LA-UR-23-30047

Los Alamos NATIONAL LABORATORY

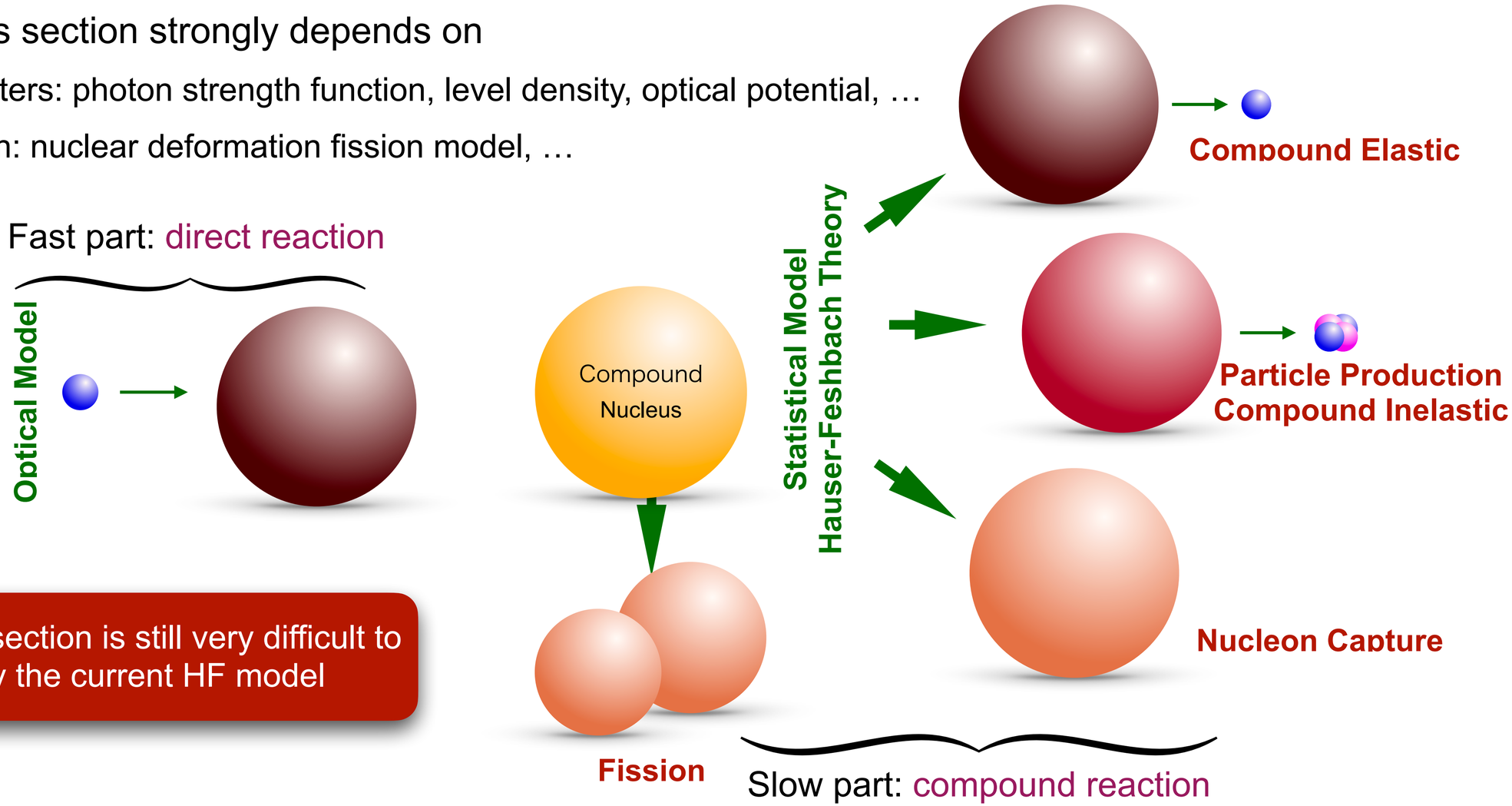
Realistic fission transmission coefficients in the statistical Hauser-Feshbach compound-nucleus reaction theory

T. Kawano
Theoretical Division

NNSA National Nuclear Security Administration

Introduction: Nuclear Reaction Rates by Statistical Hauser-Feshbach

- **Statistical Hauser-Feshbach theory for nuclear reactions in the keV to MeV energy range**
  - Widely employed for astrophysics study - nucleo-synthesis
  - Calculated cross section strongly depends on
    - model parameters: photon strength function, level density, optical potential, …
    - implementation: nuclear deformation fission model, …



Fission cross section is still very difficult to predict by the current HF model

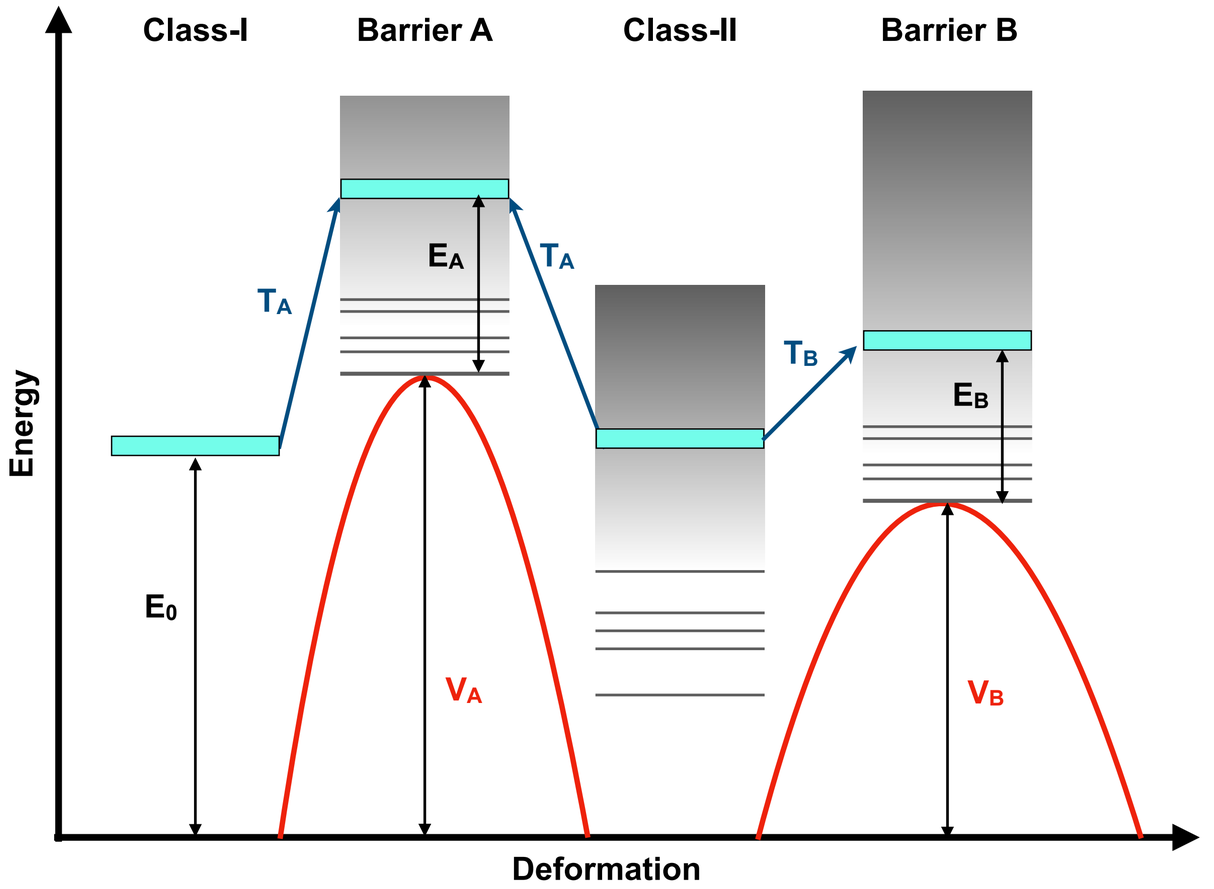
Fission Transmission Calculation - Conventional Method

- **Extremely simplified penetration model employed**
  - Hill and Wheeler expression gives an analytical expression by WKB approximation for Inverted parabolic barriers
  - Often double-humped barrier shape assumed, which are combined by

$$T_f = \frac{T_A T_B}{T_A + T_B}$$

  - Sometimes potential valleys considered (Class-II)
- **Because two barriers are fully decoupled, there is no actual fission path**
  - Number of fission channels determined by the level densities on top of each barrier
  - Includes fission level density enhancement

$$T_i(E) = \frac{1}{1 + \exp\left(2\pi \frac{V_i + E - E_0)}{C_i}\right)}, i = A, B$$



Los Alamos NATIONAL LABORATORY

Solving Schrödinger Equation for Fission Penetration

- **1-D Schroedinger Equation in the deformation coordinate**

$$\frac{d^2}{dx^2}\phi(x) + \frac{2\mu}{\hbar^2}\left\{E - (V(x) + iW(x))\right\}\phi(x) = 0$$

$$\frac{\mu}{\hbar^2} = 0.054A^{5/3} \quad \mathrm{MeV}^{-1}$$

Cramer, Nix PRC 2, 1048 (1970)

  - which satisfies the asymptotic solution

$$u^{(-)}(kx) - Su^{(+)}(kx) \quad x > x_{\max}$$

$$Au^{(-)}(kx) \quad x < x_{\min}$$

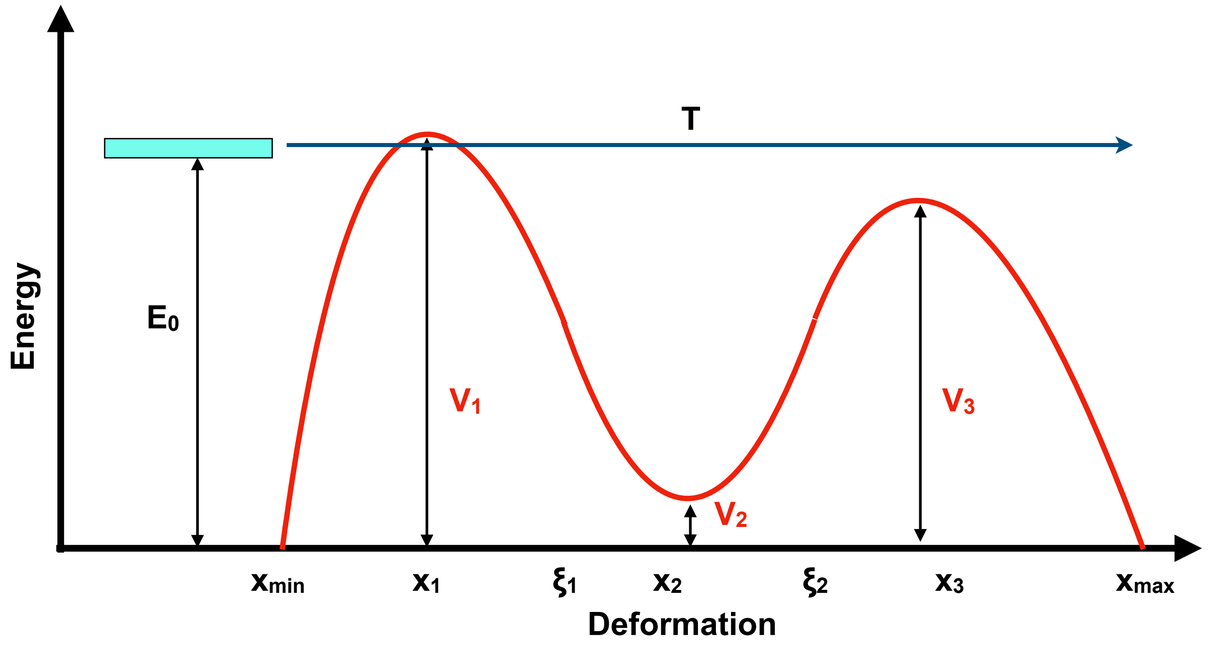
$$u^{(\pm)}(kx) = \cos(kx) \pm i\sin(kx)$$

  - transmission coefficient (penetration) is given by

$$T = 1 - |S|^2$$

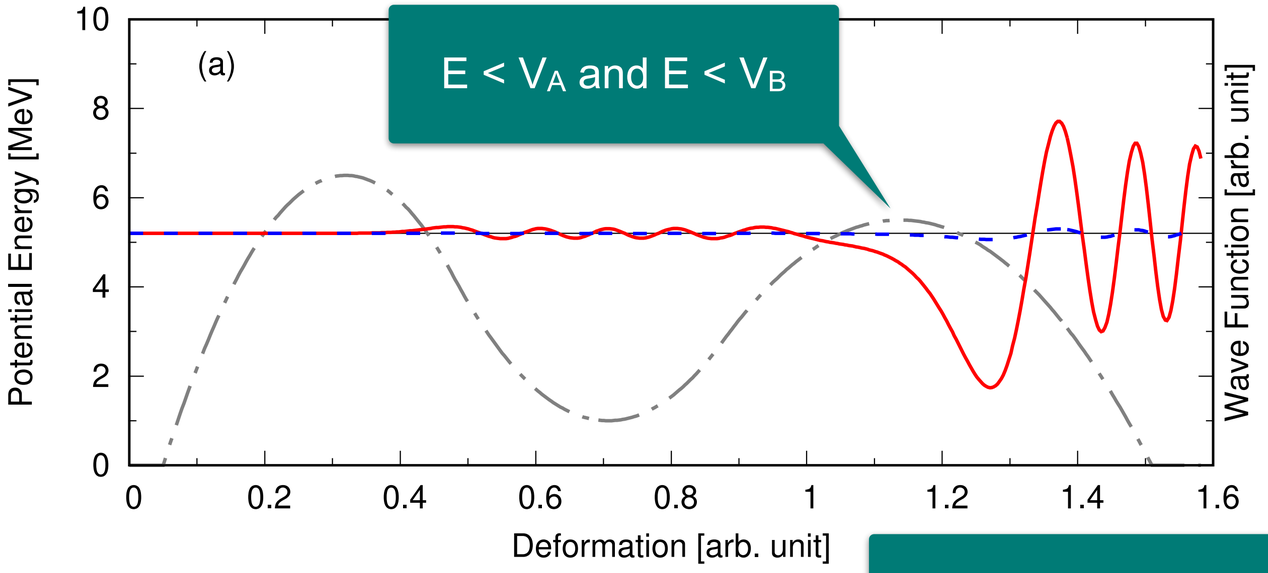
  - when complex potential

$$T = A = \left.\frac{u^{(-)} - Su^{(+)}}{\phi}\right|_{x_m}$$

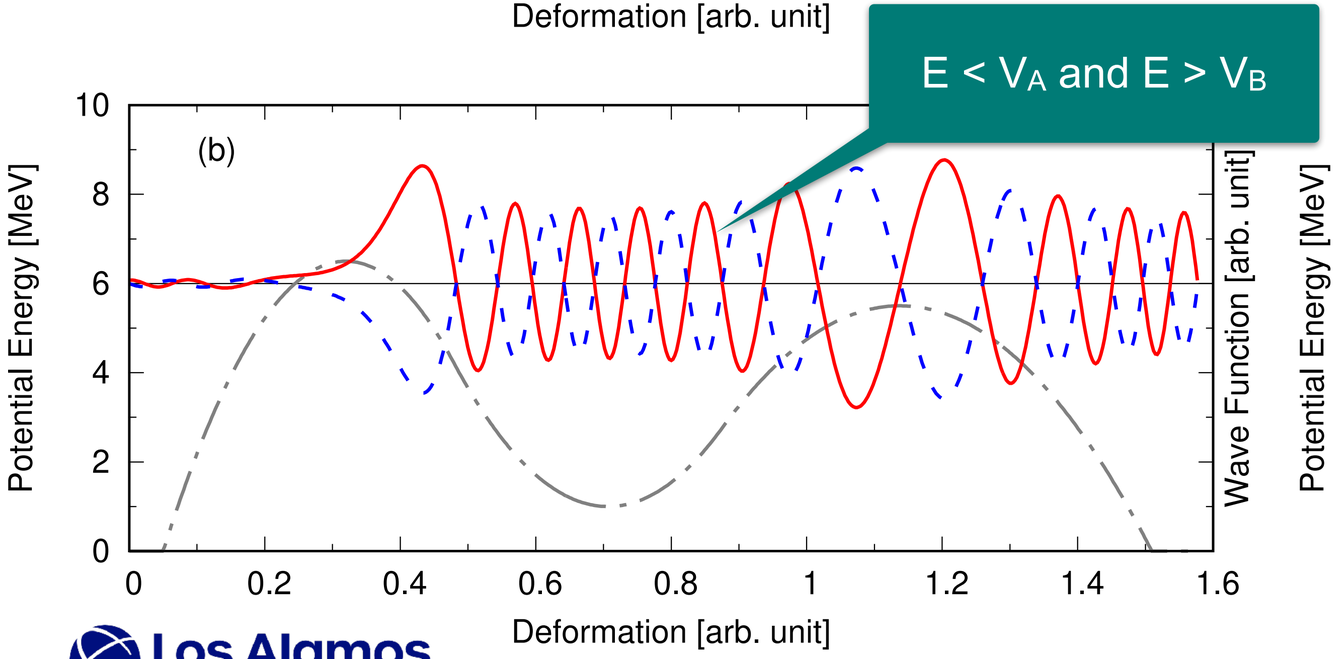


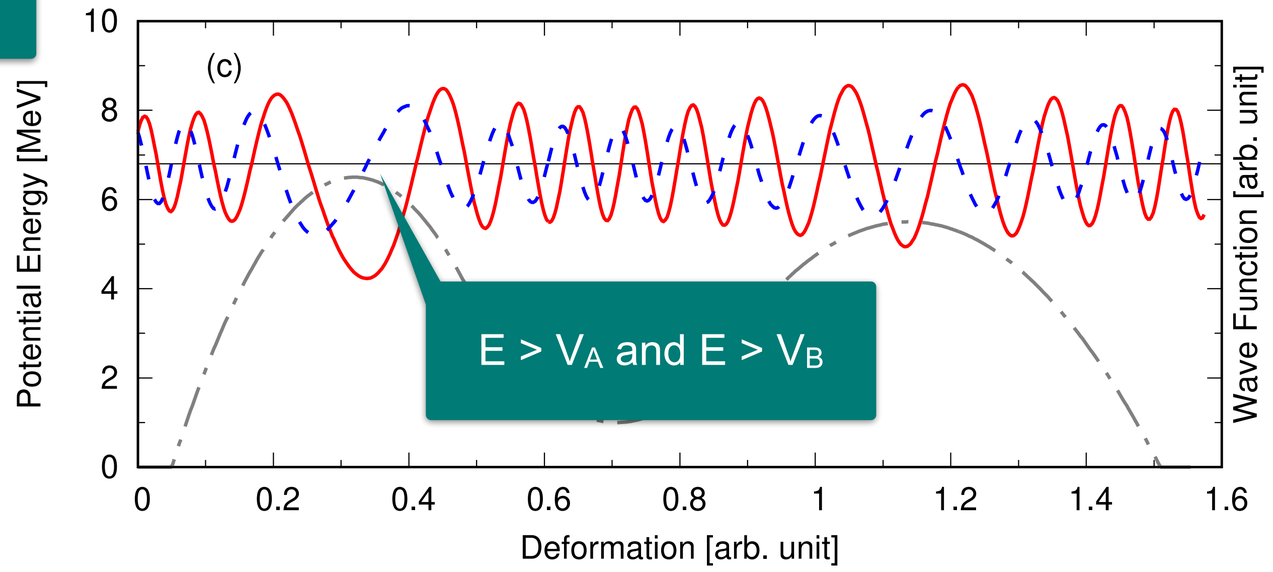
Los Alamos NATIONAL LABORATORY

1-D Fission Potential and Wave-Functions



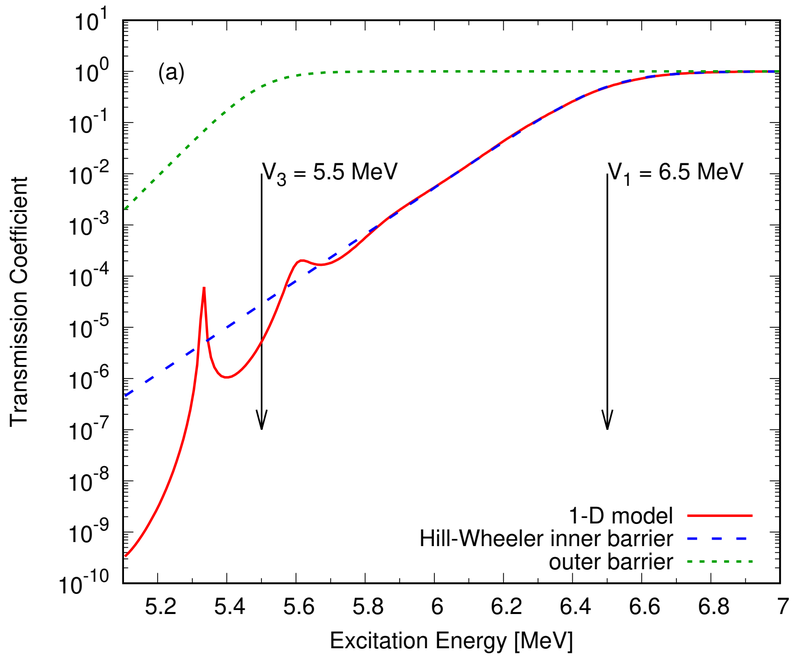
Transmission coefficient is given by the amplitude inside the barriers when unit wave in the exteror region is given

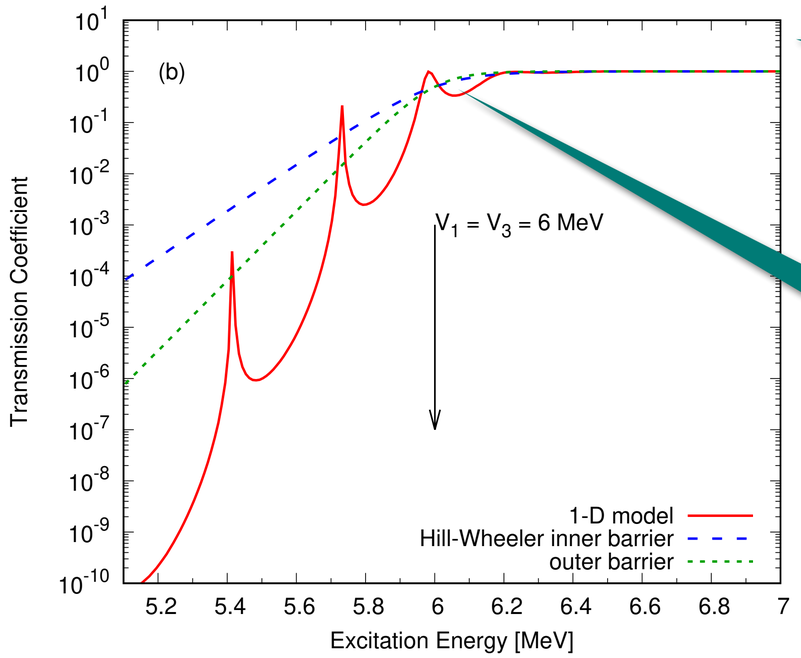




Los Alamos NATIONAL LABORATORY

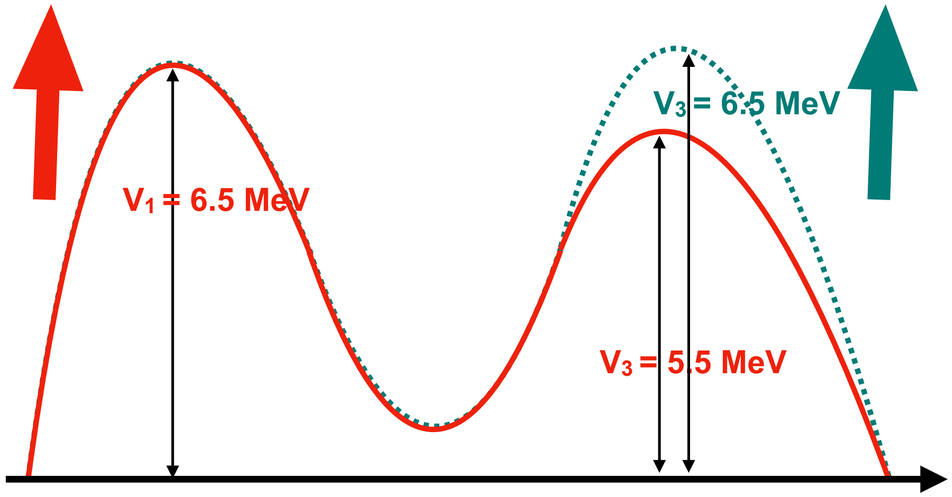
Fission Transmission Coefficient





Transmission coefficient is 1 when the system energy is higher than the all barriers

Fission cross section will be enhanced even the energy is below the fission barriers



- **Resonance-like structure appears**
  - not a CN resonance but nuclear shape effect
  - this happens when incoming and reflection waves are in phase

Los Alamos NATIONAL LABORATORY

Penetration Through Excited States - Fission Trajectory Model

- **Penetration happen many excited states**
  - Fission paths must be defined for all of the possible trajectories
  - They are distorted by the nuclear deformation, mass asymmetry, pairing effect, etc
- **We employ a simplified trajectory compression model**

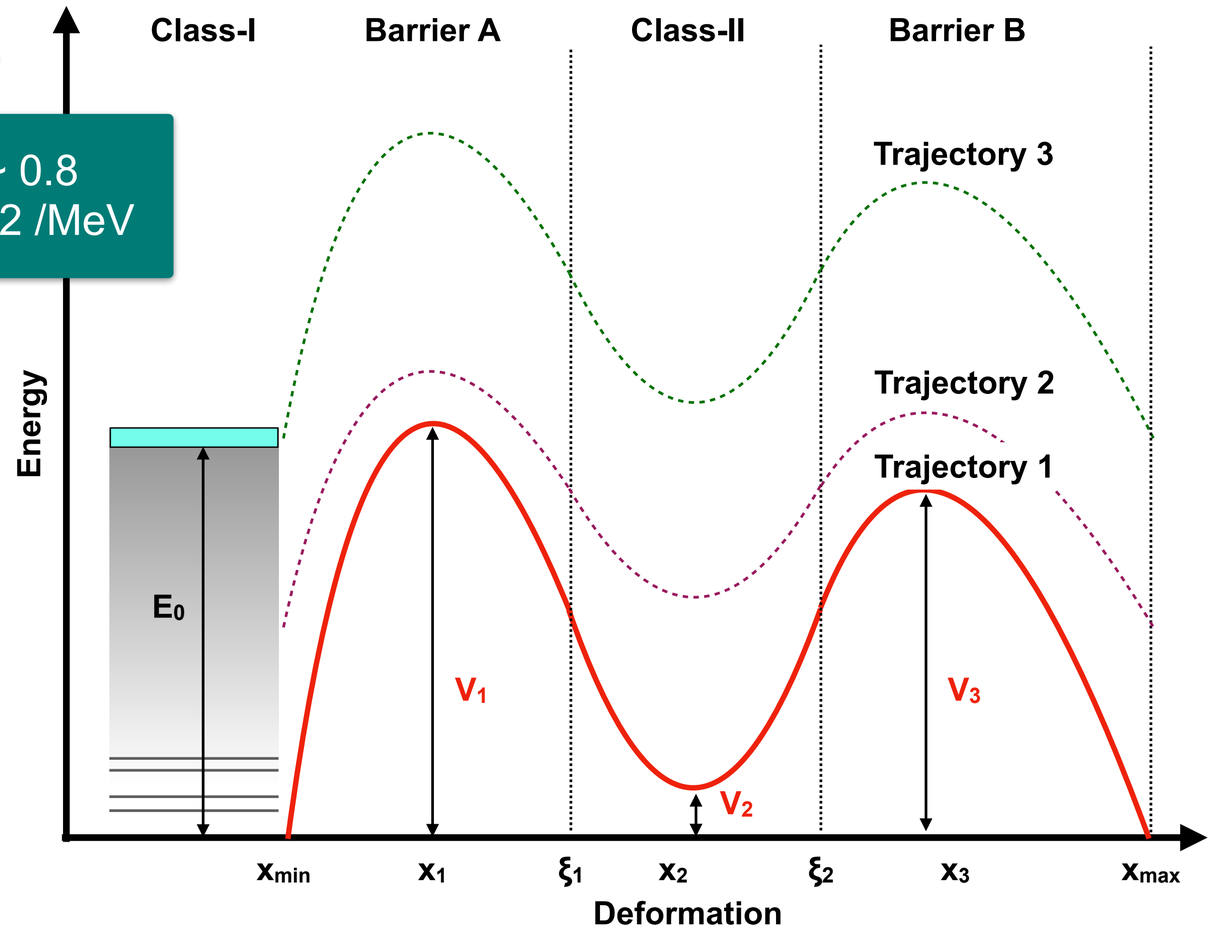
$$\varepsilon_x = \left\{ f_0 + \left(1 - e^{-f_1 E_x}\right)\left(1 - f_0\right) \right\} E_x$$

$$V(E_x, x) = V_0(E_x, x) + \varepsilon_x$$

  - where f_0 and f_1 are parameters

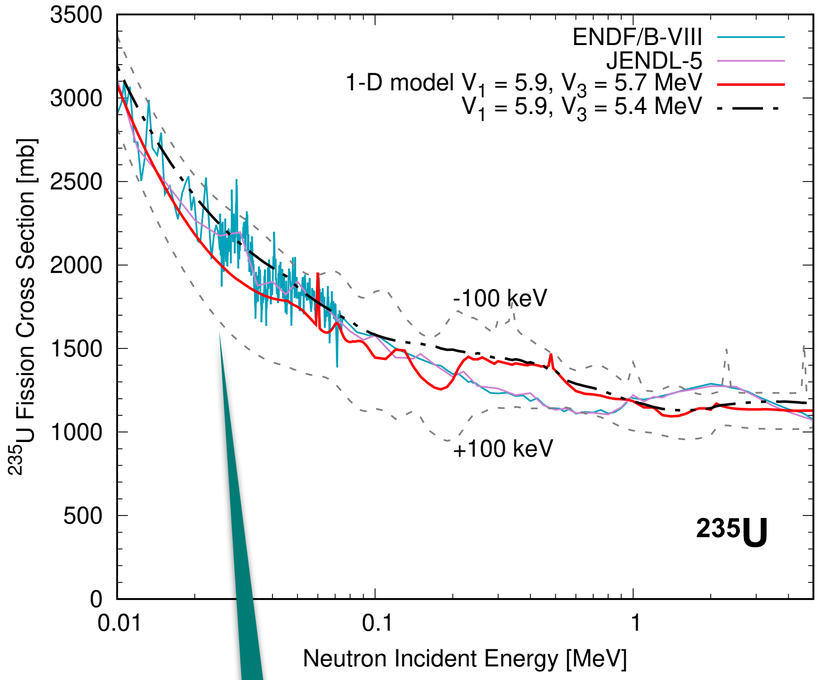
$f_0 \sim 0.8$
$f_1 \sim 0.2$ /MeV

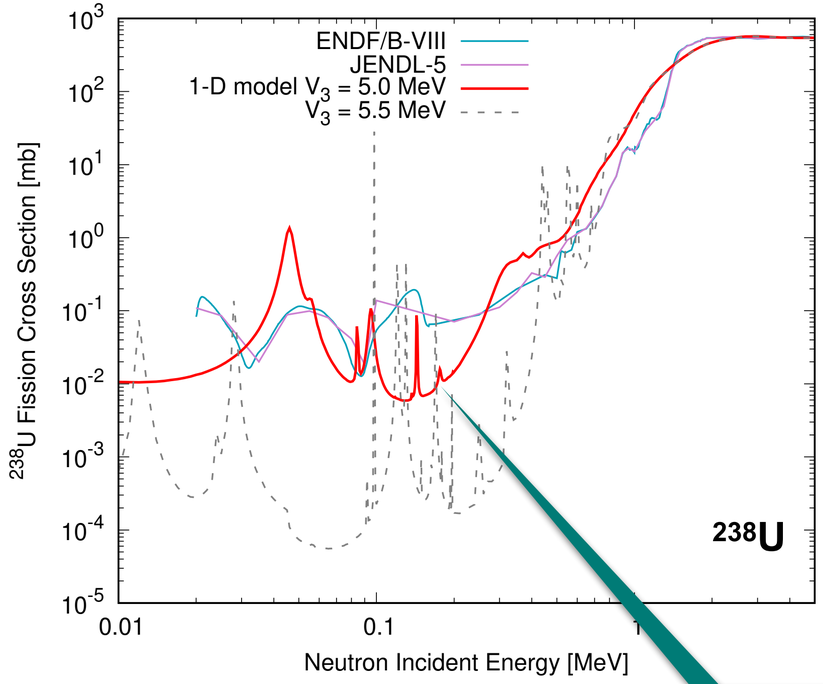
Low-lying discrete levels are lowerd, and continuum at higher energy region asymptotes CN level density

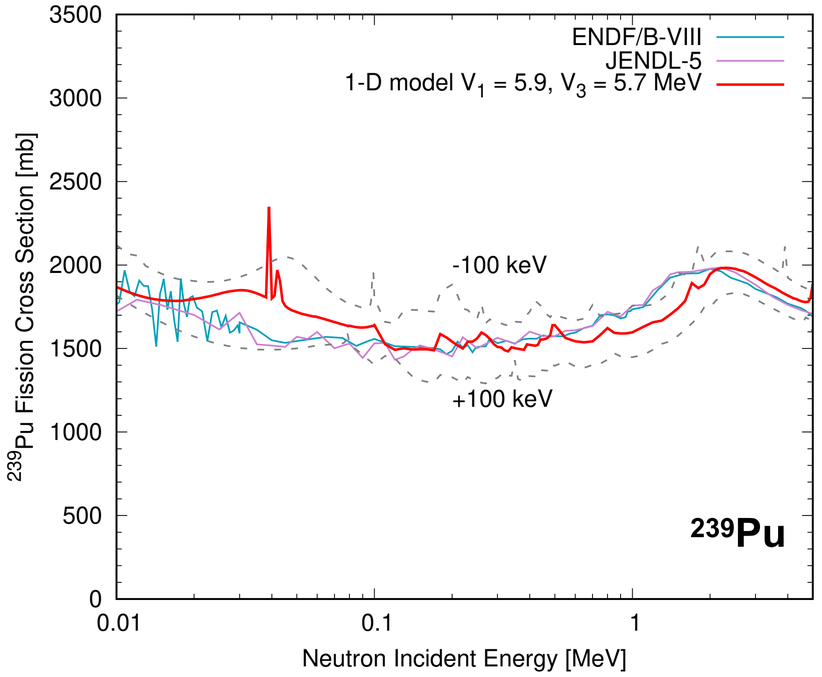


Calculated Fission Cross Sections for Major Actinides

- **Calculated cross sections compared with “evaluated data”**
  - reveals resonance-like structure, remarkable for ^{238}U which is a sub-threshold fissioner
  - calculations reasonably agree with evaluations even limited number of parameters employed







+/- 100 keV change in the fission barrier roughly cover the range of experimental data

sub-thresold fission cross sectionssignificantly increased

Los Alamos NATIONAL LABORATORY

Concluding Remarks and Perspective

- **New fission penetration model incorporated into coupled-channels Hauser-Feshbach model**
  - Fission penetration (= fission transmission coefficients) is calculated for the arbitrary-shape 1-D potential by solving Schroedinger equation numerically
    - This model ensures that the penetrability is always unity when the system energy is above all the fission barriers
  - Fission paths through excited states expressed by the fission trajectory model
    - Although very crude at this moment, only limited number of parameters involved
  - Calculated fission cross sections for major actinides compared with high-quality evaluated data
    - We demonstrate the 1-D model potential with the fission trajectory model reproduces experimental fission cross section data
- **Feasible (plausible) extensions**
  - Potential energy surface by more microscopic insights
  - Include different deformation coordinates: mass asymmetry and fragment deformation
  - Coupled-channels formalism for Schroedinger equation
  - Consistent calculation for both the fission cross section and the fission product yields