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Bosonic dark matter condensed by thermal fermions

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Based on 2109.07423

"DM from thermal fermions"
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Outline

- Ultralight boson with a huge population is a **cold dark matter** candidate.

$$\frac{n_\phi}{s} \sim 10^{19}\,\frac{10^{-20}\text{eV}}{m_\phi}$$

- It can be generated by a misaligned initial amplitude forming a **coherent state** (e.g., QCD axion with instanton potential).

- New idea: **Coherent DM state** driven by **thermal fermions**.

Bosonic DM in a coherent state

- Coherent state = Eigenstate of annihilation/creation operators

$$\hat{\phi}(x) = \int_k [a_k e^{-ik\cdot x} + a_k^+ e^{+ik\cdot x}]$$

$$a_k|\phi_c\rangle = \phi_k|\phi_c\rangle;\ a_k^+|\phi_c\rangle = \phi_k^*|\phi_c\rangle$$

$$\langle a_k\rangle_c = \phi_k;\ \ \langle a_k^+\rangle_c = \phi_k^*$$

- Coherent state = classical field

$$\hat{\phi}(x) = \int_k [a_k e^{-ik\cdot x} + a_k^+ e^{+ik\cdot x}]$$

Assuming a monochromatic state

$$\langle\hat{\phi}(x)\rangle_c = \phi_0 e^{-ik_0\cdot x} + \phi_0^+\, e^{+ik_0\cdot x}$$

Energy density: $\rho_\phi = 2m_\phi^2|\phi_0|^2$

DM amplitude around us : $\rho_{DM}^{\odot} \approx 0.3\mathrm{GeV/cm^3} \Rightarrow |\phi_0^{\odot}| \approx 10^{12}\mathrm{GeV}\,\dfrac{10^{-22}\mathrm{eV}}{m_\phi}$

Misalignment mechanism

- Evolution in the FLRW universe:

$$\langle \hat{\phi}(x) \rangle_T = \phi(t)$$

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + V'(\phi) = 0$$

- For an approximately free field:

$$\phi''(x) + \frac{3}{2x}\phi'(x) + \phi(x) \approx 0$$

$$x \equiv m_\phi t$$

- Analytic solution

$$\phi(x) = C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}}$$

$$H \gg m_\phi \ (x \ll 1): \quad \phi = \phi_i; \dot{\phi} = 0$$

$$H \ll m_\phi \ (x \gg 1): \ \phi \sim \phi_i \frac{\sin\left(m_\phi t + \frac{\pi}{8}\right)}{\left(m_\phi t\right)^{3/4}}$$

$$m_\phi \gg H_{eq} \approx 3 \cdot 10^{-27}\text{eV}$$

CDM density: $\rho_{DM}(x_{eq}) \approx 0.48\ \text{eV}^4$ $\quad \rho_\phi(x) \sim \frac{m_\phi^2 \phi_i^2}{x^{3/2}} \Rightarrow \phi_i \sim 0.01\ M_p \left(\frac{10^{-20}\text{eV}}{m_\phi}\right)^{1/4}$

Scalar field in thermal background

- Scalar field interacting with thermal fermions:

$$\mathcal{L}' = y_\phi \, \hat{\phi} \, (\bar{f}_R f_L + \bar{f}_L f_R)$$

$$\rightarrow V_{T,\mathrm{eff}}(\phi) = -\frac{g_f}{2\pi^2} T^4 J_F\left(\frac{(m_f + y_\phi \phi)^2}{T^2}\right)$$

Dolan+Jackiw,
Weinberg, 1974

- Leading thermal effects in cosmological evolution:

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + (m_\phi^2 + m_T^2)\phi(t) \approx \frac{\partial}{\partial\phi}\langle\mathcal{L}'\rangle_T$$

$$m_T^2 = \frac{g_f}{24} y_\phi^2 T^2, \quad \langle\mathcal{L}'\rangle_T = y_\phi \phi \frac{g_f m_f T^2}{24}$$

$$g_f = 4N_c \;\; (2) \;\; \text{for} \;\; f = q, l \; (\nu)$$

Esteban+Salvado, 2101.05804

Batell+Ghalsasi, 2109.04476

General features I

- Evolution from $T_{ew} \approx 100$ GeV down to $T_{eq} \approx 0.8$ eV:

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \left(1 + \frac{x_1}{x}\right)\tilde{\phi}(x) = \frac{x_S}{x}$$

$$x_1 \equiv y_\phi^2 \frac{c_t^2 g_f}{48}\frac{M_P}{m_\phi}, \qquad x_S \equiv y_\phi \frac{c_t^2 g_f}{48}\frac{m_f}{m_\phi}$$

(Notation) $T = c_t\sqrt{M_P/2t}$ $\quad c_t = 1.74/g_*^{1/4}$

$$x = m_\phi t = \frac{c_t^2 m_\phi M_P}{2T^2},$$

$$\tilde{\phi} = \phi/M_P, \quad m_{20} \equiv m_\phi/10^{-20}\text{eV}$$

- Vanishing initial condition at x_{ew}: $\phi = 0, \phi' = 0$.

- Nontrivial evolution from T_{ew} (x_{ew}) to $T_f = m_f(x_f)$, and then free evolution from $T_f(x_f)$ to T_{eq} (x_{eq}) for $f = q, l$.

- Evolution from T_{ew} to T_{eq} for $f = \nu$.

General features II

- Different types of solutions depending on the location of $T_1\ (x_1)$:

$$x_{ew} \overset{x_1}{\dashrightarrow} x_f \dashrightarrow x_{eq}$$

For $f = q, l$: i) $x_1 < x_{ew} < x_f \to x_{eq}$

ii) $x_{ew} < x_f < x_1 \to x_{eq}$

iii) $x_{ew} < x_1 < x_f \to x_{eq}$

For $f = \nu$: $x_{ew} < x_1 < x_{eq}$

- The resulting DM density is to be described by

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2}{\pi} \frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}}$$

$$\rho_\phi = \rho_{DM} \Rightarrow c_f^2 x_S^2 \approx 2.5 \times 10^{-4} m_{20}^{-1/2}$$

$$x_{ew} \approx 10^{-15} m_{20} \qquad x_{eq} \approx 2 \times 10^7 m_{20}$$

$$x_f \approx 5 \times 10^{-4} m_{20} (m_e/m_f)^2$$

$$x_1 \approx 10^{47} y_\phi^2 m_{20}^{-1} \qquad x_S \approx 2 \cdot 10^{24}\, y_\phi\, m_{20}^{-1} \left(\frac{m_f}{m_e}\right)$$

Asymptotic solutions in various regimes

$x \gg x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$$\tilde{\phi}(x) = \frac{\pi x_S}{(2x)^{1/4}}\left(\frac{x}{\Gamma(3/4)} J_{1/4}(x)\, F_{1,2}\left(\frac{1}{2};\frac{3}{4},\frac{3}{2};-\frac{x^2}{4}\right) - \frac{x^{3/2}}{3\Gamma(5/4)}\left(J_{1/4}(x) - Y_{1/4}(x)\right) F_{1,2}\left(\frac{3}{4};\frac{5}{4},\frac{7}{4};-\frac{x^2}{4}\right)\right) + C_1\frac{J_{1/4}(x)}{x^{1/4}} + C_2\frac{Y_{1/4}(x)}{x^{1/4}}$$

$x \ll x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$$\tilde{\phi}(x) = \frac{x_S}{x_1} + C_1\frac{e^{2i\sqrt{xx_1}}}{\sqrt{x}} + iC_2\frac{e^{-2i\sqrt{xx_1}}}{\sqrt{xx_1}}$$

$x > x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\tilde{\phi}(x) = C_1\frac{J_{1/4}(x)}{x^{1/4}} + C_2\frac{Y_{1/4}(x)}{x^{1/4}}$$

$\tilde{\phi}(x)$ at $x \to 0$ & ∞

Case i)

```
DSolve[y''[x] + 3/2/x y'[x] + y[x] == xS/x, y[x], x][[1]];
Series[y[x] /. %, {x, 0, 1}] // FullSimplify
Series[y[x] /. %%, {x, Infinity, 1}] // Normal // FullSimplify
```

$$-\frac{2^{1/4}\, \mathbb{C}_2\, \mathrm{Gamma}\left[\frac{1}{4}\right]}{\pi\sqrt{x}} + \frac{\mathbb{C}_1 + \mathbb{C}_2}{2^{1/4}\, \mathrm{Gamma}\left[\frac{5}{4}\right]} + \frac{2\, xS\, x}{3} + O[x]^{5/4}$$

$$\frac{xS}{x} + \frac{-\mathrm{Cos}\left[\frac{\pi}{8} + x\right]\left(2\,\mathbb{C}_2 + 2^{1/4}\sqrt{\pi}\, xS\, \mathrm{Gamma}\left[\frac{3}{4}\right]\right) + 2\,\mathbb{C}_1\, \mathrm{Sin}\left[\frac{\pi}{8} + x\right]}{\sqrt{2\pi}\, x^{3/4}}$$

Case iii)

```
DSolve[y''[x] + 3/2/x y'[x] + y[x] == 0, y[x], x][[1]];
Series[y[x] /. %, {x, 0, 1}] // FullSimplify
Series[y[x] /. %%, {x, Infinity, 1}] // Normal
```

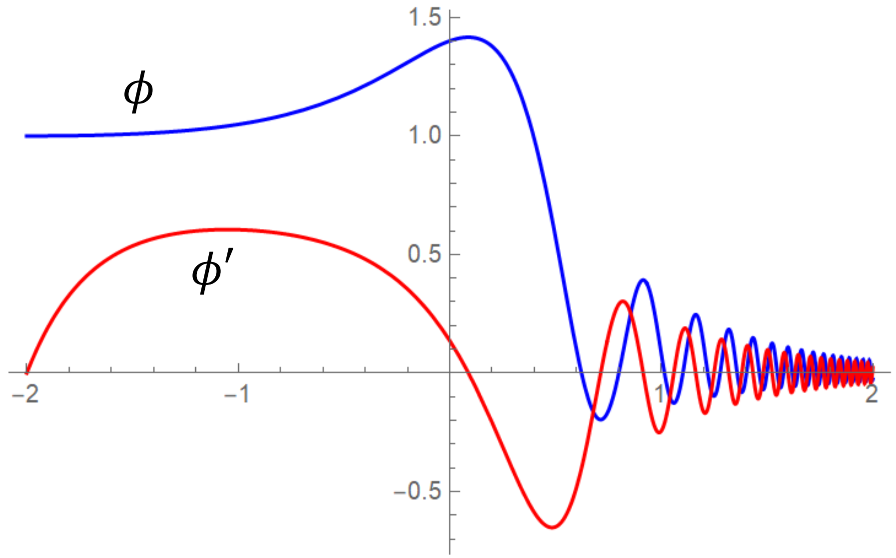
$$-\frac{2^{1/4}\, \mathbb{C}_2\, \mathrm{Gamma}\left[\frac{1}{4}\right]}{\pi\sqrt{x}} + \frac{\mathbb{C}_1 + \mathbb{C}_2}{2^{1/4}\, \mathrm{Gamma}\left[\frac{5}{4}\right]} + O[x]^{5/4}$$

$$-\frac{\sqrt{\frac{2}{\pi}}\, \mathbb{C}_2\, \mathrm{Cos}\left[\frac{\pi}{8} + x\right]}{x^{3/4}} + \frac{\sqrt{\frac{2}{\pi}}\, \mathbb{C}_1\, \mathrm{Sin}\left[\frac{\pi}{8} + x\right]}{x^{3/4}}$$

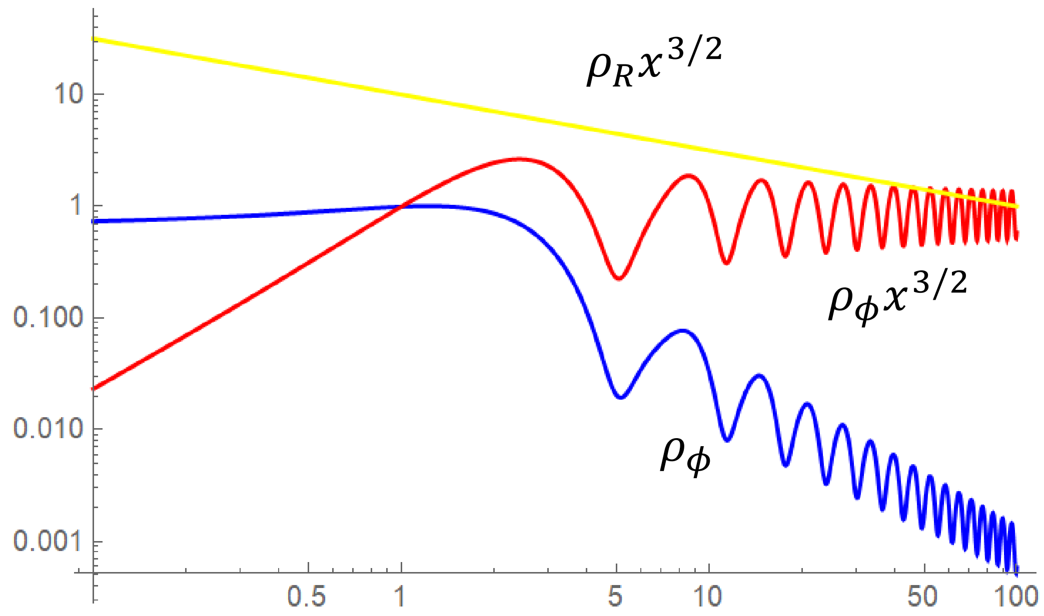
Case i)

- Behaviors of ϕ & ρ_ϕ with the vanishing initial conditions

```
yns =
  NDSolve[{y''[x] + 3 / 2 / x y'[x] + y[x] == 1 / x, y[0.01] == 1,
     y'[0.01] == 0}, y, {x, 0.01, 100}][[1]];
Plot[{y[x] /. yns /. x → 10^xx, y'[x] /. yns /. x → 10^xx},
 {xx, -2, 2}, PlotStyle → {Blue, Red}]
```



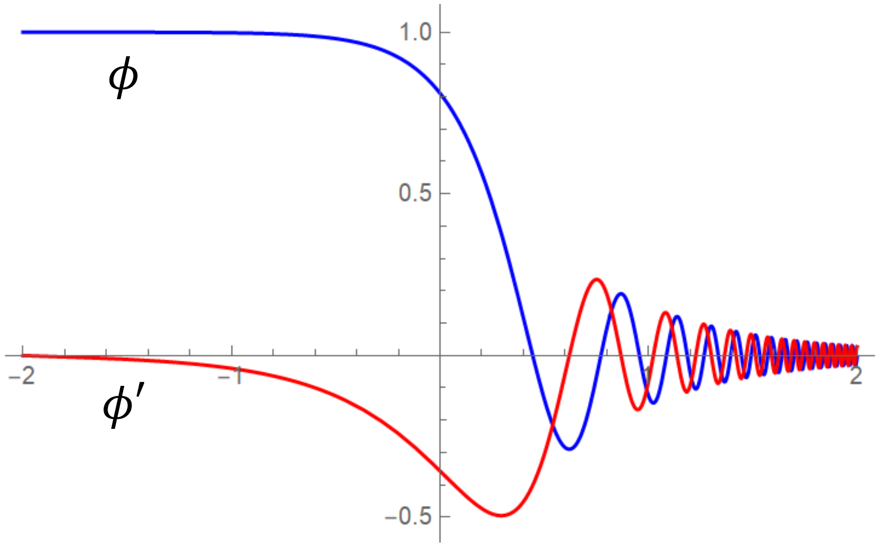

```
(y[x]^2 + y'[x]^2) / 2 /. yns;
LogLogPlot[{%, % x^(3 / 2), (100 / x)^2 (x / 100)^(3 / 2)},
 {x, 0.1, 100}, PlotStyle → {Blue, Red, Yellow}]
```



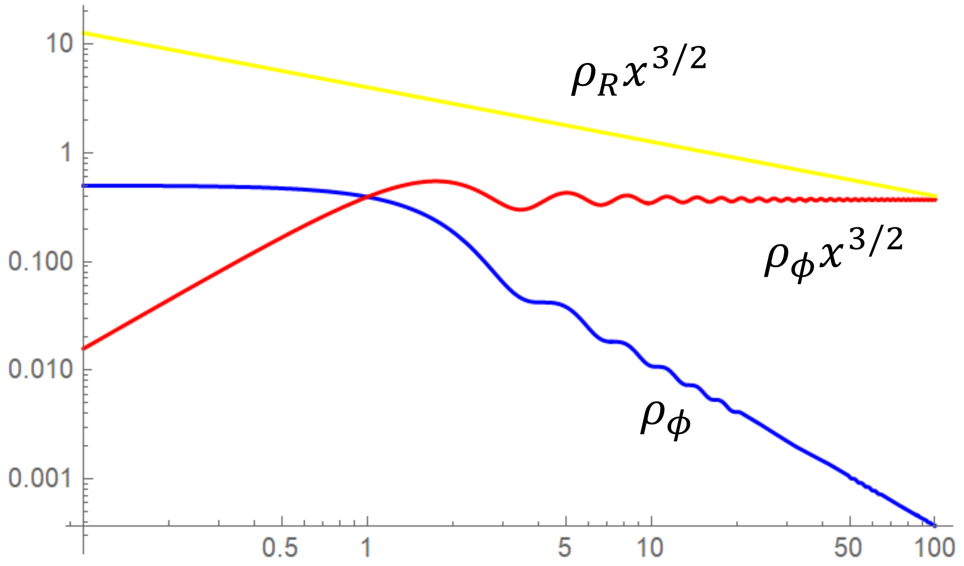
Case iii)

- Behaviors of ϕ & ρ_ϕ with the initial misalignment.

```
yns =
  NDSolve[{y''[x] + 3 / 2 / x y'[x] + y[x] == 0, y[0.01] == 1,
     y'[0.01] == 0}, y, {x, 0.01, 100}][[1]];
Plot[{y[x] /. yns /. x → 10^xx, y'[x] /. yns /. x → 10^xx},
 {xx, -2, 2}, PlotStyle → {Blue, Red}]
```




```
(y[x]^2 + y'[x]^2) / 2 /. yns;
LogLogPlot[{%, % x^(3 / 2), 0.4 (100 / x)^2 (x / 100)^(3 / 2)},
 {x, 0.1, 100}, PlotStyle → {Blue, Red, Yellow}]
```



Scalar DM condensed by q/l

i) $x_1 < x_{ew} < x_f$

$x < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$x > x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

Solution

$$\tilde{\phi}(x) = \begin{cases} x_S\left(\frac{\pi}{\Gamma\left(\frac{3}{4}\right)}\frac{J_{1/4}(x)}{(2x)^{1/4}}\left[G_1(x) - G_1(x_{ew})\right] - \frac{4\Gamma\left(\frac{3}{4}\right)}{3}\frac{J_{-1/4}(x)}{(2x)^{1/4}}\left[G_2(x) - G_2(x_{ew})\right]\right) \\ C_1\frac{J_{1/4}(x)}{x^{1/4}} + C_2\frac{Y_{1/4}(x)}{x^{1/4}} \end{cases}$$

$$G_1(x) = xF_{1,2}\left(\frac{1}{2};\frac{3}{4},\frac{3}{2};-\frac{x^2}{4}\right),\ G_2(x) = x^{3/2}F_{1,2}\left(\frac{3}{4};\frac{5}{4},\frac{7}{4};-\frac{x^2}{4}\right)$$

$$\frac{C_1}{x_S} = \frac{\pi}{2^{\frac{1}{4}}\Gamma(\frac{3}{4})}G_1(x_f) - \frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{3}G_2(x_f)$$

$$\frac{C_2}{x_S} = \frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{3}G_2(x_f)$$

Scalar DM condensed by q/l

i) $x_1 < x_{\rm ew} < x_f$

$$x < x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2}{\pi}\frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}} \quad \text{with} \quad c_f^2 = C_1^2 + C_2^2$$

$$c_f^2 = \begin{cases} 4\sqrt{2}\,\Gamma\left(\frac{5}{4}\right)^2 x_f^2 & \text{for } x_f \ll 1 \\ \dfrac{\pi\Gamma\left(\frac{3}{4}\right)^2}{2\sqrt{2}} & \text{for } 1 \ll x_f \end{cases}$$

$$y_\phi \approx 3.6\cdot 10^{-24}\,\frac{m_{20}^{-\frac{1}{4}}}{N_c}\left(\frac{m_f}{m_e}\right) \text{ with } \frac{550}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{4}{5}} \ll m_{20} \ll 2\cdot 10^3\left(\frac{m_f}{m_e}\right)^2 \quad \text{for } x_f \ll 1$$

$$y_\phi \approx 3\cdot 10^{-27}\,\frac{m_{20}^{\frac{3}{4}}}{N_c}\left(\frac{m_e}{m_f}\right) \text{ with } m_{20} \gg \text{Max}\left[2\frac{10^{16}}{N_c^2}\left(\frac{m_f}{m_e}\right)^4, 2\cdot 10^3\left(\frac{m_f}{m_e}\right)^2\right] \quad \text{for } 1 \ll x_f$$

(*) Neglecting Freeze-In:
$$m_\phi \ll 46\text{ eV } (5.7\text{ MeV}) \quad \text{for} \quad f = e\ (b),$$
$$y_\phi \ll 5.3\,(1.4)\times 10^{-11}.$$

Scalar DM condensed by q/l

ii) $x_{ew} < x_f < x_1$

$x < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$x > x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

Solution

$$\tilde{\phi}(x) = \begin{cases} \frac{x_S}{x_1}\left(1 - \sqrt{\frac{x_{ew}}{x}}\cos\left[2\left(\sqrt{x x_1} - \sqrt{x_{ew} x_1}\right)\right] - \frac{1}{2\sqrt{x x_1}}\sin\left[2\left(\sqrt{x x_1} - \sqrt{x_{ew} x_1}\right)\right]\right) \\ C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}} \end{cases}$$

$$\frac{C_1}{x_S} = -\frac{\pi x_f^{\frac{1}{4}}}{4x_1}\left[2x_f Y_{\frac{5}{4}}(x_f) - Y_{\frac{1}{4}}(x_f)\cos(2\sqrt{x_1 x_f}) + \sqrt{\frac{x_f}{x_1}} Y_{-\frac{3}{4}}(x_f)\sin(2\sqrt{x_1 x_f})\right],$$

$$\frac{C_2}{x_S} = \frac{\pi x_f^{\frac{1}{4}}}{4x_1}\left[2x_f J_{\frac{5}{4}}(x_f) - J_{\frac{1}{4}}(x_f)\cos(2\sqrt{x_1 x_f}) + \sqrt{\frac{x_f}{x_1}} J_{-\frac{3}{4}}(x_f)\sin(2\sqrt{x_1 x_f})\right].$$

Scalar DM condensed by q/l

ii) $x_{ew} < x_f < x_1$

$x < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$x > x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2}{\pi}\frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}} \quad \text{with} \quad c_f^2 = C_1^2 + C_2^2$$

$$c_f^2 = \begin{cases} 4\sqrt{2}\,\Gamma\left(\frac{5}{4}\right)^2 x_f^2 & \text{for } x_f \ll x_1 \ll 1 \\ \frac{\pi x_f^3}{2x_1^2} & \text{for } 1 \ll x_f \ll x_1 \end{cases}$$

$$y_\phi \approx 3.6\cdot 10^{-24}\,\frac{m_{20}^{-\frac{1}{4}}}{N_c}\left(\frac{m_f}{m_e}\right) \text{ with } m_{20} \ll 31\,N_c^{-\frac{2}{5}}\left(\frac{m_f}{m_e}\right)^{8/5} \quad \text{for } x_f \ll x_1 \ll 1$$

$$y_\phi \approx 5.3\cdot 10^{-24}\,m_{20}\left(\frac{m_e}{m_f}\right)^{\frac{1}{2}} \text{ with } m_{20} \gg 2\cdot 10^3\left(\frac{m_f}{m_e}\right)^2 \quad \text{for } 1 \ll x_f \ll x_1$$

(*) BBN constraint: $m_\phi \ll 10^{-6}\,(2.5\times 10^{-3})$ eV for $f = e\ (b)$,

$y_\phi \ll 5.7\times 10^{-10}\,(1.5\times 10^{-8})$.

Scalar DM condensed by q/l

iii) $x_{ew} < x_1 < x_f$

$$x < x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$$x_1 < x < x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x_f < x$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

(i) $x_1 \ll x_f \ll 1$:

$$y_\phi \approx 3.6\times 10^{-24}\frac{m_{20}^{-\frac{1}{4}}}{N_c}\frac{m_f}{m_e} \quad \text{with} \quad \frac{28}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{8}{5}} \ll m_{20} \ll \text{Min}\left[2\times 10^3\left(\frac{m_f}{m_e}\right)^2, \frac{6\times 10^5}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{4}{5}}\right]$$

(ii) $x_1 \ll 1 \ll x_f$:

$$y_\phi \approx 3\times 10^{-27}\frac{m_{20}^{\frac{3}{4}}}{N_c}\frac{m_e}{m_f} \quad \text{with} \quad 2\times 10^3\left(\frac{m_f}{m_e}\right)^2 \ll m_{20} \ll \text{Min}\left[3.2\times 10^{11}N_c^2\left(\frac{m_f}{m_e}\right)^4, \frac{2.1\times 10^{16}}{N_c^2}\left(\frac{m_e}{m_f}\right)^4\right]$$

(iii) $1 \ll x_1 \ll x_f$:

$$y_\phi \approx 1.9\times 10^{-22}\frac{m_{20}^{\frac{1}{3}}}{N_c^{\frac{1}{6}}}\left(\frac{m_f}{m_e}\right)^{\frac{2}{3}} \quad \text{with } 1.2\times 10^8 N_c^{\frac{5}{4}}\left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \ll m_{20} \ll 4.4\times 10^{11}N_c^2\left(\frac{m_f}{m_e}\right)^4 \quad \text{for } x_1^5 \ll x_f$$

$$y_\phi \approx 4.5\times 10^{-28}\frac{m_{20}}{N_c}\left(\frac{m_e}{m_f}\right)^{\frac{3}{2}} \quad \text{with} \quad m_{20} \gg 4.9\times 10^8 N_c^{\frac{5}{4}}\left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \quad \text{for } x_f \ll x_1^5.$$

(*) BBN+Feeze-In: $y_\phi \ll 5.7\,(1.2)\times 10^{-10}$,

$$m_\phi \ll 1.3\times 10^{-2}\,(6.1\times 10^3)\ \text{eV} \quad \text{for} \quad f = e\,(b)$$

Scalar DM condensed by ν

- $x_{ew} < x_1 < x_{eq} < x_\nu$

$x < x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$x > x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

Solution

$$\tilde{\phi}(x) = \begin{cases} \frac{x_S}{x_1}\left(1 - \sqrt{\frac{x_{ew}}{x}}\cos\left[2\left(\sqrt{x x_1} - \sqrt{x_{ew}x_1}\right)\right] - \frac{1}{2\sqrt{x x_1}}\sin\left[2\left(\sqrt{x x_1} - \sqrt{x_{ew}x_1}\right)\right]\right) \\ C_1\frac{J_{1/4}(x)}{x^{1/4}} + C_2\frac{Y_{1/4}(x)}{x^{1/4}} + \frac{x_S}{(2x)^{1/4}}\left(\frac{\pi}{\Gamma\left(\frac{3}{4}\right)}J_{1/4}(x)G_1(x) - \frac{4\Gamma\left(\frac{3}{4}\right)}{3}J_{-1/4}(x)G_2(x)\right) \end{cases}$$

$(*)\ G_{1,2}(x) \to$ constant for $x \to \infty$

$$\frac{C_1}{x_S} = -\frac{\pi}{2^{\frac{1}{4}}\Gamma(\frac{3}{4})}G_1(x_1) + \frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{3}G_2(x_1) - \frac{\pi}{2}\frac{Y_{\frac{1}{4}}(x_1)\sin^2(x_1) - Y_{-\frac{3}{4}}(x_1)[x_1 - \frac{1}{2}\sin(2x_1)]}{x_1^{3/4}}$$

$$\frac{C_2}{x_S} = -\frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{6}G_2(x_1) + \frac{\pi}{2}\frac{J_{\frac{1}{4}}(x_1)\sin^2(x_1) - J_{-\frac{3}{4}}(x_1)[x_1 - \frac{1}{2}\sin(2x_1)]}{x^{3/4}}$$

Scalar DM condensed by ν

- $x_{ew} < x_1 < x_{eq} < x_\nu$

$x < x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$x > x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$$\rho_\phi(x_{eq}) \approx \frac{c_\nu^2 x_S^2}{\pi}\frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}} \text{ with } c_\nu^2 = C_1^2 + C_2^2$$

$$c_\nu^2 = \frac{\pi\Gamma\left(\frac{3}{4}\right)^2}{2\sqrt{2}} + \frac{2^{\frac{3}{4}}\pi^{\frac{3}{2}} C_1}{\Gamma\left(\frac{1}{4}\right) x_S} + \frac{C_1^2 + C_2^2}{x_S^2} \approx 3.34 \text{ for } x_1 \gg 1$$

$$y_\phi \approx 4.2 \cdot 10^{-20}\, m_{20}^{\frac{3}{4}}\left(\frac{0.05\text{eV}}{m_\nu}\right) \text{ with } m_{20} \gg 710\left(\frac{0.05\text{eV}}{m_\nu}\right)^4$$

$$\frac{x_S}{x_1} \sim 4.8 \times 10^{-11}\, m_{20}^{-3/4}$$

$$\frac{x_1}{x_{eq}} \sim 8.8\, m_{20}^{-1/2}$$

(*) BBN: $y_\phi \lesssim 7 \times 10^{-6}$ $\quad m_\phi \lesssim 0.092 \text{ eV}\left(\frac{m_\nu}{0.05 \text{ eV}}\right)^{4/3}$

Conclusion

- An ultralight scalar can form a coherent DM state through its tiny coupling to SM fermions.

- Considering the ϕqq or ϕll coupling, it works for wide ranges of $m_\phi \sim (10^{-22}, 10^{6})\mathrm{eV}$ and $y_\phi \sim (10^{-28}, 10^{-8})$.

- For the $\phi \nu\nu$ coupling, DM genesis requires $m_\phi \sim (10^{-17}, 10^{-1})\mathrm{eV}$ and $y_\phi \sim (10^{-17}, 10^{-5})$, and can occur **very late** at around $T > 10T_{eq}$.