Grand Color Axion

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Outline:

- The Strong CP “problem”
- Axion quality problem and its “solutions”
- Grand Color: an explicit model
- LO Axion potential
- NLO potential and “heavy axion quality problem”
- Phenomenology

Strong CP "Problem"

- A problem for a predictive UV completion of the SM:

$$\eta \sim 0.4$$

$$|\bar{\theta}| < 10^{-10}$$

$$\bar{\theta} = \theta_{\rm QCD} - {\rm Arg}({\rm det}[Y_u^{\rm SM}]{\rm det}[Y_d^{\rm SM}])$$

- Three (spurious) symmetries act on the QCD topological angle:

anomalous U(1) Peccei-Quinn, Weinberg, Wilczek (1977-1978) Georgi, McArthur (1981)

CP Nelson (1984) Barr (1984)

P Babu-Mohapatra (1990) Barr-Chang-Senjanovic (1991)

	Pros	Cons
QCD axion	IR effect (no restrictions on FV&CPV!), Clear signature	Never been observed → Quality problem?
CP or P	Consistent EFT, Constrained UV?	Constrained UV? Involved models

Axion mechanism

PQ symmetry + SM $\Longrightarrow$ $\mathcal{Z} = \int \mathcal{D}(\text{fields})\ e^{-S_{\rm QCD} - S_{\rm weak} - i\int \frac{\bar{a}}{f_a} \frac{g_C^2}{32\pi^2} G\widetilde{G}}$ + derivative couplings

$$V_{\rm eff} = V_0(\bar{a}^2) + V_1^{\rm NP}(\bar{a}^2, \bar{a}\bar{\theta}_{\rm weak})$$

LO in 1/fa and 1/v (just QCD):
exactly minimized at a=0.

Vafa-Witten (1984)

NLO perturbations not dangerous because:
No spontaneous CP & No flat directions at LO →
only tadpole can destabilize, but explicit breaking is small

Axion “quality problem"

Beyond the Standard Model physics can introduce:

— New sources of explicit PQ breaking
— Sizable sources of CP violation

$$V_{\rm eff} = V_0(\bar{a}^2) + V_1^{\rm NP}(\bar{a}, \bar{\theta}_{\rm weak}, \bar{\theta}_{\rm NP})$$

These NLO effects are dangerous: within bounds if $|V_1^{\rm NP}| < 10^{-10}|V_0|$

WHY?! “axion quality problem”

$$|V_1^{\rm NP}| < 10^{-10}|V_0|$$

Suppress V1

Non-trivial UV completions
(String theory, accidental symmetries, etc.)

Enhance V0

"Heavy axion" models

Heavy QCD axion

Must have a potential aligned with QCD!

- Small QCD instantons
- Mirror SM
- Grand Color

Heavy QCD axion

Must have a potential aligned with QCD!

Small QCD instantons | Mirror SM | Grand Color

$$\mathcal{L}_{\text{axion}} \supset \left(\bar{\theta}_{\text{C}} + \frac{a}{f_a}\right) \frac{g_{\text{C}}^2}{32\pi^2} G\widetilde{G}$$

Holdom-Peskin (1982)
➔ sensitivity to UV effects

Agrawal-Howe (2018)
➔ multiple axions (quality needs to be checked)

Heavy QCD axion

Must have a potential aligned with QCD!

Small QCD instantons

Mirror SM

Grand Color

$$\mathcal{L}_{\rm axion} \supset \left(\bar{\theta}_{\rm C} + \frac{a}{f_a}\right) \frac{g_{\rm C}^2}{32\pi^2} G\widetilde{G} + \left(\bar{\theta}_{\rm C'} + \frac{a}{f_a}\right) \frac{g_{\rm C'}^2}{32\pi^2} G'\widetilde{G}'$$

The same by a Z2

Rubakov (1997)
(Berezhiani et al, Hook, etc.)
➔ requires an entire copy of the SM

Heavy QCD axion

Must have a potential aligned with QCD!

Small QCD instantons

Mirror SM

Grand Color

$$\mathcal{L}_{\text{axion}} \supset \left(\bar{\theta}_{\text{C}} + \frac{a}{f_a}\right)\frac{g_{\text{C}}^2}{32\pi^2}G\widetilde{G} + \left(\bar{\theta}_{\text{C}'} + \frac{a}{f_a}\right)\frac{g_{\text{C}'}^2}{32\pi^2}G'\widetilde{G}'$$

Dimopoulos (1979)
➔ proposal, but no model

Gherghetta et al (2016)
Gavela et al. (2018)
➔ not complete

Grand Color → Color x Color': the same angle at tree level… loops?

Grand Color

Main goal: finding an explicit model that
— does not spoil $\bar{\theta}_{\mathrm{C}} = \bar{\theta}_{\mathrm{C}'}$
— does not break the Standard Model gauge group at color’ confinement
— does not introduce new fine-tunings

N=odd

7<N<17

		$SU(N)_{\text{GC}}$	$SU(2)_{\text{L}}$	$U(1)_{\text{Y}'}$
Fermions (SM + exotic)	Q	$\mathbf{N}$	$\mathbf{2}$	$\frac{1}{2N}$
	U	$\overline{\mathbf{N}}$	$\mathbf{1}$	$-\frac{1}{2} - \frac{1}{2N}$
	D	$\overline{\mathbf{N}}$	$\mathbf{1}$	$+\frac{1}{2} - \frac{1}{2N}$
	ℓ	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
	e	$\mathbf{1}$	$\mathbf{1}$	$+1$
Scalars (Only H has Yukawas)	H	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$
	Φ	$\mathbf{Adj}$	$\mathbf{1}$	0
	Ξ	$\mathbf{N} \otimes_A \mathbf{N}$	$\mathbf{1}$	$\frac{1}{N}$

Same Yukawa structure as in the SM $\mathcal{L}_{\text{Yuk}} = Y_u\, QHU + Y_d\, Q\widetilde{H}D + Y_e\, \ell\widetilde{H}e + \text{hc}$

The interactions QQΞ*, UDΞ must be forbidden: gauge B-L (need RH n) or take Ξ composite.

$$SU(N)_{\mathrm{GC}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y'}}$$

$\downarrow$ f_GC $\langle \Phi \rangle$ $\langle \Xi \rangle$

$$SU(3)_{\mathrm{C}} \times Sp(N-3) \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$$

$\downarrow$ f   Sp(N-3) confinement does not break electroweak

Standard Model with θ=0

Below f_GC

Scalars decouple except H.
Fermions decompose into SM plus exotics (accidentally chiral).

$$\begin{cases} Q = q \oplus \psi_q \sim (\mathbf{3}, \mathbf{1}, \mathbf{2}_{1/6}) \oplus (\mathbf{1}, \mathbf{N-3}, \mathbf{2}_0) \\ U = u \oplus \psi_u \sim (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}_{-2/3}) \oplus (\mathbf{1}, \mathbf{N-3}, \mathbf{1}_{-1/2}) \\ D = d \oplus \psi_d \sim (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}_{+1/3}) \oplus (\mathbf{1}, \mathbf{N-3}, \mathbf{1}_{+1/2}) \end{cases}$$

The theta angles of QCD and Sp(N-3) are the same up to tiny effects!

$\bar{\theta}_{\mathrm{C}} = \bar{\theta}_{\mathrm{C}'}$ + tiny effects!!!

Why?

At renormalizable level: loops of the SM Yukawas are negligible.
New Yukawa couplings and/or masses would be a problem (as in earlier realizations).

Ellis-Gaillard (1979), Khriplovich (1986), Khriplovich-Vainshtein (1994)

At non-renormalizable level: can be suppressed.

$$\frac{\bar{c}_5}{f_{\rm UV}}\frac{g^2_{\rm GC}}{32\pi^2}\,\Phi\, G_{\rm GC}\tilde{G}_{\rm GC}$$

Dim-5 forbidden by charging Φ under B-L or making it composite.
Dim-6 are naturally suppressed provided $f_{\rm GC} < 10^{13}\ \text{GeV}$ (UV cutoff is the Planck scale).

Below f_GC:

$$\mathcal{L}_{\rm axion} \supset \left(\bar\theta_{\rm C} + \frac{a}{f_a}\right)\frac{g_{\rm C}^2}{32\pi^2}G\widetilde{G} + \left(\bar\theta_{\rm C'} + \frac{a}{f_a}\right)\frac{g_{\rm C'}^2}{32\pi^2}G\widetilde{G}$$

$$\bar\theta_{\rm C} = \bar\theta_{\rm C'}$$ + tiny effects!!!

A single axion can simultaneously relax color and Sp(N-3) angles:

f>>fπ gives a heavy axion

Is the Sp potential aligned with the QCD one?

Yes, but not obvious.

The hypothesis of Vafa-Witten do not apply:

— we have Yukawa couplings to a fundamental scalar H.

— the Nambu-Goldstone bosons of Sp(N-3) can (and do) acquire a vev. Hence, the axion potential may be affected by both explicit and spontaneous CP violation.

Need to check!!!

— At confinement, approximate SU(12)→Sp(12) $\langle\psi_q\psi_q\rangle, \langle\psi_u\psi_d\rangle \neq 0$
— The electroweak group remains unbroken.
— Nambu-Goldstone bosons (NGBs) are generated.
— The electroweak charged ones get positive masses squared → vanishing vev.
— 14 are neutral and acquire a potential from Yukawas → vacuum alignment.

Real (by CP) unknown

\# of generations

$$V_{\rm neutral} = \frac{c_{ud}}{N} f^4\, {\rm Tr}\left[Y_u \Sigma_R Y_d^t \Sigma_L\right] e^{i\frac{\bar{a}}{N_g f_a}} + {\rm hc} + \mathcal{O}(Y^4, v^2/f^2)$$

Standard Model Yukawa matrices

$$\frac{\langle \bar{a} \rangle}{f_a} = \begin{cases} 0 \text{ mod } 2\pi & \text{if } N_g = \text{even} \\ 0 \text{ mod } 2\pi & \text{if } N_g = \text{odd and } c_{ud} < 0 \\ \pi \text{ mod } 2\pi & \text{if } N_g = \text{odd and } c_{ud} > 0 \end{cases}$$

General argument. Relation confirmed explicitly for Ng=1,2,3.

Now: **What is the sign of c_ud in our model?**
It turns out that c_ud is the same as in another theory where VW can be demonstrated.
Hence c_ud<0!

Toy model: Ng=2

$$c_{ud} > 0: \quad \langle \Sigma_L \rangle = (\pm) \begin{pmatrix} i\cos\theta_c & i\sin\theta_c \\ i\sin\theta_c & -i\cos\theta_c \end{pmatrix}, \qquad \langle \Sigma_R \rangle = (\pm) \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \qquad \langle \bar{a} \rangle = 0,$$

$$c_{ud} < 0: \quad \langle \Sigma_L \rangle = (\pm) \begin{pmatrix} i\cos\theta_c & i\sin\theta_c \\ i\sin\theta_c & -i\cos\theta_c \end{pmatrix}, \qquad \langle \Sigma_R \rangle = (\mp) \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \qquad \langle \bar{a} \rangle = 0$$

As expected, minimum has vanishing axion for any sign of c_ud.
Axion effective potential:

$$V_{\text{eff}} = -2\,\frac{|c_{ud}|}{N} f^4\, \text{Tr}[\widehat{Y}_u \widehat{Y}_d] \sqrt{1 - 4\frac{\det[\widehat{Y}_u \widehat{Y}_d]}{\text{Tr}^2[\widehat{Y}_u \widehat{Y}_d]} \sin^2\left(\frac{\bar{a}}{2f_a}\right)}$$

$$V_{\rm eff} = -2\,\frac{|c_{ud}|}{N} f^4\,{\rm Tr}[\widehat{Y}_u\widehat{Y}_d]\sqrt{1-4\frac{\det[\widehat{Y}_u\widehat{Y}_d]}{{\rm Tr}^2[\widehat{Y}_u\widehat{Y}_d]}\sin^2\left(\frac{\bar{a}}{2f_a}\right)}$$

General lessons:

— non-trivial part proportional to det of Yukawas (axion is exact NGB if det=0);

— in the limit of large Ng'th generation we recover the Ng-1 potential (decoupling);

— given the large SM flavor hierarchy, the axion mass is always of order

$$m_a^2 = 2\frac{|c_{ud}|}{N}\,\frac{\det[\widehat{Y}_u\widehat{Y}_d]}{{\rm Tr}[\widehat{Y}_u\widehat{Y}_d]}\,\frac{f^4}{f_a^2}$$

$$\sim 2\,\frac{|c_{ud}|}{N}\,y_u y_d\,\frac{f^4}{f_a^2}$$

Consistent with dimensional analysis

The real thing: Ng=3

Numerical analysis

$$m^2_{\Pi_0} \simeq \begin{pmatrix} 6.4 \times 10^{-2} \\ 2.5 \times 10^{-2} \\ 2.4 \times 10^{-2} \\ 2.4 \times 10^{-2} \\ 2.4 \times 10^{-2} \\ 2.4 \times 10^{-2} \\ 2.3 \times 10^{-2} \\ 2.7 \times 10^{-5} \\ 1.2 \times 10^{-5} \\ 2.5 \times 10^{-6} \\ 1.6 \times 10^{-6} \\ 1.6 \times 10^{-6} \\ 4.0 \times 10^{-8} \end{pmatrix} \times \frac{|c_{ud}|}{N} f^2$$

$$m_a^2 \simeq 1.8 \times 10^{-10} \times \frac{|c_{ud}|}{N} \frac{f^4}{f_a^2} \simeq 2 \frac{|c_{ud}|}{N} y_u y_d \frac{f^4}{f_a^2}$$

Minimization at NLO

Can NLO corrections at the renormalizable level spoil the solution?

No, for the very same reason as in the QCD axion:

(1) the LO solution has no flat directions
(2) CP is not spontaneously broken at LO (non-trivial)
(3) the axion vev can only be controlled by explicit CP violation, which is small (same as SM!)

Can NLO corrections at the non-renormalizable level spoil the solution?

Yes, but these effects decouple and can be easily under control…

Higher-dimensional operators introduce new CP-odd, flavor-invariant phases.
Examples:

$$\frac{\bar{c}_{ijkl}}{f_{\rm UV}^2} Q_i Q_j U_k D_l$$

$$\frac{\bar{c}_{ijkl}}{f_{\rm UV}^2} (\Psi_i \Psi_j)(\Psi_k \Psi_l)^\dagger$$

They do not break PQ, but…
Affect the NGB vev, and hence axion vev.
Strongest bound is $f \lesssim 10^{-7} f_{\rm UV}$

$$\frac{\bar{c}_W}{M_{\rm UV}^2} \frac{g_{\rm GC}^3}{16\pi^2} G_{\rm GC} G_{\rm GC} \widetilde{G}_{\rm GC}$$

It does not break PQ, but…
Brings new CP-odd, flavor-invariant phases.
Weaker bound.

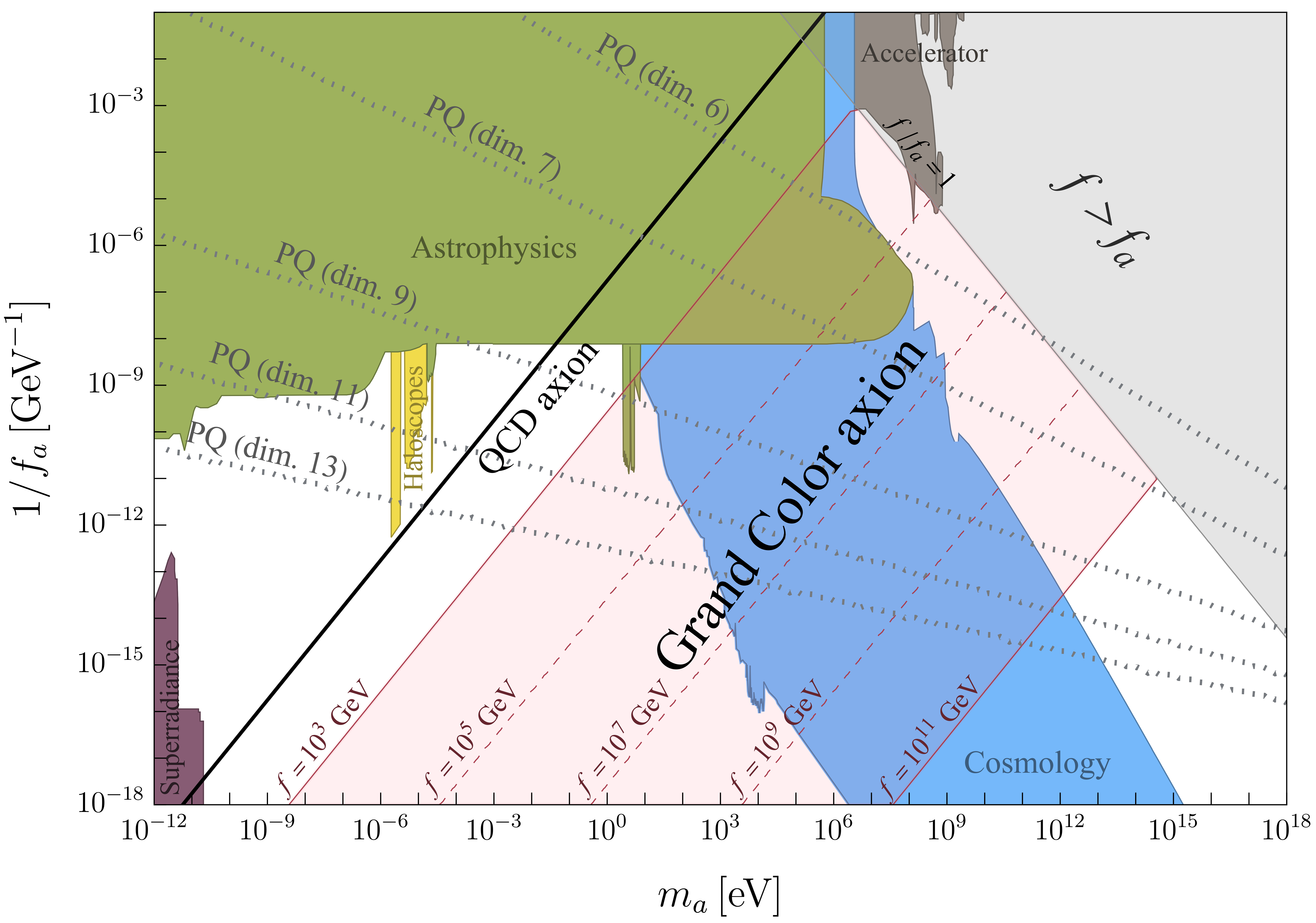
NLO stability: General Lesson

Heavy axion models suffer from a “heavy axion quality problem”:
Because f is large, CP-violating or flavor-violating higher-dimensional operators involving the d.o.f. of the strong dynamics can contribute non-negligibly to the axion potential, even if PQ-conserving.

Very rich phenomenology

New electroweak states (heavy resonances and NGBs). All unstable.
The lightest states are the neutral NGBs:

— mix with axion and acquire anomalous couplings to SM gauge bosons
— all NGBs are unstable: always decay rates larger than axion
— there is a potentially massless photophobic NGB (removed gauging B-L)
— modified axion phenomenology:
  (1) always heavier than QCD axion (f must be large because of collider bounds on charged states)
  (2) even KSVZ axion has (bare) photon coupling because QED is partially embedded in Grand Color
  (3) quality is improved



PQ (dim. 6)
PQ (dim. 7)
PQ (dim. 9)
PQ (dim. 11)
PQ (dim. 13)
Accelerator
Astrophysics
$f/f_a = 1$
$f > f_a$
Haloscopes
QCD axion
Grand Color axion
Superradiance
$f = 10^3$ GeV
$f = 10^5$ GeV
$f = 10^7$ GeV
$f = 10^9$ GeV
$f = 10^{11}$ GeV
Cosmology
$1/f_a$ [GeV^{-1}]
m_a [eV]

Conclusions

- ☑ **A new compelling heavy axion model**
  - (i) Very minimal (field content, no global symmetry besides PQ)
  - (ii) No new fine-tunings
  - (iii) Very rich phenomenology
- ☑ All heavy axion scenarios suffer a mild **“heavy axion quality problem”**
- ☑ …Worth exploring alternatives to axions

Backup…

Minimization at LO

Let us generalize the problem in order to acquire intuition (exactly the same structure in QCD).

$$V_{\rm LO} = V_0(\Pi) e^{i\frac{\bar{a}}{N_g f_a}} + {\rm hc}$$

Extremum condition:

$$\begin{cases} \frac{\delta V_0}{\delta \Pi_m} e^{i\bar{a}/f_a N_g} + \frac{\delta V_0^*}{\delta \Pi_m} e^{-i\bar{a}/f_a N_g} = 0 \\ V_0 e^{i\bar{a}/f_a N_g} - V_0^* e^{-i\bar{a}/f_a N_g} = 0. \end{cases} \implies \begin{cases} \frac{\delta |V_0|}{\delta \Pi_m} = 0 \\ \sin\left(\frac{\bar{a}}{N_g f_a}\right) = \mp \frac{{\rm Im}[V_0]}{|V_0|} \end{cases}$$

Looking at the Hessian one sees that |V0| must be maximized. **Then the axion vev follows.**

Example: QCD with 2 flavors

Rotate to positive mu,md, then

$$V_0 = C[m_u e^{i\pi_0/f_\pi} + m_d e^{-i\pi_0/f_\pi}]$$

$$|V_0|^2 = |C|^2[(m_u + m_d)^2 \cos^2 \pi_0/f_\pi + (m_u - m_d)^2 \sin^2 \pi_0/f_\pi]$$

$$= |C|^2[(m_u + m_d)^2 - 4m_u m_d \sin^2 \pi_0/f_\pi]$$

— When mu,md>0 the maximum is at π0/fπ=0,π ⟹ **V0 is real and ⟨a⟩=0**.
— When mu md=0 π0 is flat direction ⟹ V0 has arbitrary phase, axion has arbitrary vev: **exact NGB**.

In our case, as in QCD, V0 is a trace between a diagonal Yukawa and a NGB matrix:

$$V_0 = \frac{c_{ud}}{N} f^4 \, [\widehat{Y}_u]_i [A]_{ii}, \qquad\qquad A = \Sigma_R \widehat{Y}_d V^\dagger_{\rm CKM} \Sigma_L$$

— The potential tends to align A along the Yukawa: A tends to go to diagonal form.
— Then the maximum is found when V0 includes a coherent sum of the diagonal elements:
The phases of the diagonal elements of A must be the same.
— Because det[A] is real, the phases are an element of the center of SU(Ng)
Phase of A is 2πn/Ng ➔ phase of V0 is Arg[c_ud]+ 2πn/Ng ➔ axion vev is:

$$\frac{\langle \bar{a} \rangle}{f_a} = \begin{cases} 0 \text{ mod } 2\pi & \text{if } N_g = \text{even} \\ 0 \text{ mod } 2\pi & \text{if } N_g = \text{odd and } c_{ud} < 0 \\ \pi \text{ mod } 2\pi & \text{if } N_g = \text{odd and } c_{ud} > 0 \end{cases}$$