

Inflation and tachyonic preheating with twin waterfalls

Adriana Menkara*, Hyun Min Lee

Based on Phys.Lett.B 834 (2022) 137483 and arxiv [2304.08686](#)

PPC 2023, Daejeon



Classifying inflationary models

1. Initial conditions.

- Both old and new inflation used to rely on **initial thermal equilibrium**.
A.H. Guth, Phys. Rev. D23, 347 (1981)
A.D. Linde, Phys. Lett. 108B
- **Chaotic inflation** studies all possible initial conditions, even outside thermal eq.

2. Shape of the potential.

- Quasiexponential inflation, power law inflation, etc.

3. End of inflation.

- **Slow roll** (e.g. chaotic inflation in the theories ϕ^n)
D. La and P.J. Steinhardt, Phys. Rev. Lett. 62, 376 (1989)
- **Waterfall phase transition** (e.g. extended inflation scenario, hybrid inflation)
A.D. Linde, Phys.Rev.D 49 (1994) 748-754

Classifying inflationary models

1. Initial conditions.

- Both old and new inflation used to rely on **initial thermal equilibrium**.
A.H. Guth, Phys. Rev. D23, 347 (1981)
A.D. Linde, Phys. Lett. 108B
- **Chaotic inflation** studies all possible initial conditions, even outside thermal eq.

2. Shape of the potential.

- Quasiexponential inflation, power law inflation, etc.

3. End of inflation.

- **Slow roll** (e.g. chaotic inflation in the theories ϕ^n)
D. La and P.J. Steinhardt, Phys. Rev. Lett. 62, 376 (1989)
- **Waterfall phase transition** (e.g. extended inflation scenario, hybrid inflation)
A.D. Linde, Phys.Rev.D 49 (1994) 748-754

Today's topic

Natural and Hybrid inflation

Natural inflation [Phys. Rev. Lett. 65, 3233]

Inflation = pNGb

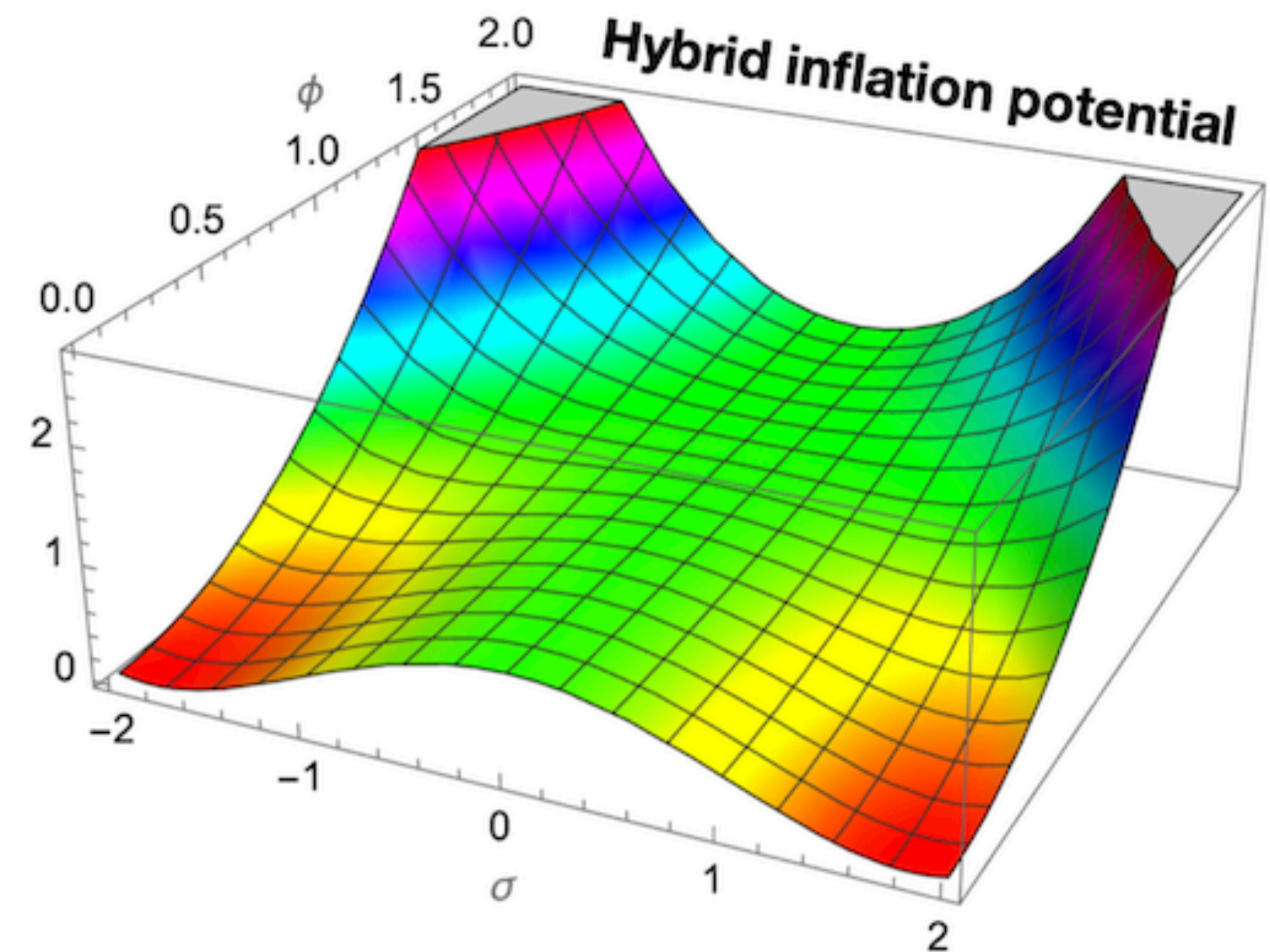
$$V(\phi) = \Lambda^4 \left(1 \pm \cos(\phi/f) \right) \quad f \sim M_P \quad \Lambda \sim M_{\text{GUT}}$$

Hybrid inflation

$$V(\sigma, \phi) = \underbrace{\frac{1}{4\lambda} (M^2 - \lambda\sigma^2)^2}_{\text{Ending stages}} + \underbrace{\frac{m^2}{2}\phi^2}_{\text{Chaotic inflation}} + \frac{g^2}{2}\phi^2\sigma^2$$

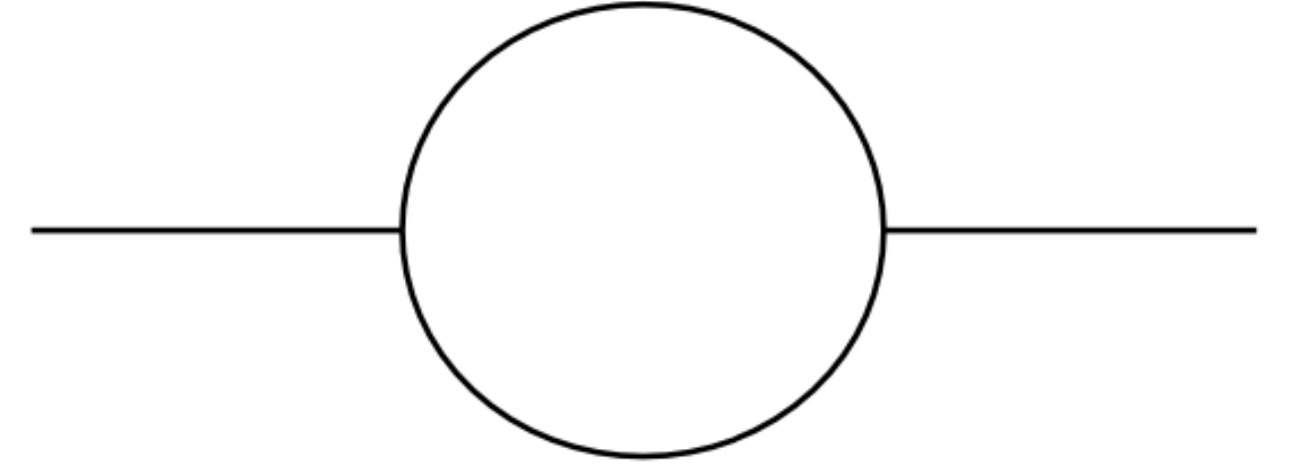
$V(0,0) = \frac{M^4}{4\lambda}$

$$m_\sigma^2 = -M^2 + g^2\phi^2 \quad \text{becomes tachyonic for } \phi < \phi_c = M/g$$



Inflation ends due to a waterfall phase transition

η problem



The coupling $\frac{g^2}{2}\phi^2\sigma^2$ badly breaks the shift symmetry $\longrightarrow$ Why is the inflation so light?

$$\left(\delta m_\phi^2\right)_{1\text{ loop}} \lesssim \left(\delta m_\phi^2\right)_{\text{tree}} \longrightarrow \Lambda^2 \lesssim (16\pi^2\eta) \frac{H^2}{g^2} \quad \text{Cut-off can not be arbitrarily large}$$

Frozen σ $M_{\sigma,\text{eff}}^2 \sim g^2\phi_0^2 \gtrsim H^2$ $\phi_0^2 \gtrsim \frac{\Lambda^2}{16\pi^2\eta}$ $\eta \sim 10^{-2}$ $\phi_0 \gtrsim \Lambda$

Excursion larger than the cutoff

Even in the marginal case $\phi_0 \sim \Lambda \implies M_{\sigma,\text{eff}}^2 \sim H^2$ $\langle\sigma\rangle \sim M_P$

EFT requires sub-Planckian fields

The model

$$V(\phi, \chi_1, \chi_2, H) = \underbrace{V_I(\phi)}_{\text{Natural inflation}} + \underbrace{V_W(\phi, \chi_1, \chi_2)}_{\text{Hybrid inflation}} + \underbrace{V_{\text{RH}}(\chi_1, \chi_2, H)}_{\text{Reheating}}$$

Slow-roll conditions and CMB set the symmetry breaking scale beyond M_P .

But the **Weak Gravity Conjecture** rules out a Trans-planckian axion decay constant f

In the presence of waterfall fields with a Z_2 **mirror symmetry** this can be avoided.

$$\phi \longrightarrow -\phi, \quad \chi_1 \longleftrightarrow \chi_2$$

The model

$$V(\phi, \chi_1, \chi_2, H) = V_I(\phi) + V_W(\phi, \chi_1, \chi_2) + V_{\text{RH}}(\chi_1, \chi_2, H)$$

Natural inflation

$$V_I(\phi) = V_0 + A(\phi)$$

Hybrid inflation

Reheating

$$V_{\text{RH}}(\chi_1, \chi_2, H) = \kappa_1(\chi_1^2 + \chi_2^2)|H|^2 + \kappa_2\chi_1\chi_2|H|^2.$$

$$A(-\phi) = A(\phi)$$

$$B(-\phi) = -B(\phi)$$

$$V_W(\phi, \chi_1, \chi_2) = B(\phi)(\chi_1^2 - \chi_2^2) + \frac{1}{2}m_\chi^2(\chi_1^2 + \chi_2^2) - \alpha^2\chi_1\chi_2$$

$$+ \beta(\chi_1^3 + \chi_2^3) + \gamma(\chi_1^2\chi_2 + \chi_1\chi_2^2)$$

$$+ \frac{1}{4}\lambda_\chi(\chi_1^4 + \chi_2^4) + \frac{1}{2}\bar{\lambda}_\chi\chi_1^2\chi_2^2 + \frac{1}{3}\lambda'_\chi(\chi_1^3\chi_2 + \chi_1\chi_2^3)$$

$$\phi^2(\chi_1^2 + \chi_2^2) \quad \text{suppressed}$$

Symmetries

$$\phi \longrightarrow -\phi, \quad \chi_1 \longleftrightarrow \chi_2$$

Z_2 protects the inflationary potential against quantum corrections

Origin of the Z_2 symmetry

$$Z_4 \times \text{CP} : \quad \phi \rightarrow -\phi, \quad \Phi \rightarrow i\Phi^*, \quad H \rightarrow H$$

Similar constructions

$$\Phi = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2)$$

[JHEP 07 (2021) 147 - Deshpande, Kumar, Sundrum]

[J.O. Gong, K.S. Jeong, Phys.Rev.D 104 (2021)]

Extra Z'_2 symmetry

$$Z'_2 : \quad \phi \rightarrow \phi, \quad \chi_1 \rightarrow \chi_1, \quad \chi_2 \rightarrow -\chi_2,$$

$$\alpha = \beta = \gamma = \lambda'_\chi = 0$$

$$V_W(\phi, \chi_1, \chi_2) = B(\phi)(\chi_1^2 - \chi_2^2) + \frac{1}{2}m_\chi^2(\chi_1^2 + \chi_2^2) - \cancel{\alpha^2 \chi_1 \chi_2} + \cancel{\beta(\chi_1^3 + \chi_2^3)} + \cancel{\gamma(\chi_1^2 \chi_2 + \chi_1 \chi_2^2)} \\ + \frac{1}{4}\lambda_\chi(\chi_1^4 + \chi_2^4) + \frac{1}{2}\bar{\lambda}_\chi \chi_1^2 \chi_2^2 + \cancel{\frac{1}{3}\lambda'_\chi(\chi_1^3 \chi_2 + \chi_1 \chi_2^3)},$$

χ_2 becomes a stable candidate for DM

Dark QCD

Microscopic model that realizes inflation

$$\mathcal{L}_{dQCD} = -m_u u_1 u_1^c - m_u u_2 u_2^c - y \Phi_1 u_1^c d - y' \Phi_1 u_1 d^c - i y \Phi_2 u_2^c d - i y' \Phi_2^* u_2 d^c + \text{h.c.}$$

Z_2 symmetry: $\Phi_1 \rightarrow i\Phi_2$ and $\Phi_2 \rightarrow -i\Phi_1$

χ_1 and χ_2 in our model are the real parts of Φ_1 and Φ_2

After QCD condensation

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \mu^2 (\chi_1^2 - \chi_2^2) \sin\left(\frac{\phi}{2f}\right) \quad \text{The } Z_2 \text{ symmetry remains } \textbf{unbroken}$$

Potential for the waterfalls arises from

$$\Delta V_W = m_\chi^2 (|\Phi_1|^2 + |\Phi_2|^2) + \lambda_\chi (|\Phi_1|^4 + |\Phi_2|^4) + 2\bar{\lambda}_\chi |\Phi_1|^2 |\Phi_2|^2 \quad Z_2 \text{ and } i \left(\Phi_1 \Phi_2^\dagger - \Phi_2 \Phi_1^\dagger \right) \text{ invariant}$$

Exit from inflation

Time dependent waterfall masses

$$m_1^2(\phi) = m_\chi^2 - \sqrt{B^2(\phi) + \alpha^4}$$

$$m_2^2(\phi) = m_\chi^2 + \sqrt{B^2(\phi) + \alpha^4}$$

Mass mixing

$$\sin 2\theta(\phi) = \frac{2\alpha^2}{m_2^2(\phi) - m_1^2(\phi)}$$

No mixing for $\alpha = 0 \longrightarrow \chi_1$ determines the end of inflation

We take the values $A(\phi) = \Lambda^4 \cos\left(\frac{\phi}{f}\right)$ $B(\phi) = -\frac{1}{2}\mu^2 \sin\left(\frac{\phi}{2f}\right) \longrightarrow$ **Shift symmetry broken to a discrete one**
 $\phi \rightarrow \phi + 4\pi f$

End of inflation

$$\phi_c = 2f \arcsin\left(\sqrt{m_\chi^4 - \alpha^4}/\mu^2\right)$$

***Slow-roll conditions still hold!**

UV insensitivity

$$V_{\text{CW}} = \frac{1}{64\pi^2} \sum_{i=1,2} \left[2m_{\chi_i}^2 M_*^2 - m_{\chi_i}^4 \ln \left(\frac{e^{\frac{1}{2}} M_*^2}{m_{\chi_i}^2} \right) \right] \simeq \frac{1}{16\pi^2} m_\chi^2 M_*^2 - \frac{1}{64\pi^2} \left(m_\chi^4 + 4B^2(\phi) + \alpha^4 \right) \ln \frac{M_*^2}{m_\chi^2}$$

**Constant vacuum
energy must be renormalized**

$$B^2(\phi) \lesssim 16\pi^2 |A(\phi)|$$

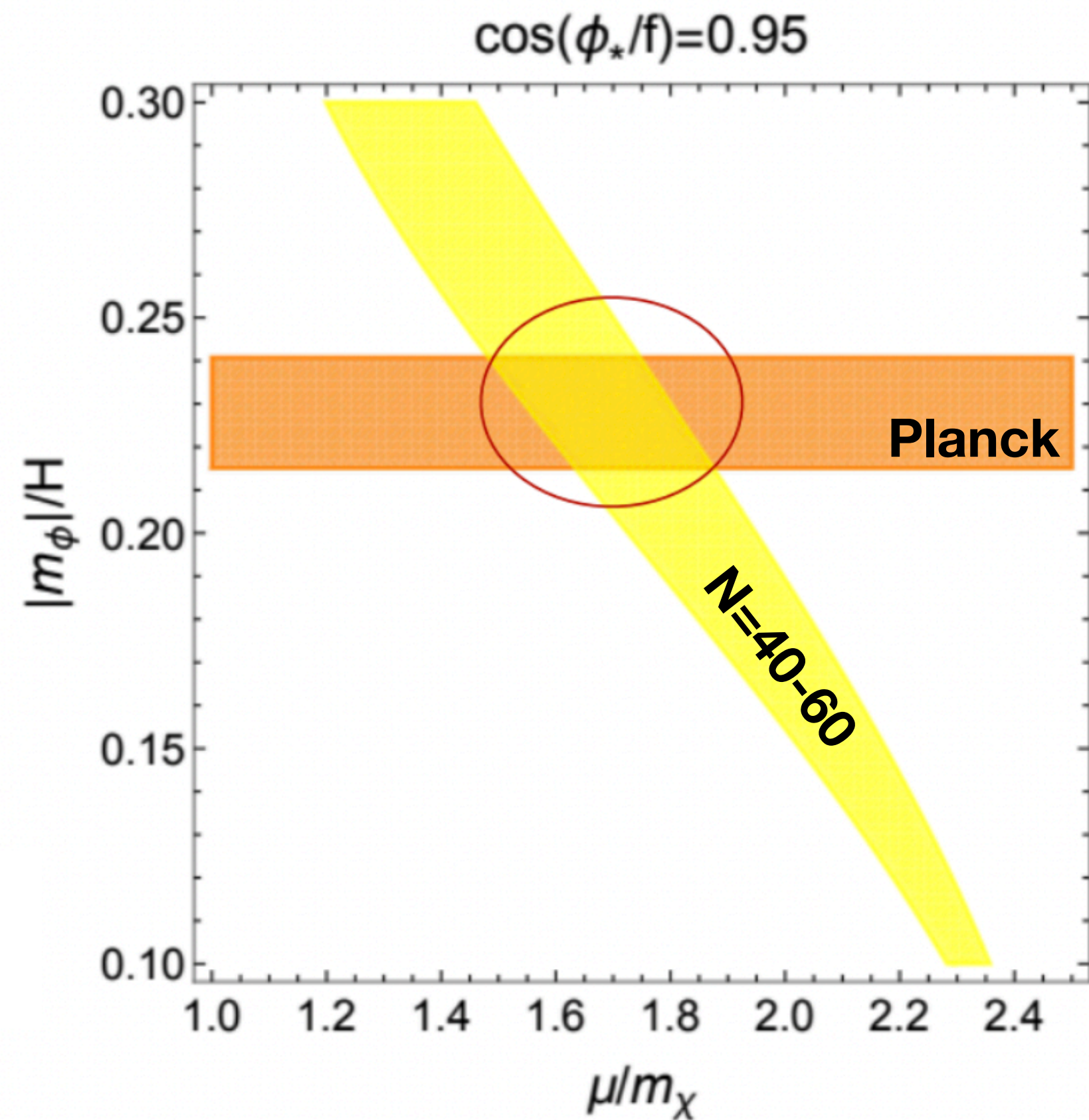
For our particular case

$$A(\phi) = \Lambda^4 \cos\left(\frac{\phi}{f}\right),$$

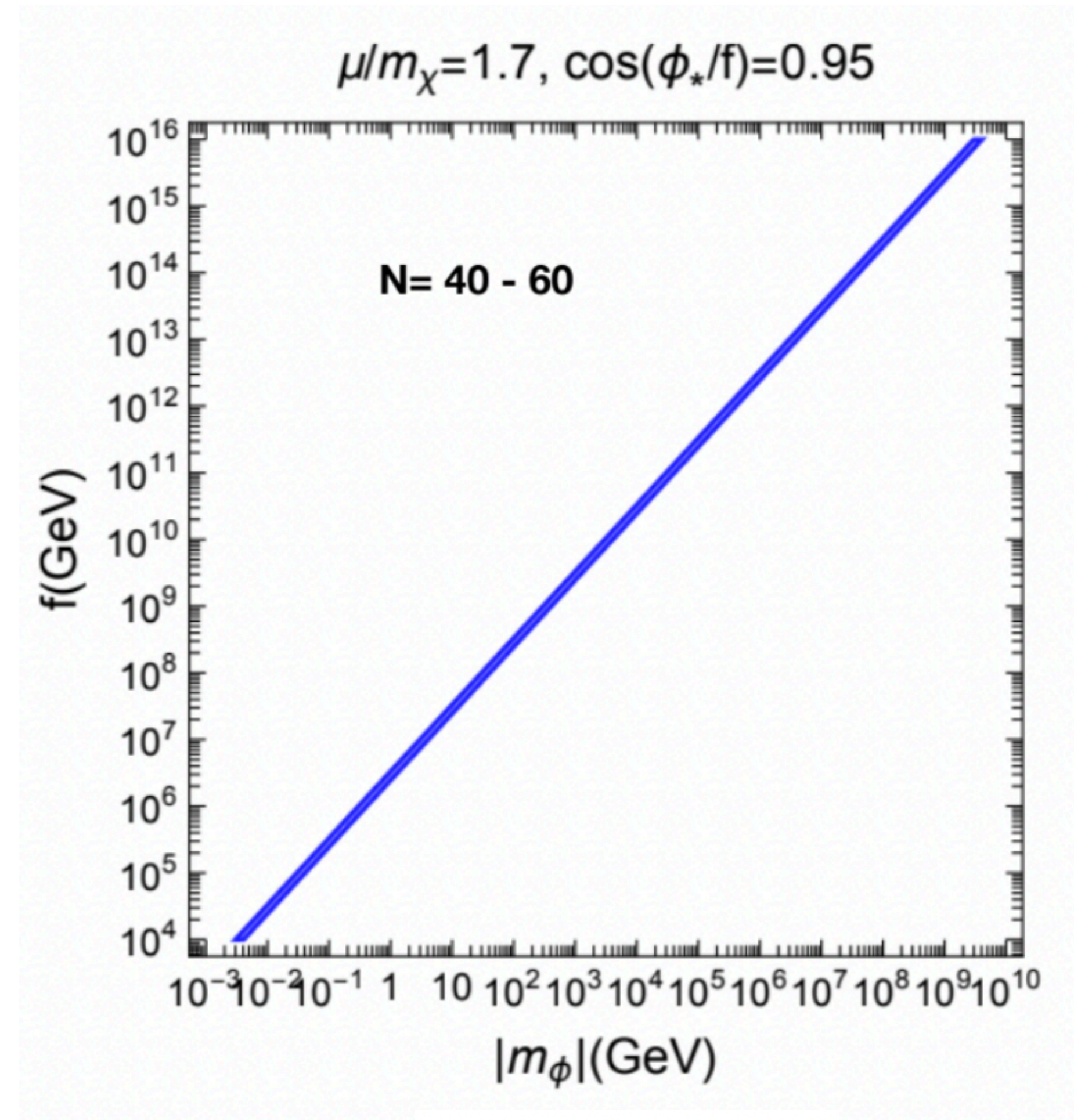
$$B(\phi) = -\frac{1}{2}\mu^2 \sin\left(\frac{\phi}{2f}\right)$$

$$\mu^2 \lesssim 8\pi\Lambda^2 \quad \text{Ensures control of the log divergences during inflation}$$

Inflationary predictions



$$r < 0.036 \Rightarrow H_I < 4.6 \times 10^{13} \text{ GeV}$$



$$f \simeq 6.4 \times 10^5 H_I$$

The symmetry responsible for the PNGB is broken during inflation

Vacuum structure

Z'_2 symmetry for χ_2

$$v_\phi = \pi f$$

$$v_\chi = \sqrt{\frac{\mu^2 - m_\chi^2}{\lambda_\chi}}$$

χ_2 is a DM candidate

$$\lambda_\chi = \frac{m_1^4}{4V_0} = 1.4 \times 10^{-20} \left(\frac{m_1}{100H_I} \right)^4 \left(\frac{H_I}{10^5 \text{ GeV}} \right)^2$$

$$v_\chi = \sqrt{\frac{4V_0}{m_1^2}} = 0.035 M_P \left(\frac{100H_I}{m_1} \right)$$

$$H_I \lesssim 1.6 \times 10^{10} \text{ GeV for } f \lesssim 10^{16} \text{ GeV}$$

$$\lambda_\chi < 10^{-10} \text{ for } m_1 \gtrsim 100H_I$$

$$v_\chi \sim 0.035 M_P$$

The inflation mass receives corrections from the waterfall couplings

$$m_a^2 = 0.0495 H_I^2 + (3 \times 10^{-9}) \left(\frac{r^2}{r^2 - 1} \right) \left(\frac{m_1}{100H_I} \right)^2 v_\chi^2$$

$m_1^2 = \lambda_\chi v_\chi^2$
 Inflation mass **much heavier** → settles fast in the minimum

Preheating

$$m_1 \gg H_I$$

Preheating takes less
than a Hubble time

Right after the end of inflation $\varphi = a \psi$ $\psi = H, \chi_2$

$$\varphi'' - \partial_i^2 \varphi + \left(m_{\psi, \text{eff}}^2 a^2 + (6\zeta_\psi - 1) \frac{a''}{a} \right) \varphi = 0$$

Non minimal coupling

Particles produced during the **waterfall transition** $n_\psi = \frac{1}{2\pi^2} \int_0^\infty dk k^2 n_k^\psi$

$$n_k^\psi = -\frac{g_\psi}{2} + \frac{1}{2\omega_k} \left(|v_k'|^2 + \omega_k^2 |v_k|^2 \right), \quad \omega_k^2 = k^2 + m_{\psi, \text{eff}}^2$$

Ansatz
$$m_{\psi, \text{eff}}^2(t) = m_{\psi, 0}^2 + \frac{1}{4} c_\psi v_\chi^2 (1 + \tanh \lambda(t - t_*))^2$$

[Garcia-Bellido, Ruiz Morales, Phys. Lett. B 536 (2002)]

Strength of the waterfall transition

$$\lambda = m_1/2 = \sqrt{\lambda_\chi} v_\chi/2$$

Preheating

$$(m_{\psi,\text{eff}}^2 a^2)(\eta) = m_{\psi,0}^2 + \frac{1}{4} c_\psi v_\chi^2 (1 + \tanh \lambda(\eta - \eta_*))^2$$

$\psi = H, \chi_2$

Analytic

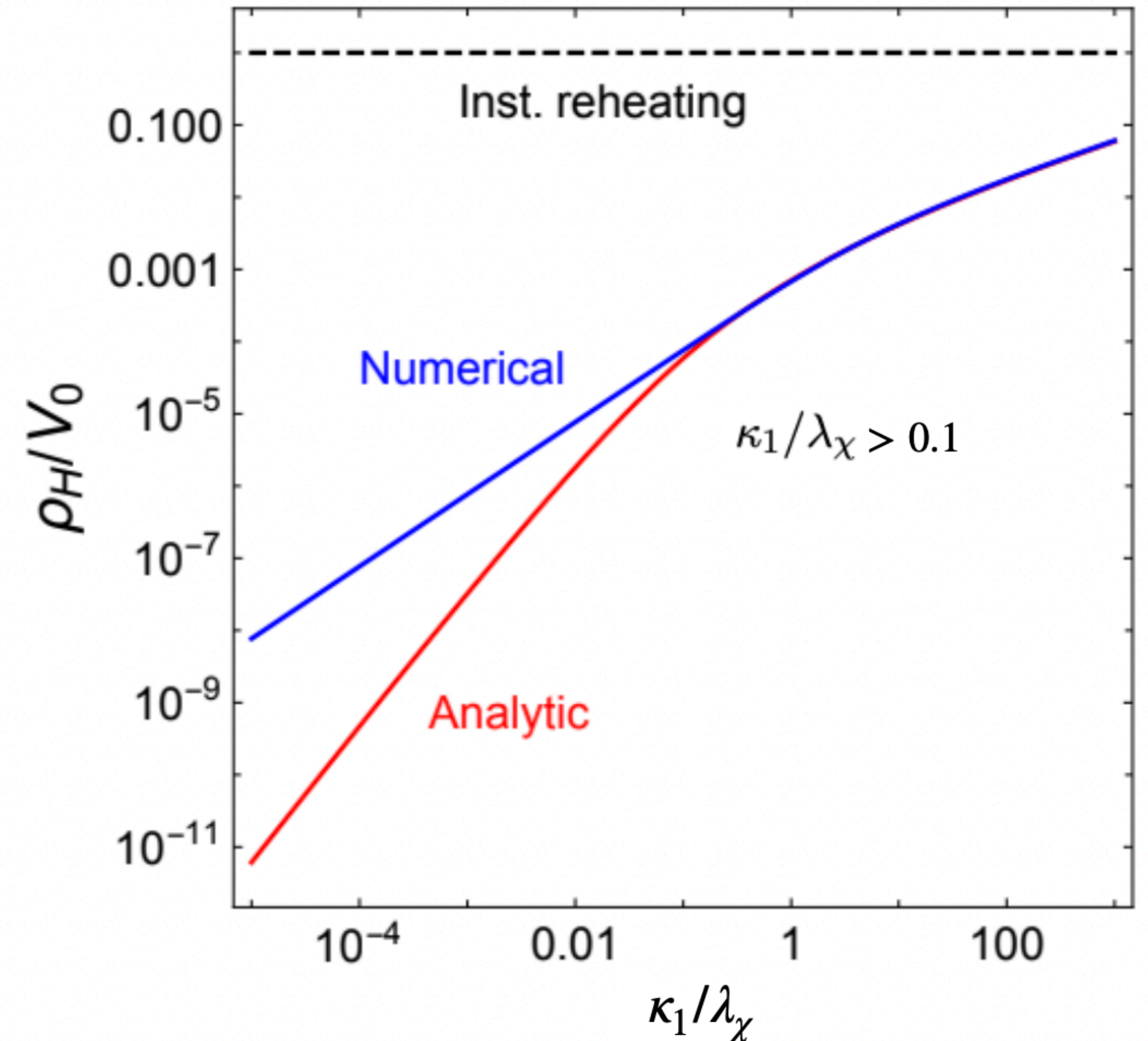
$$\frac{\rho_\psi}{V_0} = \frac{2g_\psi \lambda_\chi}{\pi^2} \int_0^\infty dk \kappa^2 n_k^\psi \sqrt{\kappa^2 + \frac{m_{\psi,0}^2}{m_1^2} + \alpha_\psi}$$

Numerical

$$\frac{\rho_\psi}{V_0} = 2 \times 10^{-3} g_\psi \lambda_\chi \underline{f(\alpha_\psi, 1.3)}$$

Fitting function

$$f(\alpha, \gamma) = \sqrt{\alpha + \gamma^2} - \gamma$$



Perturbative reheating

$$\mathcal{L}_{\text{int}} = \frac{\mu^2}{8f^2} v_\chi a^2 \tilde{\chi}_1 + \frac{\mu^2}{16f^2} a^2 (\tilde{\chi}_1^2 - \chi_2^2) - \bar{\lambda}_\chi v_\chi \tilde{\chi}_1 \chi_2^2 - \kappa_1 v_\chi \tilde{\chi}_1 h^2 + \dots$$

$$\tilde{\chi}_1 \nrightarrow aa$$

Closed channels

$$\tilde{\chi}_1 \nrightarrow \chi_2 \chi_2$$

$$\chi_1 \rightarrow hh$$

Reheating

For non instantaneous reheating

$$N = 51.3 - \frac{1}{3} \ln \left(\frac{H_I}{1.6 \times 10^{10} \text{ GeV}} \right) - \boxed{\frac{1}{12} \ln \left(1 - \frac{\rho_R(t_*)}{V_0} \right)} + \frac{1}{3} \ln \left(\frac{\boxed{T_{\text{RH}}}}{10^{14} \text{ GeV}} \right)$$

Radiation produced
during **preheating**

$$T_{\text{RH}} \ll 10^{14} \text{ GeV} \quad \text{for } f \lesssim 10^{16} \text{ GeV,}$$

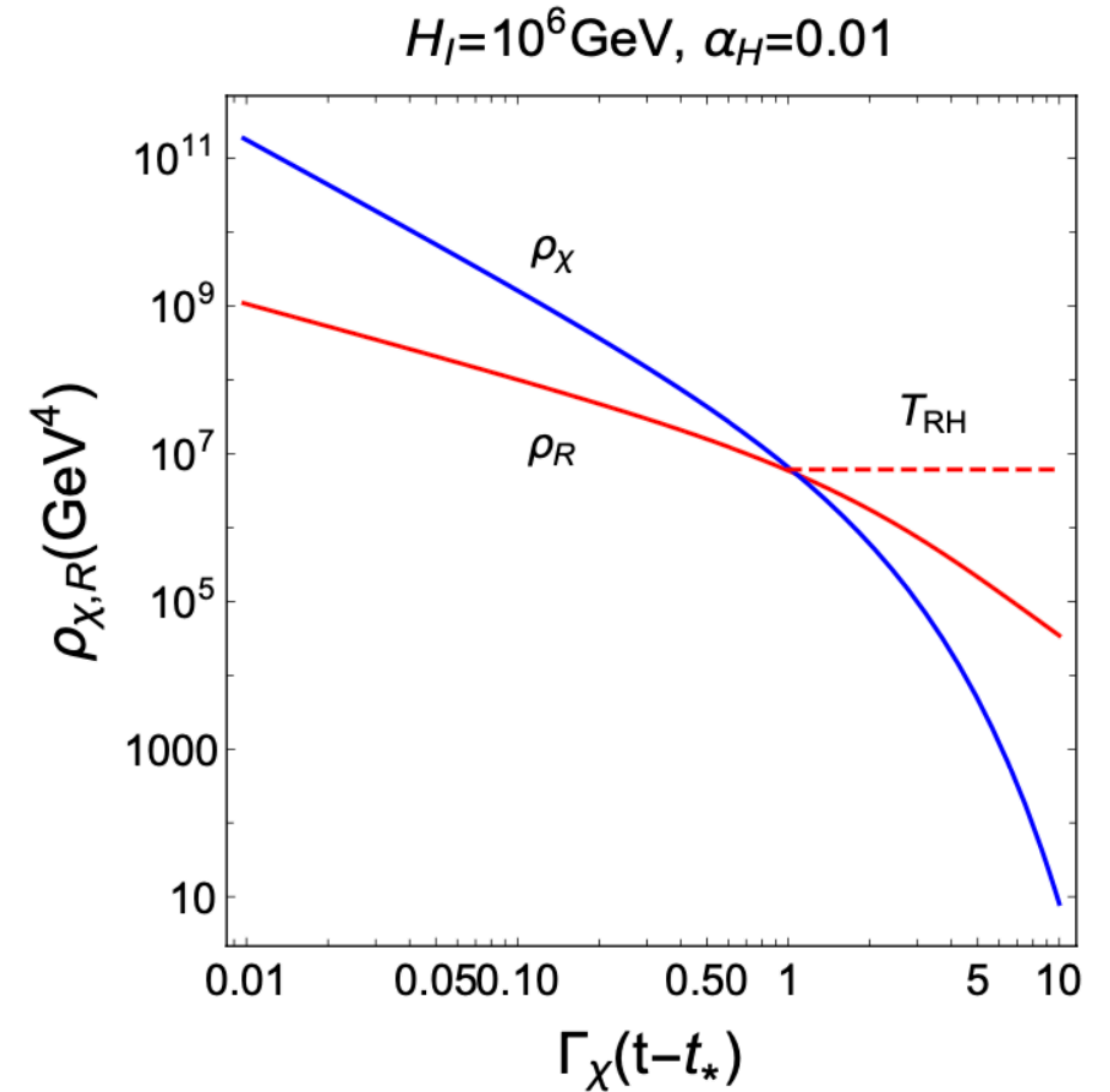
Radiation energy density

After preheating

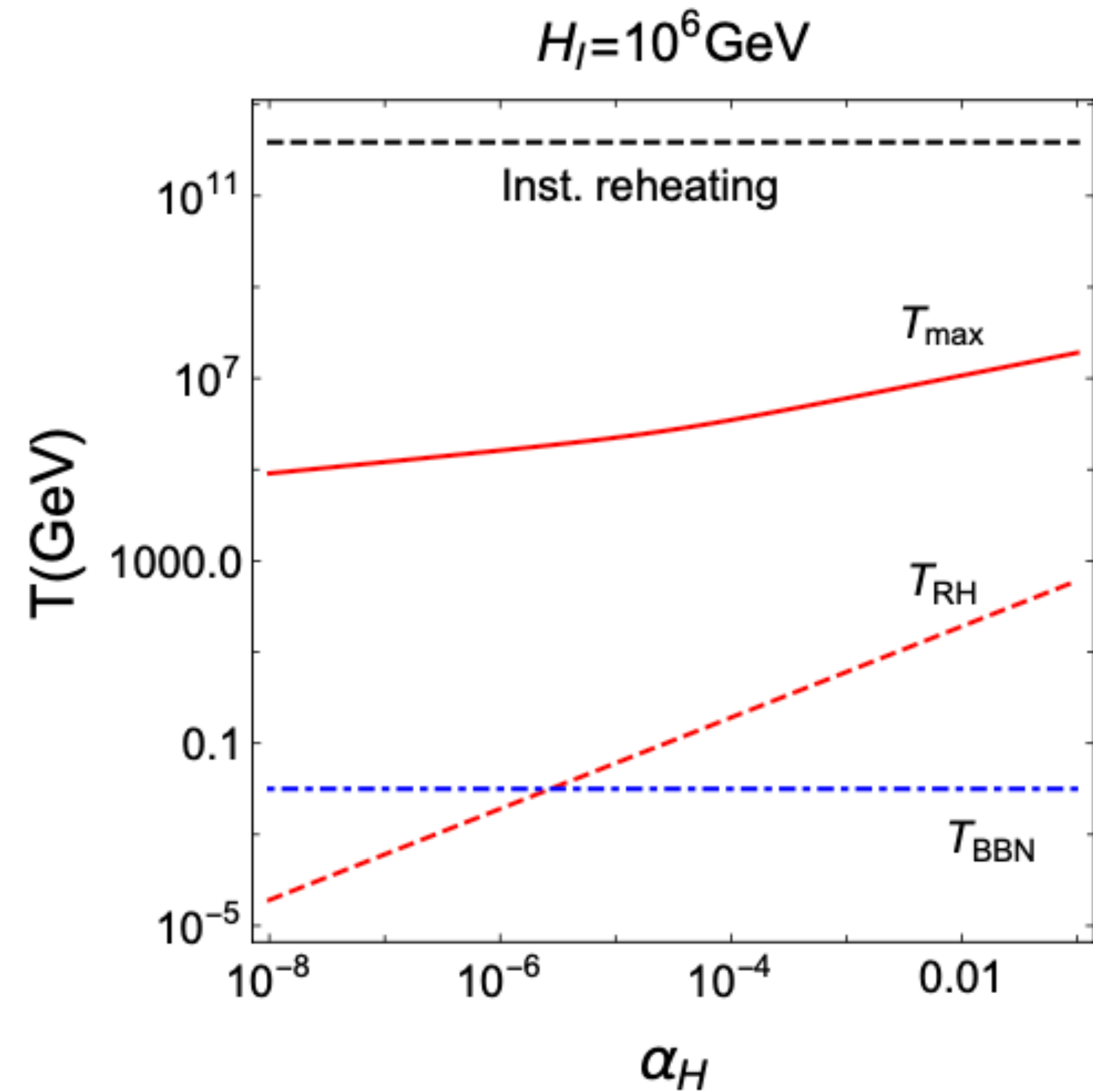
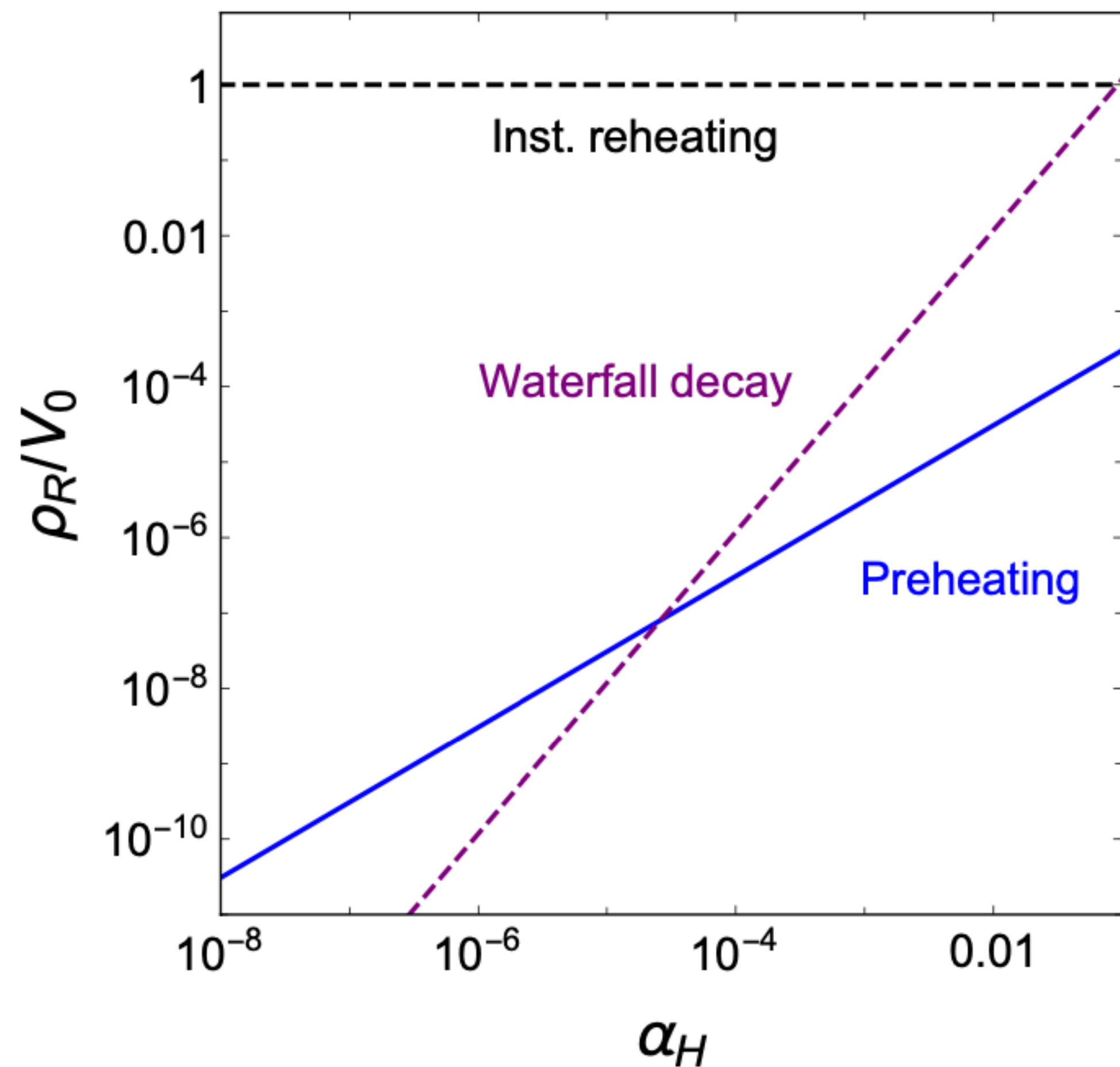
$$\rho_R(t) = \rho_{\chi_1}(t_*) \left(1 + \frac{v}{A}\right)^{-8/3} \left[\frac{\rho_R(t_*)}{\rho_{\chi_1}(t_*)} + \int_0^v \left(1 + \frac{v'}{A}\right)^{2/3} e^{-v'} dv' \right]$$

$$\rho_R(t_{\max}) = 1.8^{-8/3} \left(\rho_R(t_*) + 1.0A\rho_{\chi_1}(t_*) \right)$$

$$A = 4\sqrt{\frac{2}{3}} \frac{\Gamma_{\chi_1}}{m_{\chi_1}} \left(\frac{v_\chi}{M_P} \right)^{-1}$$

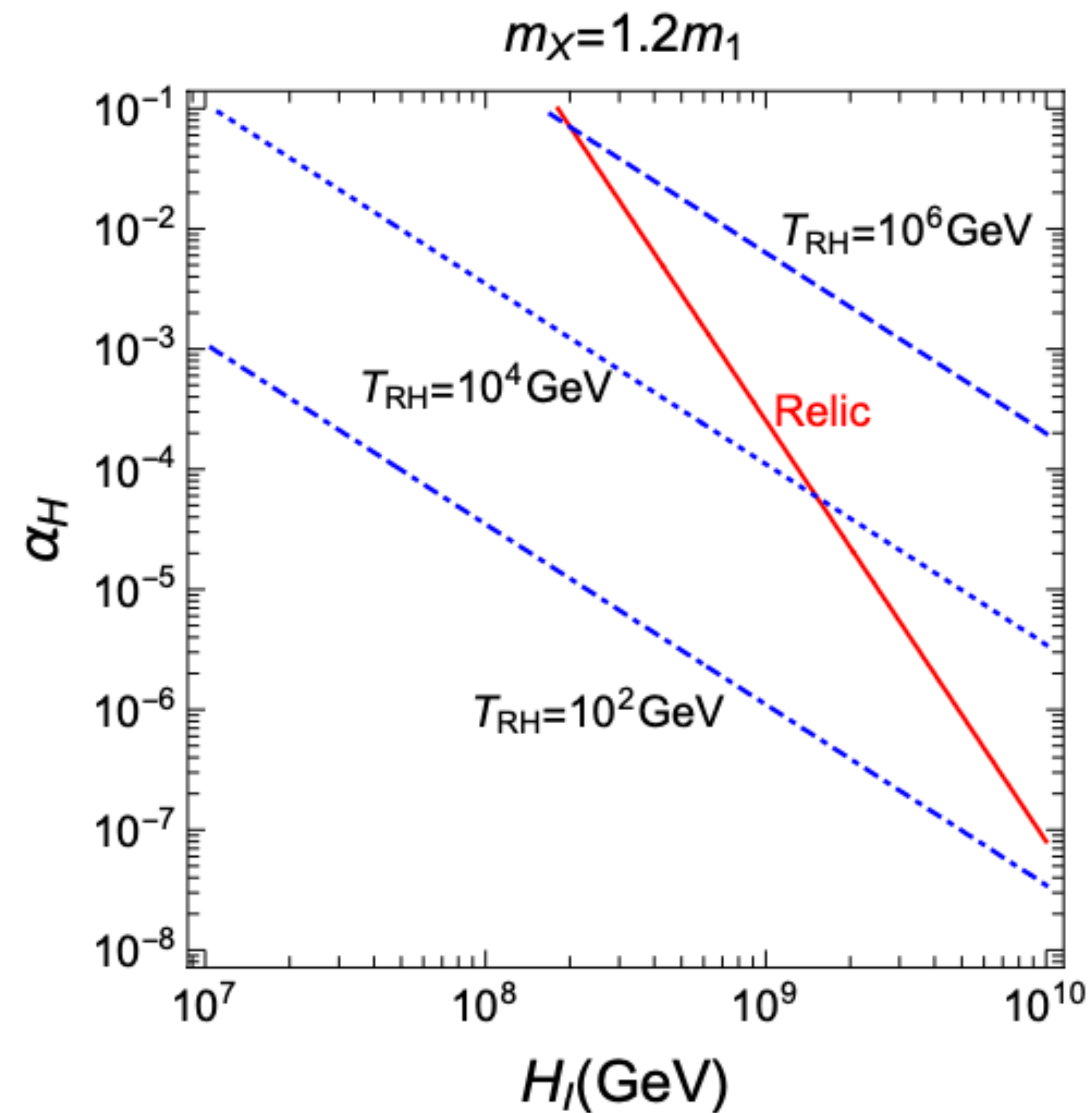


Preheating vs reheating

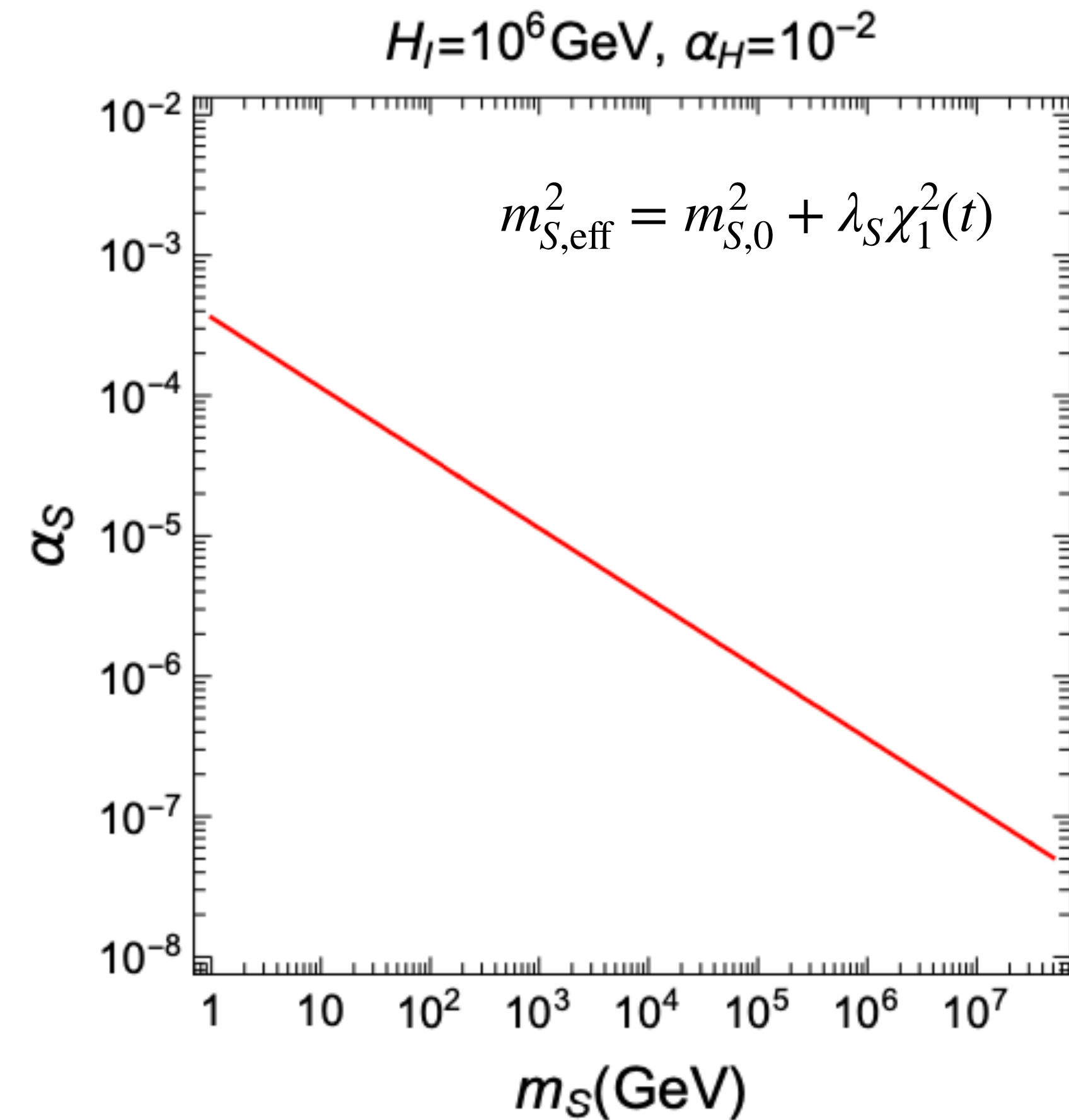


Dark Matter Production

χ_2 Dark Matter



Another scalar candidate



Summary

- We presented a natural inflation model, in which the inflation is a PNGB. **Phys.Lett.B 834 (2022)**
- We showed how the Z_2 symmetry arises from a larger Z_4 symmetry
- Large parameter space for successful inflation
- We studied the possibility of preheating, but reheating is dominant.
- We studied Dark Matter production