# Analytic expression of triple-α reaction rates by a non-adiabatic three-body model

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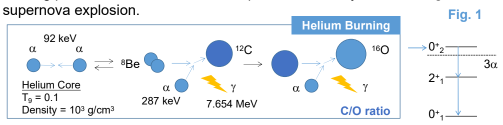
Triple-α reaction rates have been determined well with the sequential process via the narrow resonances. However, direct triple-α process at off-resonant energies still remains in unsolved problems. In this poster, the direct triple-α process is described with a non-adiabatic Faddeev HHR expansion method. The direct 3α contribution is confirmed to be $10^{-15}$_$10^{-3}$ pb order in photo-disintegration of $^{12}C(2^+_1 \to 0^+)$ for 0.15 < E < 0.35 MeV. Using a three-body type of S-factors, the rates are given in a simple analytic expression. In addition, they are converted into REACLIB format. From composition ratio in NACRE, the non-resonant sequential process between α + *bound* $^8$Be is found to dominate the NACRE rates for $0.03 < T_9 < 0.07$. I, therefore, find that the current evaluated rates could be reduced by about $10^{-4}$ at $T_9$ = 0.05, from the accurate description of $^8$Be break-up.

## 1.1 Introduction

**Triple-α reaction** plays an important role in nucleosynthesis heavier than $^{12}$C, because no stable nuclei exist in mass number A=5 and A=8 [1,2]. This reaction, followed by $^{12}C(\alpha,\gamma)^{16}O$ [3], controls C/O ratio at the end of helium burning phase in stars, and it affects up to the nucleosynthesis in *e.g.* supernova explosion.

Fig. 1




Thermal equilibrium α : $^8$Be = $10^{11}$ : 1, and $^{12}C(0^+_2)$ Γα : Γγ = $10^3$ : 1

### 1.4 Present Report

I describe **direct triple-α process** with a non-adiabatic Faddeev **HHR***, and I discuss the 3α reaction rates. After I review Faddeev HHR used in the present poster, I deduce a simple analytic form of the rates with introducing S-factors. The rates are also converted into REACLIB. I show that the evaluated rates could be reduced at $T_9$= 0.05, because of an accurate description of $^8$Be break-up.

## 1.2 Triple-α process

**Triple-α reaction via the resonances, (Fig. 1)**

- In contrast to $^{12}C(\alpha,\gamma)^{16}O$, triple-α reaction is currently well-understood through the experimental studies of the $0^+_2$ state in $^{12}$C ($E_R$ = 0.379 MeV). $E_R$ is the c.m. energy with respect to the 3α threshold in $^{12}$C.
- *i.e.* the reaction rates have been determined successfully with the **sequential process** via the narrow resonances. (*e.g.* [4,5])
- Pioneering work: **CF88** [6], Nomoto (1985) [7]

Standard Evaluated Reaction Rates

**NACRE (1999)** [5]

- Experimental update from CF88, based on Nomoto (1985) [7], Langanke (1986) [8].
- $^8$Be is assumed to be bound. The reaction proceeds via two resonances: $^8Be(0^+_1)$, $^{12}C(0^+_2)$. Recent experimental progress is found in *e.g.* [4].

**Triple-α reaction from ternary continuum states, (Fig. 2)**

- **Direct triple-α process**.
- This process is generally expected to be *very* slow, because three α-particles almost simultaneously collide and fuse into a $^{12}$C nucleus.
- The direct process is neglected or is treated in some approximations.

## 1.3 Theoretical studies of triple-α reaction

**Adiabatic** approximation of $^8$Be continuum has been applied to tackle the 3α continuum problem.

3α Continuum Problem

- Formulae in hyper-spherical coordinates (*e.g.* [9])
- Coulomb Modified Faddeev (**CMF**) method [10]
- Adiabatic channel function (**ACF**) method [11]

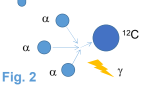



Fig. 2

However, quantum-mechanical description at off-resonant energies still seems to remain in unsolved problems.

**Non-adiabatic approaches** of 3α have also been performed, recently. [12,13] (Fig.3)

- Relax the continuum states of $^8$Be
- Faddeev Hyperspherical Harmonics and *R*-matrix (**HHR**) expansion method [13,14,15].
- My calculation is labeled with **HHR***.

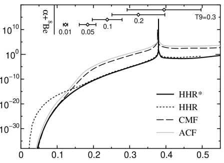



Fig. 3

## 2.1 Faddeev HH expansion

**Three-body Schrödinger equation:** $(H_{3\alpha} - E)\Psi = 0$ (1)

- Faddeev equations, consisting of three components,

$$\begin{cases} \left[\hat{T}_{(1)} + (\hat{V}_{31} + \hat{V}_{3\alpha}) - E\right]\Psi_{(1)} = -\hat{V}_{31}\Psi_{(2)} - \hat{V}_{31}\Psi_{(3)} \\ \left[\hat{T}_{(2)} + (\hat{V}_{23} + \hat{V}_{3\alpha}) - E\right]\Psi_{(2)} = -\hat{V}_{23}\Psi_{(3)} - \hat{V}_{23}\Psi_{(1)} \\ \left[\hat{T}_{(3)} + (\hat{V}_{12} + \hat{V}_{3\alpha}) - E\right]\Psi_{(3)} = -\hat{V}_{12}\Psi_{(1)} - \hat{V}_{12}\Psi_{(2)} \end{cases} \quad (2)$$

- Three identical sets of equations are found, because of the symmetric 3α system. Third component is rewritten as

$$\left[-\frac{\hbar^2}{2\mu_{12}}\nabla^2_{\bar{x}_3} - \frac{\hbar^2}{2\mu_{(12)3}}\nabla^2_{\bar{y}_3} + \hat{V}_{sym}\right]\Psi_{lm} = E\,\Psi_{lm}, \quad \hat{V}_{sym} = \sum_{i<j}^{3} V_{\alpha\alpha}(r_{ij}) + V_{3\alpha} \quad (3)$$

3α interaction; α+α interaction

**Coupled-channel (CC) equations with hyper-radial wavefunctions:**

$$\left[T_\gamma + U^l_{\gamma\gamma}(\rho) - \epsilon\right]\chi^l_\gamma(\rho) = -\sum_{\gamma'\neq\gamma} U^l_{\gamma'\gamma}(\rho)\chi^l_{\gamma'}(\rho)$$
$$T_\gamma = \frac{d^2}{d\rho^2} - \frac{(K+3/2)(K+5/2)}{\rho^2}$$
$$U^l_{\gamma'\gamma} = -\frac{2m_N}{\hbar^2}V^l_{\gamma'\gamma}(\rho),$$
$$\epsilon = -\frac{2m_N}{\hbar^2}E. \quad (5)$$

Translate Jacobi coordinates into hyper-spherical coordinates

$$\rho^2 = x_i^2 + y_i^2, \quad \theta_i = \arctan\left(\frac{x_i}{y_i}\right) \quad (4)$$

The coordinates are different from CMF.

- Introducing hyper-angular momentum *K*, I obtain the *ordinary* CC equations for inelastic scattering. *e.g.* [16,17] (if *L*= *K*+3/2).

**Basis functions:** Functions of ρ and $\Omega_5$ are separated.

$$\Psi_{lm} = \rho^{-5/2}\sum_{\gamma n} c^n_\gamma \varphi^K_n(\rho)\Phi^\gamma_{lm}(\Omega_5)$$
$$\chi^l_\gamma(\rho) = \sum_n c^n_\gamma\varphi^K_n(\rho). \quad \Phi^\gamma_{lm}(\Omega_5) = \phi^{l_{x_3}l_{y_3}}_K(\theta_3)\left[Y_{l_{x_3}}(\hat{\boldsymbol{x}}_3)\otimes Y_{l_{y_3}}(\hat{\boldsymbol{y}}_3)\right]_{lm}$$
$$\phi^{l_{x_3}l_{y_3}}_K(\theta_3) = N^{l_{x_3}l_{y_3}}_{n_\theta}(\sin\theta_3)^{l_{x_3}}(\cos\theta_3)^{l_{y_3}}G_{n_\theta}(a,b;\sin^2\theta_3) \quad (6)$$

Hyper-harmonic functions; Hyper-radial wave functions; Normalization; Jacobi polynomials

- The final results are independent of the adopted $\varphi^K_n(\rho)$, if a large number of basis functions are used so as to expand well the wave functions.

## 2.2 *R*-matrix expansion

**CC equations of Eq.(5):** $(\boldsymbol{T}+\boldsymbol{U})\boldsymbol{X} = \epsilon\boldsymbol{X}$ (7)

- Matrix diagonalization, matrix size: (8,800 x 8,800) for $0^+$ in $^{12}$C
- Eigenstates, $E(l^+_i) = -\frac{\hbar^2}{2m_N}\epsilon_i, \quad \chi^l_{\gamma i}(\rho) = \sum_n c^n_{\gamma i}\varphi^K_n(\rho)$ (8)
  - $E(l^+_i) < 0$, Bound states, $\chi^l_{\gamma i}(\rho) \to N^l_{\gamma i}W_{-\eta_\gamma,K+2}(2\kappa\rho)$ (9)
    *N*: normalization constants, *W*: Whitterker functions
  - $E(l^+_i) > 0$, Resonances corresponding to low-lying levels & mathematically orthogonal states without specific interpretation in physics.

*R*-matrix expansion: Continuum states with scattering boundary condition are expanded by the resultant eigenfunctions.

**Expansion of interior scattering waves:** $a_c$ : channel radius

$$\chi^{l\,in}_{\alpha\beta}(k,\rho) = \sum_i A_{i\beta}(k)\chi^l_{\alpha i}(\rho) \quad (10)$$
$$A_{i\beta}(k) = \frac{\hbar^2}{2m_N}\frac{1}{E(l^+_i)-E}\sum_\gamma \chi^l_{\gamma i}(a_c)\left[H^{-\,\prime}_{K+3/2}(\eta_\gamma;ka_c)\delta_{\gamma\beta} - S^l_{\gamma\beta}(E,a_c)H^{+\,\prime}_{K+3/2}(\eta_\gamma;ka_c)\right] \quad (11)$$

S-matrix; Incoming Coulomb function; Outgoing Coulomb function

- *S*-matrix is defined by *R*-matrix,

$$S^l_{\alpha\beta}(E,a_c) = [Z^*_{\alpha\beta}(E,a_c)]^{-1}Z_{\alpha\beta}(E,a_c), \quad (12)$$
$$Z_{\alpha\beta}(E,a_c) = H^-_{K+3/2}(\eta_\alpha;ka_c)\delta_{\alpha\beta} - a_c R^l_{\alpha\beta}(E,a_c)H^{-\,\prime}_{K+3/2}(\eta_\beta;ka_c) \quad (13)$$
$$R^l_{\alpha\beta}(E,a_c) = \sum_i \frac{\tilde{\gamma}_{\alpha i}\tilde{\gamma}_{\beta i}}{E(l^+_i)-E} \quad (14)$$

R-matrix

- Reduced width amplitudes are defined as $\tilde{\gamma}_{\alpha i} = \sqrt{\frac{\hbar^2}{2m_N a_c}}\chi^l_{\alpha i}(a_c)$ (15)
- To include long-range Coulomb couplings, CC equations in the external region are solved numerically from ρ = $a_c$ to ρ = $\rho_m$, $\chi^{l\,in}_{\alpha\beta}(k,a_c) \longrightarrow \chi^{l\,ext}_{\alpha\beta}(k,\rho)$. I use *R*-matrix propagation technique [13] to obtain the linearly-independent solutions.

**Expansion by the linearly-independent waves:**

$$\tilde{\chi}^l_{\gamma\gamma_0}(k,\rho) = \sum_{\gamma'} C_{\gamma'\gamma_0}(k)\chi^{l\,ext}_{\gamma\gamma'}(k,\rho) \quad (16)$$

- The coefficients are obtained by matching to the asymptotic form of w.f. [12],

$$\tilde{\chi}^l_{\gamma\gamma_0}(k,\rho) \to \frac{i}{2}\left[I^{(\gamma_0)}_{\gamma,K+3/2}(\eta_\gamma;k\rho_m) - \sum_{\gamma'}S^l_{\gamma'\gamma_0}(E)O^{(\gamma')}_{\gamma,K+3/2}(\eta_\gamma;k\rho_m)\right] \quad (17)$$

- Coupled-Coulomb waves [17]: $O^{(\gamma')}_{\gamma,K+3/2}(\eta_\gamma;k\rho) = a^{(\gamma')}_\gamma(k,\rho)H^+_{\gamma,K+3/2}(\eta_\gamma;k\rho), \ I = O^*$ (18)
  + screening, $a^{(\gamma')}_\gamma \to \delta_{\gamma\gamma'}$ Effective, if the off-diagonal part of Coulomb coupling potentials is relatively small at a matching radius $\rho_m$, compared with *E*. (*e.g.*[16])
- Interior scattering waves including long-range Coulomb couplings are obtained as

$$\tilde{\chi}^{l\,in}_{\gamma\gamma_0}(k,\rho) = \sum_{\gamma'}C_{\gamma'\gamma_0}(k)\chi^{l\,in}_{\gamma\gamma'}(k,\rho) = \sum_i D_{i\gamma_0}(k)\chi^l_{\gamma i}(\rho) \qquad D_{i\gamma_0}(k) \equiv \sum_{\gamma'}C_{\gamma'\gamma_0}(k)A_{i\gamma'}(k), \quad \chi^l_{\gamma i}(\rho) = \sum_n c^n_{\gamma i}\varphi^K_n(\rho) \quad (19)$$

← Eq.(10); Eigenfunctions, Eq.(8); Continuum states; Arbitrary orthogonal functions

- B(E2) strength between $0^+$ continuum states and $2^+_1$ are calculated with Eqs. (8), (16) and (19). Quadruple precision is required to execute stable calculations.

**B(E2) strength:**

$$\frac{dB(E2;0^+\to 2^+_1)}{dE} = \frac{5e^2}{2\pi\hbar v}\sum_{\gamma_0}\left|\sum_{\gamma'\gamma}M^{2^+_1 0^+}_{\gamma'\gamma\gamma_0}(k)\left(f^x_{\gamma'\gamma}\zeta_x + f^y_{\gamma'\gamma}\zeta_y\right)\right|^2 \quad (20)$$
$$M^{2^+_1 0^+}_{\gamma'\gamma\gamma_0}(k) = M^{2^+_1 0^+\,Int}_{\gamma'\gamma\gamma_0}(k) + M^{2^+_1 0^+\,Ext}_{\gamma'\gamma\gamma_0}(k) \quad (21)$$

Hyper-angle part; Hyper-radial part

- Internal component, $M^{2^+_1 0^+\,Int}_{\gamma'\gamma\gamma_0}(k) = i^K\sum_i D_{i\gamma_0}(k)\int_0^{a_c}\chi^{2^+}_{\gamma'1}(\rho)\chi^{0^+}_{\gamma i}(\rho)\rho^2 d\rho$ (22)
- External component, $M^{2^+_1 0^+\,Ext}_{\gamma'\gamma\gamma_0}(k) = i^K\int_{a_c}^\infty \chi^{2^+}_{\gamma'1}(\rho)\tilde{\chi}^{0^+}_{\gamma\gamma_0}(k,\rho)\rho^2 d\rho$ (23)

**Photo-disintegration cross sections:** $\sigma_g(E) = \frac{4\pi^3}{75}\left(\frac{E_g}{\hbar c}\right)^3\frac{1}{5}\frac{dB(E2;0^+\to 2^+_1)}{dE}$ (24)

**Reaction rates:**

$$R_{3\alpha}(E) = N_A^2\frac{480\pi}{(\mu_{12}\mu_{(12)3})^{3/2}}\frac{\hbar^3}{c^2}\frac{E_g^2}{E^2}\sigma_g(E) \quad (25)$$
$$\langle R_{3\alpha}\rangle = \frac{1}{2}\frac{1}{(k_BT)^3}\int_0^\infty R_{3\alpha}(E)E^2 e^{-E/(k_BT)}dE \quad (26)$$

## 3.1 Analytic form of reaction rates

**Three-body type of S-factors:**

$$S_{3\alpha}(E) \equiv E_g^2\sigma_g(E)\exp\left(\frac{2\pi\eta_0}{\sqrt{E}} + aE\right) = s_0\left(1 + s_1E + s_2E^2\right), \quad (27)$$

- If S-factors are defined by Eq. (27), Eq. (26) is given as Eqs. (28)_(30).
- Coefficients are adjusted so as to fit the results from HHR*. (Fig. 4, Table 1)

**Non- resonant contribution:** $T_a = (k_BT)/(1 + ak_BT)$ (28)

$$\langle R_{3\alpha}\rangle_{NR} = N_A^2\frac{45\pi\sqrt{\pi}\,\hbar^3}{2m_N^3\,c^2}\frac{(\pi\eta_0)^{1/3}}{(k_BT)^{13/6}(1+ak_BT)^{5/6}}\tilde{S}(T_a)\exp\left[-\frac{3(\pi\eta_0)^{2/3}}{T_a^{1/3}}\right] \quad (29)$$
$$\tilde{S}(T_a) \approx s_0\left[\left(1 + \frac{5T_a}{36E_0}\right) + s_1E_0\left(1 + \frac{35T_a}{36E_0}\right) + s_2E_0^2\left(1 + \frac{89T_a}{36E_0}\right)\right], \ E_0 = (\pi\eta_0T_a)^{2/3}. \quad (30)$$

**Resonant contribution:**

$$\langle R_{3\alpha}\rangle_R = \frac{9\sqrt{3}\pi^3}{4}\frac{N_A^2\hbar^5}{m_N^3}\frac{\Gamma_\gamma(0^+_2)}{(k_BT)^3}\exp\left(-\frac{E(0^+_2)}{k_BT}\right) \quad (31)$$
$$\approx 7.605\times 10^{-9}\frac{\Gamma_\gamma(0^+_2)}{T_9^3}\exp\left(-\frac{11.605\,E(0^+_2)}{T_9}\right) \quad (32)$$

**REACLIB [18]:**

- The coefficients are listed in Table 2.

$$\langle R_{3\alpha}\rangle = \sum_i \exp\left(a_{i0} + a_{i1}/T_9 + a_{i2}/T_9^{1/3} + a_{i3}T_9^{1/3} + a_{i4}T_9 + a_{i5}T_9^{5/3} + a_{i6}ln(T_9)\right) \quad (33)$$

## 3.2 S-factors and $\sigma_g$ for E < 0.22 MeV

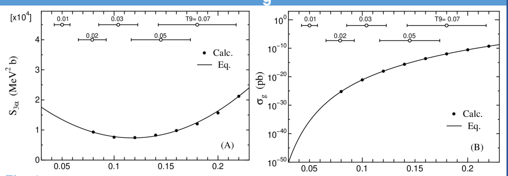



Fig. 4

- Compared with the cross sections in Fig.4(B), the energy dependence of the S-factors in Fig.4(A) is quite weak.
- The reaction rates below $T_9$= 0.02 are generated from the extrapolated S-factors.
- However, I do not think that the derived rates have large uncertainties, because the S-factors do not seem to have the considerable variation to energies.

Table 1 Coefficients, obtained from HHR* with the CD potential.

| $s_0$ (MeV$^2$b) | $s_1$ (MeV$^{-1}$) | $s_2$ (MeV$^{-2}$) | $\eta_0$ (MeV$^{1/2}$) | $a$ (MeV$^{-1}$) | $E(0^+_2)$ (MeV) | $\Gamma_\gamma(0^+_2)$ (meV) |
|---|---|---|---|---|---|---|
| 2.570×10$^4$ | -12.12 | 51.46 | 4.153 | 5.0 | 0.3796 | 3.908 |

## 3.3 Comparison between reaction rates

- The derived rates are consistent with NACRE for $0.08 < T_9 < 3$, (Figs. 5 and 6).
- In contrast, the reaction rates are reduced by $10^{-4}$ at $T_9$ ≈ 0.05.
- This is caused by the non-adiabatic description of $^8$Be break-up & continuum states.
  - $\sigma_g$ is reduced from that of ACF and CMF with the sequential process at *E* = 0.18 MeV (Fig. 3). Due to the strong influence of $0^+_2$, the difference in $\sigma_g$ for *E* > 0.2 MeV cannot be found in the reaction rates.
- Astrophysical impact of the direct triple-α process is expected to be small, because the difference is found before helium burning temperatures.

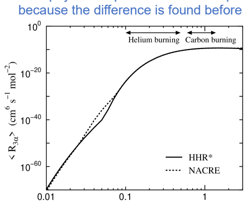



Fig. 5

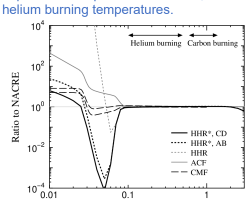



Fig. 6

## 3.4 Translation into REACLIB

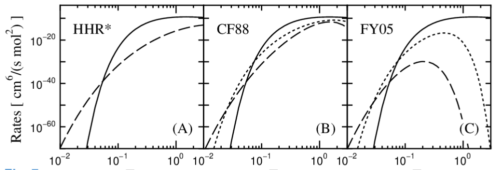



Fig. 7

- The reaction rates are defined in Eq. (33). HHR* does not have a component of non-resonant sequential process between α+$^8$Be (dotted curves in Fig.7).
- This means that the statistically generated $^8$Be is broken-up immediately by the third α-particle before its lifetime at $T_9$ ≈ 0.05.
- To update the rates in [18,6,19], this contribution may be eliminated by hand.

Table 2

| $i$ | $a_{i0}$ | $a_{i1}$ | $a_{i2}$ | $a_{i3}$ | $a_{i4}$ | $a_{i5}$ | $a_{i6}$ | (%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 2.296 | 0. | -37.25 | 0. | -1.4 | 0. | -13/6 | 19% |
| 2 | -17.33 | -4.405 | 0. | 0. | 0. | 0. | -3. | n/a |

## 4. Three components in NACRE

In NACRE, the rates are given by

$$\langle R_{3\alpha}\rangle = \frac{24\hbar N_A^2}{\pi(\mu_{12}\mu_{(12)3})^{1/2}(k_BT)^3}\int_0^\infty dE_1\int_0^\infty dE_2\frac{\sigma_{\alpha\alpha}(E_1)\sigma_{\alpha^8\mathrm{Be}}(E_2;E_1)}{\Gamma_\alpha(^8\mathrm{Be},E_1)}E_1E_2\exp\left(-\frac{E_1+E_2}{k_BT}\right)$$
$$= \overset{(1)}{\int_{E_{R1}-\epsilon_1}^{E_{R1}+\epsilon_1}\int_{E_{R2}-\epsilon_2}^{E_{R2}+\epsilon_2}}\bigcirc\,dE_1dE_2 + \overset{(2)}{\int_{E_{R1}-\epsilon_1}^{E_{R1}+\epsilon_1}\int_{other}}\bigcirc\,dE_1dE_2 + \overset{(3)}{\int_{other}\int_{E_{R2}-\epsilon_2}^{E_{R2}+\epsilon_2}}\bigcirc\,dE_1dE_2 + \overset{(4)}{\int_{other}\int_{other}}\bigcirc\,dE_1dE_2$$
$$\equiv \langle R_{3\alpha}\rangle_{DR} + \langle R_{3\alpha}\rangle_{^8\mathrm{Be}} + \langle R_{3\alpha}\rangle_{^{12}\mathrm{C}} + \langle R_{3\alpha}\rangle_{NR}. \quad (34)$$

Energy of α+$^8$Be; Energy of α+α; Res. in α+α; Res. in α+$^8$Be

The integrals of $E_1$ and $E_2$ in Eq. (34) can be divided into four terms. They are interpreted as (1) double resonances, (2) non-resonant sequential process, (3) $^{12}$C resonance, (4) non-resonant component. The composition ratio is shown in Fig. 8.

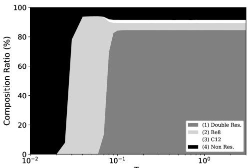



Fig. 8

- The component of (3) appears to be small over the entire region. Therefore, the standard reaction rates for $0^+$ below $T_9$ = 3 are found to generally consist of three components.
- The contribution of (2) is found to dominate the rates for $0.03 < T_9 < 0.07$. However, this term may not seem to be realistic **because $^8$Be is unbound**.

## 5. α+α and 3α interaction

Two types of interaction potentials are used in Eq. (3).

$$V^N_{\alpha\alpha}(\bar{x}_i) = (V_{c0}\mathbf{P_0} + V_{c2}\mathbf{P_2})e^{-(\bar{x}_i/a)^2} + V_0e^{-(\bar{x}_i/b)^2}, \qquad V^l_{3\alpha}(\rho) = v_{3\alpha,l}\exp\left[-\left(\frac{\rho}{r_{3\alpha}}\right)^2\right] \quad (35)$$

- **AB**: shallow [20,9,13]
- **CD**: core+deep, folding potentials [21,22], determined by phase shifts & α-width
- AB & CD have the 3α potentials to reproduce the energies of $0^+_2$ & $2^+_1$ in $^{12}$C.

Table 3 Potential parameters. The final results do not strongly depend on the adopted potentials.

| | $V_{c0}$ (MeV) | $V_{c2}$ (MeV) | $r_c$ (fm) | $V_0$ (MeV) | $r_v$ (fm) | $J_v$(MeV fm$^3$) |
|---|---|---|---|---|---|---|
| AB | 125.0 | 20.0 | 1.53 | −30.18 | 2.85 | 243 |
| CD | 295.45 | 197.8 | 1.57 | −118.7 | 2.16 | 416 |

| | $v_{3\alpha,0}$ (MeV) | $v_{3\alpha,2}$ (MeV) | $r_{3\alpha}$ (fm) |
|---|---|---|---|
| AB | 20.145 | 16.36 | 6 |
| CD | −37.15 | −21.14 | 5 |

## 6. Summary

**In the present poster, I have described the direct triple-α process with non-adiabatic Faddeev HHR*, and I have discussed the reaction rates. The derived rates are expressed in a simple analytic form, and they are converted into REACLIB format.**

- ✯ Direct triple-α contribution is confirmed to be $10^{-15}$_$10^{-3}$ pb order in the photo-disintegration cross sections of $^{12}C(2^+_1 \to 0^+)$ for 0.15 < *E* < 0.35 MeV. This is far below the values predicted by ACF and CMF that include the assumed long resonant tail of $0^+_2$. In spite of the large difference between the non-adiabatic and adiabatic cross sections, the derived rates are concordant with NACRE at the helium burning temperatures.
- ✯ The difference in $\sigma_g$ below E < 0.2 MeV can be seen in the rates for $T_9$ < 0.07. From the comparison among the calculations, I have found that the reaction rates could be reduced by about $10^{-4}$ at $T_9$ = 0.05, because of the non-adiabatic description of $^8$Be break-up and continuum states.
- ✯ The derived reaction rates have been expressed in the simple analytic form as Eqs. (28)_(32) with Table 1. The present rates do not have the component corresponding to the non-resonant sequential process between α+$^8$Be. I, therefore, find that this component could be at least eliminated by hand, to update the reaction rates in NACRE & REACLIB.

## References

[1] F. Hoyle, Astrophys. J. Suppl. Ser. 1, 121 (1954).
[2] E.E. Salpeter, Astrophys. J. **115**, 326 (1952).
[3] M. Katsuma, Phys. Rev. C **78**, 034606 (2008); Astrophys. J. **745**, 192 (2012); Springer Proc. in Physics **219**, 389 (2019); JPS Conf. Proc. **31**, 011060 (2020).
[4] D. Dell'Aquila, *et al.*, Phys. Rev. Lett. **119**, 132501 (2017); R. Smith, *et al.*, Phys. Rev. Lett. **119**, 132502 (2017).
[5] (NACRE) C. Angulo, M. Arnould, M. Rayet, *et al.*, Nucl. Phys. A **656**, 3 (1999).
[6] (CF88) G.R. Caughlan, W.A. Fowler, At. Data Nucl. Data Tables **40**, 283 (1988).
[7] K. Nomoto, F.-K. Thielemann, S. Miyaji, Astron. Astrophys. **149**, 239 (1985).
[8] K. Langanke, M. Wiescher, F.-K. Thielemann, Z. Phys. A **324**, 147 (1986).
[9] D.V. Fedorov, A.S. Jensen, Phys. Lett. B **389**, 631 (1996).
[10] S. Ishikawa, Phys. Rev. C **87**, 055804 (2013); ibid. **90**, 061604 (2014).
[11] H. Suno, Y. Suzuki, P. Descouvemont, Phys. Rev. C **94**, 054607 (2016).
[12] M. Katsuma, to appear in Proc. OMEG16, Communications in Physics, arXiv:2302.03844 (2023).
[13] N.B. Nguyen, F.M. Nunes, I.J. Thompson, Phys. Rev. C **87**, 054615 (2013); N.B. Nguyen, F.M. Nunes, I.J. Thompson, E.F. Brown, Phys. Rev. Lett., **109** , 141101 (2012); N.B. Nguyen, Ph.D. dissertation, Michigan State University (2013).
[14] I.J. Thompson, F.M. Nunes, *Nuclear Reactions for Astrophysics*, (Cambridge University Press, 2009).
[15] P. Descouvemont, J. Phys. G **37**, 64010 (2010); *Theoretical Models for Nuclear Astrophysics*, (Nova Science Publishers, 2003).
[16] M. Katsuma, Y. Sakuragi, S. Okabe, Y. Kondō, Prog. Theor. Phys. **107**, 377 (2002).
[17] G.R. Satchler, *Direct Nuclear Reactions*, (Oxford University Press, 1983)
[18] JINA Reaclib, https://reaclib.jinaweb.org/he4(aa,g)c12
[19] (FY05) H.O.U. Fynbo, C.Aa. Diget, U.C. Bergmann, *et al.*, Nature **433**, 136 (2005).
[20] S. Ali, A.R. Bodmer, Nucl. Phys. **80**, 99 (1966).
[21] B. Buck, H. Friedrich, C. Wheatley, Nucl. Phys. A **275**, 246 (1977); G.R. Satchler, W.G. Love, Phys.Rep. **55**, 183 (1979); G. Bertsch, J. Borysowicz, H. McManus, W.G. Love, Nucl. Phys. A **284**, 399 (1977).
[22] D. Baye, Phys. Rev. Lett. **58**, 2738 (1987).

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