Rapid turns in multi-field inflation: restored predictivity and resonant PBH production

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IBS-PNU Joint Workshop on Particle Physics and Cosmology, Busan

- Inflation is the leading paradigm for the generation of anisotropies of CMB
- Scalar fields suffice to derive models that fit the Planck data
- Single-field inflation phenomenologically viable
- Inflation is a cosmological collider
- Why examine multi-field models? High-energy embedding of inflation remains an open problem

The **top-down** approach

- High-energy theories include at least two scalar fields: e.g. α-attractors [Kalosh, Linde, Roest 13] derived from supergravity. Stringy models predict a plethora of scalar fields at high energies
- Higgs inflation is inherently multi-field [Bezrukov, Shaposhnikov 08][Kaiser, Sfakianakis 13]
- Usually extra fields are stabilized by giving them very large masses ($\sim \mathcal{O}(H)$)
- How is this done?

The model

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V\right) \tag{1}$$

Assuming an **FLRW** metric, **zero spatial curvature** (with $H \equiv \dot{a}/a$)

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V \tag{2}$$

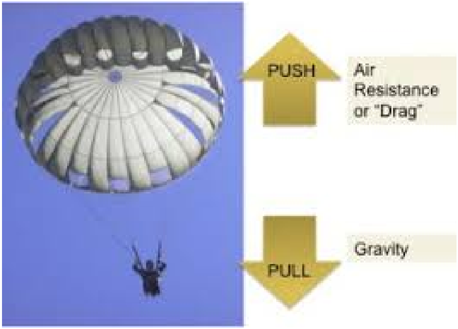
$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 \tag{3}$$

$$\underbrace{\ddot{\phi}}_{\text{acceleration}} + \underbrace{3H\dot{\phi}}_{\text{Hubble friction}} + \underbrace{V_{,\phi}}_{\text{potential gradient}} = 0 \tag{4}$$

- Similarities with parachute fall

$$\ddot{x} + b\dot{x} + mg = 0 \tag{5}$$

terminal velocity: $\ddot{x} = 0 \Leftrightarrow \dot{x} = -mg/b$



For a massive quadratic field

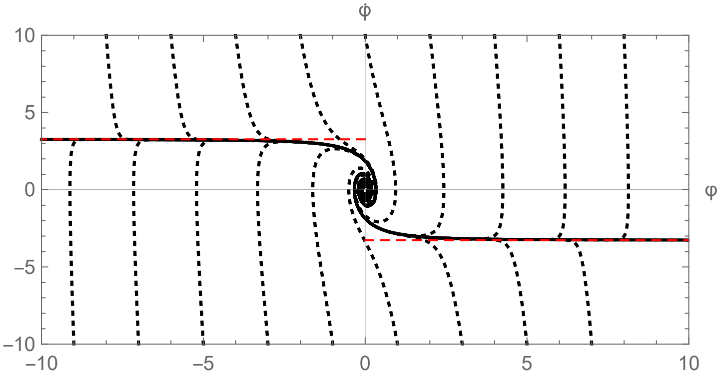


Figure: Numerical solution (dashed) versus the slow-roll approximation (red)

- Generic solutions quickly converge to particular solutions. The dependence on the velocities is lost
- The previous picture holds for other models (under certain slow-roll conditions)
- Single-field models achieve a notion of initial conditions independence. Important for self-consistency as infaltion was introduce to tackle fine-tunings of initial conditions for the ΛCDM
- What happens for multiple fields?

- Multiple scalar fields with minimal derivative couplings

$$\mathcal{L}_{\rm m} = \sqrt{-g}\left(\frac{1}{2}G_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V\right) \qquad (6)$$

where G_{ij} behaves as a **metric**

- **Non-minimal** models with $L_{\rm gr} = \sqrt{-g}f(\phi)R_{\rm ein}$ can be brought in previous form via a conformal transformation $g \to \Omega(\phi)g$ Jordan frame $\to$ Einstein frame [Kaiser 10] (Higgs inflation)

- Supergravity models typically yield flat or hyperbolic-like metrics [Kalosh, Linde, Roest...]

- Non-trivial G_{ij} quite generic

- Fields start with random initial conditions
- The heuristic picture is the following: for a 'suitable' multi-variable potential
  1. Hubble friction dissipates excess kinetic energy (slow-roll evolution)
  2. Fields move towards the minimum, where 'heavy' degrees decay first
  3. The lightest field drives evolution
- Last phase is the **attractor** solution. If it is reached fast we avoid the initial conditions dependence of multi-field models. Important for self-consistency of inflation
- Is the attractor solution equivalent to a single-field solution?

- EOM

$$D_t\dot{\phi}^i + 3H\dot{\phi}^i + V^{,i} = 0\,, \tag{7}$$

where

$$D_t A^i \equiv \frac{d}{dt}A^i + \dot{\phi}^k \Gamma^i_{km} A^m \tag{8}$$

- **One possibility**: gradient flow

$$\dot{\phi}^i \propto V^{,i} \tag{9}$$

- If more than one fields dynamical, multiple ways to inflate. Predictions depend on the values at horizon crossing ⇒ non-unique

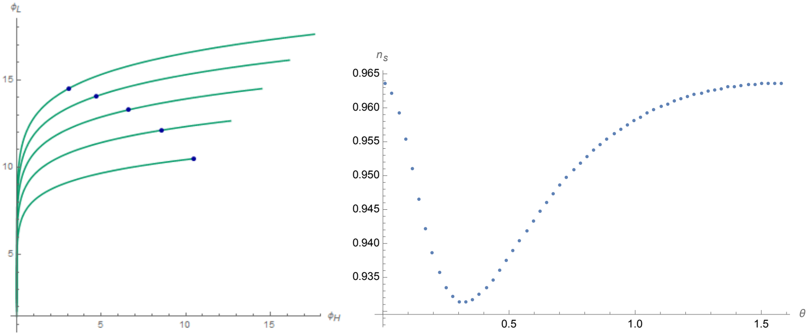


Figure: ‘Boomerang’ trajectories (left) and spectral index as a function of the angle (right) for two quadratic fields with similar masses ($V = \frac{1}{2}m_{\rm H}^2\phi_{\rm H}^2 + \frac{1}{2}m_{\rm L}^2\phi_{\rm L}^2$)

- It was claimed that predictivity is restored at the many-field limit [Easther, Frazer, Peiris, Price 14]. Predictions become sharp and approach specific values as $N \to \infty$. However, this turned out to be partly false
- Although the distribution of different values for observables becomes sharper as $N \to \infty$, the peak value depends on the assumed prior [**PC**, Roest, Rosati 20]
- Meaningful models require heavy fields to be stabilized quickly
- For gradient-flow models it's sufficient to place heavy fields at their minimum $V_{,i} = 0$. Then inflation happens along the light axis and the problem equivalent to single-field models

- The validity of gradient flow is measured by smallness of certain parameters

- During inflation e.g.

$$\epsilon \approx \epsilon_V \equiv \frac{1}{2V^2} G^{ij} V_{,i} V_{,j} \ll 1 \qquad (10)$$

  Steep potentials are excluded

- For strong field space curvature gradients tend to become steep. Gradient flow becomes unsustainable
- What happens to the heavy fields?

- Heavy fields stabilized at minima of an **effective potential** [Tolley, Wyman 09], [**PC**, Roest, Sfakianakis 20]

$$V_{\text{eff}}^{,i} = V^{,i} + \Gamma^i_{LL}\dot{\phi}_L^2 \tag{11}$$

Centrifugal forces tend to drift fields away from their minima

- Example: α-attractors for certain parameter values (e.g. for $\alpha \ll 1$) quickly deviate from gradient flow and a secondary inflationary phase begins. Due to that they yeild different predictions [**PC**, Roest, Sfakianakis 19]

- Negatively curved field spaces can lead to geometrical destabilization of usual slow-roll inflation [Turzinsky, Renaux-Petel 15]

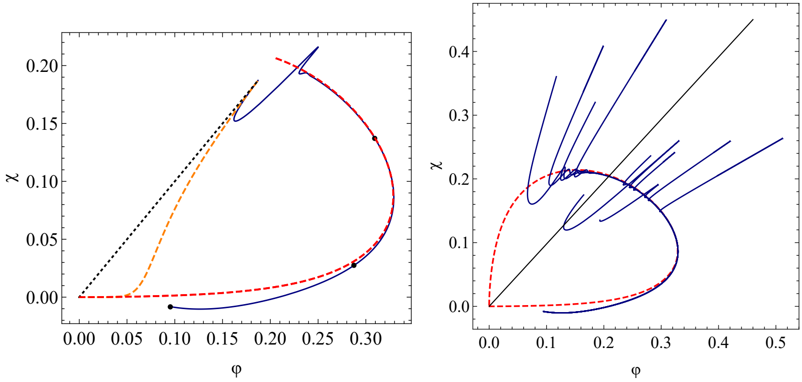


Figure: The attractor solution of two-field alpha attractors with $V = \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}m_\phi^2\phi^2$ and $G_{ij} = \frac{\alpha}{(1-\chi^2-\phi^2)^2}\delta_{ij}$.

- For 2 fields: 1 curvature and 1 isocurvature (entropic). The latter may source the curvature perturbation on superhorizon scales
- If 'heavy' then can be integrated out. However, it leaves imprints on observables [Achucarro, Gong, Hardeman, Palma, Patil 10]

$$S = \int d\tau d^3x a^2 \epsilon M_{\rm pl}^2 \left[\frac{1}{c_s^2} \left(\frac{d\mathcal{R}}{d\tau} \right)^2 - \nabla^2 \mathcal{R}^2 \right] \qquad (12)$$

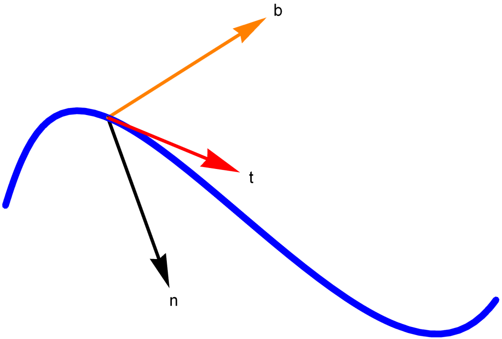
where the speed of sound of fluctuations c_s^2 depend on quantities that quantify the strength of multi-field effects

$$c_s^2 \equiv 1 + \frac{4\Omega^2}{\mathcal{M}_{nn} - \Omega^2}$$

- When $c_s^2 < 0$ prior to horizon crossing transient instability in the power spectrum [Cremonini, Lalak, Turzynski 10].
- Power spectrum grows exponentially and can account for **PBH** production [Palma, Sypsas, Zenteno 20] [Fumagalli, Renaux-Petel, Ronayne, Witkowski 20] [Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky 20] [many more...]
- PBH can be **dark matter** candidates
- An imaginary sound speed can be achieved with large turns

- Rate of change of tangent vector $t^i \equiv D_N\phi^i/|D_N\phi^i|$

$$D_N t^i \equiv \mathbf{\Omega} n^i$$



- Quantifies the deviation from following a geodesic

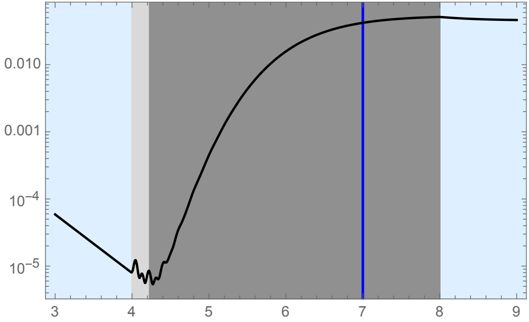


Figure: Power spectrum for one mode in the presence of a turn

- Multiple turns can have resonant effect [Boutivas, Dalianis, Kodaxis, Tetradis 22] [**PC**, Gong in progress]

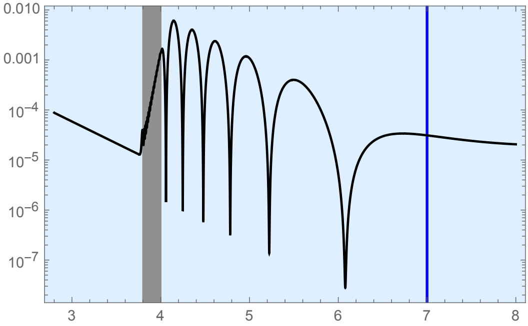


Figure: Evolution of a k-mode for a sharp turn

- During turn P_R increases exponentially and after the turn oscillates with frequency $\kappa \equiv k/(aH)$ (both positive and negative frequencies)

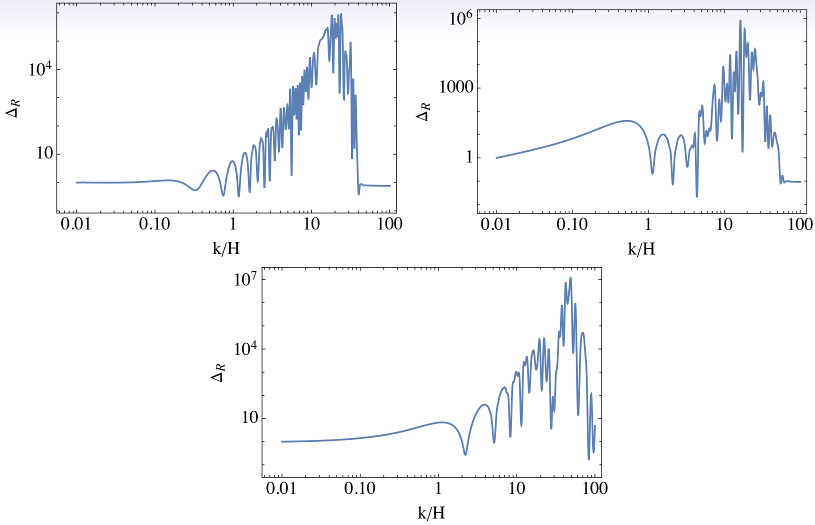


Figure: One vs two vs three sharp turns of equal angle $\theta \equiv \int dN\Omega = 10\pi$. Maximum value for each turn satisfies $\Omega_1 > \Omega_2 > \Omega_3$

- If a subsequent turn is placed at a distance $1/\kappa$ resonance can happen
- Successive smaller turns achieve the same (or larger) effect. Perturbative control requires $\Omega^n_{\rm max} P_R \ll 1$ [Fumagalli, Renaux-Petel, Ronayne, Witkowski 20]. With multiple turns the system remains under control
- For more fields one can have the same effect without extremely large turns by increasing the magnitude of higher order bending parameters (e.g. the torsion of the curve)

- Multi-field models are motivated by high-energy theories
- There is a notion on initial conditions dependence for single-field models
- To retain this in multi-field models heavy fields should be stabilized
- The attractor solution can either be gradient flow (single-field) or rapid-turn (multi-field)
- This novel behaviour can have applications in early (inflation) and late universe (dark matter and dark energy)

Thank you for your attention!