NG

- The primordial statistics may deviate significantly from the expected distribution
- The correlation function is not Gaussian
- Need to develop new techniques to analyze this type of NG

Thanks!