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Beating the Lyth bound with parametric resonance during inflation

based on arXiv: 2010.03537 (published on PRD) and 2105.12554 (published on PRL)
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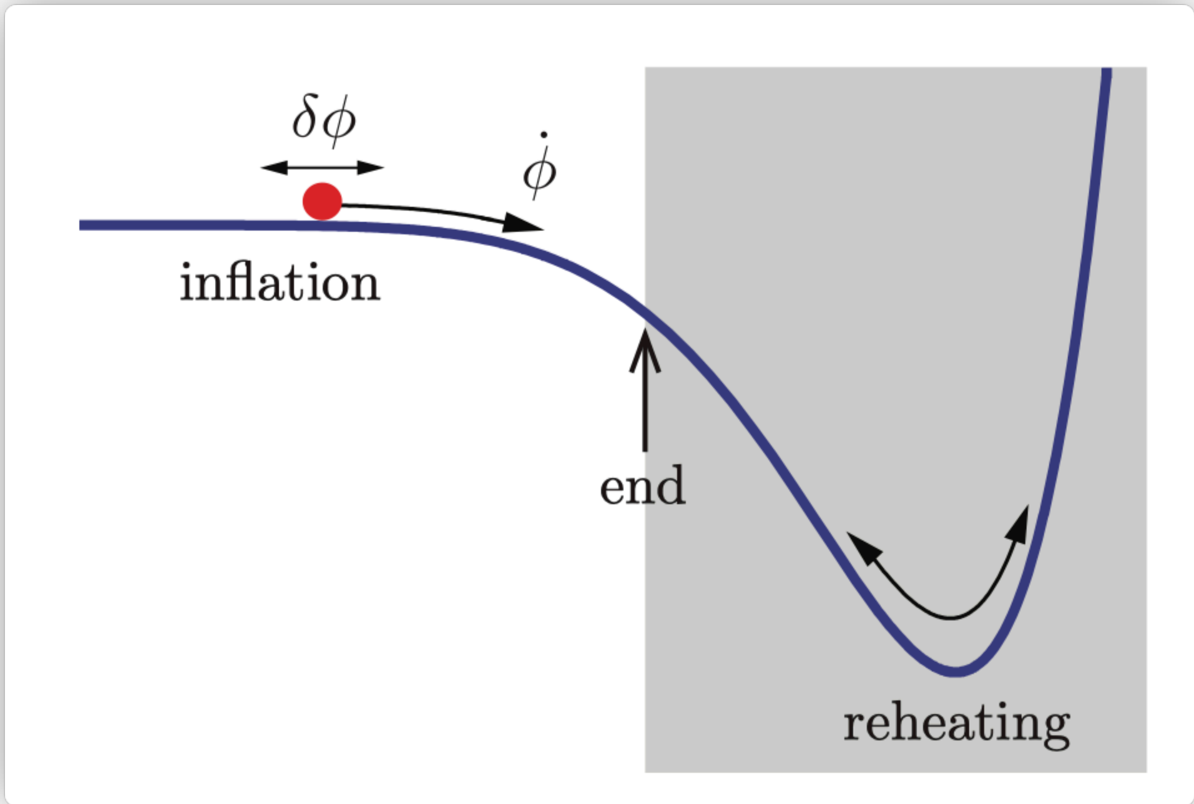
Outline

- Introduction to inflation prediction
- Lyth bound
- Model
- Conclusion

Single filed slow-roll inflation

as the Standard Model of the very early universe



$$\epsilon_V = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta_V = M_{\rm Pl}^2 \frac{V''}{V}$$

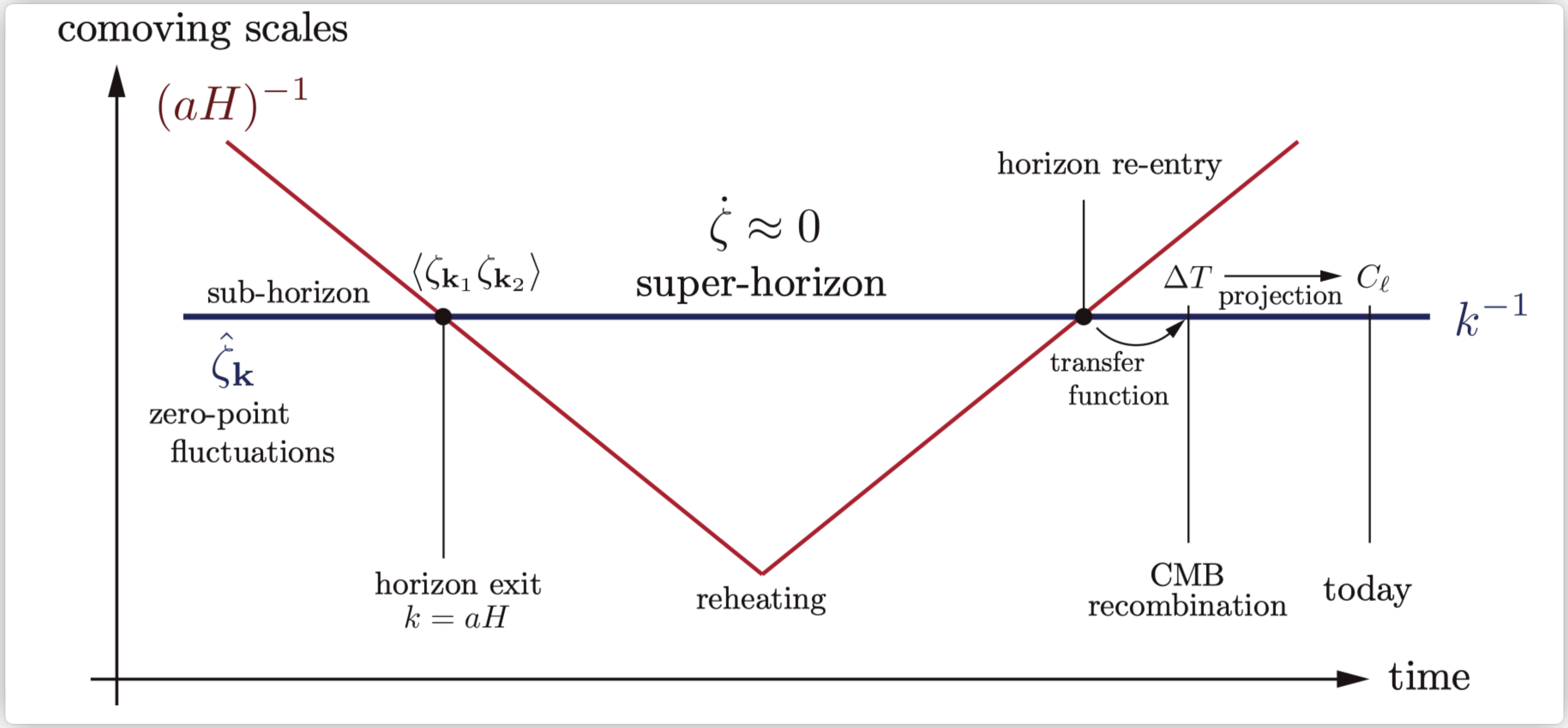
potential dominated expansion

long enough duration of inflation

Many different models: chaotic inflation, hilltop inflation, Natural inflation, Higgs inflation, Starobinsky inflation

Prediction from inflation

Scalar Fluctuations



dimensionless power spectrum

$$P_{\mathcal{R}}(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2$$

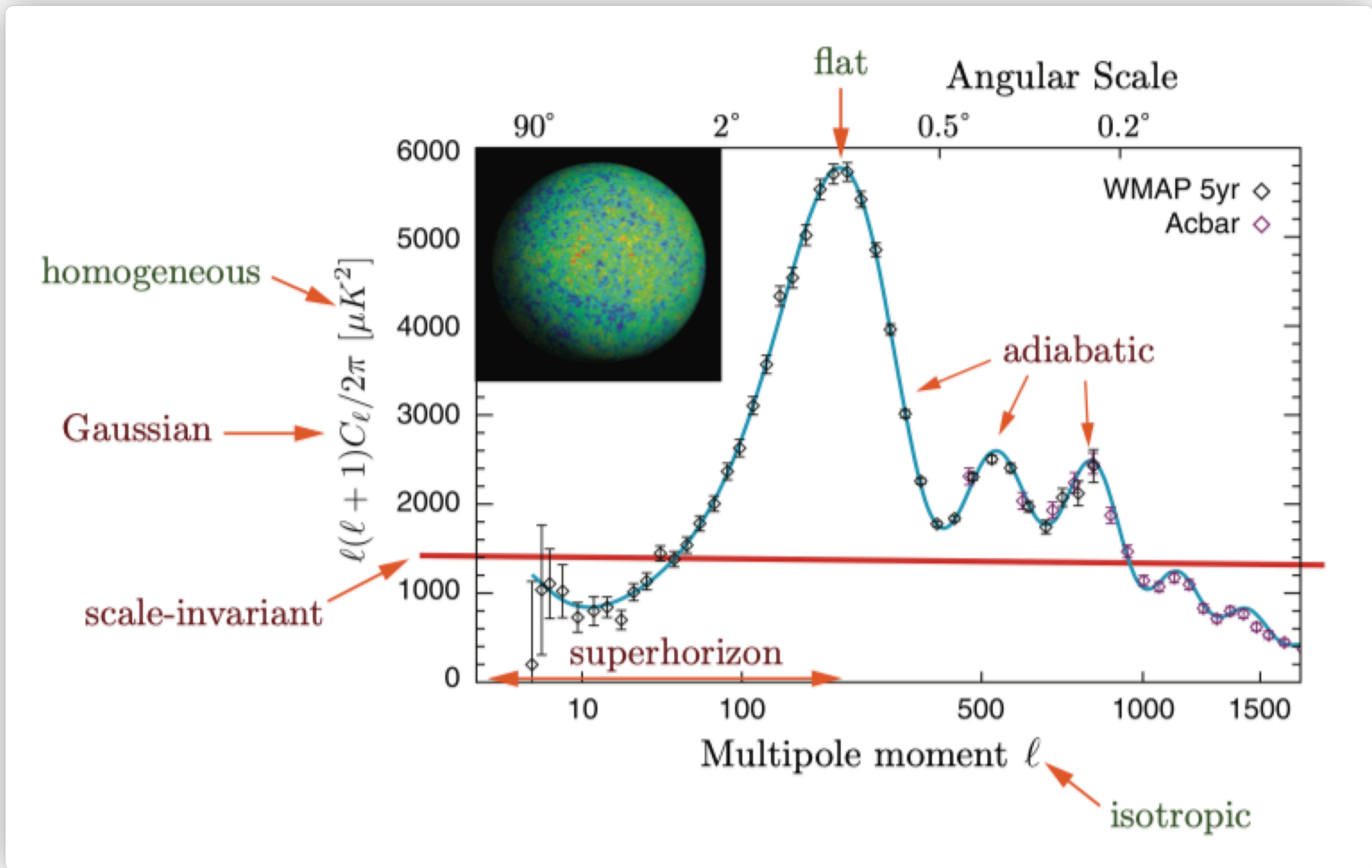
de Sitter fluctuations $\delta\phi$

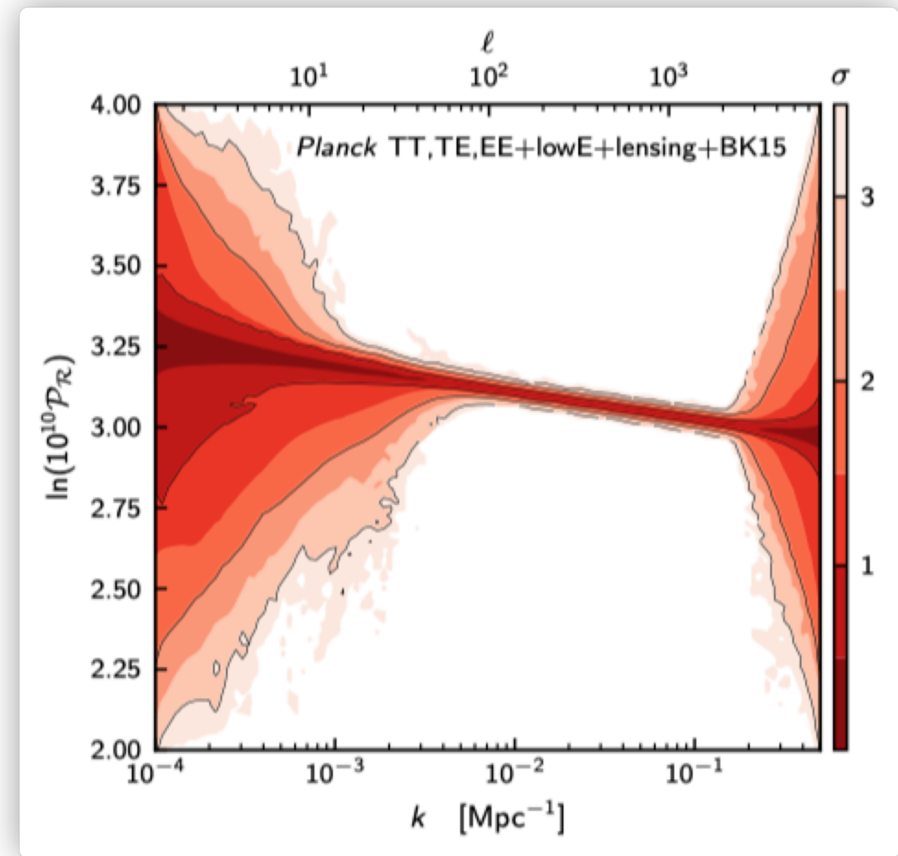
conversion $\delta\phi \to \mathcal{R}$

evaluated at horizon crossing
$k = aH$

Prediction from inflation

Scalar Fluctuations observation





Planck 2018 results.

$n_s = 0.9649 \pm 0.0042$

nearly scale-invariant

scale-dependence $P_\mathcal{R}(k) = A_s k^{n_s - 1}$

e.g. slow-roll inflation $n_s - 1 = 2\eta - 6\epsilon$

vacuum fluctuations during inflation fits observation

Prediction from inflation

Tensor Fluctuations

Besides scalar fluctuations inflation produces tensor fluctuations:

$$ds^2 = dt^2 - a^2(t)(1 + h_{ij})dx^i dx^j$$

gravitational waves

$$P_h(k) = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$$

vacuum fluctuations

Predictions from inflation

scalars

$$P_{\mathcal{R}}(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2$$

tensors

$$P_h(k) = \frac{8}{M_{\rm pl}^2} \left(\frac{H}{2\pi}\right)^2$$

$$\frac{H}{2\pi}$$ **vacuum fluctuations**

What is Lyth bound?

The Lyth bound

scalars

$$P_{\mathcal{R}}(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2$$

tensors

$$P_h(k) = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$$

tensor-to-scalar ratio

$$r \equiv \frac{P_h}{P_{\mathcal{R}}} = 8\left(\frac{d\phi}{dN_e}\frac{1}{M_{\rm pl}}\right)^2$$

where $dN_e \equiv H dt = d \ln a$

speed of field evolution

The Lyth bound

tensor-to-scalar ratio

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 8\left(\frac{d\phi}{dN_e}\frac{1}{M_{\rm pl}}\right)^2$$

field evolution over 60 e-folds

$$\Delta\phi \gtrsim \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2} M_{\rm Pl}$$

tensor-to-scalar ratio r determines the field excursion during inflation

$r > 0.01$

Lyth 1996

If we observe tensors it proves that the inflation field moved over a super-Planckian distance!

current upper limit

$r \lesssim 0.03$ (BICEP)

$$\Delta\phi \gg M_{\rm Pl}$$

EFT failed

But this only consider vacuum contribution!

There is also induce perturbation during inflation.

How to beat Lyth bound?

- The scalar power spectrum should fit the experiment result, which is nearly scale-invariant.

- In experiment, if we detect tensor perturbation, we can not distinguish whether it is vacuum fluctuations or induce perturbations.

$$\longrightarrow \Delta\phi \gtrsim \mathcal{O}(1)\left(\frac{r\uparrow}{0.01}\right)^{1/2} M_{\rm Pl}$$

- Enhance the tensor perturbations but remain small field excursion.

Fields perturbations

$\delta\chi$: massless controls the curvature perturbation;

$\delta\phi$: massive, resonant, gravitationally coupled to $\delta\chi$, controls isocurvature perturbations.

$$S_3 \supset \frac{1}{2}\int dt \frac{d^3k d^3p}{(2\pi)^6} a^3 \left(-\sqrt{2\epsilon_\chi}\frac{\boldsymbol{k}\cdot\boldsymbol{p}}{k^2}\mathcal{M}^2_{\rm eff}\delta\phi_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi_{\boldsymbol{p}}\delta\chi_{-\boldsymbol{k}}\right) - \frac{2}{M^2_{\rm pl}}\int dt \frac{d^3k d^3p}{(2\pi)^6} a^3 \left(e^\lambda_{ij}(\boldsymbol{k})\frac{p_i p_j}{a^2}\right)\delta\phi_{\boldsymbol{p}}\delta\phi_{\boldsymbol{k}-\boldsymbol{p}}h^\lambda_{-\boldsymbol{k}}$$

slow-roll suppression

$$\ddot{\delta\phi}_k + 3H\dot{\delta\phi}_k + \left(\frac{k^2}{a^2} + \mathcal{M}^2_{\rm eff}\right)\delta\phi_k = 0$$

→ parametric resonance

$$\ddot{\delta\chi}_k + 3H\dot{\delta\chi}_k + \frac{k^2}{a^2}\delta\chi_k = \frac{\sqrt{2\epsilon_\chi}}{M_{\rm Pl}}[\ddot{\phi}\delta\phi_k + \mathcal{S}_k]$$

→ $$\mathcal{S}_k = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left\{\frac{\boldsymbol{p}\cdot\boldsymbol{k}}{k^2}\left[\frac{(\boldsymbol{p}-\boldsymbol{k})^2}{a^2} + \mathcal{M}^2_{\rm eff}\right] - \frac{\boldsymbol{p}\cdot(\boldsymbol{k}-\boldsymbol{p})}{2a^2}\right\}\delta\phi_{|\boldsymbol{p}|}\delta\phi_{|\boldsymbol{k}-\boldsymbol{p}|}$$

$$\ddot{h}^\lambda_k + 3H\dot{h}^\lambda_k + \frac{k^2}{a^2}h^\lambda_k = \mathcal{T}^\lambda_k(t)$$

→ $$\mathcal{T}^\lambda_{\boldsymbol{k}}(\tau) = \frac{2}{M^2_p}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3} e^\lambda_{ij}(\boldsymbol{k})p_i p_j \delta\phi_{\boldsymbol{p}}(\tau)\delta\phi_{\boldsymbol{k}-\boldsymbol{p}}(\tau) + (\phi \leftrightarrow \chi)$$

Effective mass is important!

Basic conditions for effective mass

- Heavy field should have oscillation

$$\delta\phi \propto \exp\left(\left(\left|M_{\rm eff}\right|/H\right)\Delta N\right)$$

e.g. Narrow parametric resonance

$$\exp(\Delta N) \sim \frac{|M_{\rm eff}|}{H}$$

$$|\mathcal{M}_{\rm eff}| \sim O(10)H \qquad \ddot{\delta\phi}_k + 3H\dot{\delta\phi}_k + \left(\frac{k^2}{a^2} + \mathcal{M}_{\rm eff}^2\right)\delta\phi_k = 0$$

- Resonant field mass needs to become small right after horizon crossing.

$$\ddot{h}_k^\lambda + 3H\dot{h}_k^\lambda + \frac{k^2}{a^2}h_k^\lambda = \mathcal{T}_k^\lambda(t) \longrightarrow \mathcal{T}_{\boldsymbol{k}}^\lambda(\tau) = \frac{2}{M_p^2}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}e_{ij}^\lambda(\boldsymbol{k})p_ip_j\delta\phi_{\boldsymbol{p}}(\tau)\delta\phi_{\boldsymbol{k}-\boldsymbol{p}}(\tau) + (\phi\leftrightarrow\chi)$$

$\mathcal{M}_{\rm eff}$ **should be small** **induce GWs long enough**

- Resonant heavy field must decay far outside horizon, not to affect curvature perturbations.

$$\mathcal{M}_{\rm eff} \gtrsim O(1)H$$

Parametric Resonance

Someone can swing higher and higher with swinging legs

Parametric Resonance

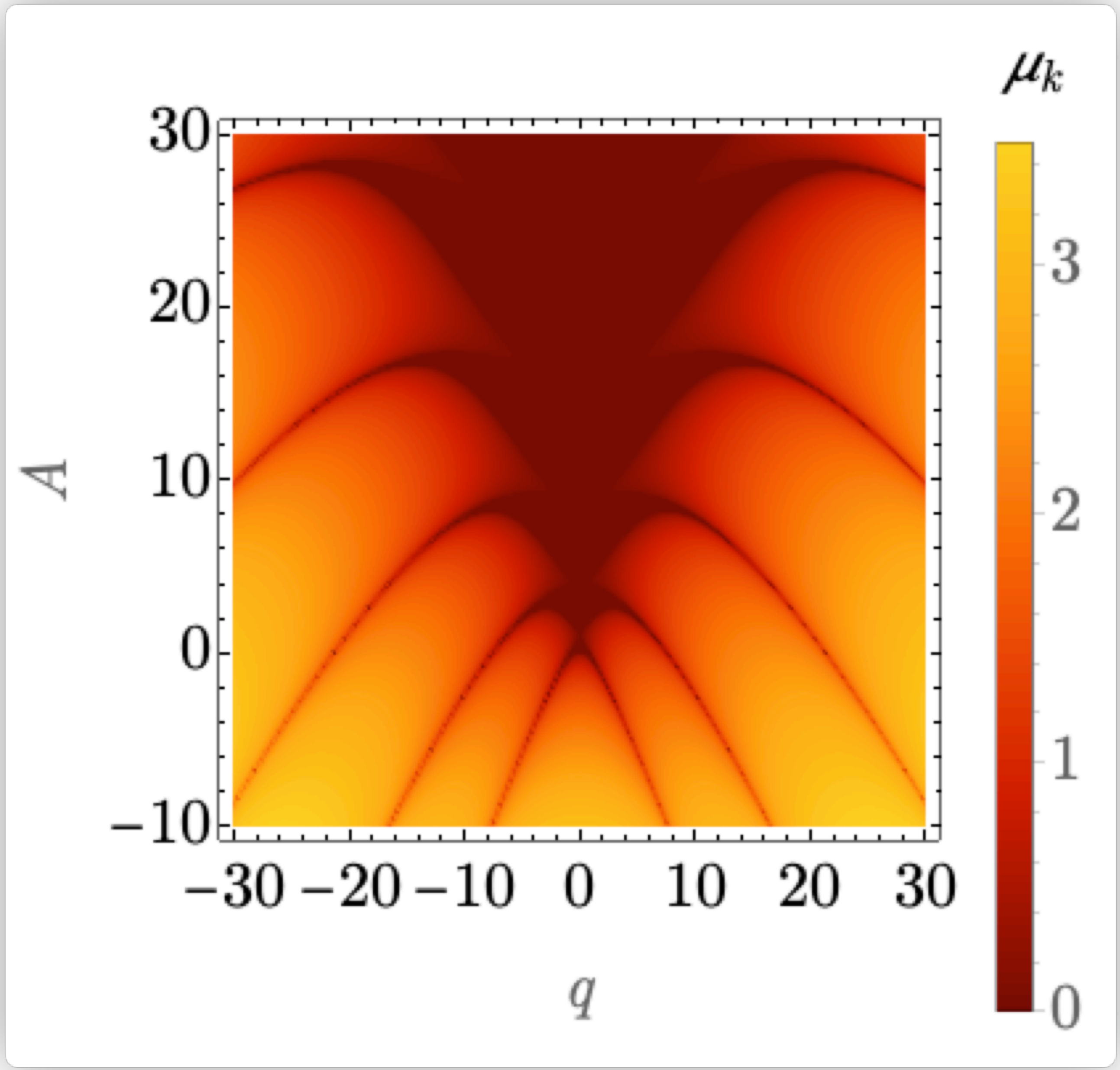
Floquet number

Mathieu equation:

$$x'' + (A - 2q\cos 2z)x = 0$$

Floquet analysis

instability: $\mu_k > 0$ $\quad \chi_k \sim e^{\mu_k z}$



Applications:
reheating,
sound speed c_s resonance to produce primordial black holes

Model realization

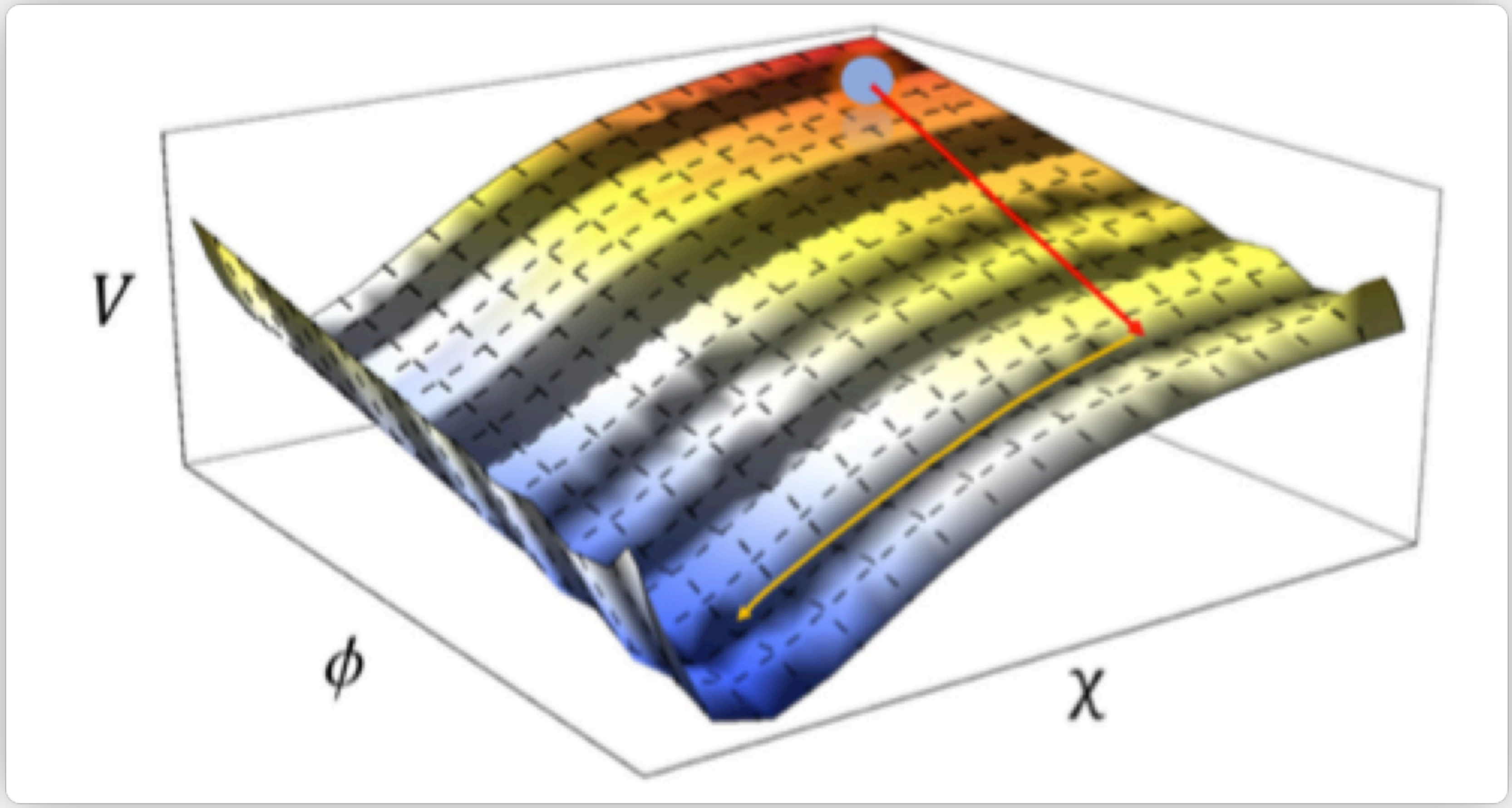
Two field inflation

vacuum energy   slope of ϕ slope of χ → 1. same as single field inflation, control curvature perturbation

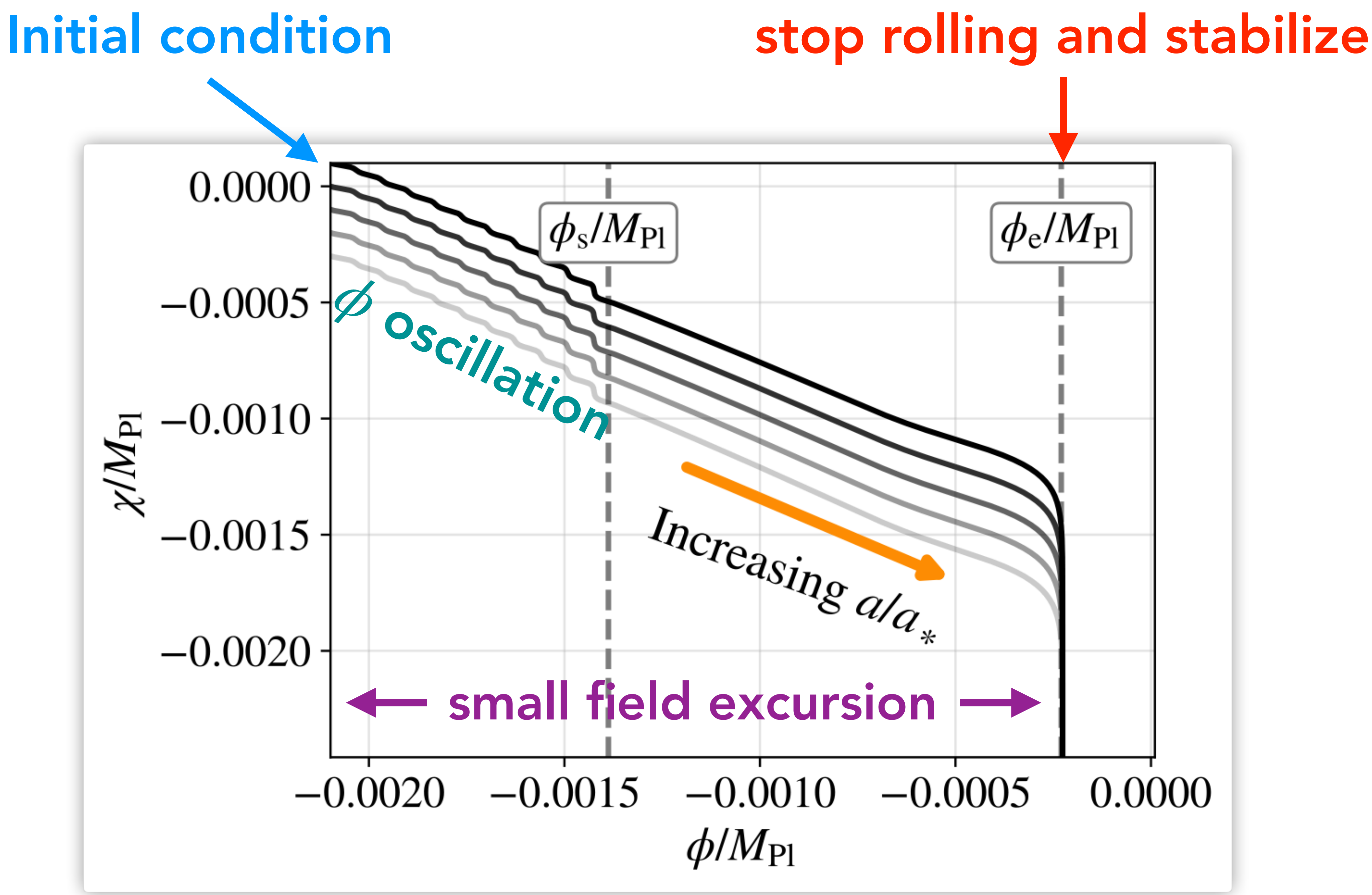
$$V(\phi,\chi) = V_0\left(1 - \sqrt{2\epsilon_\phi}\frac{\phi}{M_{\rm Pl}} + \sqrt{2\epsilon_\chi}\frac{\chi}{M_{\rm Pl}} + \eta_\chi\frac{\chi^2}{2M_{\rm Pl}^2}\right) + V_m(\phi) + \Lambda^4(\phi)\cos\left(\frac{\phi}{f_{\rm a}}\right)$$

3. Super-horizon decay

ϕ oscillation
↓
2. generate $\delta\phi$ amplification



Background field evolution



two phase inflation model

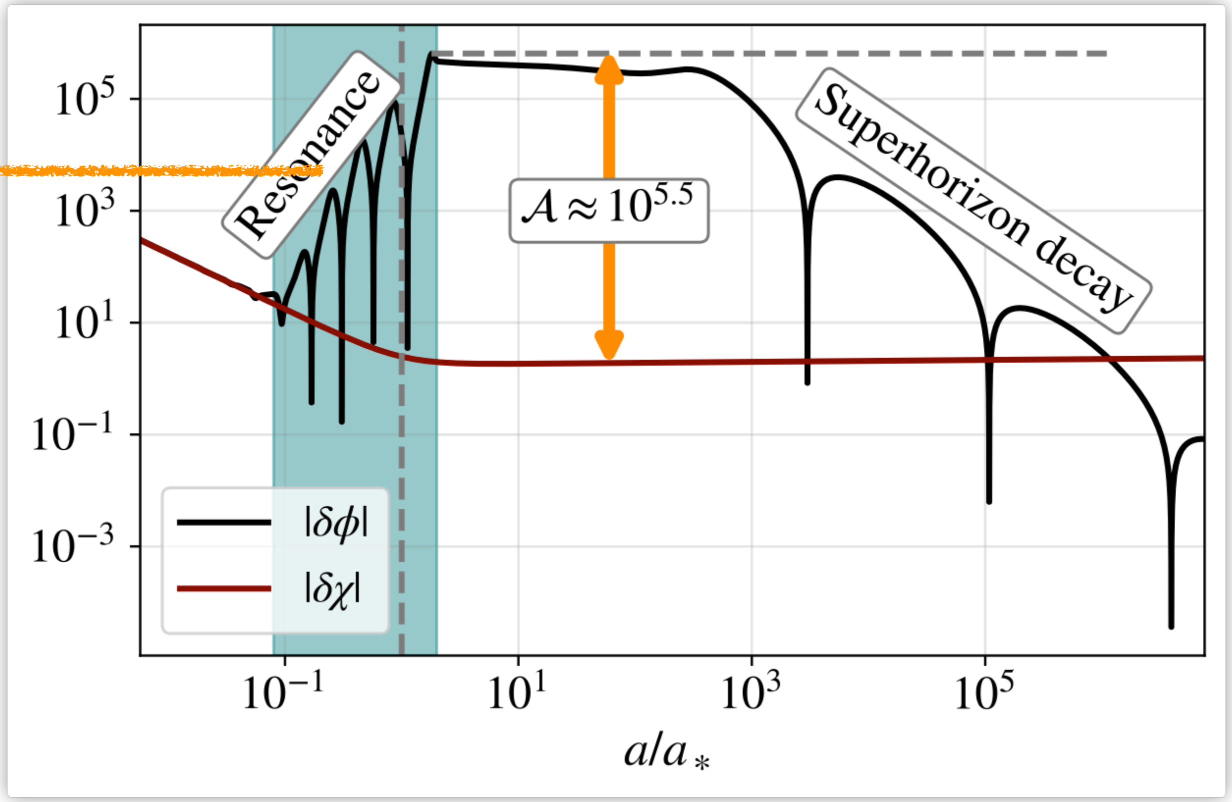
Fields perturbations

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + \mathcal{M}^2_{\rm eff}\right)\delta\phi_k = 0$$ → parametric resonance

$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \frac{k^2}{a^2}\delta\chi_k = \frac{\sqrt{2\epsilon_\chi}}{M_{\rm Pl}}[\ddot{\phi}\delta\phi_k + \mathcal{S}_k]$$ → we have parameter space to remain small

$$\ddot{h}^\lambda_k + 3H\dot{h}^\lambda_k + \frac{k^2}{a^2}h^\lambda_k = \mathcal{T}^\lambda_k(t)$$ **where:** $$\mathcal{T}^\lambda_{\boldsymbol{k}}(\tau) = \frac{2}{M_p^2}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3} e^\lambda_{ij}(\boldsymbol{k}) p_i p_j \delta\phi_{\boldsymbol{p}}(\tau)\delta\phi_{\boldsymbol{k}-\boldsymbol{p}}(\tau) + (\phi \leftrightarrow \chi)$$

1. induce gravitational waves



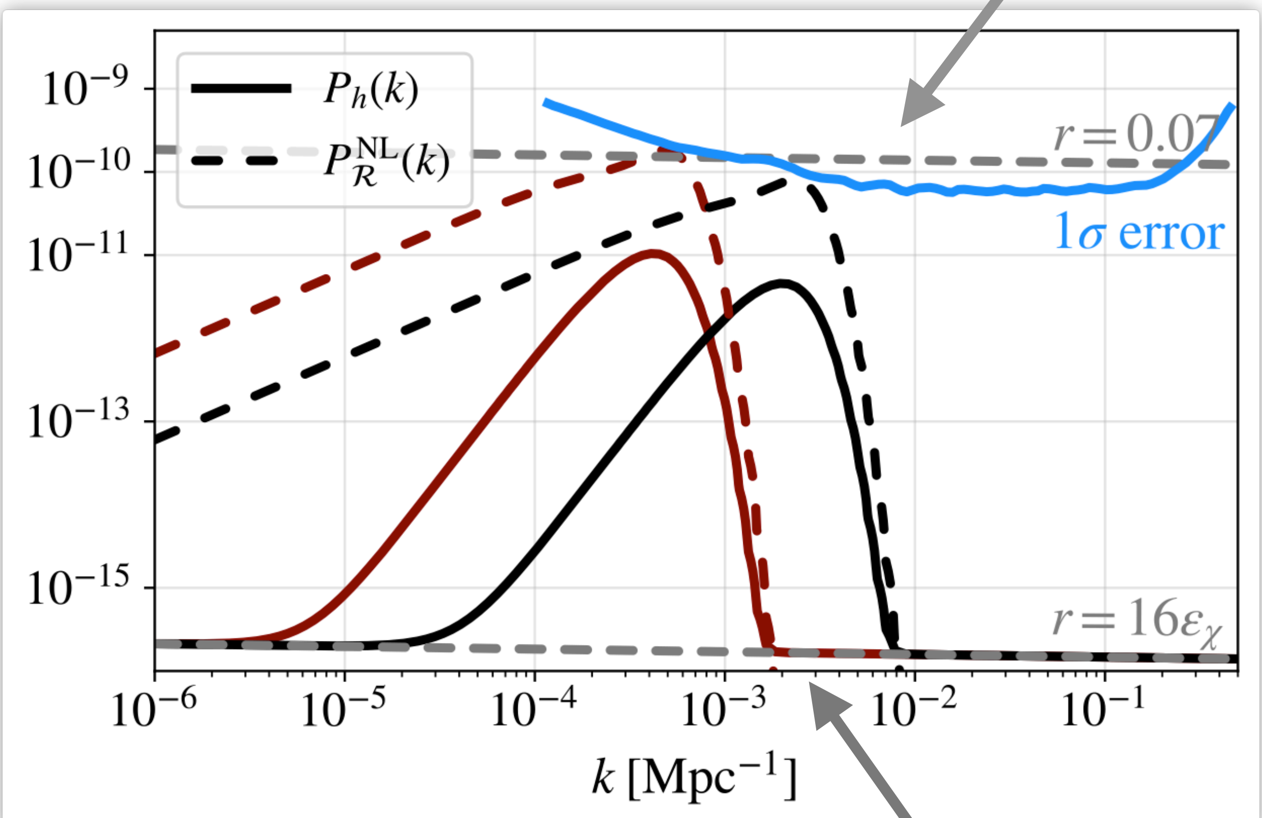
2. curvature perturbation

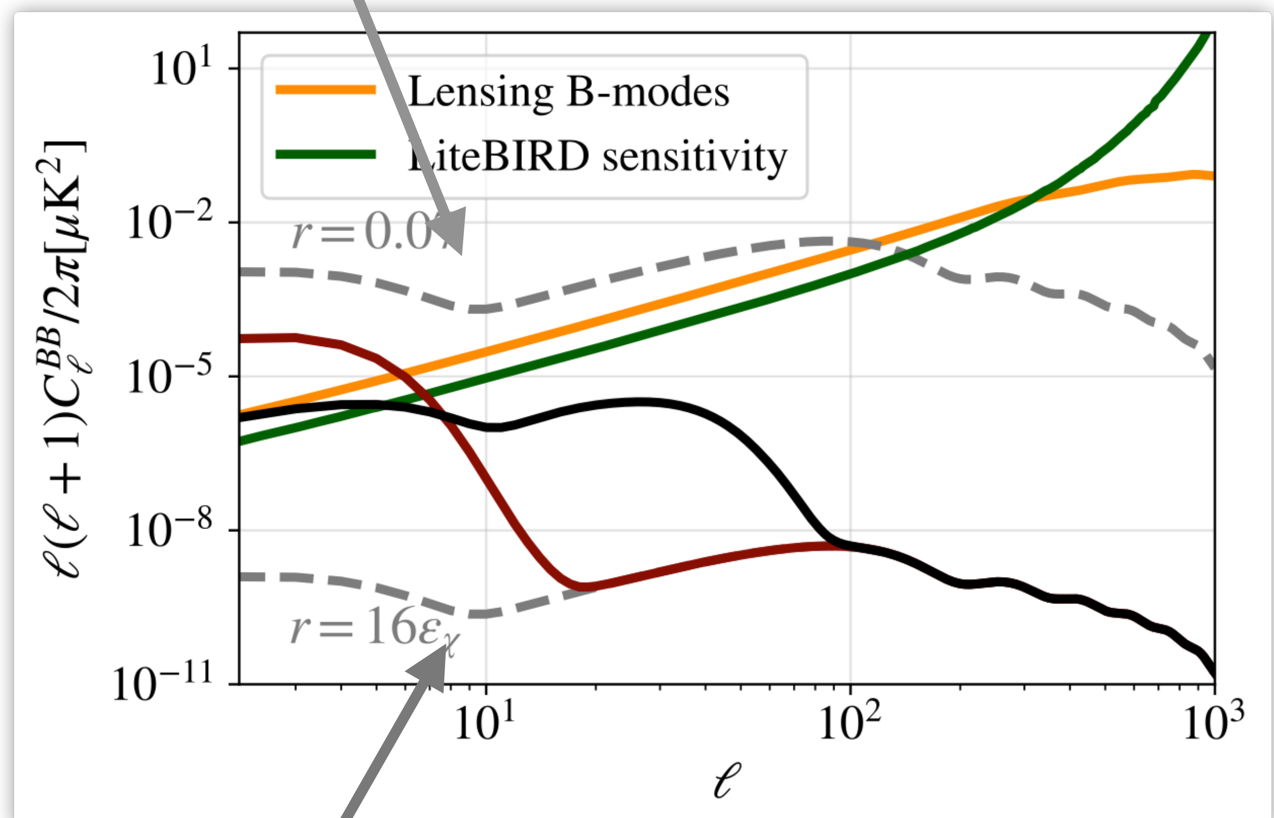
3. super-horizon decay

curvature perturbation: $\delta\chi$ ↔ (different source) ↔ tensor perturbation: $\delta\phi$

Induce gravitational waves and BB power spectrum

current upper limit





vacuum contribution

base on small field excursion

Lyth bound was beaten by considering induce perturbations.

curvature perturbation: $\delta\chi$

tensor perturbation: $\delta\phi$

different sources to break the implicit binding in Lyth bound

Summary

- Our mechanism provides a scale-dependent counter-example of the Lyth bound within slow-roll inflation.
- Our scenario can generate detectable primordial GWs for the forthcoming CMB polarization experiments such as LiteBIRD even within inflationary models with very low energy scales and limited excursion range of the inflaton field.


Thanks for your attention