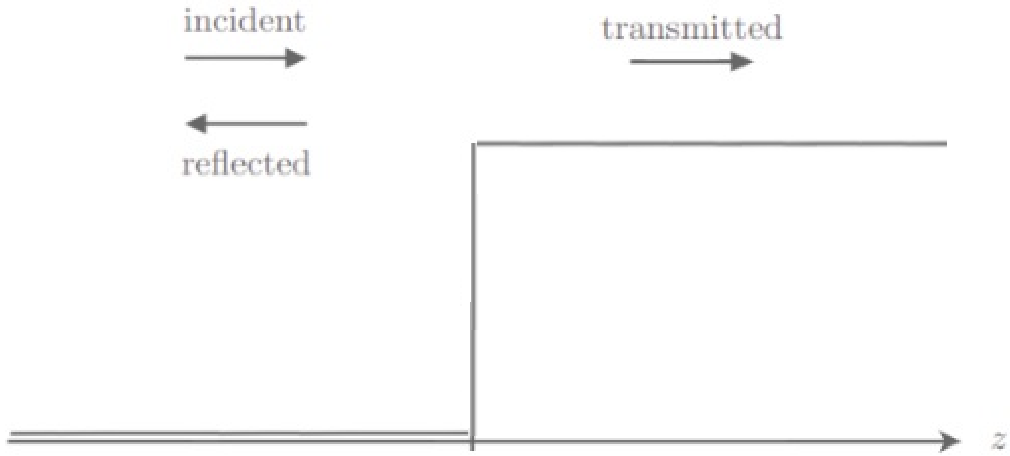
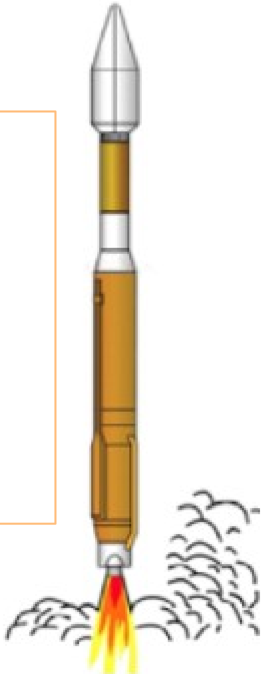




The Rocket Science of Expanding Bubbles

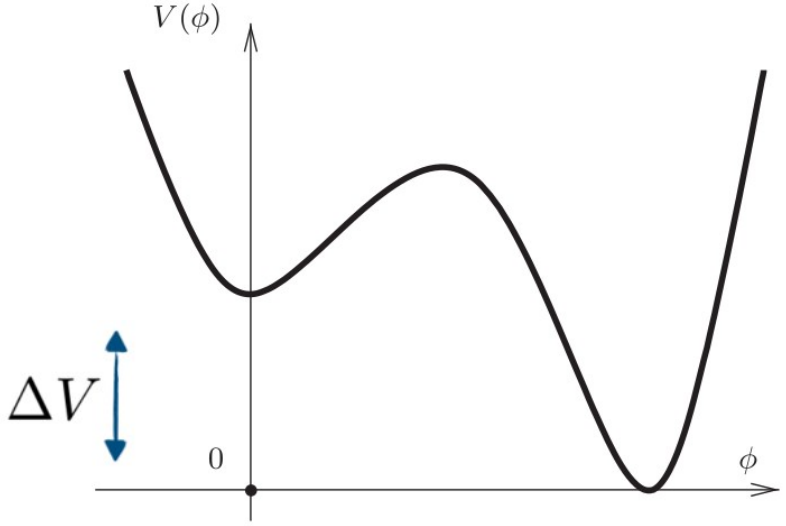
Rudin Petrossian-Byrne

ICTP Trieste

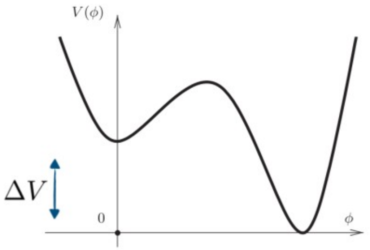
Based on 2212.xxxxx, in collaboration with
Isabel Garcia-Garcia (NYU & IAS)
& Giacomo Koszegi (UC Santa Barbara)

1st Order Phase Transitions in the Early Universe

both in (minimal) extensions of the SM and in more general hidden sectors



Motivation



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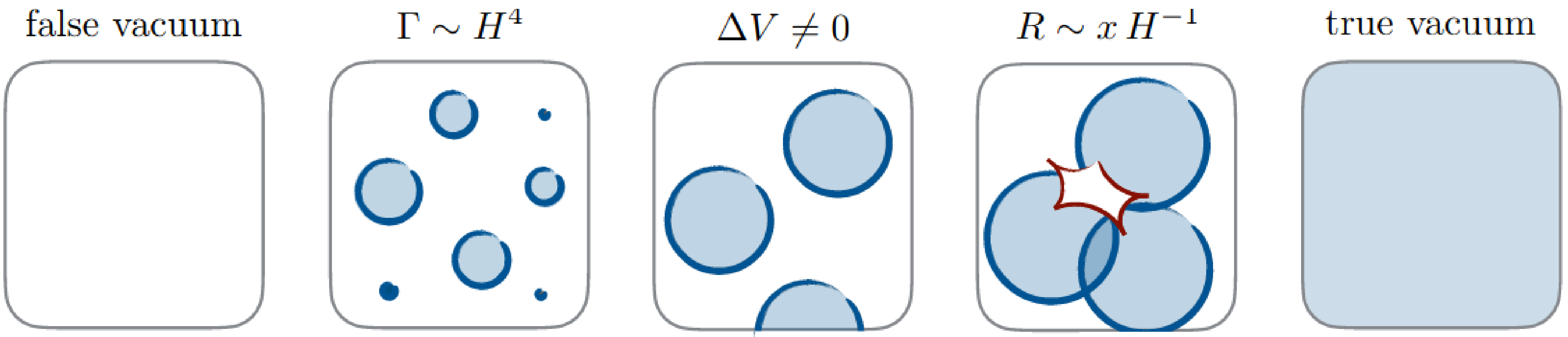

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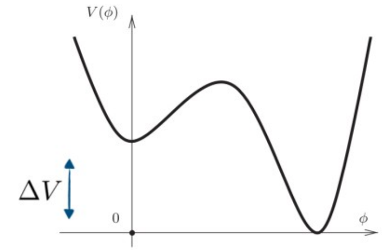

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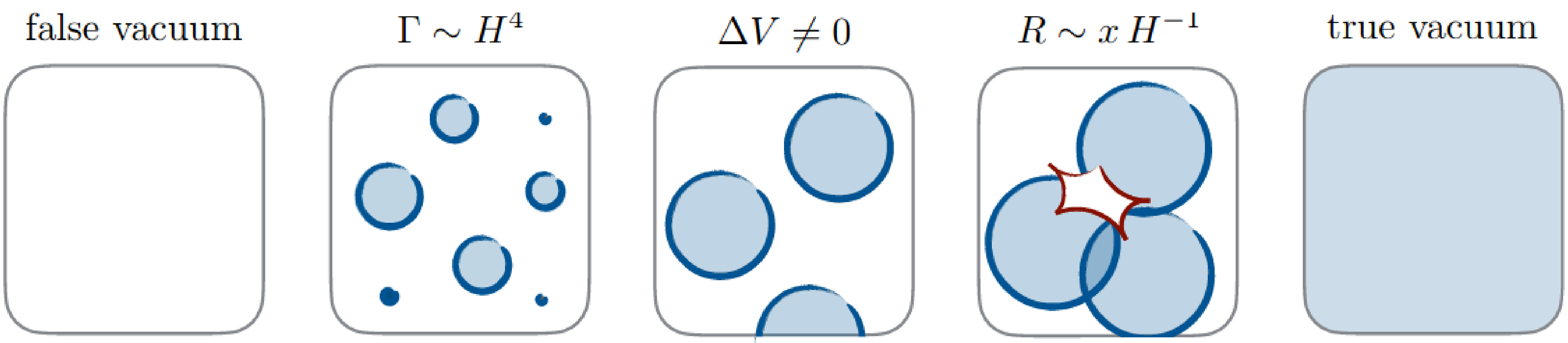

Hogan (1986); Kosowsky, Turner, Watkins (1992);
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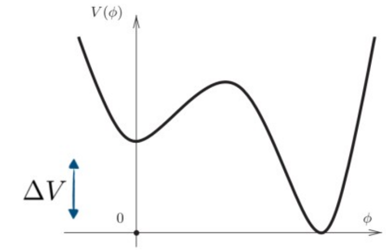
Bubble wall collisions a significant source of gravitational waves



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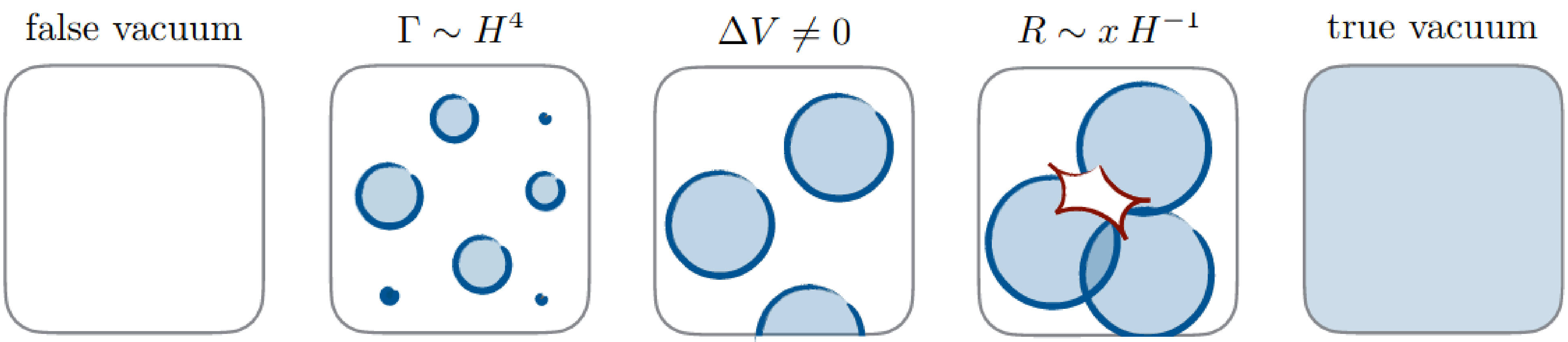

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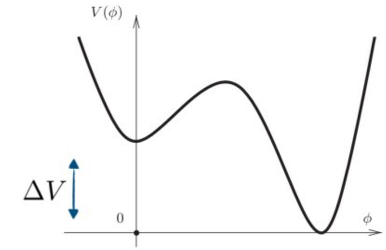

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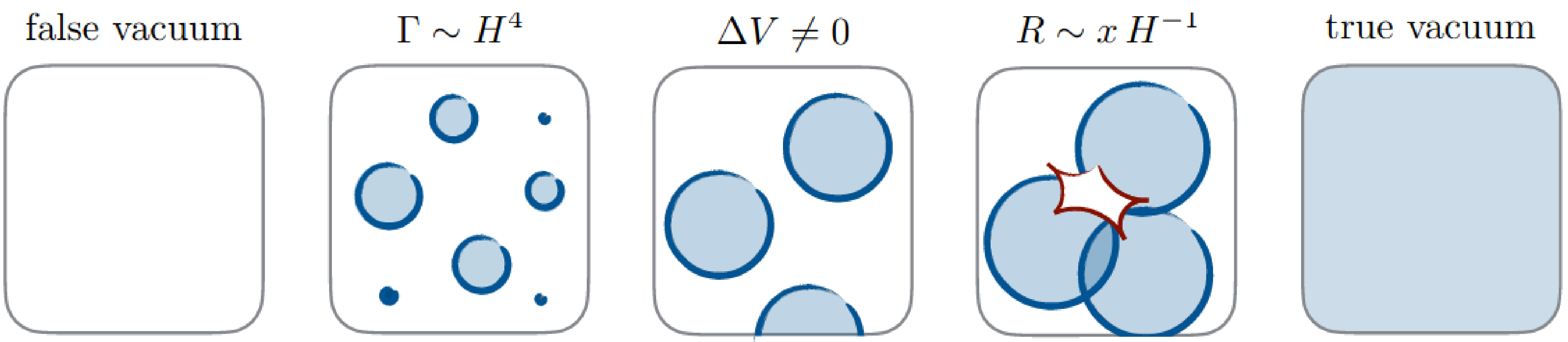

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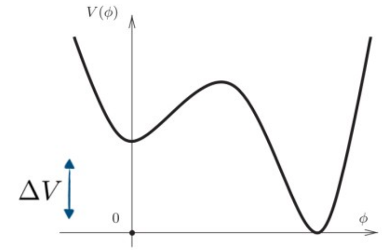

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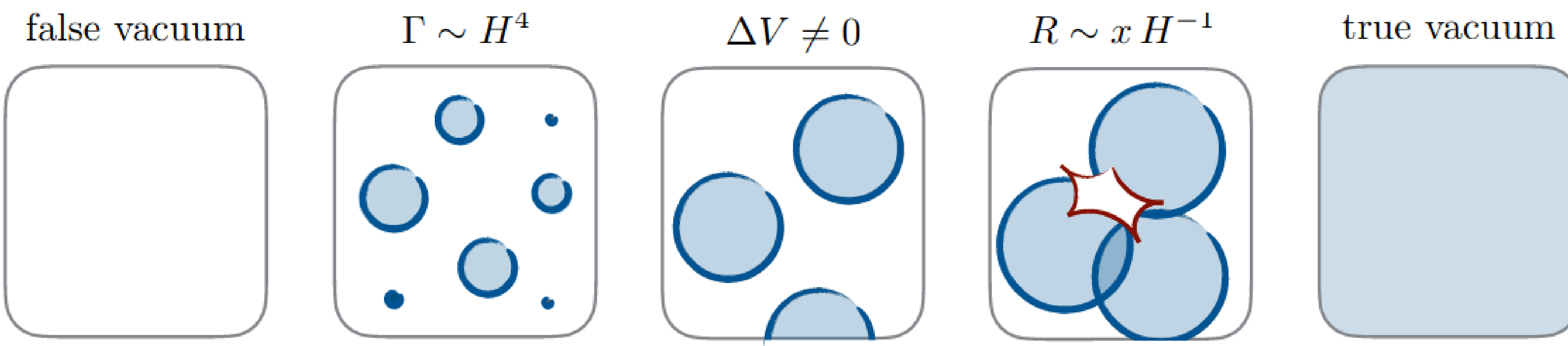

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Motivation

BBN

$$f_0 \sim x^{-1}\,\frac{T_* T_0}{M_{\rm Pl}} \sim 1\ \text{mHz} \times \frac{T_*}{100\ \text{GeV}} \sim 10\ \text{nHz} \times \frac{T_*}{1\ \text{MeV}} \qquad (\text{with } x \sim 10^{-2})$$

EW scale

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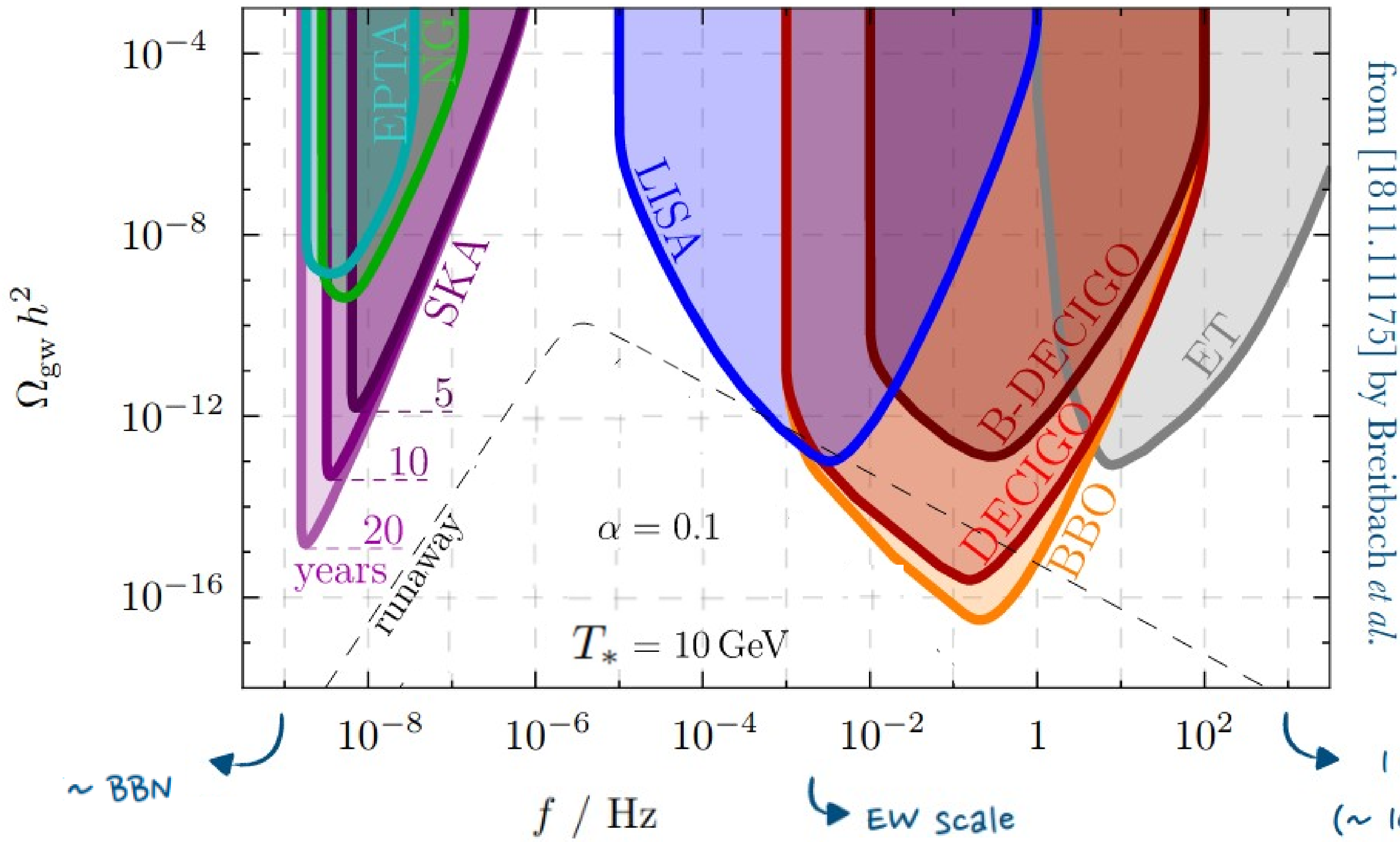

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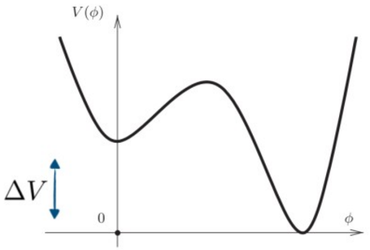
Life is Complicated



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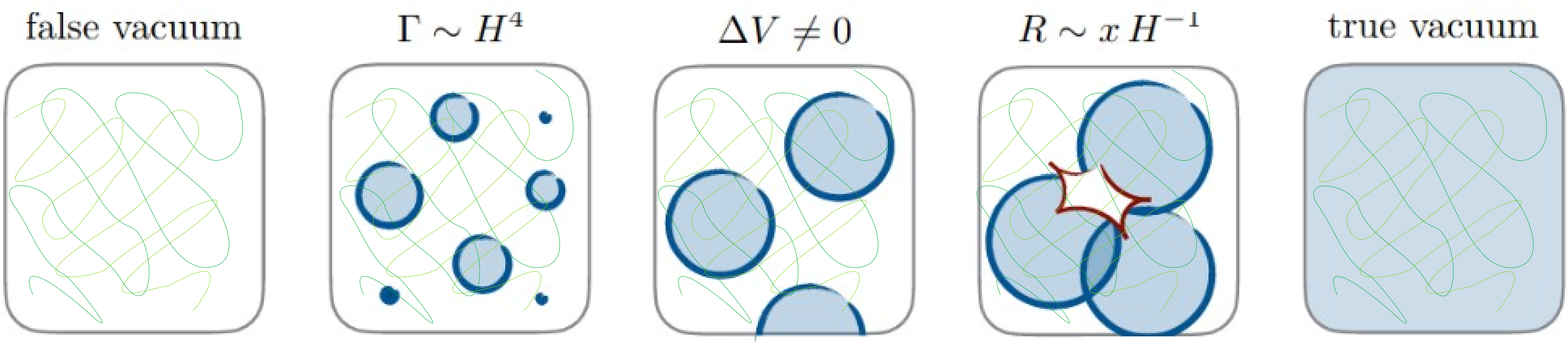

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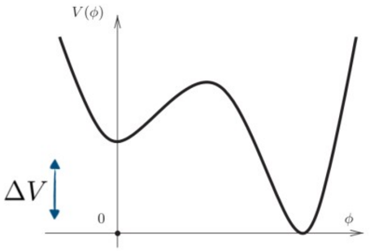

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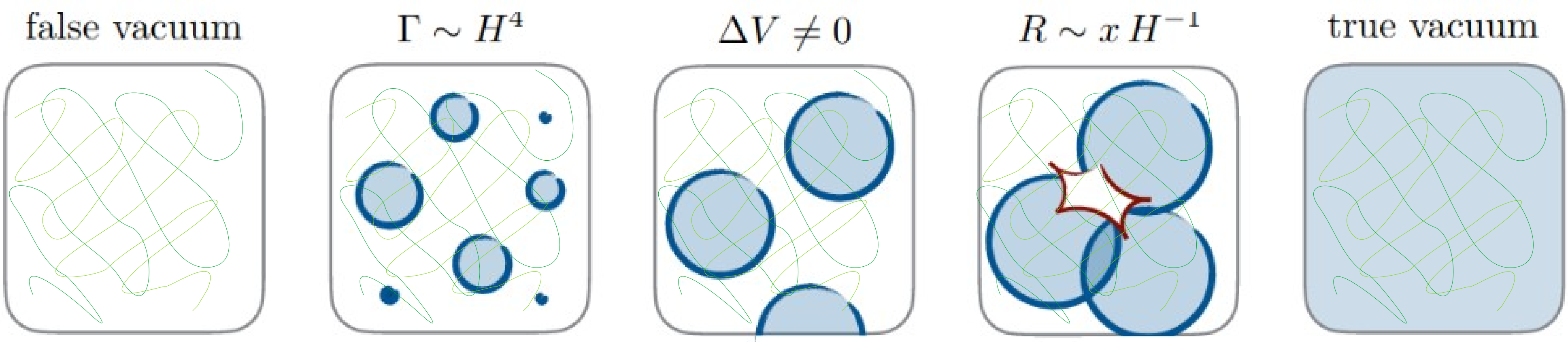

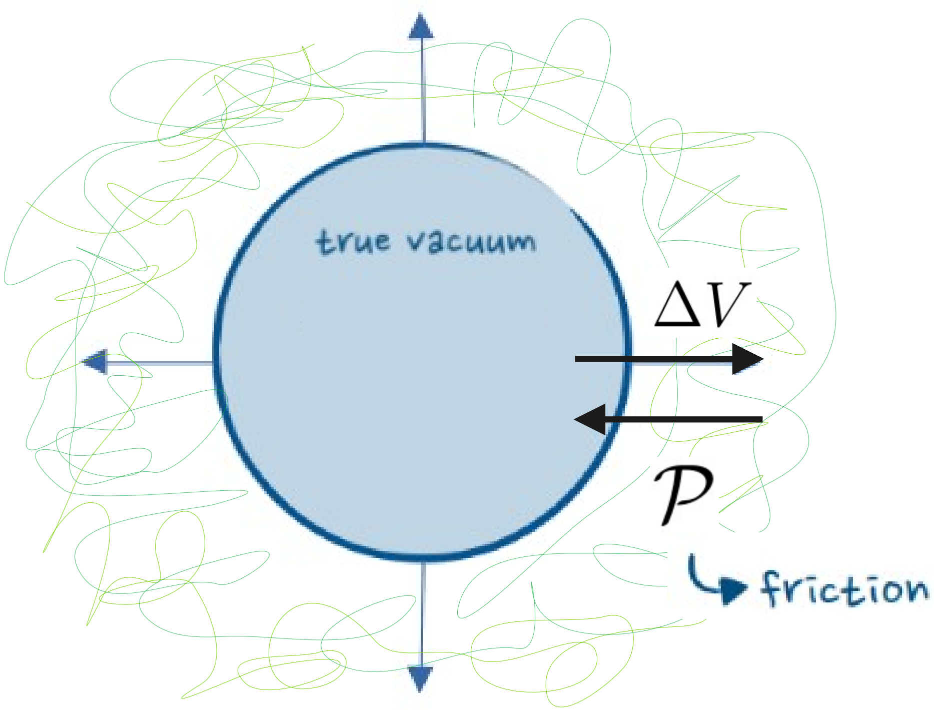

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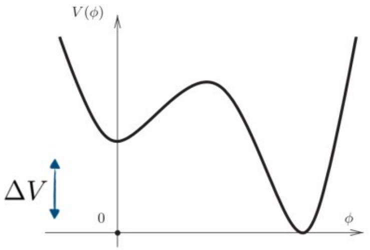

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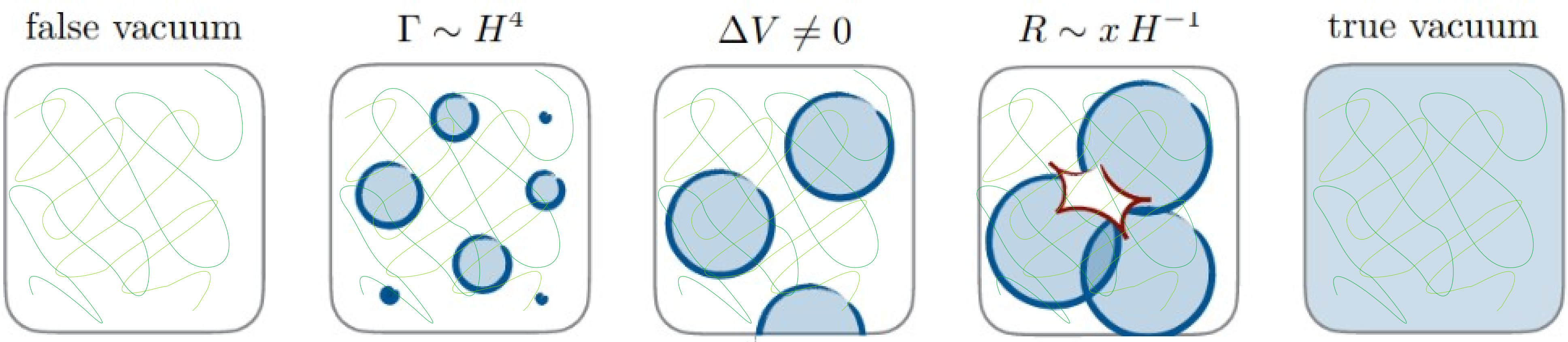

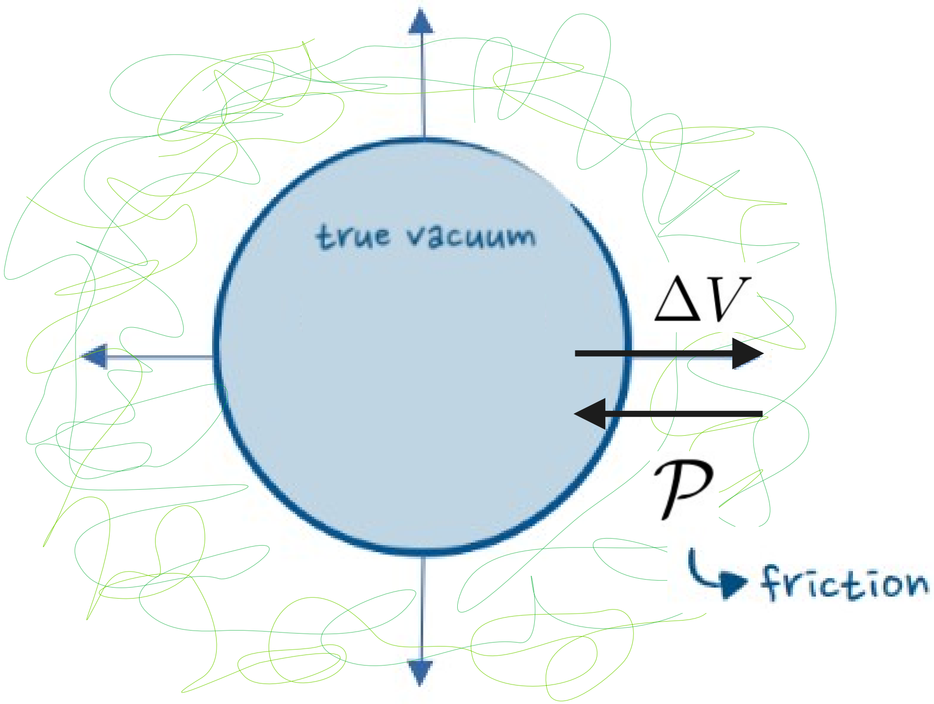

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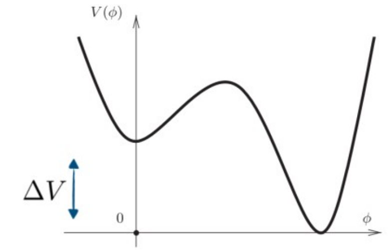

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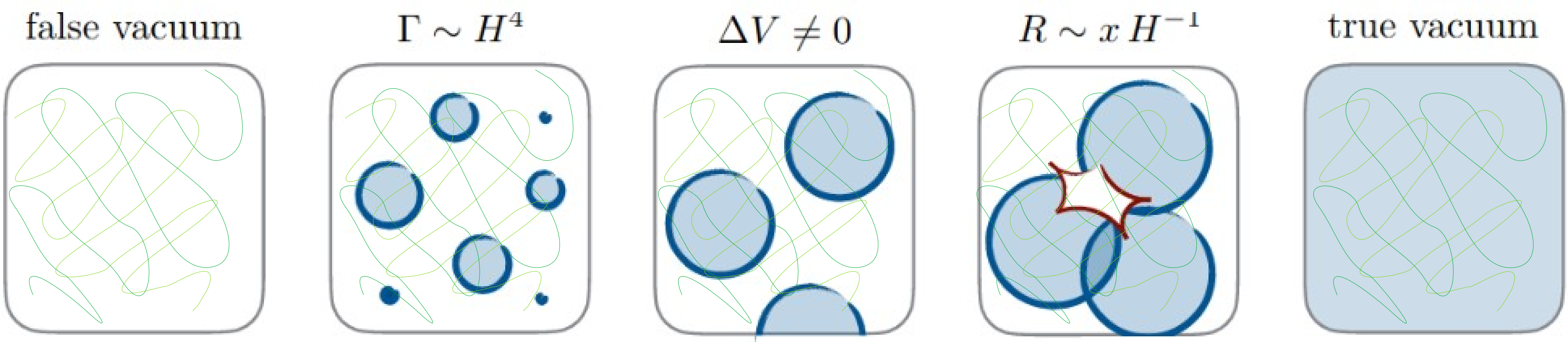

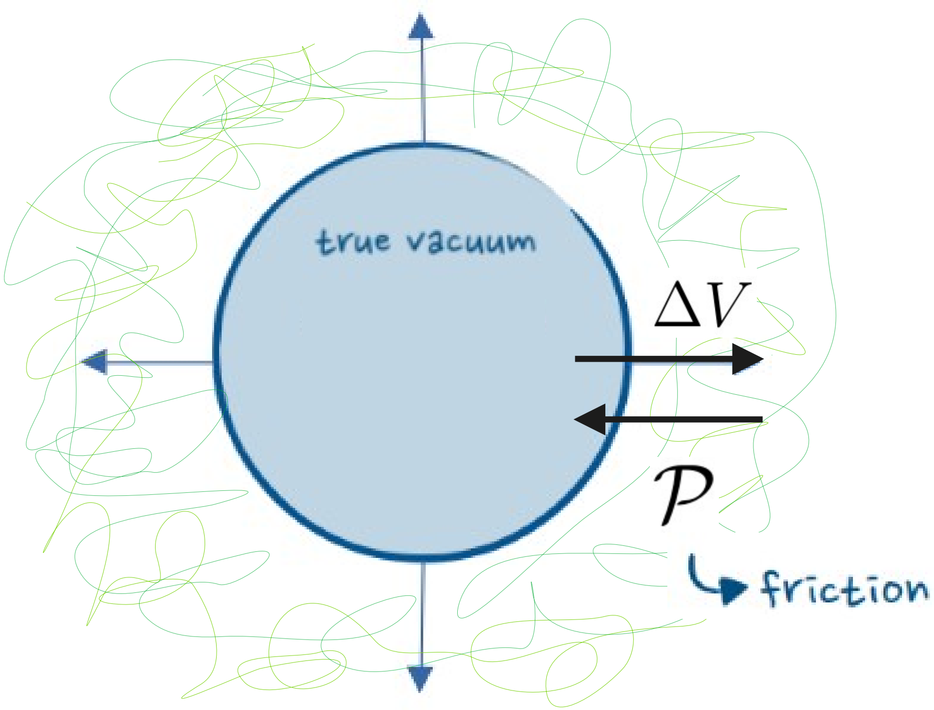

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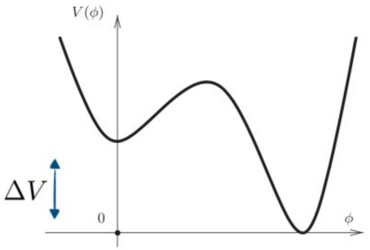

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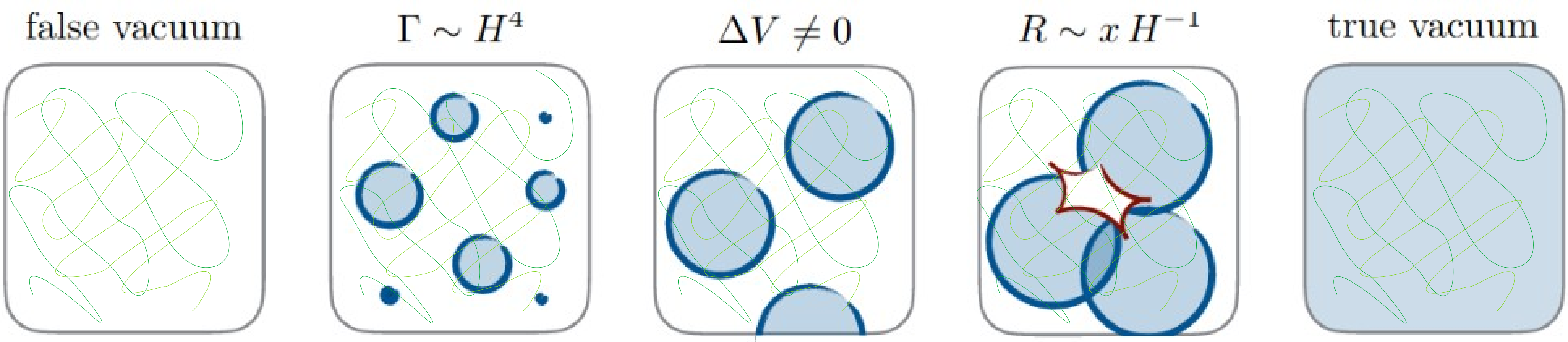

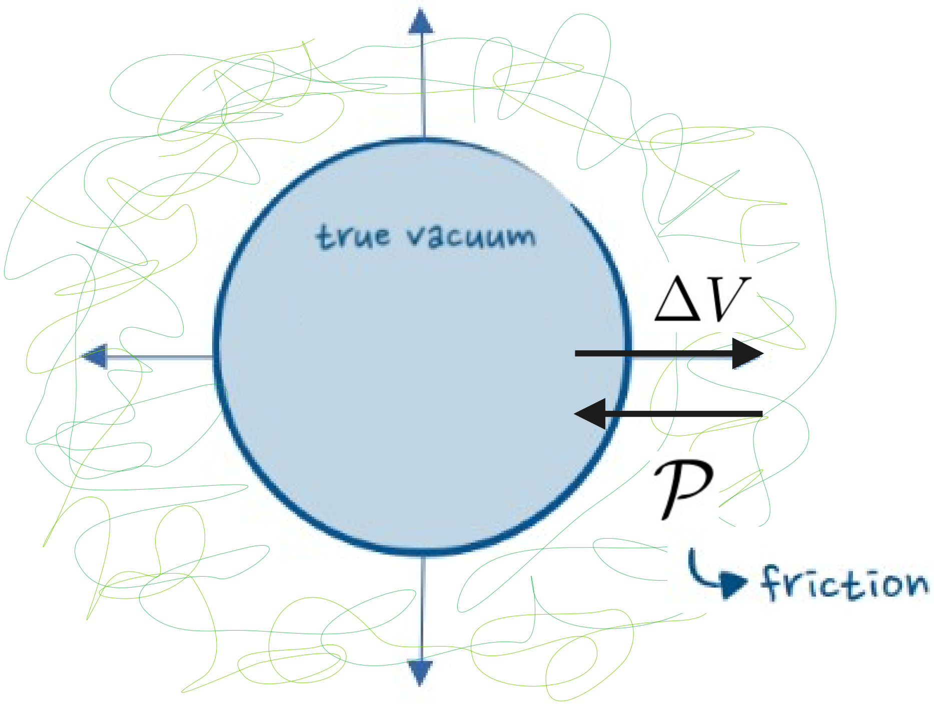

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Runaway vs. non-runaway

Strength & shape of GW signal depend on dominant source of GW,

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e.g.

$\Delta V \gg \mathcal{P}$ **'runaway'**

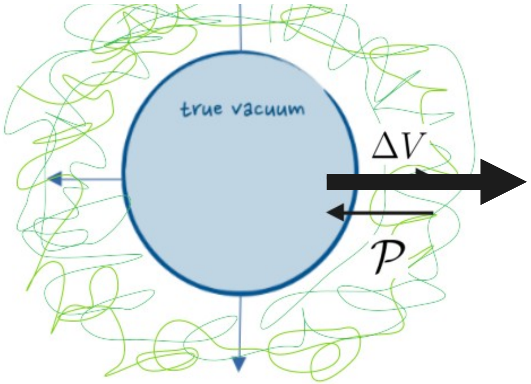
$\gamma \to \infty$

$$\frac{E_{\rm wall}}{E_{\rm total}} \simeq \frac{4\pi\, R^2\, \sigma\, \gamma}{\frac{4\pi}{3}\, R^3\, \Delta V} = 1$$

Dominated by bubble collisions

(as in vacuum)

$\Omega_{\rm gw} \propto f^{-1}$ fall-off



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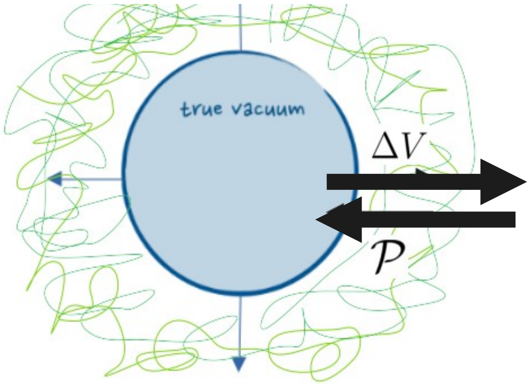

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vs

$\Delta V \approx \mathcal{P}$ **e.g. thermal plasma**

$\gamma_{\rm eq} = {\rm const.}$

$$\frac{E_{\rm wall}}{E_{\rm total}} \simeq \frac{4\pi\, R^2\, \sigma\, \gamma_{\rm eq}}{\frac{4\pi}{3}\, R^3\, \Delta V} \ll 1$$

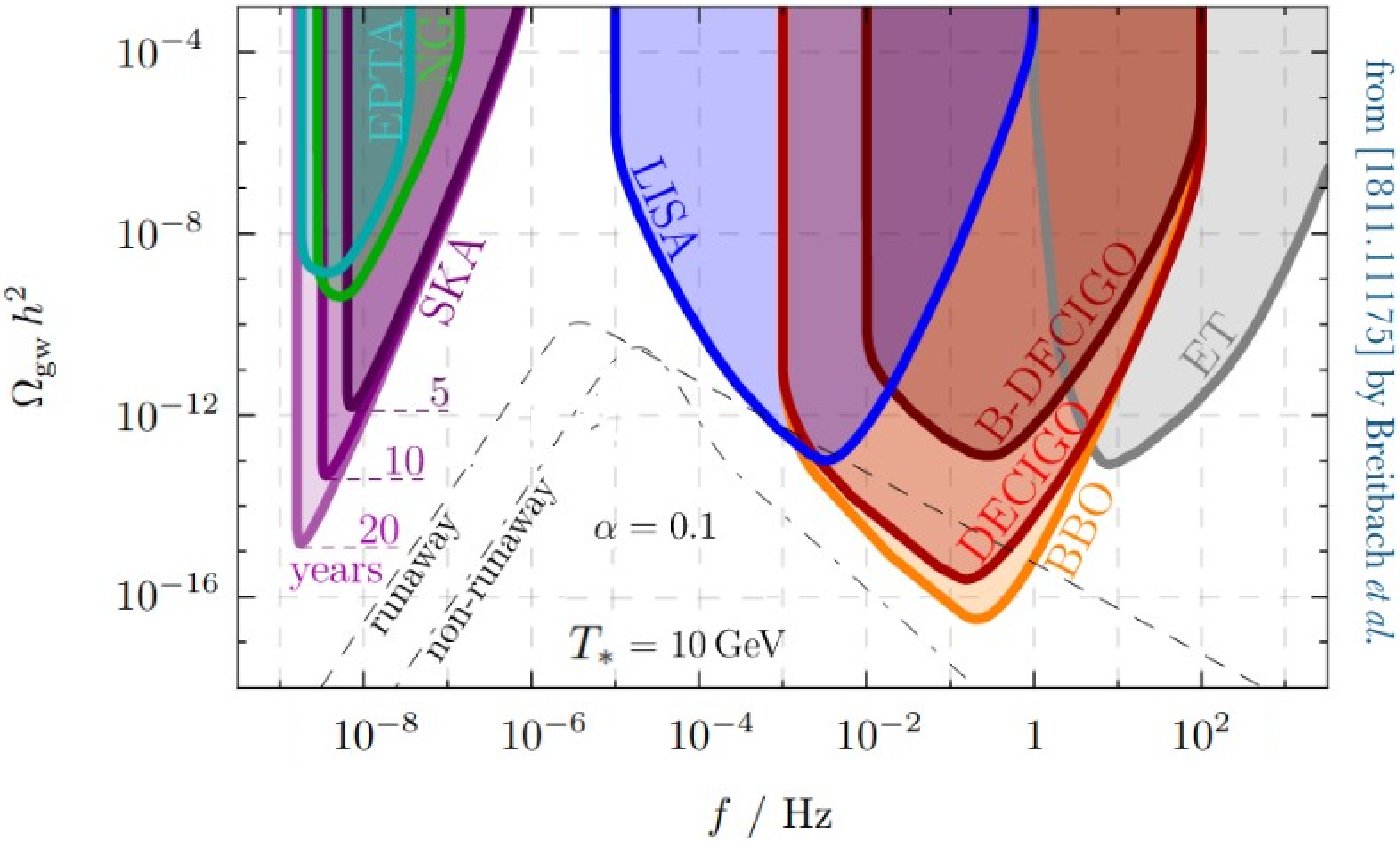
Sound waves & Turbulence in thermal plasma

$\Omega_{\rm gw} \propto f^{-3}$ fall-off

see e.g. [1512.06239] & [1910.13125] by LISA Cosmology Working Group

Runaway vs. non-runaway

Generally friction effects distort the signal!



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Calculating Pressure

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Simplification in relativistic limit $\gamma \gg 1$

Think of expanding domain wall as interacting with individual particles

difficult

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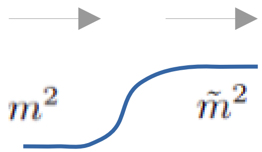

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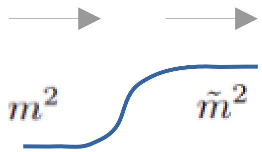

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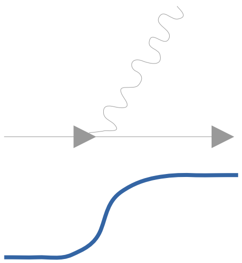

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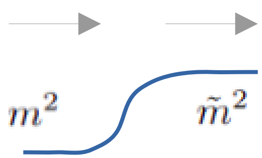

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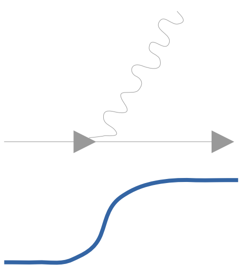

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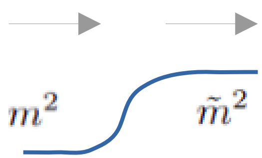

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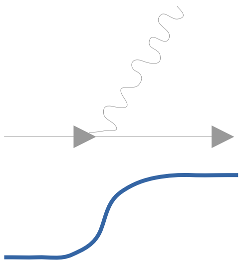

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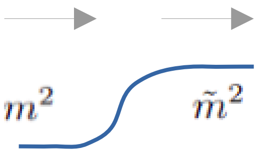

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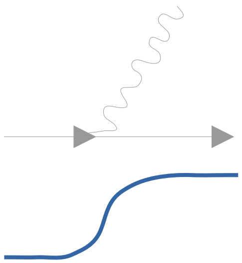

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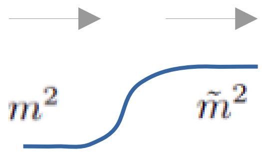

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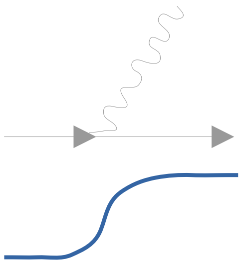

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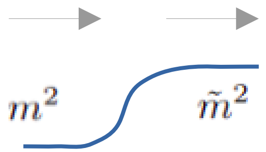

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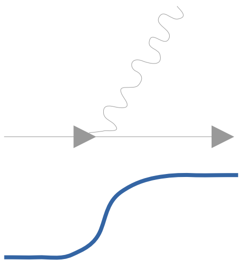

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Identify complementary signatures associated with an observable stochastic background

LO Pressure Set up

- Ignore curvature of wall and boost to its rest frame.
- Particle mass changes from m^2 to $\tilde{m}^2$ take $k^x = k^y = 0$

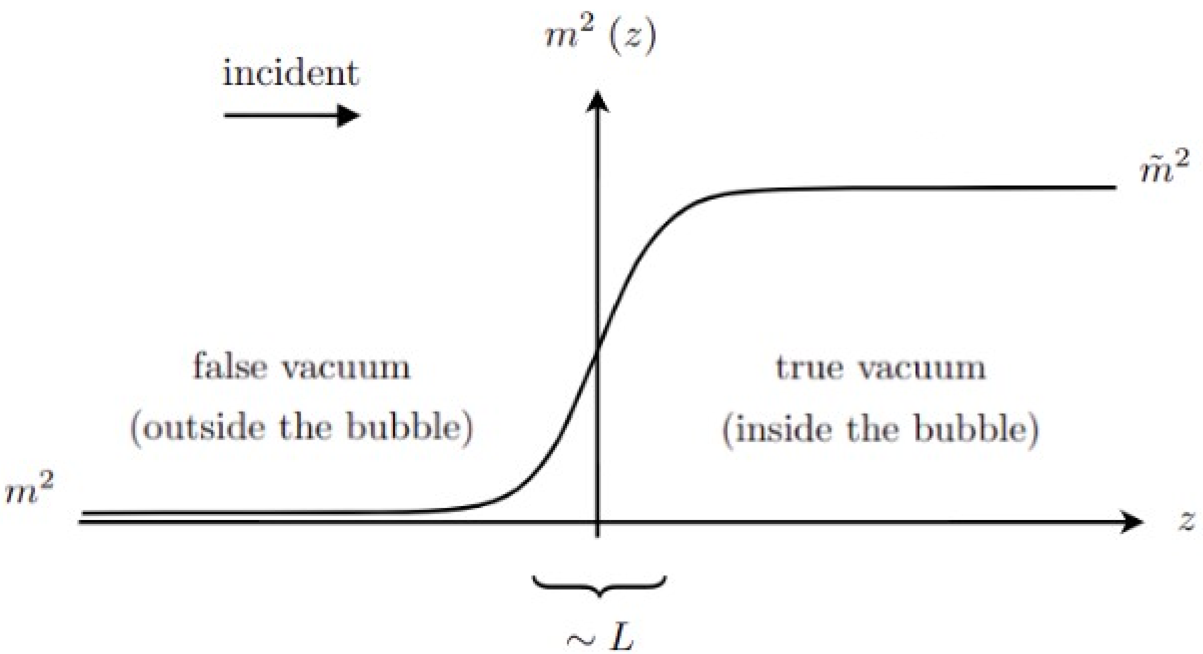
Incoming energy:

(thermal plasma)

$\omega \sim \gamma T$

(cold matter)

$\omega \sim \gamma m$



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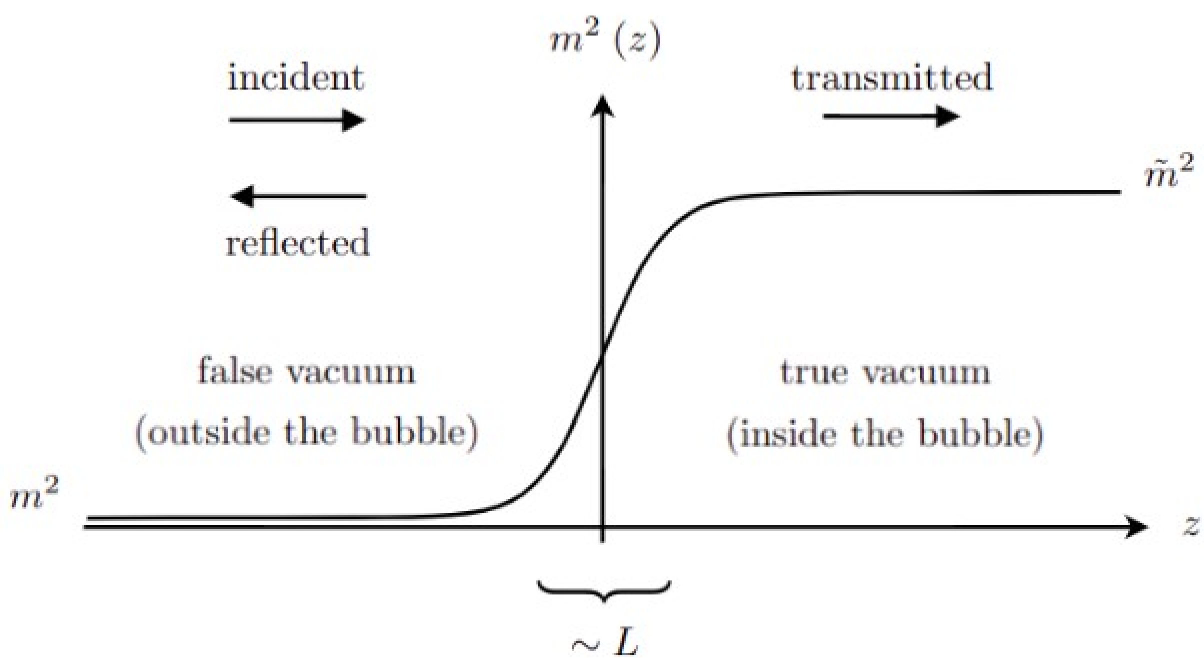
(thermal plasma)

$\omega \sim \gamma\, T$

(cold matter)

$\omega \sim \gamma\, m$



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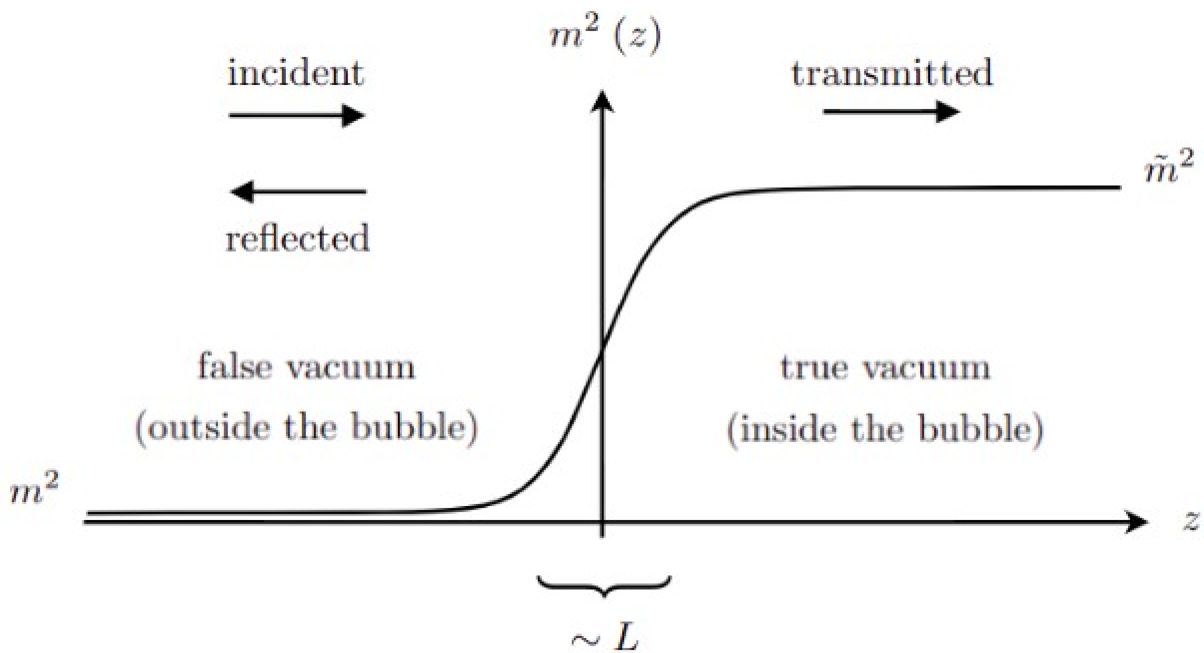

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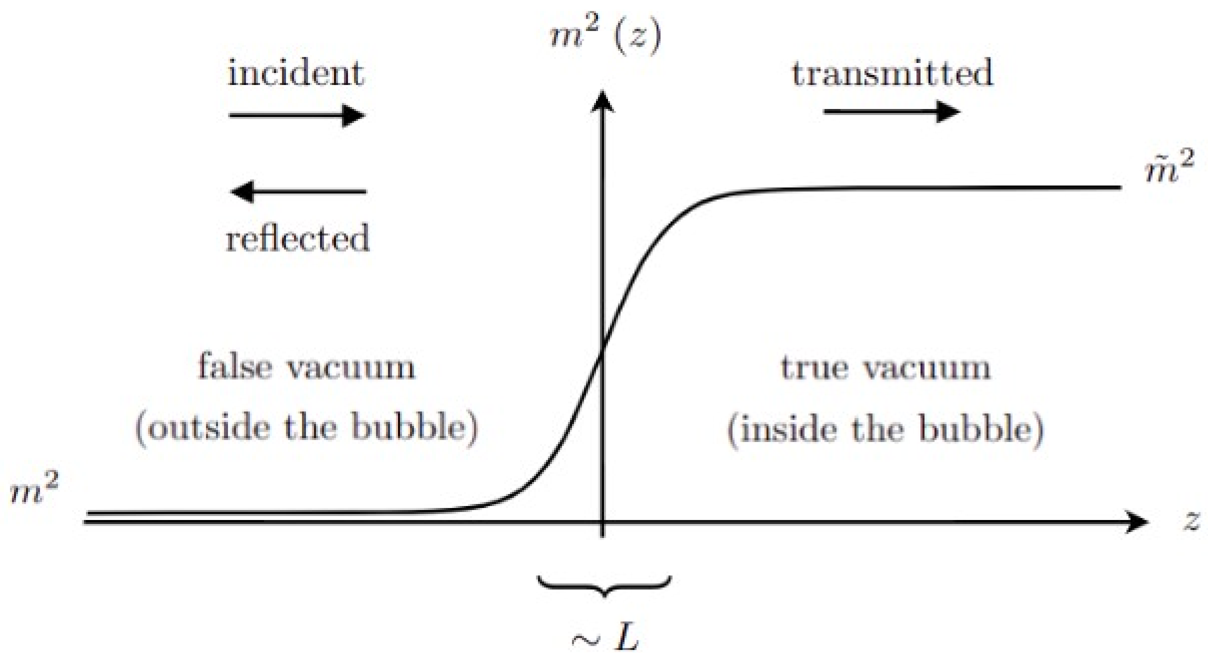

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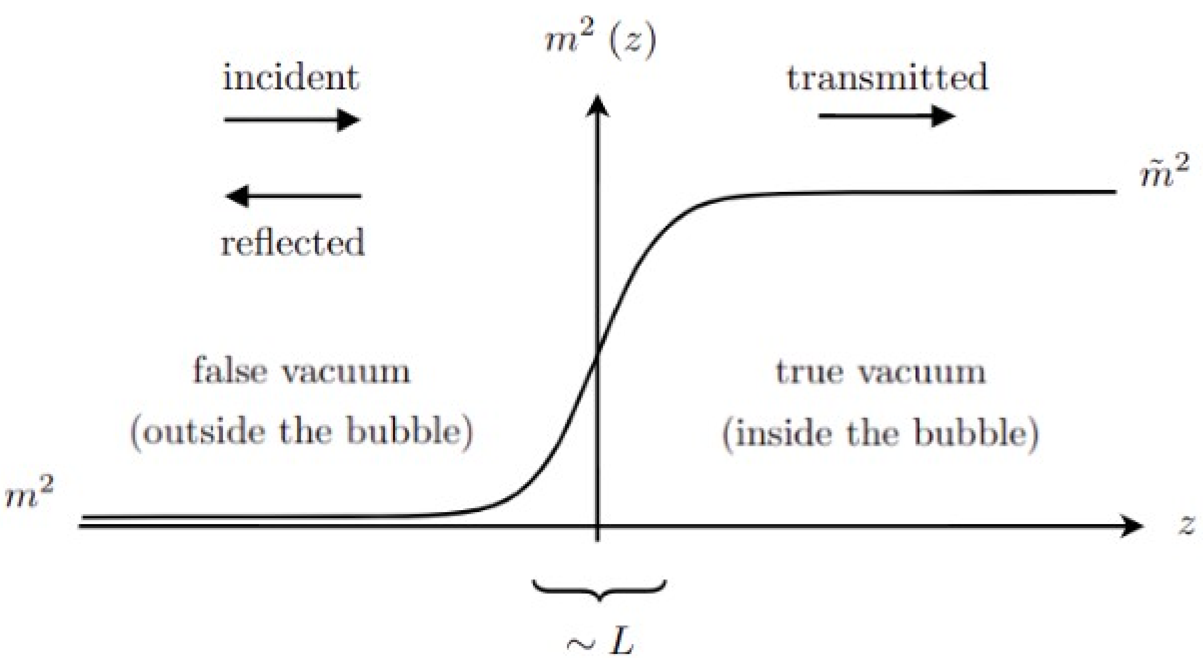

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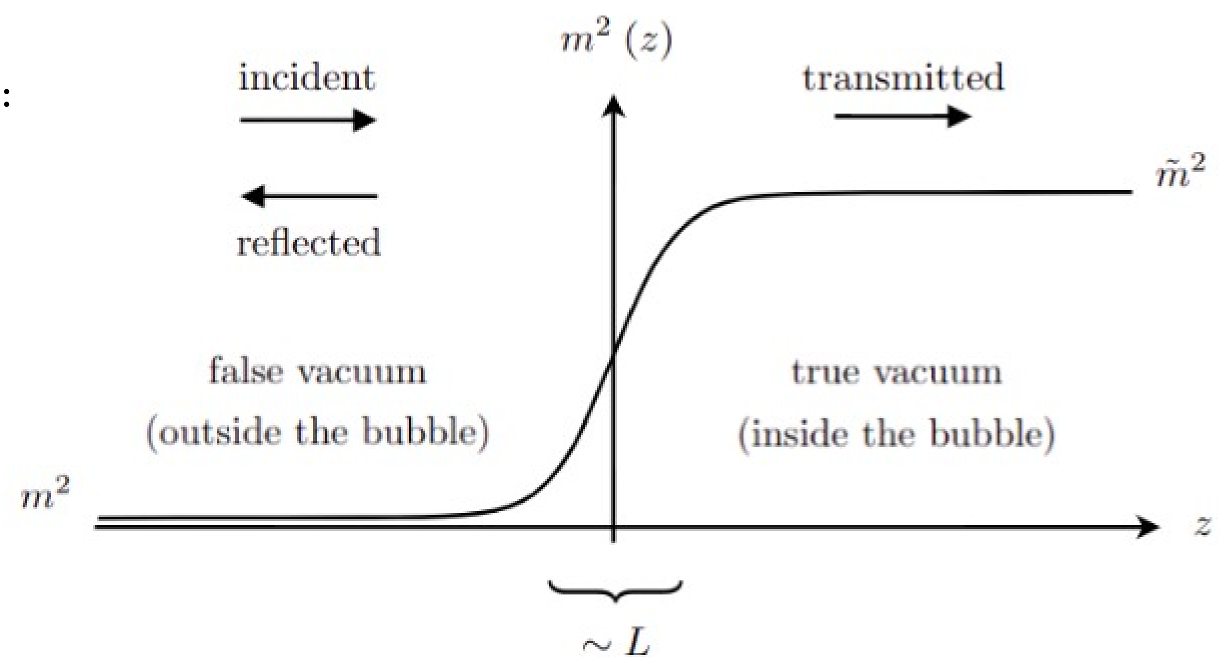

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Let's solve for R and $\overline{\mathrm{T}}$…. Use step function approximation for simplicity.

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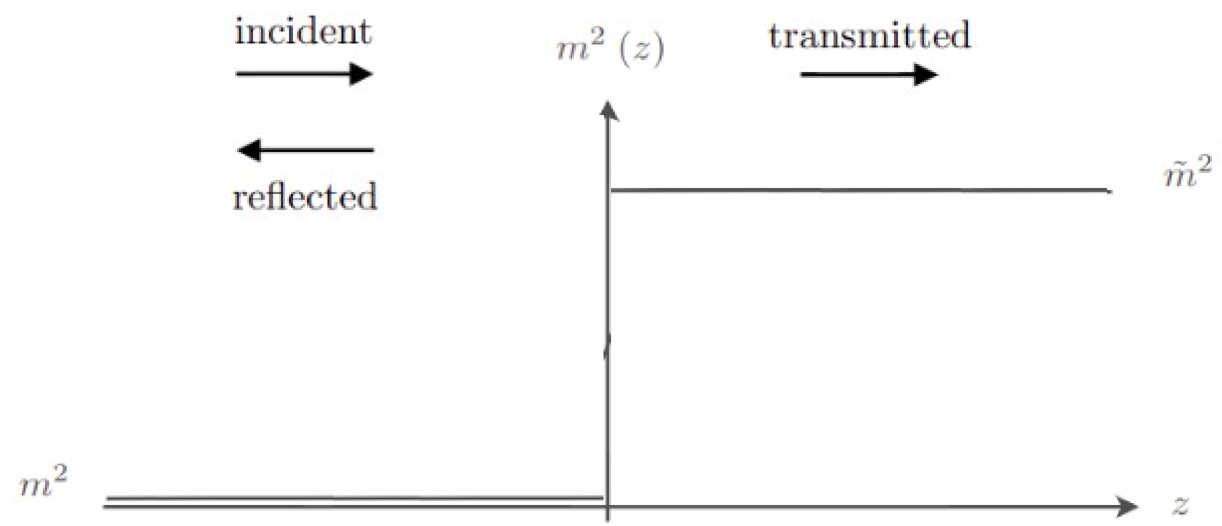

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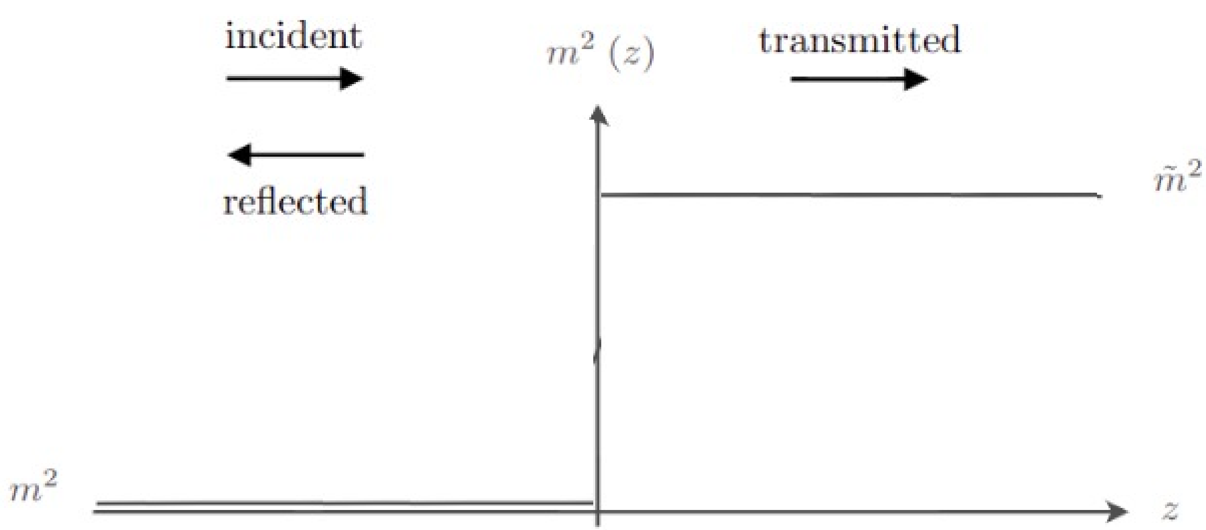

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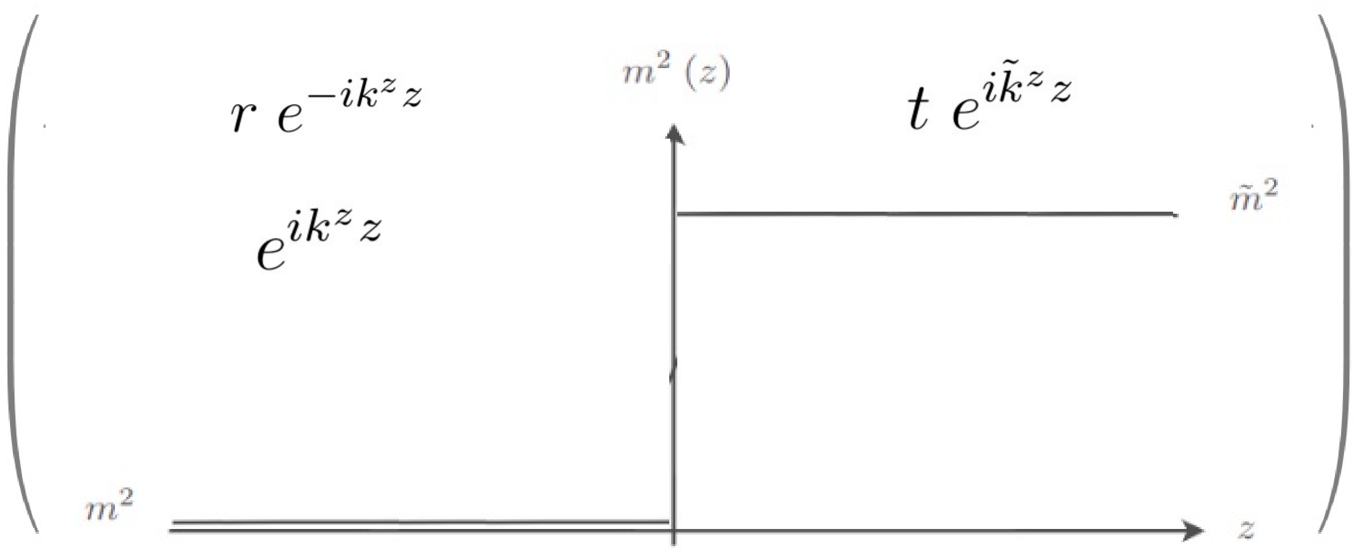

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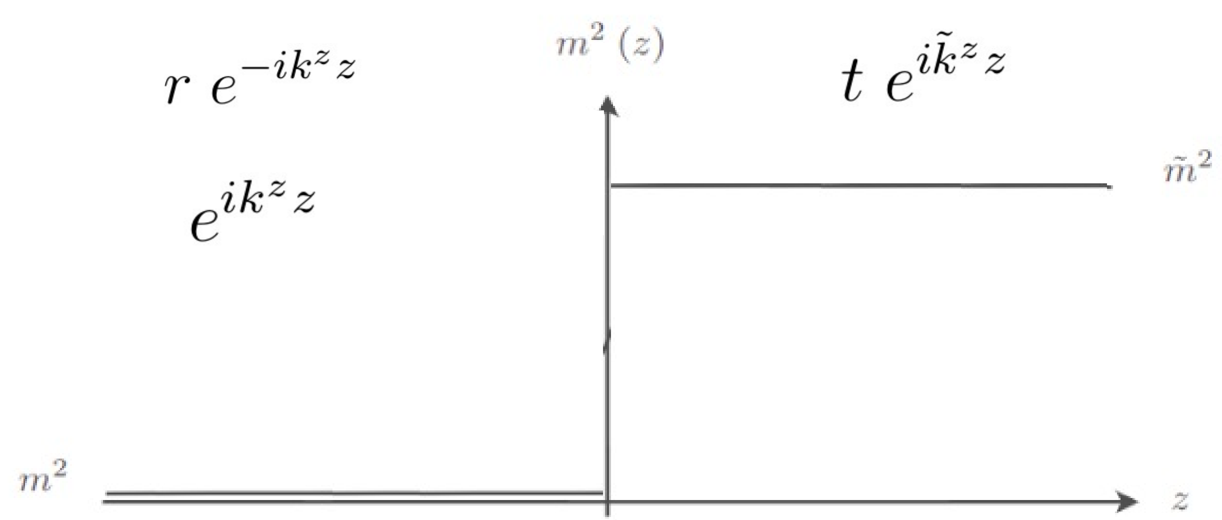

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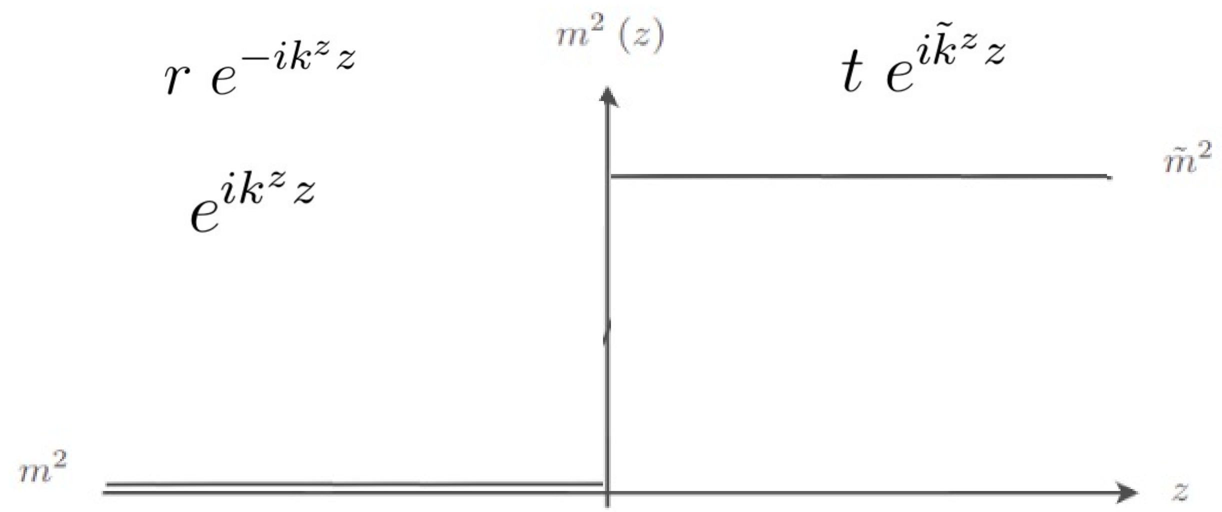

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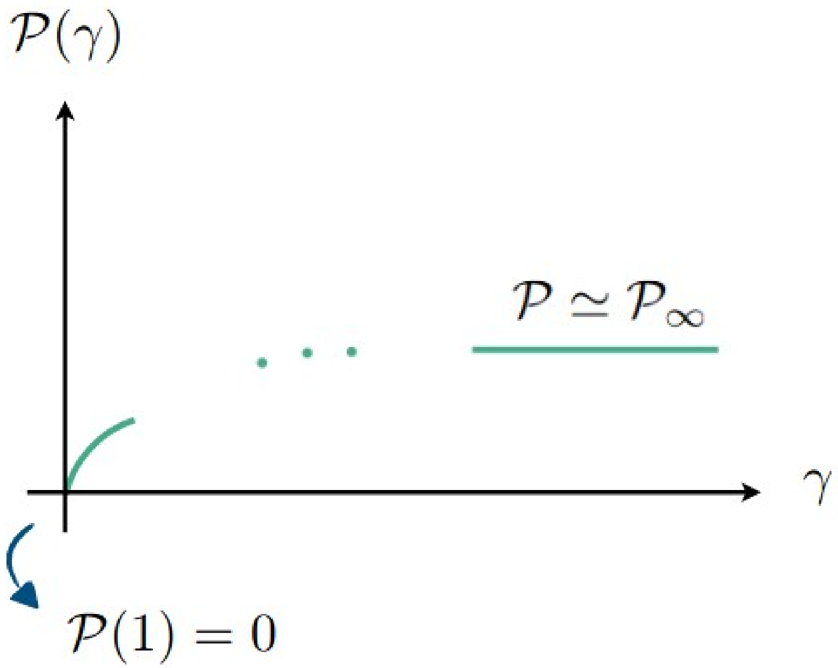
Total Pressure

$$\mathcal{P}_{\infty,\mathrm{LO}} = \sum_{\mathrm{d.o.f.}\ j} \left(\gamma\ n_j \frac{\Delta m_j^2}{2\ \omega_j} \right) \sim \sum_{\mathrm{d.o.f.}\ j} T^2 \Delta m_j^2$$

Runaway Criterion



Bödeker, Moore [0903.4099]

Run-away criterion:

$$\Delta V > \mathcal{P}_\infty \quad \Rightarrow \quad \text{run-away}$$

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implicit assumption that frictional pressure increases monotonically with bubble wall speed…

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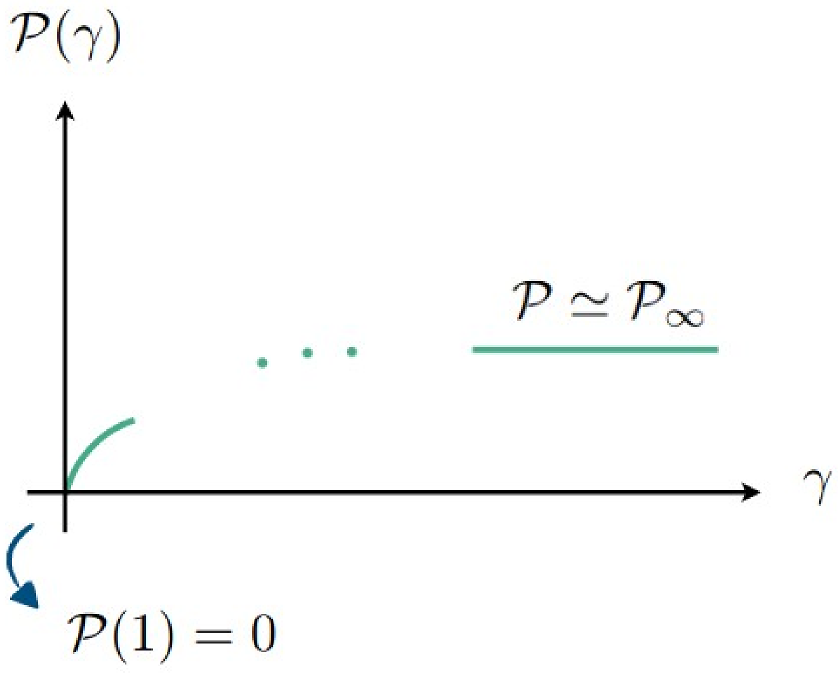

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In the presence of massive dark photons with phase-dependent mass, interesting dynamics can lead to a pressure maximum at intermediate γ-factors, and a much stronger run-away condition

Model Scenario

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)}_{\text{scalar sector}} \underbrace{- \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2 V_\mu V^\mu}_{\text{massive dark photon}} + \boxed{\frac{\kappa}{2}\phi^2 V_\mu V^\mu} + \cdots$$

cannot be forbidden by symmetries

$$\Rightarrow \quad m_V^2 = m^2 + \underbrace{\kappa \langle \phi \rangle^2}_{\Delta m^2}$$

true vacuum

$$\langle \phi \rangle \equiv v \neq 0$$

false vacuum

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<u>Here</u>: focus on $\Delta m^2/m^2 \ll 1$ so that $\Lambda \gg 4\pi v$ (more towards the end)

Massive Vector

Our problem reduces to solving the EOM for massive E&M with a varying photon mass

$$m_\gamma^2 = \text{const.}$$

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = 0 \quad \text{and} \quad \partial_\mu A^\mu = 0$$

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$$\varepsilon_x^\mu = (0,1,0,0), \;\; \varepsilon_y^\mu = (0,0,1,0) \quad \text{and} \quad \varepsilon_l^\mu = \left(\frac{k_z}{m_\gamma}, 0, 0, \frac{\omega}{m_\gamma}\right)$$

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$$\partial_\mu A^\mu = -\frac{\partial_z m_\gamma^2}{m^2} A^3 \quad \Rightarrow \quad \partial_\mu A_\perp^\mu = 0 \quad \text{and} \quad \partial_\mu A_l^\mu = -\frac{\partial_z m_\gamma^2}{m_\gamma^2} A_l^3$$

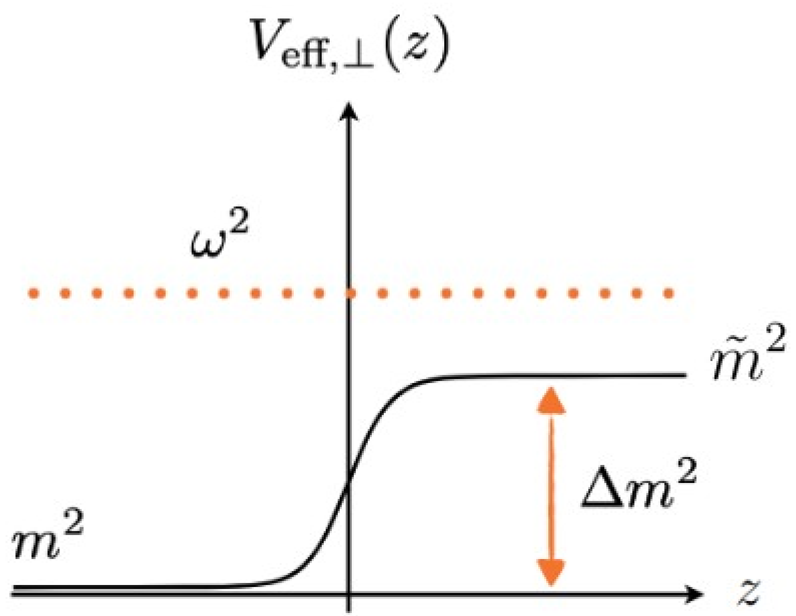
Transverse Polarizations

'Lorentz' condition $\qquad \partial_\mu A^\mu_\perp = 0$

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = 0$$

$$A^\mu_\perp(t,z) = e^{-i\omega t}\, a^\mu_\perp(z)$$

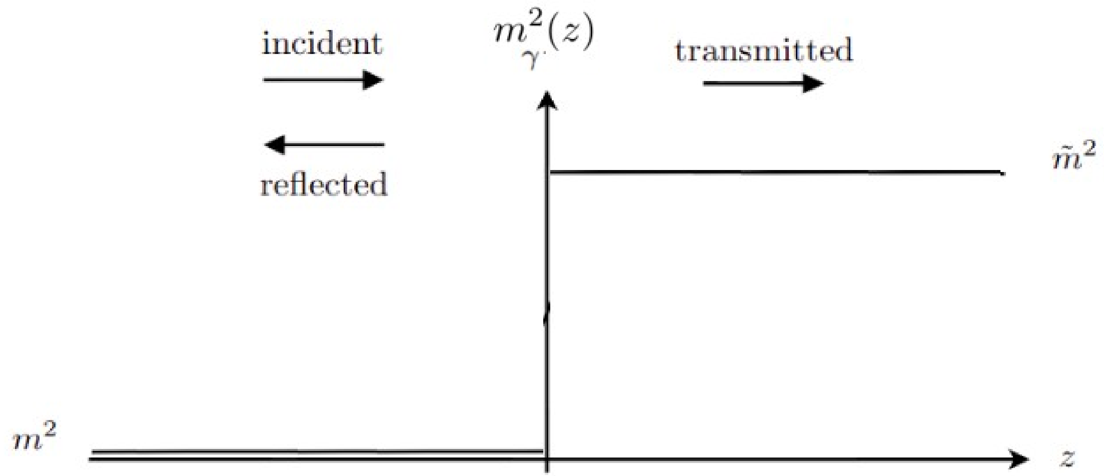
$$\Rightarrow \quad (\Box + m_\gamma^2) A^\mu_\perp = 0 \qquad \Rightarrow \qquad \boxed{\left(-\partial_z^2 + m_\gamma^2\right) a^\mu_\perp = \omega^2 a^\mu_\perp}$$



Just like scalar case!

Longitudinal Polarization: Step function

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = 0$$



Longitudinal Polarization: Step function

Write down solutions for $z < 0 \;\; \& \;\; z > 0$

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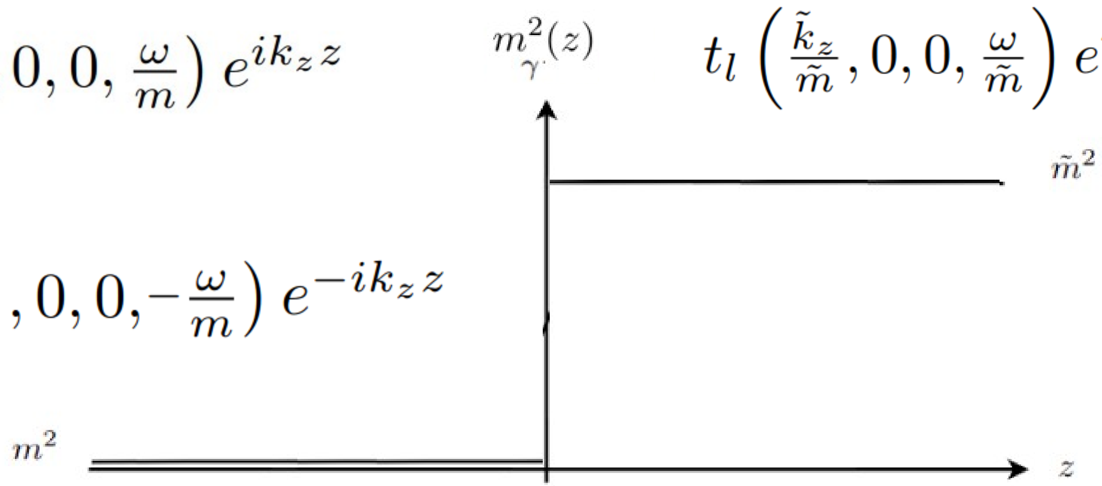
$$\partial_\mu \left(m_\gamma^2(x) A^\mu \right) = 0$$

$$A_l^\mu = e^{-i\omega t}\begin{pmatrix} \left(\frac{k_z}{m},0,0,\frac{\omega}{m}\right) e^{ik_z z} & t_l\left(\frac{\tilde{k}_z}{\tilde{m}},0,0,\frac{\omega}{\tilde{m}}\right) e^{i\tilde{k}_z z} \\ r_l\left(\frac{k_z}{m},0,0,-\frac{\omega}{m}\right) e^{-ik_z z} & \end{pmatrix}$$



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Derive matching conditions

$$\partial_\mu \left(m_\gamma^2(x) A^\mu \right) = 0$$

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{+\epsilon} dz \, \partial_\mu \left(m_\gamma^2(z) A^\mu \right) = 0$$

$m_\gamma^2(z) A^3$ remains continuous at $z = 0$

$$m(1 - r_l) = \tilde{m} \, t_l$$

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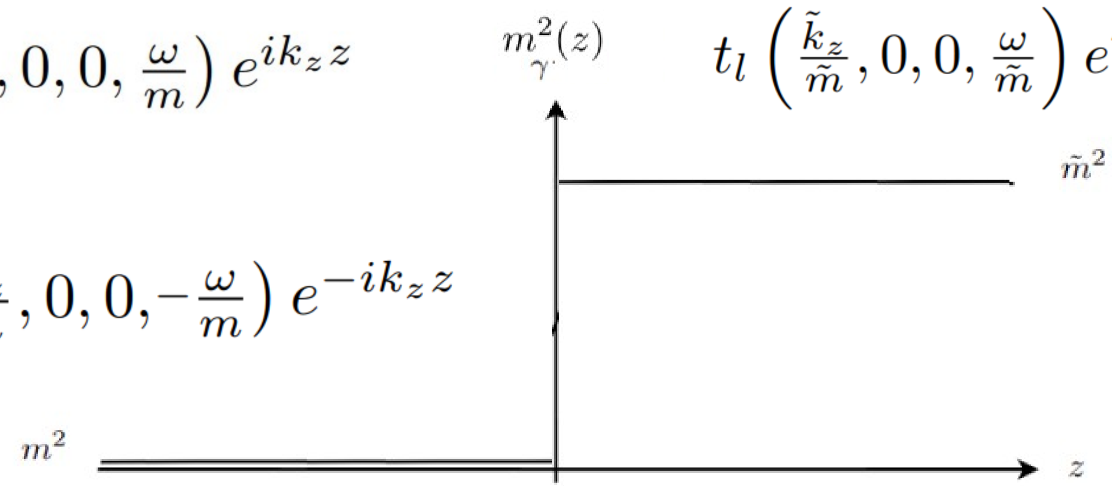

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$$\underbrace{\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = 0}_{\nu = z}$$

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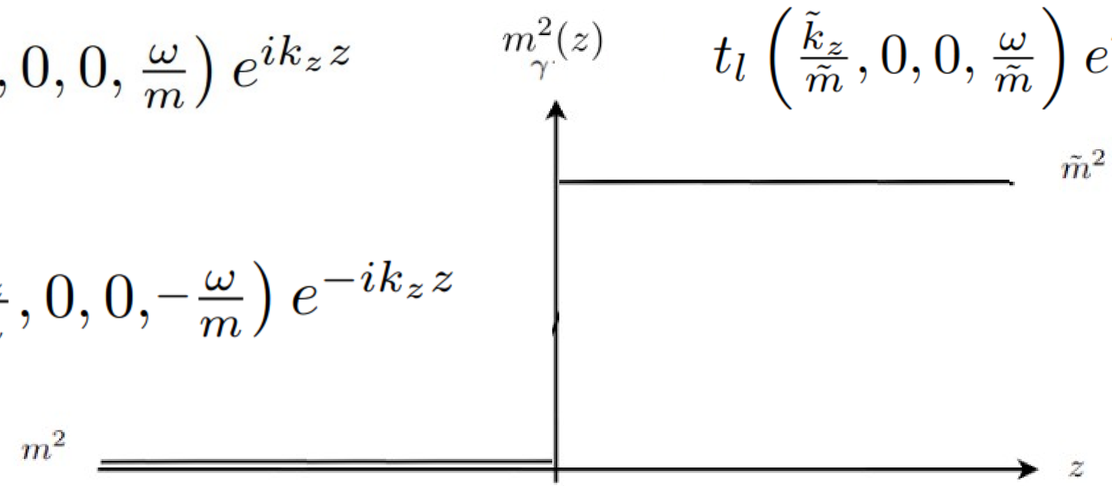
$\partial_t A^0$ remains continuous at $z = 0$

$$\frac{k^z}{m}(1 + r_l) = t_l \frac{\tilde{k}^z}{\tilde{m}}$$

$$A_l^\mu = e^{-i\omega t} \left(\begin{array}{cc} \left(\frac{k_z}{m}, 0, 0, \frac{\omega}{m} \right) e^{i k_z z} & t_l \left(\frac{\tilde{k}_z}{\tilde{m}}, 0, 0, \frac{\omega}{\tilde{m}} \right) e^{i \tilde{k}_z z} \\ r_l \left(\frac{k_z}{m}, 0, 0, -\frac{\omega}{m} \right) e^{-i k_z z} & \end{array} \right)$$



Longitudinal Polarization: Step function

Write down solutions for $z<0$ & $z>0$

$$\underbrace{\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = 0}_{\nu = z}$$

Derive matching conditions

$$\lim_{\epsilon\to 0}\int_{-\epsilon}^{+\epsilon} dz\, \partial_\mu \left(m_\gamma^2(z) A^\mu\right) = 0$$

$m_\gamma^2(z) A^3$ remains continuous at $z=0$

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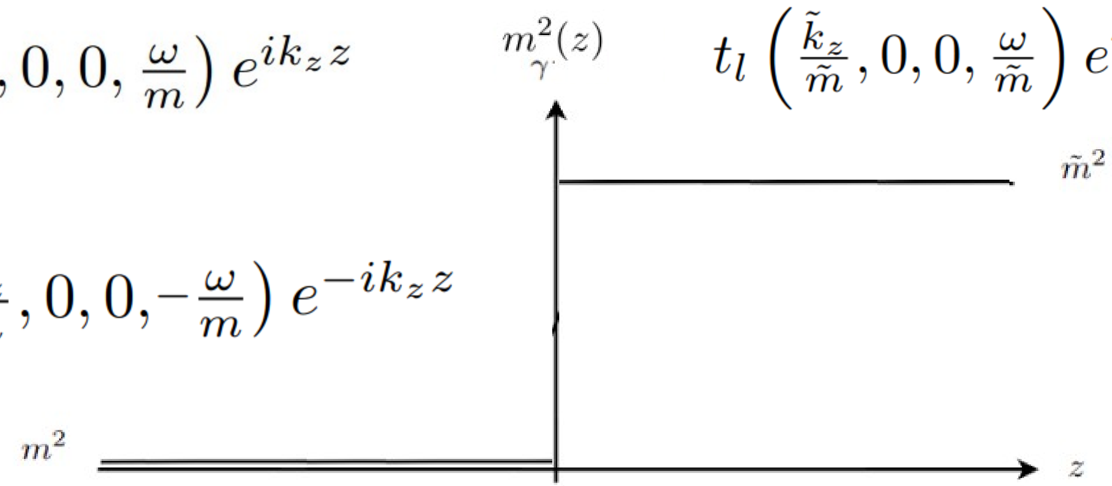
Reflection probability

$$R_l = \left|\frac{\tilde{m}^2\, k_z - m^2\, \tilde{k}_z}{\tilde{m}^2\, k_z + m^2\, \tilde{k}_z}\right|^2 \xrightarrow{\omega\gg m,\tilde{m}} \left(\frac{\tilde{m}^2 - m^2}{\tilde{m}^2 + m^2}\right)^2 \simeq \left(\frac{\Delta m^2}{2m^2}\right)^2$$

$$A_l^\mu = e^{-i\omega t}\begin{pmatrix} \left(\frac{k_z}{m}, 0, 0, \frac{\omega}{m}\right) e^{ik_z z} & t_l \left(\frac{\tilde{k}_z}{\tilde{m}}, 0, 0, \frac{\omega}{\tilde{m}}\right) e^{i\tilde{k}_z z} \\ r_l \left(\frac{k_z}{m}, 0, 0, -\frac{\omega}{m}\right) e^{-ik_z z} & \end{pmatrix}$$



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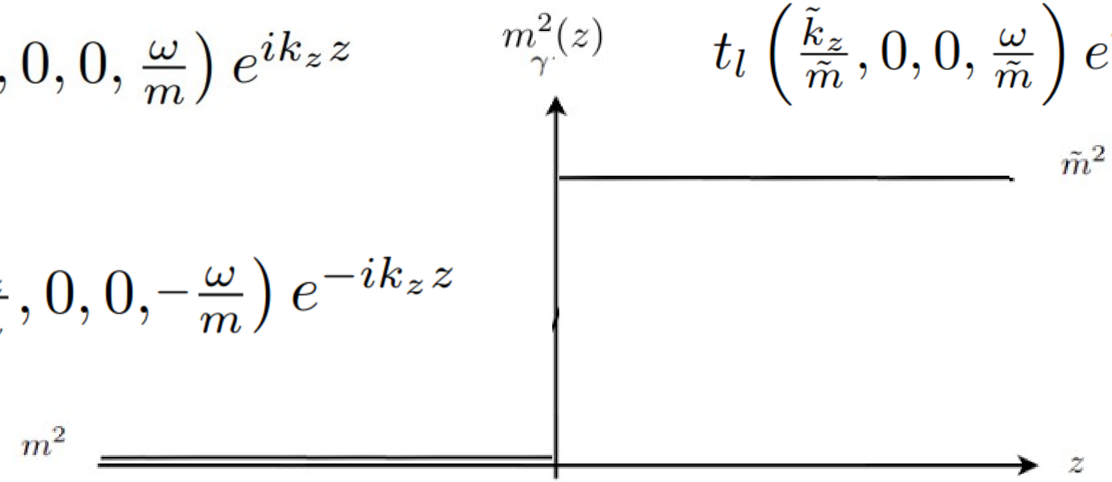
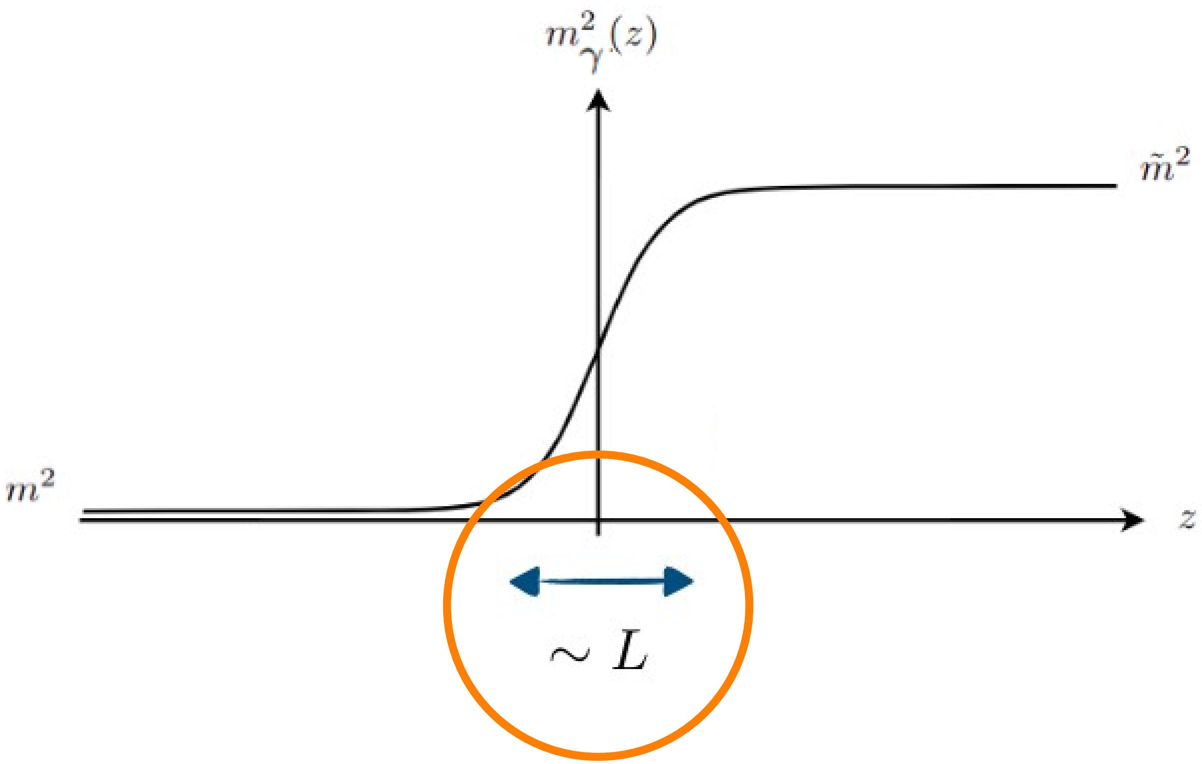
!?

$$A_l^\mu = e^{-i\omega t} \left(\left(\frac{k_z}{m}, 0, 0, \frac{\omega}{m}\right) e^{ik_z z} \quad t_l \left(\frac{\tilde{k}_z}{\tilde{m}}, 0, 0, \frac{\omega}{\tilde{m}}\right) e^{i\tilde{k}_z z} \quad r_l \left(\frac{k_z}{m}, 0, 0, -\frac{\omega}{m}\right) e^{-ik_z z} \right)$$



Longitudinal: Smooth Wall



Smooth wall: Longitudinal

$$A_l^3(t,z) = e^{-i\omega t}\,\frac{\omega}{m_\gamma(z)}\,a_l(z)$$

$$(\Box + m_\gamma^2)A_l^\mu - \partial^\mu(\partial_\nu A_l^\nu) = 0 \quad \Rightarrow$$

Effective Schrodinger equation

$$\left(-\partial_z^2 + m_\gamma^2 + \frac{3}{4}\left(\frac{\partial_z m_\gamma^2}{m_\gamma^2}\right)^2 - \frac{1}{2}\frac{\partial_z^2 m_\gamma^2}{m_\gamma^2}\right) a_l = \omega^2 a_l$$

Smooth wall: Longitudinal

$$A_l^3(t,z) = e^{-i\omega t}\frac{\omega}{m_\gamma(z)}a_l(z)$$

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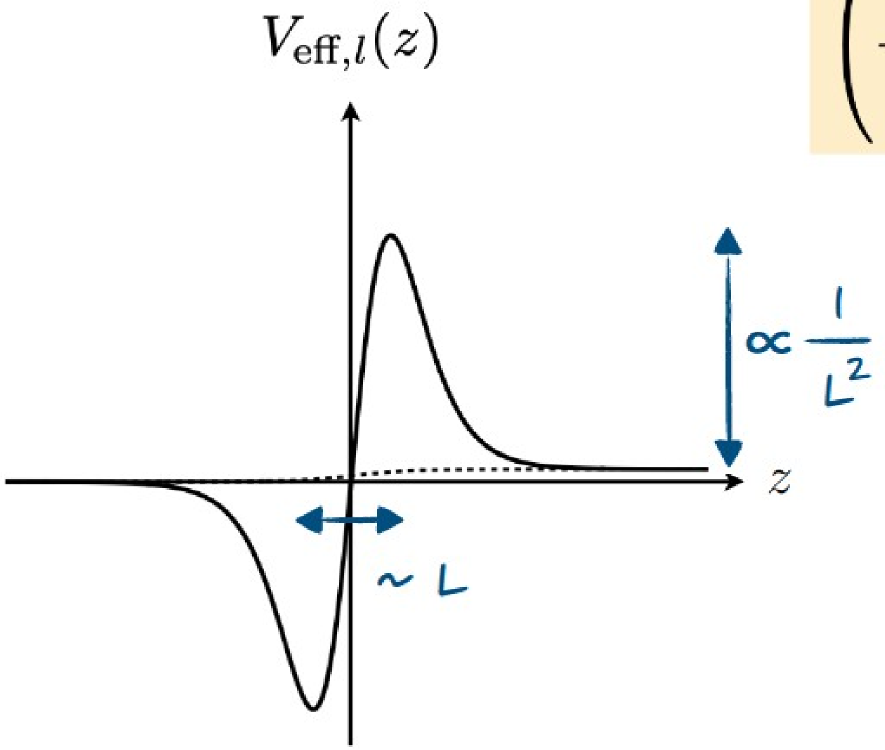
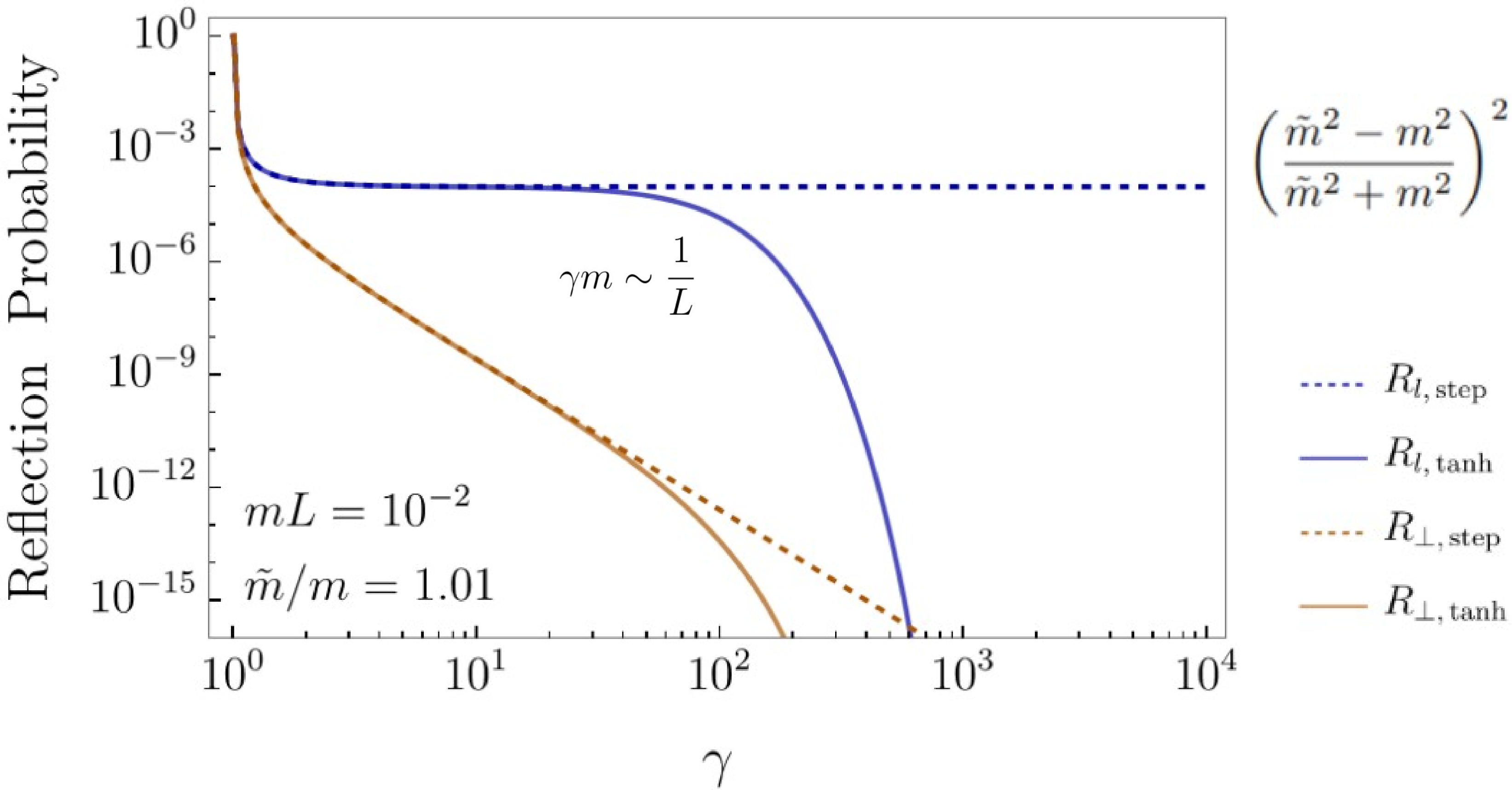


Numerical Analysis

e.g. $$m_\gamma^2(z) = \frac{1}{2}\left(m^2+\tilde{m}^2\right) - \frac{1}{2}\left(m^2-\tilde{m}^2\right)\tanh\left(z/L\right)$$



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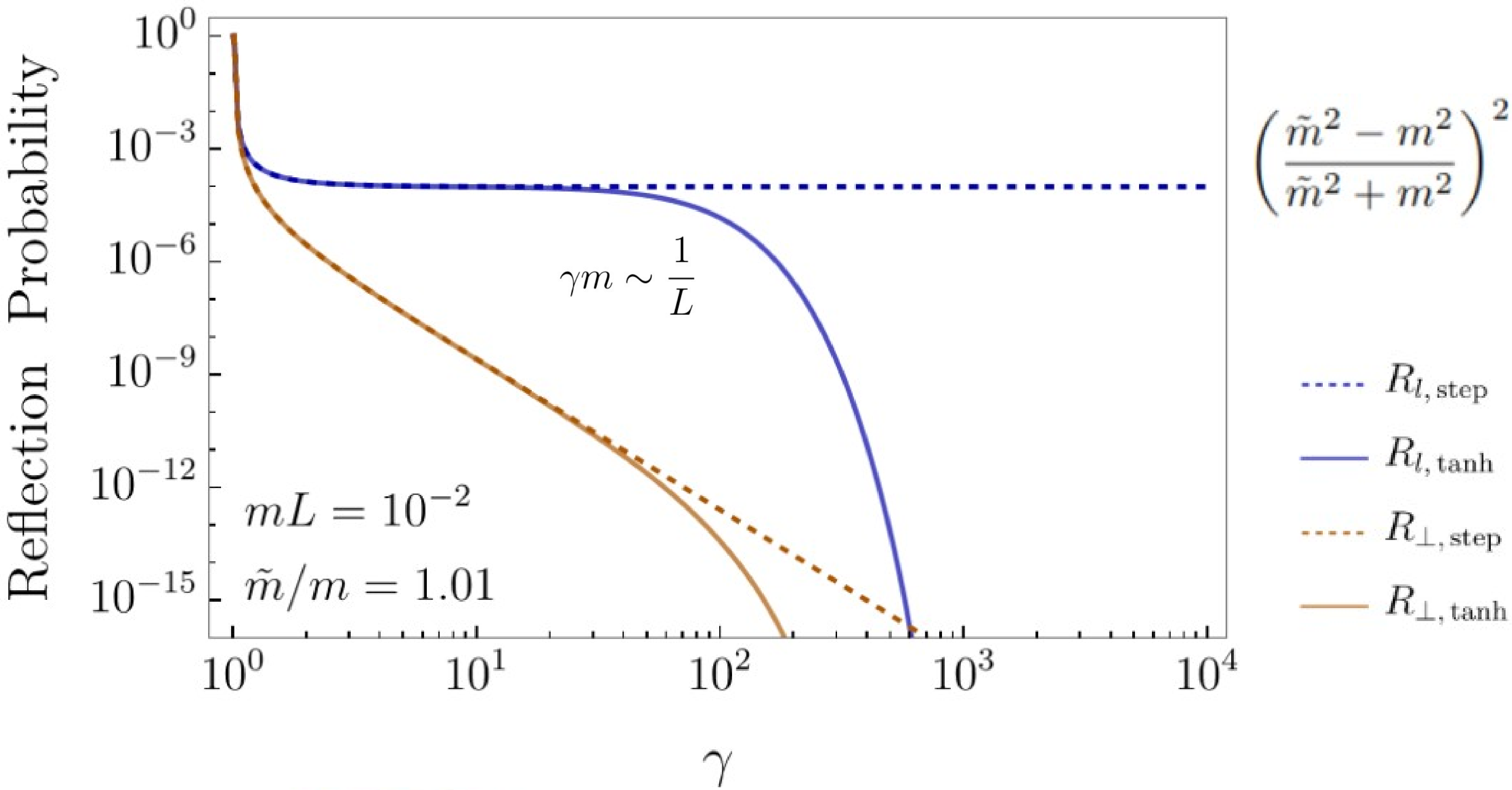


In the regime $1 \ll \gamma \ll \frac{1}{m\,L}$ $\Rightarrow$ $R_l \simeq \left(\frac{\tilde{m}^2 - m^2}{\tilde{m}^2 + m^2}\right)^2 \simeq \left(\frac{\Delta m^2}{2m^2}\right)^2$

"inter-relativistic"

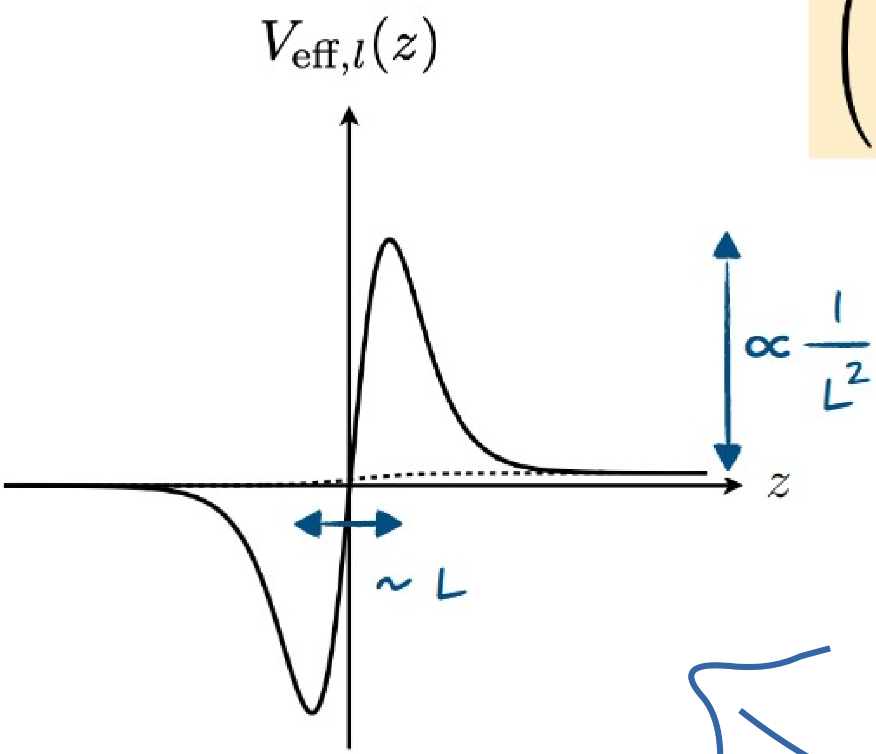
requires $m\,L \ll 1$

constant independent of γ !!

Smooth wall: Longitudinal

$$A_l^3(t,z) = e^{-i\omega t}\,\frac{\omega}{m_\gamma(z)}\,a_l(z)$$

$$(\Box + m_\gamma^2)A_l^\mu - \partial^\mu(\partial_\nu A_l^\nu) = 0 \quad \Rightarrow$$



$$\left(-\partial_z^2 + m_\gamma^2 + \frac{3}{4}\left(\frac{\partial_z m_\gamma^2}{m_\gamma^2}\right)^2 - \frac{1}{2}\frac{\partial_z^2 m_\gamma^2}{m_\gamma^2}\right) a_l = \omega^2 a_l$$

$$\sim \left(\frac{(\Delta m^2)^2}{m^4 L^2}\right) \qquad \sim \left(\frac{\Delta m^2}{m^2 L^2}\right)$$

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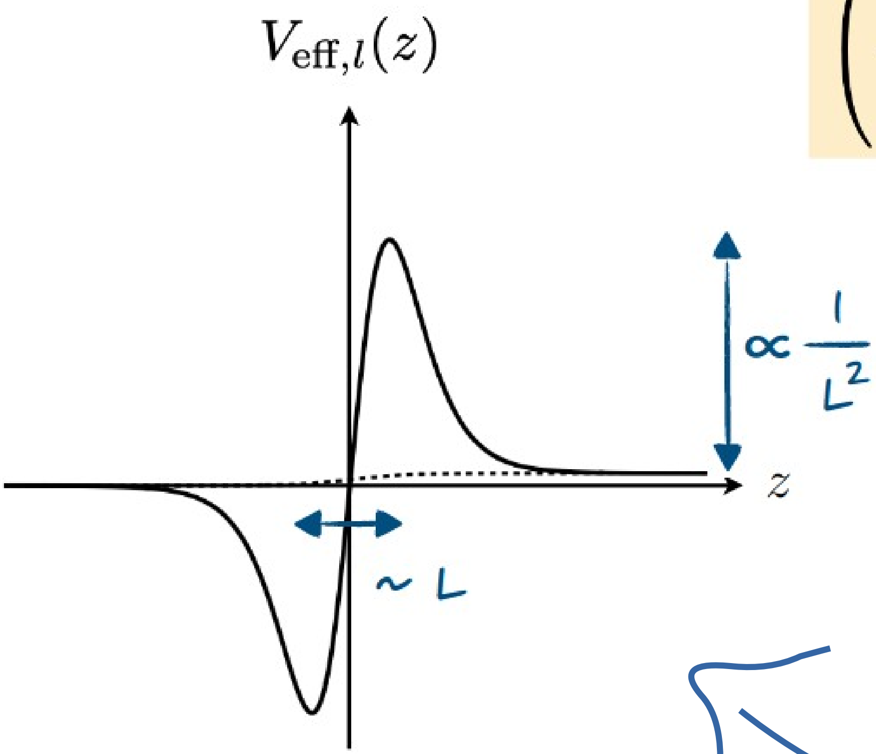

$$V_{\mathrm{eff},l} \simeq -\frac{1}{2}\left(\frac{\partial_z^2 m_\gamma^2(z)}{m_\gamma^2(z)}\right) \simeq -\frac{\Delta m^2}{2m^2}\Theta_L''(z) \qquad \Theta_L''(z) \xrightarrow{L\to 0} \delta'(z)$$

The Born Approximation

Can prove for general wall profile

$$R_{l,\,\mathrm{Born}} = \frac{1}{4k_z^2}\left|\int_{-\infty}^{\infty} dz\; e^{2ik_z z}\;\; V_{\mathrm{eff},l}\right|^2$$

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$$R_{l,\,\mathrm{Born}} = \frac{1}{4k_z^2}\left|\int_{-\infty}^{\infty} dz\ e^{2ik_z z}\ V_{\mathrm{eff},l}\right|^2$$

$$\simeq \frac{1}{4k_z^2}\left(\frac{\Delta m^2}{2m^2}\right)^2\left|\left[e^{2ik_z z}\ \Theta_L'(z)\right]_{-\infty}^{+\infty} - 2ik_z\int_{-\infty}^{\infty} dz\ e^{2ik_z z}\ \Theta_L'(z)\right|^2$$

$$\simeq \left(\frac{\Delta m^2}{2m^2}\right)^2\left|\int_{-L}^{L} dz\ (1 + 2ik_z z + \ldots\quad)\ \Theta_L'(z)\right|^2$$

$$= \left(\frac{\Delta m^2}{2m^2}\right)^2\left[1 + \mathcal{O}(k_z^2\ L^2)\right]$$

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$$= \left(\frac{\Delta m^2}{2m^2}\right)^2\left[1 + \mathcal{O}(k_z^2\;L^2)\right]$$

e.g. For the tanh profile $$R_{l,\,\mathrm{Born}} = \left(\frac{\Delta m^2}{2m^2}\right)^2 \frac{\pi^2 (k_z L)^2\,\left(1 + (k_z L)^2\right)}{\sinh^2(\pi k_z L)}\;.$$

Dark Photons Pressure

Pressure from a population of cold dark photons (e.g. dark matter)

$$\mathcal{P} = \underbrace{\gamma v n_\gamma}_{\text{flux}} \times \frac{1}{3} \overbrace{\sum_\lambda}^{\text{sum over polarizations}} (R_\lambda \, \Delta k_R + T_\lambda \, \Delta k_T) \simeq \frac{2}{3}\gamma^2 \rho_\gamma R_l + \underbrace{\frac{1}{2}\rho_\gamma \frac{\Delta m^2}{m^2}}_{=\mathcal{P}_\infty} + \cancel{\frac{4}{3}\gamma^2 \rho_\gamma R_\perp}$$

ignore

$\Delta k_R = 2\ k^z$

$R_l \propto \gamma^0$

$$\boxed{1 \ll \gamma \ll (mL)^{-1}}$$

"inter-relativistic"

Dark Photons Pressure

$$\mathcal{P} = \gamma|\vec{v}|n_V \times \frac{1}{3}\sum_\lambda (R_\lambda\,\Delta k_R + T_\lambda\,\Delta k_T)$$

$R_\lambda + T_\lambda = 1,\ \gamma \gg 1$

$$\simeq \underbrace{\frac{2}{3}\gamma^2\rho_V R_l}_{\propto\gamma^2} + \underbrace{\frac{1}{2}\rho_V\frac{\Delta m^2}{m^2}}_{\mathcal{P}_\infty} + \underbrace{\frac{4}{3}\gamma^2\rho_V R_\perp}_{\propto\gamma^{-2}}$$

for $\boxed{1 \ll \gamma \ll (mL)^{-1}}$ "inter-relativistic"

Pressure grows $\propto \gamma^2$, and reaches maximum at $\gamma \sim (mL)^{-1}$

$\Rightarrow$ **Maximum Dynamic Pressure**

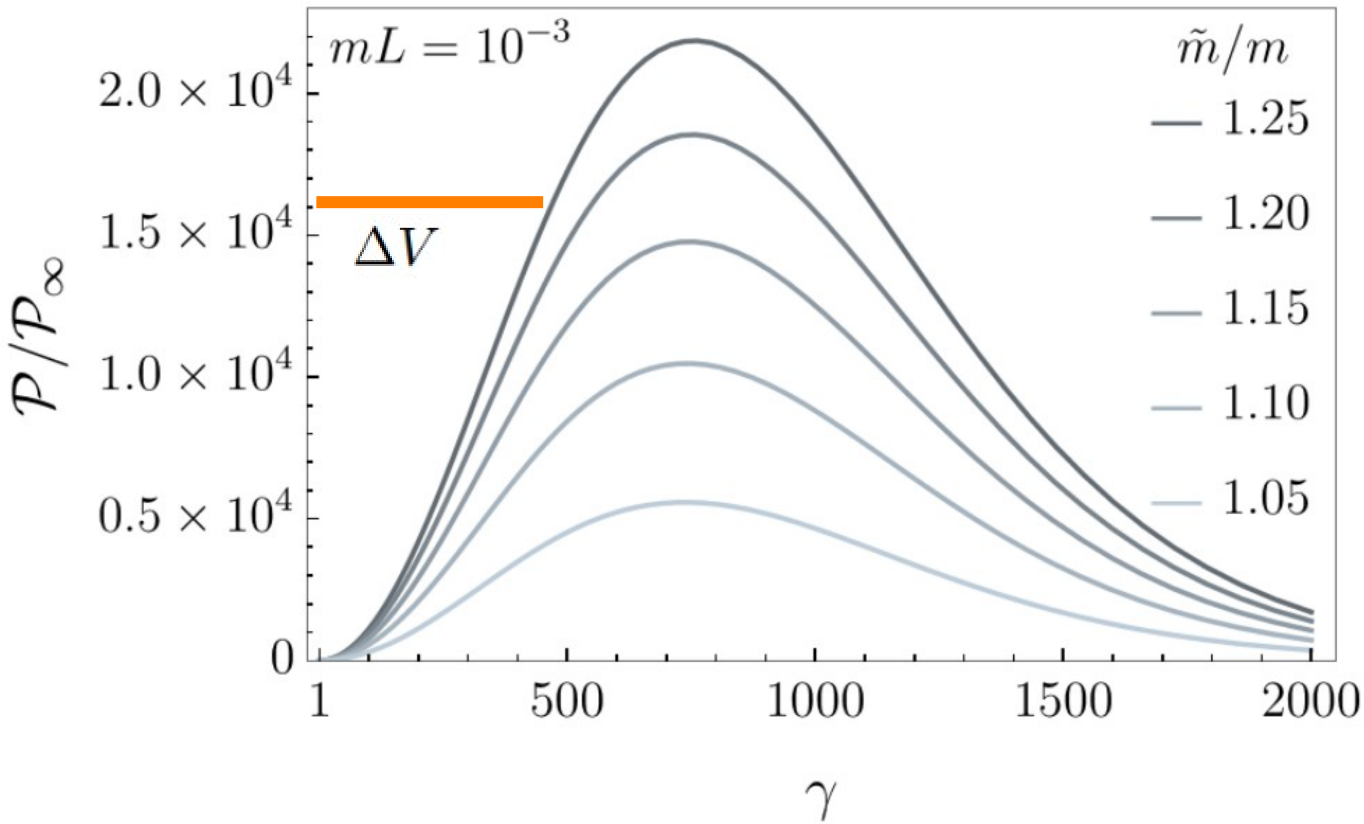
$$\mathcal{P}_{\rm mdp} \sim \frac{\rho_V}{(mL)^2}\left(\frac{\Delta m^2}{m^2}\right)^2 + \rho_V\frac{\Delta m^2}{m^2} \simeq \frac{\rho_V}{(mL)^2}\left(\frac{\Delta m^2}{m^2}\right)^2 \gg \mathcal{P}_\infty$$

$mL \ll \sqrt{\Delta m^2/m^2}$

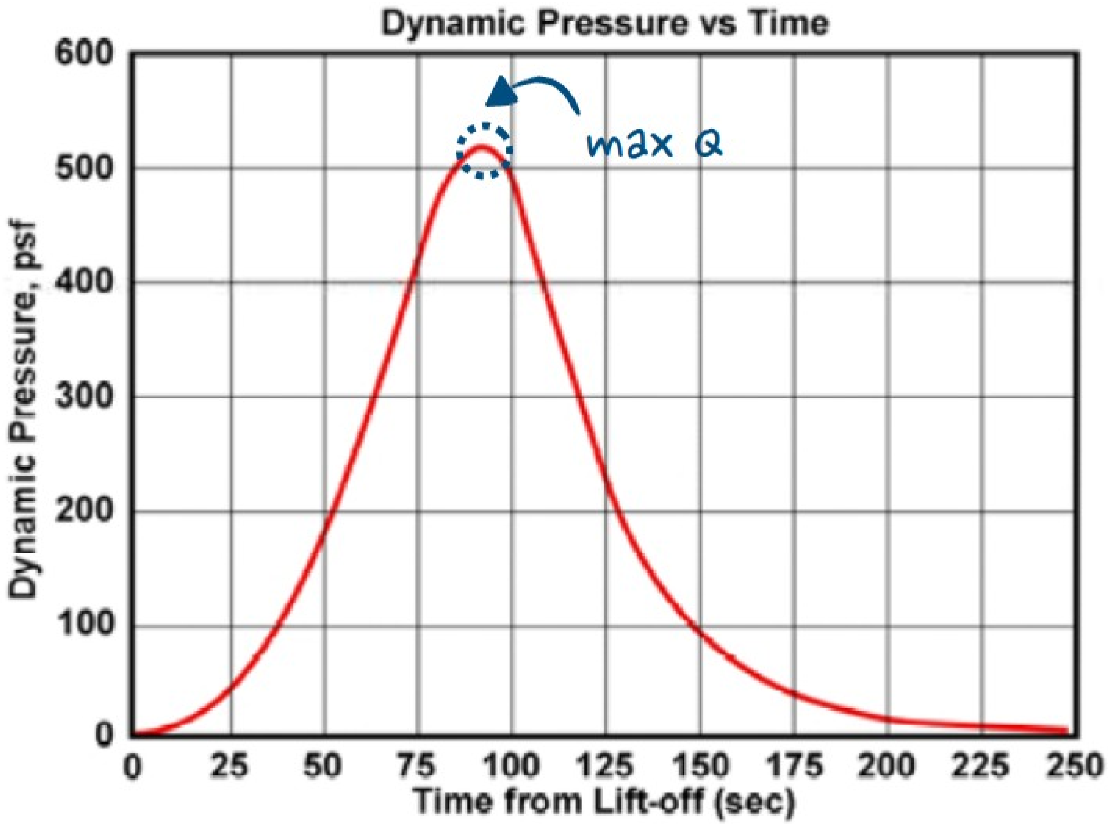
Maximum Dynamic Pressure

Run-away criterion: $\cancel{\Delta V > \mathcal{P}_\infty}$ $\longrightarrow$ $\Delta V > \mathcal{P}_{\rm mdp}$

$$\sim \frac{\rho_V}{(mL)^2}\left(\frac{\Delta m^2}{m^2}\right)^2$$



Rocket Science!



Dynamic pressure vs time for a typical ULA Atlas V launch

Credit: Atlas V user's guide

www.ulalaunch.com/docs/default-source/rockets/atlasvusersguide2010.pdf

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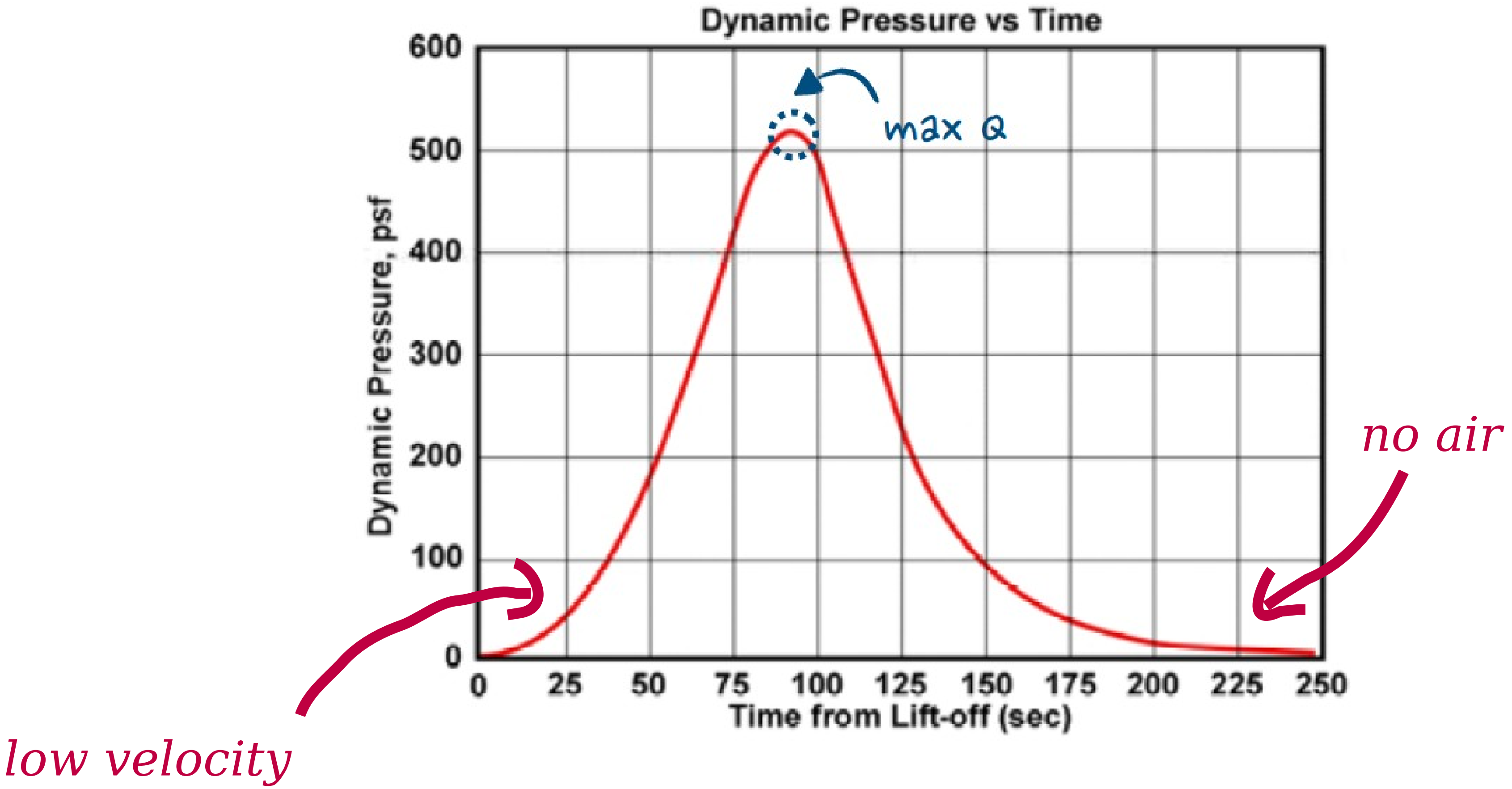

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Max Q Condition

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Max Q condition

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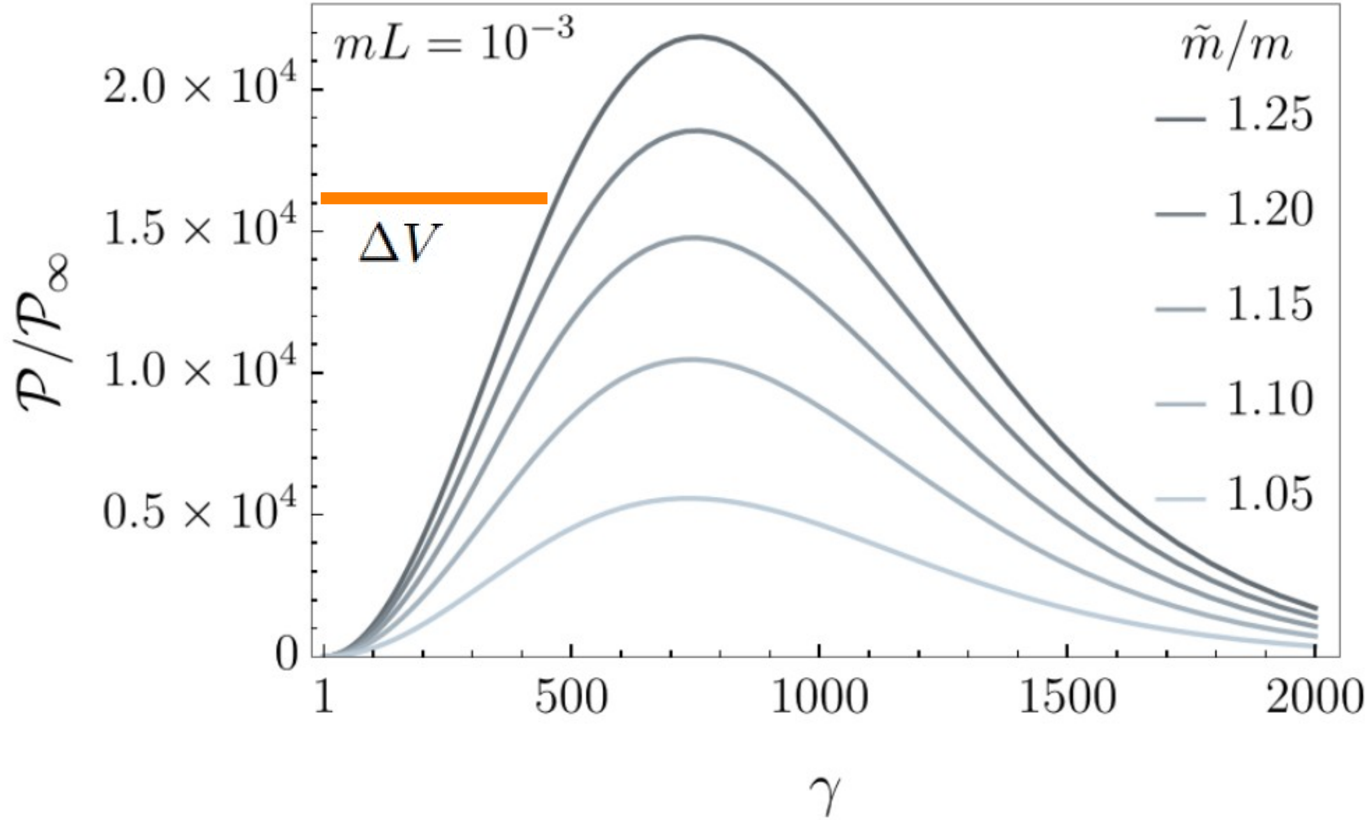

Equilibrium

If $\Delta V < \mathcal{P}_{\rm mdp}$, bubble walls will reach a terminal velocity:

$$\Delta V - \mathcal{P}(\gamma_{\rm eq}) = 0 \qquad \Rightarrow \qquad \gamma_{\rm eq} \simeq \left(\frac{3\Delta V}{2\rho_V R_l}\right)^{1/2} \sim \frac{m^2}{\Delta m^2}\left(\frac{\Delta V}{\rho_V}\right)^{1/2} \gg 1$$

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e.g.

$$\gamma_{\rm eq} \simeq 3 \times 10^6 \frac{0.1}{\Delta m^2/m^2}\left(\frac{\alpha}{0.1}\right)^{1/2}\left(\frac{T_*}{100~\text{GeV}}\right)^{1/2}\left(\frac{\rho_V}{\rho_{\rm dm}}\right)^{-1/2}$$

$$\alpha \equiv \frac{\Delta V}{\rho_{\rm SM}(T_*)} = \frac{\Delta V}{\frac{\pi^2}{30} g_*(T_*) T_*^4}$$

temperature of SM when transition takes place

Normalise to dark matter density at T_*. Conservative!

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temperature of SM when transition takes place

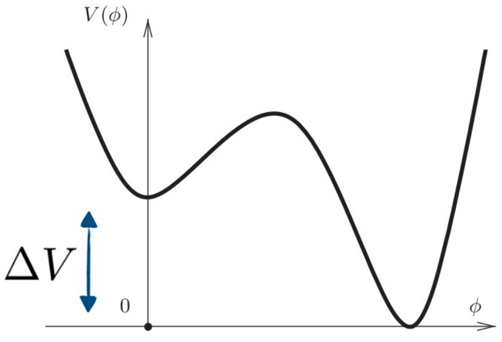
Normalise to dark matter density at T_*. Conservative!

c.f.

$$\gamma_{\rm coll.} \sim \frac{H_{\rm coll.}^{-1}}{R_c} \sim \frac{m_{\rm Pl}}{T_*^2 R_c} = 10^{16} \times \frac{100\ \rm GeV}{T_*} \times \frac{R_c^{-1}}{T_*}$$

What does it all mean?

Given a phase transition: T_* , $\alpha \equiv \frac{\Delta V}{\rho_{\rm SM}(T_*)}$, L , R_0



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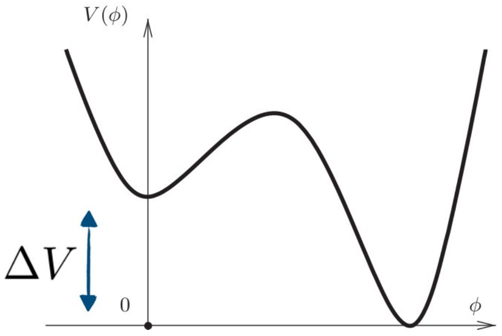
0.01



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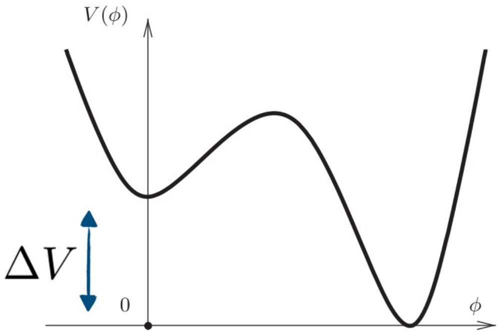

e.g.

$$L \sim T_*^{-1}$$

$$R_0 \sim 10^2 L$$

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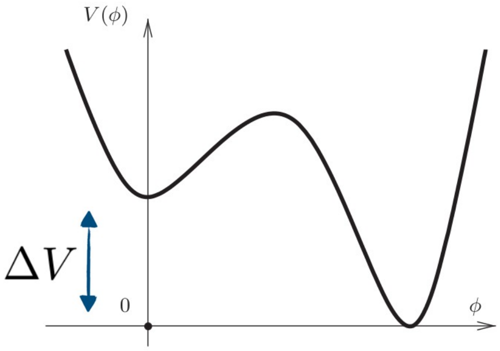


e.g.

$$L \sim T_*^{-1}$$

$$R_0 \sim 10^2 L$$

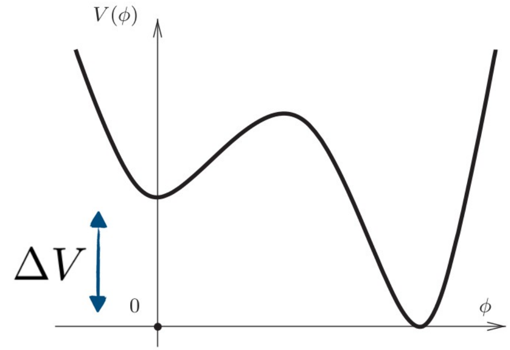
$$\gamma_{\rm eq} \sim \frac{m^2}{\Delta m^2}\left(\frac{\Delta V}{\rho_V}\right) \overset{!}{\lesssim} \min\left\{\frac{1}{mL},\ x\underbrace{\frac{H(T_*)^{-1}}{R_0}}_{\gamma_{\rm coll.}}\right\}$$

$x = 0.1$

What does it all mean?

Given a phase transition: $T_*\,,\quad \alpha \equiv \dfrac{\Delta V}{\rho_{\rm SM}(T_*)}\,,\, L\,,\, R_0$

$\alpha \to 0.01$



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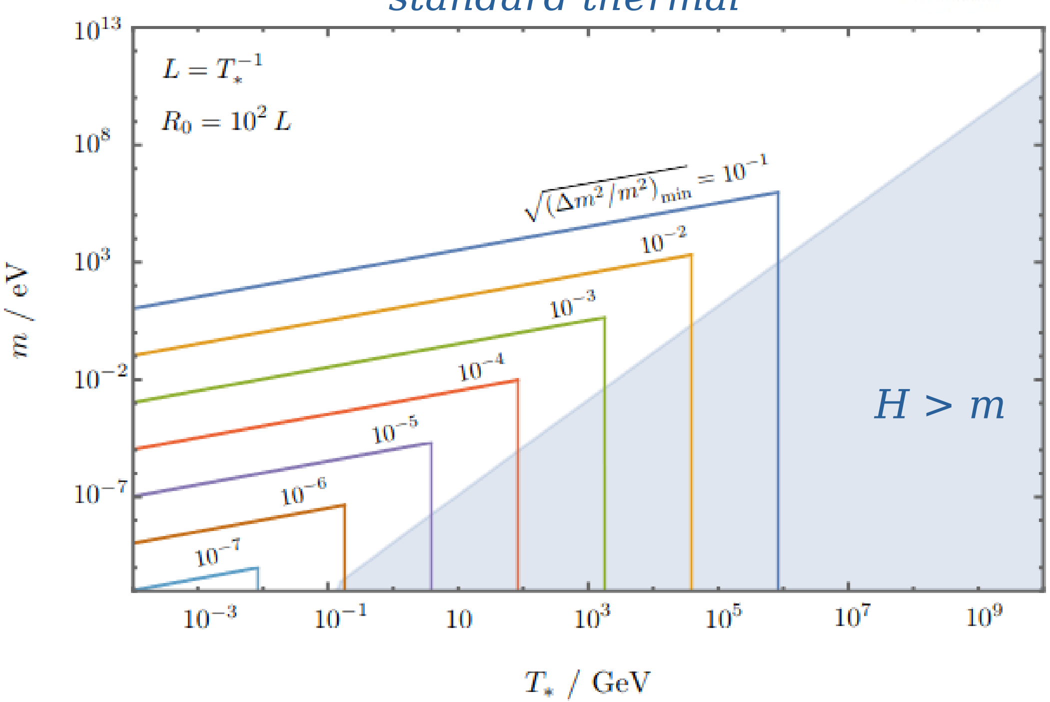
'standard thermal'



What does it all mean?

Given a phase transition: $T_*\,,\quad \alpha \equiv \dfrac{\Delta V}{\rho_{\rm SM}(T_*)}\quad, L\,, R_0$

0.01

e.g.

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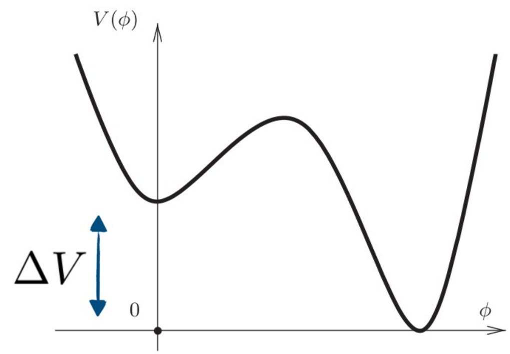

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$x = 0.1$

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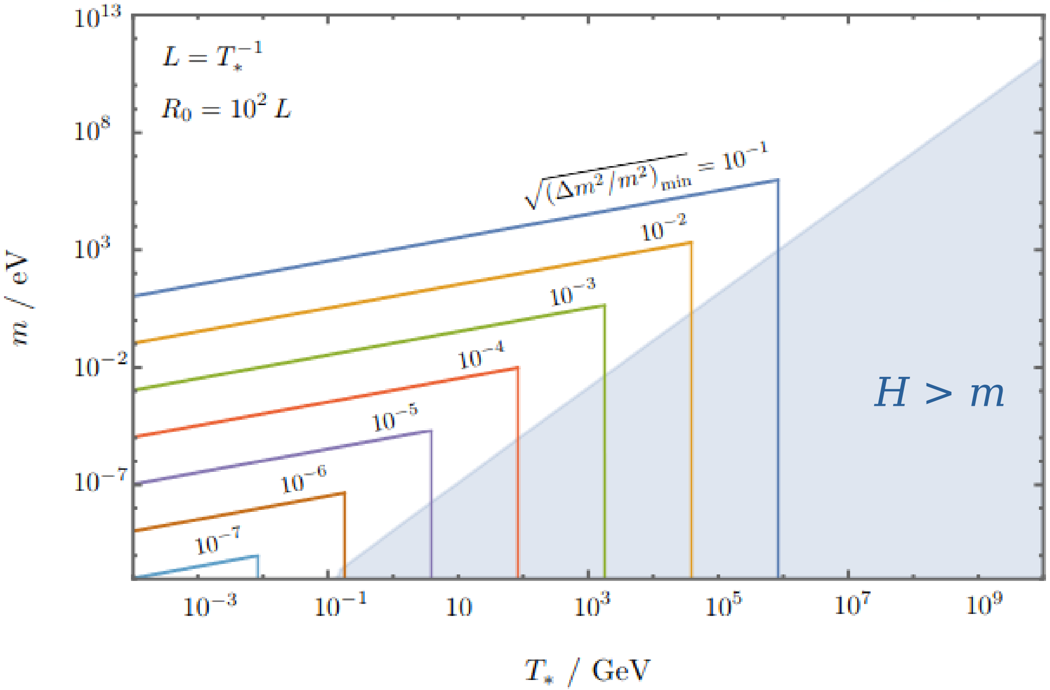
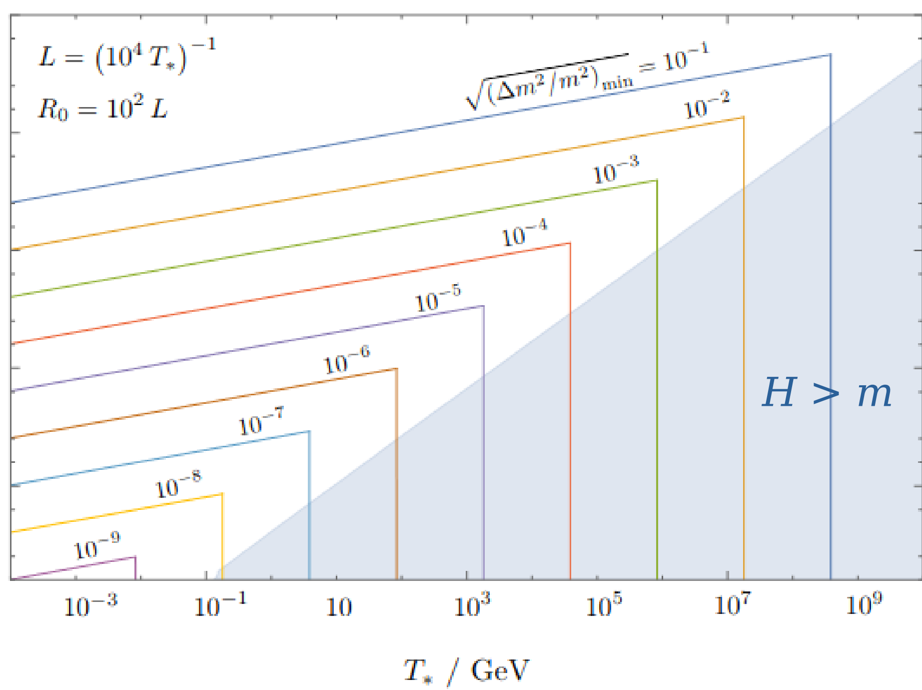


'super-cooled thermal / cold'

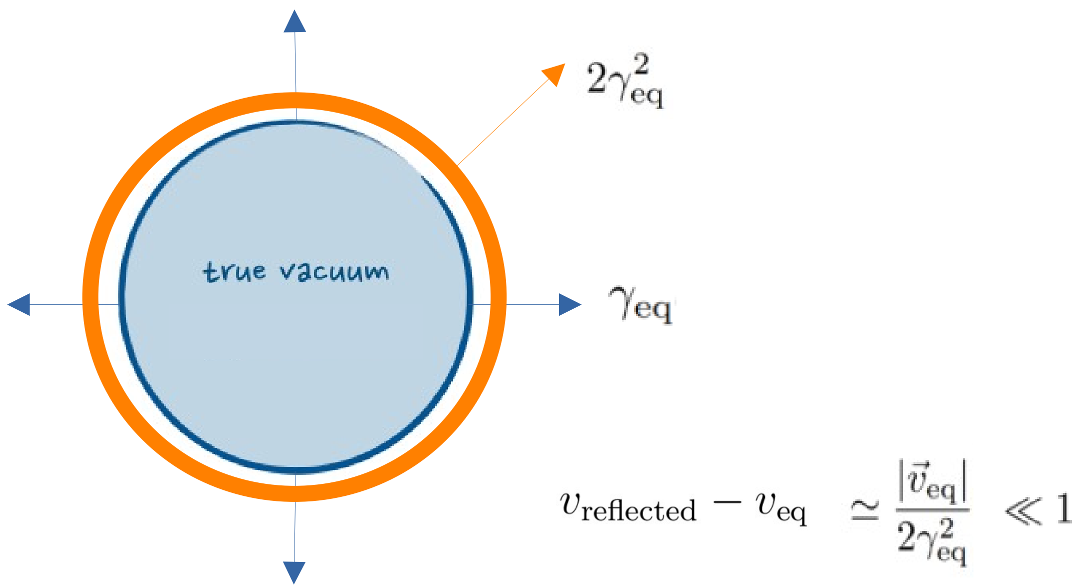


Equilibrium Dynamics

$$\frac{E_{\rm wall}}{E_{\rm total}} \simeq \frac{4\pi R(t)^2 \gamma_{\rm eq}\sigma}{\frac{4\pi}{3} R(t)^3 \Delta V} \sim \frac{\gamma_{\rm eq}\sigma}{\Delta V\, R(t)}$$

Energy goes into reflected (now relativistic) longitudinal dark photons.

Back in the dark matter frame, simple relativistic kinematics give:



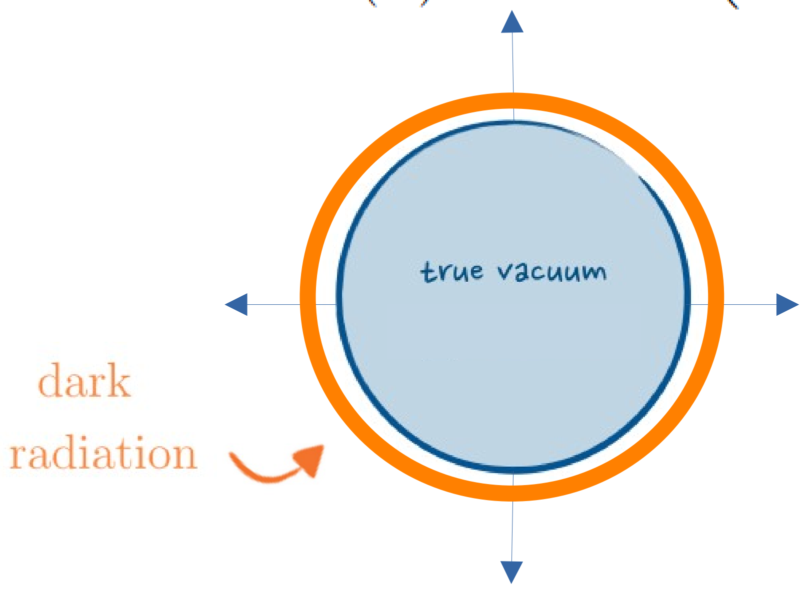
Reflected longitudinals concentrated in a thin shell

Dark Matter → Dark Radiation

A fraction of the dark photons become relativistic after collision with the bubble walls, potentially turning a fraction of the dark matter into dark radiation

BBN

$$\gamma_{\rm dr}(T \leq T_*) \simeq 2\gamma_{\rm eq}^2 \frac{a(T_*)}{a(T)} \approx 2\times 10^8 \left(\frac{0.1}{\Delta m^2/m^2}\right)^2 \left(\frac{\alpha}{0.1}\right)\left(\frac{\rho_V}{\rho_{\rm dm}}\right)^{-1}\left(\frac{T}{1\ \rm MeV}\right) \overset{!}{\gtrsim} 1$$



Dark Matter → Dark Radiation

A fraction of the dark photons become relativistic after collision with the bubble walls, potentially turning a fraction of the dark matter into dark radiation

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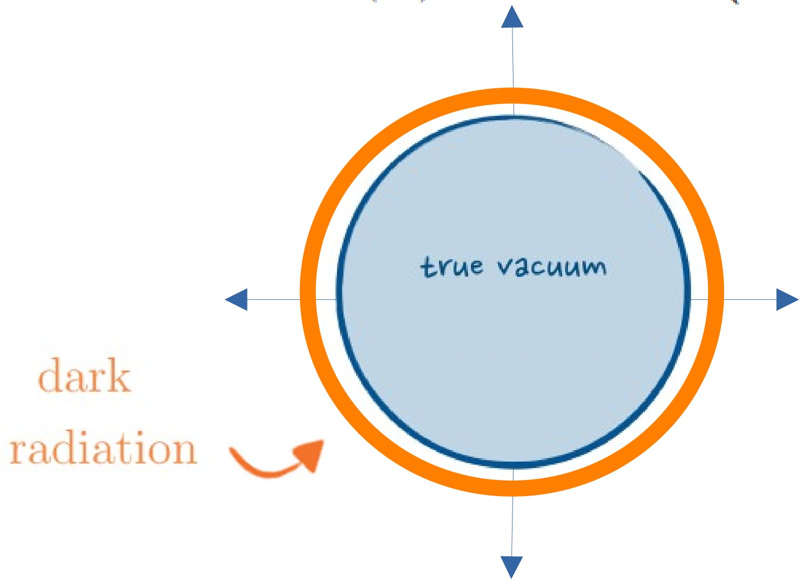
Recombination (CMB)



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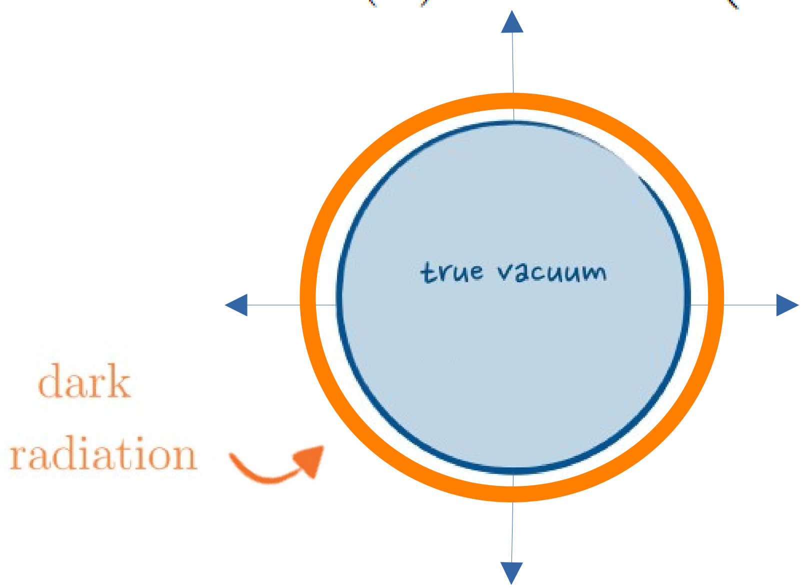

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \left(\frac{\rho_{\rm dr}(T_*)}{\rho_{\gamma,0}}\right) \left(\frac{a(T_*)}{a(T_0)}\right)^4$$

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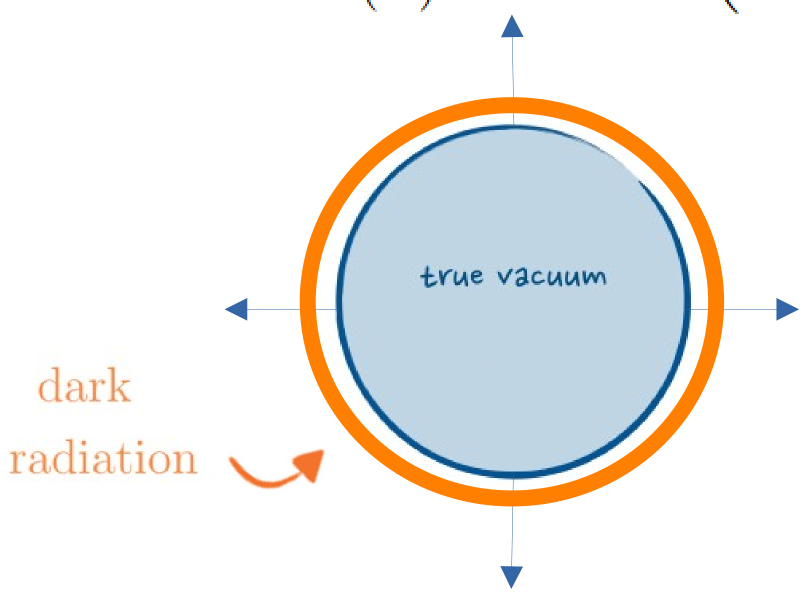

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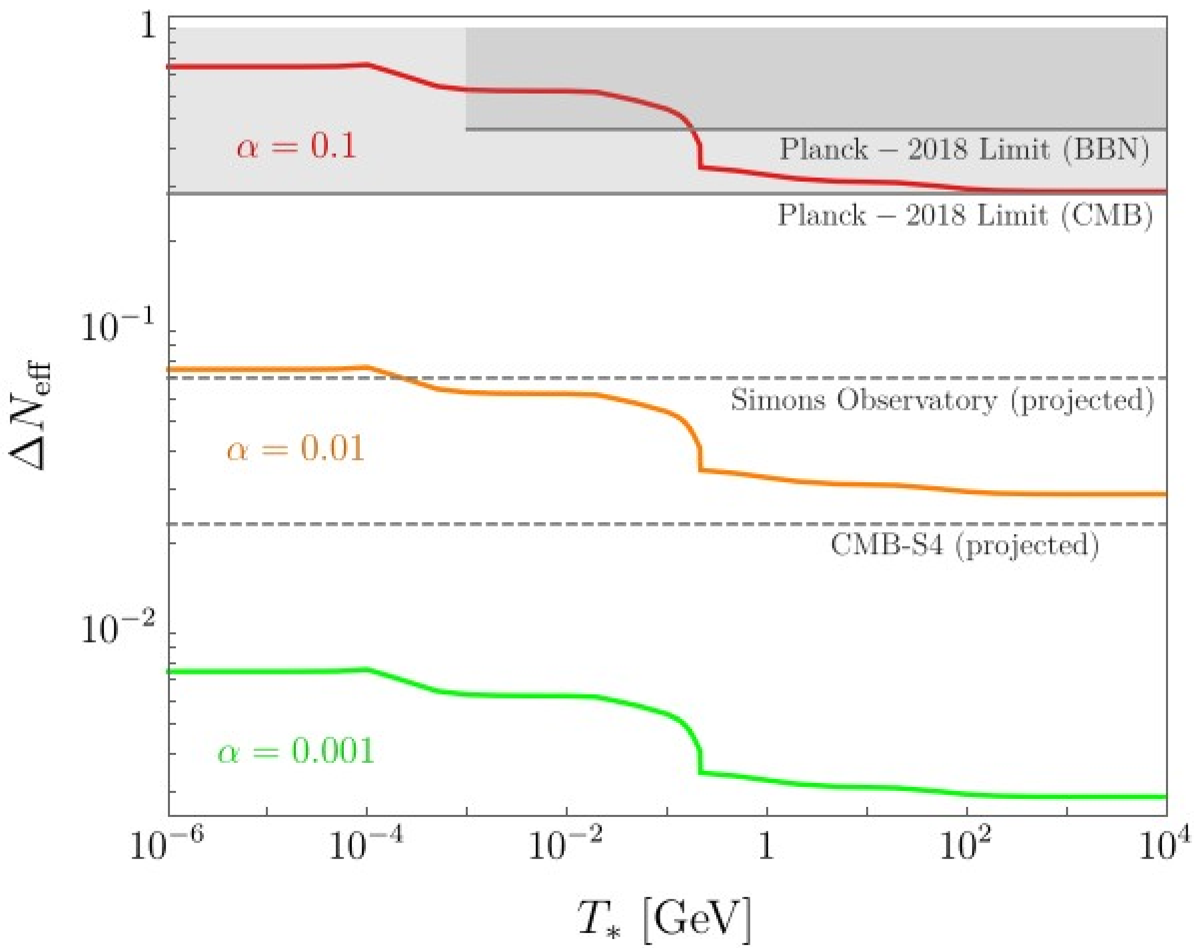

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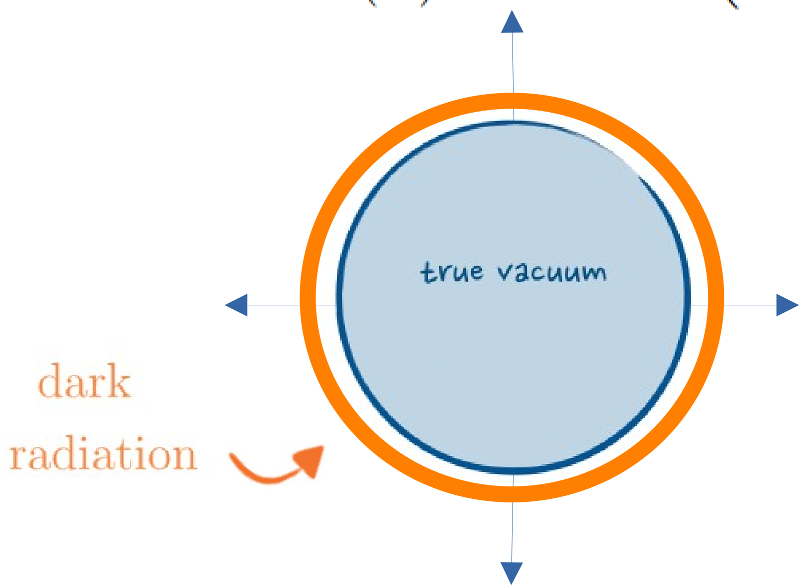

Recombination (CMB)

'Hot/warm' dark matter today !

Conclusions and Outlook

Conclusions

- Highlighted an interesting physical effect at LO:
  domain walls can act as 'longitudinal mirrors'
- Explicitly demonstrated friction on bubble can be have a maximum at intermediate γ
- Complementary signal of FOPT when dark photons around

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- Highlighted an interesting physical effect at LO:
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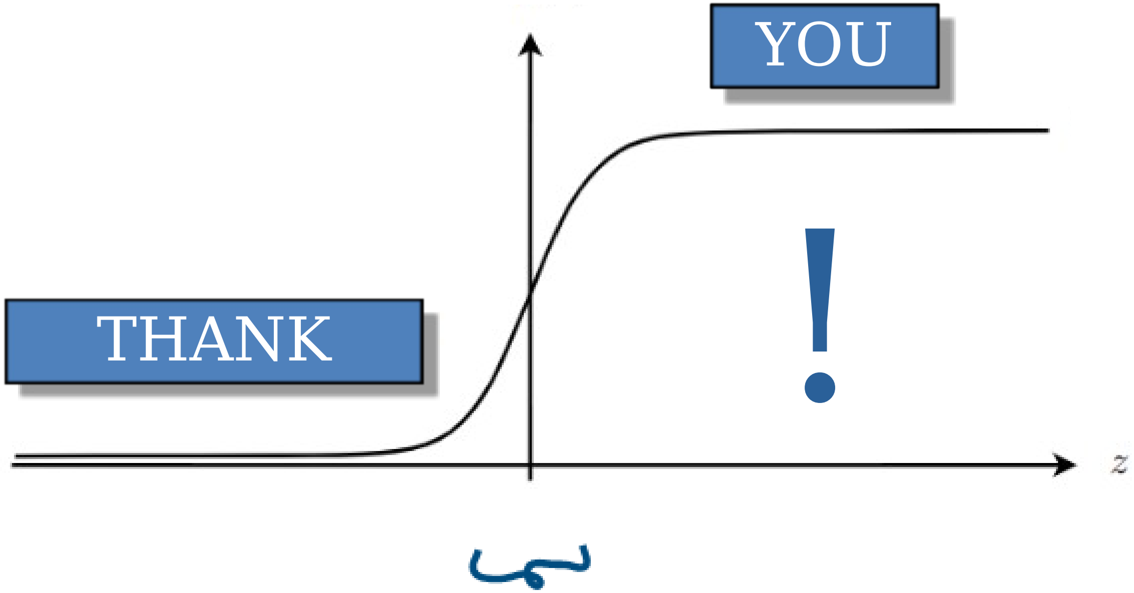
Outlook General (quite rich!)

- e.g. Several aspects of NLO calculations are still not settled.

Outlook Specific

- Possibility and dynamics of $\Delta m^2 \gg m^2, \quad R \to 1$?
- Essentially a Nambu-Goldstone boson effect?
- Do thermal / medium masses count?
- Applications to dark photon detection?



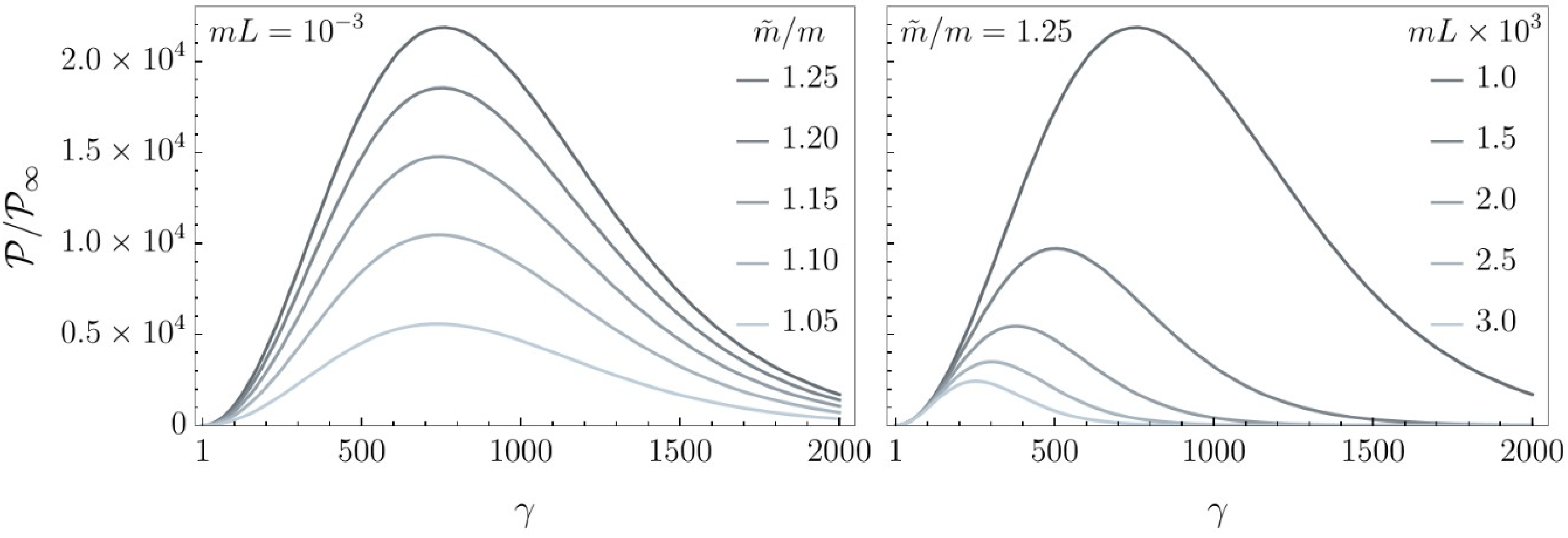
Questions?

Spare Slides

Maximum Dynamic Pressure

Run-away criterion: $\cancel{\Delta V > \mathcal{P}_\infty}$ $\longrightarrow$ $\Delta V > \mathcal{P}_{\rm mdp}$

Max Q condition



Equilibrium Dynamics: Details

Once the bubble's speed is constant the wall carries a decreasing fraction of the total energy

$$\frac{E_{\rm wall}}{E_{\rm total}} \simeq \frac{4\pi R(t)^2 \gamma_{\rm eq}\sigma}{\frac{4\pi}{3} R(t)^3 \Delta V} \sim \frac{\gamma_{\rm eq}\sigma}{\Delta V\, R(t)}$$

Energy goes into reflected (now relativistic) longitudinal dark photons

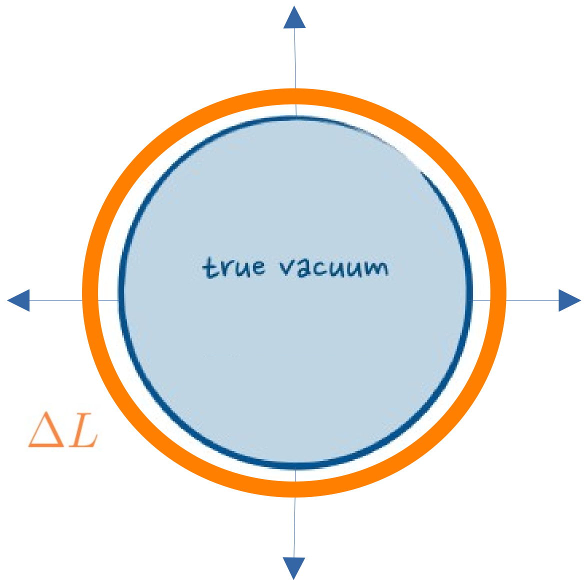
$$|\vec{v}_{\rm dr}| = \frac{2|\vec{v}_{\rm eq}|}{1+|\vec{v}_{\rm eq}|^2} \qquad \text{and} \qquad \gamma_{\rm dr} = \frac{1+|\vec{v}_{\rm eq}|^2}{1-|\vec{v}_{\rm eq}|^2} \simeq 2\gamma_{\rm eq}^2$$

Reflected dark photons speed only *slightly* larger than wall speed

$$\Delta v_{\rm dr} \equiv |\vec{v}_{\rm dr}| - |\vec{v}_{\rm eq}| = |\vec{v}_{\rm eq}| \frac{1-|\vec{v}_{\rm eq}|^2}{1+|\vec{v}_{\rm eq}|^2} \simeq \frac{|\vec{v}_{\rm eq}|}{2\gamma_{\rm eq}^2} \ll 1$$

Dark photons distributed on shell of thickness

$$\Delta L \sim \Delta v_{\rm dr} \times \Delta t \simeq \frac{|\vec{v}_{\rm eq}|\Delta t}{2\gamma_{\rm eq}^2} \lesssim \frac{R_{\rm coll.}}{2\gamma_{\rm eq}^2} \ll R_{\rm coll.}$$



$\Delta m^2 \gg m^2, \quad R \to 1\ ?$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(\partial_\mu - igA)\Phi|^2 + \frac{1}{2}(\partial_\mu\phi)^2 - V(\rho,\phi),$$

where $\rho \equiv |\Phi|$. We imagine the potential having two minima, both symmetry breaking so that A is always massive. An example potential is

$$V(\rho,\phi) = \frac{\lambda'}{4}\left(\rho^2 - v'^2\right)^2 + \frac{\lambda}{4}\phi^2\left(\phi - v\right)^2 - \frac{1}{2}k\rho^2\phi^2$$

Notice stability requires $k^2 < \lambda\lambda'$. In order for $(\rho,\phi) = (v',0)$, where we wish to start, to be a minimum we need $\lambda v^2 \gg 2kv'^2$. Thus there are two small numbers

$$\alpha = \frac{k^2}{\lambda\lambda'} \ll 1, \qquad \beta = \frac{2kv'^2}{\lambda v^2} \ll 1.$$

A second minimum exists (from mathematica) at

$$\phi_0 = \frac{v}{4}\left(3 + \sqrt{1 + 8\left(\beta - \alpha\beta + \alpha\right)}\right)\left(1-\alpha\right)^{-1} = v\left(1 + \beta + 2\alpha\right) + \ldots$$

$$\rho_0 = v'\sqrt{1 + 2\alpha\left(\frac{\lambda\phi_0^2}{2kv'^2}\right)} \approx v'\sqrt{1 + \frac{2\alpha}{\beta}} + \ldots$$

Goldstone Equivalence theorem

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_V^2(z)V_\mu V^\mu$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}m_V^2(z)(V_\mu - \partial_\mu\alpha)^2 - \frac{1}{2\xi}\left(\partial_\mu V^\mu - m^2\xi\alpha\right)^2$$

$$\partial_\mu\left(m_V^2(z)\partial^\mu\alpha\right) + \xi m^4\alpha = 0$$

$$\alpha = e^{-i\omega t}\begin{cases} e^{ik_z} + r\, e^{-ik_z}, & z < 0 \\ t\, e^{i\tilde{k}_z}, & z > 0 \end{cases},$$

$$k_z = \sqrt{\omega^2 - \xi m^2}\ , \quad \tilde{k}_z = \sqrt{\omega^2 - \xi m^4/\tilde{m}^2}\ ,$$

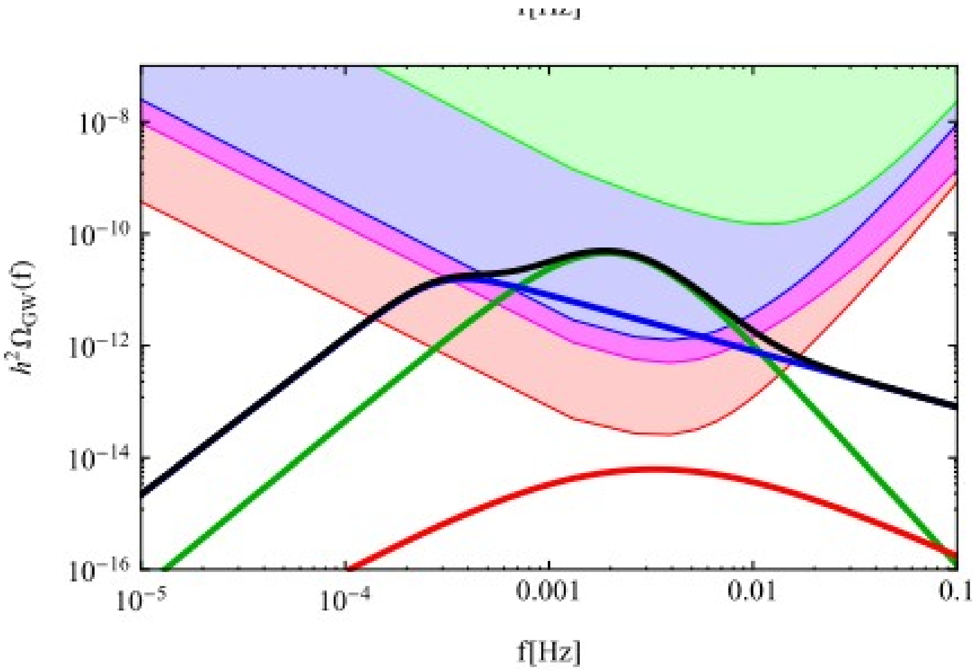
$$m_V^2\partial_z\alpha \quad \text{and} \quad \alpha \quad \text{are continuous at } z = 0,$$

$$R = |r|^2 = \left(\frac{k_z m^2 - \tilde{k}_z\tilde{m}^2}{k_z m^2 + \tilde{k}_z\tilde{m}^2}\right)^2 \xrightarrow{\omega\gg m,\tilde{m}} \left(\frac{m^2 - \tilde{m}^2}{m^2 + \tilde{m}^2}\right)^2 + \mathcal{O}\left(\tilde{m}^2/\omega^2\right)$$

$$\boxed{m \longrightarrow 0}$$

Self Interactions

1st Order Phase Transitions



bubble collisions

turbulence

sounds waves