Non-minimally Assisted Inflation : A General Analysis

Sang Chul Hyun

in collaboration with

Jinsu Kim(Tongji U.), **Tatsuki Kodama**(Saga U.),
Seong Chan Park(Yonsei U.), **Tomo Takahashi**(Saga U.)

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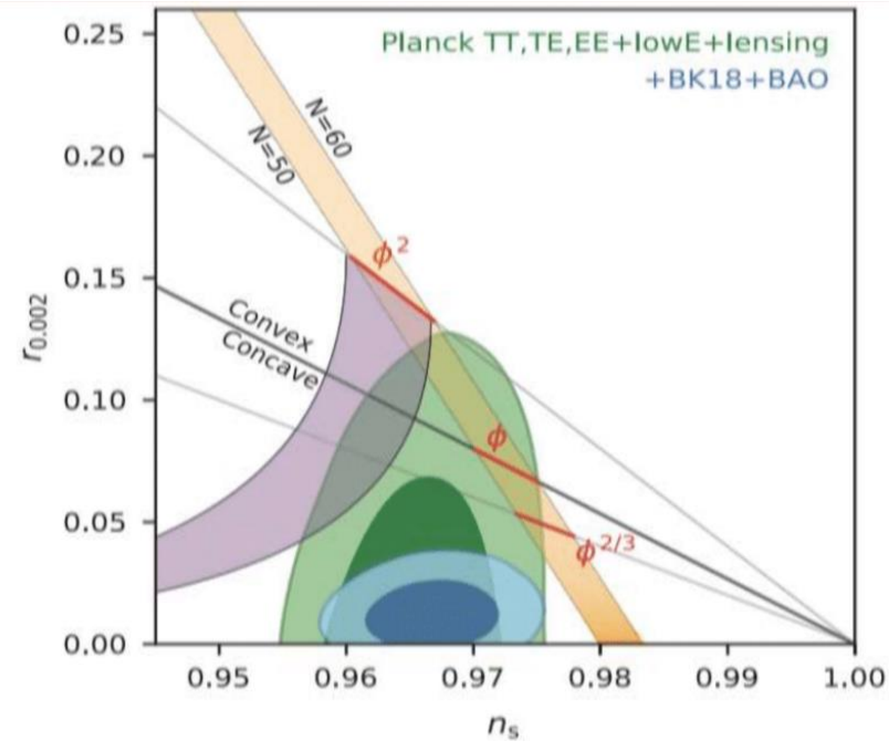
Invalidity of Single-Field Inflation

- The latest observational constraints jointly analyzed by the Planck and BICEP/Keck (BK) array have come out. $\rightarrow$ Experimental bounds regarding scalar spectral index n_s and tensor-to-scalar ratio r became more stringent. ($0.958 \leq n_s \leq 0.975, r \leq 0.036$ for 2σ bound)

- Consequently, a plethora of single-field inflationary models that had been viable before latest results have been ruled out.
    - Power-law chaotic inflation $V(\phi) \sim \phi^p$
    - Power-law exponential inflation $V(\phi) \sim \exp(-\lambda\phi)$
    - Hybrid Inflation $V(\phi) = 1 + (\phi/\mu)^2$
    - else..



Y. Akrami et al. (2020)
P. A. R. Ade et al. (2021)

Our Idea

- To solve aforementioned problems that single-field inflationary scenario has, we aim to modify that scenario. There are lots of mechanisms to do that.
  - Introduction of non-minimal coupling of the inflaton field to gravity sector *F. L. Bezrukov (2008)*
  - Modification of inflaton kinetic term *S. Ferrara (2013)*

- In this work, we took different approach. We planned to add another scalar field named ***assistant field***, denoted by s, to build multi-field inflationary Lagrangian.

Our Idea

- The most general Lagrangian containing original inflaton ϕ and newly introduced field s would be written as follows :

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$\rightarrow S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 f(s) R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(s) - V(\phi)\right]$$

- Non-minimal coupling with gravity f can be generally expressed with respect to ϕ and s field.

The Assistant Field

- We assume that newly introduced field s is assistant field. It is defined as a field where the energy density of that field is extremely small comparing with the original inflaton field ϕ. In other words,

$$|\rho_s| \ll \rho_\phi$$

- However, not all multi-field inflationary scenario satisfy this condition. Several assumptions between two fields are necessary.

Assumptions

- Assumption #1 : $V_J(\phi)$ is negligible comparing with $V_J(s)$.

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 f(s)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V_J(s) - V_J(\phi)\right]$$

$$\rightarrow S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 f(s)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V_J(\phi)\right]$$

Total energy density

Slow-roll assumption

$$\rho = T^{00} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{s}^2 + V_J(\phi) + V_J(s) - 3\frac{df}{ds}H\dot{s} \simeq \underbrace{V_J(\phi)}_{\rho_\phi} + \underbrace{V_J(s) - 3\frac{df}{ds}H\dot{s}}_{\rho_s}$$

First assumption naturally leads to a massless assistant field.

Assumptions

- Assumption #2 : During inflation, the amplitude of s field is also negligible comparing with Planckian mass.

- Taylor Series expansion of non-minimal coupling function $f(s)$

$$f(s) = 1 + \xi_2 \left(\frac{s}{M_P}\right)^2 + \xi_4 \left(\frac{s}{M_P}\right)^4 + \cdots$$

results in a multi-field inflationary scenario with *minimal* coupling such that it contains effective mass term of assistant field inside an action.

$$\frac{1}{2}M_{Pl}^2 f(s)R = \frac{1}{2}M_{Pl}^2 R + \frac{1}{2}M_{Pl}^2 \xi_2 \left(\frac{s}{M_{\mathrm{Pl}}}\right)^2 R + \cdots$$

$$m_s^2 = -M_{Pl}^2 \xi_2 \frac{R}{M_{Pl}^2} = -\xi_2 R \simeq -\xi_2 (12H^2)$$

Assumptions

- Assumption #2 : During inflation, the amplitude of s field is also negligible comparing with Planckian mass.

- Calculation of ratio of an energy density of assistant field $\rho_s \approx \frac{1}{2}m_s^2 s^2$ to an energy density of inflaton field $\rho_\phi \approx V_J(\phi)$ naturally

$$\frac{|\rho_s|}{\rho_\phi} \approx \frac{6\xi_2 H^2 s^2}{V_J(\phi)} \simeq \frac{2\xi_2 \frac{s^2}{M_{Pl}^2}}{\left(1 + \xi_2 \frac{s^2}{M_{Pl}^2}\right)^2}$$

if

$$\xi_2 \frac{s^2}{M_{Pl}^2} \ll 1$$

More on Assumption #2

- Thanks to the second assumption, higher order terms of non-minimal coupling term beyond the first order can be safely negligible.

$$f(s) \simeq 1 + \xi_2 \left(\frac{s}{M_P} \right)^2$$

- Consequently, our action becomes

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{Pl}^2 \left(1 + \xi_2 \left(\frac{s}{M_{Pl}} \right)^2 \right) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s - V_J(\phi) \right]$$

- One advantage of using second approximation is to express general mathematical form of non-minimal coupling by using only one parameter : ξ_2 .

Comparison with Our Previous Work

- Previously, we first studied non-minimal assisted inflationary scenario, in the case of chaotic inflation, where the potential form is given as follows :

$$V(\phi) = \lambda_\phi M_{Pl}^4 \left(\frac{\phi}{M_{Pl}} \right)^n$$

Sang Chul Hyun, Jinsu Kim, Seong Chan Park, Tomo Takahashi (2022)

- We can study other general single-field inflation models other than chaotic one. Extension beyond chaotic inflation is the main motivation of this work.

Weyl Transformation

- Converting Jordan-frame action into Einstein-frame one,

$$S_J = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 f(s)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(\phi)\right]$$

$$\Big\downarrow \quad g_{E,\mu\nu} = f(s)g_{\mu\nu},\, f(s) = 1+\xi_2(s/M_{Pl})^2$$

$$R = f(s)[R_E - 6g^{E,\mu\nu}\nabla_\mu\nabla_\nu \ln\sqrt{f(s)} - 6g^{E,\mu\nu}\nabla_\mu \ln\sqrt{f(s)}\nabla_\nu \ln\sqrt{f(s)}]$$

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2}\mathcal{K}_1 g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\mathcal{K}_2 g_E^{\mu\nu}\partial_\mu s\partial_\nu s - V_E(\phi,s)\right]$$

$$V_E(\phi,s) = \frac{V(\phi)}{f(s)^2} = \frac{V(\phi)}{(1+\xi_2 s^2/M_{Pl}^2)^2} \qquad \mathcal{K}_1 \equiv \frac{1}{f(s)}, \quad \mathcal{K}_2 \equiv \frac{f(s)+6\xi_2^2(s/M_{Pl})^2}{f(s)^2}$$

Einstein-frame potential

Canonical Field

- Introducing another scalar field σ to canonically normalize kinetic term of assistant field,

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2}\mathcal{K}_1 g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\mathcal{K}_2 g_E^{\mu\nu}\partial_\mu s\partial_\nu s - V_E(\phi, s)\right]$$

$$= \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2f(s)}(\partial\phi)^2 - \frac{1}{2}(\partial\sigma)^2 - V_E(\phi, \sigma(s))\right]$$

$$V_E(\phi, s) = \frac{V(\phi)}{f(s)^2} = \frac{V(\phi)}{(1+\xi_2 s^2/M_{Pl}^2)^2}$$

Einstein-frame potential

$$\left(\frac{\partial\sigma}{\partial s}\right)^2 = \mathcal{K}_2 = \frac{f(s) + 6\xi_2^2(s/M_{Pl})^2}{f(s)^2}$$

Slow-roll analysis & δN formalism

- We transformed action in Jordan frame to Einstein frame and earned approximated equation of motions by using slow-roll assumption.

Slow-roll assumption : $\{\epsilon^i, |\eta^{ij}|, \epsilon^b\} \ll 1$

$$\epsilon^\sigma \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\sigma}}{V}\right)^2, \epsilon^\phi \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\phi}}{V}e^{-b}\right)^2, \eta^{\sigma\sigma} \equiv M_{Pl}^2\frac{V_{,\sigma\sigma}}{V},$$

$$\eta^{\phi\phi} \equiv M_{Pl}^2\frac{V_{,\phi\phi}}{V}e^{-2b}, \eta^{\phi\sigma} \equiv M_{Pl}^2\frac{V_{,\phi\sigma}}{V}e^{-b}, \epsilon^b \equiv 8M_{Pl}^2 b_{,\sigma}^2$$

- We calculated three CMB observables : spectral index n_s, tensor-to-scalar ratio r and local-type nonlinearity parameter $f_{NL}^{(local)}$ to match with latest constraints. We used δN formalism to calculate these and plotted them numerically.

· *A. A. Starobinsky, PLB117, 175. (1982),*
· *D. S. Salopek, J. R. Bond, PRD42, 3936 (1990),*
· *M. Sasaki and E. D. Stewart, PTP 95, 71. (1996)*

Field Space metric

$$G_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2b(s)} \end{pmatrix}, b(s) \equiv \frac{1}{2} \ln \mathcal{K}_1 = \frac{1}{2} \ln \left(\frac{1}{1 + \xi_m s^m / M_{Pl}^m} \right)$$

Cosmological observables

Expressions for the cosmological observables in the δN formalism ($N_{,i} \equiv \frac{\partial N}{\partial \varphi^i} \quad (\varphi^i = \{\sigma, \phi\})$)

❖Curvature power spectrum : $\mathcal{P}_\zeta = \left(\frac{H}{2\pi} \right)^2 G^{ij} N_{,i} N_{,j}$

❖Scalar Spectral Index : $n_s = 1 + 2\frac{\dot{H}}{H^2} - 2\frac{1 + N_{,k}\left(\frac{M_{Pl}^6}{3} R^{kmnl} V_{,m} V_{,n} / V^2 - M_{Pl}^4 V^{;kl} / V \right) N_{,l}}{G^{ij} N_{,i} N_{,j} M_{Pl}^2}$

❖Tensor-to-Scalar Ratio : $r = \frac{8}{M_{Pl}^2 G^{ij} N_{,i} N_{,j}}$

❖Local-type nonlinearity parameter : $f_{NL}^{local} = -\frac{5}{6} \frac{G^{ij} G^{mn} N_{,i} N_{,m} N_{,jn}}{(G^{kl} N_{,k} N_{,l})^2}$

- We set the number of e-folds from horizon crossing point($\star$) to end-of-inflation point(e) to be equal to 60 . ($N = \int_\star^e H dt = 60$)

J. Kim, Y. Kim and S. C. Park, CQG31, 135004 (2014)

Model Independent Analysis

- Class I : End of inflation via slow-roll violations
  - Chaotic Inflation : $V(\phi) = \lambda_\phi M_{Pl}^4 \left(\frac{\phi}{M_{Pl}}\right)^n$
  - Loop Inflation : $V_J(\phi) = \Lambda^4(1 + \alpha \log \phi)$
- Class II : End of inflation via a separate sector
  - Power-law Inflation : $V_J(\phi) = \Lambda^4 \exp(-\lambda\phi)$ →
  - Hybrid Inflation : $V_J(\phi) = \Lambda^4 \left[1 + \left(\frac{\phi}{\mu}\right)^2\right]$

Slow-roll Parameters

$$\epsilon^{(0)} \equiv \frac{1}{2}\left(\frac{V_{J,\phi}}{V_J}\right)^2 = \frac{1}{2}\lambda^2$$

$$\eta^{(0)} \equiv \frac{V_{J,\phi\phi}}{V_J} = \lambda^2$$

Remark on Hybrid Inflation

The original scalar potential for the hybrid inflation model is given as follows :

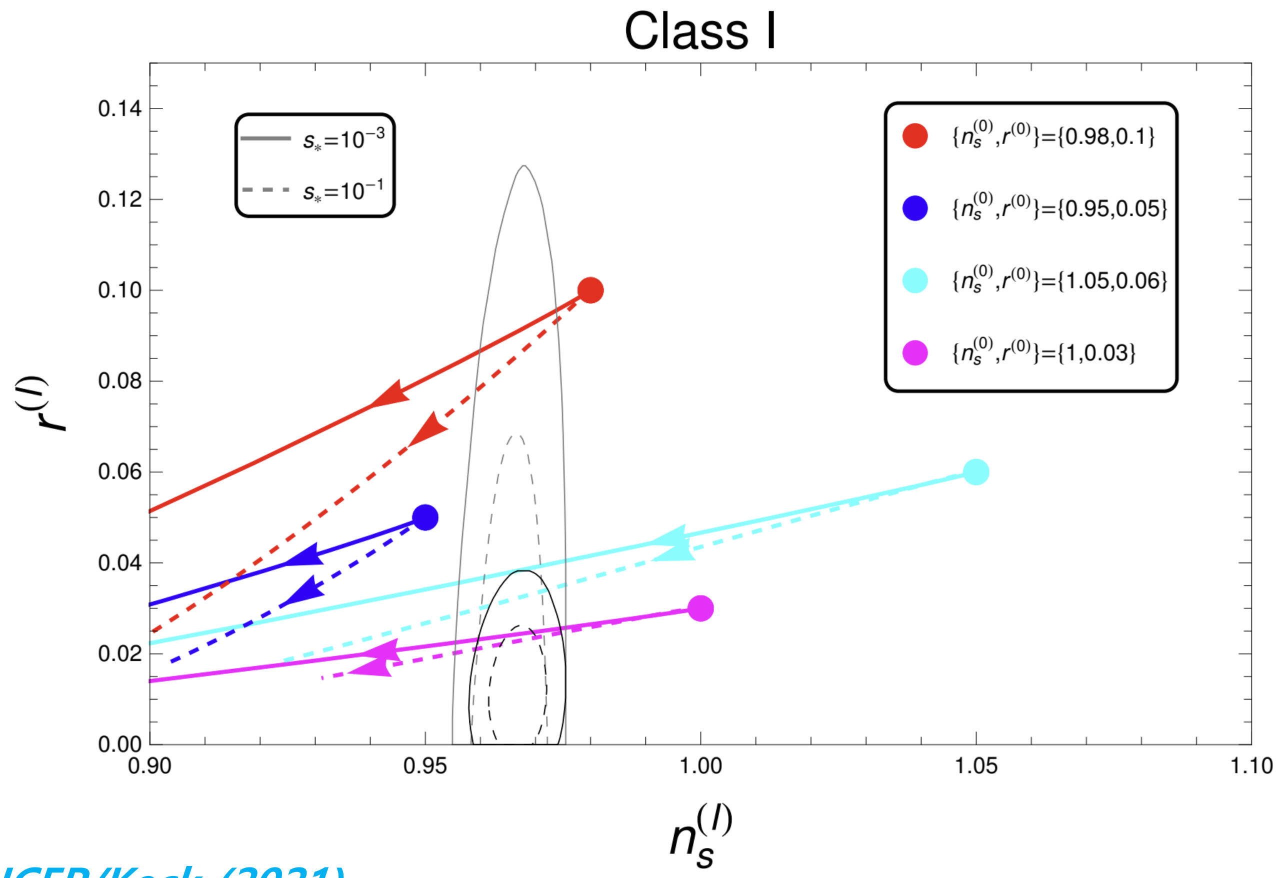
$$V_J(\phi, s) = \frac{1}{2}m^2\phi^2 + \frac{\lambda'}{4}(s^2 - \Delta^2)^2 + \frac{\lambda}{2}\phi^2 s^2$$

Since we assumed that s field is extremely small($\xi_2(s/M_{\rm Pl})^2 \to 0$), the inflationary trajectory mainly goes through the valley($s = 0$) of the potential. Hence, it can be effectively described during inflation by

$$V_J(\phi) = \Lambda^4 \left[1 + \left(\frac{\phi}{\mu}\right)^2\right] \qquad \Lambda \equiv (\lambda'/4)^{1/4}\Delta, \mu \equiv \sqrt{\lambda'/2}\Delta^2/m$$

Model Independent Analysis

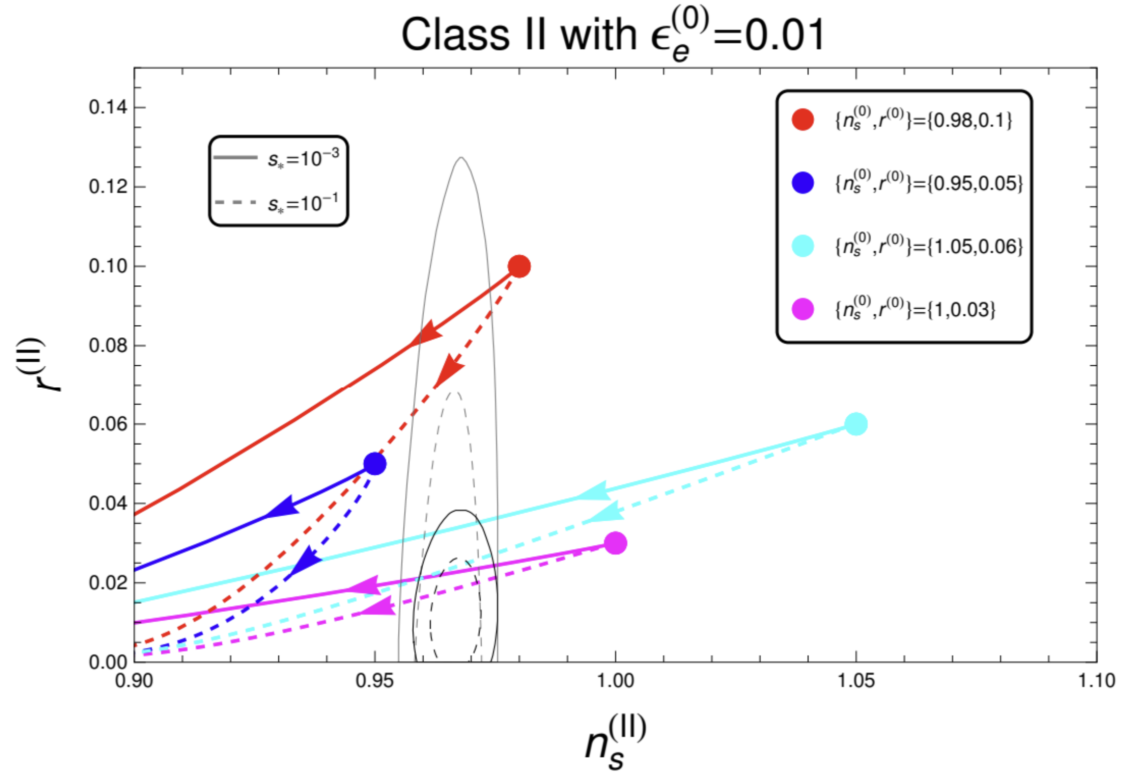
Class I : End of inflation via slow-roll violations

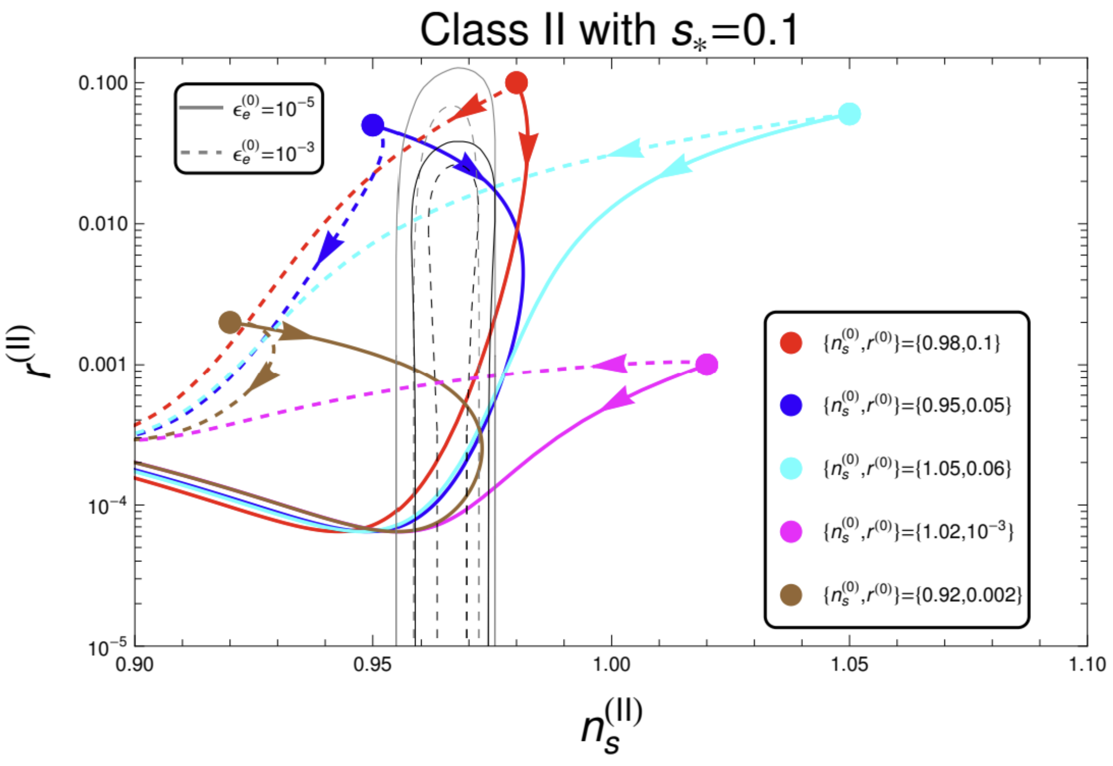


Planck (2020), Planck + BICEP/Keck (2021)

Model Independent Analysis

Class II : End of inflation via a separate sector





Planck (2020), Planck + BICEP/Keck (2021)

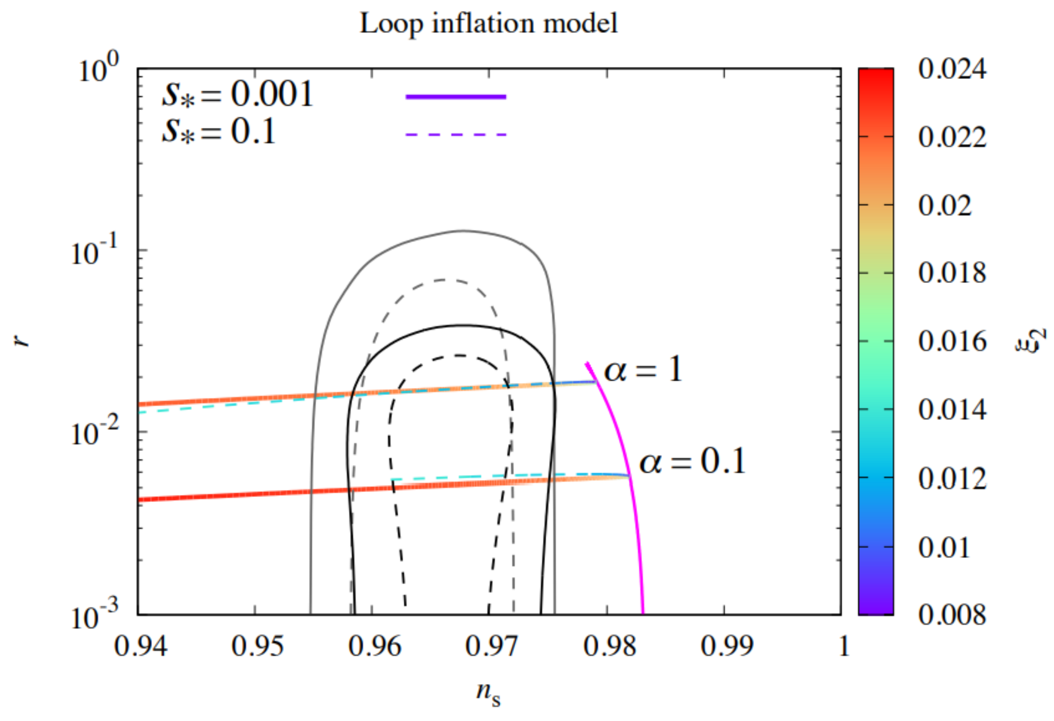
Model Dependent Analysis

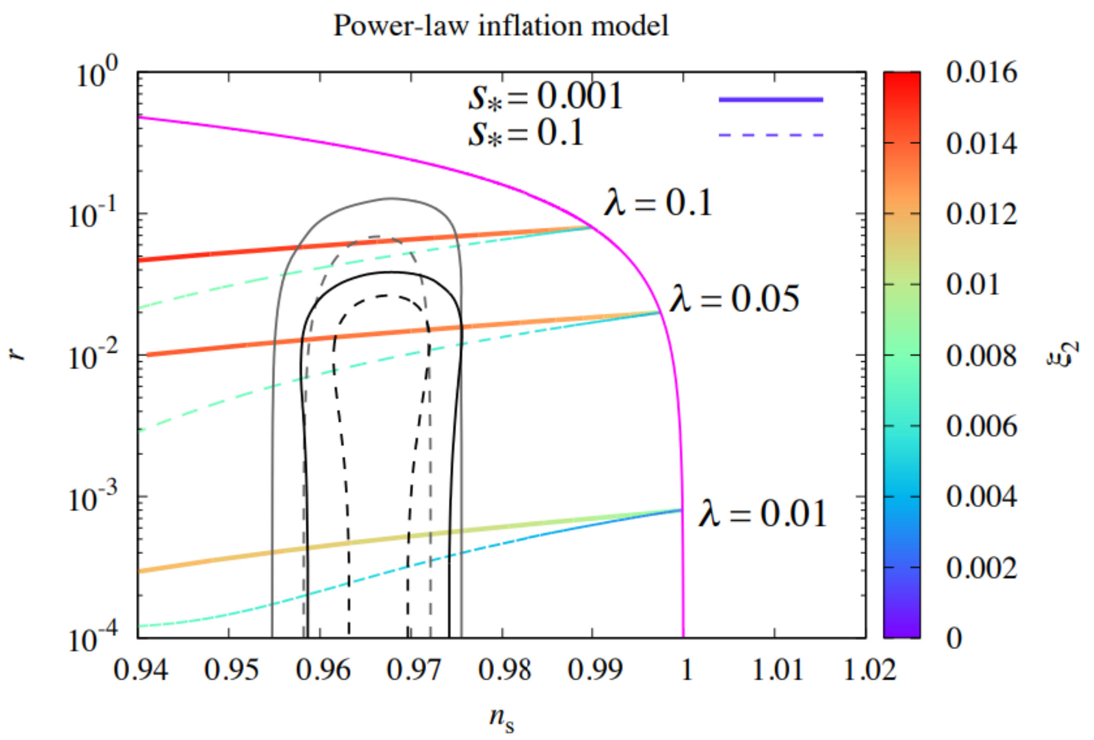
- Class I : End of inflation via slow-roll violations
  - Chaotic Inflation : $V(\phi) = \lambda_\phi M_{Pl}^4 \left(\frac{\phi}{M_{Pl}}\right)^n$

    Sang Chul Hyun, Jinsu Kim, Seong Chan Park, Tomo Takahashi (2022)

  - Loop Inflation : $V_J(\phi) = \Lambda^4(1 + \alpha \log \phi)$
- Class II : End of inflation via a separate sector
  - Power-law Inflation : $V_J(\phi) = \Lambda^4 \exp(-\lambda\phi)$
  - Hybrid Inflation : $V_J(\phi) = \Lambda^4 \left[1 + \left(\frac{\phi}{\mu}\right)^2\right]$

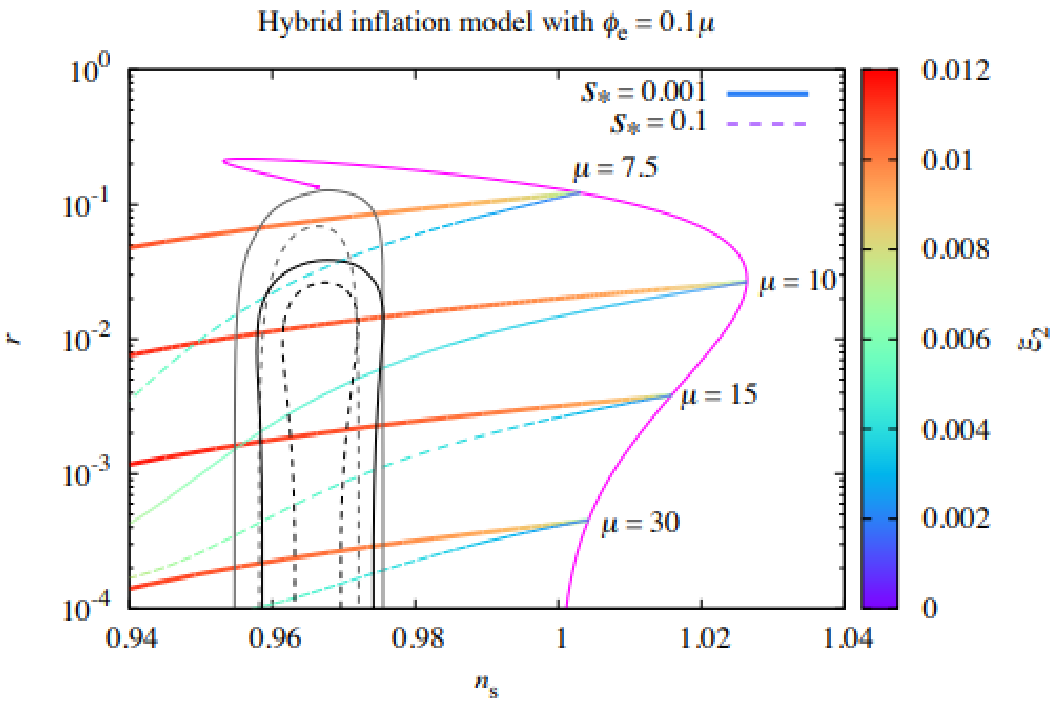
Loop Inflation / Power-law Inflation

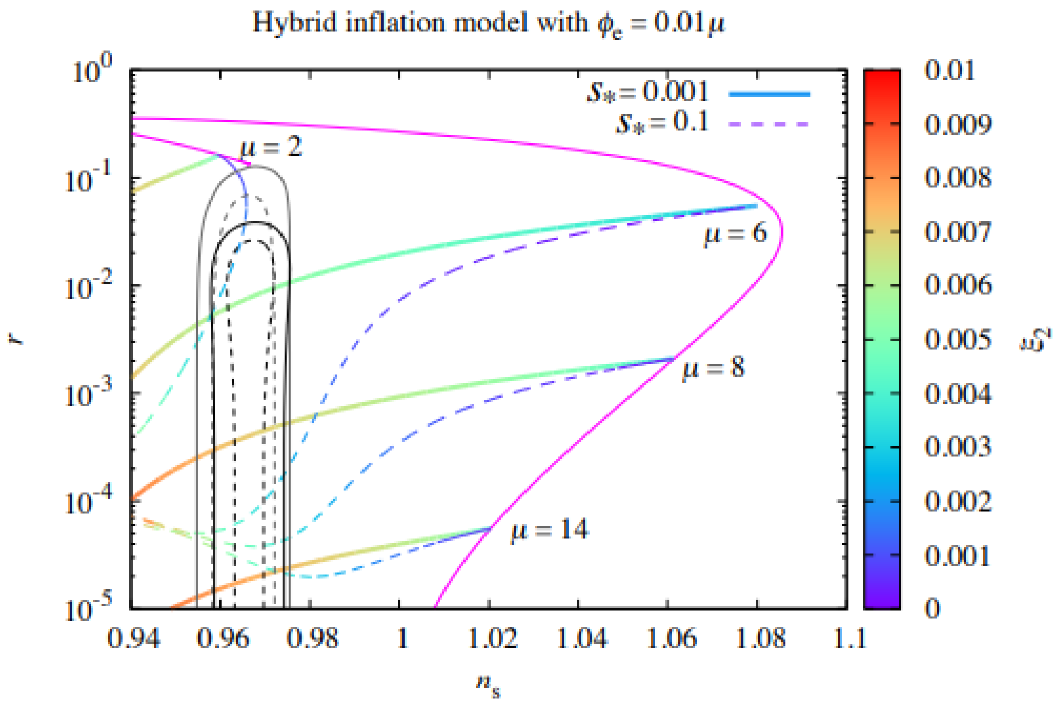




Planck (2020), Planck + BICEP/Keck (2021)

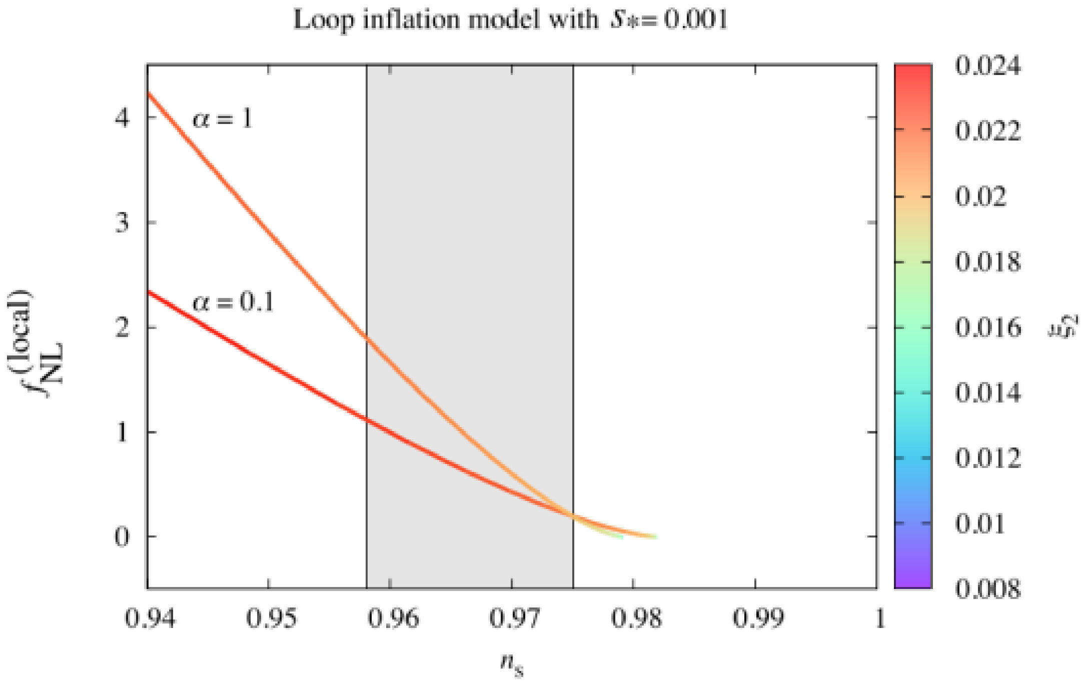
Hybrid Inflation

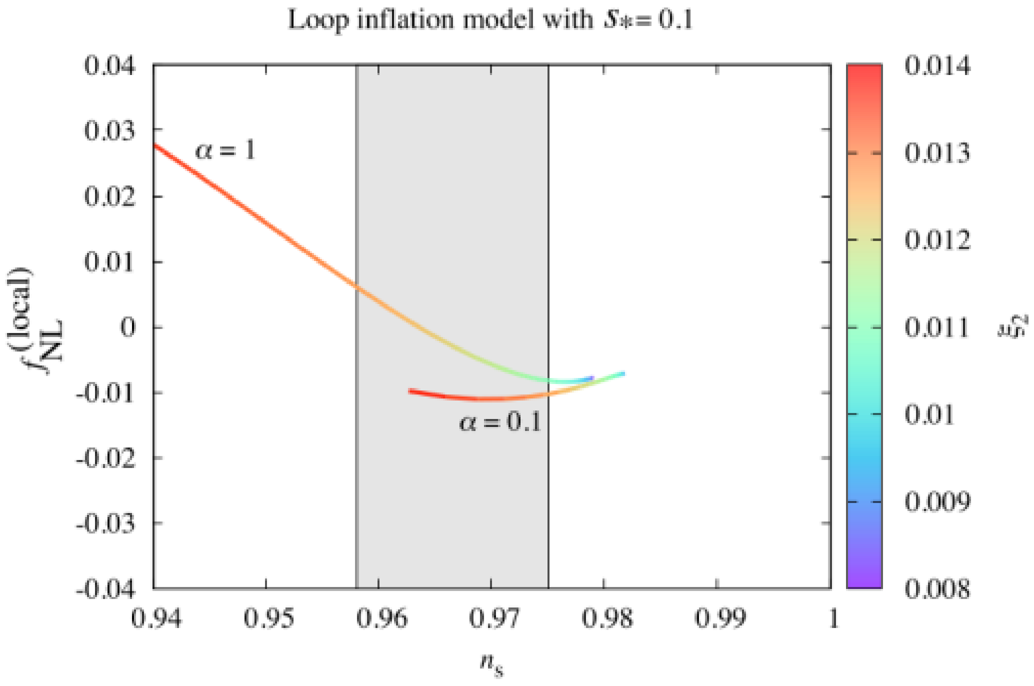




Planck (2020), Planck + BICEP/Keck (2021)

Loop Inflation

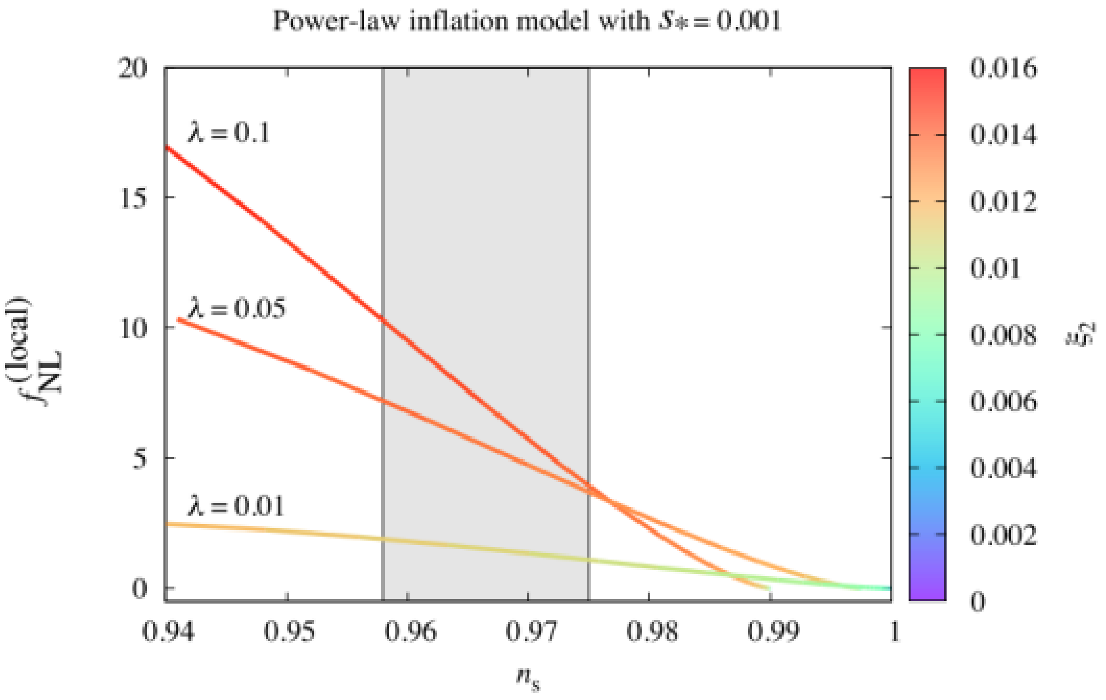


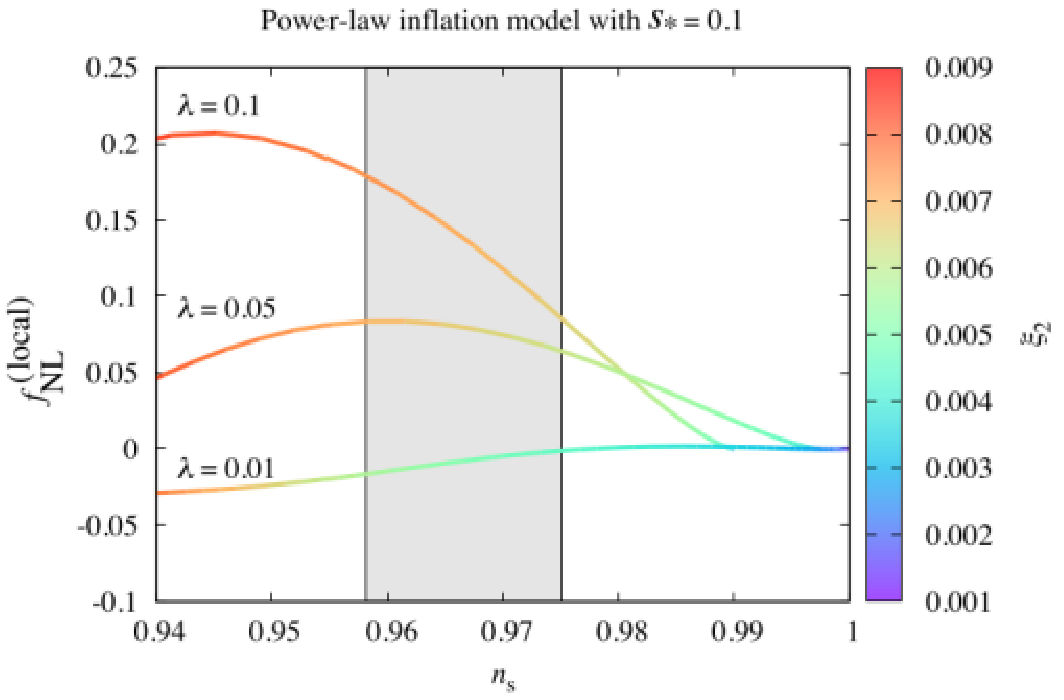


Planck (2020), Planck + BICEP/Keck (2021)

2σ bound for $f_{NL}^{(local)}$: $-11.1 < f_{NL}^{(local)} < 9.3$

Power-law Inflation

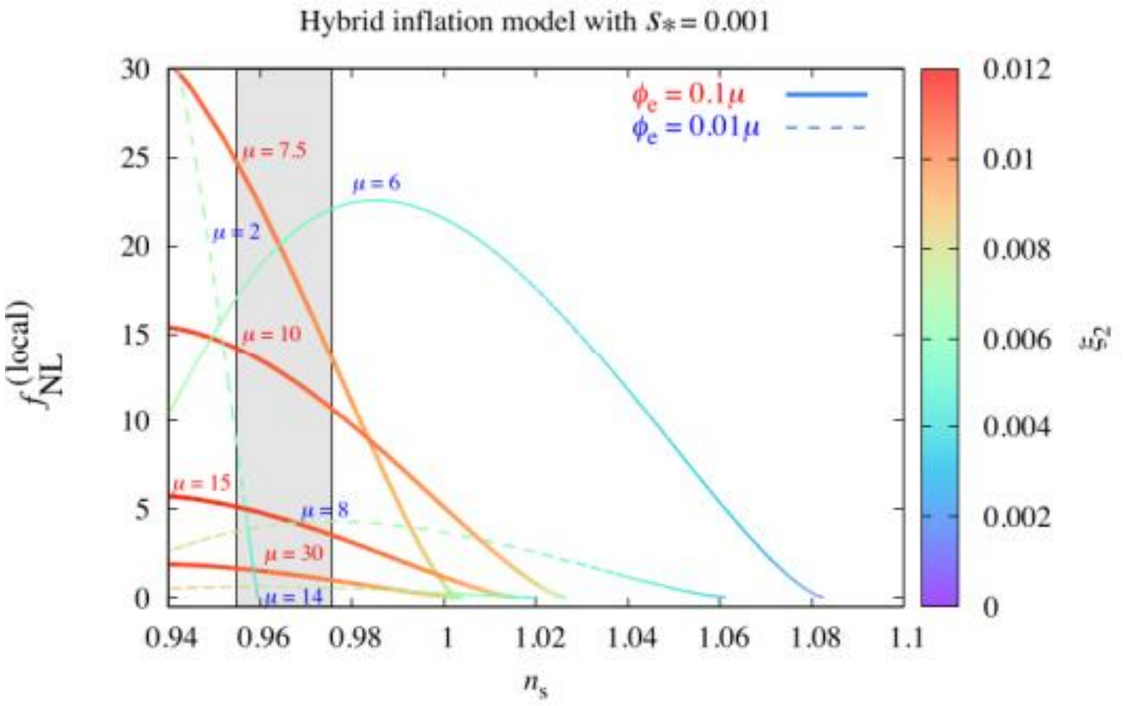


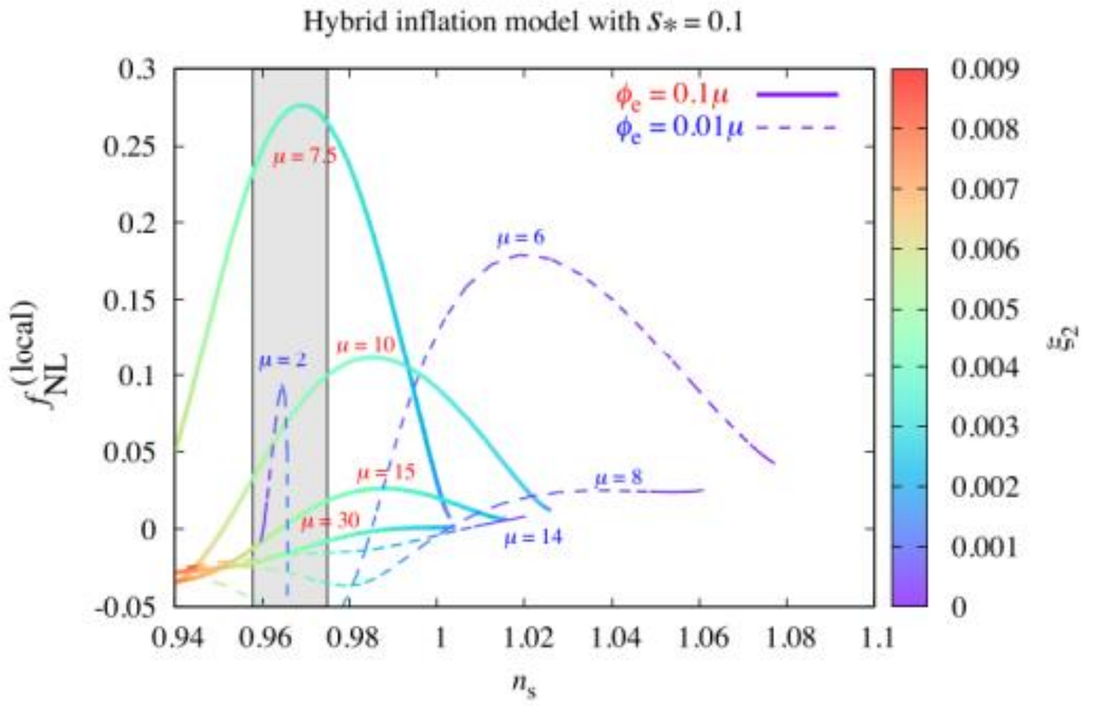


Planck (2020), Planck + BICEP/Keck (2021)

2σ bound for $f_{NL}^{(local)}$: $-11.1 < f_{NL}^{(local)} < 9.3$

Hybrid Inflation





Planck (2020), Planck + BICEP/Keck (2021)

2σ bound for $f_{NL}^{(local)}$: $-11.1 < f_{NL}^{(local)} < 9.3$

Conclusions

- As a way to rescue single-field inflationary scenario, we added additional scalar field to build multi-field inflationary Lagrangian.
- Newly introduced field is assumed to satisfy assistant field, whose energy density is extremely small comparing with energy density of inflaton.
- For Class I, both tensor to scalar ratio and the scalar spectral index decreases as the size of non-minimal coupling increases, reviving many single-field models which were previously ruled out.
- For Class II, a small $\epsilon_e^{(0)}$ value may bring a small scalar spectral index into the observationally favored region.
- Not all single-field models can be revived with the help of the assistant field, like a model which predicts a small n_s, and so on.

Thank you!

Backup : Running of Scalar Spectral Index

In a general multi-field inflationray scenario, the running of scalar spectral index is calculated by applying δN formalism.

· *Gong, J. O. (2015)*

$$\alpha_s \equiv \frac{dn_s}{d\ln k} \simeq 4\epsilon^2 - 2\frac{\dot{\epsilon}}{H} + 2\frac{N_{,i}N_{,j}}{G^{mn}N_{,m}N_{,n}}\left(4\epsilon w^{ij} + 2w^{i}{}_{k}w^{jk} - w^{ij}{}_{;k}\frac{\dot{\varphi}_0^k}{H}\right) - (n_s - 1)^2$$

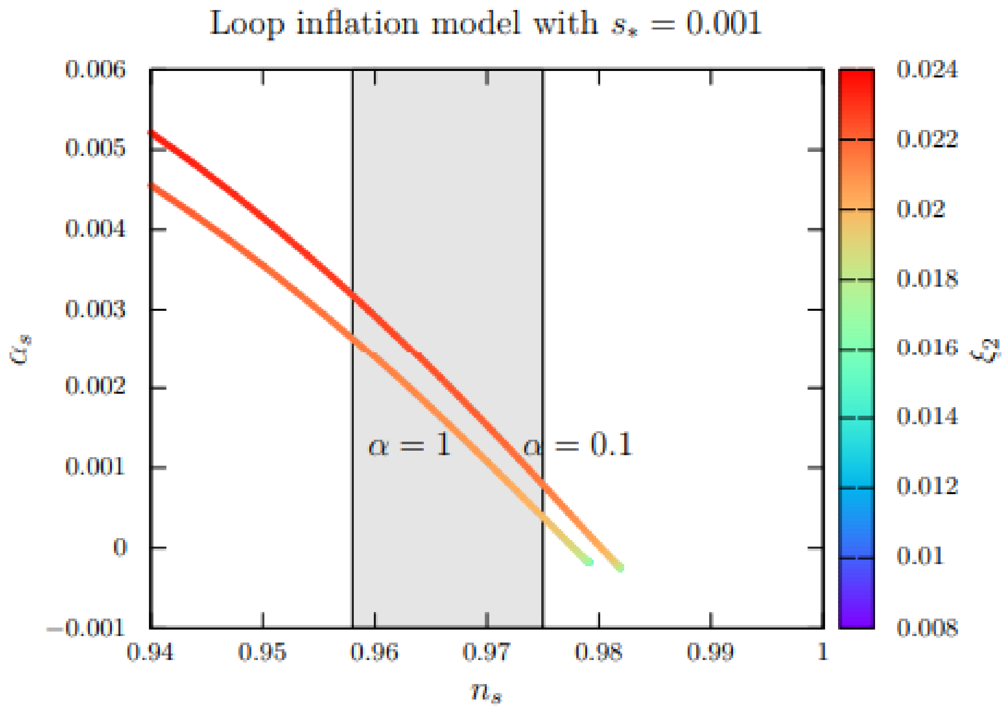
evaluated at the horizon crossing, denoted by the subscript *, where

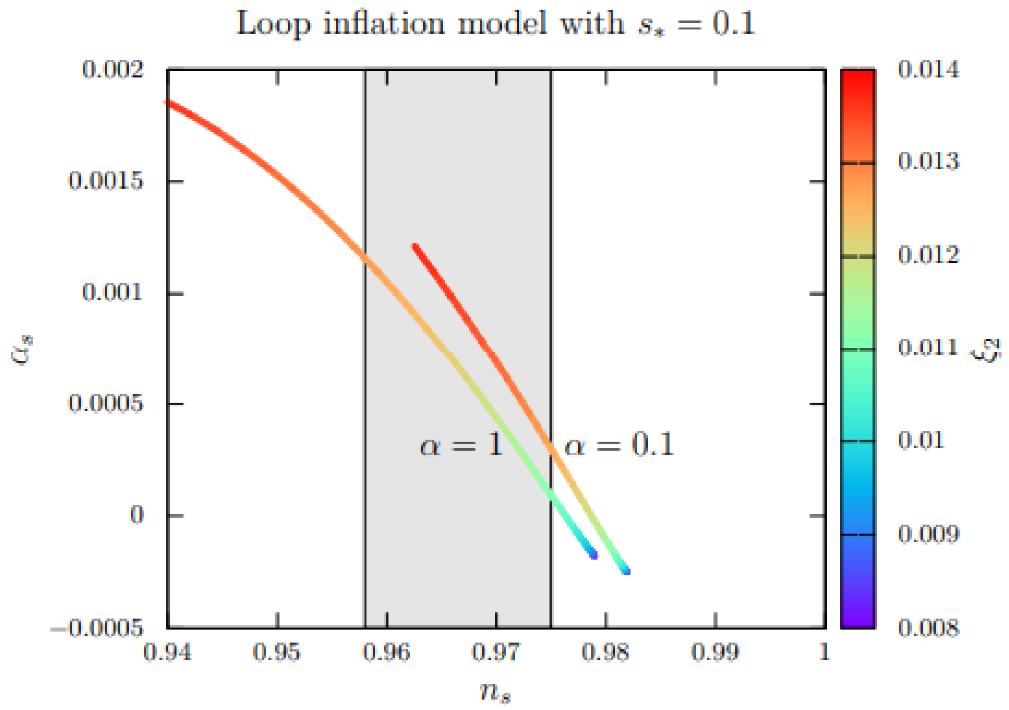
$$w_{ij} = w_{(i;j)} + \tfrac{1}{3}R_{m(ij)n}\frac{\dot{\varphi}_0^m\dot{\varphi}_0^n}{H^2}, u_i = -\frac{V_{,i}}{3H^2}$$

Constraints :

$$0.001 \le \alpha_s \le 0.025 \text{ (Planck TT+lowE+lensing,}1\sigma\text{ bound)}$$
$$-0.008 \le \alpha_s \le 0.012 \text{ (Planck TT,TE,EE+lowE+lensing,}1\sigma\text{ bound)}$$

Backup : Running of Scalar Spectral Index





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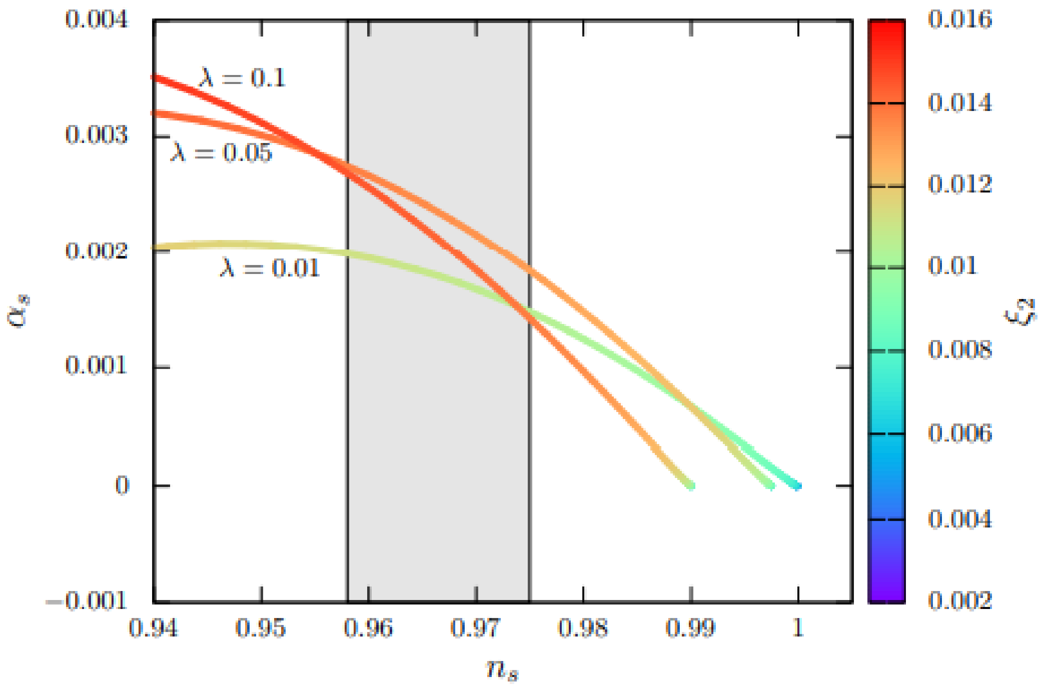

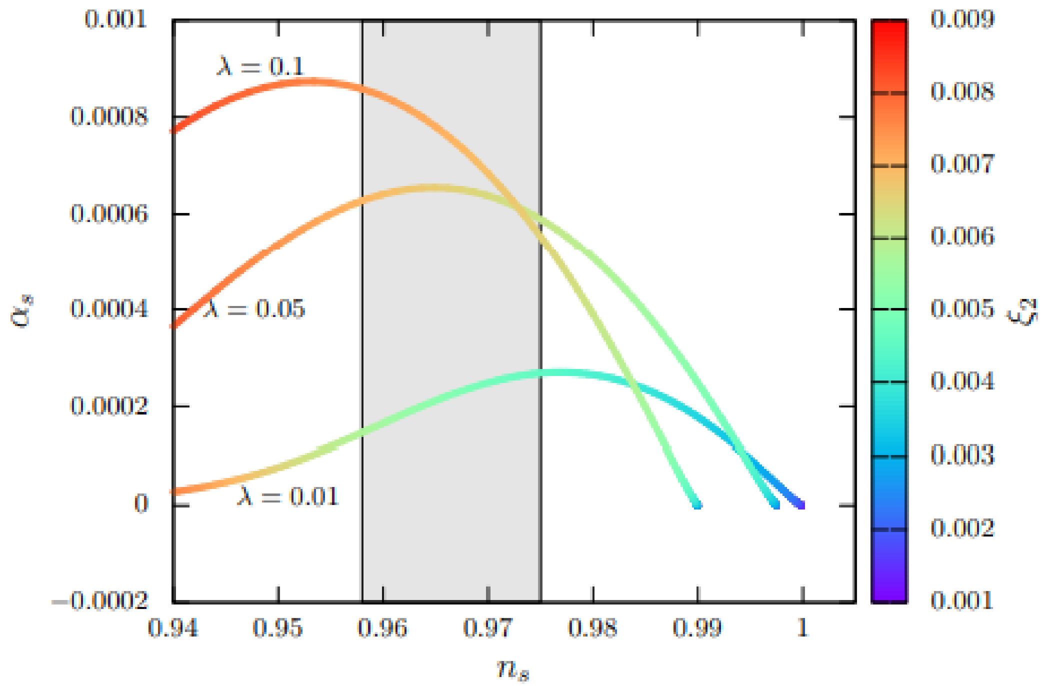

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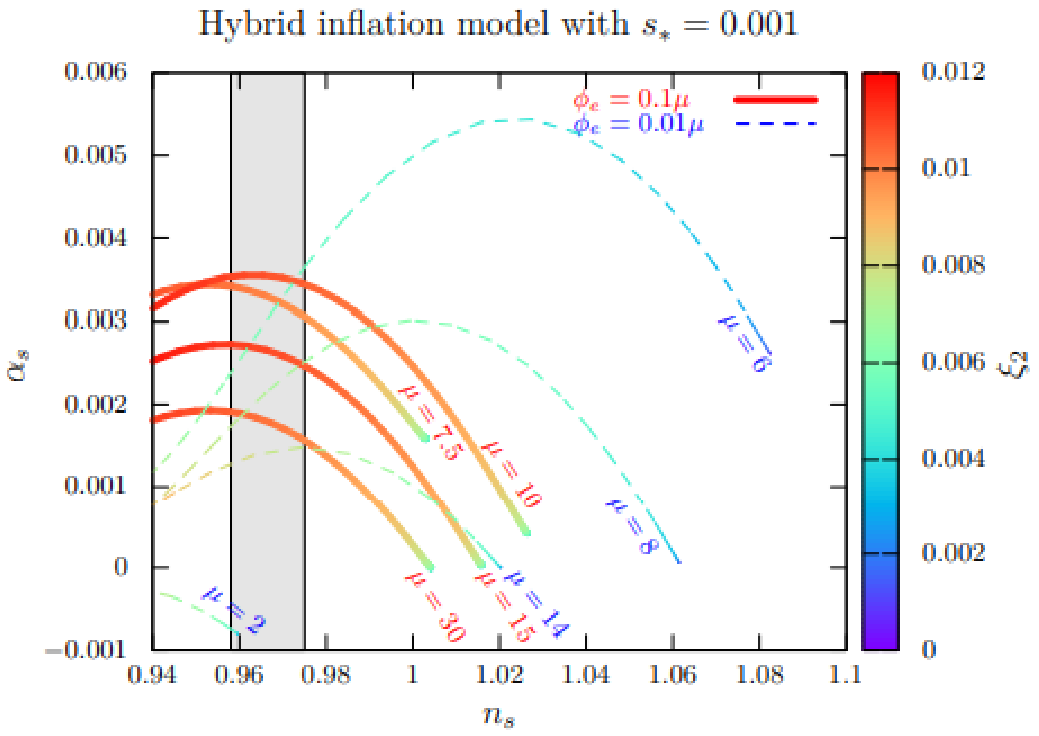

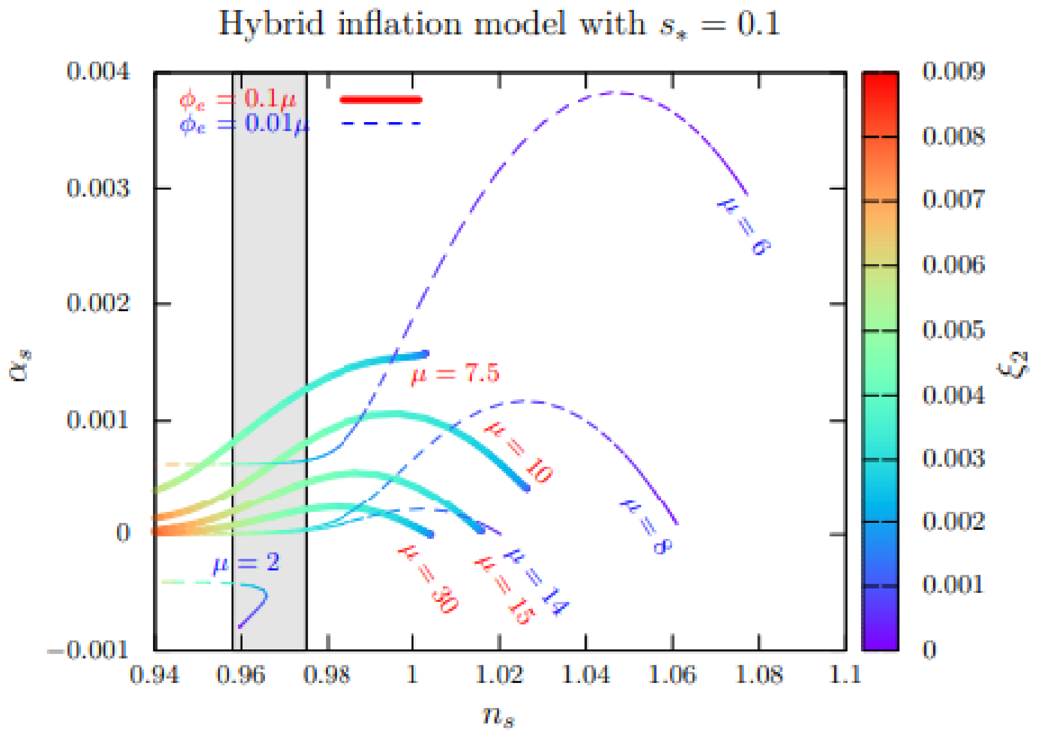

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Backup : Field Trajectories