Perspectives on understanding the substructure of multiquark hadrons in lattice QCD

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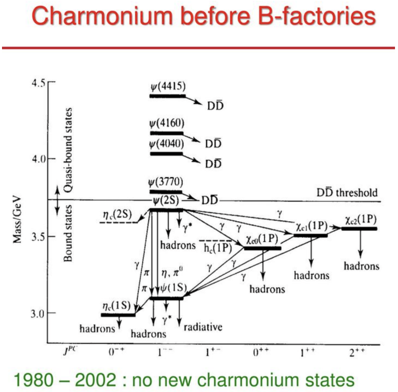
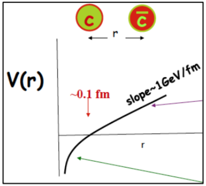
Phys.Rev.D 99 (2019) 5, 054505 [1810.10550]
Phys.Rev.Lett. 118 (2017) 14, 142001 [1607.05214]

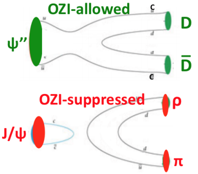
Heavy spectrum pre *B*-factories - A success story





Before the advent of *B*-factories the study of heavy particles, in particular charmonia, can be seen as success story:

- predicted and measured masses agree
- potential model works well
- OZI-rule applies, no exceptions

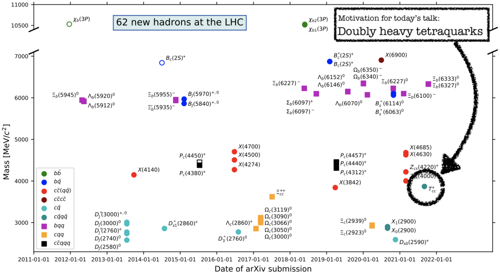


(Olsen, Mitchell, 2019)

Heavy spectrum today - a situational overview

Many new heavy hadrons observed, e.g. 62 at LHCb ($\sim$12 tetra-/pentaquarks)

- In 2003: $X(3872) \rightsquigarrow c\bar{c}d\bar{u}$ discovered at Belle
- 4-/5-quark states **not expected** in quark models.
- Many predicted quark model **states not found**.



(LHCb collaboration, LHCb-FIGURE-2021- 001, 2021)

... many not explained in theory

QCD often approximated in models
$\rightsquigarrow$ many extensions possible
$\rightsquigarrow$ many interpretations
$\rightsquigarrow$ often contradictory statements

model	building blocks
"plain"	$q_{(i,c)}$, $\bar{q}_{(i,c)}$
diquark	$[qq]_{(i,j,c)}$ & $q/\bar{q}$
triquark	$[qq\bar{q}]_{(i,j,k,c)}$ & $q/\bar{q}$
hydro-onium	$[Q\bar{Q}]_{(i,j)}$, $[q\bar{q}]_{(i,j)}$, $[qqq]_{(i,j,k)}$
molecular	$[Q\bar{q}]_{(i,j)}$, $[q\bar{Q}]_{(i,j)}$, $[qqQ]_{(i,j,k)}$, ...

In the following:

- **Goal: Non-perturbative insights into exotic hadrons in full QCD**
- ...

A new family of tetraquarks? - observation of T_{cc}^{+} at LHCb

Narrow state observed in $D^0 D^0 \pi^+$

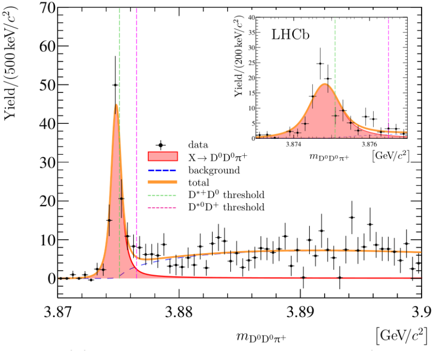
- Fitted to P-wave BW
- $\delta m = -273 \pm 61 \pm 5^{+11}_{-14} keV/c^2$ below $D^0 D^{*+}$ threshold
- $\Gamma = 410 \pm 165 \pm 43^{+18}_{-38} keV$

consistent with $cc\bar{u}\bar{d}$ tetraquark

- Possible family of states: $bc\bar{u}\bar{d}$, $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$, ...
- QN: $I(J^P) = 0(1^+)$
- Recent discussion in theory, both in pheno and lattice ⇝ predictions, binding mechanism

$B_{T_{cc}} = 0.3\text{MeV}$

⇝ LHCb-PAPER-2021-031



In the following:

- **Goal: Non-perturbative insights into exotic hadrons in full QCD**
- Doubly heavy tetraquarks as new QCD states and **diquarks** as effective d.o.f's in QCD

Diquarks - (possible) attractive building blocks for ordinary and exotic hadrons

Diquarks - an attractive concept

"The concept of diquarks is almost as old as the quark model, and actually predates QCD [1]" $\rightsquigarrow$ arXiv:2203.16583; [1] PR 155, 1601 (1967)

- Successful for low-lying baryons and exotic hadrons.
  - Well founded in QCD with many predictions.
  - But, experimental evidence has been elusive.
- Light diquarks:
  - special "good" $(\bar{3}_F, \bar{3}_c, J^P = 0^+)$ configuration
  - quarks on "good" diquarks attract each other
  - large mass splitting in good, bad and not-even-bad
  - non-vanishing size or compact?
- HQSS-limit: A diquark acts as an antiquark $[QQ] \leftrightarrow \bar{Q}$.

$\rightsquigarrow$ currently one motivation for T_{QQ}-type hadrons, next slide

3 types of diquark:
good, bad and not-even bad

Diquark operator:

$$\boxed{D_\Gamma = q^c C \Gamma q'}$$

$\rightsquigarrow$ c, C =charge conjugation

$\rightsquigarrow$ Γ acts on Dirac space

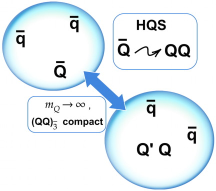
J^P	C	F	Op: Γ
0^+	$\bar{3}$	$\bar{3}$	$\gamma_5,\ \gamma_0\gamma_5$
1^+	$\bar{3}$	6	$\gamma_i,\ \sigma_{i0}$
0^-	$\bar{3}$	6	$\mathbb{1},\ \gamma_0$
1^-	$\bar{3}$	$\bar{3}$	$\gamma_i\gamma_5,\ \sigma_{ij}$

Revisit ideas for stable multiquarks based on diquarks

$\rightsquigarrow$ Ader et al. ('82); Manohar, Wise ('93); ...

- Effective $q-q$ interaction in "good" diquarks
- HQS ($Q \sim b$) relates $[\bar{Q}\bar{Q}]_3 \leftrightarrow Q$
- $[\bar{Q}\bar{Q}]_3^{m_Q\to\infty}$ becomes compact
- Combine (HH)+(ll) diquarks into tetraquarks:

$$\boxed{\{qq'\}[\bar{Q}\bar{Q}'] = (qC\gamma_5 q')(\bar{Q}C\gamma_i\bar{Q}') := T_{QQ'}}$$

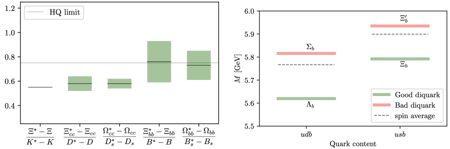


Model expectations:

- Wave-fct, prefer $J^P(T_{QQ'}) = 1^+$
- HQS, prefer $\bar{b}\bar{b}$
  $\Rightarrow$ heavier $[\bar{Q}\bar{Q}']$ more binding
- Diquark, prefer $\{ud\}$ type
  $\Rightarrow$ lighter $\{qq'\}$ more binding

Binding opportunity in model

- PDG mesons/baryons provide constraints



Doubly heavy tetraquarks - deeply bound $J^P = 1^+$ T_{bb} and $T_{bb}^{\ell s}$

HQ spin symmetry - good diquark (HQS-GDQ) picture predictions:

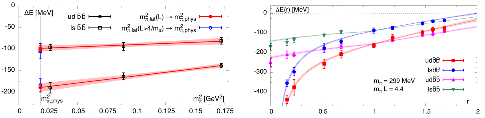
- $J^P = 1^+$ *ground state* tetraquark
- Deeper binding with:
  $\rightarrow$ heavier Q in $[\bar{Q}'\bar{Q}]$
  $\rightarrow$ lighter q in $\{qq'\}$

All observed on the Lattice!

***$bb\bar{q}\bar{q}'$* are a focal point** $\rightarrow$ All efforts observe deeply bound $bb\bar{u}\bar{d}$

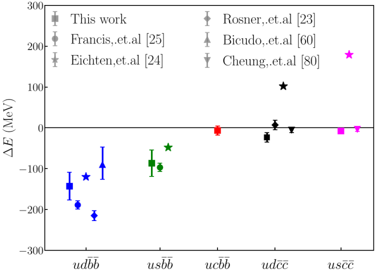
.
· Junnarkar, Mathur, Padmanath ('18)
· Leskovec, Meinel, Plaumer, Wagner ('19)
· HadronSpectrum Coll. ('17)
· Mohanta, Basak ('20)
· Colquhoun, AF, Hudspith, Lewis, Maltman ('17, '18, '20)

Lattice energies give robust predictions



⇝AF et al. ('17, '18)

Community overview



⇝Mathur et al. ('19)

Tetraquarks on the lattice

$qq'\bar{Q}'\bar{Q}$ ground state tetraquarks? Do they exist in QCD? What would their binding mechanism/properties be?

Goal: Test HQS-GDQ picture predictions

- Calculate $\Delta E = E_{\text{tetra}} - E_{\text{meson-meson}}$ e.g. in $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$
- Perform survey in m_{quark}, flavor, quantum number, ...
- Verify, quantify predictions of binding mechanism

2 main lattice approaches followed

1. Based on potential

- Static quarks ($m_Q = \infty$)
  Ansatz and Schrödinger equation to predict energies

  $\rightsquigarrow bb\bar{u}\bar{d}$, Bicudo et al. ('17,'19)
- HAL QCD method
  Lattice potentials used to determine scattering properties

  $\rightsquigarrow$HAL QCD ('16,'18)

2. Based on spectrum

- Finite volume energy levels
  Lattice energies equated to (un)observed states.

  $\rightsquigarrow$AF et al. ('17,'18, '20), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)
- Scattering analysis
  Lattice energies converted to scattering phase shifts

  $\rightsquigarrow$HadSpec ('18,'20)

Spectrum based methods a more direct and systematics are easier to control.

Lattice tetraquarks - 3+1 step recipe

The main tool is to adopt a variational approach

Lattice GEVP gives access to finite volume energy states (masses, overlaps).

Beware: Operator overlaps do not necessarily connect to the naively expected structures. Be careful when equating lattice correlators with trial-wave functions.

Step I: Set up a basis of operators, here $J^P = 1^+$

Diquark-Antidiquark:

$$D = \Big((q_a)^T (C\gamma_5) q'_b \Big) \times \Big[\bar{Q}_a (C\gamma_i)(\bar{Q}'_b)^T - a \leftrightarrow b \Big]$$

Dimeson: $$M = (\bar{b}_a \gamma_5 u_a)\,(\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a)\,(\bar{b}_b \gamma_i u_b)$$

Step II: Solve the GEVP and fit the energies

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu \;,$$

$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t) C_{VV}(t)} \;,\; \lambda(t) = A e^{-\Delta E (t - t_0)} \;.$$

$\rightsquigarrow$ $\Delta E = E_{\text{tetra}} - E_{\text{thresh}}$ in case of binding correlator $(C_{\mathcal{O}_1\mathcal{O}_2}(t))/(C_{PP}(t)C_{VV}(t))$.

Most use these operators, but a larger basis has been worked out.
$\Rightarrow$ Need to be used by more groups. $\rightsquigarrow$ HadronSpectrum Coll. ('17)

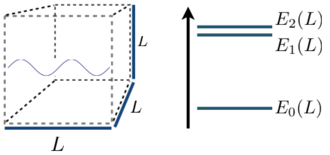
Current state-of-the-art

Step III: Finite volume corrections

Large energy shifts are possible due to the finite lattice volume.

Scenario I: Scattering state
The finite volume energy belongs to a scattering state, the corrections go as

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + \frac{a}{L^3} + \mathcal{O}(\frac{1}{L^4})\right]$$



⇝ M. Hansen

Scenario II: Stable state
The corrections are exponentially suppressed with $\kappa = \sqrt{E_{b,\infty}^2 + p^2}$

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + Ae^{-\kappa L}\right]$$

With a single volume available:

- In a bound state corrections are $\sim$ exp(binding momentum) ⇝ strong supp. m_{had} =heavy
- In a scattering state expect large deviation around threshold

With multiple volumes available:

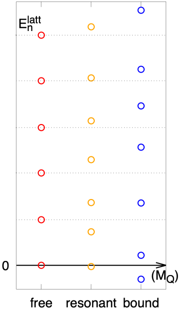
- Track mass dependence ⇝ decide bound/scatt. state
- Power law corrections might be too small to resolve

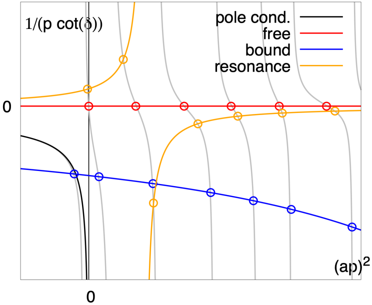
Future step for community

Step IV: Finite volume / Scattering analysis

Limitation: Small GEVP without f.vol analysis ok for deeply bound states.
Insufficient to tell apart free, resonant or virtual bd. states.

Extension: Connect energies to scattering phase shifts via finite volume quantisation conditions (Lüscher-formalism).





- ◦ connect (many) f.vol states to scattering parameters (sketch: BW)
- ◦ resonance: extra state(s) appear, lowest state close to threshold

A first step towards the full program

⇝ Padmanath, Prelovsek, arxiv:2202.10110

A lattice study of T_{cc} with unphysical quark masses

- **Idea:** Scattering analysis enables the extraction of the pole properties in the complex plane. ⇝ spectrum shown in extra info slides
- **Caveat:** $E_B(T_{cc}) < 1\text{MeV}$ requires highly precise calculations at the physical point with control over extra systematics (e.g. isospin breaking)
- *Possible solution:* Mapping of the pole trajectory with quark mass
- *Milestone:* Virtual bound state in T_{cc} at $m_\pi = 280\text{MeV}$ found.
- *Alternative paths?* Not clear. Mapping from GEVP overlaps is difficult due to ambiguous identification of trial operators.

Binding energy

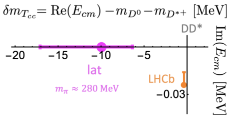


FIG. 3. The pole in the scattering amplitude related to T_{cc} in the complex energy plane: our lattice result at the heavier charm quark mass (magenta) and the LHCb result (orange).

A virtual bound state

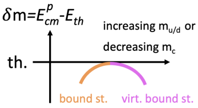


FIG. 4. Sketch of the binding energy for the (virtual) bound state dominated by the molecular component. It is based on a purely attractive potential $V(r)$ and partial wave $l = 0$ within quantum mechanics.

Diquarks - Towards understanding $T^{qq'}_{QQ'}$ and other hadrons

With the state-of-the-art it is already clear:

HQS-GDQ predictions all observed on the Lattice!

Many open questions*

- Binding mechanism? Flavor dependence?
- Role of diquarks?
- Structure of $T^{qq'}_{QQ'}$?
- **Consequences for other hadrons?**

HQS-GDQ picture in $T^{qq'}_{QQ'}$ is just one example where diquarks play a crucial role in understanding the hadron spectrum. ⇝ [2203.16583][2203.03230]

Need for fully non-perturbative insight

Towards a clearer understanding and footing in QCD using lattice calculations

1. **diquark formalism:** Find gauge invariant probe
2. **diquark spectrum:** Fundamental properties
3. **diquark structure:** Probe $q-q$ interaction

Surveyed $T^{qq'}_{QQ'}$ candidates

observed (>1 group)
no deep binding
observed (1 group)
not clear (>1 group)

channel	deeply bound
$J^P=1^+$	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$cc\bar{u}\bar{d}$ $cc\bar{\ell}\bar{s}$
	$bb\bar{b}\bar{b}$
$J^P=0^+$	$bb\bar{u}\bar{u}$ $cc\bar{u}\bar{u}$
	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bb\bar{s}\bar{s}$ $cc\bar{s}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$bb\bar{c}\bar{c}$ $cc\bar{u}\bar{d}$
	$bb\bar{b}\bar{b}$

*some answers via step IV

Diquark spectroscopy

"[Diquark] mass differences are fundamental characteristics of QCD"
$\rightsquigarrow$ Jaffe, arXiv:hep-ph/0409065 (2005)

Diquarks on the lattice - a gauge invariant probe

- A problem for the lattice is that diquarks are colored, i.e. not-gauge invariant.
  - Could fix a gauge, but then properties are gauge-dependent (masses, sizes,...)

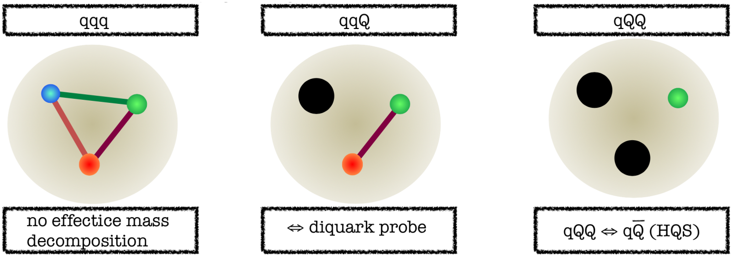
⇝ lattice and Dyson-Schwinger, see e.g. [15-20] in 2106.09080

- **Alternative:** Static spectator quark Q ($m_Q \to \infty$) cancels in mass differences.
  - Diquark properties exposed in a gauge-invariant way.

⇝ hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353

$$\boxed{C_\Gamma(t) \sim \exp\left[-t\left(m_{D_\Gamma} + m_Q + \mathcal{O}(m_Q^{-1})\right)\right]}$$

⇒ **Lattice osbervable:** Diquark embedded in a static-light-light baryon!



⇝ picture of baryons from Hosaka, 2013

Lattice spectroscopy - diquark-(di)quark differences

We consider mass differences of $qq'Q$ baryons:

$$\boxed{C_{\Gamma}^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)}$$

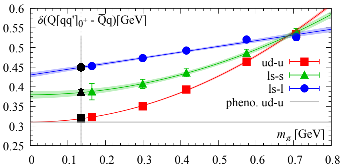
Special status of good diquark observed

- Good ud diquark lowest in spectrum
- Pattern repeated in ℓs and ss'

... and of a $qq'Q$ baryon with a light-static meson

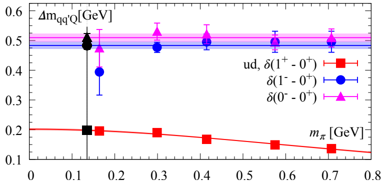
$$\boxed{C_{\Gamma=\gamma_5}^{qq'Q}(t) - C_{\gamma_5}^{q'\bar{Q}}(t)}$$

$Qqq' - \bar{Q}q'$ splittings

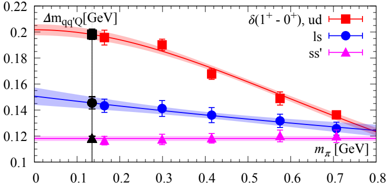


⇝ useful for 4-/5-quark states.

ud 0^+ versus 1^+, 0^- and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



Diquark spectroscopy - comparing results

- We want to compare our results with phenomenology

$\rightsquigarrow$ more details in extra info slides

  - Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
  - For pheno estimates combine charm and bottom hadron masses such that leading $\mathcal{O}(1/m_Q)$ $(Q = c, b)$ cancel

- The main spectroscopy results are summarised as:

All in [MeV]	$\delta E_{\mathrm{lat}}(m_\pi^{\mathrm{phys}})$	$\delta E_{\mathrm{pheno}}$	$\delta E^{\mathrm{bottom}}_{\mathrm{pheno}}$	$\delta E^{\mathrm{charm}}_{\mathrm{pheno}}$
$\delta(1^+ - 0^+)_{ud}$	198(4)	206(4)	206	210
$\delta(1^+ - 0^+)_{\ell s}$	145(5)	145(3)	145	148
$\delta(1^+ - 0^+)_{ss'}$	118(2)			
$\delta(Q[ud]_{0^+} - \bar{Q}u)$	319(1)	306(7)	306	313
$\delta(Q[\ell s]_{0^+} - \bar{Q}s)$	385(9)	397(1)	397	398
$\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$	450(6)			

$\rightsquigarrow$ use the bottom estimate for static

$\rightsquigarrow$ use charm-bottom difference as estimate for deviation from static

$\Rightarrow \lesssim \mathcal{O}(7)$MeV deviation

- Overall, very good agreement observed.

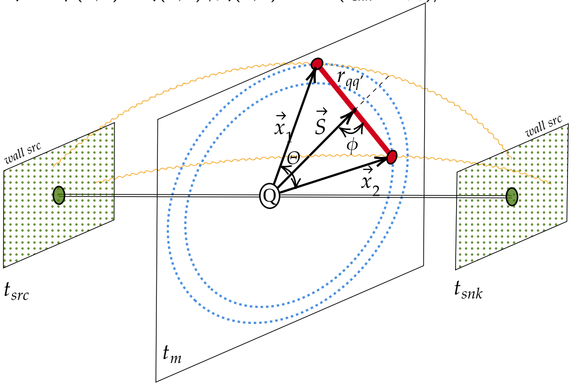
Diquark structure

Diquarks - spatial correlations

We access (good) diquark structure information through density-density correlations:

$$\boxed{C_\Gamma^{dd}(\vec{x}_1,\vec{x}_2,t)=\left\langle \mathcal{O}_\Gamma(\vec{0},2t)\ \rho(\vec{x}_1,t)\rho(\vec{x}_2,t)\ \mathcal{O}_\Gamma^\dagger(\vec{0},0)\right\rangle := \rho_2(r_{ud},S,\phi;\Gamma)}$$

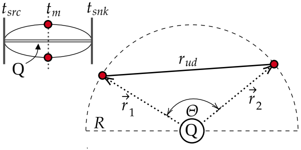
$\rightsquigarrow$ $\mathcal{O}_\Gamma = q^c C\Gamma q$ and $\rho(\vec{x},t)=\bar{q}(\vec{x},t)\gamma_0 q(\vec{x},t)$, $t_m=(t_{snk}+t_{src})/2$ to minimize excited states



Main tool: Correlations between two light quarks' relative positions to the static quark.
S, r_{ud} fixed: Distance between static quark Q and closer of the two light quarks q, q' is

- Minimized for $\phi=\pi$, possible disruption due to Q is largest
- Maximized for $\phi=\pi/2$, possible disruption due to Q is smallest

Good diquark attraction



Setting $\phi = \pi/2$:

- $|\vec{x}_1| = |\vec{x}_2| = R$, use R, Θ:

$$\rho_2^\perp(R,\Theta) = \rho_2(r_{ud}, S, \pi/2)$$

- Attraction visible through increase in $\rho_2^\perp$ for small Θ at any fixed R

Two limiting cases for the two quarks:

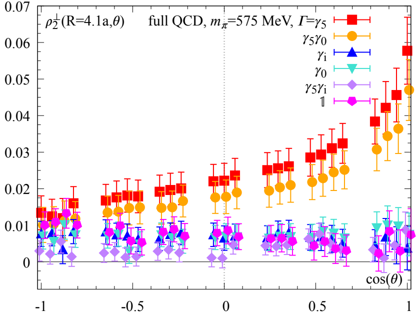
- $\cos(\Theta) = 1$ on top of each other
- $\cos(\Theta) = -1$ opposite each other

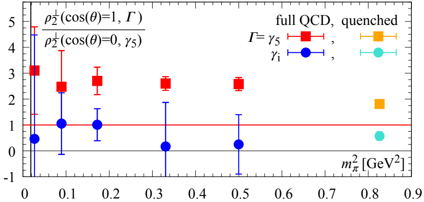
"Lift" as qualitative criterion:

$$\frac{\rho_2^\perp(R,\Theta=0,\Gamma)}{\rho_2^\perp(R,\Theta=\pi/2,\gamma_5)}$$

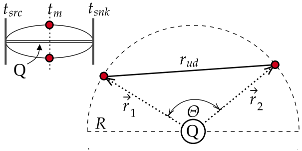
Increase observed in good diquark only

Spatial correlation over Θ





Good diquark size



- Distance between quarks:

  $r_{ud} = R\sqrt{2\,(1-\cos(\Theta))}$

  $\rightsquigarrow$ different visualisation

- $\rho_2^{\perp}(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$

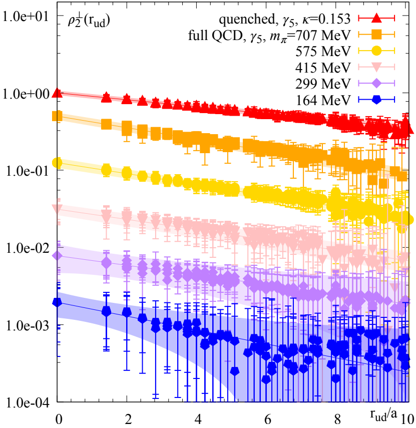
  $\rightsquigarrow$ "characteristic size" r_0

- Need to control:
  - interference from Q
    $\rightsquigarrow$ we limit analysis to $r_{ud} < R$
  - periodicity effects
    $\rightsquigarrow$ in practice we find $L = 5r_0$

- Further checks:
  $A(R, r_{ud} = 0) \sim \exp(-R/R_0)$

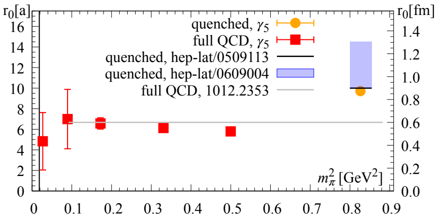
Data well described by (single) exponential Ansatz

Spatial correlation over r_{ud}



- $r_{ud} = 0$ normalised, offset for each m_π
- all R shown simultaneously
- combined fits over $\forall R$ with shared r_0

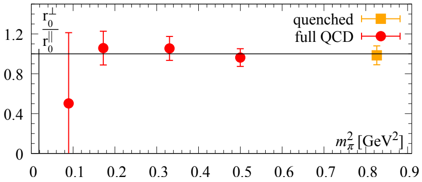
Size dependence $r_0(m_\pi)$



Good diquark size:

- Agreement w/ prev. quenched and dynamical studies
- Refinement through our results
- $r_0 \simeq \mathcal{O}(0.6)$fm weak m_π dependence
- $r_{\rm diquark} \sim r_{\rm hadron}$, (using: arXiv:1604.02891)

Shape dependence $r_0^\perp / r_0^{||}(m_\pi)$



Good diquark shape:

- Get radial and tangential radii $r_0^{||}$, $r_0^\perp$
- Ratio $r_0^\perp / r_0^{||}$ sensitive to distortions

  $= 1$, spherical

  $\neq 1$, prolate/oblate
- Ratio $\simeq 1$ for all $m_\pi \Rightarrow$ spherical
- Consistent w/ scalar, $J = 0$, shape

Summary - Understanding heavy multiquarks

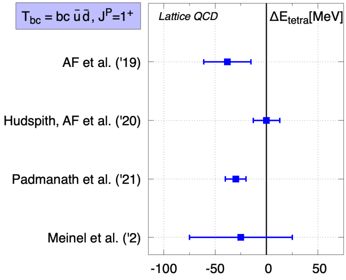
Lattice QCD approach to exotic hadrons, tetraquarks and diquarks

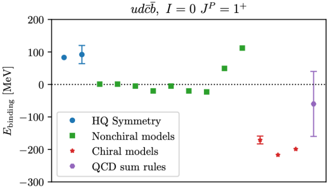
- QCD interactions without approximations
- Firm lattice evidence for doubly heavy tetraquarks, esp. $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$
- Broad agreement with a description based on a diquark+HQS model
- Gauge invariant approach to diquarks
- Special status of "good" diquark confirmed, δE =198(4) MeV
- $q - q$ attraction in good diquark observed, $r_0 \simeq \mathcal{O}(0.6)$fm $\sim r_{\text{hadronic}}$

Outlook

- Pin down T_{bc} with high precision
- Refine diquark and tetraquark models
- Tetraquark diquark content / structure? Diquarks in light(er) baryons? ...

Exciting discoveries ahead?





Thank you for your attention.

Further material

Summary - Understanding heavy multiquarks

Lattice QCD approach to exotic hadrons, tetraquarks and diquarks

- QCD interactions without approximations, gauge invariant approach to diquarks

Doubly heavy tetraquarks

- Lattice evidence for doubly heavy tetraquarks, esp. $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$
- Broad agreement with a description based on a diquark+HQS model
- Lattice studies focussing on consolidating and estimating systemtatics
- First studies of tetraquark structure using scattering phase shifts ongoing

Diquark spectroscopy

- Special status of "good" diquark confirmed, attraction of 198(4)MeV over "bad"
- Chiral and flavor dependence modelled through simple Ansatz
- Very good agreement with phenomenological estimates

Diquark structure

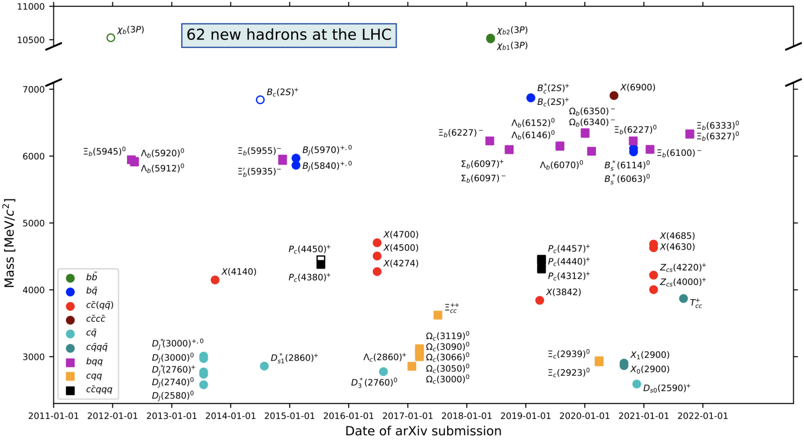
- $q - q$ attraction in good diquark induces compact spatial correlation
- Good diquark size $r_0 \simeq \mathcal{O}(0.6)$fm $\sim r_{\text{meson, baryon}}$, weakly m_π dependent
- Good diquark shape appears nearly spherical

Outlook

- Results provide support for the good diquark picture
- Hope to refine diquark and tetraquark model parameters
- Refinement towards diquarks in light baryons? Tetraquark diquark content? ...

Heavy spectrum today - a success story turned challenge to theory

- In 2003: $X(3872) \rightsquigarrow c\bar{c}d\bar{u}$ discovered at Belle
- $X(3872)$ is the first (heavy) **tetraquark** signal that has been confirmed persistently
- Since then many exotic states and especially $\mathcal{O}(12)$ heavy 4-/5-quark states observed



(LHCb collaboration, LHCb-FIGURE-2021- 001, 2021)

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

$$C_\Gamma(t) = \sum_{\vec{x}} \left\langle [D_\Gamma Q](\vec{x}, t)\ [D_\Gamma Q]^\dagger(\vec{0}, 0) \right\rangle$$

$\rightsquigarrow$ static quark=Q and $D_\Gamma = q^c C\Gamma q$
$\rightsquigarrow$ flavor combinations ud, ℓs, ss'
$\rightsquigarrow$ static-light mesons $[\bar{Q}\Gamma q]$

setting up on the lattice - we recycle

- $n_f = 2 + 1$ full QCD, $32^3 \times 64$, $a = 0.090$fm, $a^{-1} = 2.194$GeV (PACS-CS gauges)
- $\boxed{m_\pi = 164, 299, 415, 575, 707\,\mathrm{MeV}}$, $m_s \simeq m_s^{\mathrm{phys}}$, propagators re-used from before
- Quenched gauge $a \simeq 0.1$fm, $\boxed{m_\pi^{\mathrm{valence}} = 909\,\mathrm{MeV}}$, to match `hep-lat/0509113`

Diquark spectroscopy - phenomenological estimates

We want to compare our results with phenomenology

- Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
- For pheno estimates use charm and bottom hadron masses where leading $\mathcal{O}(1/m_Q)$ $(Q = c, b)$ can be cancelled

Four estimates considered:

- $\delta(1^+ - \ 0^+)_{ud}$: $\boxed{\frac{1}{3}\left(2M(\Sigma_Q^*) + M(\Sigma_Q)\right) - M(\Lambda_Q)}$
- $\delta(1^+ - 0^+)_{us}$: $\boxed{\frac{2}{3}\left(M(\Xi_Q^*) + M(\Sigma_Q) + M(\Omega_Q)\right) - M(\Xi_Q) - M(\Xi_Q')}$
- $\delta(Q[ud]_{0^+} - \bar{Q}u)$: $\boxed{M(\Lambda_Q) - \frac{1}{4}\left(M(P_{Qu}) + 3M(V_{Qu})\right)}$

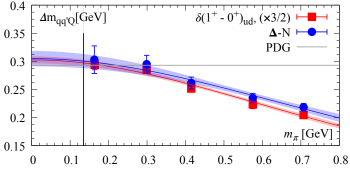
  $\rightsquigarrow$ P_{Qu},V_{Qu} are the ground-state, heavy-light mesons
- $\delta(Q[us]_{0^+} - \bar{Q}s)$:

$$\boxed{M(\Xi_Q) + M(\Xi_Q') - \frac{1}{2}(M(\Sigma_Q) + M(\Omega_Q)) - \frac{1}{4}(M(P_{Qs}) + 3M(V_{Qs}))}$$

$\rightsquigarrow$ P_{Qs},V_{Qs} are the ground-state, heavy-strange mesons

Δ-Nucleon mass difference

$[\Delta - N](m_\pi)$



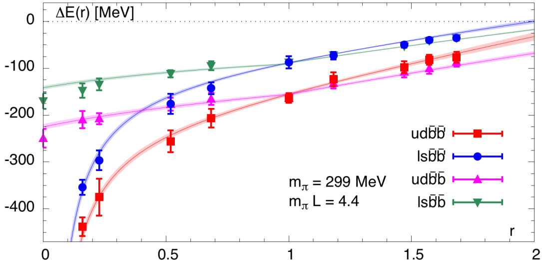
Measured the mass difference of $\Delta - N$

- Prediction: $\delta(\Delta - N) = 3/2 \times \delta(1^+ - 0^+)_{ud}$
- Same Ansatz as before
- Prediction holds well, even at fairly large m_π

A tunable system - opportunity together with pheno

AF et al. ('18)

*5 parameter pheno-Ansatz in Appendix



◦ E.g. scans in $m_{b'}$ map out the heavy quark mass dependence.

◦ Away from physical masses the binding mechanism can be probed.
→ Mass dependence can be confronted with model predictions.
→ System can be tuned continuously from the bound to the resonant or non-interacting regimes.

→ Requires robust control of finite volume spectrum.

Review of doubly heavy tetraquarks in lattice QCD

Confirm and predict doubly heavy tetraquarks non-perturbatively

Tetraquarks as ground states? What would their binding mechanism/properties be?

HQS-GDQ picture, consequences for $qq'\bar{Q}'\bar{Q}$ tetraquarks:

- ◦ $J^P = 1^+$ *ground state* tetraquark below meson-meson threshold
- ◦ Deeper binding with heavier quarks in the $\bar{Q}'\bar{Q}$ diquark
- ◦ Deeper binding for lighter quarks in the qq' diquark

Ideal for lattice: Diquark dynamics and HQS could enable $J^P = 1^+$ *ground state doubly heavy tetraquarks* with flavor content $qq'\bar{Q}\bar{Q}'$.

Goal: $\Delta E = E_{\text{tetra}} - E_{\text{meson}-\text{meson}}$, e.g. in $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$ and others
$\Rightarrow$ Verify, quantify predictions of binding mechanism in mind.

Lattice point of view

- Hidden flavor $qQ\bar{q}'\bar{Q}$ are tetraquark candidates as excitations of $Q\bar{Q}'$.
  $\rightsquigarrow$ technical difficulty for lattice calculations, need to resolve many f.vol states.
  $\rightsquigarrow$ $qq'\bar{Q}\bar{Q}'$, i.e. ground state candidates would be better to handle.

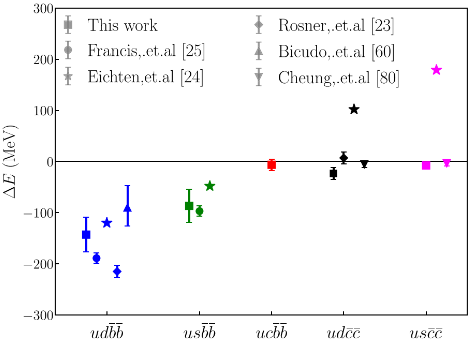
In the following

- ◦ Tetraquarks with two heavy (c, b) and two light (ℓ, s) quarks.
- ◦ Lattice evidence for $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$.
- ◦ Recent updates on systematics.
- ◦ Survey of candidates status.

What we know: A review of recent lattice studies

What we know: Deeply bound $J^P = 1^+$ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ tetraquarks

Community overview



⇝Mathur et al. ('19)

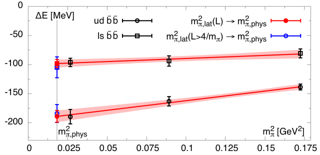
Qualitative agreement with pheno

○ All three predictions met:
→ $J^P = 1^+$ bound ground state.
→ deeper binding with $m_Q \uparrow$.
→ deeper binding with $m_q \downarrow$.

○ $bb\bar{q}\bar{q}'$ are a focal point → All efforts observe deeply bound $bb\bar{u}\bar{d}$

.

AF et al. ('17)



· Junnarkar, Mathur, Padmanath ('18)
· Leskovec, Meinel, Plaumer, Wagner ('19)
· HadronSpectrum Coll. ('17)
· Mohanta, Basak ('20)
· Colquhoun, AF, Hudspith, Lewis, Maltman ('17, '18, '20)

Overview -possible doubly heavy tetraquark candidates

Surveying candidates

observed (>1 group)
no deep binding
observed (1 group)
not confirmed (>1 group)

channel	deeply bound
$J^P = 1^+$	$bb\bar{u}\bar{d}$ $\;$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $\;$ $bc\bar{\ell}\bar{s}$
	$bs\bar{u}\bar{d}$ $\;$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $\;$ $bb\bar{s}\bar{c}$
	$cc\bar{u}\bar{d}$ $\;$ $cc\bar{\ell}\bar{s}$
	$bb\bar{b}\bar{b}$
$J^P = 0^+$	$bb\bar{u}\bar{u}$ $\;$ $cc\bar{u}\bar{u}$
	$bb\bar{u}\bar{d}$ $\;$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $\;$ $bc\bar{\ell}\bar{s}$
	$bb\bar{s}\bar{s}$ $\;$ $cc\bar{s}\bar{s}$
	$bs\bar{u}\bar{d}$ $\;$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $\;$ $bb\bar{s}\bar{c}$
	$bb\bar{c}\bar{c}$ $\;$ $cc\bar{u}\bar{d}$
	$bb\bar{b}\bar{b}$

Deeply bound states

Focus: strong interaction stable

$\rightarrow$ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ in $J^P = 1^+$.
$\rightarrow$ $cc\bar{q}\bar{q}'$ not deep.
$\rightarrow$ $bc\bar{q}\bar{q}'$ not clear.
$\rightarrow$ further candidates not observed.
$\rightarrow$ none observed in $J^P = 0^+$.

$\rightsquigarrow$Bicudo et al. ('17), AF et al. ('17,'18, '20), HadSpec Coll. ('18), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)

States above threshold, resonances?

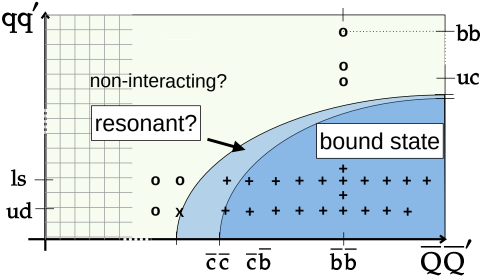
$\rightarrow$ $bb\bar{u}\bar{d}$ in $J^P = 1^+$ /w static quarks find a resonance just above threshold. $\rightsquigarrow$Bicudo et al. ('19)
$\rightarrow$ No results from other approaches.
$\rightarrow$ What about $cs\bar{u}\bar{d}$?

$\rightsquigarrow$ under investigation Hudspith, AF et al.('20), HadSpec ('20)

Shallow binding?

$\circ$ $cc\bar{u}\bar{d}$ now observed by LHCb, robust lattice post-diction?
$\rightarrow$ Work to remove current limitations.

A tunable system - binding diagram



◦ Mapping out the flavor/mass binding diagram.
→ (Un-)binding transition?
→ Connecting resonance?

◦ Surveying more J^{PC} candidates
→ Other binding mechanisms?
→ More exotica? ($cs\bar{u}\bar{d}$, $cc\bar{c}\bar{c}$,. . .)

Task: Establish the finite volume spectra and perform scattering analysis
→ What is the resonant/bound nature of the tetraquark candidates?

Recent lattice updates - a glimpse at the community trends

Chiral limit

Majority of studies have performed extrapolations to m_{phys}.

Continuum limit

Few studies have taken (partial) continuum limits.

Finite volume

∘ Initial volume scaling in one study. → **More work needed!**

Operator choice

∘ One study uses non-local sinks, but local sources.
∘ Two studies use a large basis in w-l approach. → **More work needed!**

Ground state systematics

∘ The systematic due to the approach-from-below in w-l correlators is assessed through a box-sink construction. ⇝Hudspith, AF et al. ('20)
∘ Corrections to energies ($\propto$ 25MeV) in w-l approach. → **Need careful re-evaluation!**

Structure properties

∘ Study in potential approach. ⇝Wagner et al. ('21)
∘ Studies using overlaps *caution required*. ⇝Mohanta,Basak('20); Wagner et al. ('21)

Deeper dive into recent updates: Structure properties

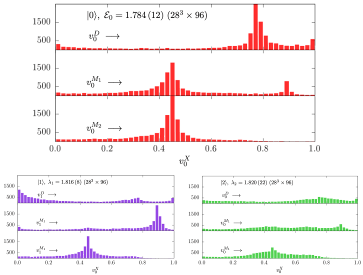
Structure properties - estimating overlaps from GEVPs

in principle: overlaps from GEVP give structure insight

- **Idea:** Overlaps give relative strengths of interpolating operator structures
- **Caveat:** Need well-defined operator structures.
  $\rightsquigarrow$ Combining local sources with non-local sinks makes this ambiguous.
- *Possible solution:* Hermitian GEVP, e.g. via distillation approach

GEVP structure I

$\rightsquigarrow$ Mohanta, Basak ('20)

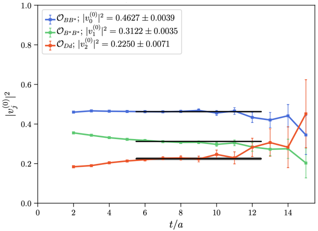


$\rightsquigarrow$ NRQCD-HISQ, 3×3 GEVP, all local

- Diquark-type structure dominant

GEVP structure II

$\rightsquigarrow$ Pflaumer ('21)



$\rightsquigarrow$ local source, non-local sink

- Relative weights:
$\sim 77\%$ Dimeson vs. $\sim 23\%$ Diquark-type

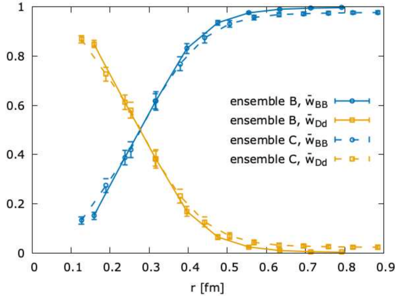
Structure properties - from the static potential

in principle: optimal trial states give structure insight

- **Idea:** Read off structure from weights of optimised trial states in Schrödinger Equation with lattice potential
- **Caveat:** Operator normalisation not trivial. Only clear connection when using static quarks. Potential needs to be interpolated
  ⇝ Estimating systematics can be difficult.

Static potential structure

⇝Wagner ('21)



- $bb\bar{u}\bar{d}$ structure mixture
- Distance dependence:
  - $r \lesssim 0.2$fm: diquark-type dominance
  - $r \gtrsim 0.3$fm: dimeson dominance
- Relative weights:
  $\sim 60\%$ Dimeson vs. $\sim 40\%$ Diquark-type

The Full Program: A first lattice study of T_{CC}

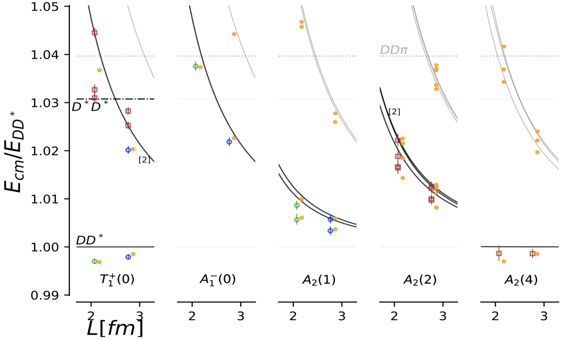
recall: performing the full finite volume analysis enables deeper insight

- **Idea:** Many lattice determined energy eigenstates are converted to scattering phase shifts via finite volume quantisation conditions.
- **Goal:** The extraction of the pole properties in the complex plane
- **Caveat:** The $E_B < 1$MeV of T_{CC} requires highly precise calculations at the physical point with many extra systematics under control (e.q. isospin breaking)
- *Possible solution:* Mapping of the pole trajectory with quark mass
- *Milestone:* The study of Padmanath, Prelovsek ('22) is a first step in this direction. They find a virtual bound state in T_{CC} at $m_\pi = 280$MeV.

A virtual bound state? - A lattice study of T_{CC} with unphysical quark masses

Finite volume / scattering analysis - spectrum results

⇝Padmanat, Prelovsek ('22)



⇝distillation, only meson-meson operators used

- One lattice spacing $a = 0.086$ fm
- Two lattice volumes available, $\simeq 2$ fm and $\simeq 3$ fm
- One $m_\pi = 280$ MeV with 2 possible valence charm quark probes, one slightly below and one slightly above the physical charm quark mass.

A virtual bound state? - A lattice study of T_{CC} with unphysical quark masses

recall: performing the full finite volume analysis enables deeper insight

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Binding energy

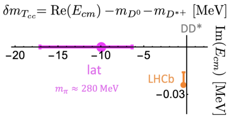


FIG. 3. The pole in the scattering amplitude related to T_{cc} in the complex energy plane: our lattice result at the heavier charm quark mass (magenta) and the LHCb result (orange).

A virtual bound state

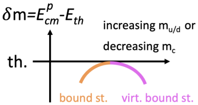


FIG. 4. Sketch of the binding energy for the (virtual) bound state dominated by the molecular component. It is based on a purely attractive potential $V(r)$ and partial wave $l = 0$ within quantum mechanics.

Lattice QCD calculates strongly coupled QFT using supercomputers

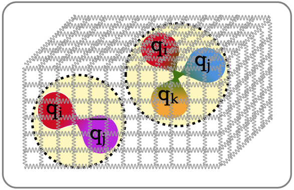
Connection to physics

($\rightarrow$ renormalisation Z_{NP})

$\rightarrow$ chiral/phys.point limit $m_\pi \rightarrow m_{\text{phys}}$

$\rightarrow$ volume limit $L \rightarrow \infty$

$\rightarrow$ continuum limit $a \rightarrow 0$



Consistent approach at all energies

$\rightarrow$ time is made imaginary $t \rightarrow it$

$\rightarrow$ lattice space-time, cut-off a^{-1}

$\rightarrow$ importance sampling, HMC

Systematic effects to control[!]

$\rightarrow$ cut-off $\mathcal{O}(a, a^2)$

$\rightarrow$ heavy quarks $\mathcal{O}(aM_Q)$

$\rightarrow$ finite volume effects $\mathcal{O}(m_\pi L)$

∘ Spectrum encoded in hadron correlators, e.g. masses and decay constants:

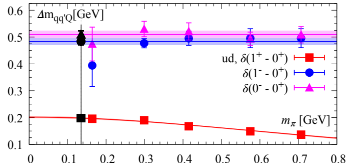
$$C_{\mathcal{O}_1\mathcal{O}_2}(t, \mathrm{p}=0) = \sum_x \langle \mathcal{O}_1(x,t)\mathcal{O}_2^\dagger(0,0)\rangle \rightsquigarrow \sum_i \frac{\langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle}{2m_i} e^{-m_i t}$$

$\rightarrow$ $m_0 = m_{\text{ground}}$ ground state, approached asymptotically $t \rightarrow$ *large*

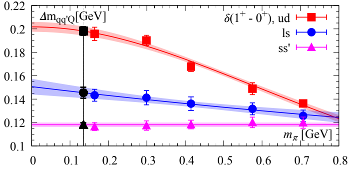
$\rightarrow$ f_i from $\langle 0|\mathcal{O}_i|n\rangle$

Lattice spectroscopy - diquark-diquark differences

ud 0^+ versus 1^+, 0^- and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



We consider mass differences of $qq'Q$ baryons:

$$\boxed{C_\Gamma^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)}$$

$\rightsquigarrow$ Q drops out

$\rightsquigarrow$ measures diquark-diquark mass difference

Bad-good diquark splitting:

- Special status of good diquark observed
- Good 0^+ ud diquark lies lowest in the spectrum
- Bad 1^+ ud diquark 100-200 MeV above
- 0^- and 1^- ud diquarks ~ 0.5 GeV above
- Pattern repeated in ℓs and ss'

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral limit: $\sim$ const
- Heavy-quark limit: decreases $\sim 1/(m_{q_1} m_{q_2})$, with $m_\pi \sim (m_{q_1} + m_{q_2})$

$$\boxed{\delta(1^+ - 0^+)_{q_1 q_2} = A / \left[1 + (m_\pi / B)^{n \in 0,1,2}\right]}$$

Lattice spectroscopy - diquark-quark differences

We consider mass differences of a $qq'Q$ baryon and a light-static meson:

$$\boxed{C^{qq'Q}_{\Gamma=\gamma_5}(t) - C^{q'\bar{Q}}_{\gamma_5}(t)}$$

$\rightsquigarrow$ Q drops out

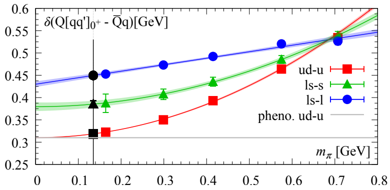
$\rightsquigarrow$ diquark-quark mass difference

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral vs. heavy-quark limiting behaviours, as before

$$\boxed{\delta(Q[q_1 q_2]_{0^+} - \bar{Q} q_2) = C\,\left[1 + (m_\pi/D)^{n\in 0,1,2}\right]}$$

$Qqq' - \bar{Q}q'$ splittings



Diquark-quark splitting:

- Established mass differences between a good diquark and an [anti]quark
- May prove useful in identifying favourable tetra-, pentaquark channels
- Omits possible distortions through additional light quarks, Pauli-blocking, spin-spin interactions ...

Diquarks on the lattice - a gauge invariant probe

- A problem for the lattice is that diquarks are colored, i.e. not-gauge invariant.
  - Could fix a gauge, but then properties are gauge-dependent (masses, sizes,...)

$\rightsquigarrow$ lattice and Dyson-Schwinger, see e.g. [15-20] in 2106.09080

- **Alternative:** Static spectator quark Q ($m_Q \to \infty$) cancels in mass differences.
  - Diquark properties exposed in a gauge-invariant way.

$\rightsquigarrow$ hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353

$$\boxed{C_\Gamma(t) \sim \exp\left[-t\left(m_{D_\Gamma} + m_Q + \mathcal{O}(m_Q^{-1})\right)\right]}$$

$\rightsquigarrow$ $t \to$ large, $m_Q \to$ large

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

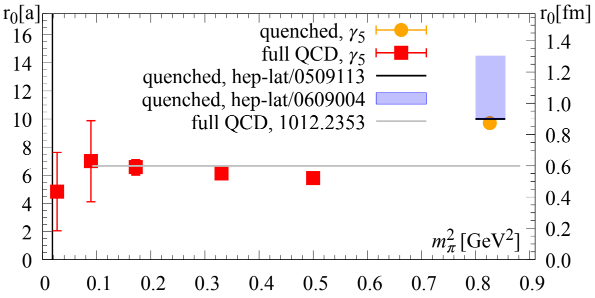
$$C_\Gamma(t) = \sum_{\vec{x}} \left\langle [D_\Gamma Q](\vec{x},t)\ [D_\Gamma Q]^\dagger(\vec{0},0)\right\rangle$$

$\rightsquigarrow$ static quark$=Q$ and $D_\Gamma = q^c C\Gamma q$
$\rightsquigarrow$ flavor combinations ud, ℓs, ss'
$\rightsquigarrow$ static-light mesons $[\bar{Q}\Gamma q]$

Clearer understanding by studying the diquark ...

1. **spectrum:** *[diquark] mass differences are fundamental characteristics of QCD* (Jaffe ’05, arXiv:hep-ph/0409065)
2. **spatial correlations:** *study attraction and special status of the "good" diquark*
3. **structure:** *estimate size and shape of the "good" diquark*

Size dependence $r_0(m_\pi)$



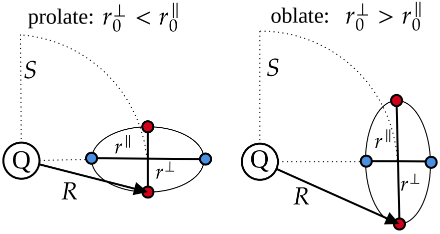
Good diquark size:

- Agreement w/ prev. quenched and dynamical
- Refinement through our results
- $r_0 \simeq \mathcal{O}(0.6)$fm weak m_π dependence
  $\rightsquigarrow \sim r_{\text{meson, baryon}}$, arXiv:1604.02891

$r_0(m_\pi)$ dependence:

- $m_{q,q'} \uparrow$ should produce more compact object
- But, diquark attraction$\downarrow$ works opposite
- Former effect dominates at large m_π?
- But, in quenched diquarks definitely larger...

Shape of good diquarks - studying wavefunction "oblateness"



Tangential and radial spatial correlation decay

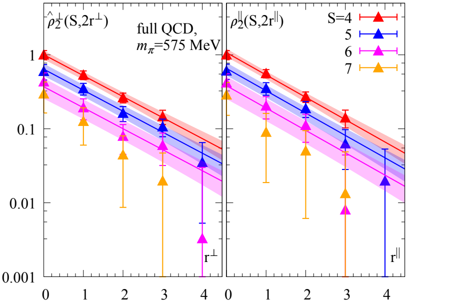
As opposed to before $R \neq$fixed:

- $\phi = \pi$: radial correlation,
  size $\rightsquigarrow r_0^\parallel$
- $\phi = \pi/2$: tangential correlation,
  size $\rightsquigarrow r_0^\perp$

- $r_0^\perp / r_0^\parallel$ gives information on shape:
  $= 1$, spherical
  $\neq 1$, prolate/oblate

- Probe $J = 0$ nature of good diquark (spherical, S-wave expectation)
- Diquark polarisation through static quark?

Oblateness results at $m_\pi = 575\text{MeV}$

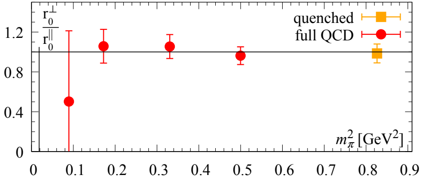


- Goal:
  - $r_0^\perp$, $r_0^{||}$ at fixed S

  Technical issue:
  - $(||)$ as before:
    $R = S$
  - $(\perp)$ different: $R = \sqrt{(r^\perp)^2 + S^2}$

  Solution:
  - Introduce "nuisance" parameter R_0
  - Adjusted in figure
  - Parallel lines $\rightsquigarrow r_0^\perp = r_0^{||}$

Shape dependence $r_0^\perp / r_0^{||}(m_\pi)$



- $r_0^\perp / r_0^{||}(m_\pi)$ dependence:
  - Ratio $\simeq 1$ for all $m_\pi \Rightarrow$ spherical
  - Consistent w/ scalar, $J = 0$, shape
  - No diquark polarisation through Q observed