Post-inflationary ALPS:

Gravitational waves, targets for haloscopes, and substructure

Ed Hardy

UNIVERSITY OF LIVERPOOL

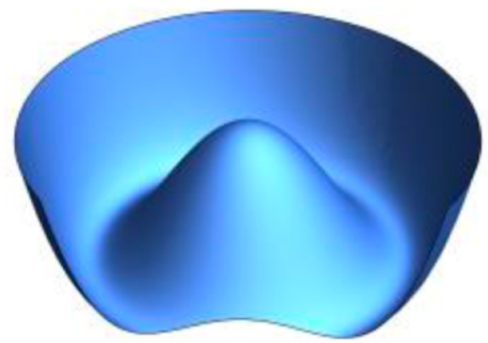
Mostly based on:

2101.11007 Gorghetto, EH, Nicolaescu; JCAP

2007.04990 Gorghetto, EH, Villadoro; SciPost

Axions (=ALP)

- Axion a
- Shift symmetry $a \to a + c$



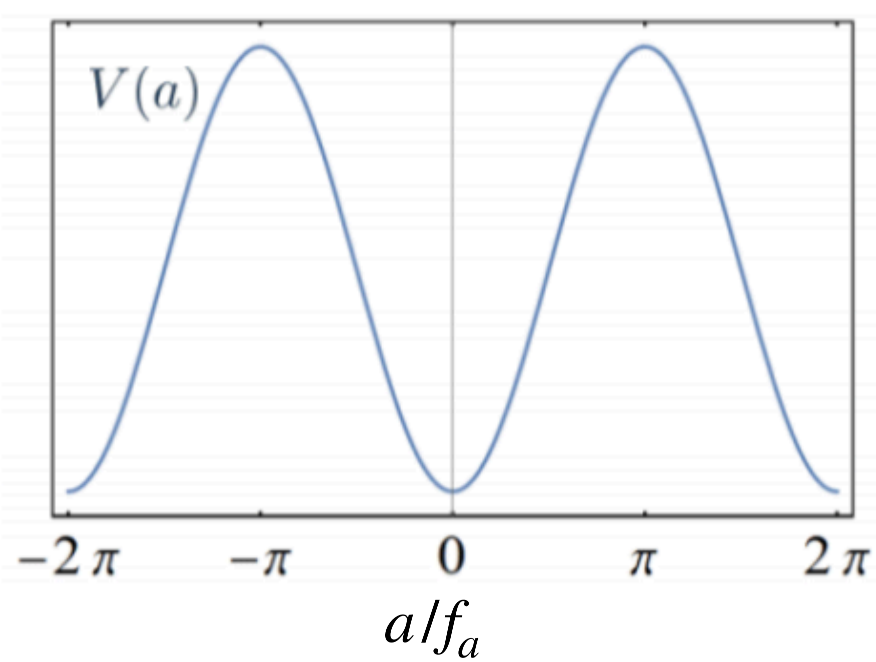
$$\mathcal{L} = |\partial_\mu \phi|^2 - \frac{m_r^2}{2v^2}\left(|\phi|^2 - \frac{v^2}{2}\right)^2$$

$$\phi = \frac{(v+r)}{\sqrt{2}} e^{ia/v}$$

(for now $N = 1$)

Define the axion decay constant f_a such that $a \cong a + 2\pi f_a$

Assume $m_r \sim f_a$

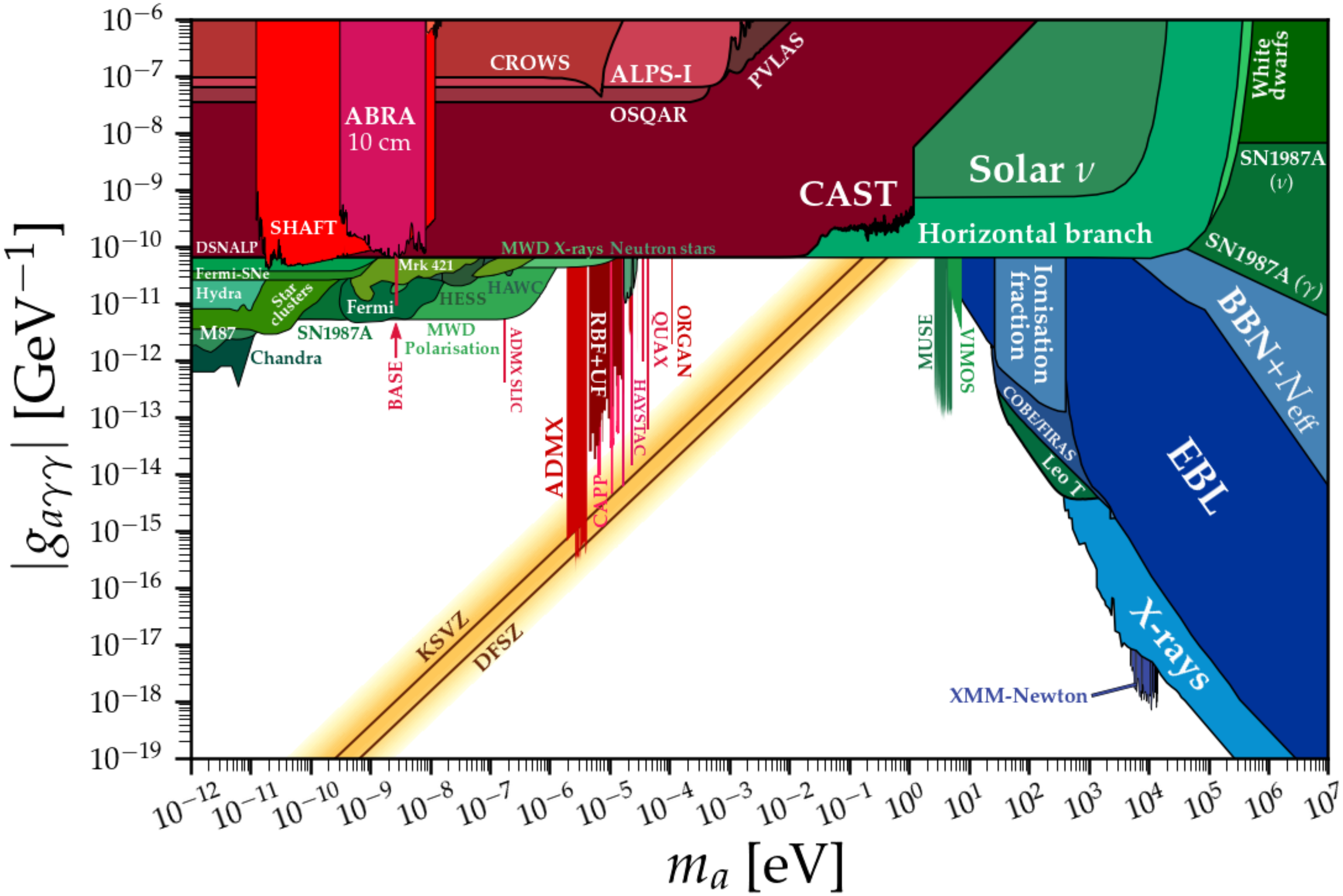


$\theta = a/f_a$

Axion mass m_a

Searches

$$\mathscr{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a\ F_{\mu\nu}\tilde{F}^{\mu\nu}$$



https://cajohare.github.io/AxionLimits

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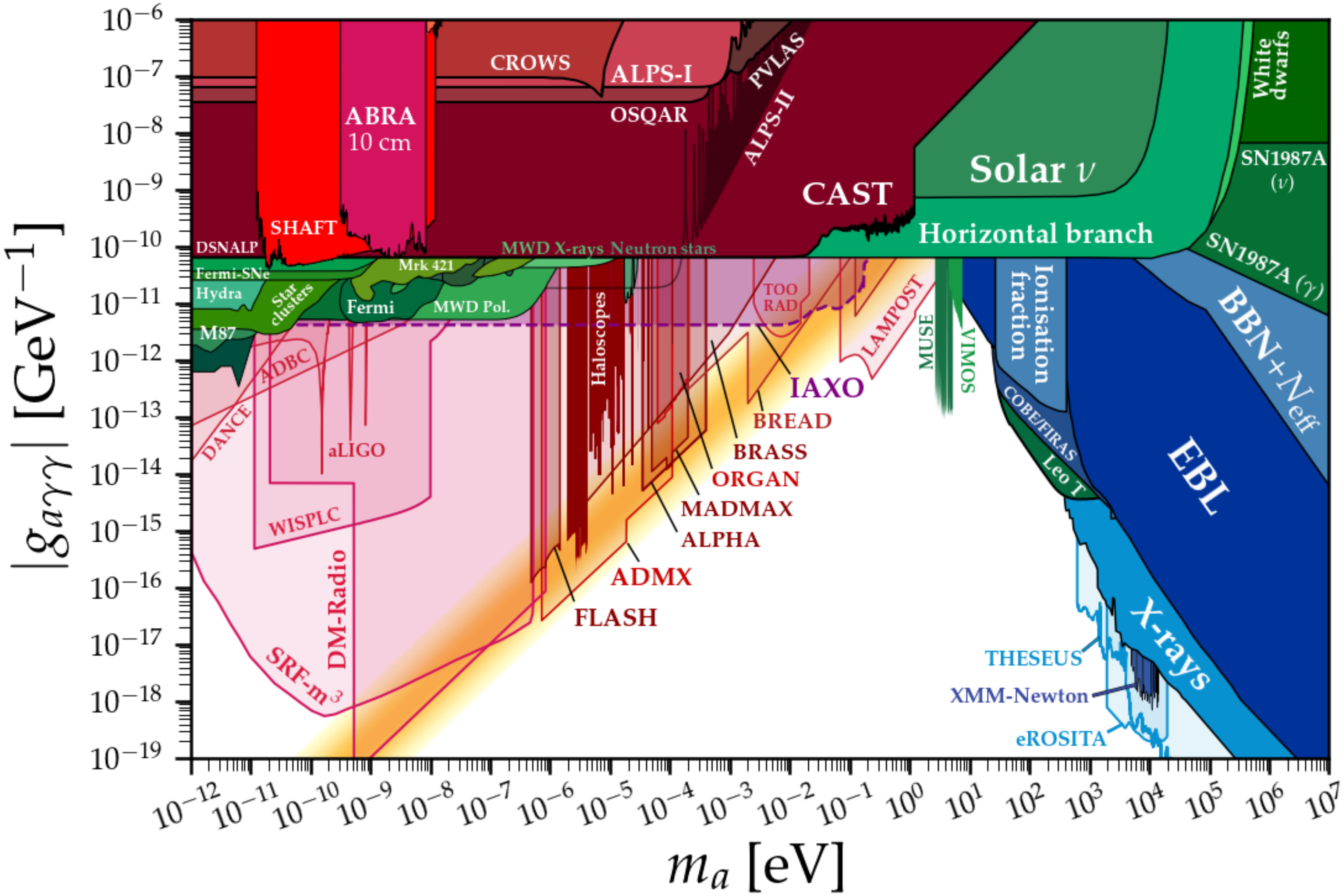

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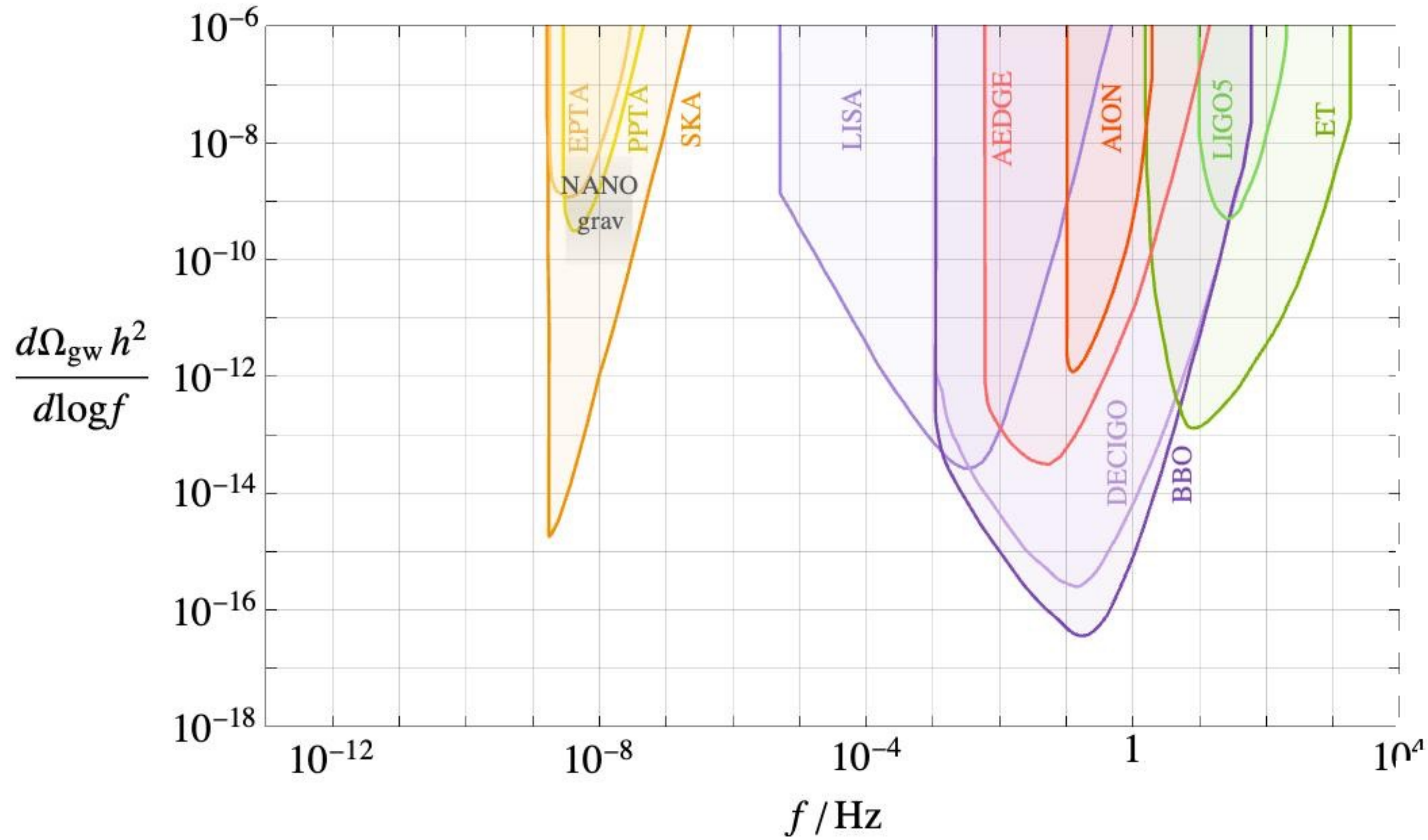
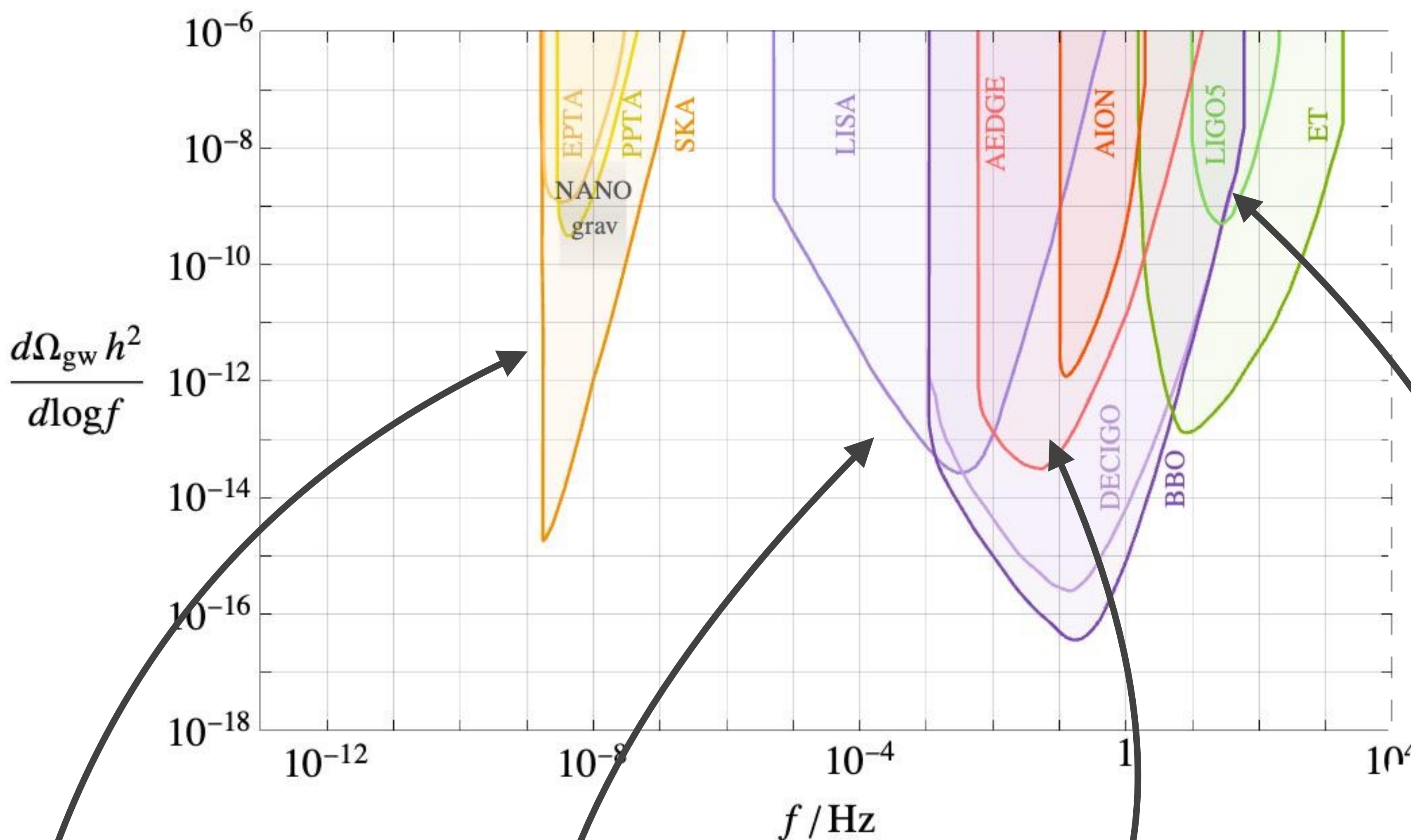
Gravitational wave searches



Caution:

Many experimental challenges to overcome

Gravitational wave searches



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 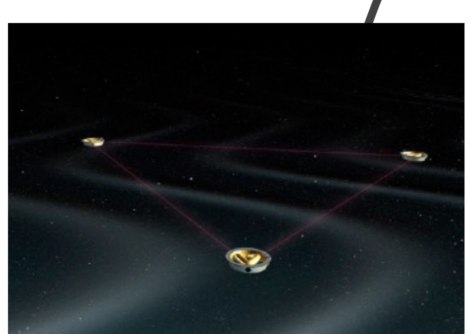 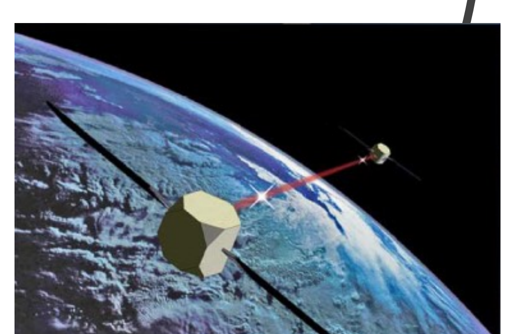 

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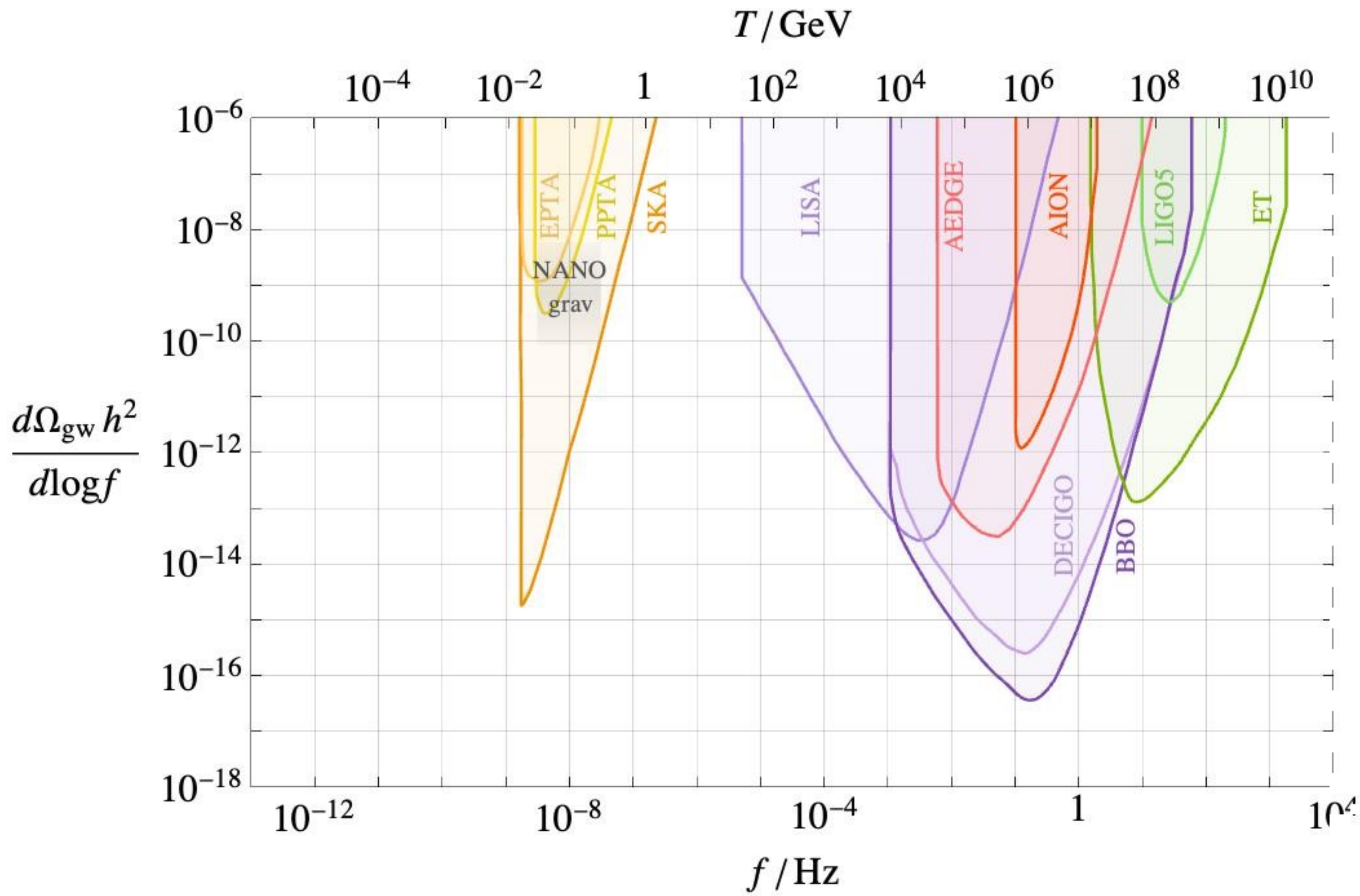


Caution:
Many experimental challenges to overcome

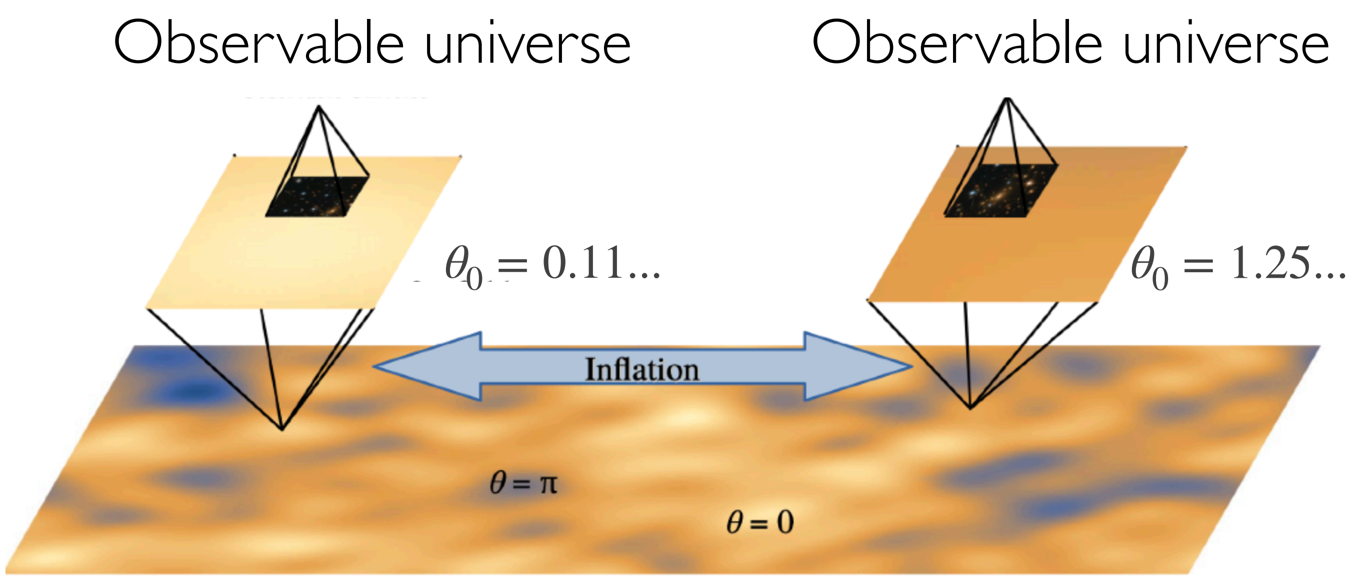
The early universe

This talk

- Post-inflationary axion-like-particles (ALPs)
- Gravitational waves
- Dark matter substructure

Initial conditions

Pre-inflationary

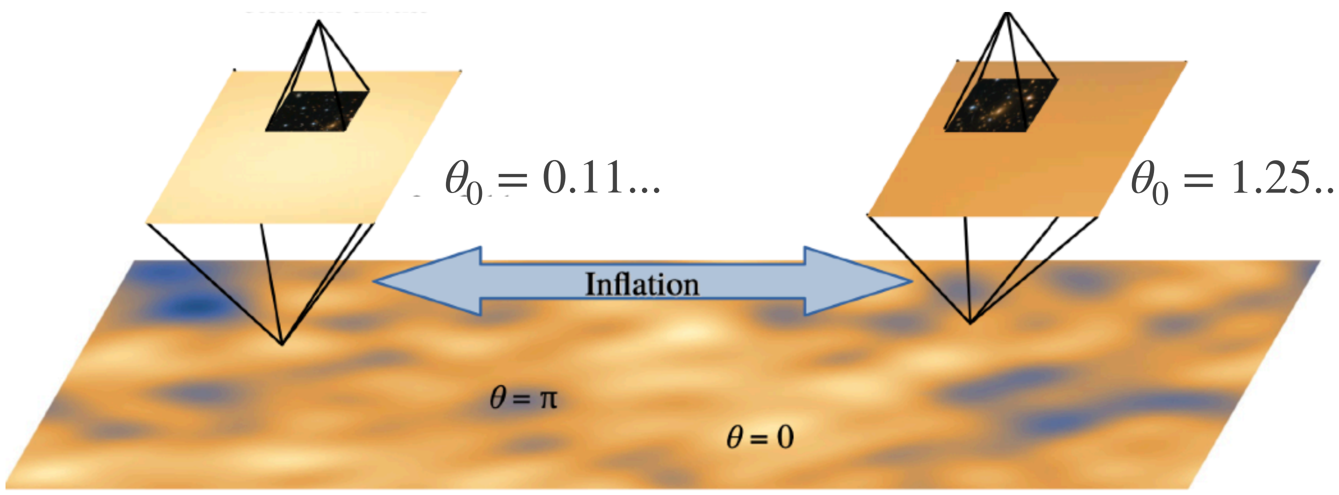


Initial conditions

Pre-inflationary

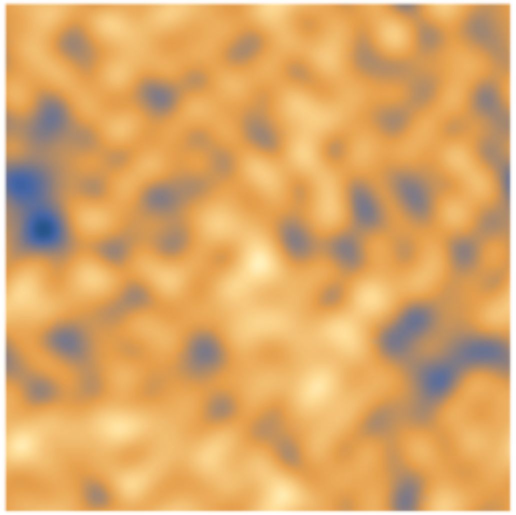
Observable universe

Observable universe

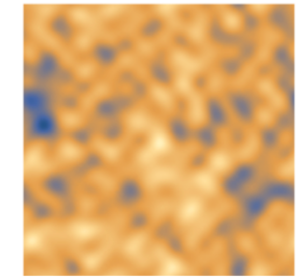


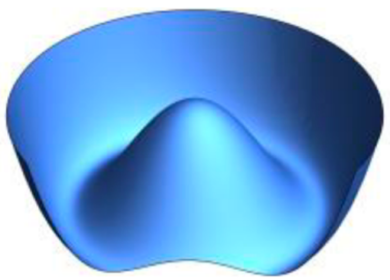
Post-inflationary

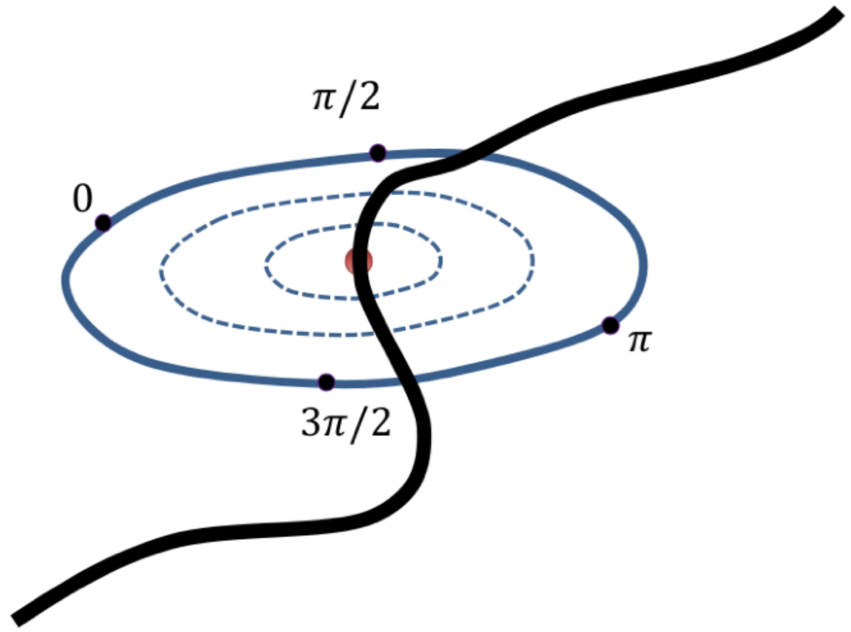
Observable universe



Topological strings







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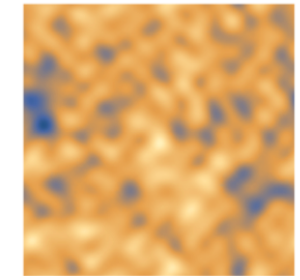
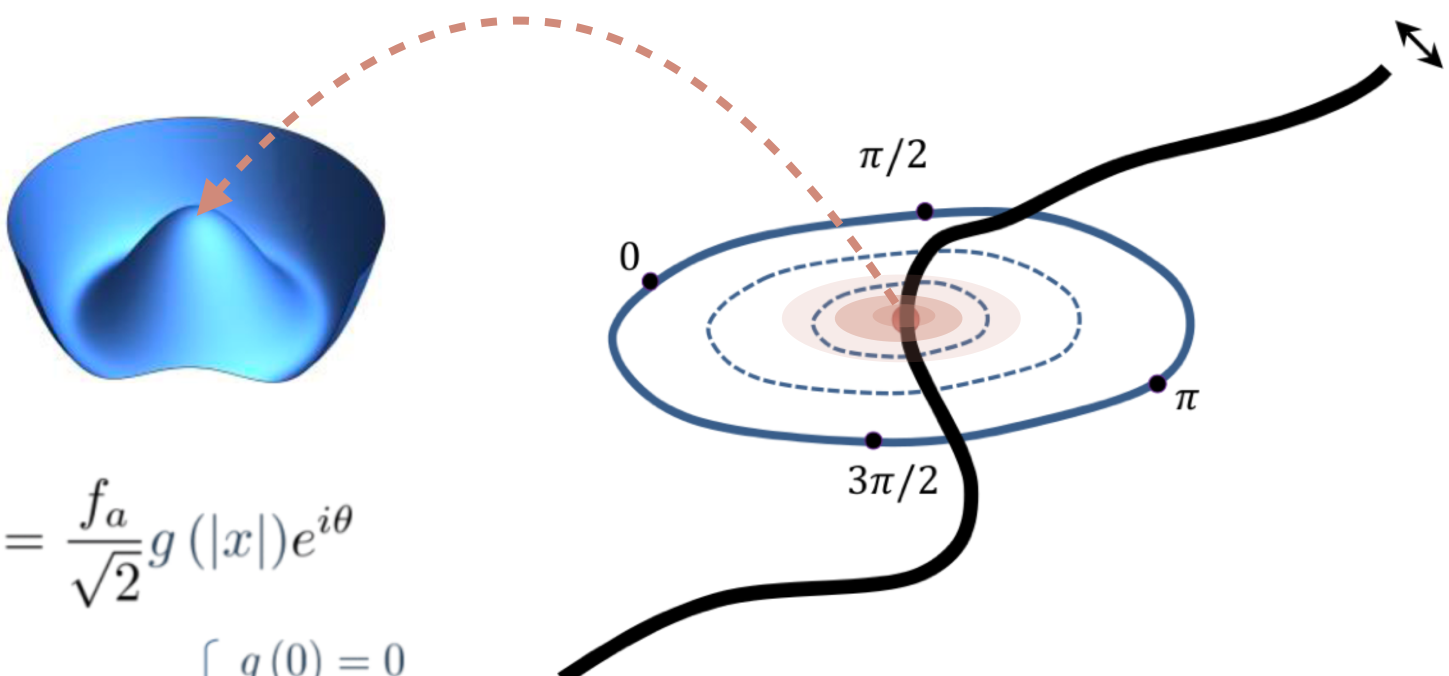


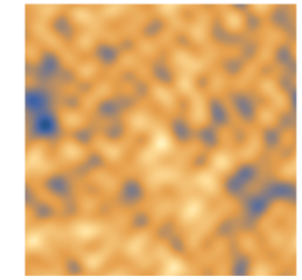


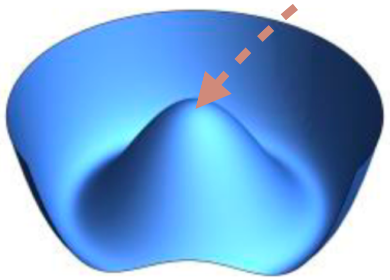
$$\delta_s \simeq \frac{1}{m_r}$$

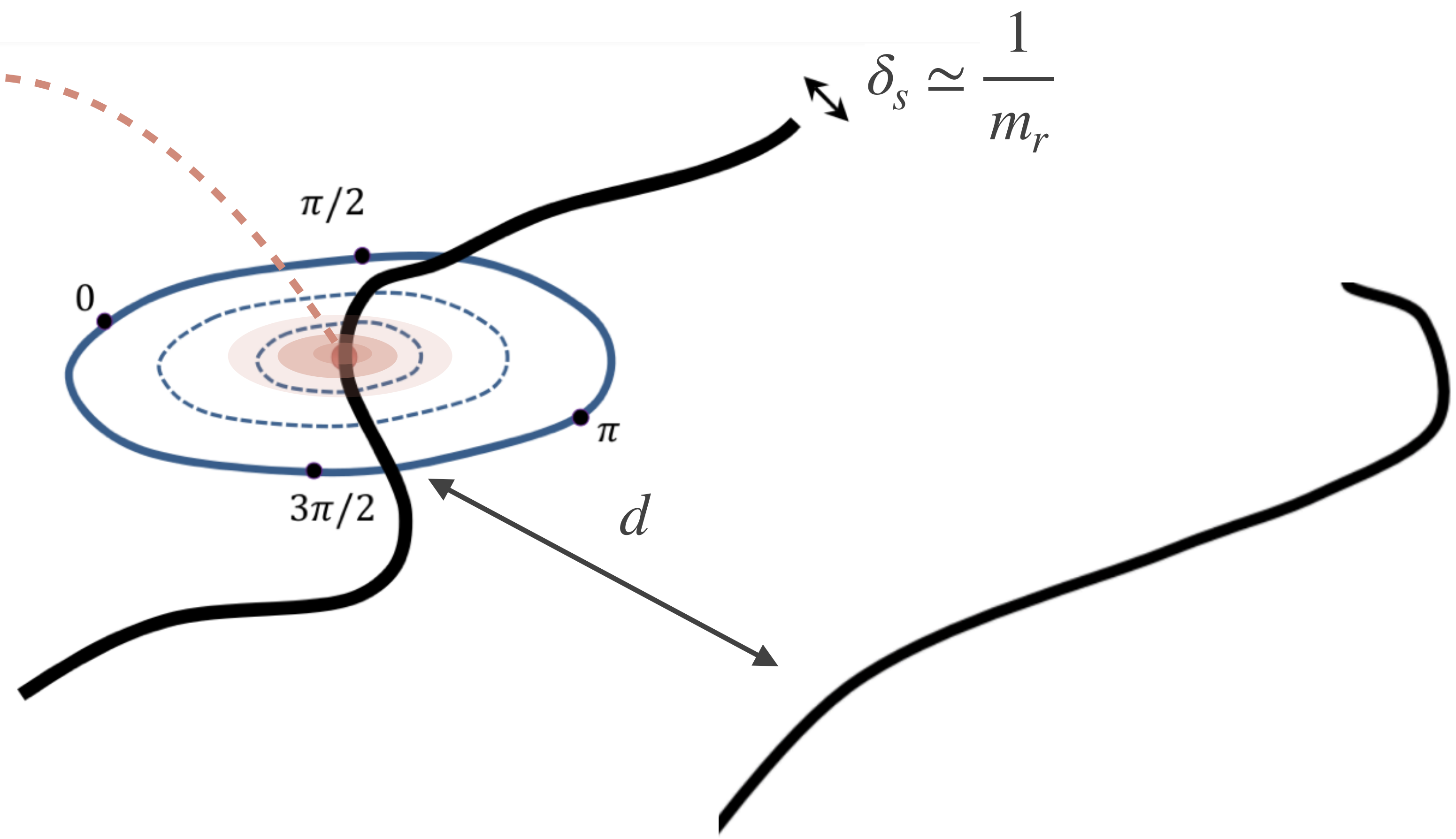
$$\phi = \frac{f_a}{\sqrt{2}} g\left(|x|\right) e^{i\theta}$$

$$\begin{cases} g\left(0\right) = 0 \\ g\left(\infty\right) = 1 \end{cases}$$

Topological strings







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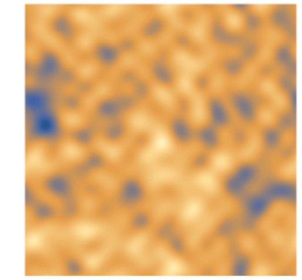
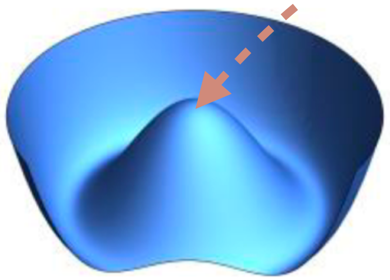
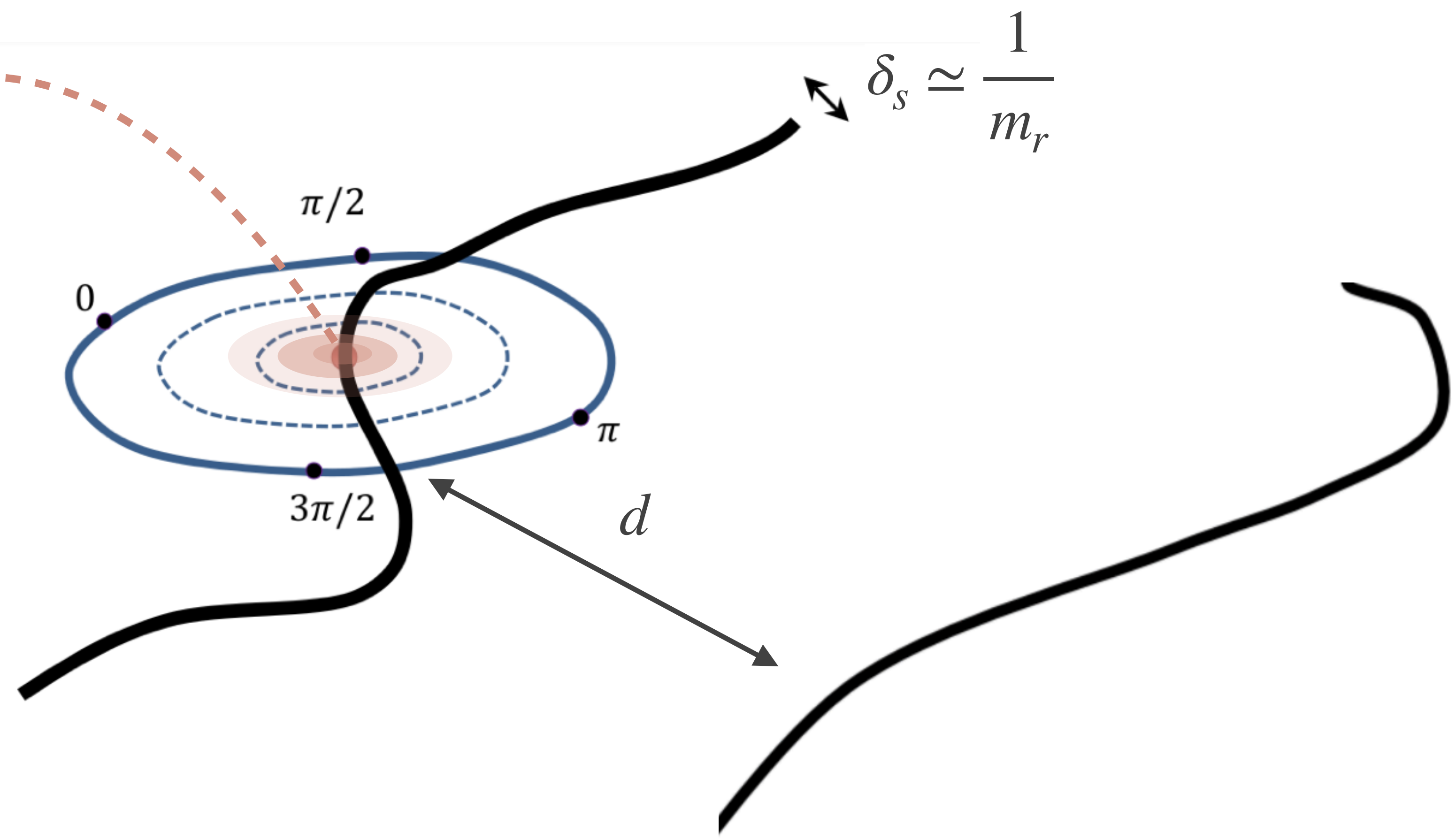
Topological strings







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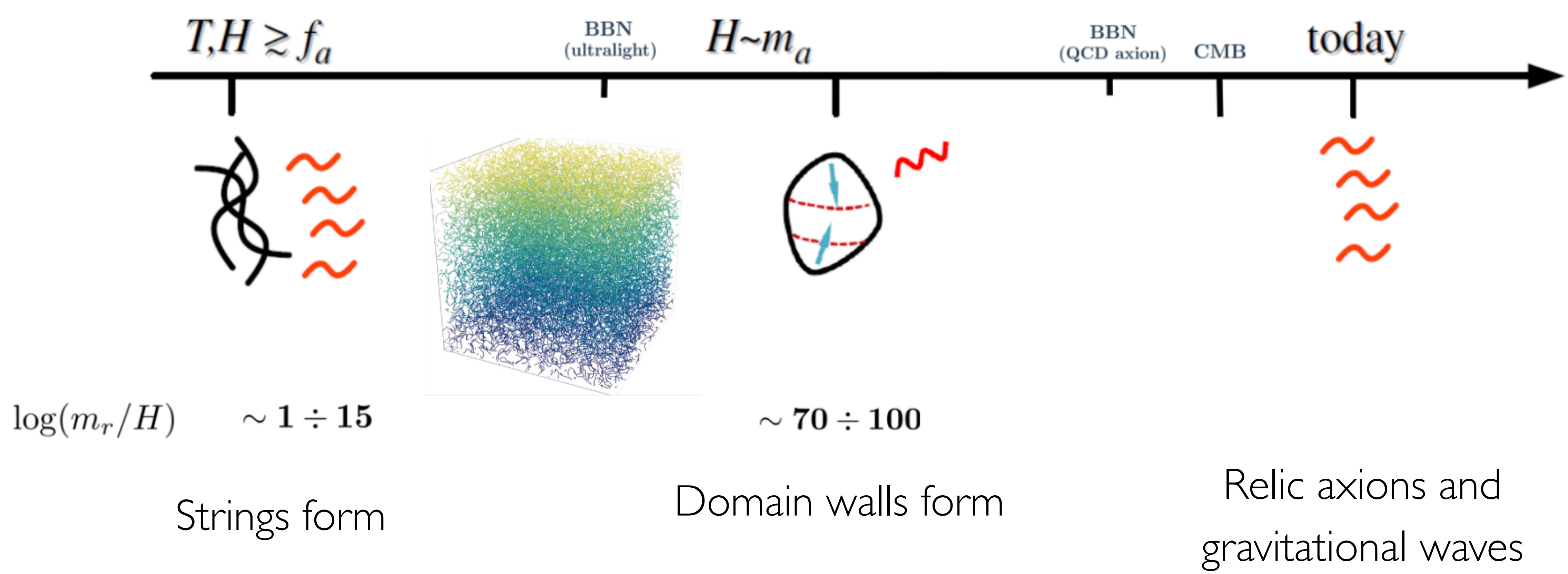
$H \sim T^2/M_{\rm Pl}$

Grows logarithmically with time

Full evolution



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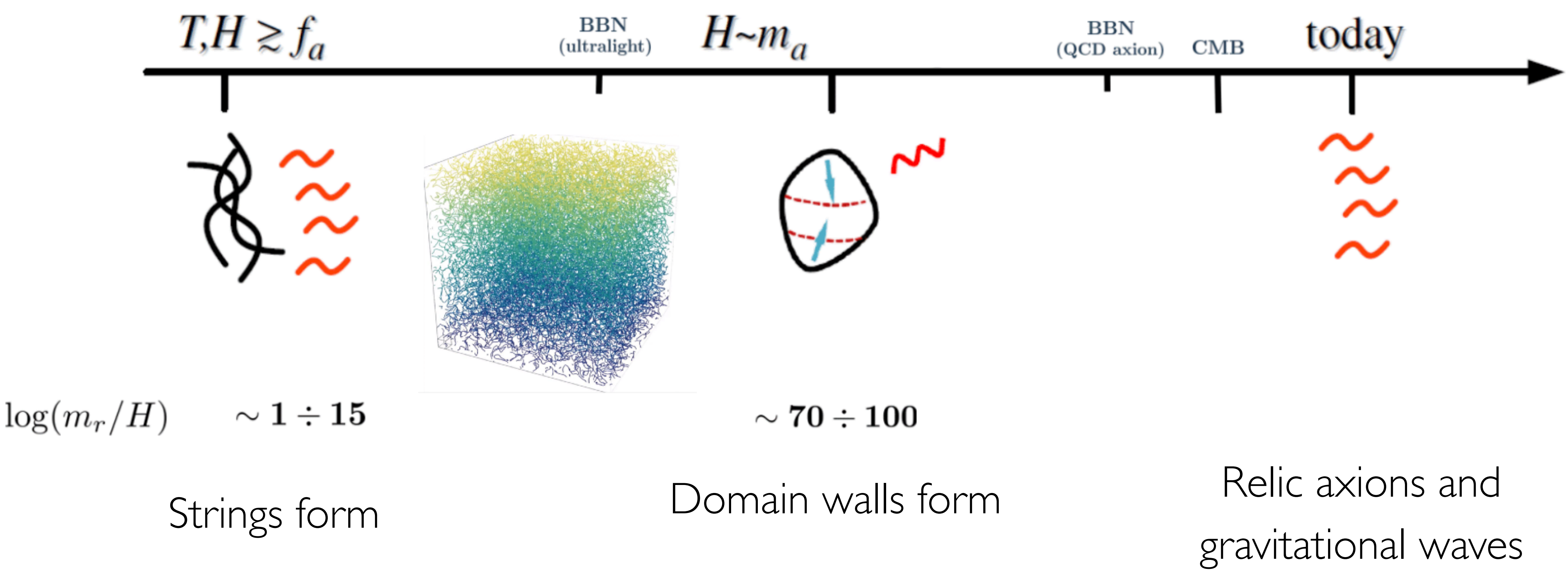
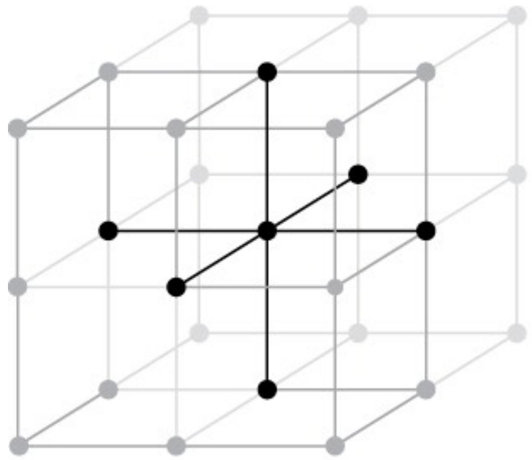


Dynamics:

- nonlinear

- large scale separation

Simulations



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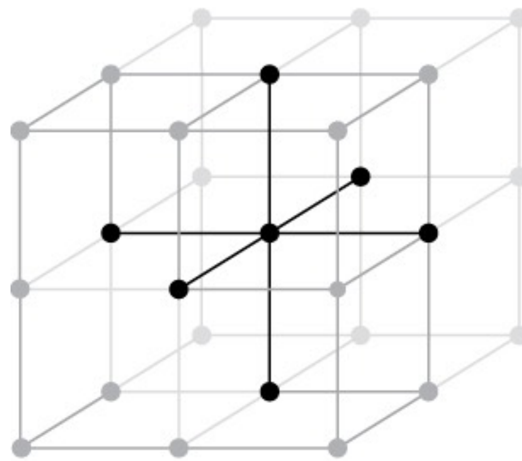
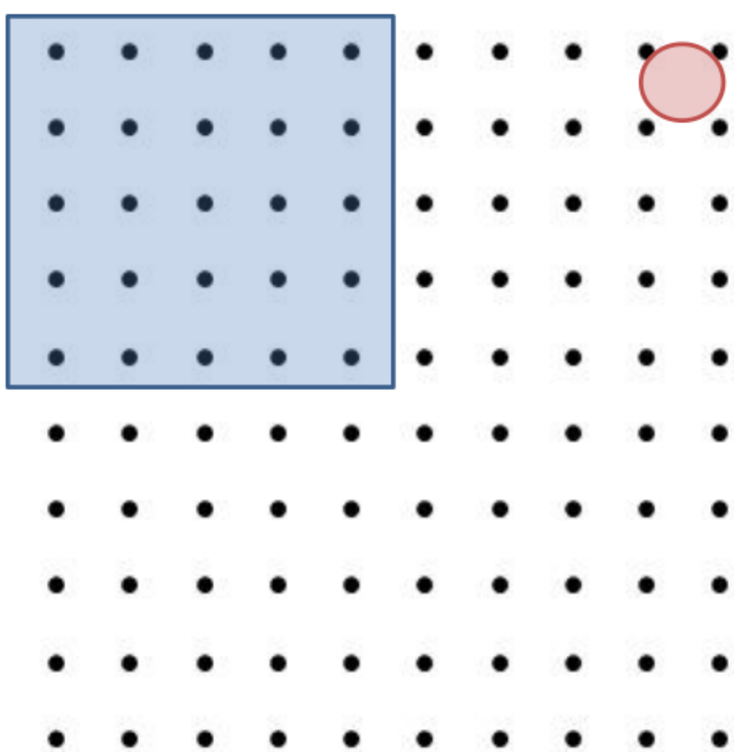
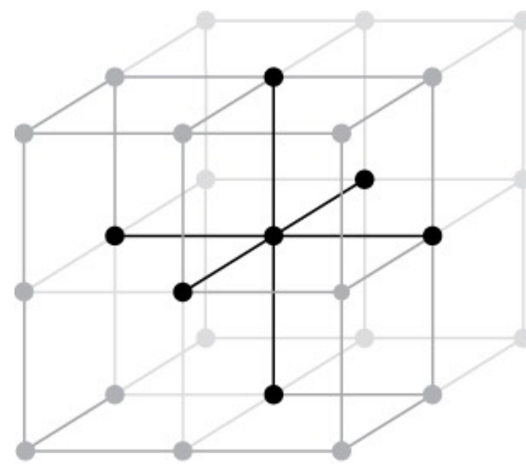


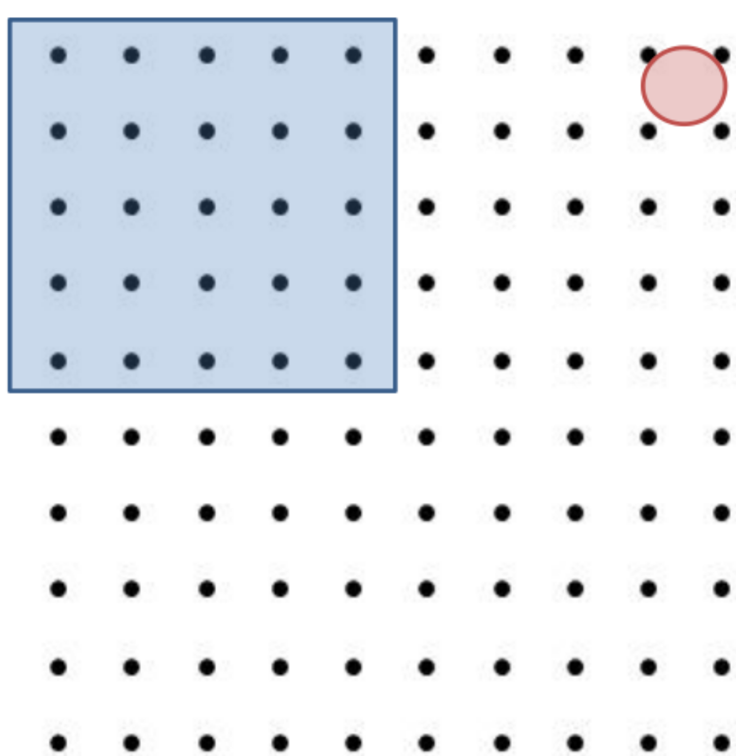


- a few lattice points per string core
- a few Hubble patches

Memory constraints $\rightarrow$ max 5000^3 grid points

Simulations





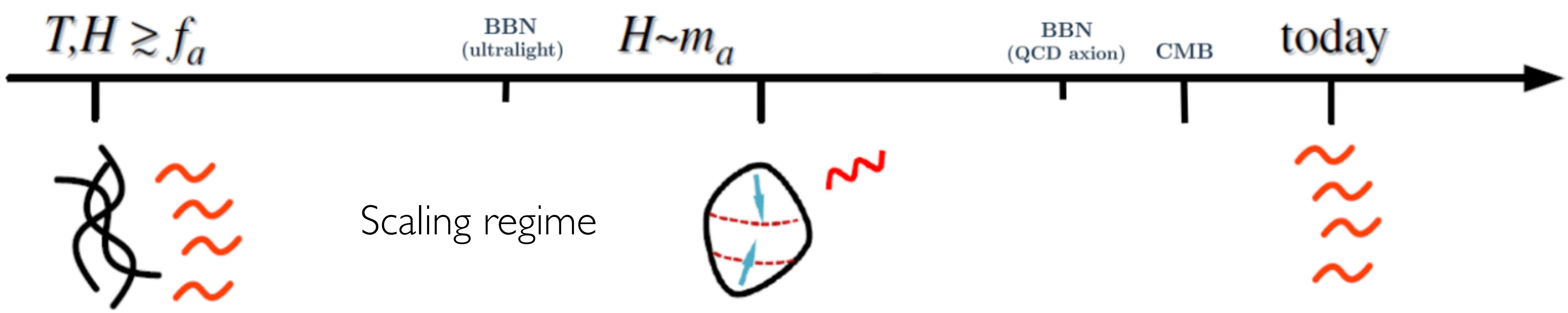
- a few lattice points per string core
- a few Hubble patches

Memory constraints → max 5000^3 grid points

Simulations $\log(m_r/H) \leq \log\left(\frac{\square}{\bigcirc}\right) \lesssim 8$

Physical $\log(m_r/H) \sim 70$

Scaling regime

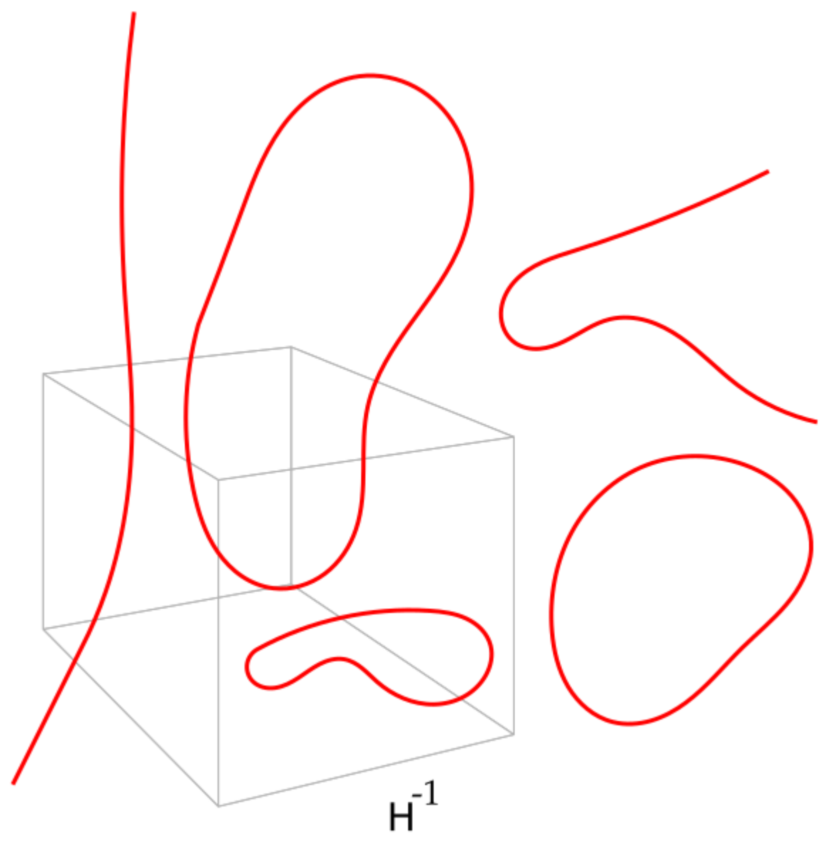


$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70 \div 100}$

Simulate →

Extrapolate

Scaling



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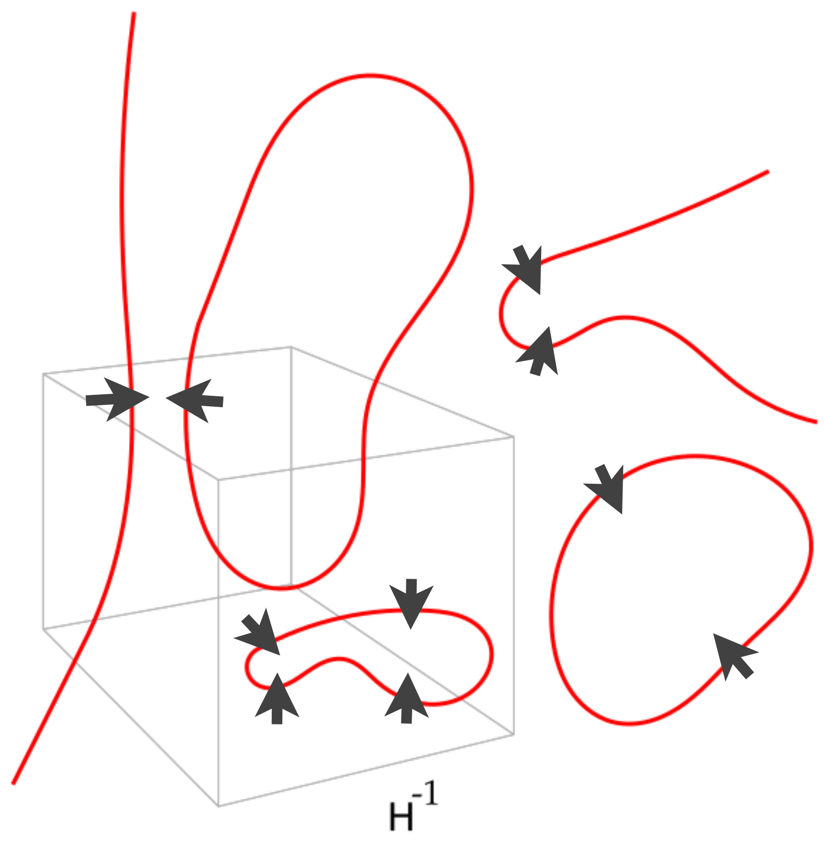

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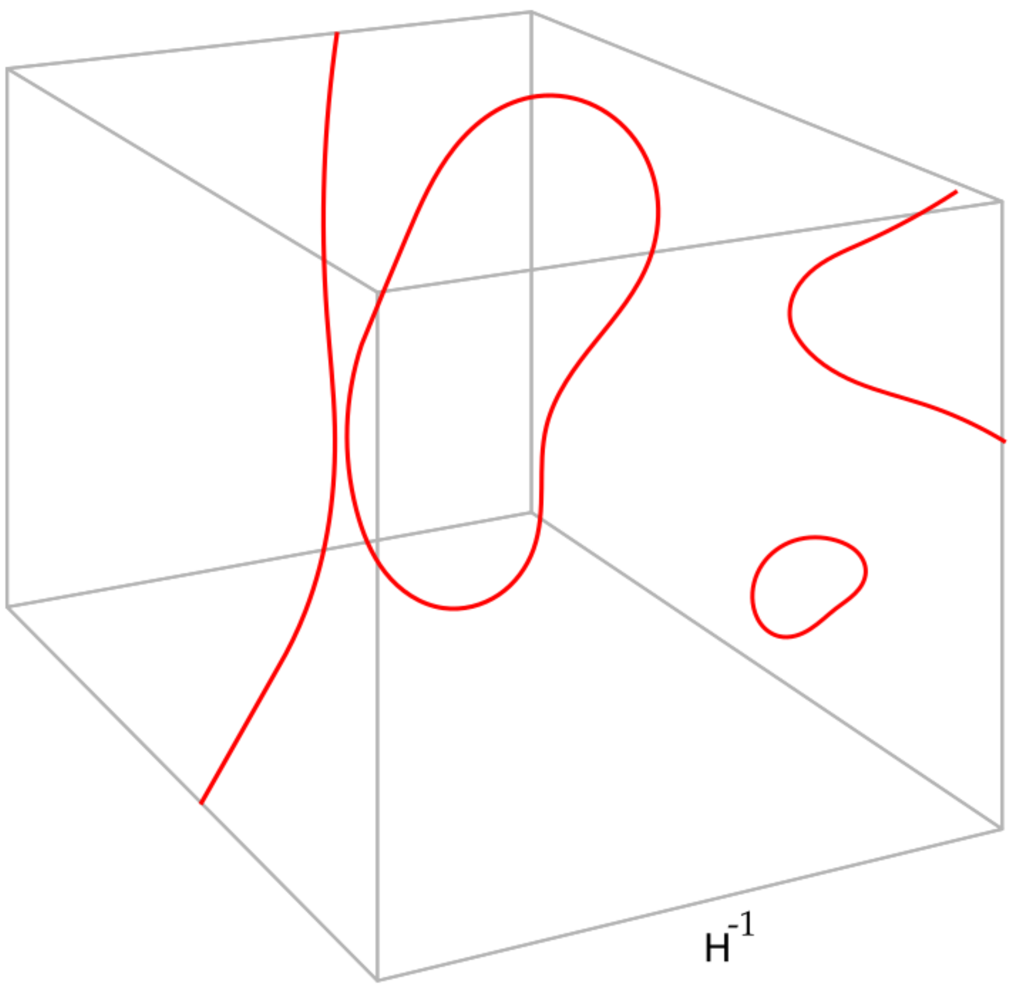
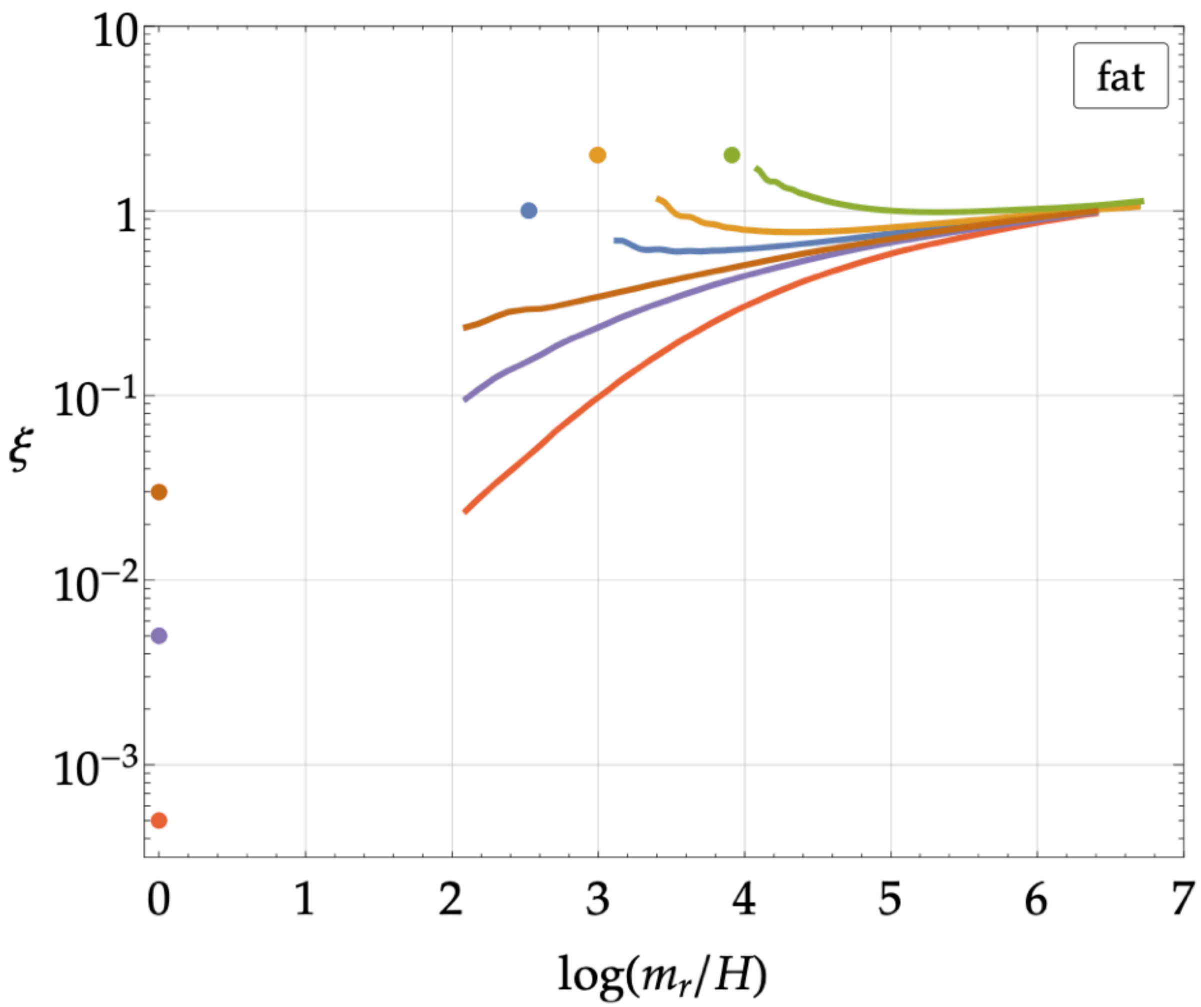


$\xi(t)$ = Length of string in Hubble lengths per Hubble volume

$\mu(t)$ = string tension $\simeq \pi f_a^2 \log(m_r/H)$

Scaling



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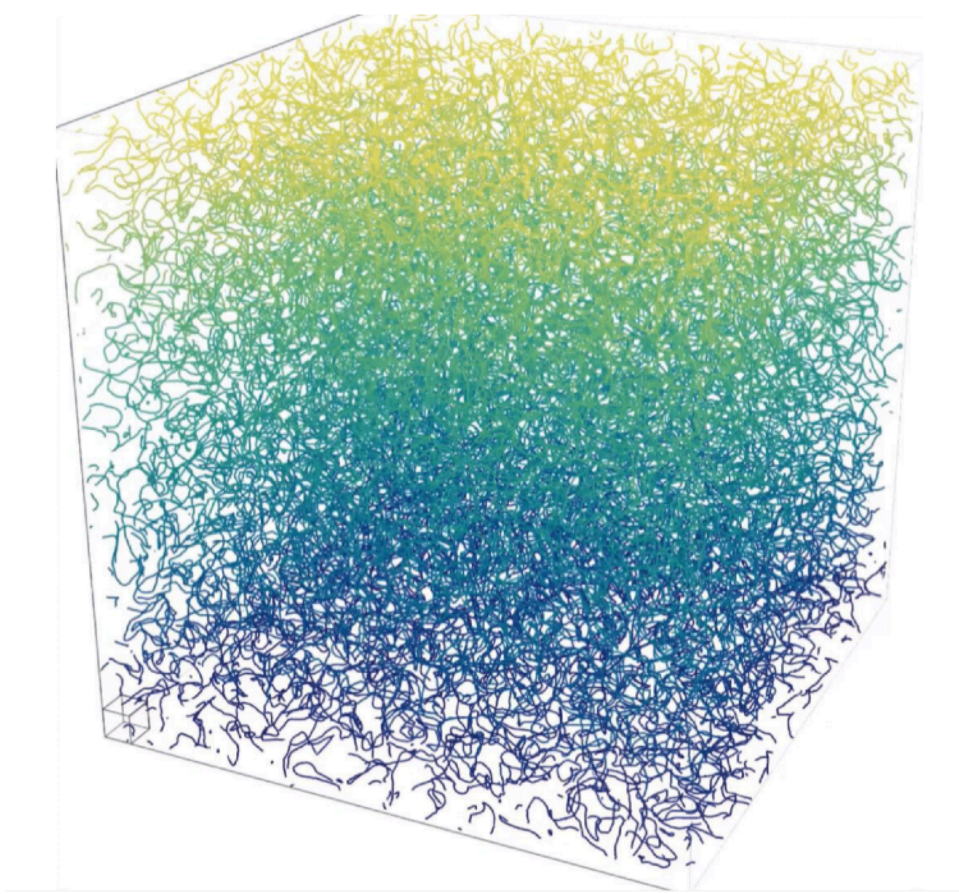

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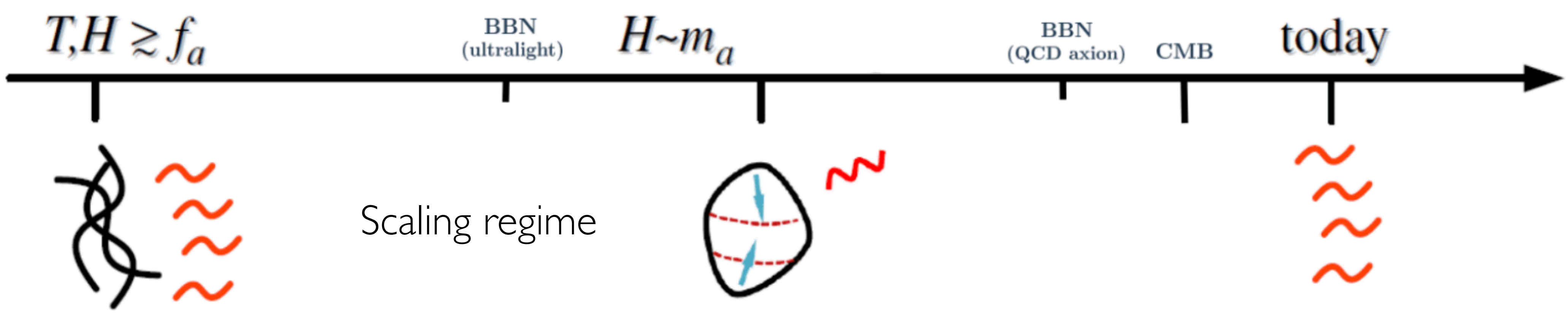

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Simulate →

Extrapolate

Need:

1. Energy emitted into GWs
2. Momentum distribution

Energy emitted

1. **Energy emitted into GWs**
2. Momentum distribution

$$\Gamma \simeq \frac{\xi(t)\mu(t)}{t^3}$$

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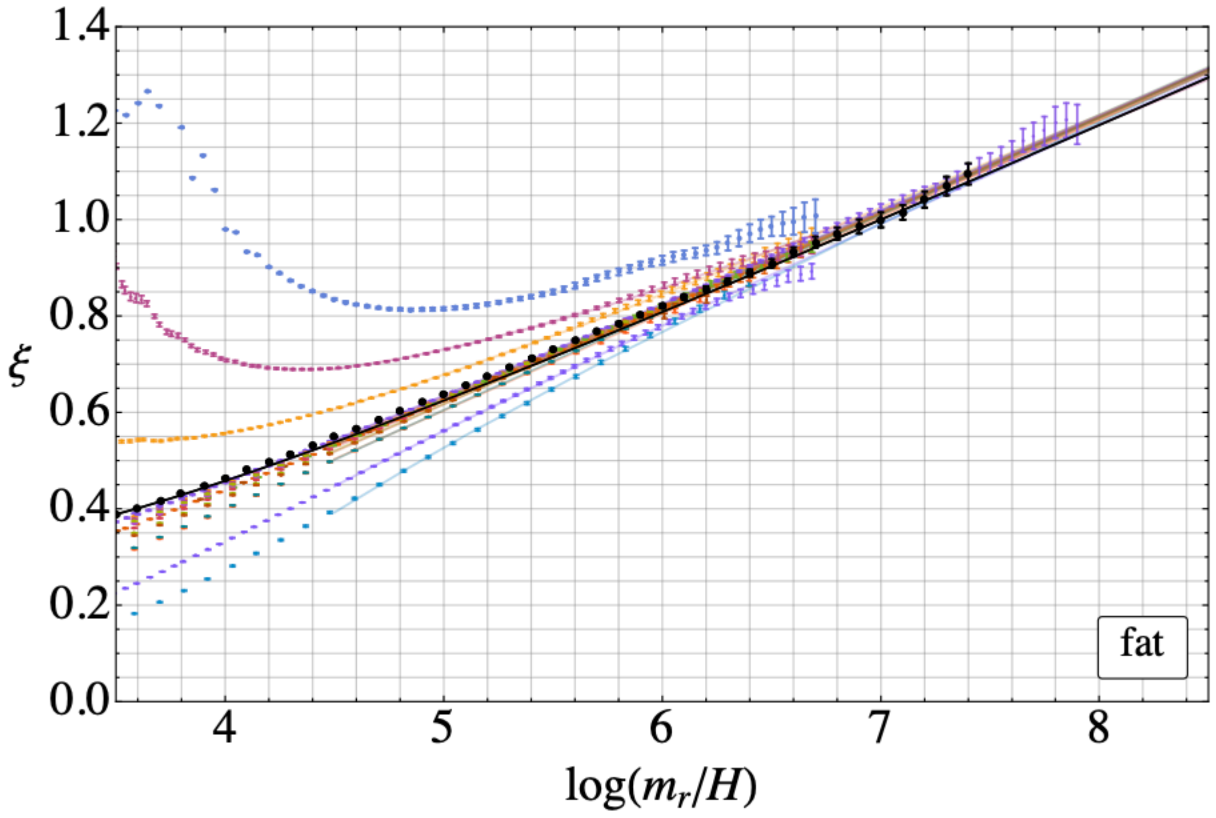

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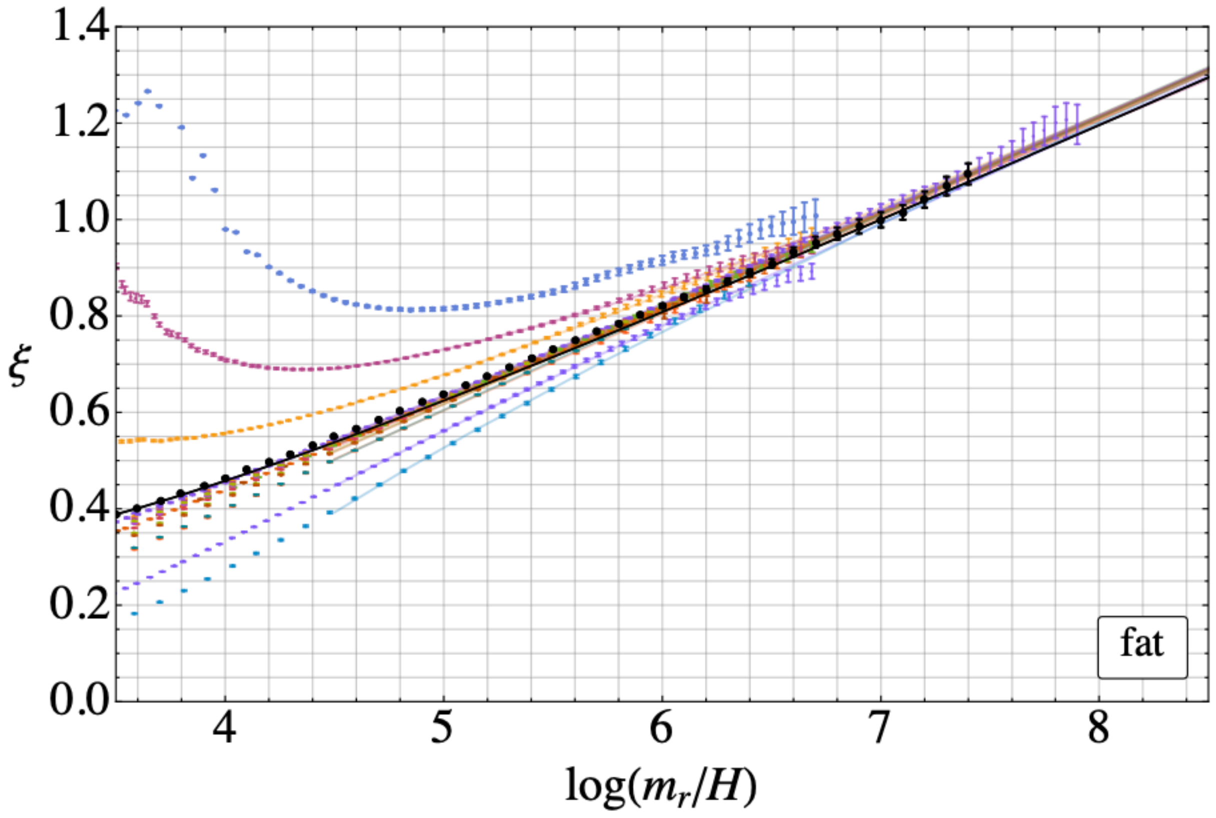

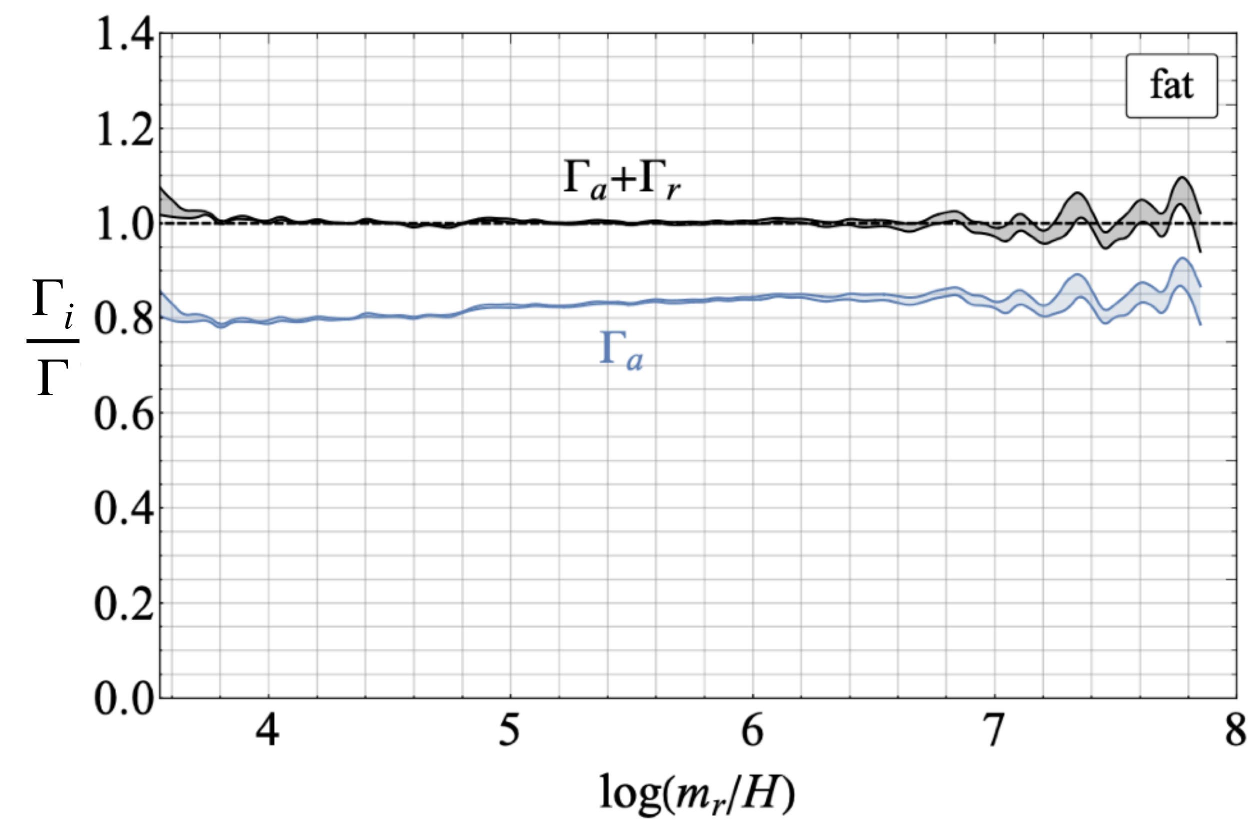

String EFT

1. **Energy emitted into GWs**
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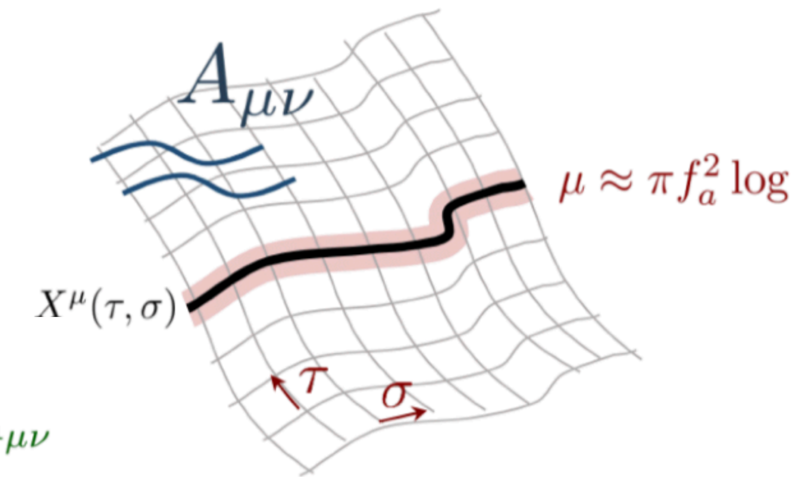
$$a \quad \longleftrightarrow \quad A_{\mu\nu} \qquad\qquad \partial A \sim F^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma}\partial_\sigma a$$



Axion-string coupling

$$S_{\mathrm{EFT}} = \underbrace{-\mu \int d\tau d\sigma\, \sqrt{-\gamma}}_{\text{Nambu-Goto action}} \underbrace{-\tfrac{1}{6}\int d^4x\, (\partial A)^2}_{\text{Axion kinetic term}} \underbrace{+\boxed{2\pi f_a}\int d\tau d\sigma\, \partial_\tau X^\mu \partial_\sigma X^\nu A_{\mu\nu}}_{\text{Axion-string interaction (Kalb-Ramond action)}}$$

$\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu$

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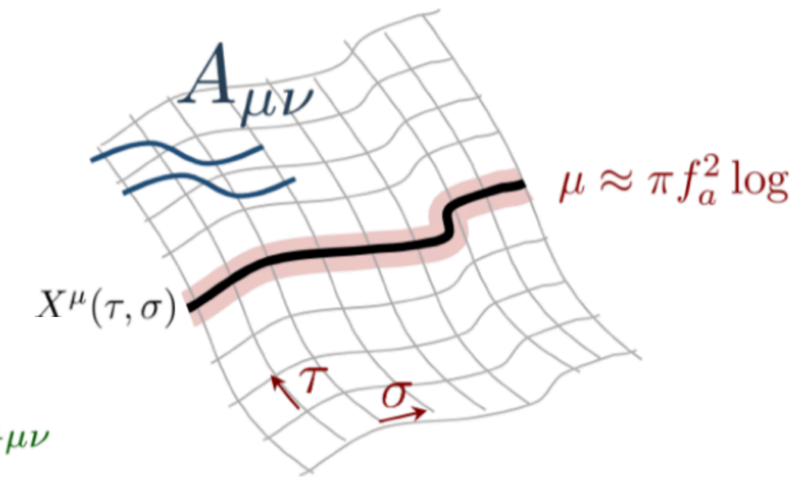

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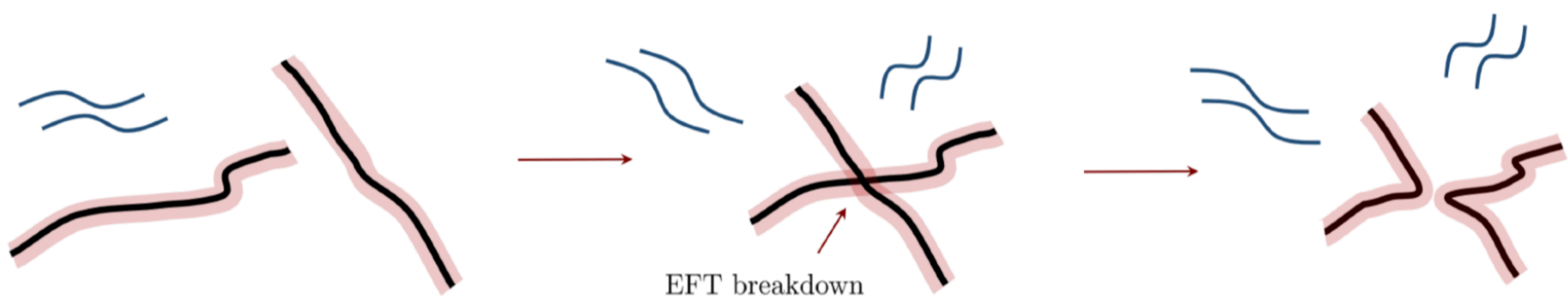
$$\text{EoM:} \quad \begin{cases} \mu(\ddot{X}^\mu - X''^\mu) = 2\pi f_a F^{\mu\nu\rho}\dot{X}_\nu X'_\rho \\ \Box_x A^{\mu\nu} = 2\pi f_a \int d\sigma\, \dot{X}^{[\mu} X'^{\nu]} \delta^3(\vec{x} - \vec{X}) \end{cases}$$



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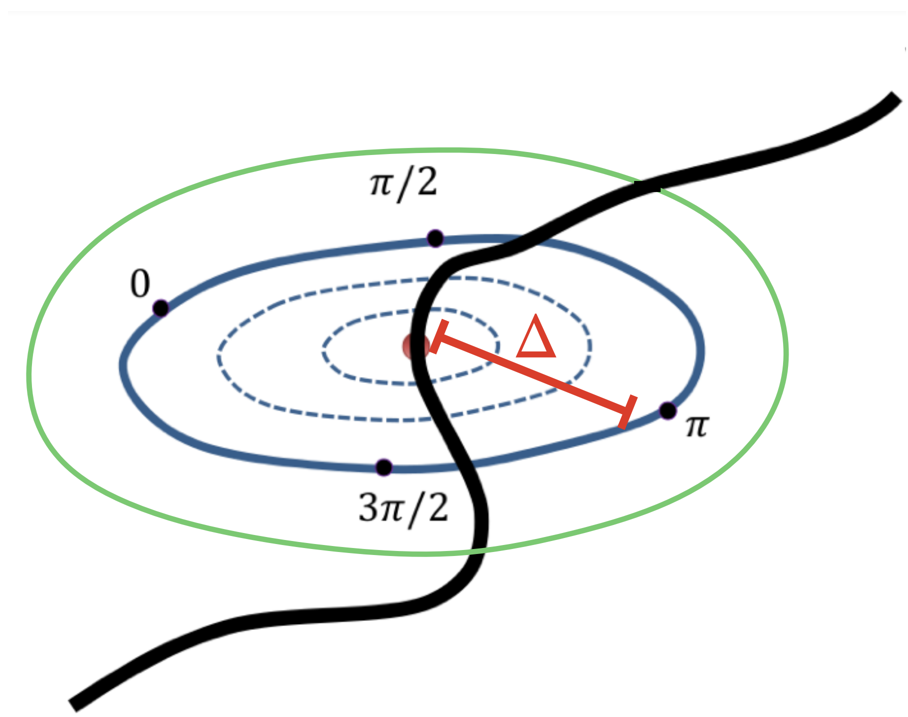

$$\mu(\Delta) = \pi f_a^2 \log\left(\Delta / m_r^{-1}\right)$$

[Lund & Regge, 1976]
also [Horn, Nicolis, Penco] in the context of superfluids

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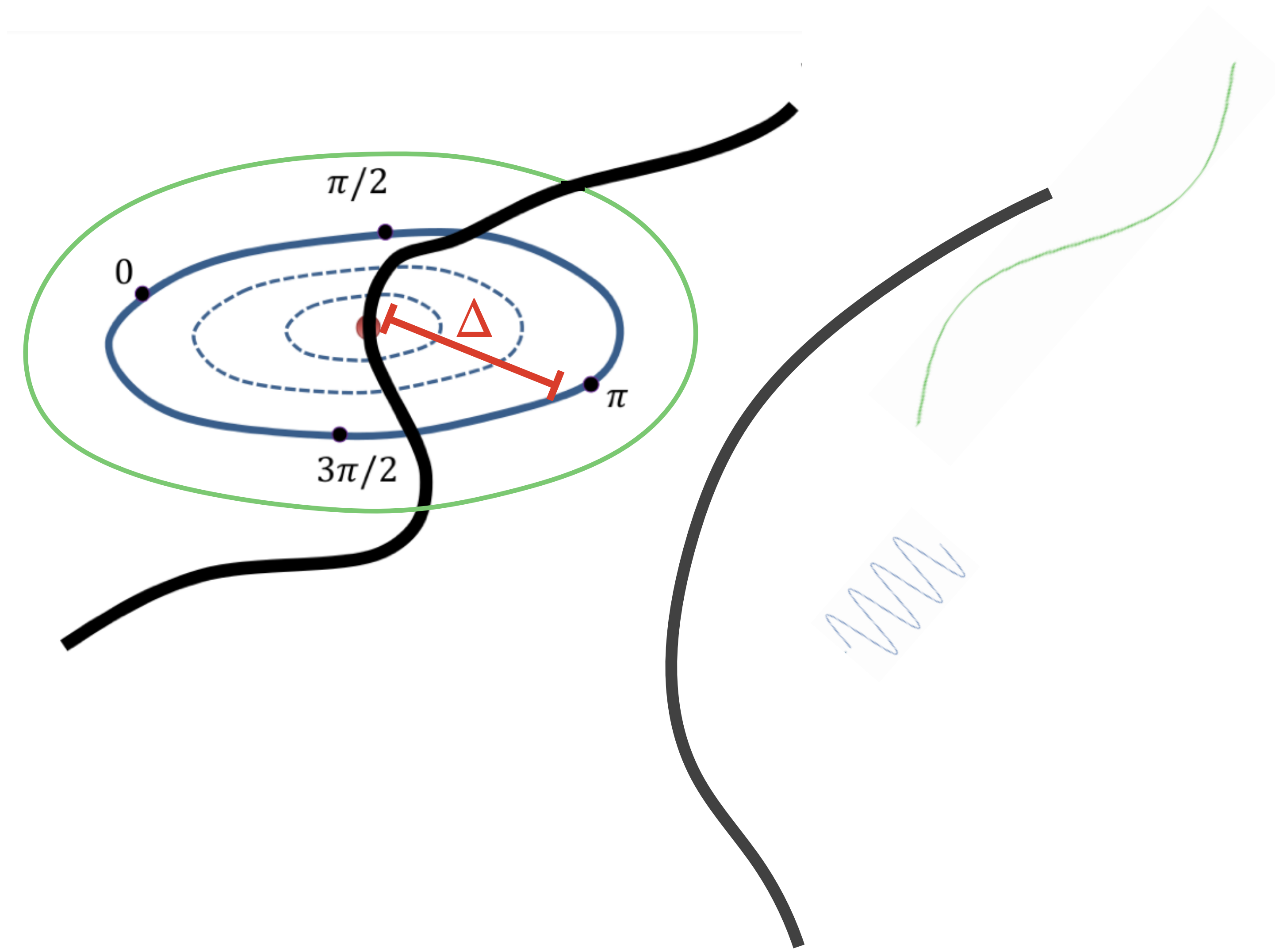

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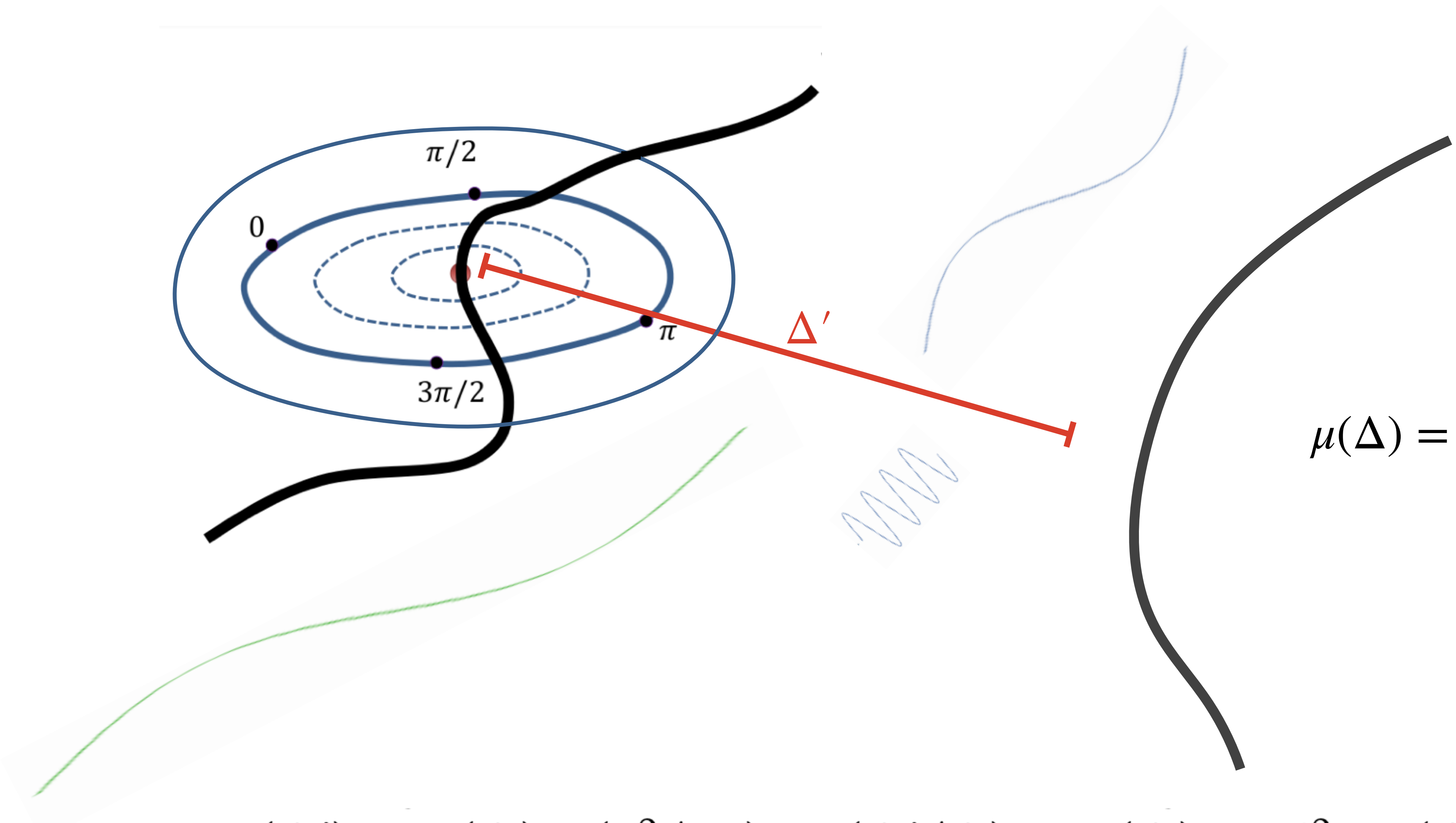

$$\mu(\Delta) = \pi f_a^2 \log\left(\Delta/m_r^{-1}\right)$$

$$\mu(\Delta') = \mu(\Delta) + (g^2/2\pi)\log(\Delta'/\Delta) = \mu(\Delta) + \pi f_a^2 \log(\Delta'/\Delta)$$

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$$\Box_x A^{\mu\nu} = 2\pi f_a \int d\sigma \, \dot{X}^{[\mu} X'^{\nu]} \, \delta^3(\vec{x} - \vec{X})$$

Einstein Eq. $$\Box_x h^{\mu\nu} = 16\pi G \left(T_s^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} T^{\lambda}_{s\lambda} \right)$$

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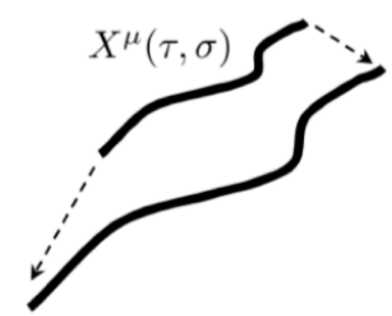
$$\frac{dE_a}{dt} = \underbrace{r_a[X]}_{\uparrow}\, f_a^2 \qquad \frac{dE_g}{dt} = \underbrace{r_g[X]}_{\uparrow}\, G\mu^2$$

Dimensionless functionals of shape of string trajectory



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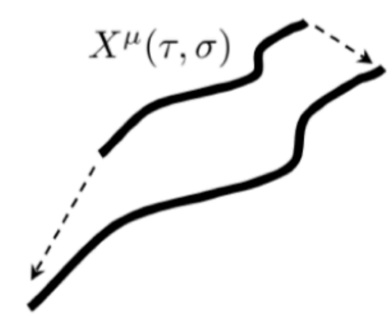

$$\frac{\Gamma_g}{\Gamma_a} = \underbrace{\frac{r_g[X]}{r_a[X]}}_{\equiv r} \frac{G\mu^2}{f_a^2} = \text{const}$$

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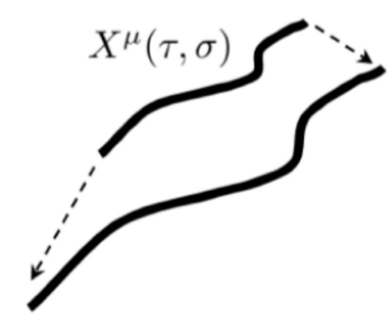
$$\Box_x A^{\mu\nu} = 2\pi f_a \int d\sigma\, \dot{X}^{[\mu} X'^{\nu]}\, \delta^3(\vec{x} - \vec{X})$$

Einstein Eq. $$\Box_x h^{\mu\nu} = 16\pi G \left(T_s^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} T^{\lambda}_{s\lambda} \right)$$

$$T_s^{\mu\nu} = \mu \int d\sigma \, \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \, \delta^3(\vec{x} - \vec{X})$$

$$\frac{dE_a}{dt} = \underbrace{r_a[X]}_{\uparrow}\, f_a^2 \qquad \frac{dE_g}{dt} = \underbrace{r_g[X]}_{\uparrow}\, G\mu^2$$

Dimensionless functionals of shape of string trajectory



$$\frac{\Gamma_g}{\Gamma_a} = \underbrace{\frac{r_g[X]}{r_a[X]}}_{\equiv r} \frac{G\mu^2}{f_a^2} = \text{const}$$

$$\boxed{\Gamma_g = r \frac{G\mu^2}{f_a^2} \Gamma} \propto \frac{\log^4}{t^3}$$

$\xi\mu/t^3$

Simulations

1. **Energy emitted into GWs**
2. Momentum distribution

$$\boxed{\Gamma_g = r\frac{G\mu^2}{f_a^2}\Gamma} \quad \propto \frac{\log^4}{t^3}$$

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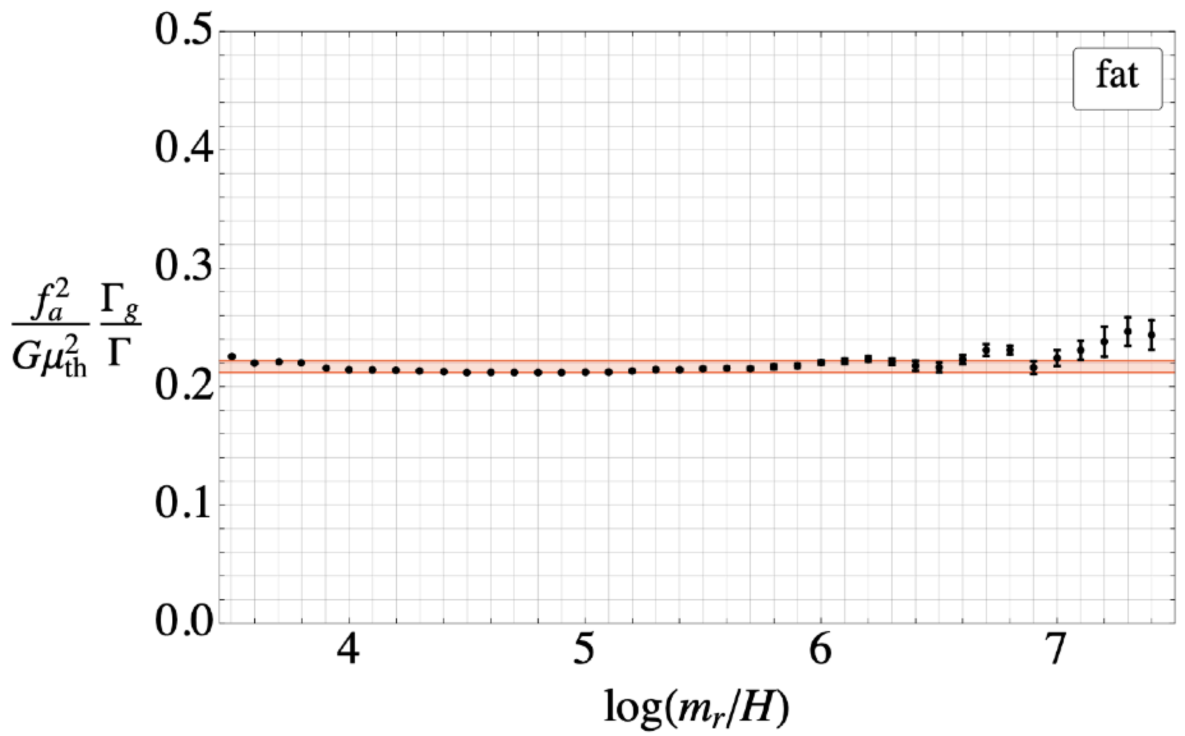

Instantaneous emission

Simulations

1. **Energy emitted into GWs**
2. Momentum distribution

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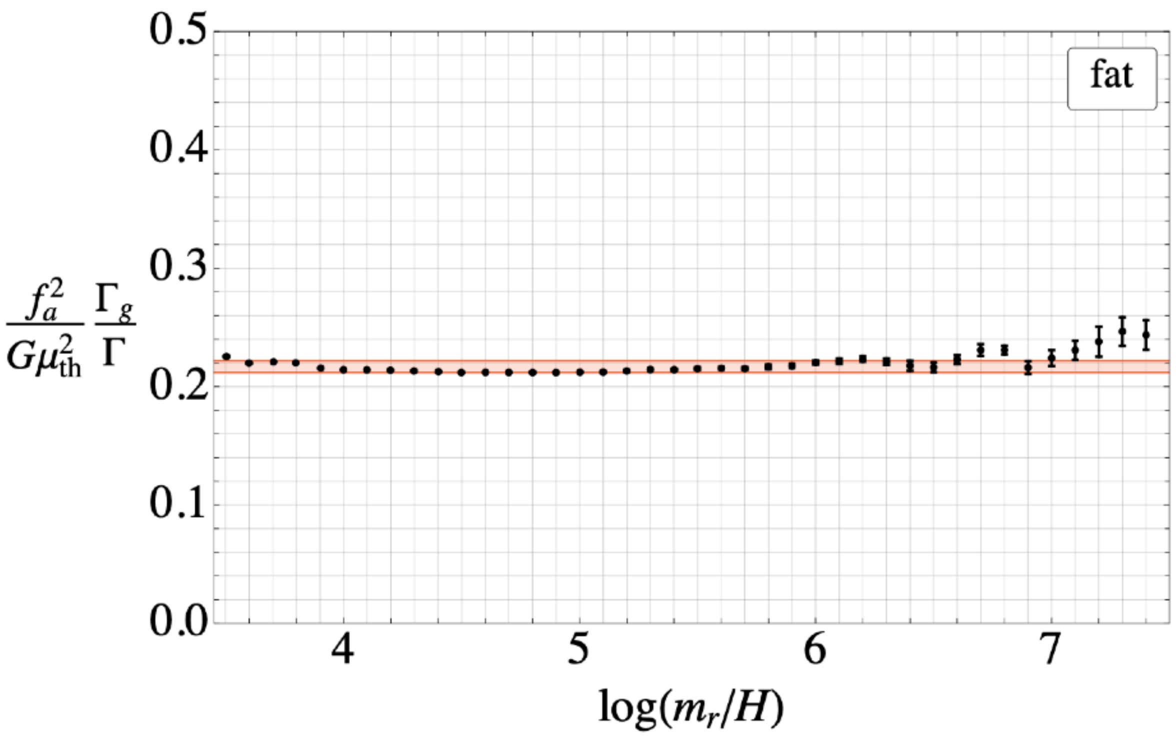
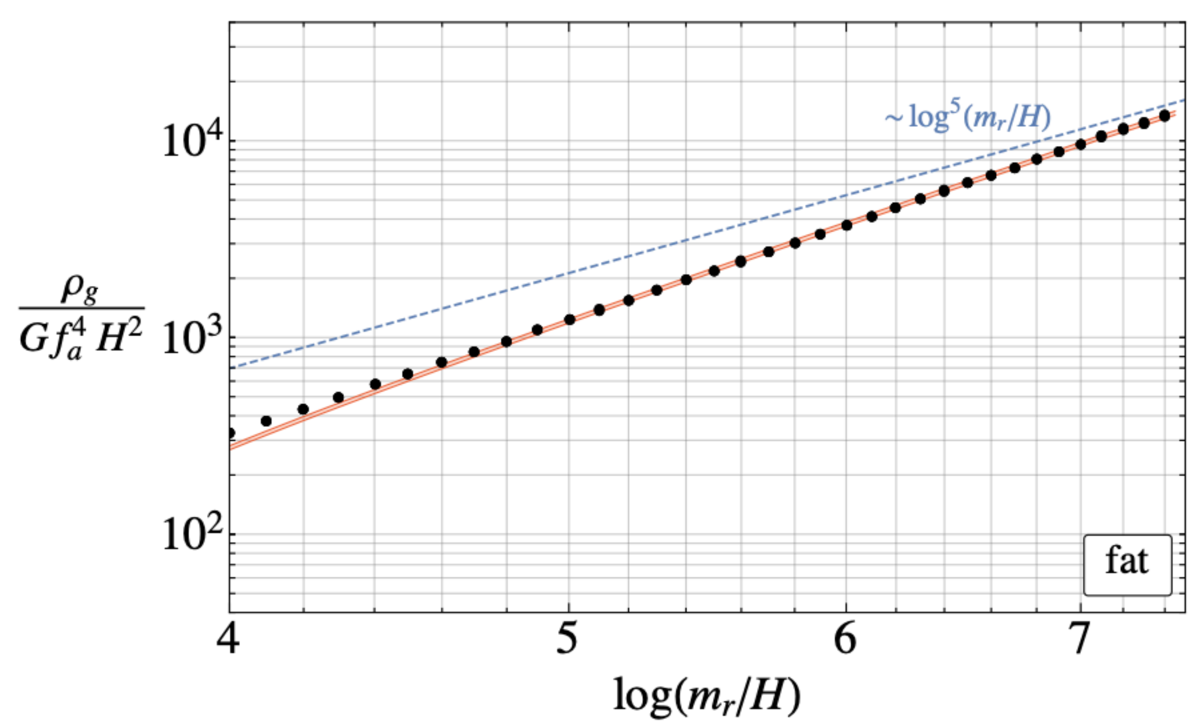


Instantaneous emission

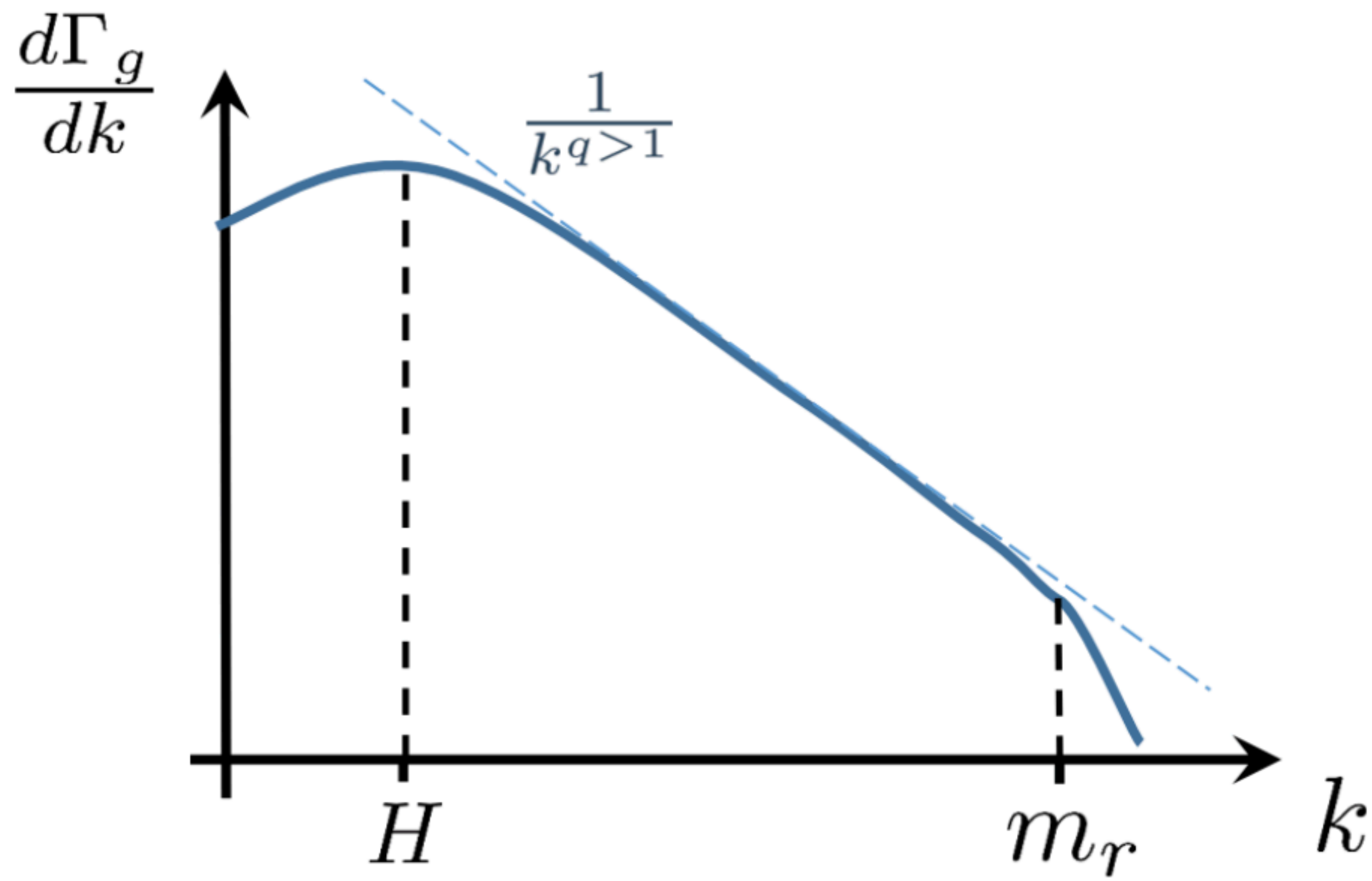


Total energy

$$\rho_g(t) \propto \int dt' \frac{\log'^4}{t'^3}\left(\frac{R(t')}{R(t)}\right)^4 \to \log^5$$

GW spectrum

1. Energy emitted into GWs
2. **Momentum distribution**



GW spectrum

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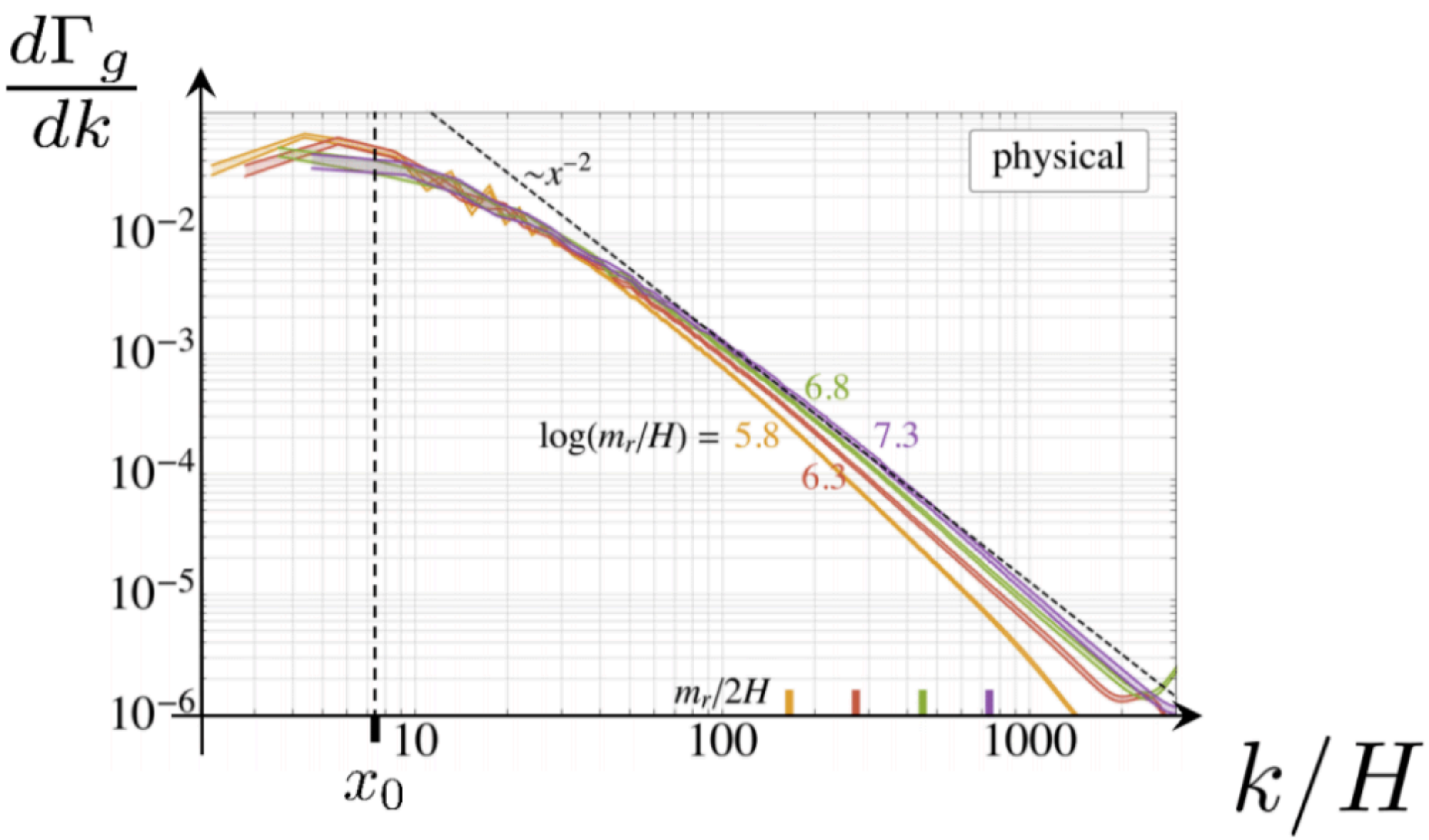
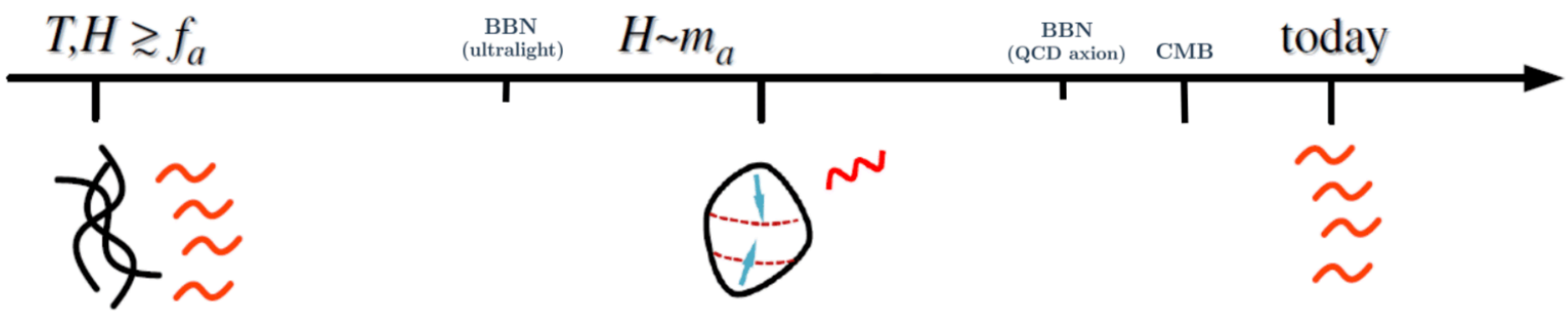
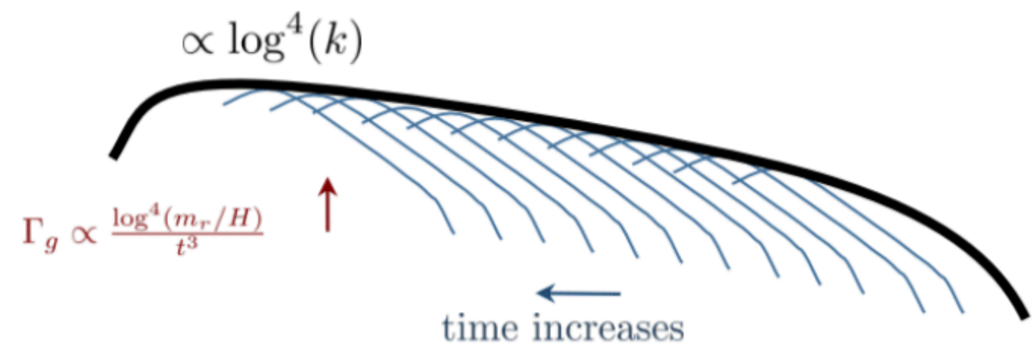


The spectrum today



$\log(m_r/H)$ $\sim 1 \div 15$ $\sim 70 \div 100$

$$\frac{\partial \rho_g}{\partial \log k} = \int dt' \, \frac{d\Gamma'}{d \log k} \left(\frac{R'}{R}\right)^4$$



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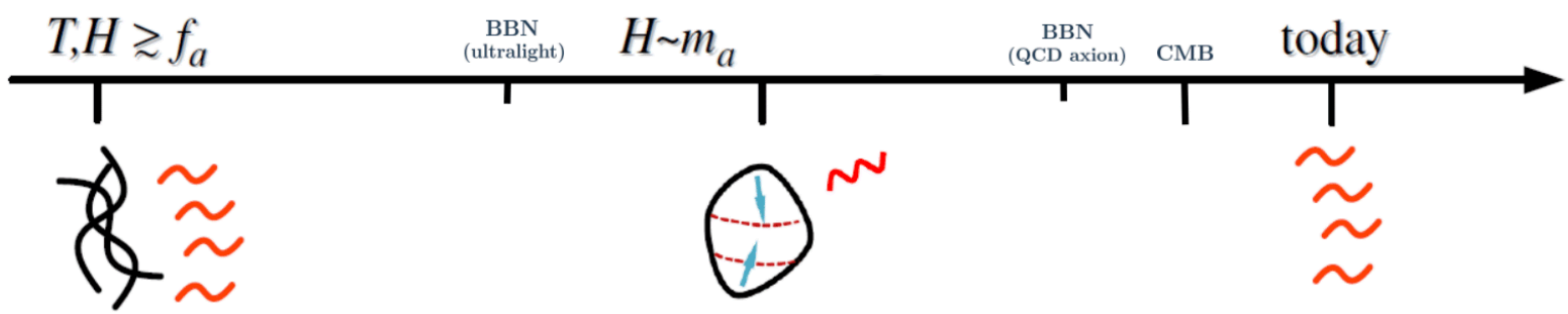

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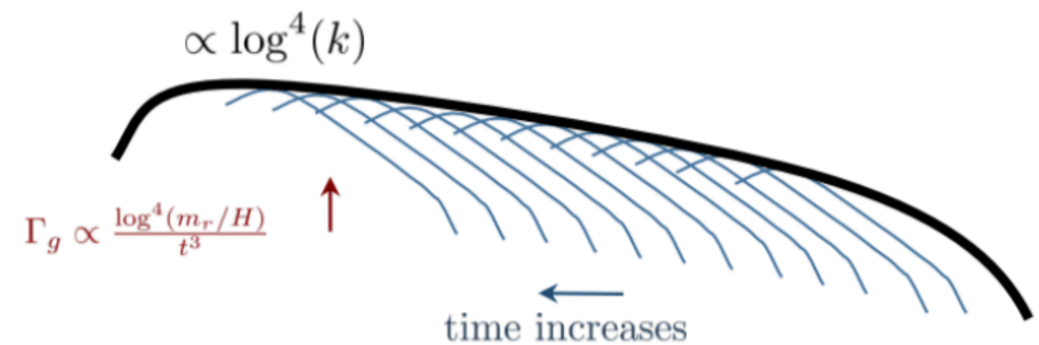
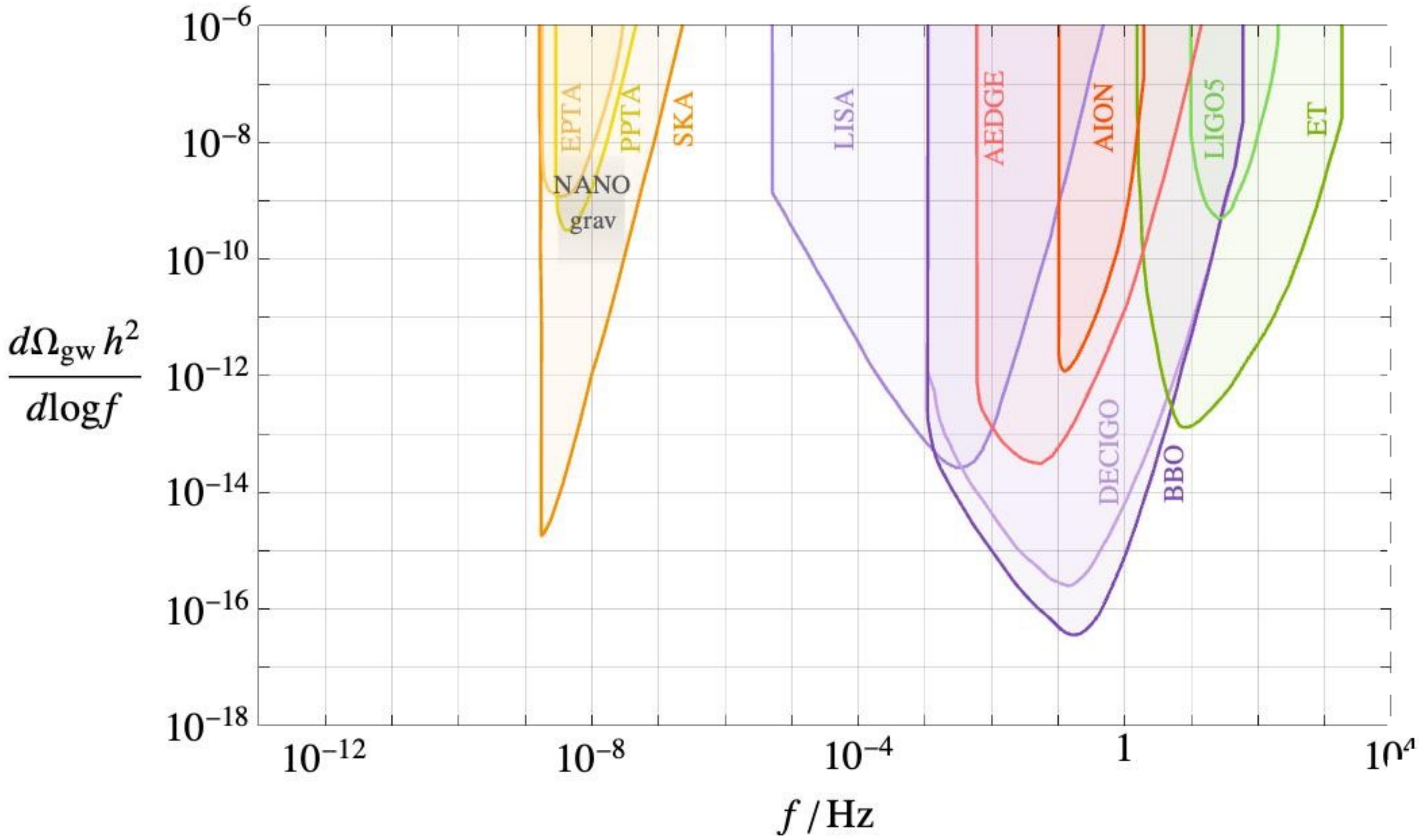


$$\boxed{\frac{d\Omega_{\rm gw} h^2}{d\log f} \simeq 10^{-15} \left(\frac{r}{0.26}\right) \left(\frac{f_a}{10^{14}\mathrm{GeV}}\right)^4 \left(\frac{10}{g_f}\right)^{\frac{1}{3}} \left\{1 + 0.12 \log\left[\left(\frac{m_r}{10^{14}\mathrm{GeV}}\right)\left(\frac{10^{-8}\mathrm{Hz}}{f}\right)^2\right]\right\}^4}$$

The spectrum today



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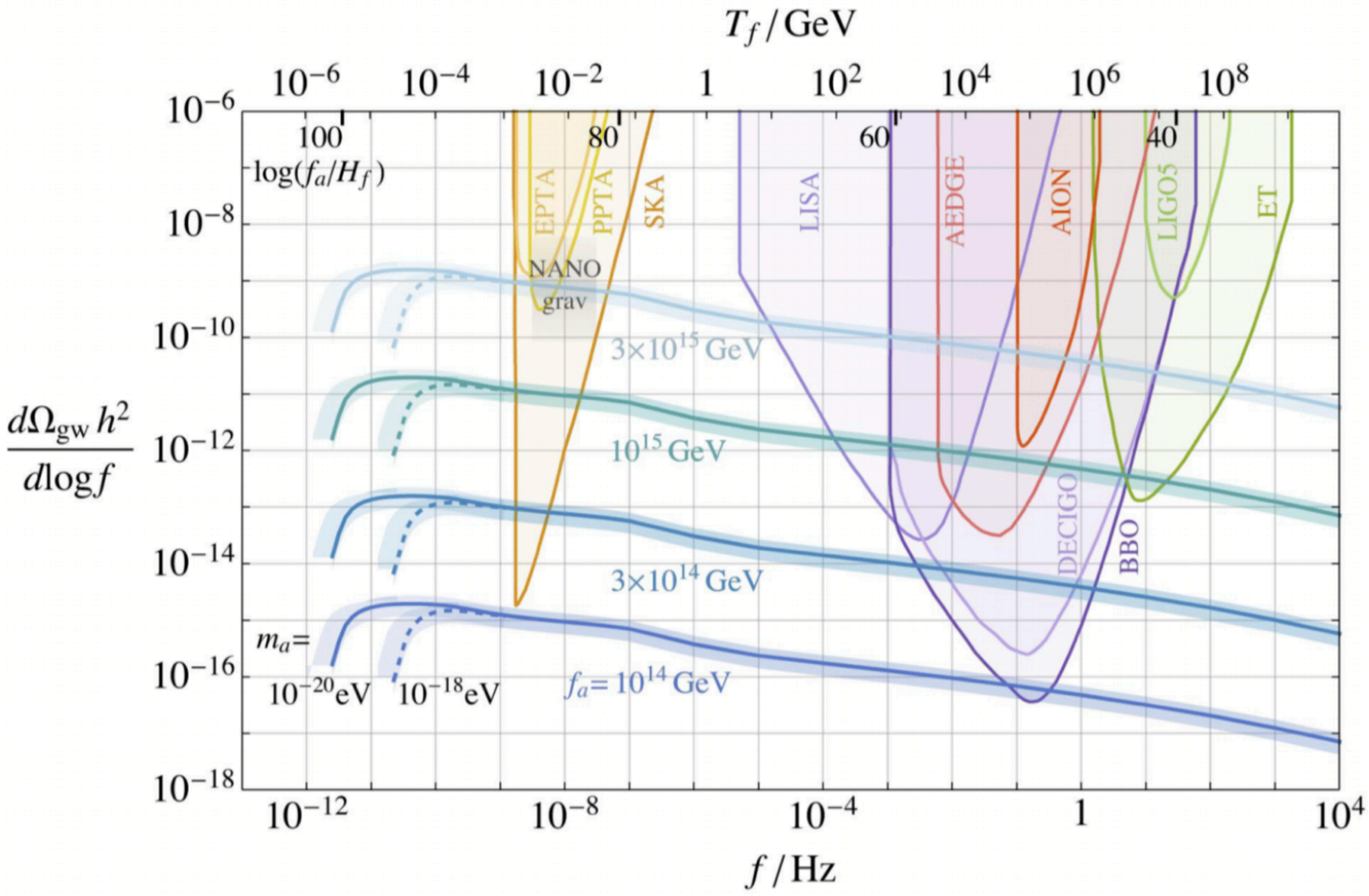

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Targets for haloscopes

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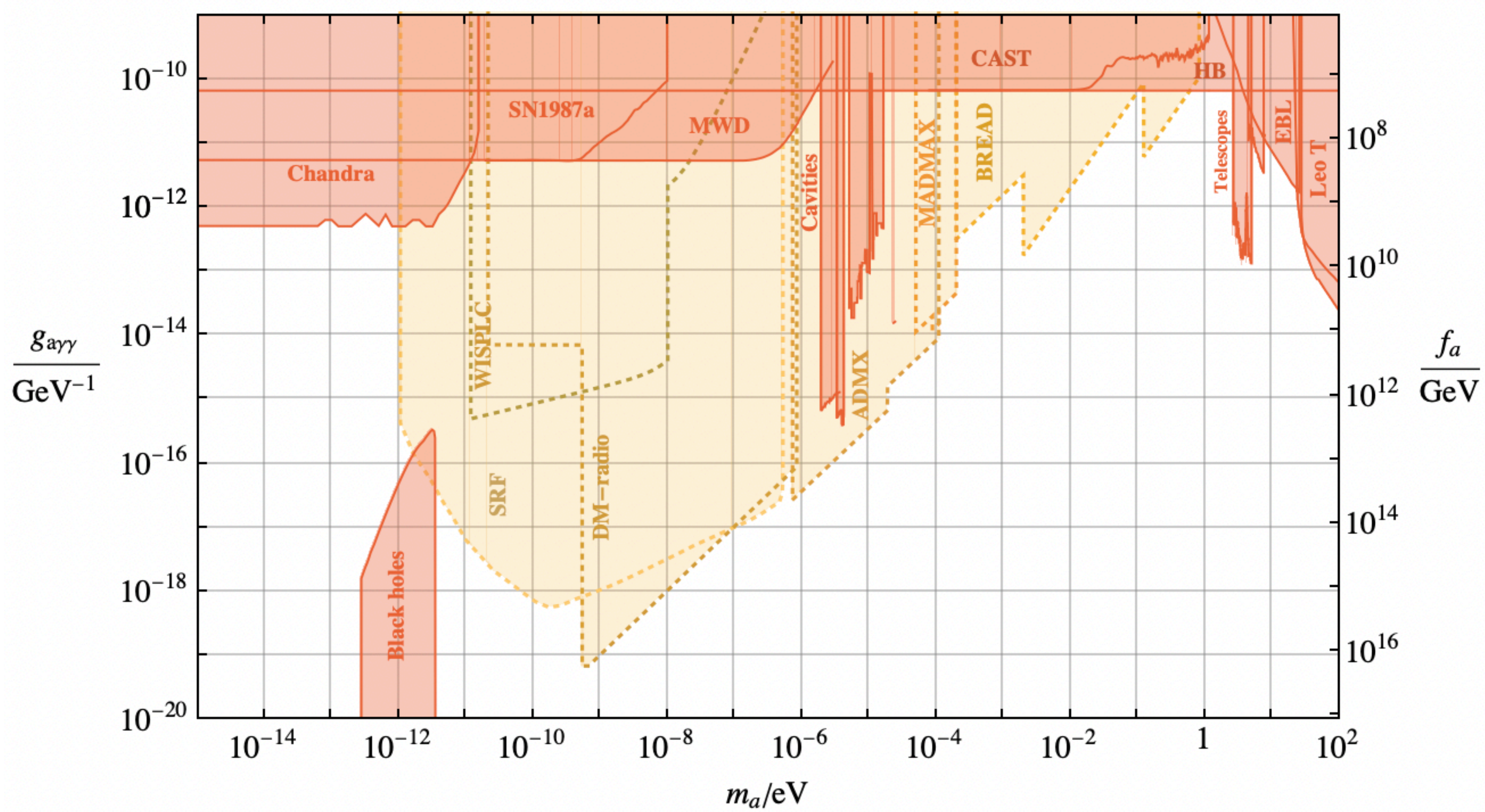

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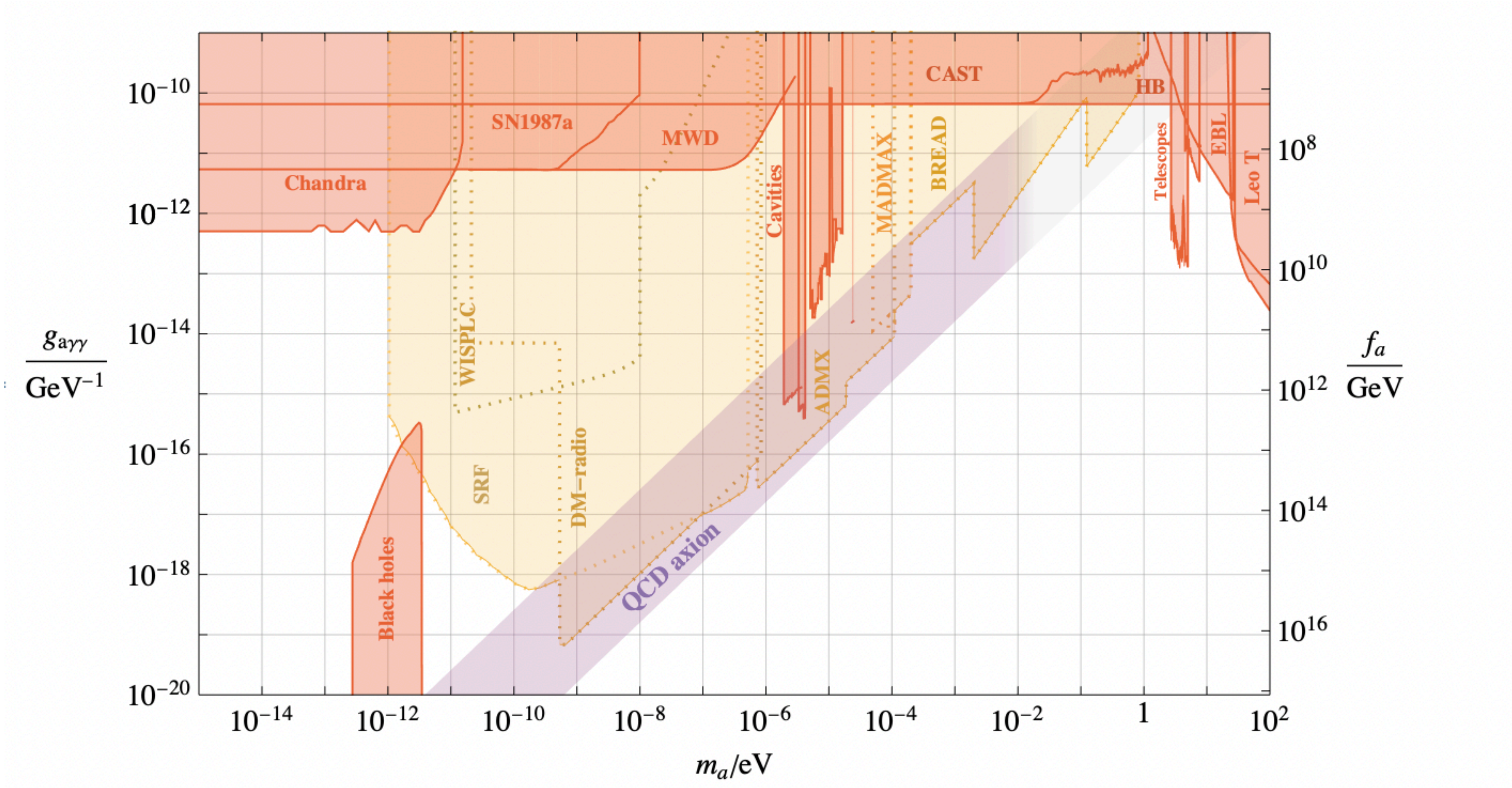


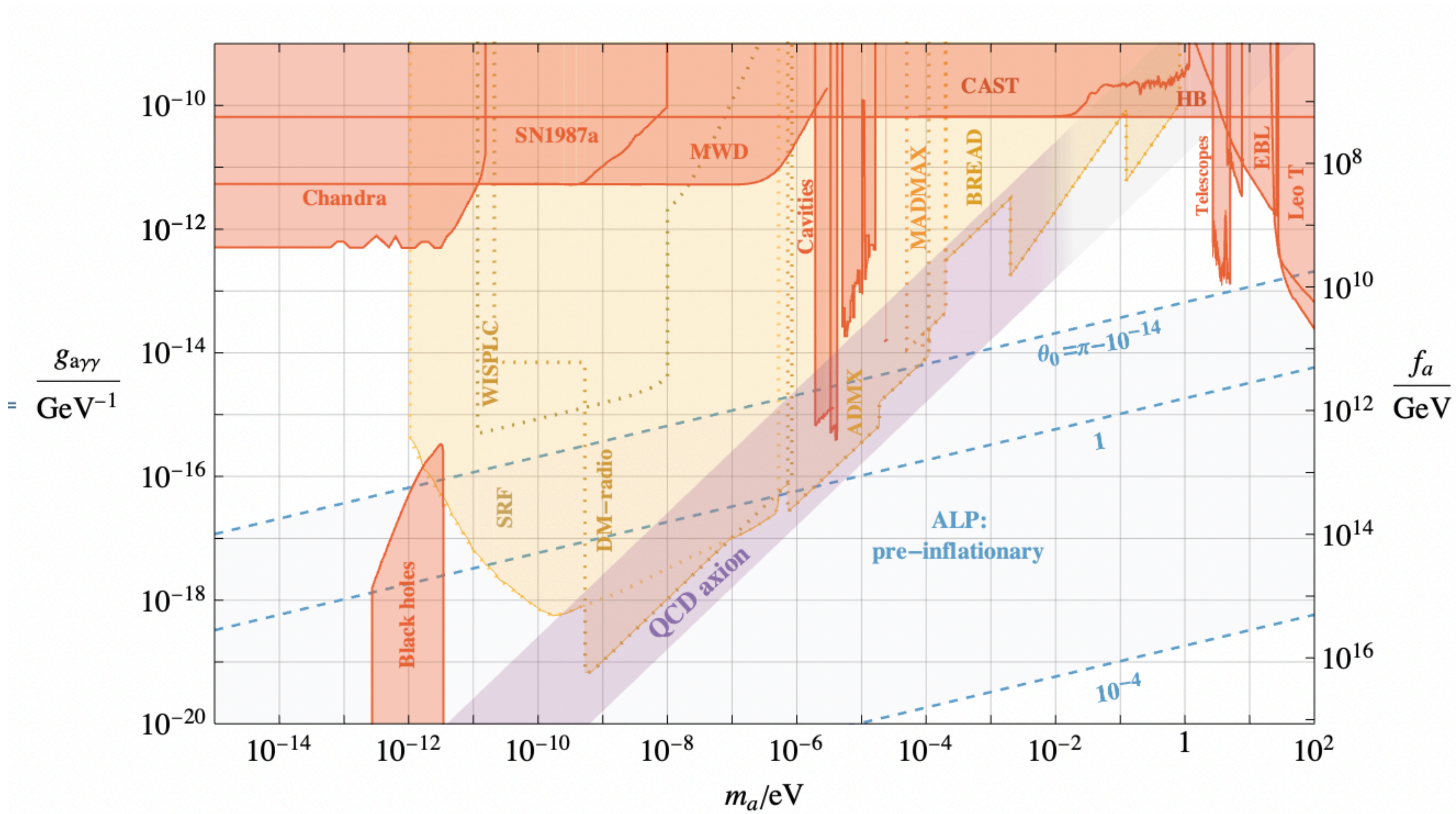
$$m_a = 5.70(6)(4)\,\mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)$$

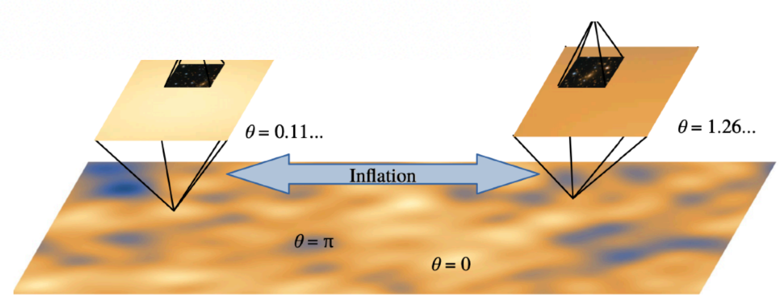
$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - 1.92(4)\right]$$

$$\boxed{g_{a\gamma\gamma} = \left[0.203(3)\frac{E}{N} - 0.39(1)\right]\frac{m_a}{\mathrm{GeV}^2}}$$

[Grilli di Cortona, EH, Pardo Vega, Villadoro]

Targets for haloscopes



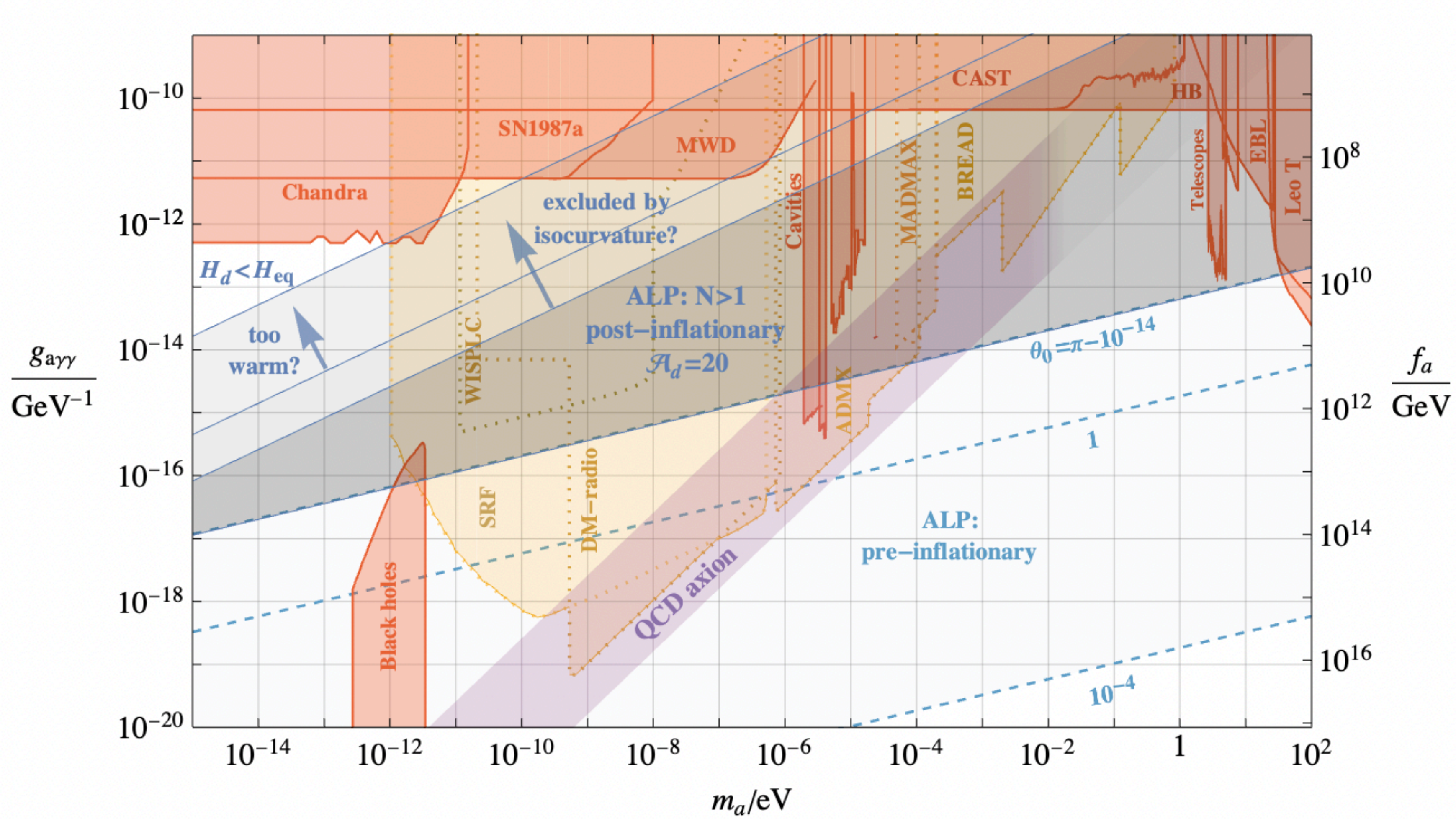


$$g_{a\gamma\gamma} \simeq \alpha_{\rm EM}/(2\pi f_a)$$

$$\frac{\Omega_a}{\Omega_{\rm DM}} \simeq 2.7 \cdot 10^{-3}\ \theta_0^2 \left(\frac{f_a}{10^{12}\ \ {\rm GeV}}\right)^2 \left(\frac{m_a}{10^{-6}\ \ {\rm eV}}\right)^{1/2}$$

$$\boxed{g_{a\gamma\gamma} \lesssim 5 \cdot 10^{-17}\,{\rm GeV}^{-1}\ \left(\frac{m_a}{10^{-6}\,{\rm eV}}\right)^{1/4} \theta_0}$$

N>1



$$\frac{\Omega_a}{\Omega_{\rm DM}} \simeq 2\left(\frac{\mathcal{A}_d}{20}\right)\left(\frac{m_a}{H_d}\right)^{1/2}\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^2\left(\frac{m_a}{10^{-6}\ \mathrm{eV}}\right)^{1/2}$$

$$g_{a\gamma\gamma} \simeq \alpha_{\rm EM}/(2\pi f_a)$$

$$\boxed{g_{a\gamma\gamma} \simeq 2\cdot 10^{-15}\,\mathrm{GeV}^{-1}\left(\frac{\mathcal{A}_d}{20}\right)^{1/2}\left(\frac{m_a}{H_d}\right)^{1/4}\left(\frac{m_a}{10^{-6}\ \mathrm{eV}}\right)^{1/4}}$$

Summary

- Scaling regime produces an approximately scale invariant GW spectrum

  $\Gamma_g \propto \log^4$ gives log violations, enhances the spectrum at low frequencies

- Observable by multiple experiments for $f_a > 10^{14}$ GeV

- At larger m_a and smaller f_a post-inflationary ALPS are good targets for haloscops

Much more to do

- Effects of friction on the strings
- Local strings
- QCD axion substructure
- QCD axion N>1
- Similar dynamics in other theories of dark matter with mass $\lesssim$ eV

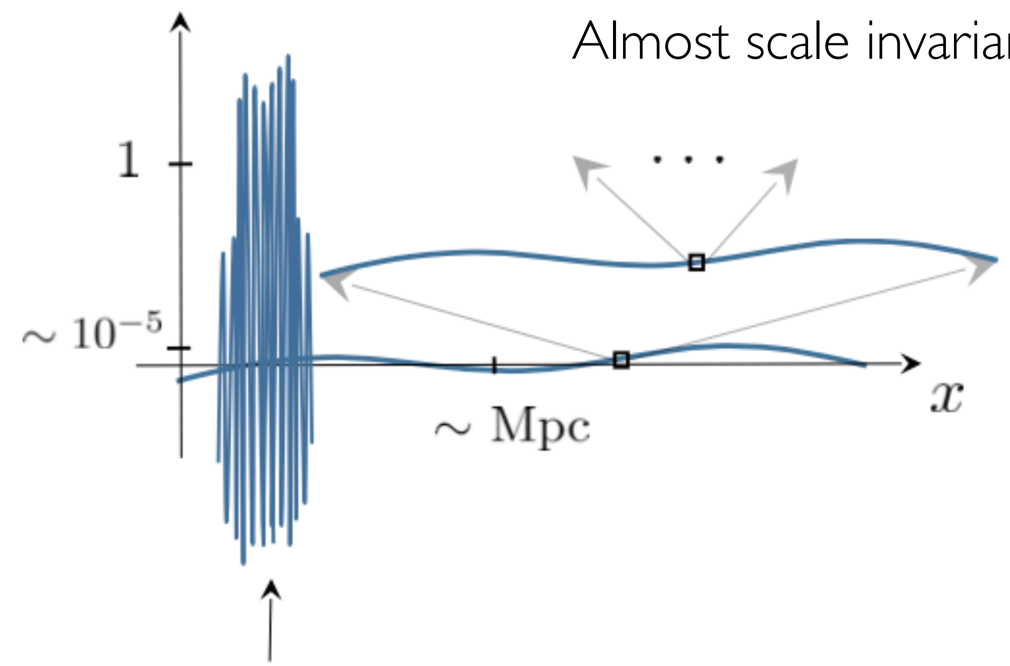
Thanks

Substructure

Expect order one fluctuations on scales set by H_d^{-1}

$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$

Almost scale invariant

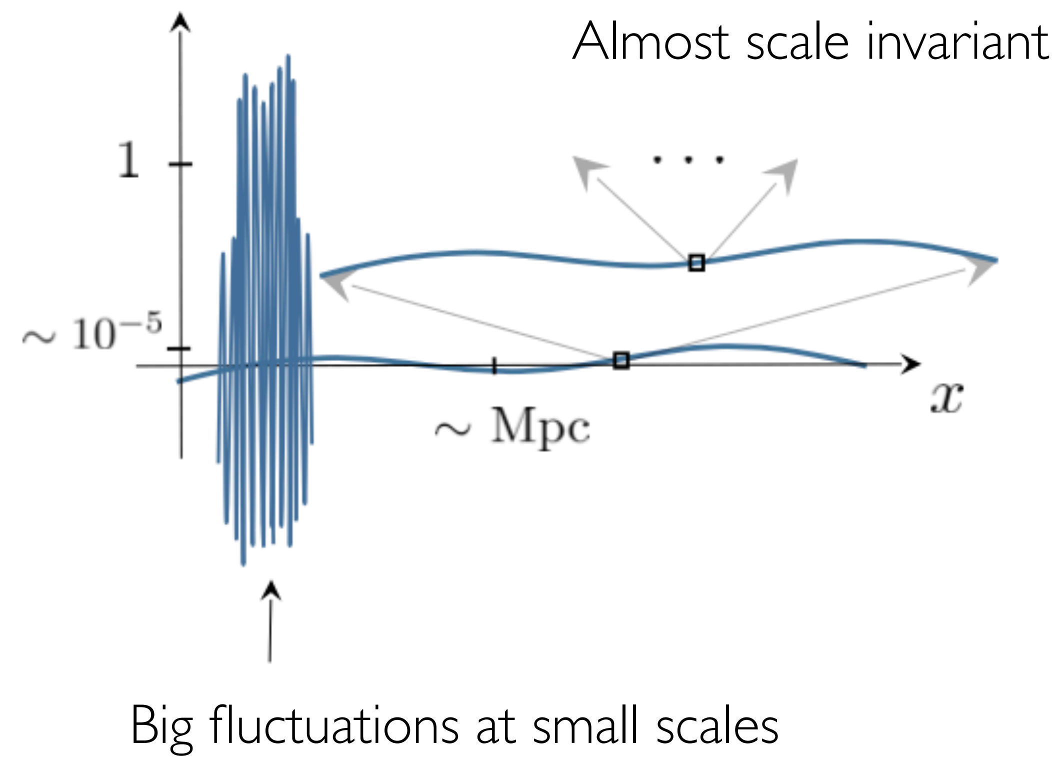


Big fluctuations at small scales

Substructure

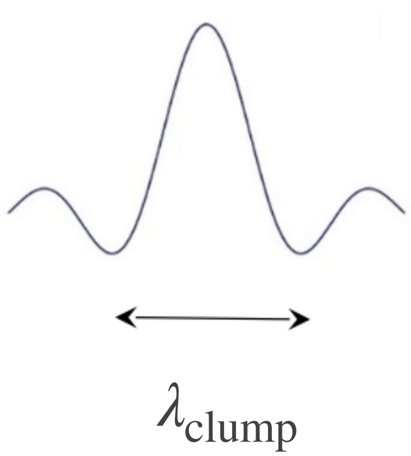
$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$

Expect order one fluctuations on scales set by H_d^{-1}



Order one over-density $\delta = \delta\rho / \bar{\rho}$

becomes nonlinear at MRE



Size of the clump $\lambda_{\text{clump}} \simeq H_d^{-1} \dfrac{R_{\text{eq}}}{R_d}$

Typical density $\rho_{\text{clump}} \simeq (1 \div 10^4)\ \rho_{\text{eq}} \ \simeq \ (1 \div 10^4)\ \text{eV}^4 \ \simeq \ (10^6 \div 10^{10})\ \rho_{\text{local}}$

Substructure

$$\frac{\lambda_{\text{clump}}}{\lambda_{\text{wave}}} \simeq \left(\frac{m_a}{H_d}\right)^{1/2}$$

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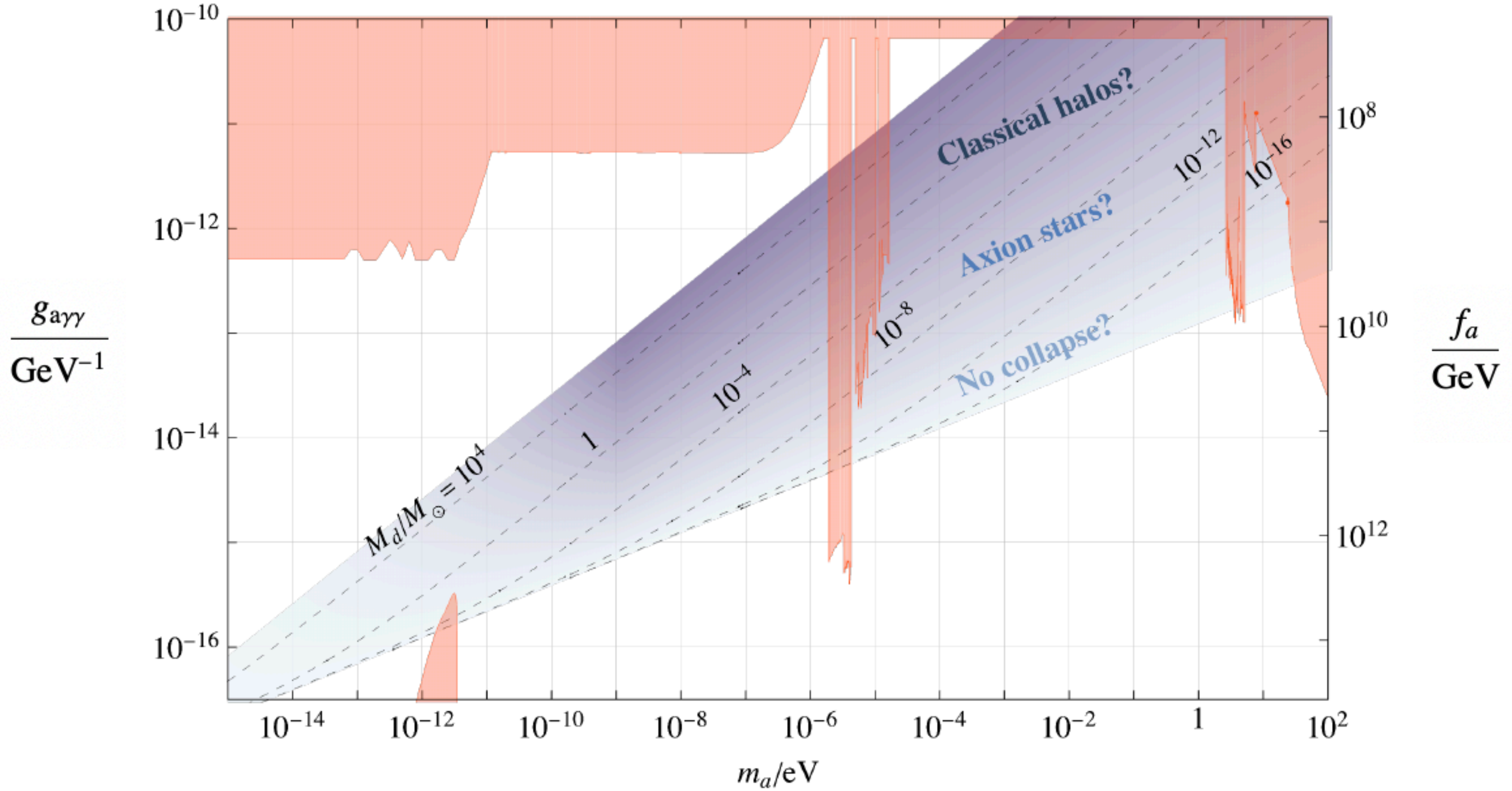
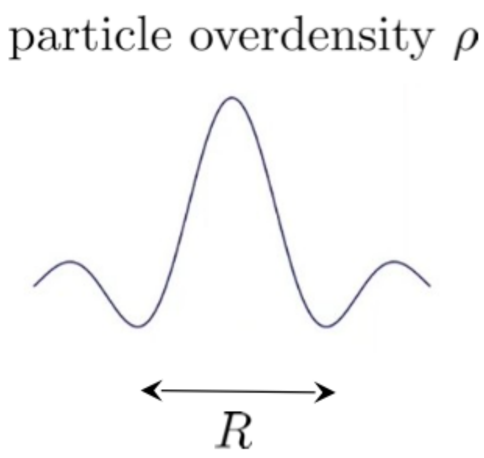


Gravitational collapse and wave effects



de Broglie wavelength of a particle in the resulting clump

$$\lambda_{\mathrm{dB}} = \frac{1}{mv} = \frac{1}{m(GM/\lambda_{\mathrm{clump}})^{1/2}} = \frac{1}{\lambda_{\mathrm{clump}}(4\pi G\rho m^2)^{1/2}}$$

$$\boxed{\lambda_{\mathrm{crit}} = (4\pi G\rho m^2)^{-1/4}}$$

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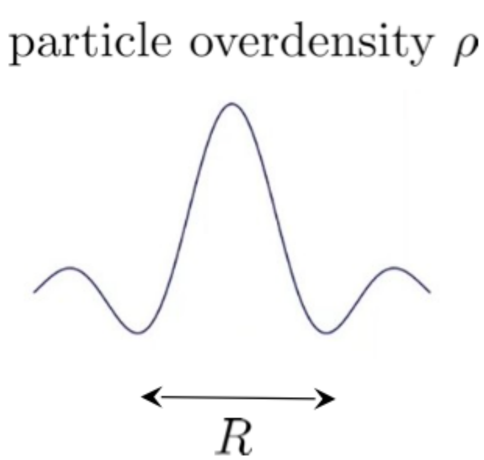

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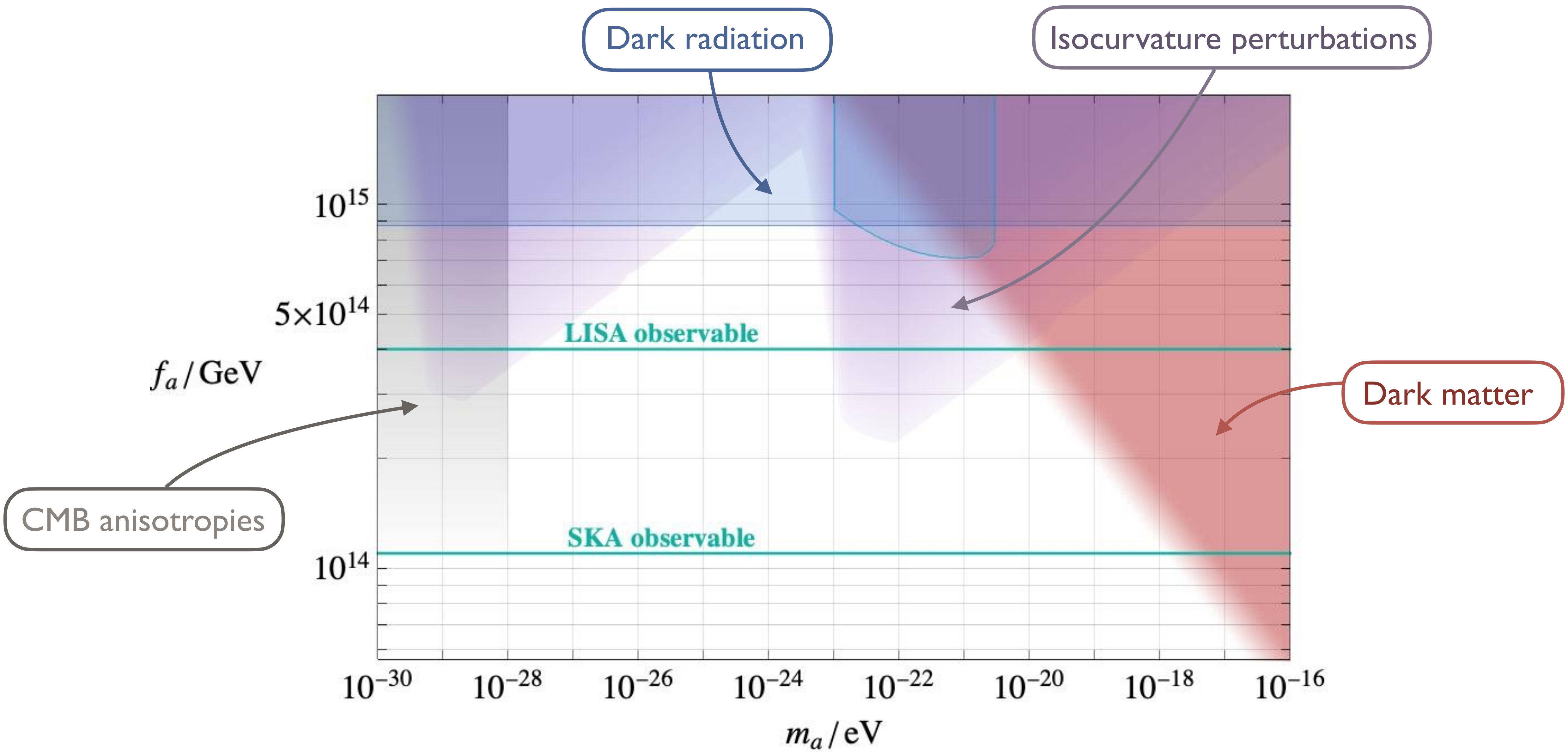
$$\frac{\lambda_{\rm clump}}{\lambda_{\rm crit}} = \frac{H_d^{-1}R_{\rm eq}/R_d}{(4\pi G\rho_{\rm clump}m^2)^{-1/4}} \simeq \left(\frac{m_a}{H_d}\right)^{1/2}$$

For H_d not too much smaller than m_a

quantum pressure cannot be neglected

Other constraints



Relic abundance

Domain wall tension

$$\sigma = \beta m_a f_a^2$$

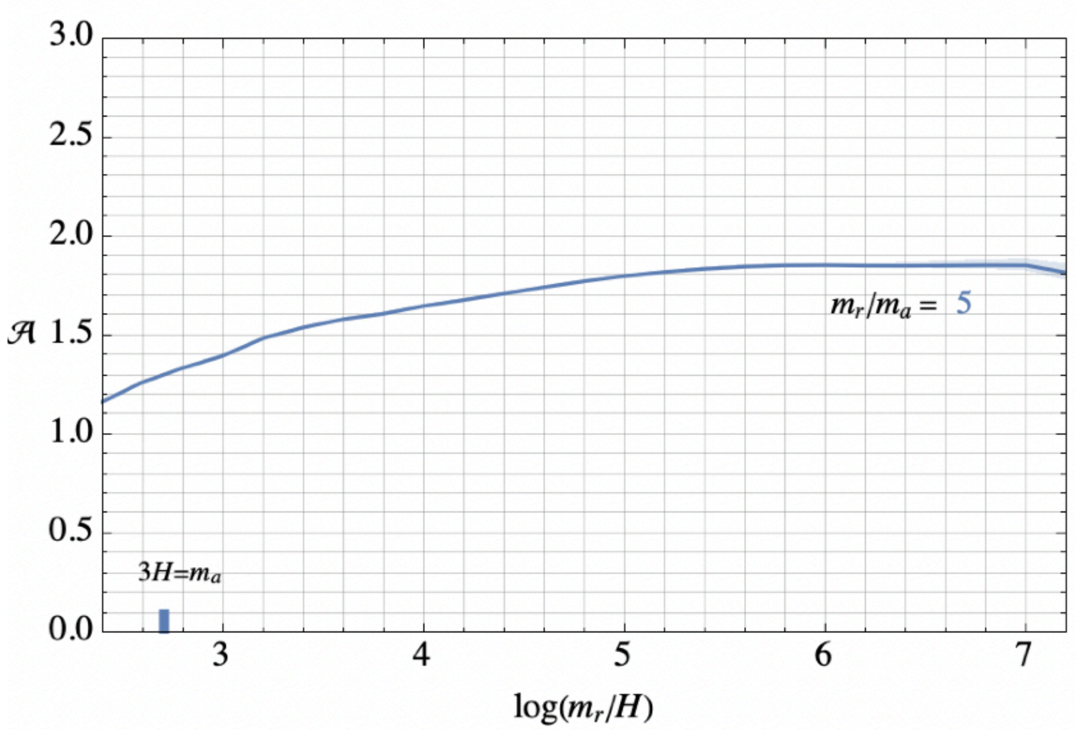
Number of domain walls per Hubble patch

$$\mathcal{A} = \lim_{V\to\infty} At/V$$

Energy density

$$\rho_{\rm w} = \sigma\mathcal{A}/t = 2\sigma\mathcal{A}H$$

$$\Gamma_a^{\rm w} \simeq \rho_{\rm w}/(2t)$$



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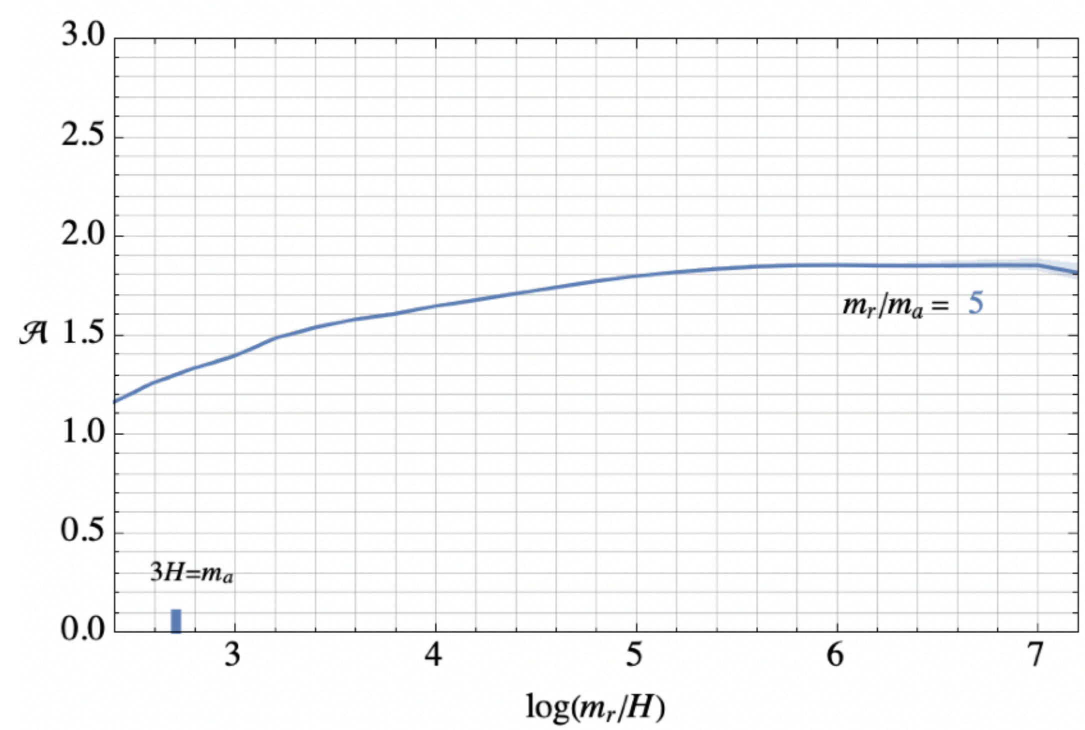


$$n_a^{\rm w}(t) \simeq \int_{\log t_\star}^{\log t_d} \mathcal{A}\frac{\beta f_a^2}{2t'}\left(\frac{R(t')}{R(t)}\right)^3 d\log t' \simeq \int_{\log t_\star}^{\log t_d} \mathcal{A}\beta f_a^2 \frac{t'^{1/2}}{2t^{3/2}}\, d\log t'$$

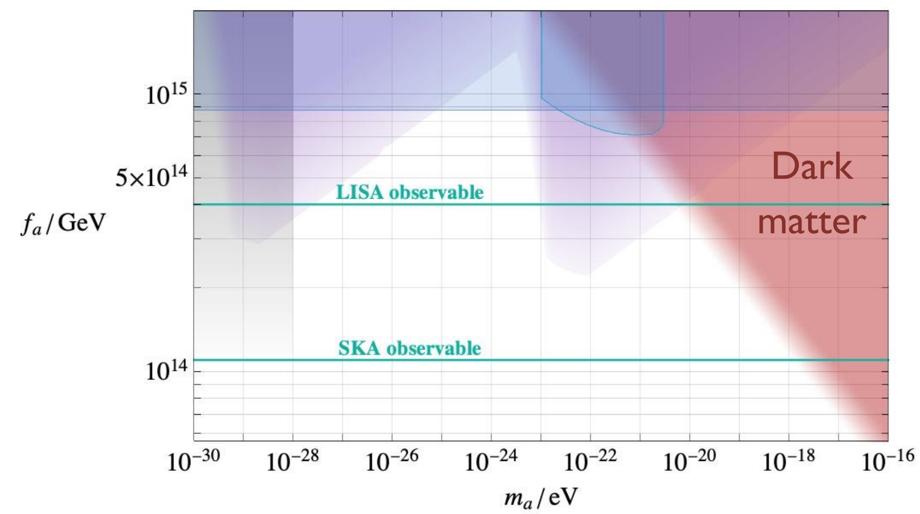
$$\frac{\Omega_a}{\Omega_{\rm DM}} \simeq \frac{\beta\mathcal{A}_d m_a f_a^2}{T_{\rm eq}H_d^{1/2}M_{\rm Pl}^{3/2}} \simeq 2\left(\frac{\mathcal{A}_d}{20}\right)\left(\frac{m_a}{H_d}\right)^{1/2}\left(\frac{f_a}{10^{12}\ \ {\rm GeV}}\right)^2\left(\frac{m_a}{10^{-6}\ \ {\rm eV}}\right)^{1/2}$$

Dark matter

Most axions emitted during scaling non-relativistic today

$$n_a^{\rm st} \equiv \int \frac{dk}{\omega_k} \underbrace{\frac{\partial \rho_a}{\partial k}}_{\uparrow}$$

$$\int dt' \frac{\partial \Gamma'}{\partial k} \left(\frac{R'}{R}\right)^3 \qquad \sim \frac{\xi\mu}{t^3}$$



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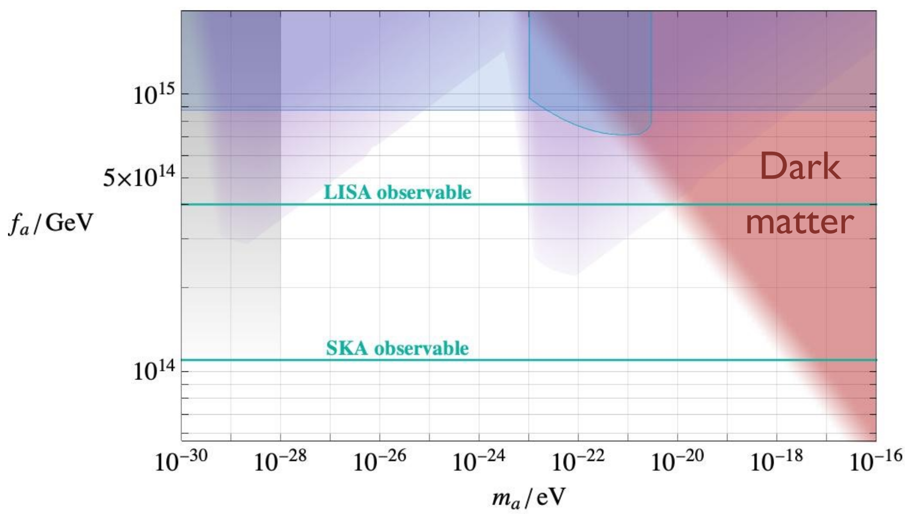


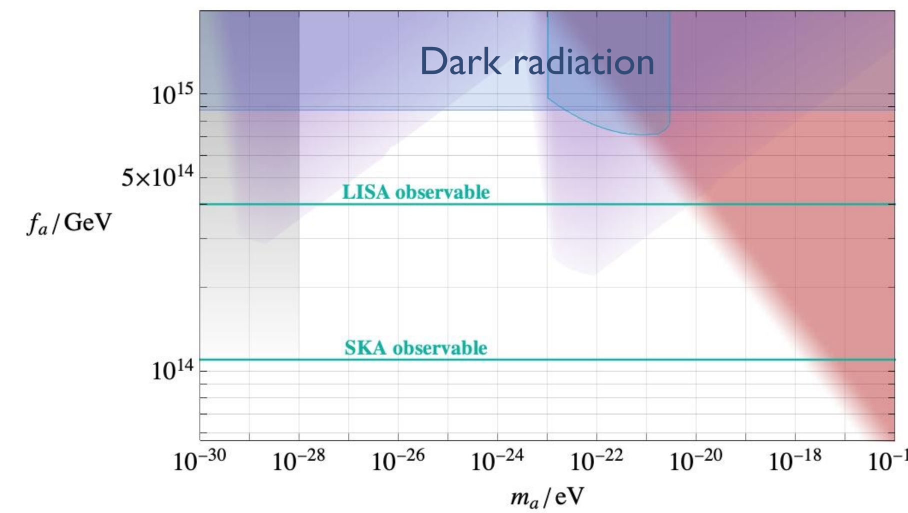
$$\boxed{\Omega_a^{\rm st} \simeq 0.1 \left(\frac{\xi_\star \log_\star}{3\times 10^3}\right) \left(\frac{f_a}{10^{14}\,{\rm GeV}}\right)^2 \left(\frac{m_a}{10^{-18}\,{\rm eV}}\right)^{\frac{1}{2}} \left(\frac{10}{x_{0,a}}\right) \left(\frac{3.5}{g_\star(T_\star)}\right)^{\frac{1}{4}}}$$

Pre-inflationary misalignment: $\quad \Omega^{\rm mis}_{\theta_0=1} \simeq 10^{-5} \left(\frac{f_a}{10^{14}\ {\rm GeV}}\right)^2 \left(\frac{m_a}{10^{-18}\ {\rm eV}}\right)^{\frac{1}{2}}$

Dark radiation

BBN happens when $H > m_a$

$$\rho_a = \int^t dt' \overset{\frac{\xi\mu}{t^3}}{\boxed{\Gamma'_a}} \left(\frac{R'}{R}\right)^4 \qquad \simeq H^2 f_a^2 \log^3$$



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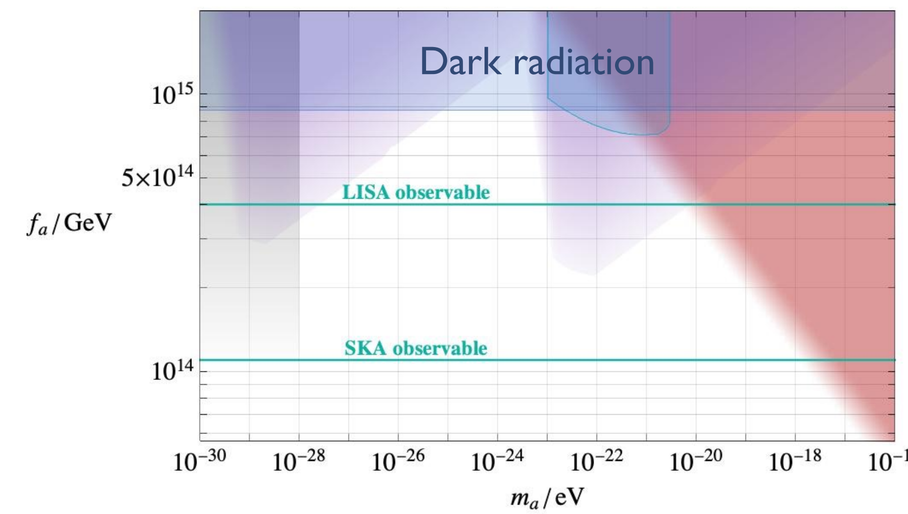


$$\Delta N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}} \frac{\rho_a}{\rho_\gamma} \overset{\text{at BBN}}{=} 0.6\left(\frac{\overset{\xi = c_1 \log}{\boxed{c_1}}}{0.24}\right)\left(\frac{f_a}{10^{15}\ {\rm GeV}}\right)^2 \left(\frac{\log(\frac{m_r}{H_{\rm BBN}})}{90}\right)^3 < 0.46$$

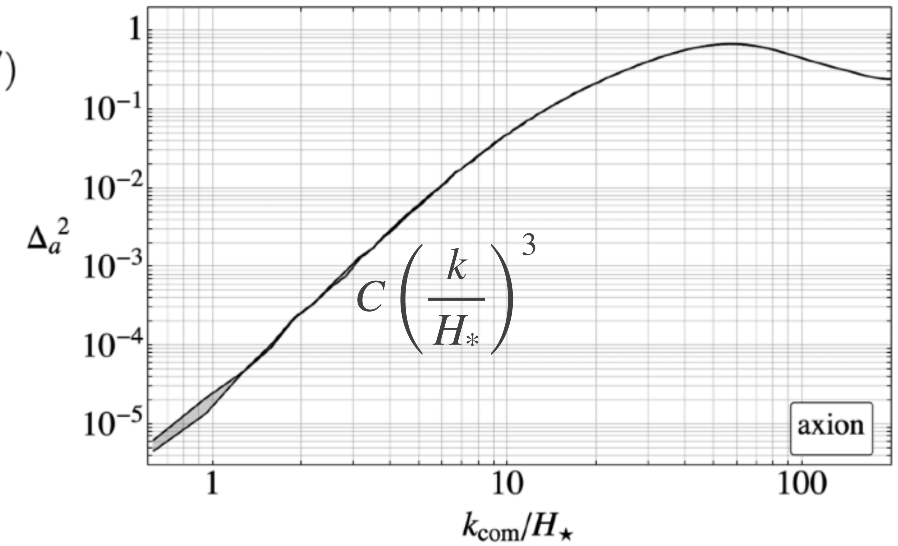
[Planck 2018]

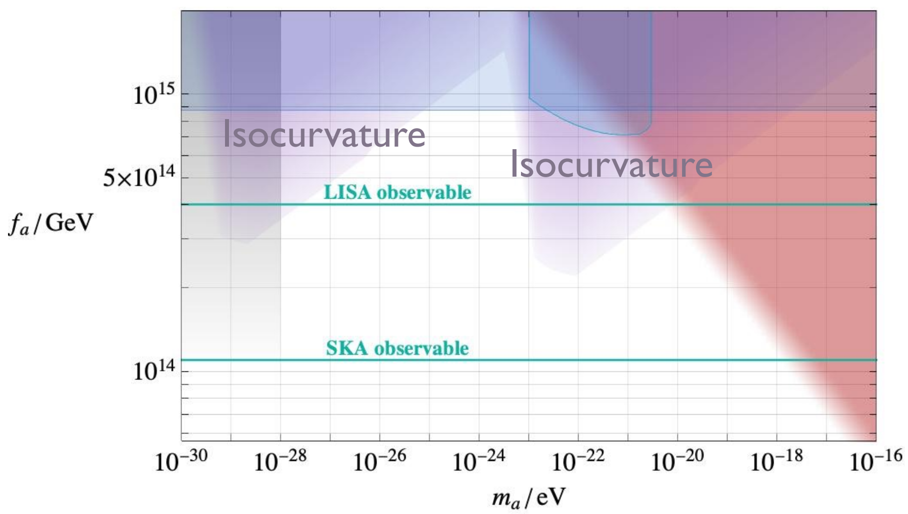
Isocurvature

The dark matter axions are inhomogeneous

$$\delta_a(x) \equiv (\rho_a(x) - \langle\rho_a\rangle)/\langle\rho_a\rangle$$

$$\langle\tilde{\delta}_a(\mathbf{k})\tilde{\delta}_a(\mathbf{k}')\rangle = \frac{2\pi^2}{k^3}\Delta_a^2(k)\delta^3(\mathbf{k}-\mathbf{k}')$$





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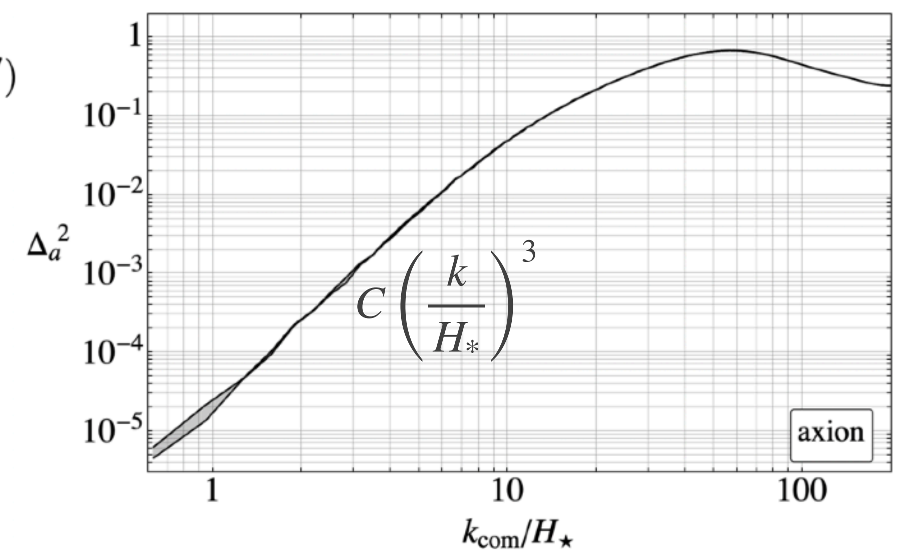
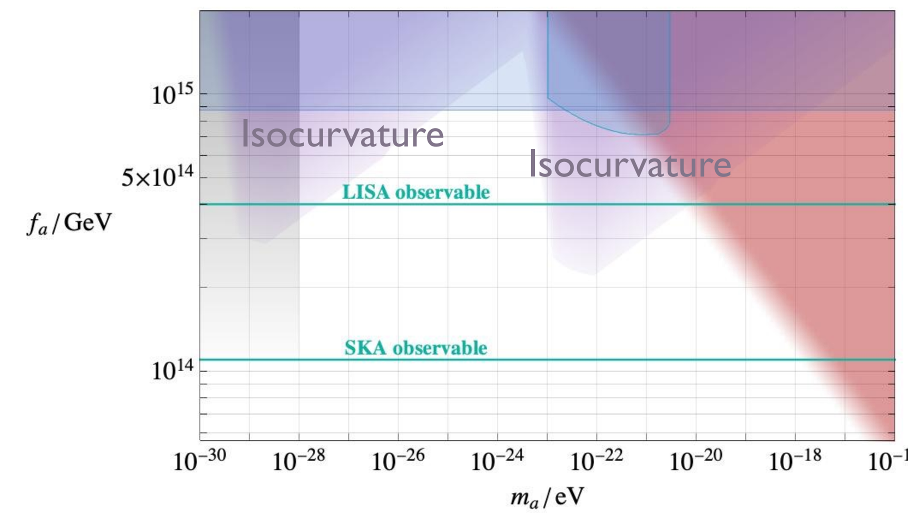


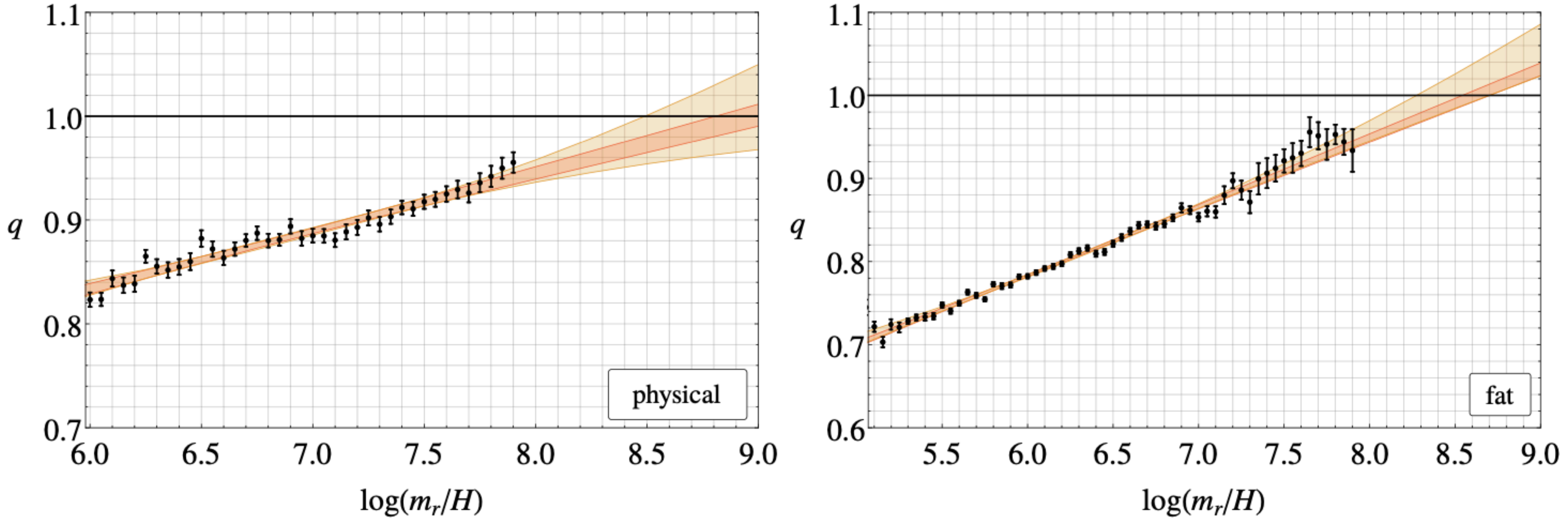


- $f_{\rm iso} < 0.64$ at $k = k_{\rm CMB}$ [Feix et al. '20,...]

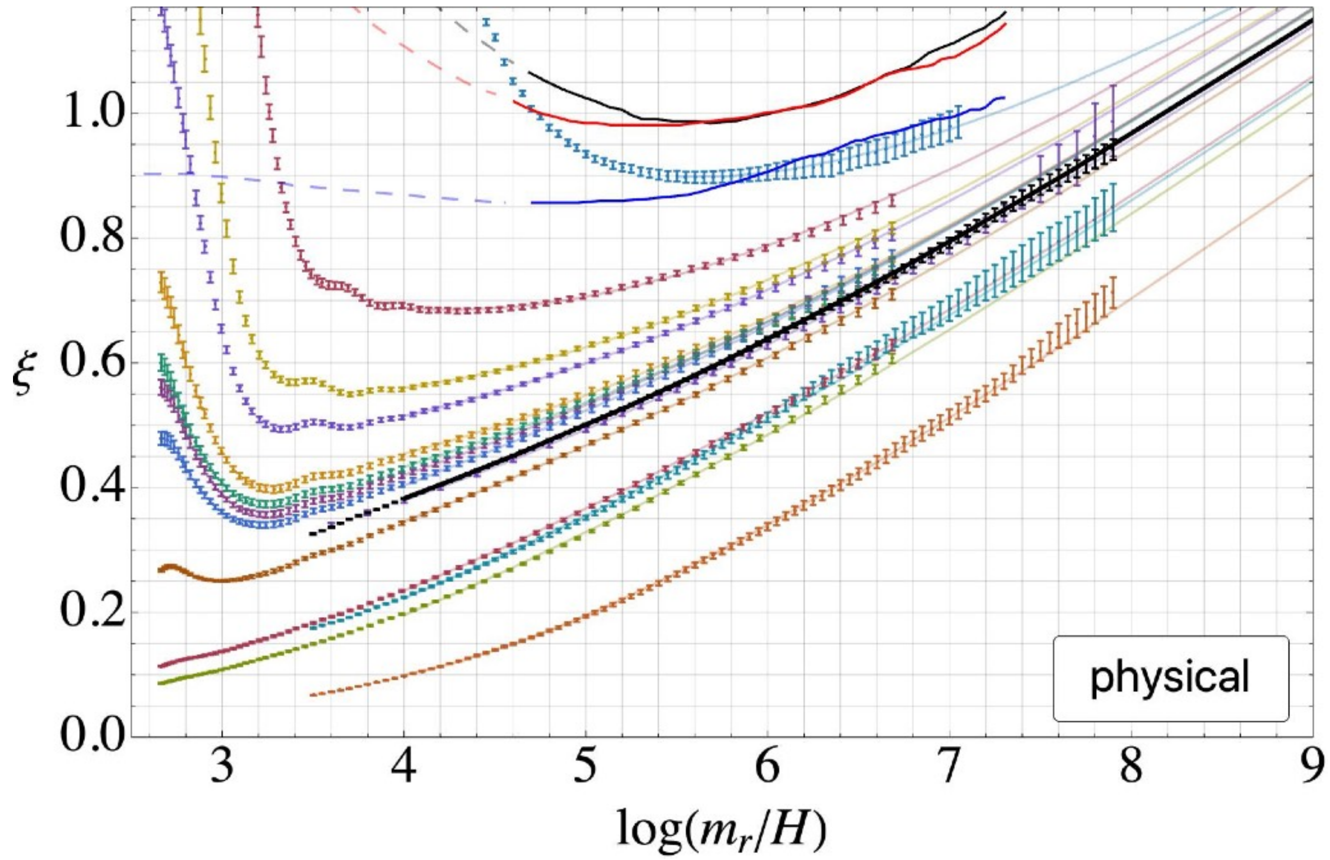
$$f_{\rm iso} = \frac{\Omega_a^{\rm st}}{\Omega_{\rm DM}}\sqrt{\frac{C\,k_{\rm CMB}^3}{A_s k_\star^3}}$$
$$\simeq 0.2\left(\frac{\xi_\star \log_\star}{3\times10^3}\right)\left(\frac{f_a}{5\times10^{14}\ \mathrm{GeV}}\right)^2\left(\frac{m_a}{10^{-28}\ \mathrm{eV}}\right)^{-1/4}\left(\frac{C}{2\times10^{-5}}\right)^{1/2}$$

- $f_{\rm iso} < 0.004$ at $k = k_{\rm L\alpha} = 10{\rm Mpc}^{-1}$ [Murgia et al. '19,...]

Axion emission spectrum



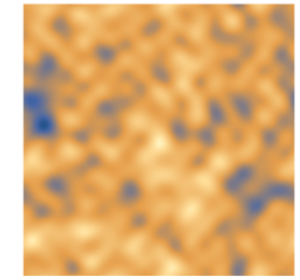
String density



Increase consistent with:

- Large string densities for NG networks
- Large density when tension artificially boosted
- Log increase is a good fit over multiple e-foldings
- Results from altering the string thickness to keep constant log

Post-inflationary



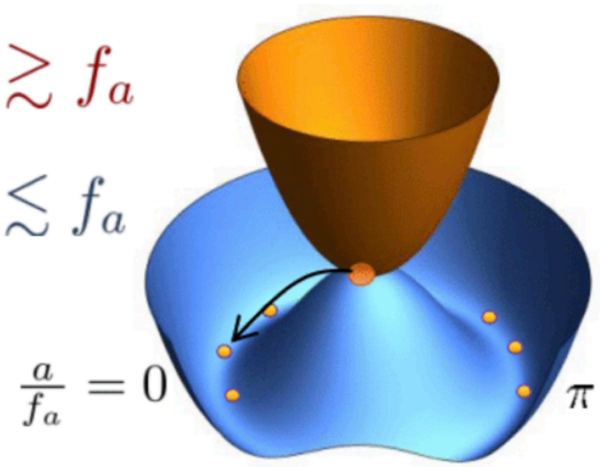
- Hubble parameter during inflation

$$\langle \sigma_a^2 \rangle = \left(\frac{H_I}{2\pi}\right)^2 \qquad\qquad \frac{H_I}{2\pi} > f_a$$

- Finite temperature

$T \gtrsim f_a$

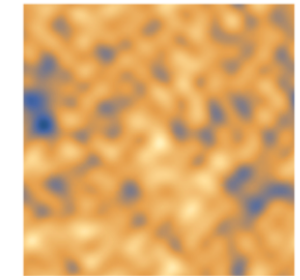
$T \lesssim f_a$



- Coupling to the inflation

$$V_\phi \supset g\varphi^2|\phi|^2 \xrightarrow{\text{during inflation}} \underbrace{g\langle\varphi\rangle^2}_{\textbf{effective mass for } \phi}|\phi|^2$$

Post-inflationary



- Hubble parameter during inflation

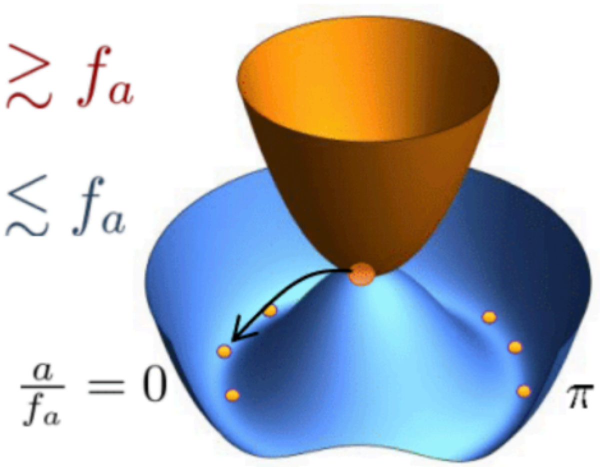
$$\langle \sigma_a^2 \rangle = \left(\frac{H_I}{2\pi}\right)^2 \qquad \frac{H_I}{2\pi} > f_a$$

$$\boxed{\frac{H_I}{2\pi} < 9.6 \times 10^{12}\ \text{GeV}}$$

- Finite temperature

$$T \gtrsim f_a$$

$$T \lesssim f_a$$



$$T_{\rm RH} \lesssim \left(\frac{H_I}{H_{\rm max}}\right)^{1/2} 10^{14}\ \text{GeV}$$

$$T_{\rm early} \lesssim \left(\frac{H_I}{H_{\rm max}}\right)^{1/2} 10^{15}\ \text{GeV}$$

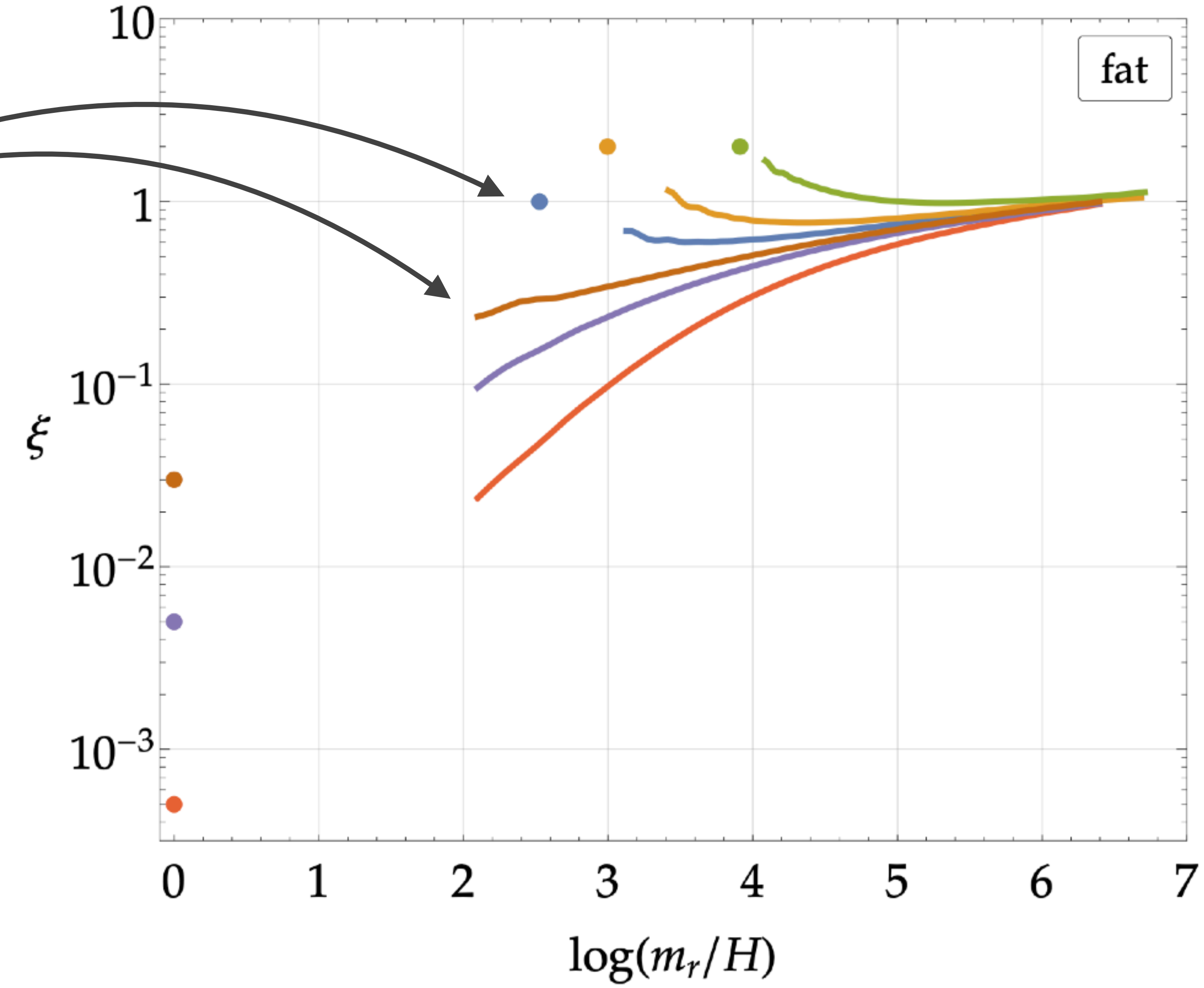
- Coupling to the inflation

$$V_\phi \supset g\varphi^2|\phi|^2 \xrightarrow{\text{during inflation}} \underbrace{g\langle\varphi\rangle^2}_{\text{effective mass for } \phi}|\phi|^2$$

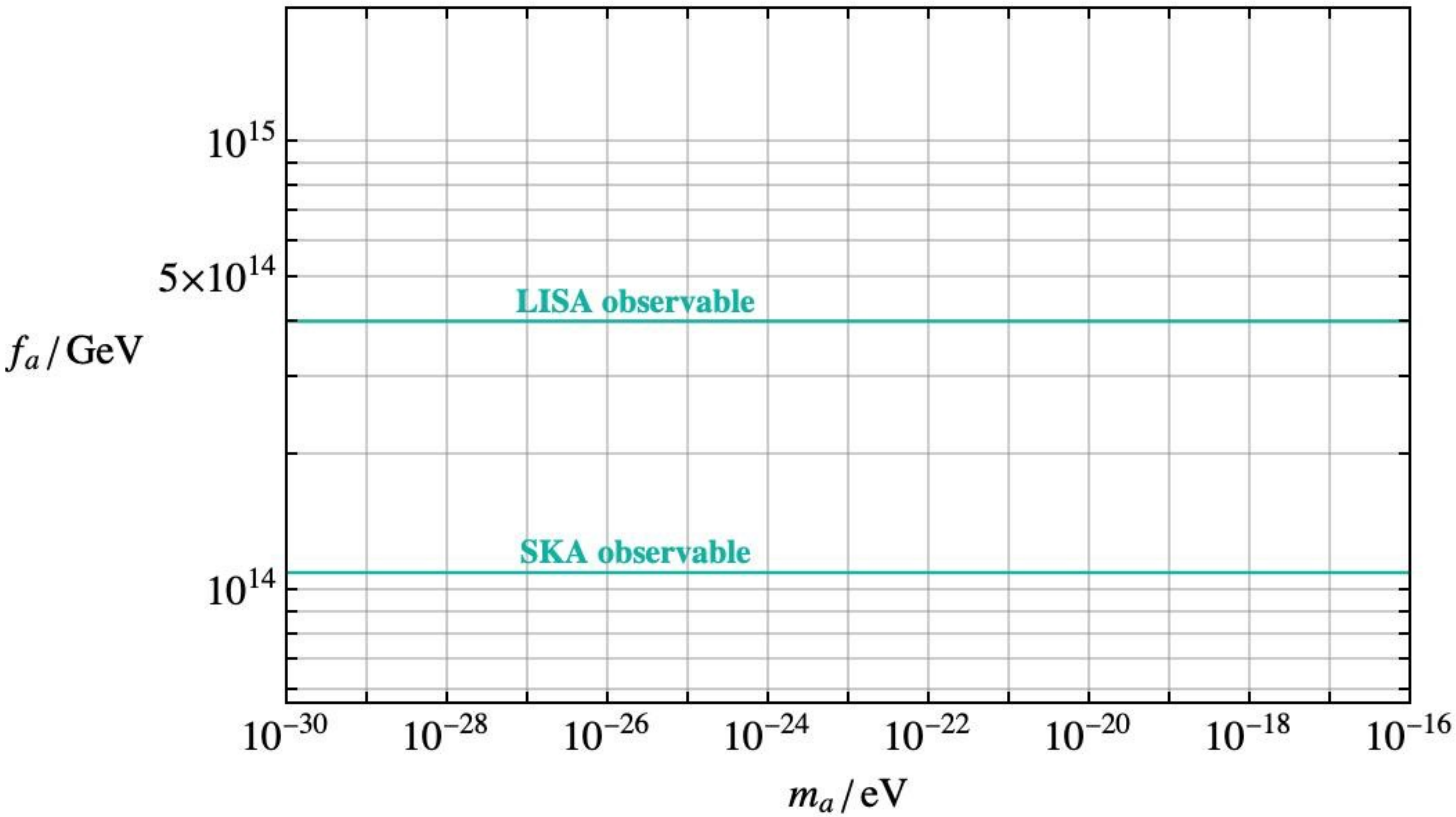
$$\boxed{g\left\langle \varphi^2 \right\rangle \gtrsim f_a^2}$$

Scaling

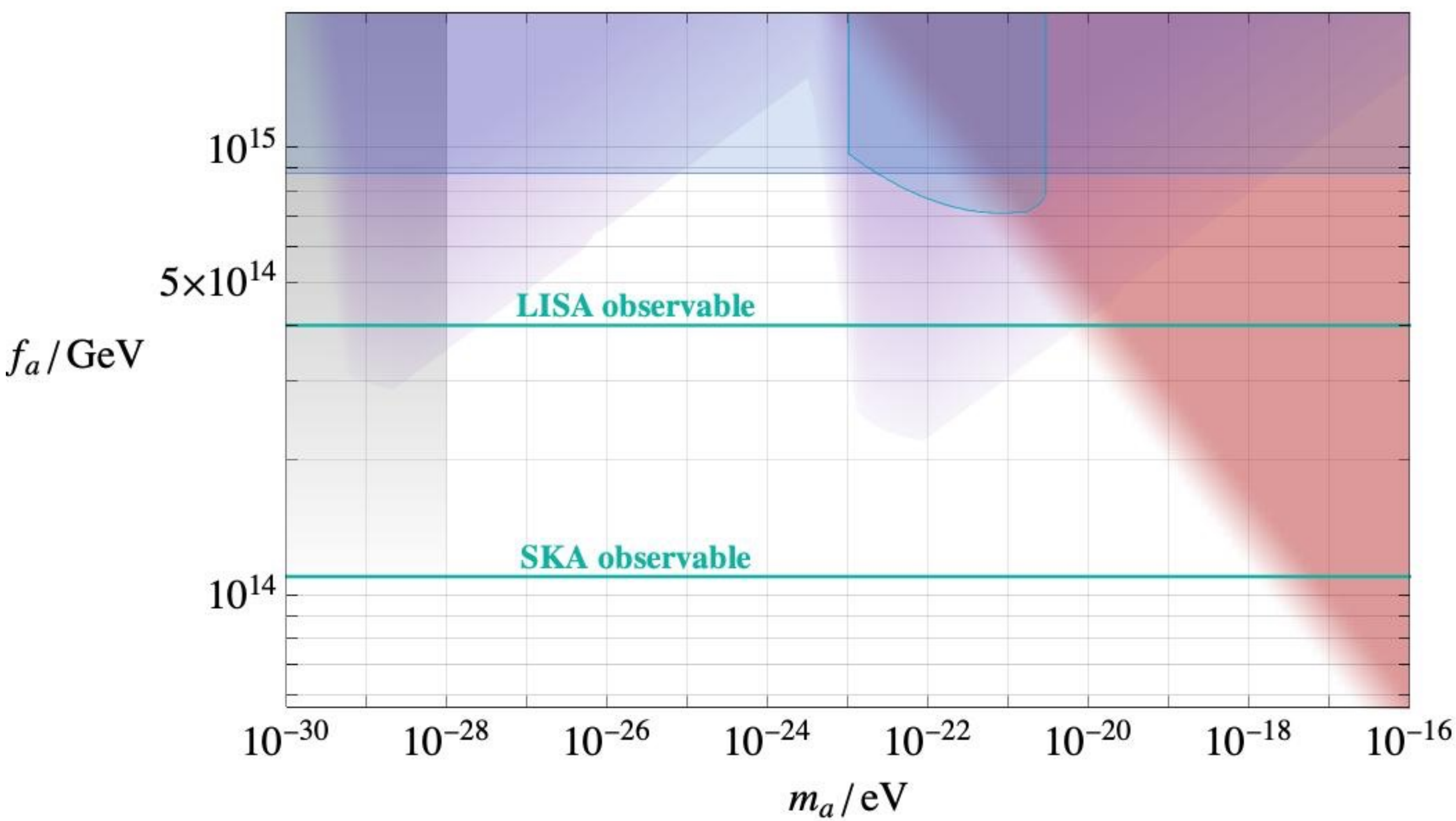
Different
initial conditions



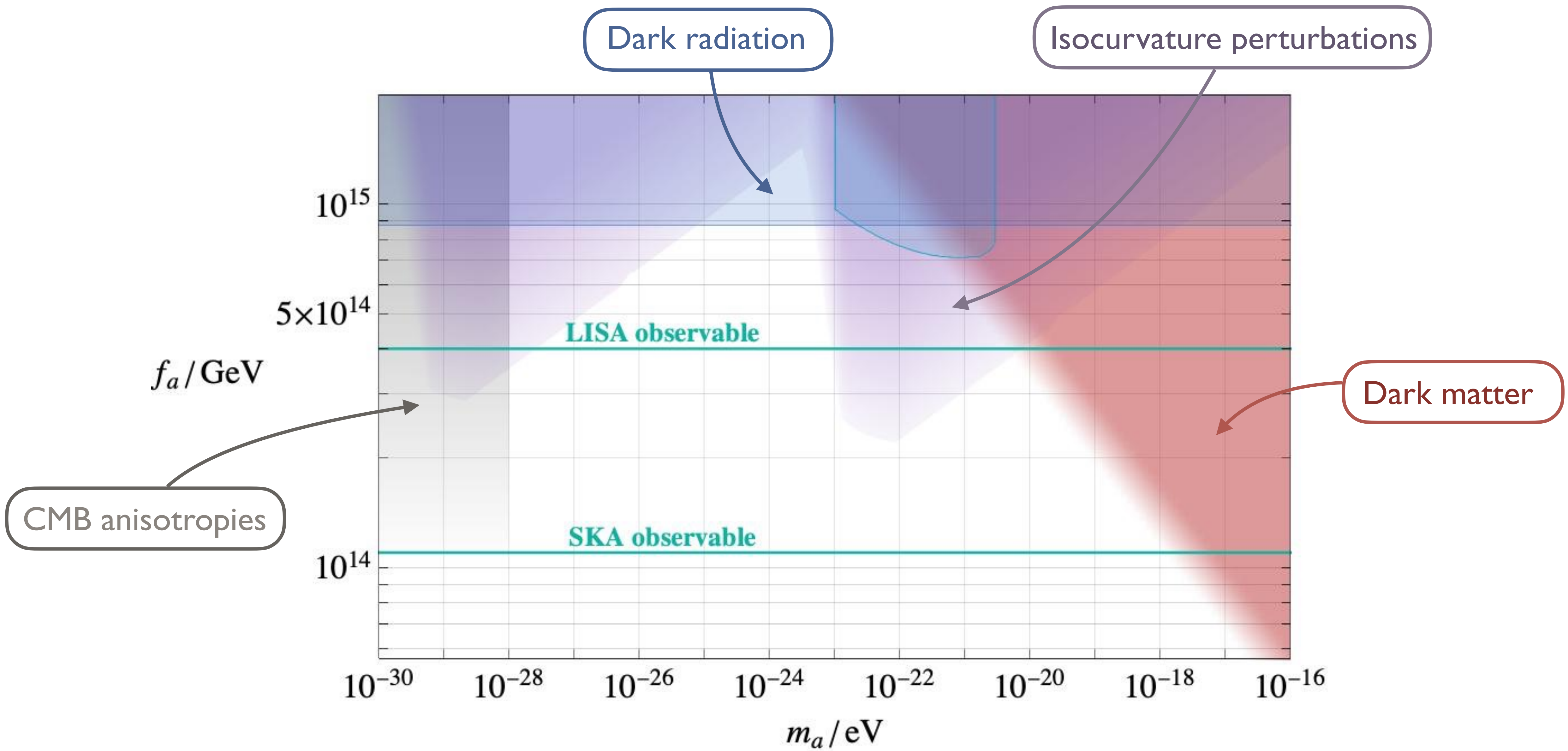
Other constraints



Other constraints

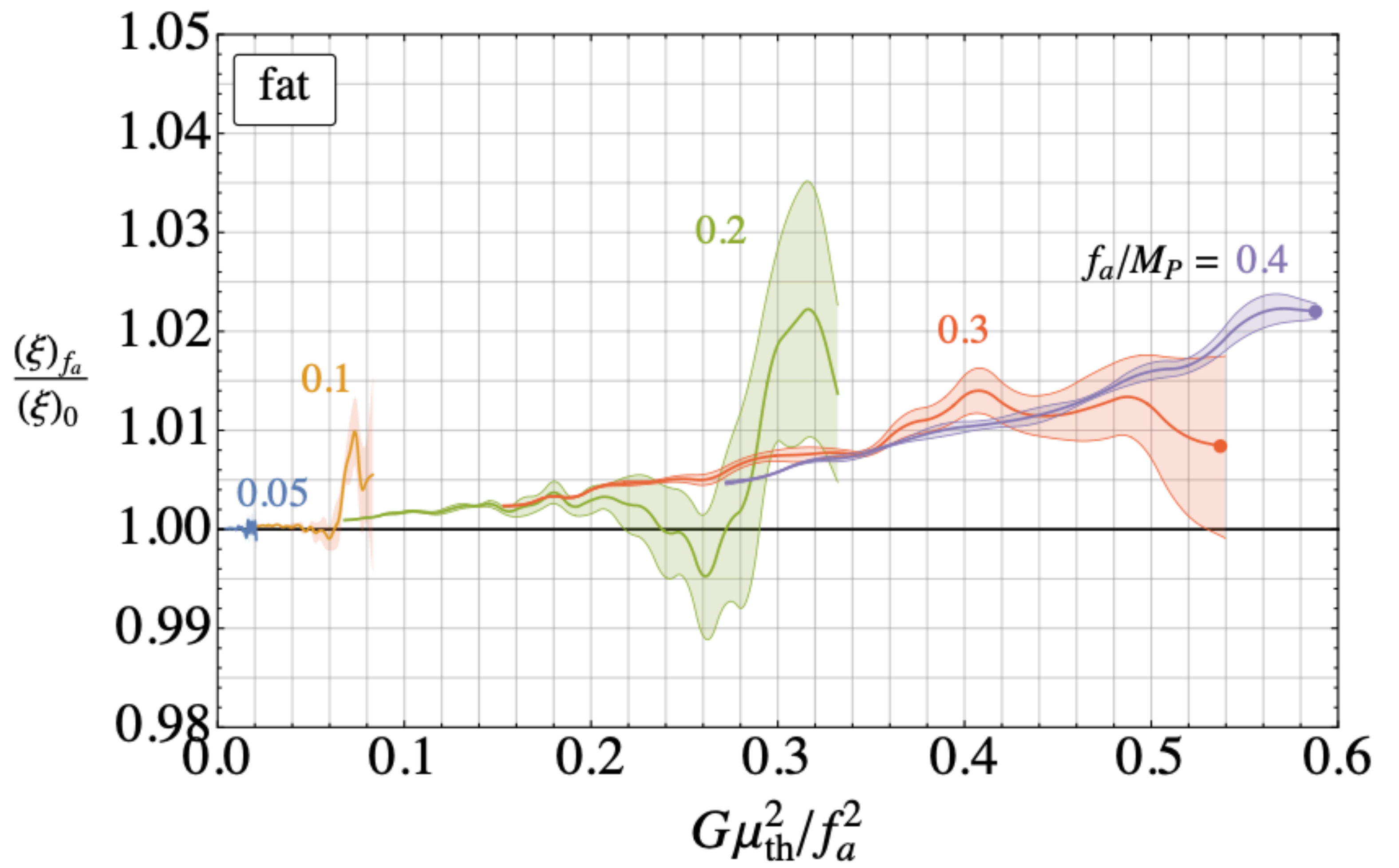


Other constraints



Backreaction

1. **Energy emitted into GWs**
2. Momentum distribution



Negligible for:

$$G\mu_{\rm eff}^2/f_a^2 \lesssim 0.5 \quad \longrightarrow \quad \log \lesssim M_{\rm P}/f_a$$