On the mass spectrum of heavy Higgs bosons in 2HDM: CDF W -mass anomaly

Jae Sik Lee (Chonnam Nat'l Univ.)

IBS-PNU Joint Workshop on Particle Physics and Cosmology December 7-10, 2022 , Busan, Korea

Main Refs.:

e-Print: 2212.07620 [hep-ph], Dong-Won Jung, Yongtae Heo, JSL

e-Print: 2204.05728 [hep-ph], PLB, Yongtae Heo, Dong-Won Jung, JSL

e-Print: 2110.03908 [hep-ph], PRD, JSL, Jubin Park

- Most general CPV 2HDM potential in the Higgs basis

$$
\begin{aligned}
V_{\mathcal{H}} = {} & Y_1(\mathcal{H}_1^\dagger\mathcal{H}_1) + Y_2(\mathcal{H}_2^\dagger\mathcal{H}_2) + Y_3(\mathcal{H}_1^\dagger\mathcal{H}_2) + Y_3^*(\mathcal{H}_2^\dagger\mathcal{H}_1) \\
& + Z_1(\mathcal{H}_1^\dagger\mathcal{H}_1)^2 + Z_2(\mathcal{H}_2^\dagger\mathcal{H}_2)^2 + Z_3(\mathcal{H}_1^\dagger\mathcal{H}_1)(\mathcal{H}_2^\dagger\mathcal{H}_2) + Z_4(\mathcal{H}_1^\dagger\mathcal{H}_2)(\mathcal{H}_2^\dagger\mathcal{H}_1) \\
& + Z_5(\mathcal{H}_1^\dagger\mathcal{H}_2)^2 + Z_5^*(\mathcal{H}_2^\dagger\mathcal{H}_1)^2 + Z_6(\mathcal{H}_1^\dagger\mathcal{H}_1)(\mathcal{H}_1^\dagger\mathcal{H}_2) + Z_6^*(\mathcal{H}_1^\dagger\mathcal{H}_1)(\mathcal{H}_2^\dagger\mathcal{H}_1) \\
& + Z_7(\mathcal{H}_2^\dagger\mathcal{H}_2)(\mathcal{H}_1^\dagger\mathcal{H}_2) + Z_7^*(\mathcal{H}_2^\dagger\mathcal{H}_2)(\mathcal{H}_2^\dagger\mathcal{H}_1) \,,
\end{aligned}
$$

$$
\mathcal{H}_1 = c_\beta\Phi_1 + \mathrm{e}^{-i\xi}s_\beta\Phi_2 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + \varphi_1 + iG^0\right) \end{pmatrix}
$$

$$
\mathcal{H}_2 = -s_\beta\Phi_1 + \mathrm{e}^{-i\xi}c_\beta\Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(\varphi_2 + ia\right) \end{pmatrix} ,
$$

$$
Y_1 \ + \ Z_1 v^2 \ = 0 \, ; \quad Y_3 \ + \ \frac{1}{2} Z_6 v^2 \ = 0
$$

$$V_{\mathcal{H}\,,\mathrm{mass}} = M_{H^\pm}^2 H^+ H^- \;+\; \frac{1}{2}(\varphi_1 \;\varphi_2 \;a)\,\mathcal{M}_0^2 \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ a \end{pmatrix}$$

$$\mathcal{M}_0^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_A^2 & 0 \\ 0 & 0 & M_A^2 \end{pmatrix} \;+\; \begin{pmatrix} 2Z_1 & \Re\mathrm{e}(Z_6) & -\Im\mathrm{m}(Z_6) \\ \Re\mathrm{e}(Z_6) & 2\Re\mathrm{e}(Z_5) & -\Im\mathrm{m}(Z_5) \\ -\Im\mathrm{m}(Z_6) & -\Im\mathrm{m}(Z_5) & 0 \end{pmatrix} v^2$$

$$M_{H^\pm}^2 = Y_2 + \frac{1}{2} Z_3 v^2 \qquad\qquad M_A^2 = M_{H^\pm}^2 + \left[\tfrac{1}{2} Z_4 - \Re\mathrm{e}(Z_5)\right] v^2$$

$$(\varphi_1, \varphi_2, a)_\alpha^T = O_{\alpha i}(H_1, H_2, H_3)_i^T$$

$$O^T \mathcal{M}_0^2 O = \mathrm{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2)$$

- Input parameters

$$\{Y_1, Y_2, Y_3; Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}$$

$$\{v, Y_2; M_{H^\pm}, Z_1, Z_4, Z_5, Z_6; Z_2, Z_7\} \quad Y_1 + Z_1 v^2 = 0; \quad Y_3 + \frac{1}{2} Z_6 v^2 = 0$$

$$\mathcal{I}_Z = \{v, M_{H^\pm}; Z_1, Z_4, Z_5, Z_6; Z_3; Z_2, Z_7\} \quad M_{H^\pm}^2 = Y_2 + \frac{1}{2} Z_3 v^2$$

$$\mathcal{I}_P = \{v, M_{H^\pm}; M_{H_1}, M_{H_2}, M_{H_3}, \{O\}_{3\times 3}; Z_3; Z_2, Z_7\}$$

$$Z_1 = \frac{1}{2v^2} \left(M_{H_1}^2 O_{\varphi_1 1}^2 + M_{H_2}^2 O_{\varphi_1 2}^2 + M_{H_3}^2 O_{\varphi_1 3}^2 \right) ,$$

$$Z_4 = \frac{1}{v^2} \left[M_{H_1}^2 (O_{\varphi_2 1}^2 + O_{a1}^2) + M_{H_2}^2 (O_{\varphi_2 2}^2 + O_{a2}^2) + M_{H_3}^2 (O_{\varphi_2 3}^2 + O_{a3}^2) - 2M_{H^\pm}^2 \right] ,$$

$$Z_5 = \frac{1}{2v^2} \left[M_{H_1}^2 (O_{\varphi_2 1}^2 - O_{a1}^2) + M_{H_2}^2 (O_{\varphi_2 2}^2 - O_{a2}^2) + M_{H_3}^2 (O_{\varphi_2 3}^2 - O_{a3}^2) \right]$$
$$- \frac{i}{v^2} \left(M_{H_1}^2 O_{\varphi_2 1} O_{a1} + M_{H_2}^2 O_{\varphi_2 2} O_{a2} + M_{H_3}^2 O_{\varphi_2 3} O_{a3} \right) ,$$

$$Z_6 = \frac{1}{v^2} \left(M_{H_1}^2 O_{\varphi_1 1} O_{\varphi_2 1} + M_{H_2}^2 O_{\varphi_1 2} O_{\varphi_2 2} + M_{H_3}^2 O_{\varphi_1 3} O_{\varphi_2 3} \right)$$
$$- \frac{i}{v^2} \left(M_{H_1}^2 O_{\varphi_1 1} O_{a1} + M_{H_2}^2 O_{\varphi_1 2} O_{a2} + M_{H_3}^2 O_{\varphi_1 3} O_{a3} \right) ,$$

- Conditions for the potential to be bounded from below (BFB)

We consider the following 5 necessary conditions for the Higgs potential to be bounded-from-below in a *marginal* sense [72, 103]: [14]

$$
\begin{aligned}
Z_1 \geq 0\,, \quad Z_2 \geq 0\,; \\
2\sqrt{Z_1 Z_2} + Z_3 \geq 0\,, \quad 2\sqrt{Z_1 Z_2} + Z_3 + Z_4 - 2|Z_5| \geq 0\,; \\
Z_1 + Z_2 + Z_3 + Z_4 + 2|Z_5| - 2|Z_6 + Z_7| \geq 0\,.
\end{aligned}
\tag{68}
$$

Note that though the quartic couplings Z_2 and Z_7 have no direct relations to the masses and mixing of Higgs bosons but they are interrelated with the other five quartic couplings of $Z_{1,3-6}$ through the UNIT and BFB conditions as shown.

- Perturbative Unitarity (UNIT)

$$\mathcal{M}_1^S = \begin{pmatrix} \eta_{00} - I & \eta^T \\ \eta & E + I \times \mathbf{1}_{3\times 3} \end{pmatrix} ; \quad \mathcal{M}_2^S = \begin{pmatrix} 3\eta_{00} - I & 3\eta^T \\ 3\eta & 3E + I \times \mathbf{1}_{3\times 3} \end{pmatrix} ,$$

$$\eta_{00} = Z_1 + Z_2 + Z_3 \text{ and } I = Z_3 - Z_4. \quad \eta^T = (\Re\mathrm{e}(Z_6 + Z_7), -\Im\mathrm{m}(Z_6 + Z_7), Z_1 - Z_2)$$

$$E = \begin{pmatrix} Z_4 + 2\Re\mathrm{e}(Z_5) & -2\Im\mathrm{m}(Z_5) & \Re\mathrm{e}(Z_6 - Z_7) \\ -2\Im\mathrm{m}(Z_5) & Z_4 - 2\Re\mathrm{e}(Z_5) & -\Im\mathrm{m}(Z_6 - Z_7) \\ \Re\mathrm{e}(Z_6 - Z_7) & -\Im\mathrm{m}(Z_6 - Z_7) & Z_1 + Z_2 - Z_3 \end{pmatrix}$$

The third 3×3 scattering matrix $\mathcal{M}_3^S$ is Hermitian which takes the form of

$$\mathcal{M}_3^S = \begin{pmatrix} 2Z_1 & 2Z_5 & \sqrt{2}Z_6 \\ 2Z_5^* & 2Z_2 & \sqrt{2}Z_7^* \\ \sqrt{2}Z_6^* & \sqrt{2}Z_7 & Z_3 + Z_4 \end{pmatrix} . \tag{64}$$

And then, the unitarity conditions are imposed by requiring that the 11 eigenvalues of the three scattering matrices $\mathcal{M}_{1,2,3}^S$ and the quantity I should have their moduli smaller than 4π.

- Simplified UNIT constraints [when Z_{6,7}=0 (left) or Z_{1,2,3,4,5}=0(right)]

$$|Z_3 \pm Z_4| < 4\pi ,$$

$$|Z_3 \pm 2|Z_5|| < 4\pi ,$$

$$|Z_3 + 2Z_4 \pm 6|Z_5|| < 4\pi ,$$

$$\left| Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + 4|Z_5|^2} \right| < 4\pi ,$$

$$\left| Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + Z_4^2} \right| < 4\pi ,$$

$$\left| 3Z_1 + 3Z_2 \pm \sqrt{9(Z_1 - Z_2)^2 + (2Z_3 + Z_4)^2} \right| < 4\pi .$$

$$\sqrt{|Z_6|^2 + |Z_7|^2} < 2\sqrt{2}\pi ,$$

$$\sqrt{|Z_6|^2 + |Z_7|^2 + |Z_6^2 + Z_7^2|} < \frac{4\pi}{3} .$$

- Combining ...

$$|Z_{1,2,5}| < 2\pi/3 , \quad |Z_{6,7}| < 2\sqrt{2}\pi/3 ,$$

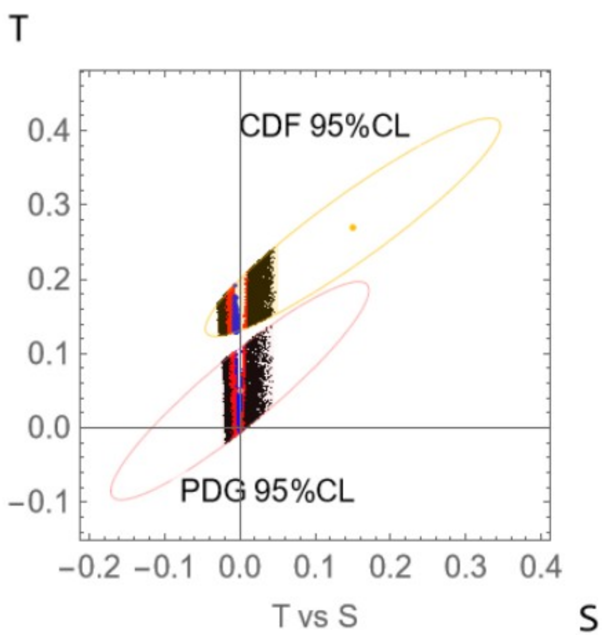
$$|Z_3 - Z_4| < 4\pi \ \cup\ |2Z_3 + Z_4| < 4\pi \ \cup\ |Z_3 + 2Z_4| < 4\pi$$

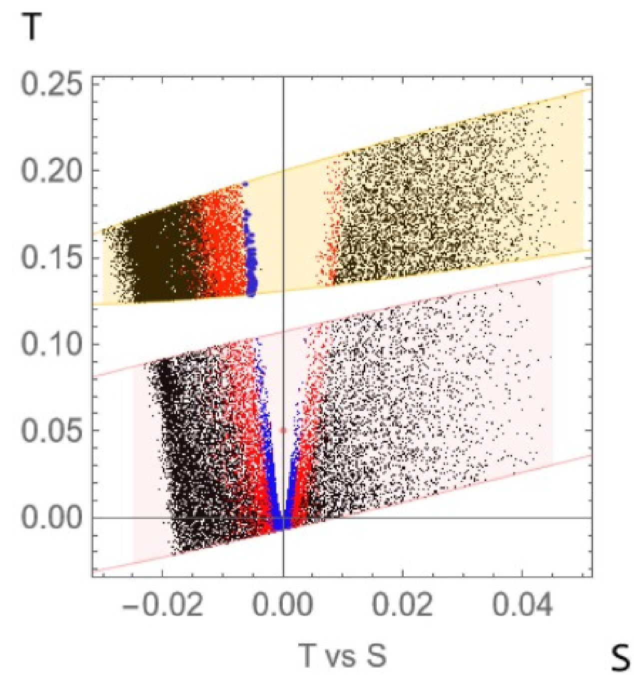
- Electroweak (ELW) Precision Observables: S and T parameters

$$\frac{(S-\widehat{S}_0)^2}{\sigma_S^2} + \frac{(T-\widehat{T}_0)^2}{\sigma_T^2} - 2\rho_{ST}\frac{(S-\widehat{S}_0)(T-\widehat{T}_0)}{\sigma_S\sigma_T} \leq R^2\,(1-\rho_{ST}^2)\,,$$

with $R^2 = 2.3,\ 4,61,\ 5.99,\ 9.21,\ 11.83$ at 68.3%, 90%, 95%, 99%, and 99.7% confidence levels (CLs)

$$\textbf{PDG [2]}: \quad (\widehat{S}_0\,,\ \sigma_S) = (0.00\,,\ 0.07)\,, \quad (\widehat{T}_0\,,\ \sigma_T) = (0.05\,,\ 0.06)\,, \quad \rho_{ST} = 0.92\,,$$
$$\textbf{CDF [3]}: \quad (\widehat{S}_0\,,\ \sigma_S) = (0.15\,,\ 0.08)\,, \quad (\widehat{T}_0\,,\ \sigma_T) = (0.27\,,\ 0.06)\,, \quad \rho_{ST} = 0.93\,.$$





$M_{H^\pm} > M_{H_1}$ *(black+red+blue)*, $M_{H^\pm} >$ *500 GeV (red+blue), and* $M_{H^\pm} >$ *900 GeV (blue)*

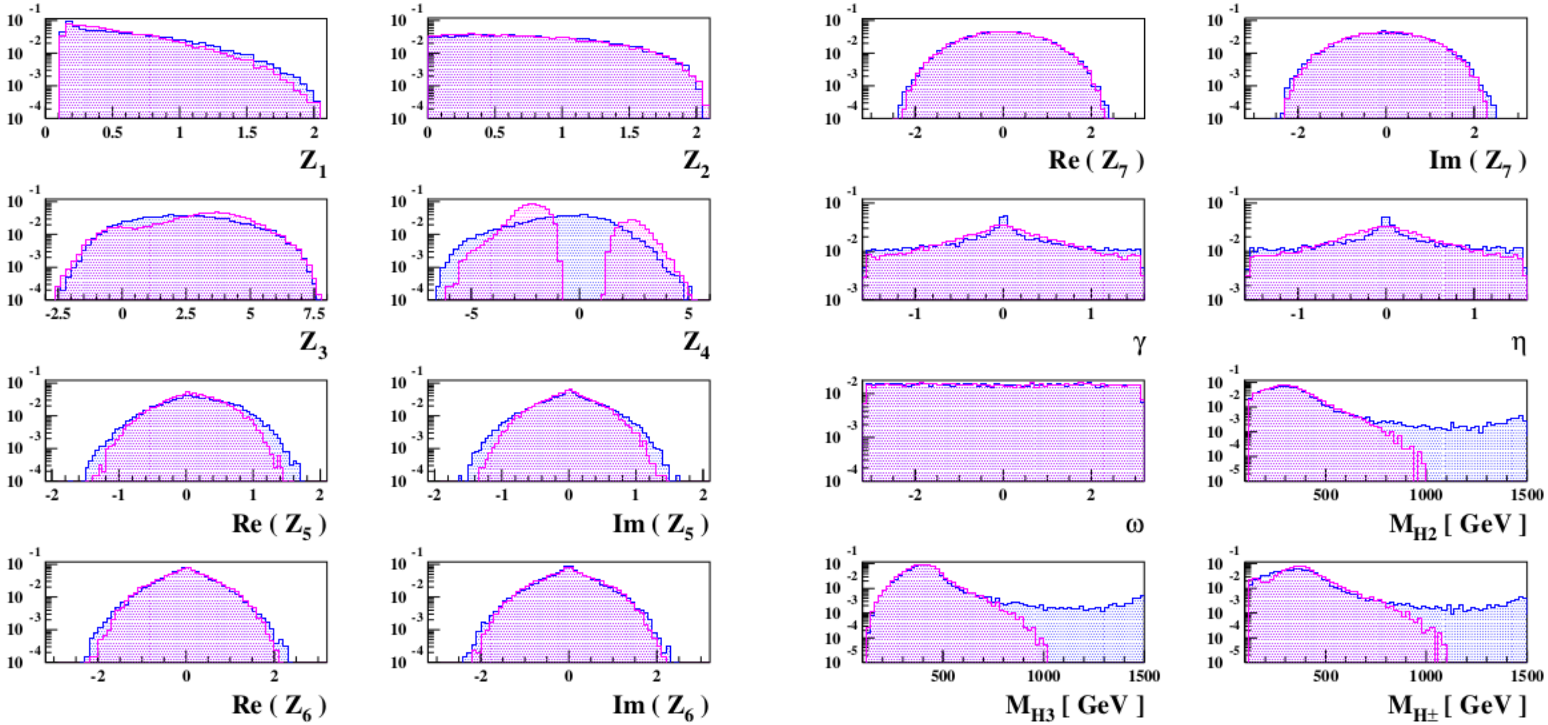


FIG. 3. **CDF**: *The normalized distributions of the quartic couplings, the mixing angles, and the masses of heavy Higgs bosons imposing the combined UNIT⊕BFB⊕ELW*$_{95\%}$ *constraints (magenta) and using the set* $\mathcal{I}_P$ *of the imput parameters, see Eq. (19). For comparisons, we also show the results applying only the UNIT⊕BFB constraints (blue).*

$$\mathcal{I}_P = \{v, M_{H^\pm}; M_{H_1}, M_{H_2}, M_{H_3}, \{O\}_{3\times 3}; Z_3; Z_2, Z_7\}$$

$$O = O_\gamma O_\eta O_\omega \equiv \begin{pmatrix} c_\gamma & s_\gamma & 0 \\ -s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\eta & 0 & s_\eta \\ 0 & 1 & 0 \\ -s_\eta & 0 & c_\eta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\omega & s_\omega \\ 0 & -s_\omega & c_\omega \end{pmatrix}$$

$$g_{H_1VV} = c_\gamma c_\eta \qquad \gamma = \eta = 0 \text{ in the alignment limit}$$

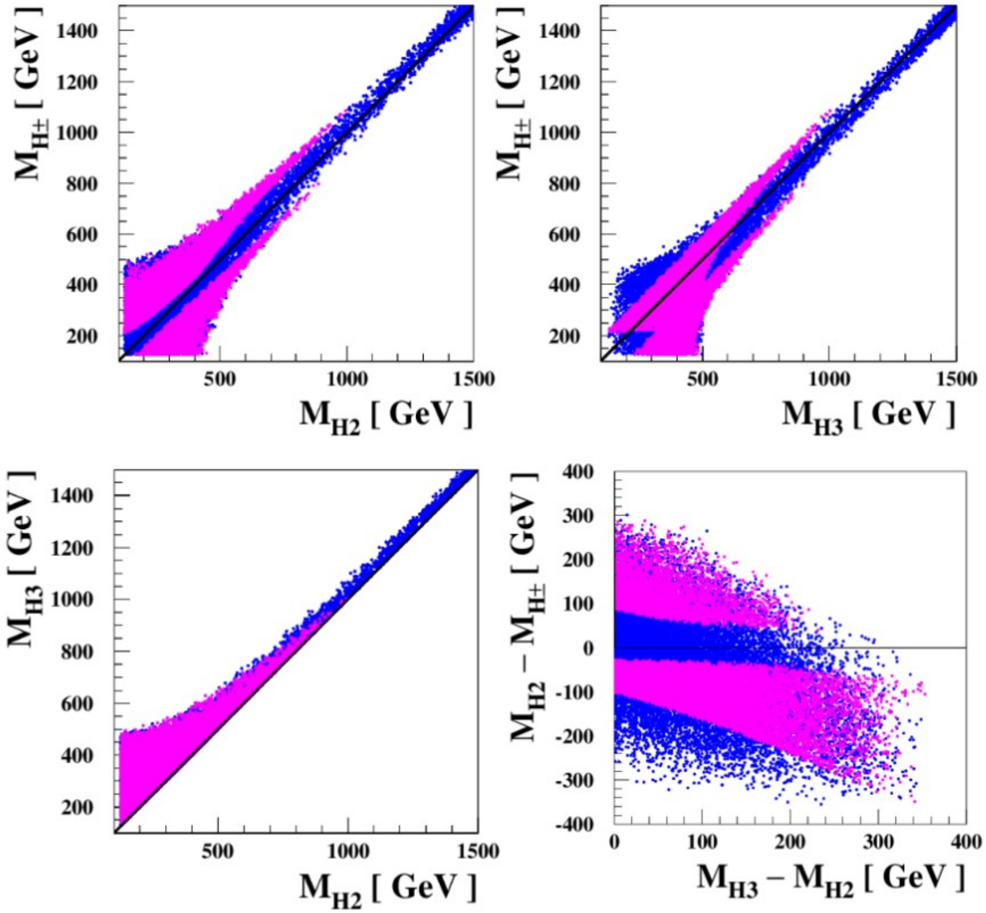


FIG. 4. **CDF**: *Scatter plots of $M_{H^\pm}$ versus M_{H_2} (upper left), $M_{H^\pm}$ versus M_{H_3} (upper right), M_{H_3} versus M_{H_2} (lower left), and $M_{H_2} - M_{H^\pm}$ versus $M_{H_3} - M_{H_2}$ (lower right) imposing the combined UNIT⊕BFB⊕ELW$_{95\%}$ constraints (magenta) and using the set $\mathcal{I}_P$ of the imput parameters, see Eq. (19). For comparisons, we also show the results applying only the UNIT⊕BFB constraints (blue).*

When the masses of the heavy Higgs bosons saturate their upper limits:

$$M_{H^\pm} > M_{H_2} \ : \ M_{H_2} \simeq M_{H_3} \simeq 1000\,\mathrm{GeV}\,, \quad M_{H^\pm} \simeq 1100\,\mathrm{GeV}\,;$$

$$M_{H^\pm} < M_{H_2} \ : \ M_{H_2} \simeq M_{H_3} \simeq 900\,\mathrm{GeV}\,, \quad M_{H^\pm} \simeq 800\,\mathrm{GeV}\,.$$

ANALYSIS

$$g_{H_1VV} \equiv 1 - \epsilon$$

$$g_{23}^2 \equiv \frac{g_{H_2VV}^2 + g_{H_3VV}^2}{2} = \frac{1 - g_{H_1VV}^2}{2} = \epsilon - \frac{\epsilon^2}{2}$$

the quantity ϵ is required to be smaller than about 0.05 at 1σ level

$$M_{H^\pm} = M - \Delta/2\,, \quad M_{H_2} = M + \Delta/2\,, \quad M_{H_3} = M + \Delta/2 + \delta$$

$$M = (M_{H^\pm} + M_{H_2})/2$$

$$\Delta \equiv M_{H_2} - M_{H^\pm}\,; \quad \delta \equiv M_{H_3} - M_{H_2}$$

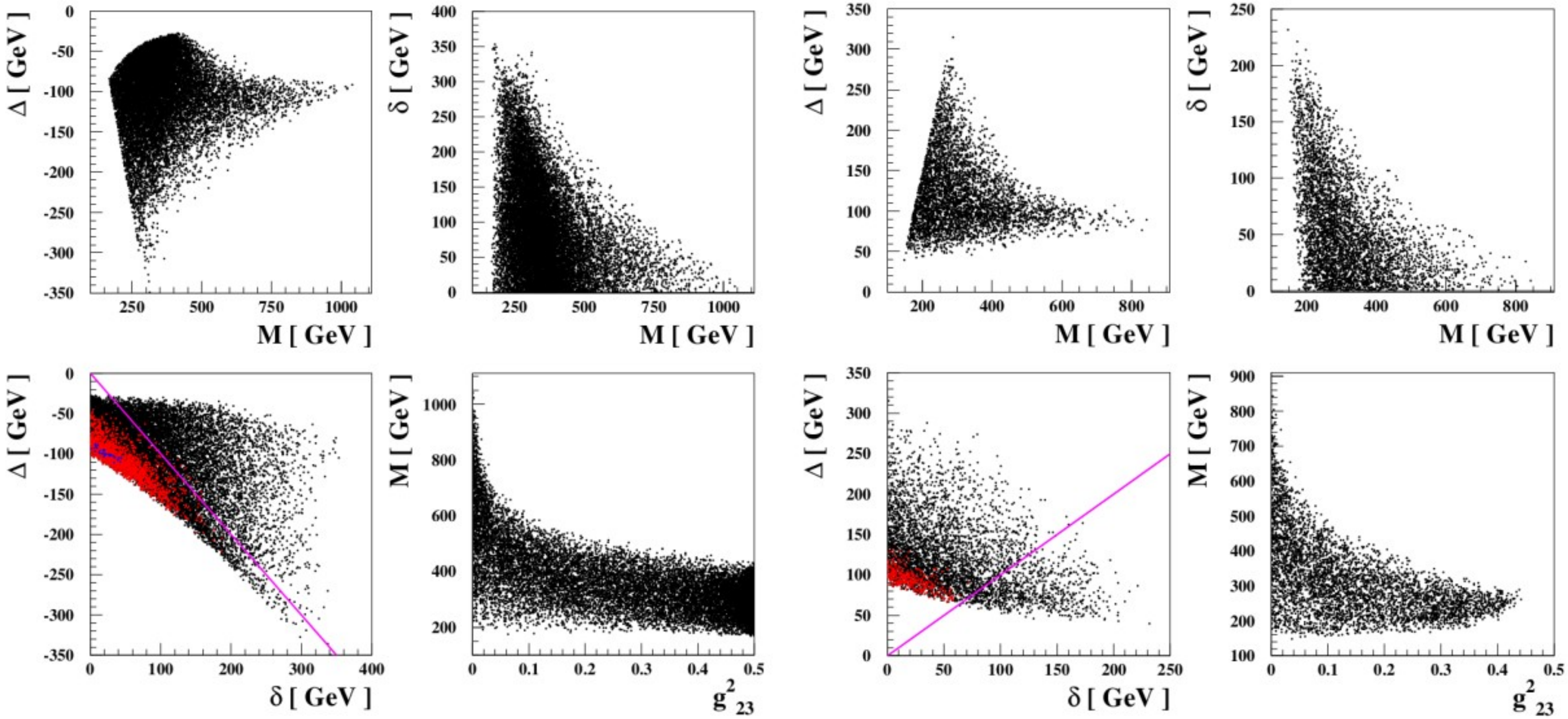


FIG. 5. **CDF**: *[Left] Scatter plots of Δ versus M (upper left), δ versus M (upper right), Δ versus δ (lower left), and M versus g_{23}^2 (lower right) for $\Delta < 0$. [Right] The same as in the left panel but for $\Delta > 0$. The magenta lines in the lower-left frames in each panel are for $\Delta = -\delta$ (Left) and $\Delta = \delta$ (Right). And the red and blue dots are for $M > 500$ GeV (red+blue) and $M > 900$ GeV (blue), respectively. The combined UNIT$\oplus$BFB$\oplus$ELW$_{95\%}$ constraints are imposed.*

$$M \gg M_{H_1} \sim |\Delta| \gtrsim \delta \text{ where } M \gtrsim 500 \text{ GeV}$$

$$\textbf{23C}: \quad M_{H_2} < M_{H_3} < M_{H^\pm} \quad \text{when} \quad \Delta < 0 \quad \text{and} \quad \delta < |\Delta|\,;$$

$$\textbf{C23}: \quad M_{H^\pm} < M_{H_2} < M_{H_3} \quad \text{when} \quad \Delta > 0\,.$$

Why non-vanishing $\Delta \equiv M_{H_2} - M_{H^\pm}$ for sizable and positive T ?

$$\frac{T}{\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\rm EM}}} \simeq \frac{4}{3}(\Delta^2 + \delta\Delta) + \left[-\left(2g_{23}^2\Delta + g^2_{_{H_2VV}}\delta\right)M - \frac{4}{3}g_{23}^2\Delta^2 - \left(\frac{1}{2}g^2_{_{H_2VV}} + \frac{4}{3}g^2_{_{H_3VV}}\right)\delta\Delta + \frac{1}{6}g^2_{_{H_2VV}}\delta^2\right]$$

$$\frac{T}{\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\rm EM}}} \simeq \frac{4}{3}(\Delta^2 + \delta\Delta) - \left(2g_{23}^2\Delta + g^2_{_{H_2VV}}\delta\right)M$$

$$\left[\Delta - \left(\frac{3}{4}g_{23}^2\,M - \frac{\delta}{2}\right)\right]^2 \simeq T' + \frac{3}{4}g^2_{_{H_2VV}}\,\delta\,M + \left(\frac{3}{4}g_{23}^2\,M - \frac{\delta}{2}\right)^2$$

$T' \equiv \frac{T}{0.18}\,(100\,{\rm GeV})^2$ using $\sqrt{2}G_F/16\pi^2\alpha_{\rm EM} \simeq 1.337\times 10^{-5}/{\rm GeV}^2$.

:=> the region of Δ with a radius smaller than $\sqrt{T'}$ around the point $\Delta = 3g_{23}^2\,M/4 - \delta/2$ should not be allowed

For example, if $T \gtrsim 0.18$, the region of $|\Delta| \lesssim 100$ GeV should be excluded in the limit of $\delta = g_{23}^2 = 0$

Why upper limit on $M = (M_{H^\pm} + M_{H_2})/2$?

$$Z_4 v^2 = 2(2\Delta+\delta)M + (\Delta+\delta)\delta - \left[2g_{23}^2 M^2 + 2(g_{23}^2\Delta + g_{H_3VV}^2\delta)M + g_{23}^2\left(\frac{\Delta^2}{2} - 2M_{H_1}^2\right) + g_{H_3VV}^2(\Delta+\delta)\delta\right]$$

$$\frac{T}{\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\rm EM}}} \simeq \frac{4}{3}(\Delta^2+\delta\Delta) + \left[-\left(2g_{23}^2\Delta + g_{H_2VV}^2\delta\right)M - \frac{4}{3}g_{23}^2\Delta^2 - \left(\frac{1}{2}g_{H_2VV}^2 + \frac{4}{3}g_{H_3VV}^2\right)\delta\Delta + \frac{1}{6}g_{H_2VV}^2\delta^2\right]$$

$$4\pi\, S \simeq \frac{1}{3}\left(2\frac{\Delta}{M} + \frac{\delta}{M}\right) + \left[-\frac{5}{9}g_{23}^2 + \frac{1}{3}(g_{H_2VV}^2 - g_{H_3VV}^2)\frac{\delta}{M}\right],$$

When $g_{23}^2 = 0$ and $\delta = 0$,

$$T \simeq 0.18\left(\frac{\Delta}{100\,\mathrm{GeV}}\right)^2 \quad \text{or} \quad |\Delta| \simeq 100\sqrt{\frac{T}{0.18}}\,\mathrm{GeV},$$

$$S \simeq 0.0053\left(\frac{\Delta}{100\,\mathrm{GeV}}\right)\left(\frac{1\,\mathrm{TeV}}{M}\right),$$

$$Z_4 \simeq 4\frac{\Delta\, M}{v^2} \simeq 6.6\left(\frac{\Delta}{100\,\mathrm{GeV}}\right)\left(\frac{M}{1\,\mathrm{TeV}}\right),$$

:=> $|Z_4| \lesssim 6.6$ leads to the upper limit of about 1 TeV on the heavy-Higgs mass scale M when $|\Delta| \simeq 100$ GeV

More details on the upper limit...

$$Z_4 v^2 \simeq (4\Delta + 2\delta)\ M - 2g_{23}^2 M^2 .$$

It is interesting and illuminating to observe that each term of the above equation involves its own physics origin. The term containing Z_4 is constrained by the UNIT$\oplus$BFB condition. The first term in the right-hand side involves the mass splittings among the heavy Higgs bosons and the high-precision CDF measurement of the W boson mass implies non-vanishing value for, especially, the mass splitting Δ between the charged and the second heaviest neutral Higgs bosons. The second term in the right-hand side is proportional to M^2 and it quickly increases when g_{23}^2 deviates from the alignment limit of $g_{23}^2 = 0$ or when the coupling g_{H_1VV} deviates from its SM value of 1.

$$g_{23}^2 M^2 - 2\widehat{\Delta}\, M + 2\widehat{\Delta}\, \widehat{M_0} \simeq 0$$

$$\widehat{\Delta} \equiv \Delta + \frac{\delta}{2}\,, \quad \widehat{M_0} \equiv \frac{Z_4 v^2}{4\Delta + 2\delta} = \frac{Z_4 v^2}{4\widehat{\Delta}}$$

$$g_{23}^2 M^2 - 2\widehat{\Delta}\, M + 2\widehat{\Delta}\, \widehat{M_0} \simeq 0$$

When $-2g_{23}^2 \widehat{\Delta} \widehat{M_0} + \widehat{\Delta}^2 > 0$, the equation can be solved for M:

$$\mathbf{23C}: \quad M_{Z_4} \simeq \frac{|\widehat{\Delta}|}{g_{23}^2}\left[-1 + \left(1 - 2g_{23}^2 \frac{\widehat{M_0}}{\widehat{\Delta}}\right)^{1/2}\right] \quad \text{when} \quad \widehat{\Delta} < 0 \quad \text{and} \quad 2g_{23}^2 \widehat{M_0} > -|\widehat{\Delta}|\,;$$

$$\mathbf{C23}: \quad M_{Z_4} \simeq \frac{\widehat{\Delta}}{g_{23}^2}\left[1 - \left(1 - 2g_{23}^2 \frac{\widehat{M_0}}{\widehat{\Delta}}\right)^{1/2}\right] \quad \text{when} \quad \widehat{\Delta} > 0 \quad \text{and} \quad 2g_{23}^2 \widehat{M_0} < \widehat{\Delta}\,;$$

$$M_{Z_4} \simeq \widehat{M_0}\left[1 + \frac{g_{23}^2}{2}\frac{\widehat{M_0}}{\widehat{\Delta}} + \frac{g_{23}^4}{2}\left(\frac{\widehat{M_0}}{\widehat{\Delta}}\right)^2 + \mathcal{O}\left(g_{23}^2 \frac{\widehat{M_0}}{\widehat{\Delta}}\right)^3\right]$$

$$M_{Z_4}^{(1)} \equiv \widehat{M_0}\left[1 + \frac{g_{23}^2}{2}\frac{\widehat{M_0}}{\widehat{\Delta}}\right]$$

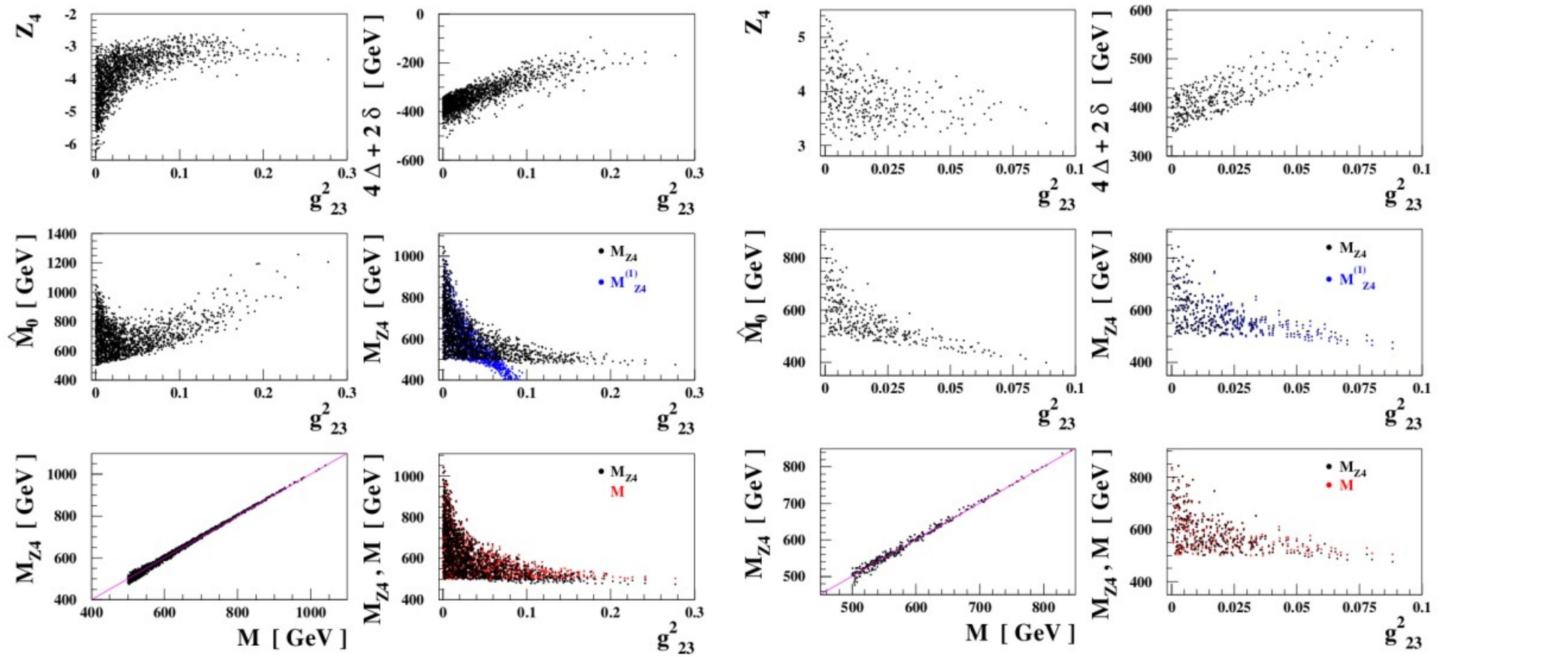


FIG. 6. **CDF**: *Behaviors of* Z_4 *(upper-left),* $4\Delta+2\delta$ *(upper-right),* $\widehat{M}_0$ *(middle-left), and* M_{Z_4} *and* $M_{Z_4}^{(1)}$ *(middle-right) according to the variation of* g_{23}^2 *and comparisons of* M_{Z_4} *and* M *(lower) for the 23C (Left) and C23 (Right) hierarchies with the magenta lines for* $M_{Z_4}=M$*. Note that* M_{Z_4} *and* $M_{Z_4}^{(1)}$ *are given by Eq. (47) and Eq. (49), respectively. The combined UNIT⊕BFB⊕ELW*$_{95\%}$ *constraints are imposed and* $M>500$ *GeV is required.*

$$\widehat{M}_0 \equiv \frac{Z_4 v^2}{4\Delta+2\delta} = \frac{Z_4 v^2}{4\widehat{\Delta}} \qquad M_{Z_4}^{(1)} \equiv \widehat{M}_0 \left[1+\frac{g_{23}^2}{2}\frac{\widehat{M}_0}{\widehat{\Delta}}\right]$$

To conclude, for a given value of g_{23}^2, the upper limit on the average mass scale $M = (M_{H^\pm} + M_{H_2})/2$ is given by the maximum value of M_{Z_4} which could be approximated by $M_{Z_4}^{(1)}$ when $g_{23}^2 \lesssim 0.05$:

$$M \lesssim \max\left(M_{Z_4}\right) \simeq \max\left[M_{Z_4}^{(1)}\right], \tag{50}$$

and then

$$M_{H_2} = M + \Delta/2 \lesssim \max\left[M_{Z_4}^{(1)}\right] + \Delta/2\,, \quad M_{H^\pm} = M - \Delta/2 \lesssim \max\left[M_{Z_4}^{(1)}\right] - \Delta/2\,, \tag{51}$$

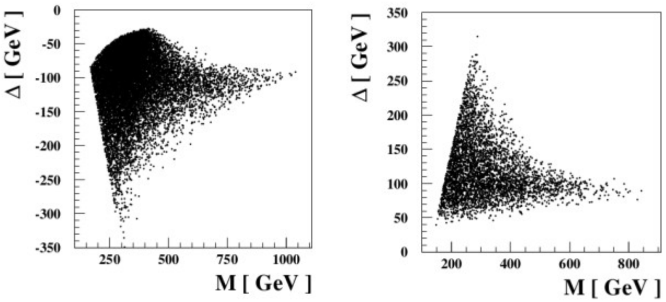
for the both 23C ($\Delta < 0$) and C23 ($\Delta > 0$) hierarchies. In this section, taking the CDF values given in Eq. (25), we find

$$\max\left[M_{Z_4}^{(1)}\right] = \max\left(\widehat{M}_0\right) \simeq \begin{cases} 1050 \text{ GeV} & \text{for } 23\mathrm{C}\,(\Delta < 0) \\ 850 \text{ GeV} & \text{for } \mathrm{C}23\,(\Delta > 0) \end{cases}, \tag{52}$$

in the alignment limit which precisely reproduces the highest masses of the heavy neutral and chaged Higgs bosons shown in Fig. 4 together with $|\Delta|_{M=\max(M)} \simeq 100$ GeV, see Eq. (31). Note that the 23C hierarchy leads to the heavier spectrum.

$$M_{H^\pm} > M_{H_2} \;:\; M_{H_2} \simeq M_{H_3} \simeq 1000\,\text{GeV}\,, \quad M_{H^\pm} \simeq 1100\,\text{GeV}\,;$$

$$M_{H^\pm} < M_{H_2} \;:\; M_{H_2} \simeq M_{H_3} \simeq 900\,\text{GeV}\,, \quad M_{H^\pm} \simeq 800\,\text{GeV}\,.$$



Between PDG and CDF $\widehat{T}_0 = 0.22\, t_{EW} + 0.05\,, \quad \widehat{S}_0 = 0.15\, t_{EW}$

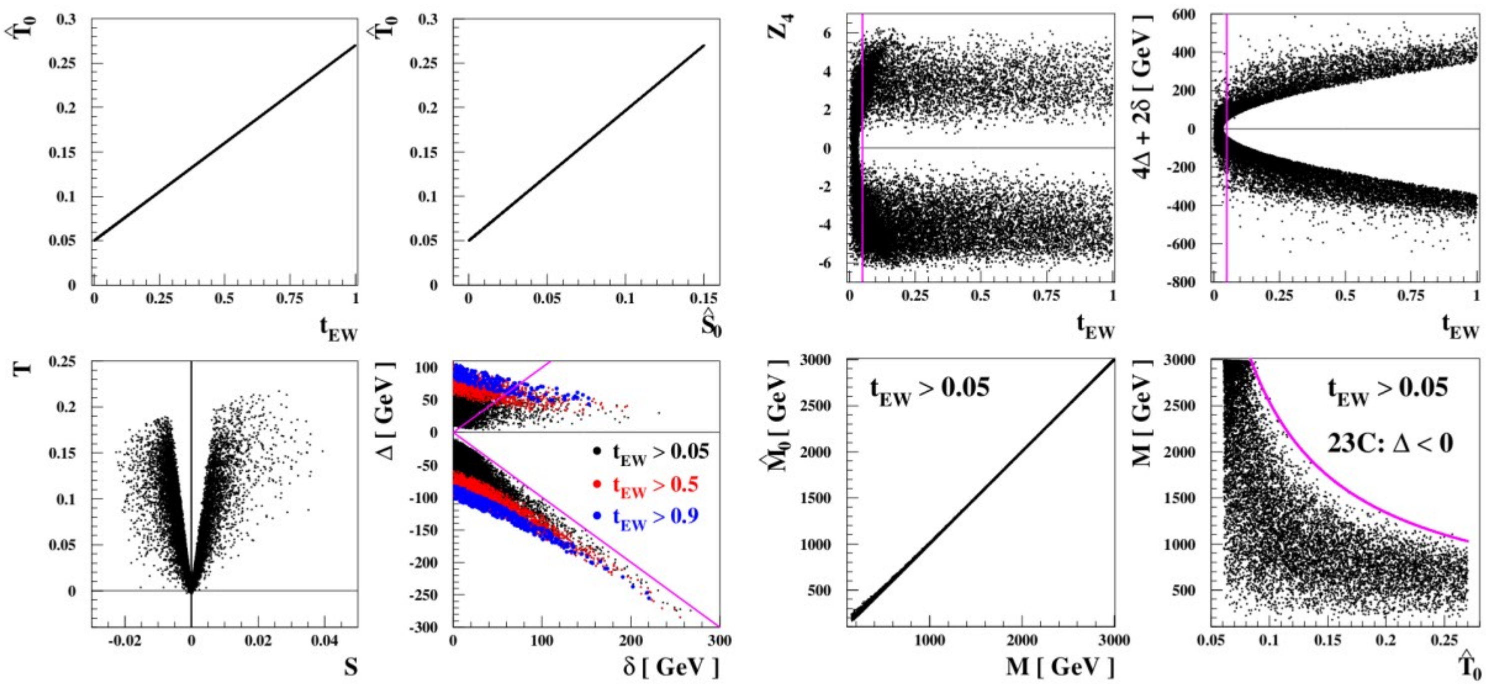


FIG. 7. *Between* **PDG** *and* **CDF***:* $\widehat{T}_0 = 0.22\, t_{EW} + 0.05$ *and* $\widehat{S}_0 = 0.15\, t_{EW}$ *with* $0 \leq t_{EW} \leq 1$ *taking the CDF values for* $\sigma_S = 0.08$, $\sigma_T = 0.06$, *and* $\rho_{ST} = 0.93$. *The alignment limit of* $\gamma = \eta = 0$ *has been taken, the heavy Higgs masses squared are scanned up to* $(3\ \mathrm{TeV})^2$, *and the other parameters are scanned as described at the end of Section* $\boxed{II}$. *The combined* $UNIT \oplus BFB \oplus ELW_{95\%}$ *constraints have been imposed. (Left) The magenta lines in the lower-right frame are for* $|\Delta| = \delta$ *and the colored dots are for* $t_{EW} > 0.05$ *(black+red+blue),* $t_{EW} > 0.5$ *(red+blue), and* $t_{EW} > 0.9$ *(blue). (Right) The vertical magenta lines in the upper frames denotes the position* $t_{EW} = 0.05$ *and the magenta curve in the lower-right frame represents the case* $M = 240/\widehat{T}_0 + 140$ *GeV.*

$$M \lesssim \left(\frac{240}{\widehat{T}_0} + 140 \right) \ \mathrm{GeV}$$

Summary

0 We analyze the implication of the large positive deviation of the S and T parameters from the SM values of zero indicated by the high-precision CDF measurement of the W boson mass on the mass spectrum of heavy Higgs bosons considering the most general CP-violating 2HDM potential. We show that the mass splitting between the charged Higgs boson and the second heaviest neutral one is necessary to accommodate the sizable value of the T parameter especially. Combining with the theoretical constraints on the quartic couplings from the perturbative unitarity and for the Higgs potential to be bounded from below, we provide the comprehensive analysis why the masses of the heavy Higgs bosons should be bounded from above.

1 By imposing the combined UNIT$\oplus$BFB$\oplus$ELW$_{95\%}$ constraints and taking the CDF values for the S and T parameters, we find

$$
\begin{aligned}
&0.1 \lesssim Z_1 \lesssim 2.0\,, \quad 0 \lesssim Z_2 \lesssim 2.1\,, \quad -2.6 \lesssim Z_3 \lesssim 8.0\,, \\
&-6.3 \lesssim Z_4 \lesssim -0.8 \,\cup\, 1.1 \lesssim Z_4 \lesssim 5.4\,, \\
&|Z_5| \lesssim 1.7\,, \quad |Z_6| \lesssim 2.4\,, \quad |Z_7| \lesssim 2.7\,.
\end{aligned}
$$

Note that the ranges of the quartic couplings appearing in the Higgs potential are largely independent of the ELW constraint except that the region with $|Z_4| \lesssim 1$ is excluded by the large value of the T parameter.

2 We figure out that there could be the following two types of mass spectrum

$$\mathbf{23C}: \quad M_{H_2} \; < \; M_{H_3} \; < \; M_{H^\pm} \quad \text{when} \quad \Delta < 0 \quad \text{and} \quad \delta < |\Delta|\,;$$

$$\mathbf{C23}: \quad M_{H^\pm} \; < \; M_{H_2} \; < \; M_{H_3} \quad \text{when} \quad \Delta > 0 \quad \text{independently of} \quad \delta\,,$$

with $\Delta = M_{H_2} - M_{H^\pm}$ and $\delta = M_{H_3} - M_{H_2}$. When $\Delta < 0$, we observe that one can achieve the hierarchy $M_{H_3} < M_{H^\pm}$ if $M = (M_{H_2} + M_{H^\pm})/2 \gtrsim 500$ GeV or in the alignment limit.

3 The upper limit on the masses of heavy Higgs bosons becomes stronger when the effects of deviation from the alignment limit are taken into account. In the CDF case, we show that the upper limit on M changes from about 1 TeV to about 700 GeV as g_{23}^2 deviates from 0 to 0.05 for the 23C hierarchy.

4 We have fully figured out the physics origins relevant for the mass scale of heavy Higgs bosons by showing that the heavy-Higgs mass scale M could be excellently represented by M_{Z_4} obtained by solving Eq. (45) which incorporates. (*i*) the UNIT⊕BFB condition, (*ii*) the mass splittings among the heavy Higgs bosons, and (*iii*) the deviation from the alignment limit.

$$g_{23}^2 M^2 - 2\widehat{\Delta}\, M + 2\widehat{\Delta}\, \widehat{M_0} \simeq 0 \quad \text{with} \quad \widehat{\Delta} = \Delta + \delta/2 \text{ and } \widehat{M_0} = Z_4 v^2/4\widehat{\Delta}$$

5 When $M \lesssim 3$ TeV, we provide the following useful and convenient empirical $\widehat{T}_0$-dependent upper limit:

$$M \;\lesssim\; \left(\frac{240}{\widehat{T}_0} + 140\right)\ \text{GeV}\,,$$

obtained by varying the central values of the S and T parameters between PDG and CDF given by Eq. (25).

$$\begin{array}{lllll} \textbf{PDG}\ [2]: & (\widehat{S}_0\,,\ \sigma_S) = (0.00\,,\ 0.07)\,, & (\widehat{T}_0\,,\ \sigma_T) = (0.05\,,\ 0.06)\,, & \rho_{ST} = 0.92\,, \\ \textbf{CDF}\ [3]: & (\widehat{S}_0\,,\ \sigma_S) = (0.15\,,\ 0.08)\,, & (\widehat{T}_0\,,\ \sigma_T) = (0.27\,,\ 0.06)\,, & \rho_{ST} = 0.93\,. \end{array}$$

6 We propose some benchmarking scenarios such as the max-CDF one in which the masses of the heavy Higgs bosons saturate their upper limits obtained by using the CDF values:

$$\mathbf{23C}_{\rm max-CDF}: \quad M_{H_2} \simeq M_{H_3} \simeq 1000\,\mathrm{GeV}\,, \quad M_{H^\pm} \simeq 1100\,\mathrm{GeV}\,;$$
$$\mathbf{C23}_{\rm max-CDF}: \quad M_{H^\pm} \simeq 800\,\mathrm{GeV}\,, \quad M_{H_2} \simeq M_{H_3} \simeq 900\,\mathrm{GeV}\,,$$

supplemented by the $\widehat{T}_0$-dependent upper limit given by Eq. ($\boxed{54}$).

$$M \;\lesssim\; \left(\frac{240}{\widehat{T}_0} + 140\right)\;\mathrm{GeV}$$

BACK UP

In the alignment limit:

$$Z_1\big|_{\gamma=\eta=0} = \frac{M_{H_1}^2}{2v^2}\,, \qquad Z_4\big|_{\gamma=\eta=0} = \frac{M_{H_2}^2 + M_{H_3}^2 - 2M_{H^\pm}^2}{v^2}\,, \qquad Z_6\big|_{\gamma=\eta=0} = 0\,,$$

$$Z_5\big|_{\gamma=\eta=0} = -\frac{M_{H_3}^2 - M_{H_2}^2}{2v^2}\,c_{2\omega} \;-\; i\,\frac{M_{H_3}^2 - M_{H_2}^2}{2v^2}\,s_{2\omega}\,.$$

- Interactions with massive vector bosons

$$\mathcal{L}_{HVV} = g\, M_W \left(W_\mu^+ W^{-\mu} \ + \ \frac{1}{2c_W^2}\, Z_\mu Z^\mu \right) \sum_i g_{H_iVV}\, H_i\,,$$

$$\mathcal{L}_{HHZ} = \frac{g}{2c_W} \sum_{i>j} g_{H_iH_jZ}\, Z^\mu (H_i \overset{\leftrightarrow}{\partial}_\mu H_j)\,,$$

$$\mathcal{L}_{HH^\pm W^\mp} = -\frac{g}{2} \sum_i g_{H_iH^+W^-}\, W^{-\mu} (H_i\, i \overset{\leftrightarrow}{\partial}_\mu H^+) + \text{h.c.}\,,$$

$$g_{H_iVV} = O_{\varphi_1 i}\,,$$

$$g_{H_iH_jZ} = \text{sign}[\det(O)]\ \epsilon_{ijk}\, g_{H_kVV} \ = \text{sign}[\det(O)]\ \epsilon_{ijk}\, O_{\varphi_1 k}\,,$$

$$g_{H_iH^+W^-} = -O_{\varphi_2 i} + iO_{ai}\,,$$

$$\sum_{i=1}^{3} g_{H_iVV}^2 \ = \ 1 \quad \text{and} \quad g_{H_iVV}^2 + |g_{H_iH^+W^-}|^2 \ = \ 1 \quad \text{for each } i = 1, 2, 3\,.$$

In 2HDM, the S and T parameters might be estimated using the following expressions [108, 109]

$$S_\Phi = -\frac{1}{4\pi}\left[\left(1+\delta_{\gamma Z}^{H^\pm}\right)^2 F'_\Delta(M_{H^\pm}, M_{H^\pm}) - \sum_{(i,j)=(1,2)}^{(1,3),(2,3)} \left(g_{_{H_iH_jZ}} + \delta_Z^{H_iH_j}\right)^2 F'_\Delta(M_{H_i}, M_{H_j})\right], \tag{71}$$

$$T_\Phi = -\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\rm EM}}\left[-\sum_{i=1}^{3}\left|g_{_{H_iH^-W^+}} + \delta_W^{H_i}\right|^2 F_\Delta(M_i, M_{H^\pm}) + \sum_{(i,j)=(1,2)}^{(1,3),(2,3)} \left(g_{_{H_iH_jZ}} + \delta_Z^{H_iH_j}\right)^2 F_\Delta(M_{H_i}, M_{H_j})\right].$$

We observe that $g^2_{_{H_iH_jZ}} = |\epsilon_{ijk}|g^2_{_{H_kVV}} = |\epsilon_{ijk}|O^2_{\varphi_1 k}$ and $|g_{_{H_iH^-W^+}}|^2 = 1 - g^2_{_{H_iVV}} = 1 - O^2_{\varphi_1 i}$ and, therefore, all the relevant couplings are determined by the three physical couplings of $g_{_{H_iVV}}$. In this work, we ignore the vertex corrections $\delta_{\gamma Z}^{H^\pm}$, $\delta_Z^{H_iH_j}$, and $\delta_W^{H_i}$ since the size of the most of the quartic couplings are smaller than 3 and the quantum corrections proportional to $\sim Z_i^2/16\pi^2$ are negligible. On the other hand, the one-loop functions are given by [17]

$$F_\Delta(m_0, m_1) = F_\Delta(m_1, m_0) = \frac{m_0^2 + m_1^2}{2} - \frac{m_0^2 m_1^2}{m_0^2 - m_1^2}\ln\frac{m_0^2}{m_1^2},$$

$$F'_\Delta(m_0, m_1) = F'_\Delta(m_1, m_0) = -\frac{1}{3}\left[\frac{4}{3} - \frac{m_0^2\ln m_0^2 - m_1^2\ln m_1^2}{m_0^2 - m_1^2} - \frac{m_0^2 + m_1^2}{(m_0^2 - m_1^2)^2}F_\Delta(m_0, m_1)\right]. \tag{72}$$

We note that $F_\Delta(m,m) = 0$ and $F'_\Delta(m,m) = \frac{1}{3}\ln m^2$. [18] When $g^2_{_{H_1VV}} = 1$, neglecting the Z^2-dependent vertex correction factors $\delta_{\gamma Z}^{H^\pm}$, $\delta_W^{H_i}$ and $\delta_Z^{H_iH_j}$, S_Φ and T_Φ are symmetric under the exchange $M_{H_2} \leftrightarrow M_{H_3}$ and they are identically vanishing when $M_{H_2} = M_{H_3} = M_{H^\pm}$. [19]

$$\frac{T}{\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\rm EM}}} = (1-g^2_{H_1VV})F_\Delta(M_{H_1},M_{H^\pm}) + (1-g^2_{H_2VV})F_\Delta(M_{H_2},M_{H^\pm}) + (1-g^2_{H_3VV})F_\Delta(M_{H_3},M_{H^\pm})$$
$$-g^2_{H_3VV}F_\Delta(M_{H_1},M_{H_2}) - g^2_{H_2VV}F_\Delta(M_{H_1},M_{H_3}) - g^2_{H_1VV}F_\Delta(M_{H_2},M_{H_3}) \,,$$
$$4\pi\, S = -F'_\Delta(M_{H^\pm},M_{H^\pm}) + g^2_{H_3VV}F'_\Delta(M_{H_1},M_{H_2}) + g^2_{H_2VV}F'_\Delta(M_{H_1},M_{H_3}) + g^2_{H_1VV}F'_\Delta(M_{H_2},M_{H_3})$$

When $m_0 \sim m_1$
$$F_\Delta(m_0,m_1) = \frac{2(m_0-m_1)^2}{3} - \frac{2\,(m_0-m_1)^4}{15\,(m_0+m_1)^2} + \mathcal{O}\left[\frac{(m_0-m_1)^6}{(m_0+m_1)^4}\right] \,,$$
$$F'_\Delta(m_0,m_1) = \frac{1}{3}\ln\left[\frac{(m_0+m_1)^2}{4}\right] + \frac{(m_0-m_1)^2}{5\,(m_0+m_1)^2} + \mathcal{O}\left[\frac{(m_0-m_1)^4}{(m_0+m_1)^4}\right]$$

for $m_1 \gg m_0$
$$F_\Delta(m_0,m_1) = \frac{m_1^2}{2} + \left(\frac{1}{2}+\ln\frac{m_0^2}{m_1^2}\right)\, m_0^2 + \mathcal{O}\left[\left(\frac{m_0^4}{m_1^2}\right)\ln\frac{m_0^2}{m_1^2}\right]$$
$$F'_\Delta(m_0,m_1) = \frac{\ln m_1^2}{3} - \frac{5}{18} + \frac{2}{3}\frac{m_0^2}{m_1^2} + \mathcal{O}\left[\left(\frac{m_0^4}{m_1^4}\right)\ln\frac{m_0^2}{m_1^2}\right] \,.$$

- Yukawa couplings in the Higgs basis

$$-\mathcal{L}_Y = \sum_{k=1,2} \overline{Q_L^0}\,\mathbf{y}_k^u\,\widetilde{\mathcal{H}}_k\,u_R^0 \;+\; \overline{Q_L^0}\,\mathbf{y}_k^d\,\mathcal{H}_k\,d_R^0 \;+\; \overline{L_L^0}\,\mathbf{y}_k^e\,\mathcal{H}_k\,e_R^0 \;+\; \text{h.c.}$$

$$\mathcal{H}_1 = \left(G^+\,,\,\frac{1}{\sqrt{2}}(v+\varphi_1+iG^0)\right)^T, \quad \mathcal{H}_2 = \left(H^+\,,\,\frac{1}{\sqrt{2}}(\varphi_2+ia)\right)^T, \quad \widetilde{\mathcal{H}}_1 = i\tau_2\mathcal{H}_1^* = \left(\frac{1}{\sqrt{2}}(v+\varphi_1-iG^0)\,,\,-G^-\right)^T, \quad \widetilde{\mathcal{H}}_2 = i\tau_2\mathcal{H}_2^* = \left(\frac{1}{\sqrt{2}}(\varphi_2-ia)\,,\,-H^-\right)^T.$$

$$-\mathcal{L}_{Y,\,\text{mass}} = \frac{v}{\sqrt{2}}\left(\overline{u_L^0}\,\mathbf{y}_1^u\,u_R^0 \;+\; \overline{d_L^0}\,\mathbf{y}_1^d\,d_R^0 \;+\; \overline{e_L^0}\,\mathbf{y}_1^e\,e_R^0 \;+\; \text{h.c.}\right)$$

$$u_L^0 = \mathcal{U}_{u_L}\,u_L\,, \quad d_L^0 = \mathcal{U}_{d_L}\,d_L\,, \quad e_L^0 = \mathcal{U}_{e_L}\,e_L\,;$$
$$u_R^0 = \mathcal{U}_{u_R}\,u_R\,, \quad d_R^0 = \mathcal{U}_{d_R}\,d_R\,, \quad e_R^0 = \mathcal{U}_{e_R}\,e_R\,,$$

$$\mathbf{M}_u = \frac{v}{\sqrt{2}}\,\mathcal{U}_{u_L}^\dagger\,\mathbf{y}_1^u\,\mathcal{U}_{u_R} = \text{diag}(m_u, m_c, m_t)\,,$$
$$\mathbf{M}_d = \frac{v}{\sqrt{2}}\,\mathcal{U}_{d_L}^\dagger\,\mathbf{y}_1^d\,\mathcal{U}_{d_R} = \text{diag}(m_d, m_s, m_b)\,,$$
$$\mathbf{M}_e = \frac{v}{\sqrt{2}}\,\mathcal{U}_{e_L}^\dagger\,\mathbf{y}_1^e\,\mathcal{U}_{e_R} = \text{diag}(m_e, m_\mu, m_\tau)\,,$$

$$-\mathcal{L}_{Y,\,\text{mass}} = \overline{u_L}\,\mathbf{M}_u\,u_R \;+\; \overline{d_L}\,\mathbf{M}_d\,d_R \;+\; \overline{e_L}\,\mathbf{M}_e\,e_R \;+\; \text{h.c.}$$

* Couplings of the neutral Higgs bosons

$$
\begin{aligned}
-\mathcal{L}_{H\bar{f}f} = & \frac{1}{v}\left[\bar{u}\,\mathbf{M}_u\,u\right]\varphi_1 + \left[\bar{u}\left(\mathbf{h}_u^H + \mathbf{h}_u^A\gamma_5\right)u\right]\varphi_2 + \left[\bar{u}\left(-i\mathbf{h}_u^A - i\mathbf{h}_u^H\gamma_5\right)u\right]a \\
& + \frac{1}{v}\left[\bar{d}\,\mathbf{M}_d\,d\right]\varphi_1 + \left[\bar{d}\left(\mathbf{h}_d^H + \mathbf{h}_d^A\gamma_5\right)d\right]\varphi_2 + \left[\bar{d}\left(i\mathbf{h}_d^A + i\mathbf{h}_d^H\gamma_5\right)d\right]a \\
& + \frac{1}{v}\left[\bar{e}\,\mathbf{M}_e\,e\right]\varphi_1 + \left[\bar{e}\left(\mathbf{h}_e^H + \mathbf{h}_e^A\gamma_5\right)e\right]\varphi_2 + \left[\bar{e}\left(i\mathbf{h}_e^A + i\mathbf{h}_e^H\gamma_5\right)e\right]a
\end{aligned}
$$

$$\mathbf{h}_f^H \equiv \frac{\mathbf{h}_f + \mathbf{h}_f^\dagger}{2}, \qquad \mathbf{h}_f^A \equiv \frac{\mathbf{h}_f - \mathbf{h}_f^\dagger}{2}, \qquad \text{with} \qquad \mathbf{h}_f \equiv \frac{1}{\sqrt{2}}\,\mathcal{U}_{f_L}^\dagger\,\mathbf{y}_2^f\,\mathcal{U}_{f_R}\,.$$

If $\mathbf{y}_2^f = \zeta_f\,\mathbf{y}_1^f\,, \qquad \mathbf{h}_f = \zeta_f\,\frac{\mathbf{M}_f}{v} \quad \mathbf{h}_f^H = \Re\mathrm{e}(\zeta_f)\,\frac{\mathbf{M}_f}{v}\,, \quad \mathbf{h}_f^A = i\,\Im\mathrm{m}(\zeta_f)\,\frac{\mathbf{M}_f}{v}\,.$

Alignment of the two Yukawa matrices in the flavor space

A. Pich and P. Tuzon, "Yukawa Alignment in the Two-Higgs-Doublet Model," Phys. Rev. D **80** (2009), 091702 doi:10.1103/PhysRevD.80.091702 [arXiv:0908.1554 [hep-ph]].

When $\Im\mathrm{m}(\zeta_f) = 0$, the conventional 2HDMs based on the Glashow-Weinberg condition [101] can be obtained by choosing ζ_f as shown in Table I. Otherwise, the couplings of the mass eigenstates of the neutral Higgs bosons $H_{i=1,2,3}$ to two fermions are given by

$$-\mathcal{L}_{H_i\bar{f}f} = \sum_{i=1}^{3} \sum_{f=u,d,c,s,t,b,e,\mu,\tau} \frac{m_f}{v}\, \bar{f}\left(g^S_{H_i\bar{f}f} + i g^P_{H_i\bar{f}f}\gamma_5\right) f\, H_i \tag{49}$$

with the scalar and pseudoscalar couplings given by

$$\begin{aligned} g^S_{H_i\bar{f}f} &= O_{\varphi_1 i} + \Re\mathrm{e}(\zeta_f) O_{\varphi_2 i} \pm \Im\mathrm{m}(\zeta_f) O_{ai}\,, \\ g^P_{H_i\bar{f}f} &= \qquad\quad \Im\mathrm{m}(\zeta_f) O_{\varphi_2 i} \mp \Re\mathrm{e}(\zeta_f) O_{ai}\,, \end{aligned} \tag{50}$$

where the upper and lower signs are for the up-type fermions $f = u, c, t$ and the down-type fermions $f = d, s, b, e, \mu, \tau$, respectively. The simultaneous existence of the scalar $g^S_{H_i\bar{f}f}$ and pseudoscalar $g^P_{H_i\bar{f}f}$ couplings signals the CP violation in the neutral Higgs sector. We figure out that there are two different sources of the neutral Higgs-sector CP violation: (i) one is the CP-violating mixing between the CP-even and CP-odd states arising in the presence of non-vanishing $\Im\mathrm{m}(Z_{5,6})$ in the Higgs potential and (ii) the other one is the complex alignment parameters of ζ_f's. Note that the second source is absent in the conventional four types of 2HDMs since ζ_f's are assigned to be real in those models.

TABLE I. Classification of the conventional 2HDMs satisfying the Glashow-Weinberg condition [101] which guarantees the absence of tree-level Higgs-mediated flavor-changing neutral current (FCNC). For the four types of 2HDM, we follow the conventions found in, for example, Ref. [102].

	2HDM I	2HDM II	2HDM III	2HDM IV
ζ_u	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$
ζ_d	$1/t_\beta$	$-t_\beta$	$1/t_\beta$	$-t_\beta$
ζ_e	$1/t_\beta$	$-t_\beta$	$-t_\beta$	$1/t_\beta$
	$\zeta_d = \zeta_e = \zeta_u$	$\zeta_d = \zeta_e = -1/\zeta_u$	$\zeta_d = -1/\zeta_e = \zeta_u$	$\zeta_d = -1/\zeta_e = -1/\zeta_u$

* Couplings of the charged Higgs bosons

$$-\mathcal{L}_{H^\pm \bar{f}_\uparrow f_\downarrow} = -\sqrt{2}\left[\overline{u_R}(\mathbf{h}_u^\dagger V)d_L\right]H^+ + \sqrt{2}\left[\overline{u_L}(V\mathbf{h}_d)d_R\right]H^+ + \sqrt{2}\left[\overline{\nu_L}\,\mathbf{h}_e\,e_R\right]H^+ \;+\; \text{h.c.}\,.$$

with $\mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L} = V_{\rm CKM} \equiv V$ and $\mathbf{h}_f = \zeta_f \dfrac{\mathbf{M}_f}{v}$

- Constraints on the alignments parameters from |g^S_H1ff ± 1| < 0.1

$$g^S_{H_1\bar{f}f} = O_{\varphi_1 1} + \zeta_f O_{\varphi_2 1} = c_\gamma - \zeta_f s_\gamma$$

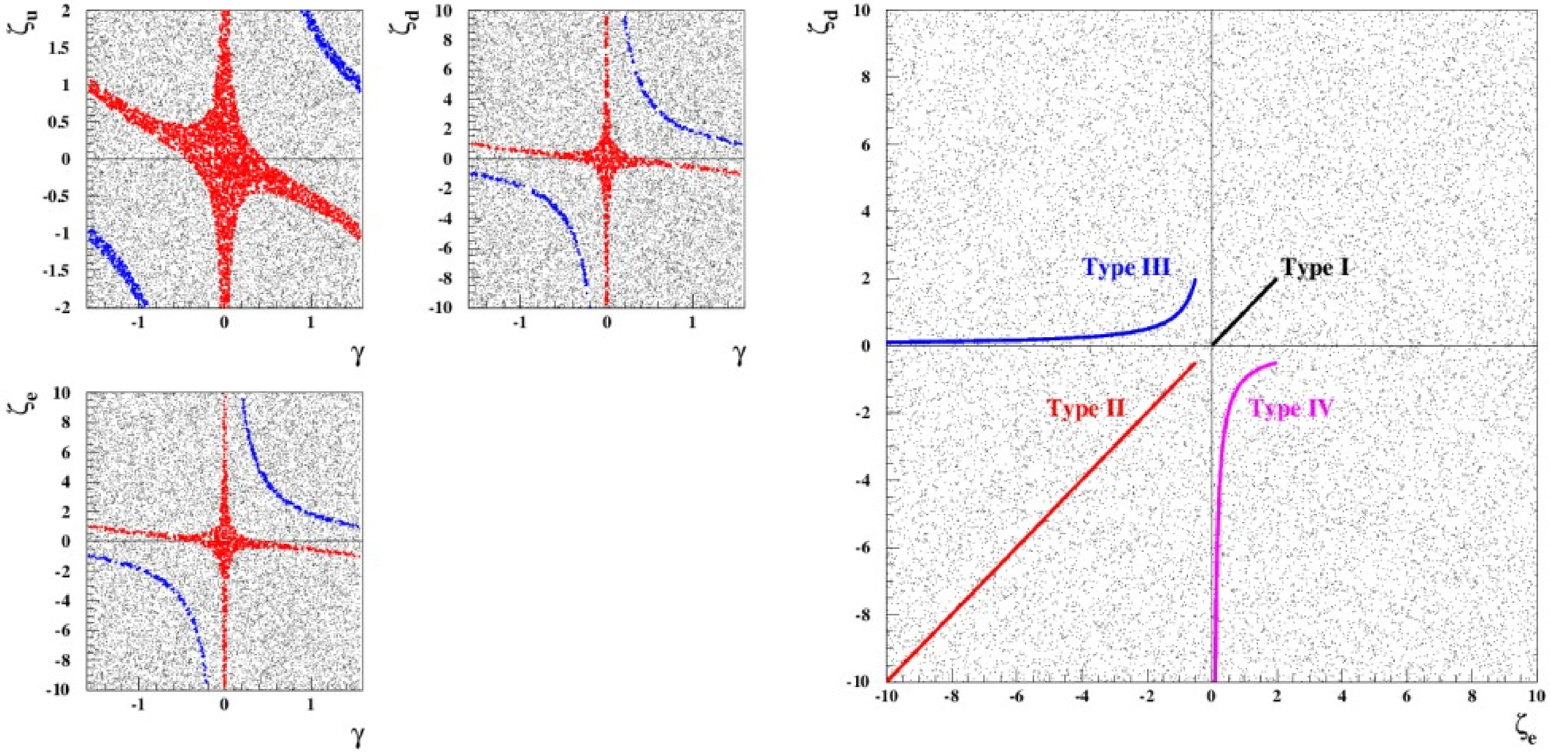


FIG. 4. *(Left) Scatter plots of ζ_u versus γ (upper left), ζ_d versus γ (upper right), and ζ_e versus γ (lower left) obtained by scanning $-\pi/2 \le \gamma \le \pi/2$ and the three real parameters of the set $\mathcal{I}^{\zeta}_{\rm CPC}$ in the ranges of $-2 < \zeta_u < 2$ and $-10 < \zeta_{d,e} < 10$. On each ζ_f-γ plane, the regions satisfying $|g^S_{H_1\bar{f}f} - 1| < 0.1$ and $|g^S_{H_1\bar{f}f} + 1| < 0.1$ are denoted in red and blue, respectively. (Right) Scatter plot of ζ_d versus ζ_e with $1/100 < \zeta_u < 2$. The four lines represent the four conventional 2HDMs as denoted taking $1/2 < t_\beta < 100$.*