

**IBS-ICTP 7th MultiDark workshop**

**The meso-inflationary QCD axion**

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**with Michele Redi**

[Peccei-Quinn '77  
Weinberg-Wilczek '78]

“the axion solves more problems than it causes” [cit??]

$$\left(\theta - \frac{a}{f_a}\right) \frac{\alpha_s}{8\pi} G \tilde{G} \quad \circ \text{ strong CP conservation}$$

$$V(a) = \frac{1}{2} m_a^2(T) a^2 + \dots \quad \circ \text{ behaves as cold dark matter}$$

**really is that good?**

## ingredients:

- **Peccei-Quinn potential**  
(must be only broken by QCD anomaly!)

$$V = \lambda \left( |\Phi|^2 - \frac{f_a^2}{2} \right)^2$$

- **Possibly avoid stable domain walls**  
(KSVZ-type with quarks U and D)

$$N_{\text{DW}} = 1 \quad f_a = f_a$$

- **Plagued by isocurvatures for  $H_I > f_a$**   
(also, 'anthropic' selection of initial field value?)

- **Relic abundance from the network?**  
(out of experimental - admx - window?)

**it's sensitive to the UV. what changes if we twist the UV?**

# cosmology of the qcd axion

axion relic abundance depends on cosmological history  
with **two main scenarios** determined by the size of  $f_a$

- pre-inflationary scenario

relic abundance from **misalignment**

inflation makes the axion field homogeneous

**incalculable** initial field value  $a_0 = \theta_0 f_a$

large (calculable) quantum fluctuations: isocurvatures

$$f_a > \max\left[\frac{H_I}{2\pi}, T_{\max}\right]$$

Peccei-Quinn broken when the scale factor:  $a_{\text{PQ}} \ll a_e \exp(-60)$



# cosmology of the qcd axion

axion relic abundance depends on cosmological history  
with **two main scenarios** determined by the size of  $f_a$

- post-inflationary scenario

relic abundance **uncertain**: **string/wall-network**

domain wall **problem**, only KSVZ-type model

large (?) power at small scales, **miniclusters**

**no isocurvature** at cosmological scales

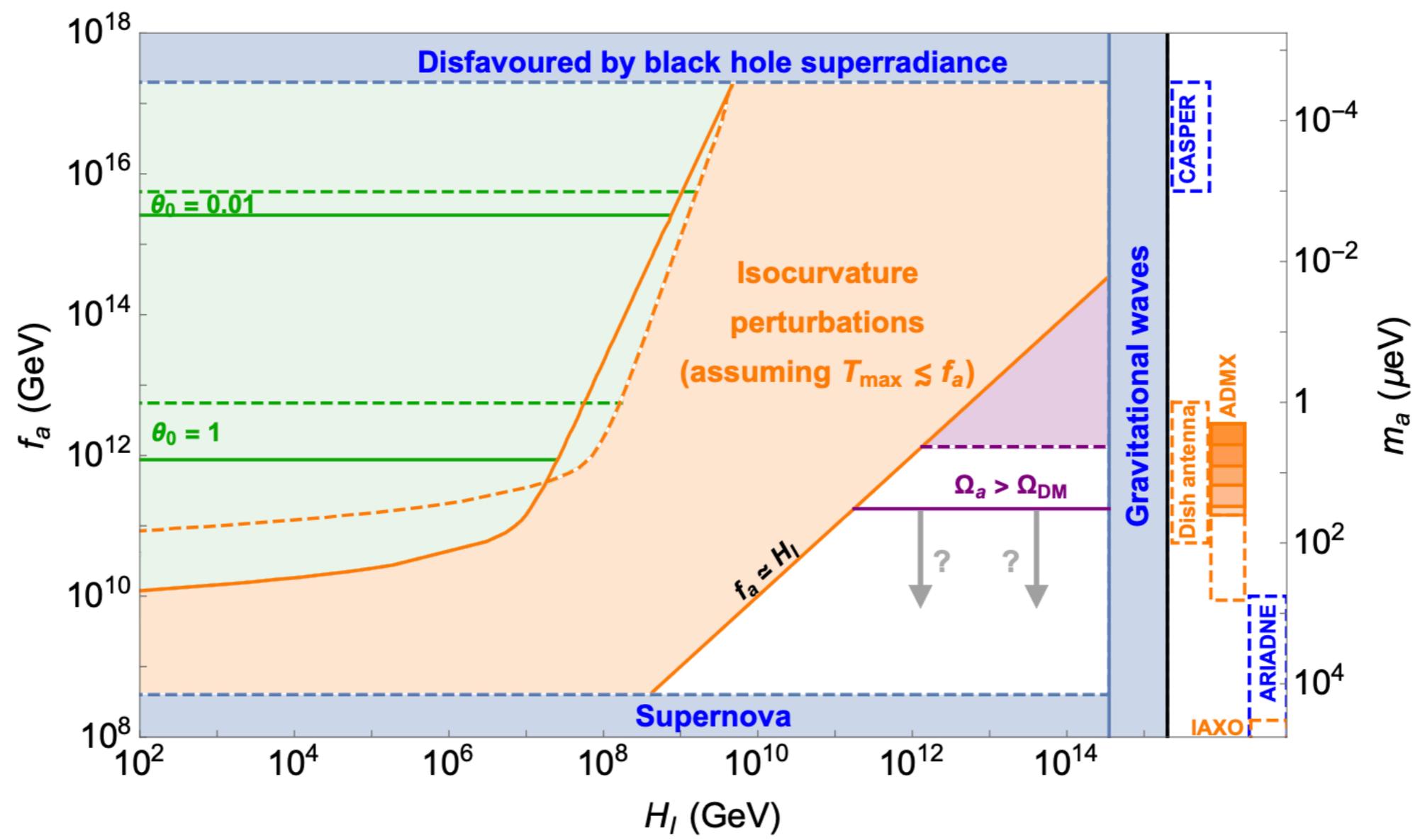
$$f_a < \max\left[\frac{H_I}{2\pi}, T_{\max}\right]$$

Peccei-Quinn broken when the scale factor:

$$T(a_{\text{PQ}}) \approx f_a$$

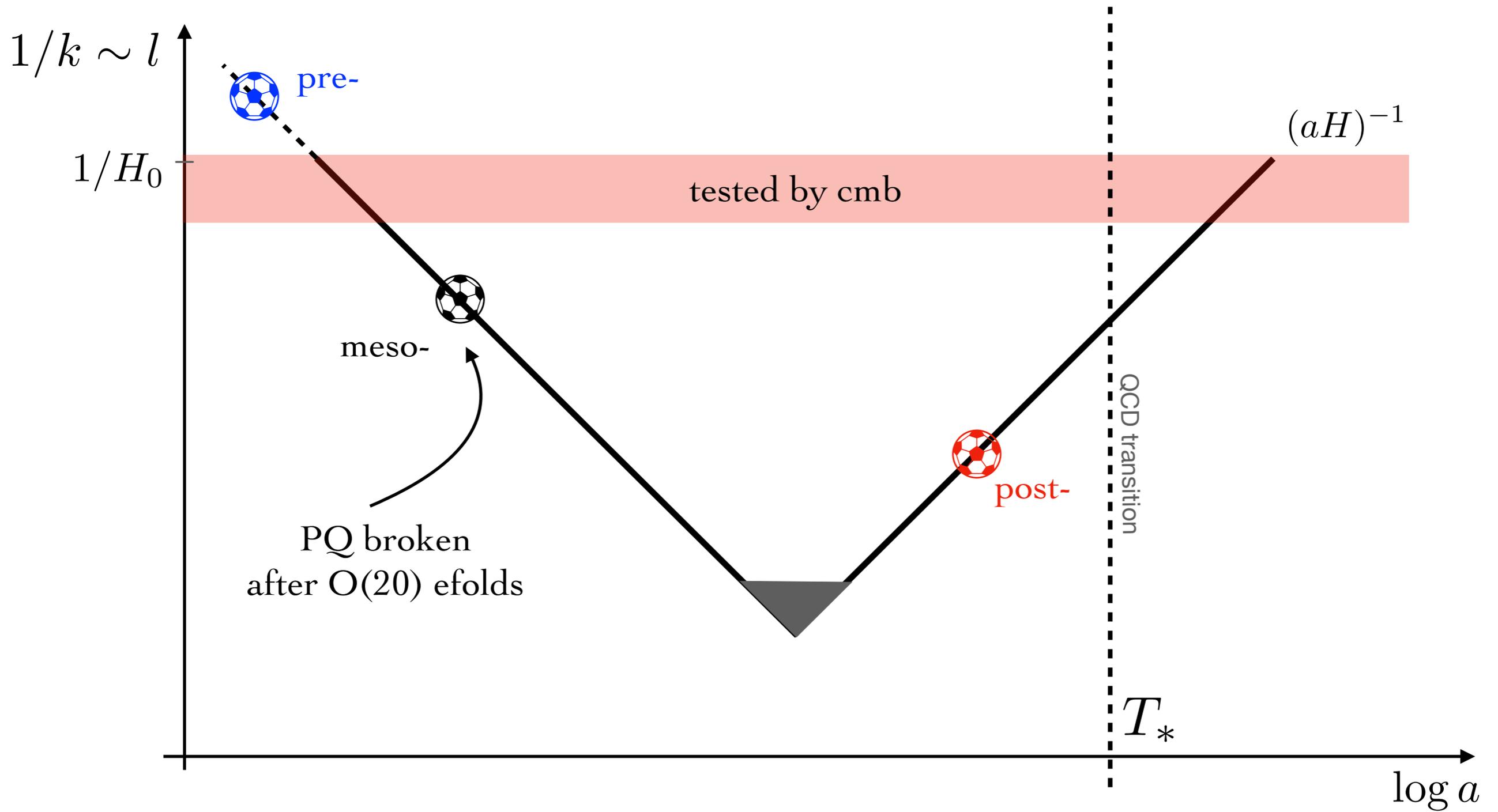


# cosmology of the qcd axion



[Grilli di Cortona, Hardy, Villadoro]

# meso-inflationary qcd axion



# meso-inflationary qcd axion

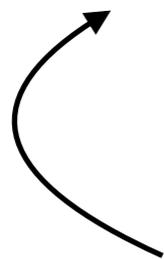
Peccei-Quinn is broken after the start of the last 60 e-folds of inflation

- meso-inflationary scenario

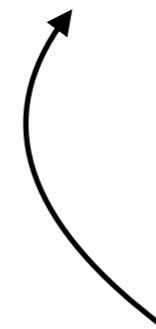
no isocurvatures at large scales (no axion initially)

large quantum fluctuations: new contribution to abundance and power spectrum

potentially long-lived string/wall network



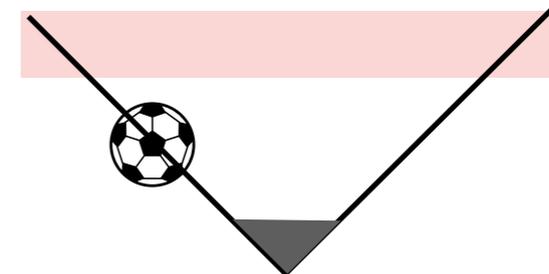
**more difficult  
to compute**



**calculable**

axion emerges at PQ transition,  
homogenous today on scales

$$d \sim \frac{1}{k_{\text{PQ}}} \equiv \frac{1}{H_0} \exp[-N_{\text{PQ}}]$$



## meso-inflationary realizations

Peccei-Quinn mass term modified by coupling to other dynamics

$$V = \lambda \left( |\Phi|^2 - \frac{f_a^2}{2} \right)^2 + \mathcal{O}|\Phi|^2$$

operator  $\mathcal{O}$  has a vev during inflation

- symmetry restoration for  $f_a > H_I/(2\pi)$

$$\langle \mathcal{O} \rangle \approx \lambda f_a^2$$

for example coupling to inflaton

[Linde '91]

field  $\Phi$  heavy before the transition, no fluctuations  
unlikely to justify  $H_I \approx f_a$

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$$V = \lambda \left( |\Phi|^2 - \frac{f_a^2}{2} \right)^2 + \mathcal{O}|\Phi|^2$$

operator  $\mathcal{O}$  has a vev during inflation

- symmetry breaking starting with  $f_a < H_I/(2\pi)$

requires drop in Hubble during PQ transition (**non trivial inflationary scenario**)

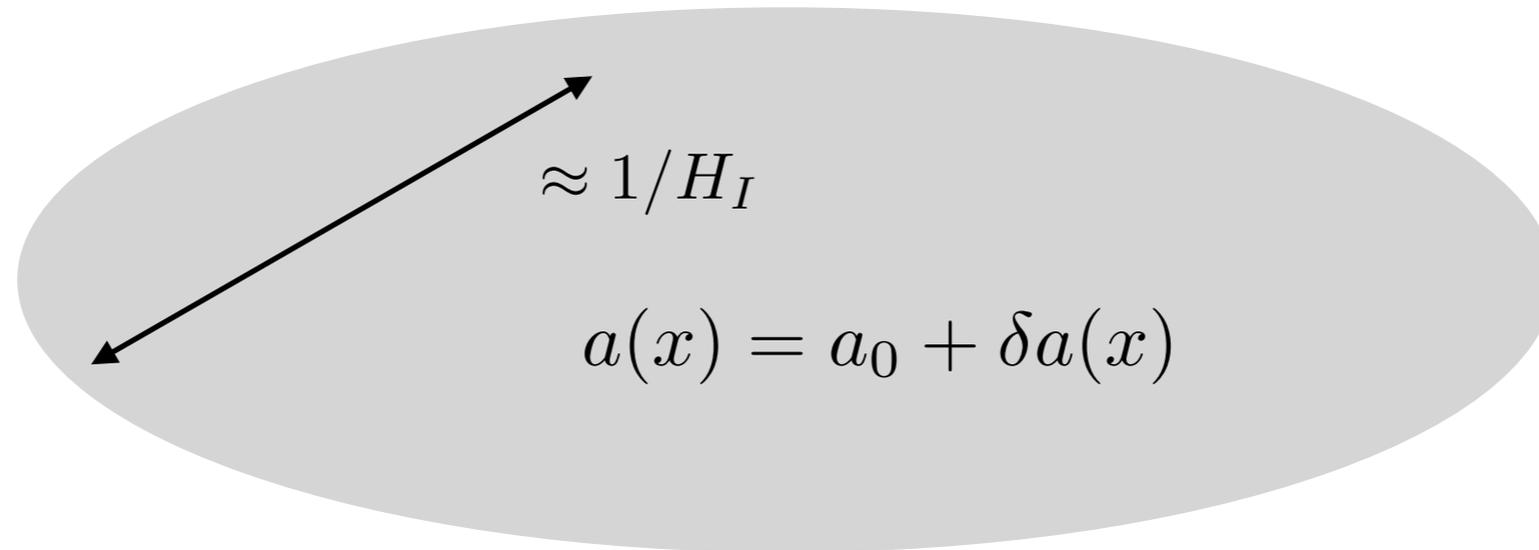
can explain parameter space  $H_I \approx f_a$

special choice by conformal coupling to Ricci  $\mathcal{O} = -\frac{1}{6}R$

[i will not give more details than these, let's see what are the consequences of the scenario]

## at the PQ phase transition during inflation

$$k_{\text{PQ}} \approx a_{\text{PQ}} H_I$$



field homogenous on these patches

$$\langle a_0^2 \rangle \approx 2.15 f_a^2$$

$\Omega_{a,\text{mis.}}$

unconstrained quantum fluctuations

$$\langle \delta a \delta a \rangle \approx \frac{H_I^2}{(2\pi)^2} \quad \text{for } k \gtrsim k_{\text{PQ}}$$

$\Omega_{a,\text{inf.}}$

one string per patch, that soon leaves the horizon

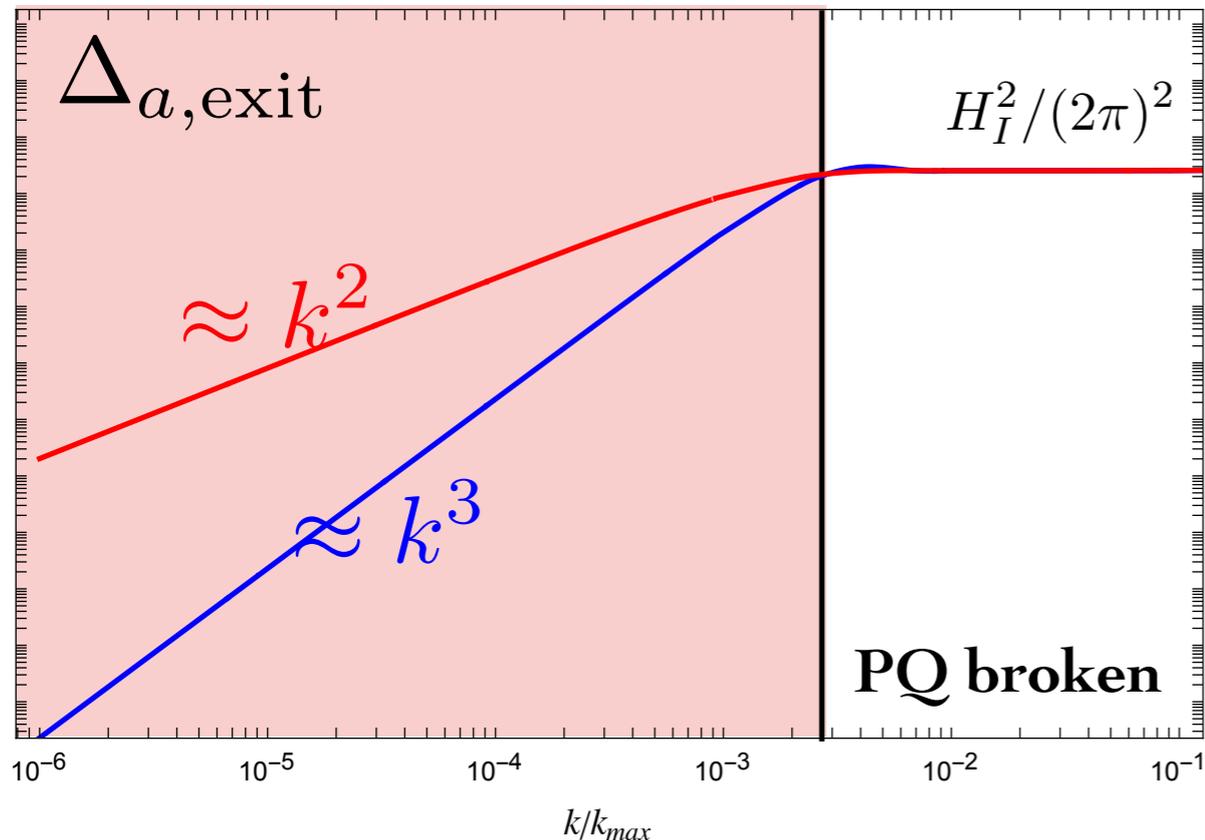
$\Omega_{a,\text{net.}}$

**need to estimate how much energy is stored in the axion field in these three contribution**

# axion abundance from inflationary fluctuations

mechanism of inflationary production of minimally coupled scalars  
(scalar/tensor perturbations, production of Stueckelberg vector from inflation...)

$$\Delta_a(\eta_e, k) = \frac{k^3}{2\pi^2} \int d^3x e^{-i\vec{k}\cdot\vec{x}} \langle a(\eta_e, \vec{x}) a(\eta_e, 0) \rangle = \frac{H_I^2}{4\pi^2} \min\left[1, \frac{k^n}{k_{\text{PQ}}^n}\right].$$

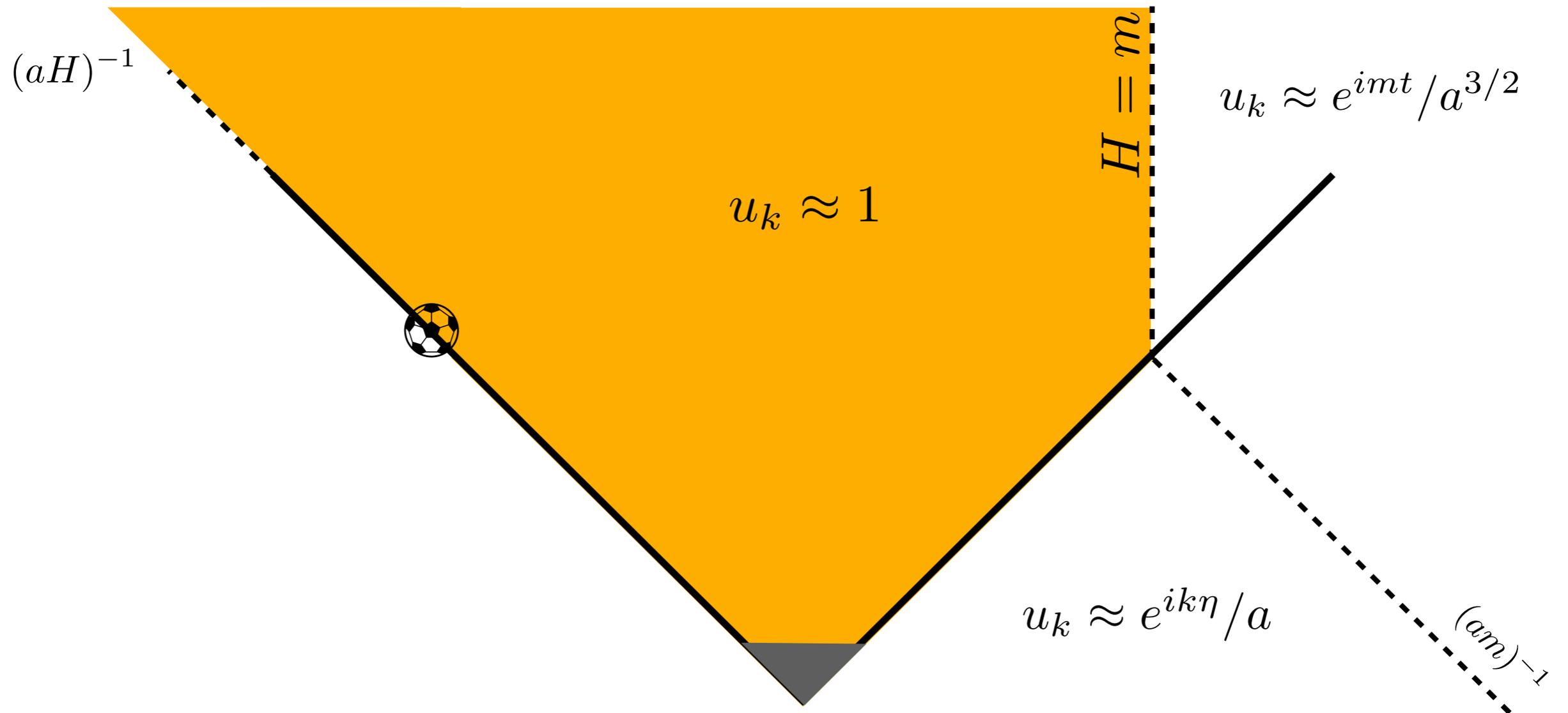


○ classical evolution after horizon exit

$$\frac{d\rho_a^{\text{inf}}}{d \log k} = \frac{\Delta_a(\eta_e, k)}{2} [g^{00} |\dot{u}_k|^2 - (g^{ij} k_i k_j - m_a^2) |u_k|^2]$$

$$u_k|_{\text{exit}} = 1, \quad \dot{u}_k|_{\text{exit}} = 0$$

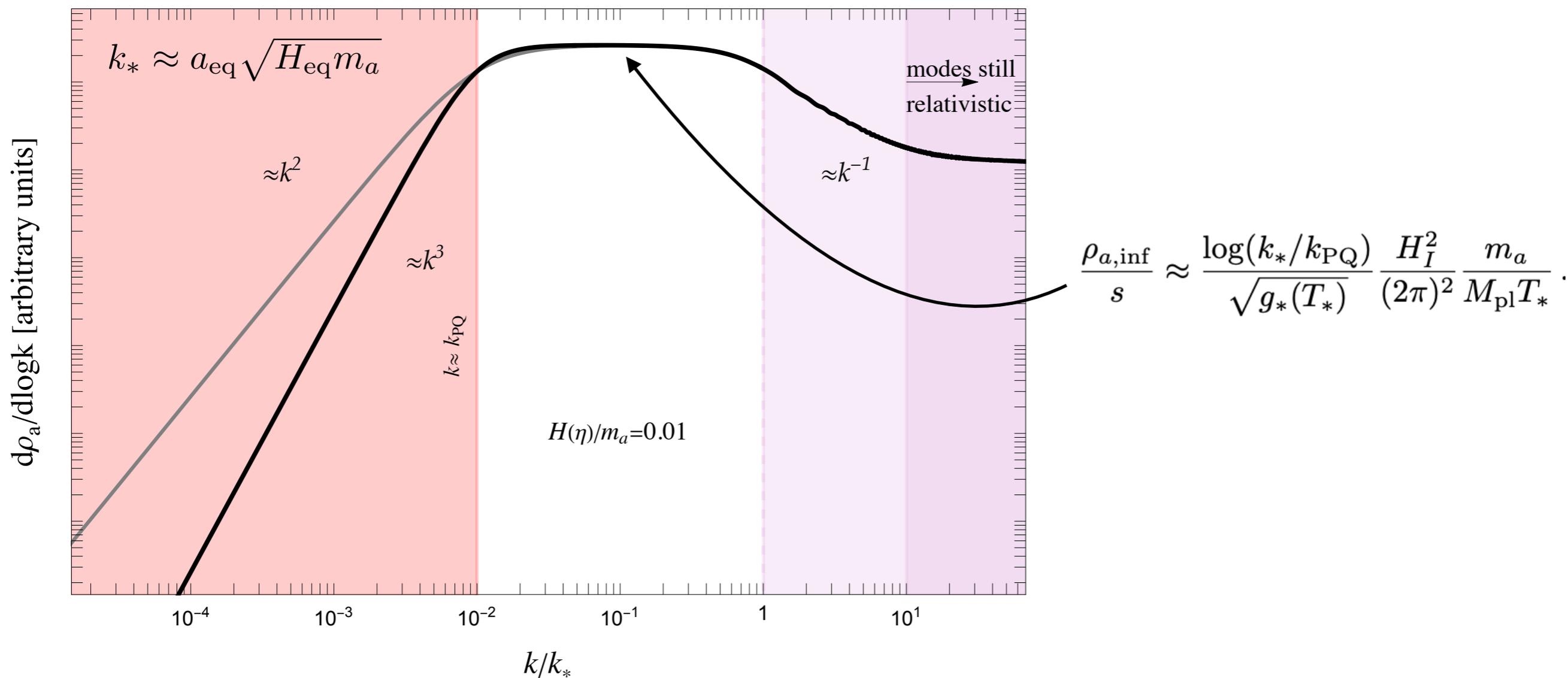
# axion abundance from inflationary fluctuations



classical evolution determines the spectral shape of the energy density

[as in Graham, Mardon, Rajendran]

# axion abundance from inflationary fluctuations



sizeable contribution to relic abundance from quantum fluctuations

$$\frac{\Omega_a^{\text{inf}}}{\Omega_a^{\text{mis}}} \approx \log\left(\frac{k_*}{k_{\text{PQ}}}\right) \times \left(\frac{H_I}{2\pi f_a}\right)^2$$

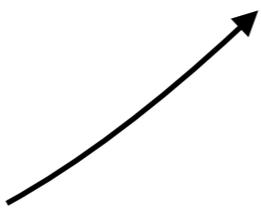
# isocurvature power spectrum

isocurvature power spectrum **calculable** knowing as a function of time  
**it is confined to short scales**

$$\left\langle \frac{\delta\rho_a}{\bar{\rho}_a} \frac{\delta\rho_a}{\bar{\rho}_a} \right\rangle \Big|_{\text{iso}} \approx \frac{\langle a(\eta, x)a(\eta, 0) \rangle^2 + 4a_0^2 \langle a(\eta, x)a(\eta, 0) \rangle}{(a_0^2 + \langle a(\eta)^2 \rangle)^2}$$

$$\Delta_{\delta_a}^{\text{iso}}(\eta, k) = \frac{\Omega_{a,\text{inf}}^2}{\Omega_a^2 \langle a^2 \rangle^2} \frac{k^3}{4\pi} \int d^3q \frac{\Delta_a(\eta, q) \Delta_a(\eta, |\vec{k} - \vec{q}|)}{q^3 |\vec{k} - \vec{q}|^3} + 4 \frac{\Omega_{a,\text{inf}} \Omega_{a,\text{mis}}}{\Omega_a^2 \langle a^2 \rangle} \Delta_a(k)$$

new contribution  
(non-gaussian)

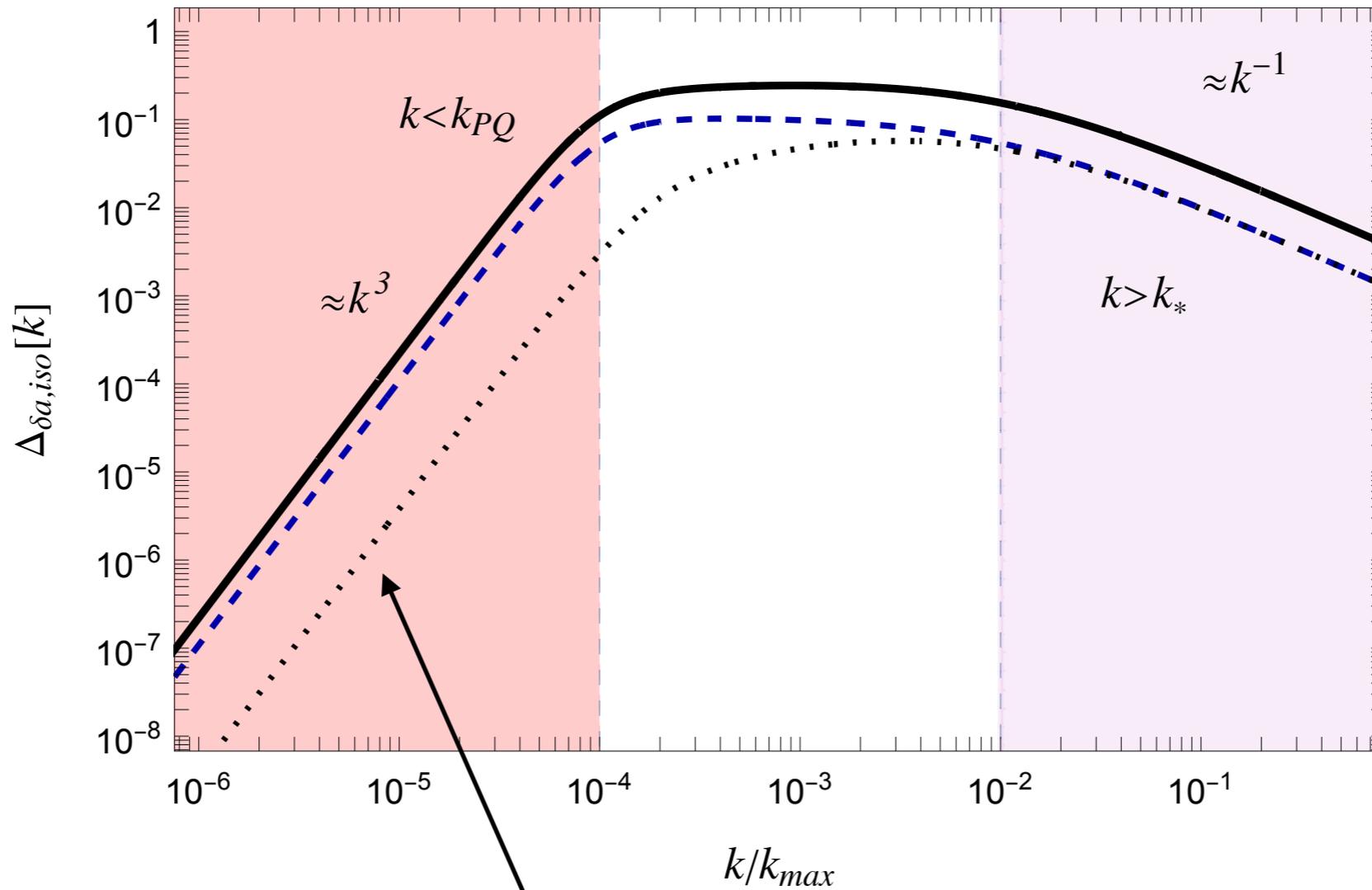


usual term for axion DM



[similar to dark photon from inflation]

# isocurvature power spectrum

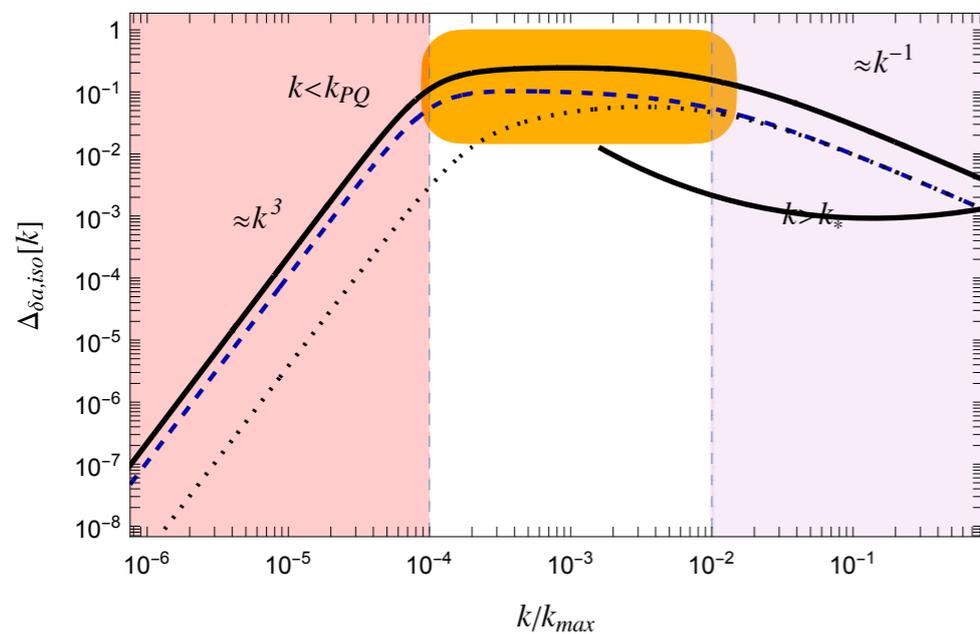


$$\Delta_{\delta_a}^{\text{iso}}(\eta, k) \approx \frac{\Omega_{a,\text{inf}}^2}{\Omega_a^2} \frac{k^3}{3 \log^2(k_*/k_{\text{PQ}}) k_{\text{PQ}}^3}$$

the pure quantum contribution  
(present also for no misalignment)  
**is universal and goes as  $k^3$**

# isocurvature power spectrum

the  $O(1)$  overdensity on the plateau leads to the formation of mini-halos and mini-clusters



$$M_{\text{cluster}} \approx 10^{-16} M_{\odot} (k_*/k)^3$$

[Gorghetto, Hardy et al]

○ also bounds from CMB, spectral distortions and structures

a generic isocurvature power spectrum  $\Delta_{\text{iso}} = A_{\text{iso}} (k/k_0)^3$   $k_0 = 0.05 \text{Mpc}^{-1}$

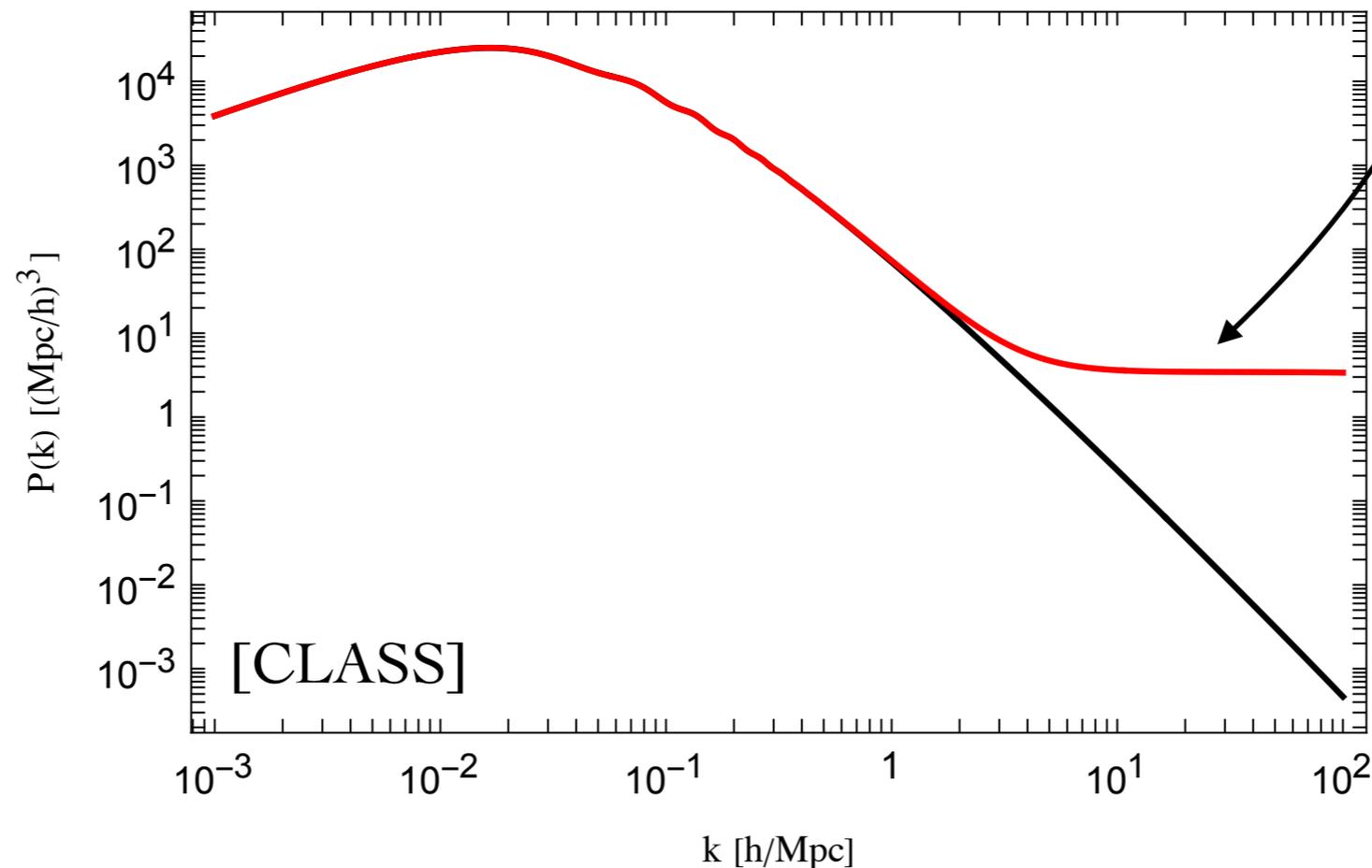
$$A_{\text{iso}}|_{\text{cmb}} \lesssim 0.8 \cdot 10^{-10} \quad A_{\text{iso}}|_{\text{lyman-}\alpha} \lesssim 10^{-14} \quad A_{\text{iso}}|_{\text{pixie}} \lesssim 10^{-11}$$

# [matter power spectrum]

the cdm linear matter power spectrum has a drop  $1/k^3$   
new effects grow with  $k$  (like SMEFT at colliders...)

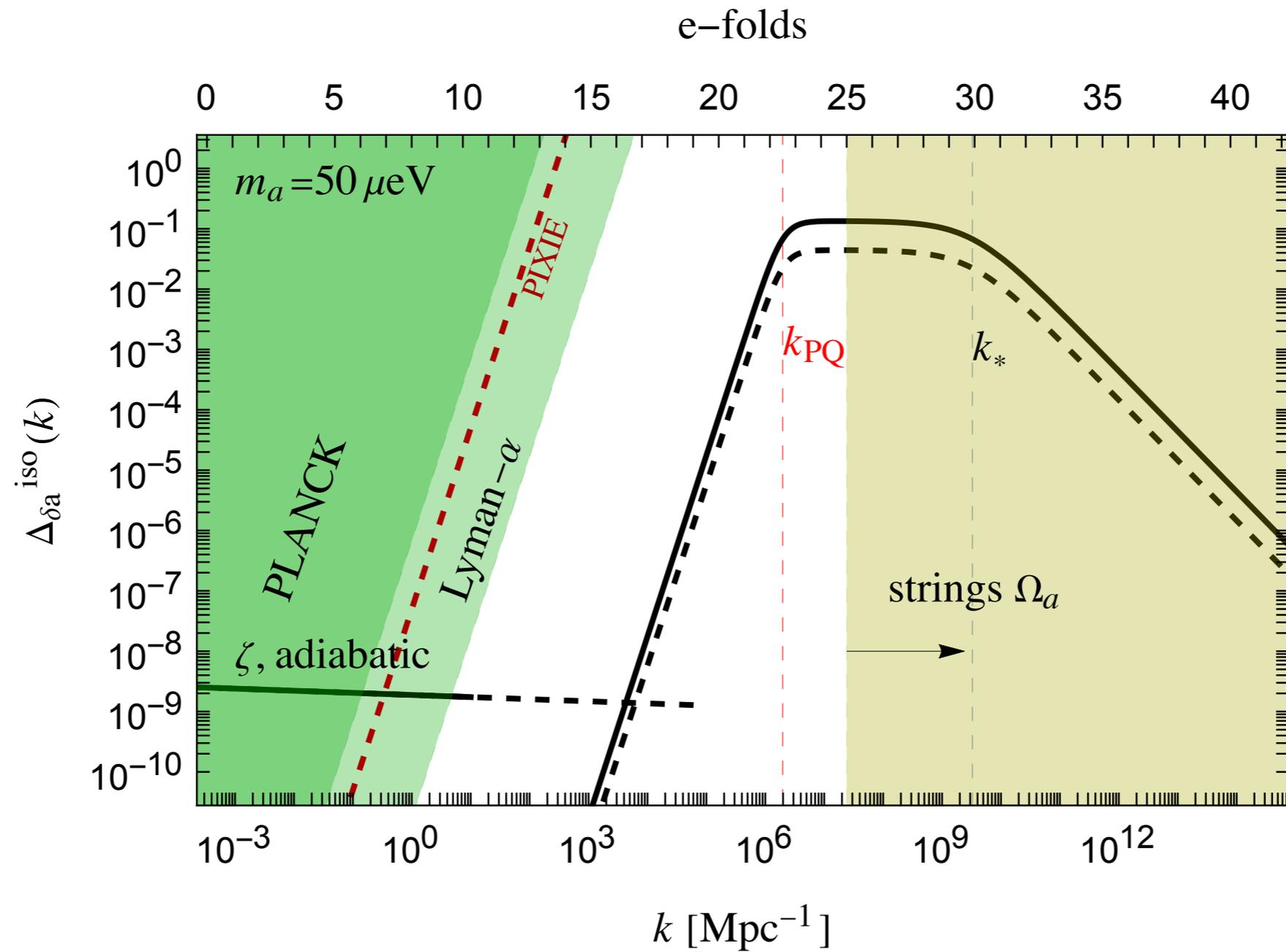
$$P_m(k) \propto \frac{1}{k^3} (\Delta_\zeta(k) + \beta \Delta_{\text{iso}}(k)) \propto \beta \frac{A_{\text{iso}}}{k_0^3}$$

(for uncorrelated iso)



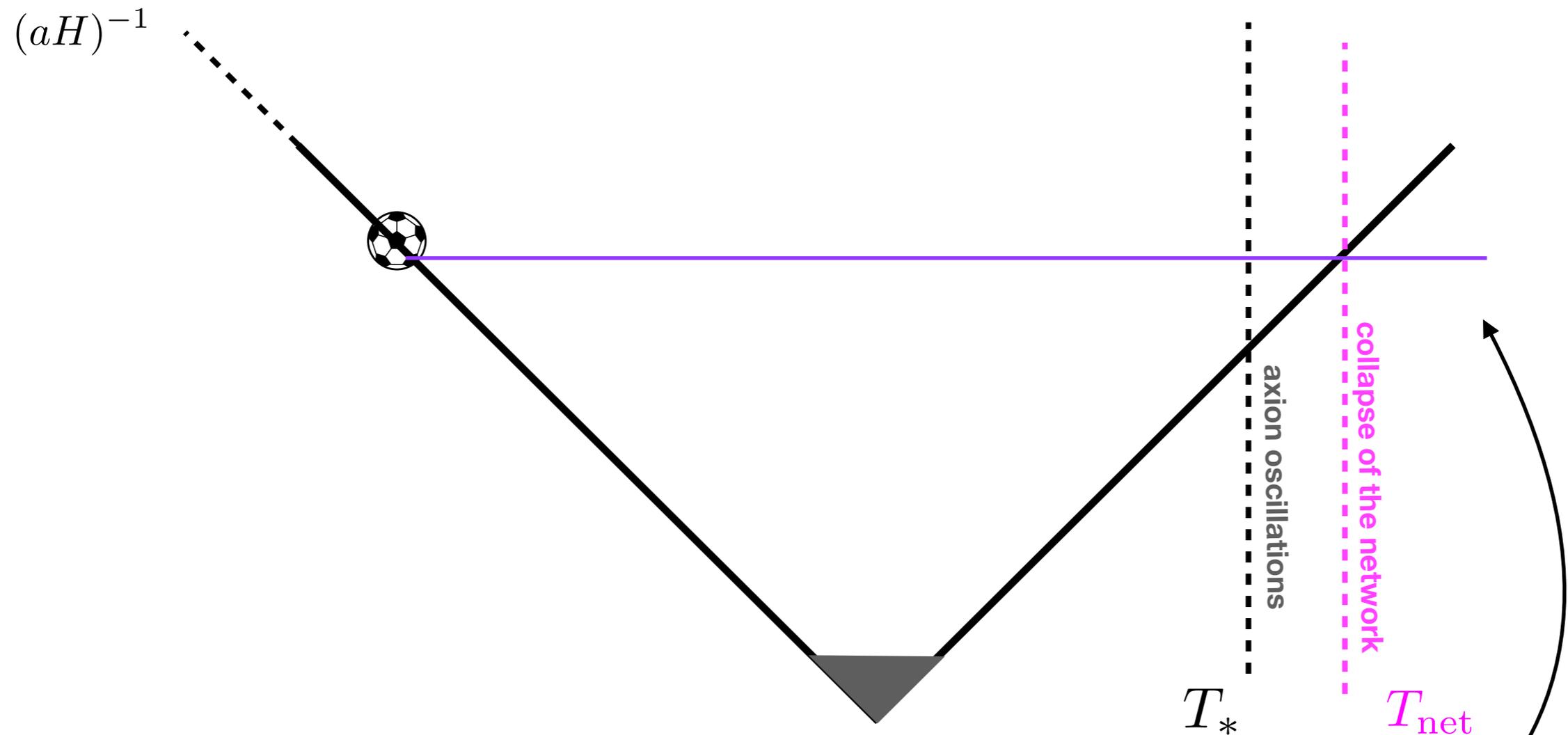
(ongoing work!)

# qcd axion without strings and isocurvatues



# axion abundance from the string network

inhomogeneities generated at the PQ phase transition leave the horizon immediately



they re-enter the horizon at  $T_{\text{net}}$ , before or after the QCD transition  
when they start to evolve and annihilate

## axion abundance from the string network

if they enter the horizon after the axion starts to oscillate,  
**they behave like a long lived network**

$$T_{\text{net}} \lesssim T_*$$

the domain wall energy density can easily become dominant  $\rho_{\text{dw}} \approx 9m_a(T)f_a^2 H(T)$

when they enter the horizon, they immediately decay/annihilate to axions

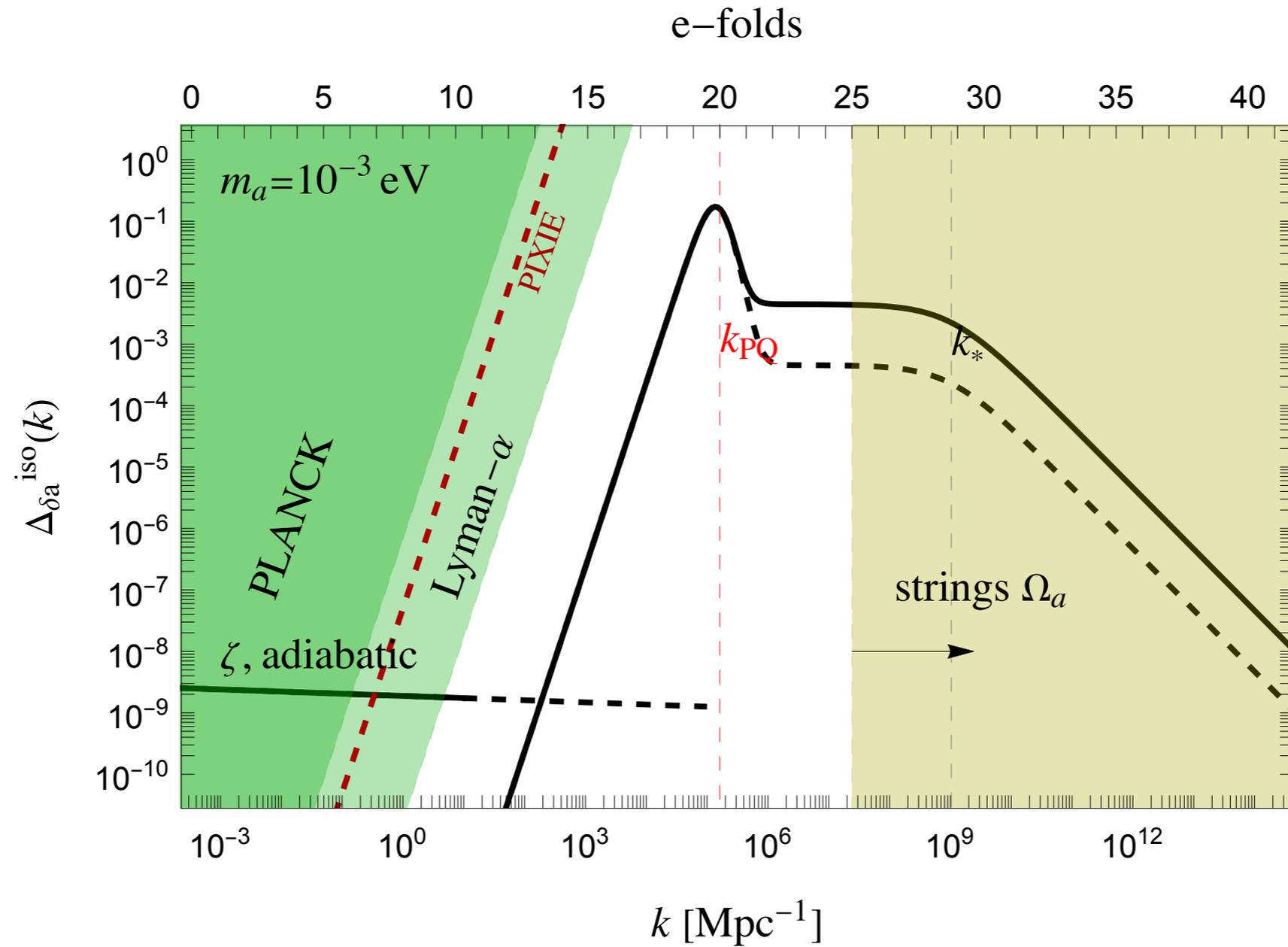
$$\Omega_a^{\text{net.}} \lesssim \Omega_a^{\text{mis}} e^{(23 - N_{\text{PQ}})}$$

final abundance depends on details, **hard to compute**

**lower bound given by astrophysical constraints on the axion decay constant**

$$f_a \approx 10^9 \text{ GeV} \leftrightarrow N_{\text{PQ}} \approx 18 - 20$$

# axion abundance from the string network



sketch!!!

**conclusions**

## **axion + isocurvature**

so far less explored scenario for axion dark matter,  
with signatures  
**lyman-alpha + miniclusters/minihalos/stars..**

## **outlook**

interesting to explore the cases:

- of ALP dark matter
- of phase transitions during inflation (heavy to light, conformal to minimal,..)

**THANK YOU!**