Electromagnetic Couplings of Axions

based on

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CTPU-PTC Seminar

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OUTLINE OF THE TALK

- Why considering axions?
- Electric-magnetic duality symmetry & axion electrodynamics
- Quantum Electro-Magneto-Dynamics (QEMD)
- Electromagnetic interactions of axions
- Theoretical motivations for new couplings
- Hadronic axion models with magnetic monopoles
- Novel experimental signatures

WHY AXIONS

- Dark matter

ΛCDM

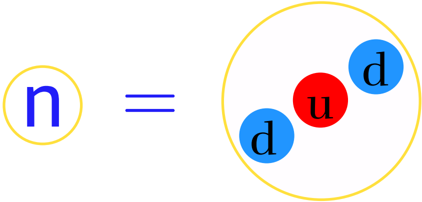
NASA image

ESO/Illustris Collaboration

What makes Cold Dark Matter ?!

⇒ axions

- CP conservation in strong interactions



$$d_n = 2.4\,(1.0)\cdot 10^{-16}\,\bar{\theta}\,e\cdot\text{cm}$$
$$|d_n| < 1.8\cdot 10^{-26} e\cdot\text{cm} \quad \Rightarrow \quad |\bar{\theta}| \lesssim 10^{-10}$$

Why is there no electric dipole moment ?!

Why $\bar{\theta} \simeq 0$?!

ELECTRIC-MAGNETIC DUALITY SYMMETRY

- SO(2) invariance of free Maxwell equations:

$$\begin{aligned} &\boldsymbol{\nabla}\times\mathbf{H}-\dot{\mathbf{E}}=0\,,\\ &\boldsymbol{\nabla}\times\mathbf{E}+\dot{\mathbf{H}}=0,\\ &\nabla\cdot\mathbf{H}=0\,,\\ &\boldsymbol{\nabla}\cdot\mathbf{E}=0 \end{aligned}$$

inv. wrt

$$\begin{aligned} \mathbf{E} &\to \mathbf{E}\cos\theta+\mathbf{H}\sin\theta\,,\\ \mathbf{H} &\to \mathbf{H}\cos\theta-\mathbf{E}\sin\theta \end{aligned}$$

- In the Lagrangian approach:

$$S_{\mathrm{EM}} = \frac{1}{2}\int d^4x\,(\mathbf{E}^2-\mathbf{H}^2)$$

breaks SO(2)?! No, since $S_{\mathrm{EM}} = S_{\mathrm{EM}}[\mathbf{A}^{\mathbf{T}}]$

where $\mathbf{E}=-\dot{\mathbf{A}}^{\mathrm{T}}\,,\quad \mathbf{H}=\boldsymbol{\nabla}\times\mathbf{A}^{\mathrm{T}}$.

$$S_{\mathrm{EM}}[\mathbf{A}^{\mathbf{T}}] = \frac{1}{2}\int d^4x\left\{\left(\dot{\mathbf{A}}^{\mathrm{T}}\right)^2-(\boldsymbol{\nabla}\times\mathbf{A}^{\mathrm{T}})^2\right\}$$

preserves SO(2):

inv. wrt $\delta\mathbf{A}^{\mathrm{T}} = -\theta\,\boldsymbol{\nabla}^{-2}\,\boldsymbol{\nabla}\times\dot{\mathbf{A}}^{\mathrm{T}}$ as $\mathcal{L}\to\mathcal{L}+df/dt$

AXION-PHOTON COUPLING

$$\mathcal{L}_{a\gamma} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \vec{E}\vec{H}$$

$$g_{a\gamma\gamma} = \frac{e^2}{8\pi^2 f_a} \cdot \left(\frac{E}{N} - 1.92\right)$$

KSVZ: $E/N = 5/3 - 44/3$

DFSZ: $E/N = 8/3$

- Equations of motion $\delta S[A_\mu] = 0$:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho - g_{a\gamma\gamma} \mathbf{H} \cdot \boldsymbol{\nabla} a \,,$$

$$\boldsymbol{\nabla} \cdot \mathbf{H} = 0 \,,$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t} \,,$$

$$\boldsymbol{\nabla} \times \mathbf{H} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} - g_{a\gamma\gamma} \left(\mathbf{E} \times \boldsymbol{\nabla} a - \frac{\partial a}{\partial t} \mathbf{H} \right) .$$

$\longrightarrow$ break SO(2)

- (Pseudo)scalar fields coupled to $F_{\mu\nu} F^{\mu\nu}$ or $F_{\mu\nu} \tilde{F}^{\mu\nu}$ break duality symmetry. Can they break it differently?

CHANGE OF A VIEWPOINT

- Due to the duality invariance of the free EM field, absolute directions in the electric-magnetic plane have no physical meaning $\longrightarrow$ one can think of the SM particles as “magnetic monopoles” of the dual potential.
- In such a dual picture, the EM field is derived from a dual four-potential:

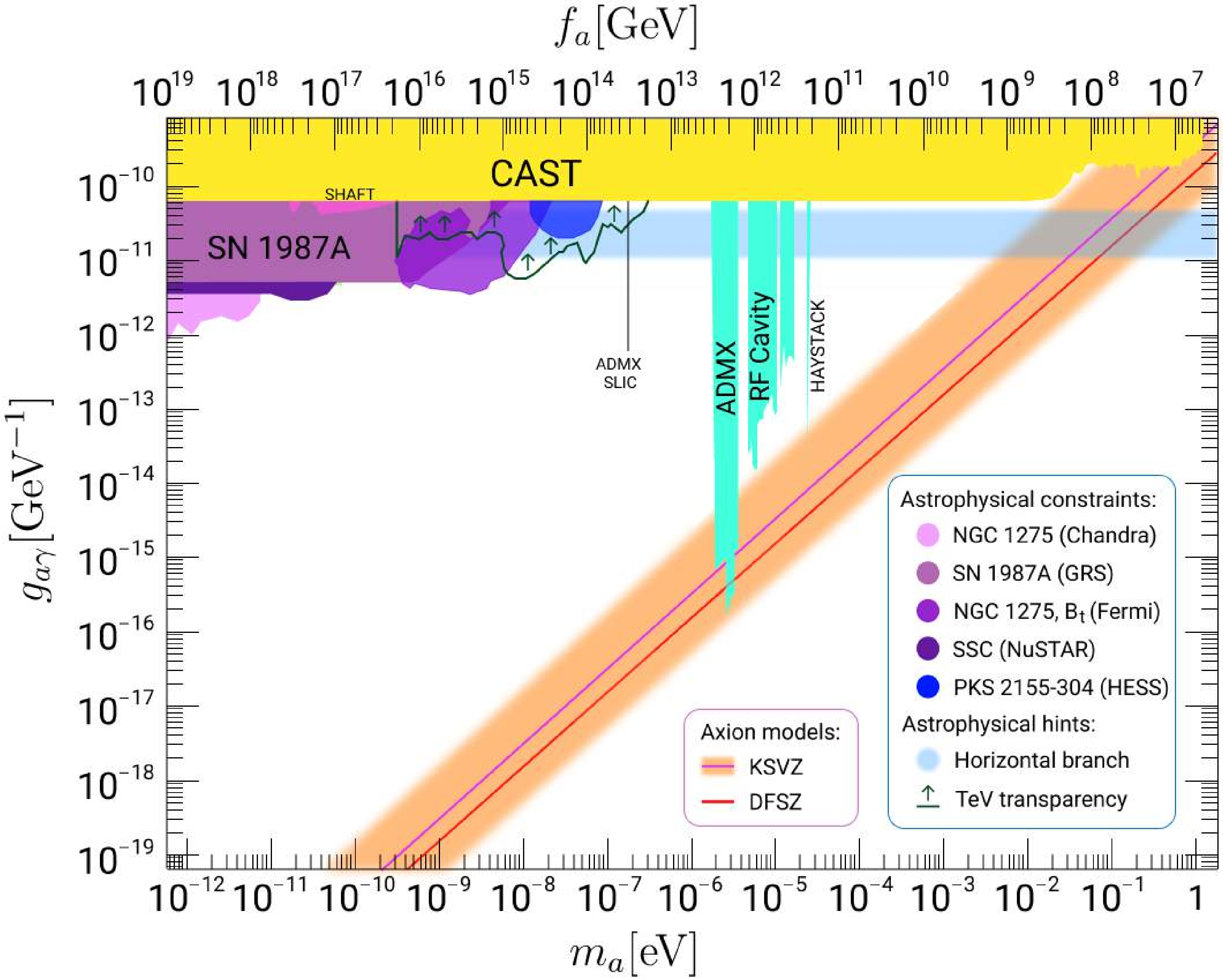
$$\mathbf{E} = -\boldsymbol{\nabla}\times\mathbf{B}\,, \quad \mathbf{H} = -\dot{\mathbf{B}} - \boldsymbol{\nabla} B_0$$

- Consider again $\mathcal{L}_{a\gamma} \;=\; -\frac{1}{4}\bar{g}_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu} \;=\; \bar{g}_{a\gamma\gamma} a \vec{E}\vec{H}$

- Equations of motion $\delta S[B_\mu] = 0$:

$$\boldsymbol{\nabla}\cdot\mathbf{E} = 0\,, \;\; \boldsymbol{\nabla}\cdot\mathbf{H} = \bar{g}_{a\gamma\gamma}\mathbf{E}\cdot\boldsymbol{\nabla} a\,, \;\; \boldsymbol{\nabla}\times\mathbf{E} = -\frac{\partial\mathbf{H}}{\partial t} + \bar{g}_{a\gamma\gamma}\left(\mathbf{H}\times\boldsymbol{\nabla} a + \frac{\partial a}{\partial t}\mathbf{E}\right), \;\; \boldsymbol{\nabla}\times\mathbf{H} = \frac{\partial\mathbf{E}}{\partial t}$$

AXION-PHOTON COUPLING: PARAMETER SPACE

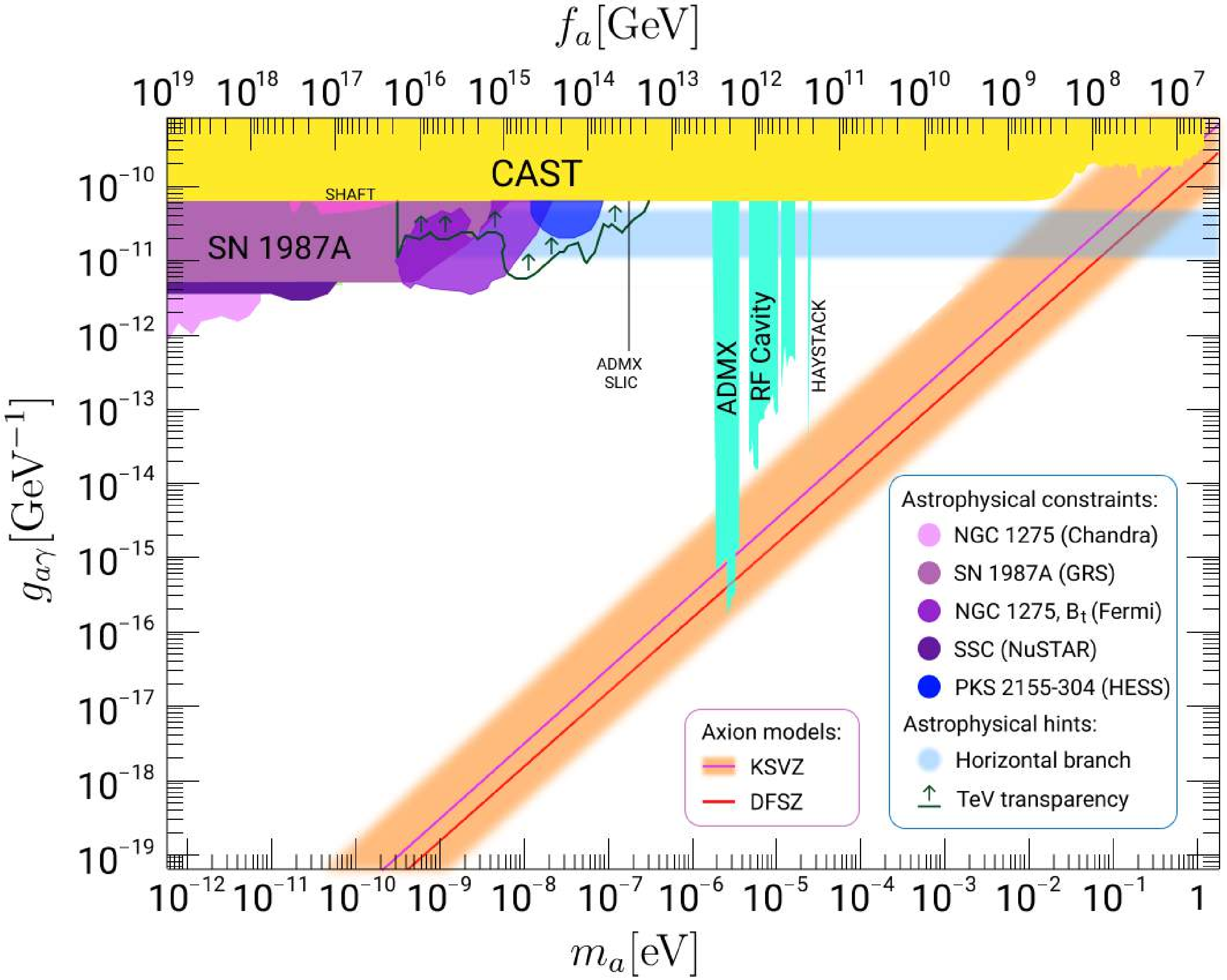


$$\mathcal{L}_{a\gamma} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$= g_{a\gamma\gamma} a \vec{E}\vec{H}$$

- axion-photon conversion in external magnetic field (astro, LSW, high-mass-axion haloscopes)
- Primakoff effect: axion production in stars (helioscopes, HB stars)
- extra axion-induced magnetic field component in external magnetic field (low-mass-axion haloscopes)

AXION-PHOTON COUPLING: THEORY



$$\mathcal{L}_{a\gamma} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$= g_{a\gamma\gamma} a \vec{E}\vec{H}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Is this the most general axion-photon Lagrangian consistent with the symmetries?

Axion shift symmetry:

$$a \to a + 2\pi n\, v_a\,,\; n \in \mathbb{Z}$$

Locality of $\mathcal{L}_{a\gamma}\,[A_\mu]$ selects one particular direction in the E-M plane

ZWANZIGER THEORY

- Adopt simplified notations: $(\partial \wedge A)_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \;\; , \quad (G)^d = \tilde{G}$
- Consider the Lagrangian by Zwanziger which makes the duality symmetry obvious:

$$\mathcal{L}_{\text{kin}} = \frac{1}{2n^2}\Big\{[n\cdot(\partial \wedge B)]\cdot[n\cdot(\partial \wedge A)^d] \; - \; [n\cdot(\partial \wedge A)]\cdot[n\cdot(\partial \wedge B)^d]$$

$$- \; [n\cdot(\partial \wedge A)]^2 \; - \; [n\cdot(\partial \wedge B)]^2\Big\}$$

- The electric-magnetic duality transformations are rotations in (A, B) plane of four-potentials:

$$n\cdot F = n\cdot(\partial \wedge A) \qquad \text{and} \qquad n\cdot F^d = n\cdot(\partial \wedge B)$$

- n^μ is a fixed four-vector, which does not enter physical observables

QUANTUM ELECTROMAGNETODYNAMICS

1971 ZWANZIGER

A_μ **and** $B_\mu \longleftrightarrow$ photon

$$\mathcal{L} = \mathcal{L}_{\text{kin}}\left(A_\mu, B_\mu, n_\mu\right) - j_e^\nu A_\nu - j_m^\nu B_\nu$$

1977 ZBN

$$Z\left(a, b, \cancel{n_\mu}\right) = \int \exp\left\{i\left(\mathcal{S}[\mathbf{A}_\mu, \mathbf{B}_\mu, \mathbf{n}_\mu, \chi, \bar{\chi}] + j_e a + j_m b\right)\right\} \times \mathcal{D}\mathbf{A}_\mu \, \mathcal{D}\mathbf{B}_\mu \, \mathcal{D}\chi \, \mathcal{D}\bar{\chi}$$

- TWO vector-potentials describe ONE particle - photon
- theory is Lorentz-invariant, kinetic part is dual-invariant
- theory is generally not CP-invariant

EQUATIONS OF MOTION

$$\mathcal{L} \;=\; \frac{1}{2n^2}\Big\{[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^d] \;-\; [n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d]$$

$$-\,[n\cdot(\partial\wedge A)]^2 \;-\; [n\cdot(\partial\wedge B)]^2\Big\} \;-\; j_e^\nu A_\nu \;-\; j_m^\nu B_\nu \;+\; \mathcal{L}_G$$

Gauge-fixing part: $\mathcal{L}_G = -\frac{1}{2n^2}\Big\{[\partial(n\cdot A)]^2 + [\partial(n\cdot B)]^2\Big\}$

EOMs:

$$\frac{n\cdot\partial}{n^2}\left(n\cdot\partial A^\mu \;-\; \partial^\mu n\cdot A \;-\; n^\mu\partial\cdot A \;-\; \epsilon^\mu{}_{\nu\rho\sigma}n^\nu\partial^\rho B^\sigma\right) \;=\; j_e^\mu \;,$$

$$\frac{n\cdot\partial}{n^2}\left(n\cdot\partial B^\mu \;-\; \partial^\mu n\cdot B \;-\; n^\mu\partial\cdot B \;-\; \epsilon^\mu{}_{\nu\rho\sigma}n^\nu\partial^\rho A^\sigma\right) \;=\; j_m^\mu \;.$$

Differential operator factorizes ⟶ effectively 1st order system!

Impose boundary conditions ⟶ $\partial_\mu F^{\mu\nu} = j_e^\nu\,,\;\; \partial_\mu F^{d\,\mu\nu} = j_m^\nu\,.$

GENERIC AXION-PHOTON EFT

$$(\partial \wedge A)_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \qquad (G)^d = \tilde{G}$$

All dimension-five operators consistent with the symmetries:

$\mathcal{L} \; = \; \mathcal{L}_{\text{kin}}\,(A\,,B\,,n\,)$	Kinetic part
$- \dfrac{1}{4}\, g_{aAA}\, a\, \mathrm{tr}\Big\{(\partial \wedge A)\,(\partial \wedge A)^d\Big\} \; - \dfrac{1}{4}\, g_{aBB}\, a\, \mathrm{tr}\Big\{(\partial \wedge B)\,(\partial \wedge B)^d\Big\}$	Anomalous axion-photon interactions, CP-conserving
$- \dfrac{1}{2}\, g_{aAB}\, a\, \mathrm{tr}\Big\{(\partial \wedge A)\,(\partial \wedge B)^d\Big\}$	Anomalous axion-photon interaction, CP-violating
$- \left(\bar{j}_e + \dfrac{e^2 a}{4\pi^2 v_a}\, j_m^\phi\right) \cdot (A - \partial\phi) - j_m \cdot B$	Witten effect induced axion-photon interaction, includes j_m^ϕ - current of 't Hooft-Polyakov monopoles

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- This Effective Field Theory is valid for any axion or axion-like particle.
- The Witten-effect induced interaction is generalized to the case of dynamical magnetic charges and is no longer given by $aF\cdot F^d$ term.
- Axions need not get extra mass in the theories with monopoles, although such mass can arise

WITTEN-EFFECT INDUCED INTERACTION

- Dyons cause CP-violation in QEMD, if there exists a dyon with charges (q,g), but no dyon with (-q,g) charges. In this case, the absolute charges of dyons allowed by the quantization condition introduce a CP-violating parameter θ into the theory:

$$q_i = \left(n_i^e + \frac{\theta}{2\pi} n_i^m\right) \cdot e \,, \quad n_i^e, n_i^m \in \mathbb{Z} \,. \qquad \longrightarrow \qquad \theta \in [0, 2\pi)$$

- The interaction part of the Zwanziger Lagrangian becomes:

$$\mathcal{L}_{int} \supset -\left(\bar{j}_e + \frac{e^2\theta}{4\pi^2}\, j_m\right) \cdot A \;-\; j_m \cdot B \,.$$

- Due to gauge invariance, θ cannot be promoted to a field, unless we introduce new degrees of freedom into the theory.
- ‘t Hooft-Polyakov magnetic monopoles carry an instanton degree of freedom ϕ , which allows one to introduce a variable θ in agreement with the high-energy phase of the theory:

$$\mathcal{L}_{int} \supset \; -\left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a}\, j_m^{\phi}\right) \cdot (A - \partial\phi) - j_m \cdot B$$

- Automatically account for the Rubakov-Callan effect due to the $\bar{j}_e \cdot \partial\phi$ term.

AXION MAXWELL EQUATIONS

Classical equations of motion corresponding to our Lagrangian are the axion Maxwell equations:

$$
\begin{aligned}
&\partial_\mu F^{\mu\nu} - g_{a\mathrm{AA}}\,\partial_\mu a\,F^{d\,\mu\nu} + g_{a\mathrm{AB}}\,\partial_\mu a\,F^{\mu\nu} - \frac{e^2 a}{4\pi^2 v_a}\,j_m^{\phi\,\nu} \;=\; \bar{j}_e^{\,\nu}\;,\\
&\partial_\mu F^{d\,\mu\nu} + g_{a\mathrm{BB}}\,\partial_\mu a\,F^{\mu\nu} - g_{a\mathrm{AB}}\,\partial_\mu a\,F^{d\,\mu\nu} \;=\; j_m^{\,\nu}\;,\\
&\left(\partial^2 - m_a^2\right) a \;=\; -\frac{1}{4}\left(g_{a\mathrm{AA}} + g_{a\mathrm{BB}}\right) F_{\mu\nu}F^{d\,\mu\nu} - \frac{1}{2}\,g_{a\mathrm{AB}}F_{\mu\nu}F^{\mu\nu}
\end{aligned}
$$

In the experimentally relevant case, in terms of electric and magnetic fields:

$$
\begin{aligned}
&\boldsymbol{\nabla}\times\mathbf{B}_a - \dot{\mathbf{E}}_a = g_{a\mathrm{AA}}\left(\mathbf{E}_0\times\boldsymbol{\nabla}a - \dot{a}\mathbf{B}_0\right) + g_{a\mathrm{AB}}\left(\mathbf{B}_0\times\boldsymbol{\nabla}a + \dot{a}\mathbf{E}_0\right),\\
&\boldsymbol{\nabla}\times\mathbf{E}_a + \dot{\mathbf{B}}_a = -g_{a\mathrm{BB}}\left(\mathbf{B}_0\times\boldsymbol{\nabla}a + \dot{a}\mathbf{E}_0\right) - g_{a\mathrm{AB}}\left(\mathbf{E}_0\times\boldsymbol{\nabla}a - \dot{a}\mathbf{B}_0\right),\\
&\boldsymbol{\nabla}\cdot\mathbf{B}_a = -g_{a\mathrm{BB}}\,\mathbf{E}_0\cdot\boldsymbol{\nabla}a + g_{a\mathrm{AB}}\,\mathbf{B}_0\cdot\boldsymbol{\nabla}a\,,\\
&\boldsymbol{\nabla}\cdot\mathbf{E}_a = g_{a\mathrm{AA}}\,\mathbf{B}_0\cdot\boldsymbol{\nabla}a - g_{a\mathrm{AB}}\,\mathbf{E}_0\cdot\boldsymbol{\nabla}a\,,\\
&\left(\partial^2 - m_a^2\right) a \;=\; \left(g_{a\mathrm{AA}} + g_{a\mathrm{BB}}\right)\mathbf{E}_0\cdot\mathbf{B}_0 + g_{a\mathrm{AB}}\left(\mathbf{E}_0^2 - \mathbf{B}_0^2\right),
\end{aligned}
$$

where we separated external fields sustained in the detector and axion-induced fields.

THEORETICAL MOTIVATION FOR NEW COUPLINGS

- Let us construct the UV-complete axion models which yield the new electromagnetic couplings g_{aBB} and g_{aAB} .
- We will see that such models are theoretically well-motivated due to the quantization of the electric charge observed in nature.
- We will calculate the couplings g_{aBB} and g_{aAB} in terms of the parameters of the models and see that they dominate the conventional g_{aAA} coupling.
- We will infer phenomenological consequences of the new couplings.

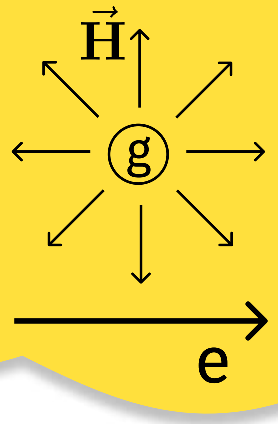
MAGNETIC MONOPOLES

- Quantization of charge

ⓤ $+\frac{2}{3}$ ⓓ $-\frac{1}{3}$ ⓔ -1 ?!

- Explained if there exist magnetic monopoles

1931 DIRAC



QM:

$$eg = 2\pi n,\ n \in \mathbb{Z}$$

Arise naturally
in Grand Unified theories

[‘t Hooft, Polyakov '74]

↓

Unification of forces

Arise in any consistent quantum gravity theory with quantized charges

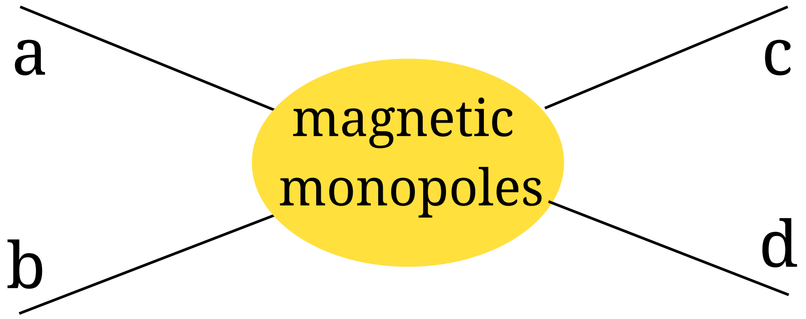
[Banks, Seiberg '11]

INDIRECT EFFECTS OF MONOPOLES

- Polchinski: “existence of magnetic monopoles seems like one of the safest bets that one can make about physics not yet seen” [Polchinski '02]

- But: the mass is not known and many models predict superheavy monopoles

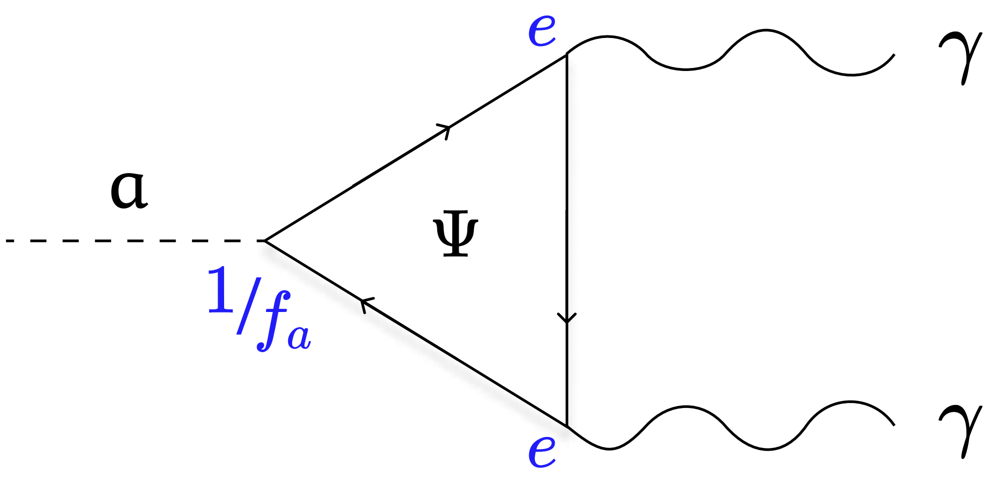
- Solution: look for the indirect effects of monopoles



- What about axion-photon interactions?

AXION-PHOTON COUPLING — MODELS

$$g_{a\gamma\gamma} = C_{a\gamma\gamma} \cdot \frac{e^2}{8\pi^2 f_a}$$



- DFSZ-like models:
  Ψ is from Standard model

- KSVZ-like (hadronic) models:
  Ψ is a new heavy particle
  carrying charge Z

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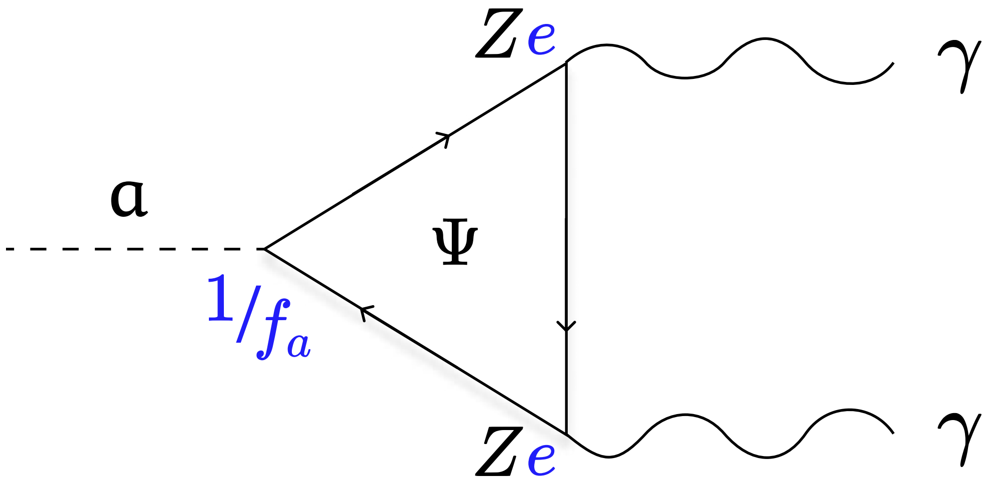

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Adler-Bardeen theorem $\Rightarrow$ no higher order corrections in e

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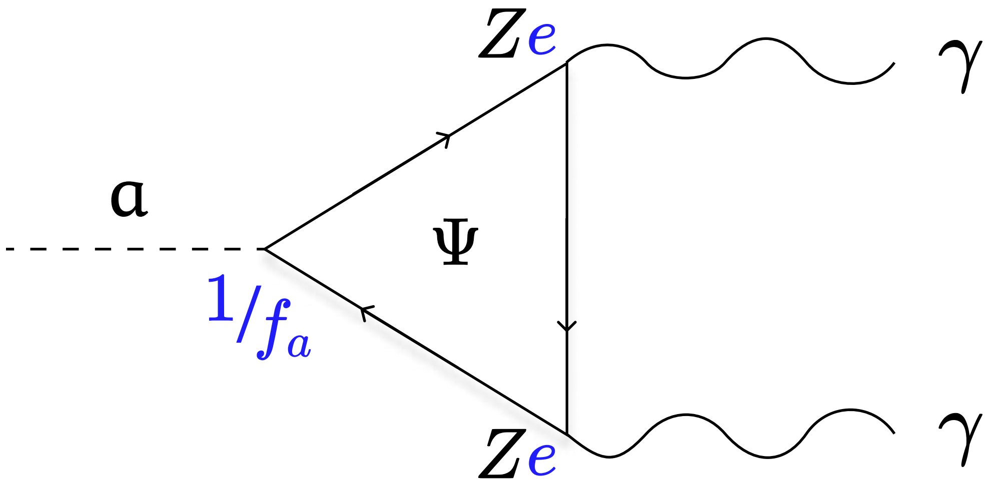

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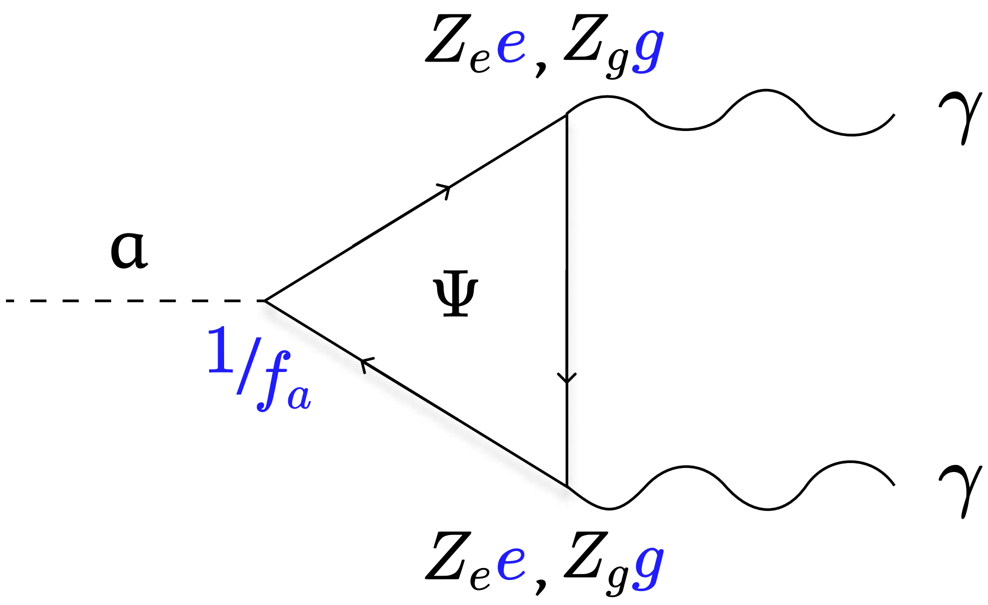
AXION-PHOTON COUPLING — GENERAL HADRONIC MODEL

$$g_{a\gamma\gamma} = C_{a\gamma\gamma} \cdot \frac{g^2}{8\pi^2 f_a}$$



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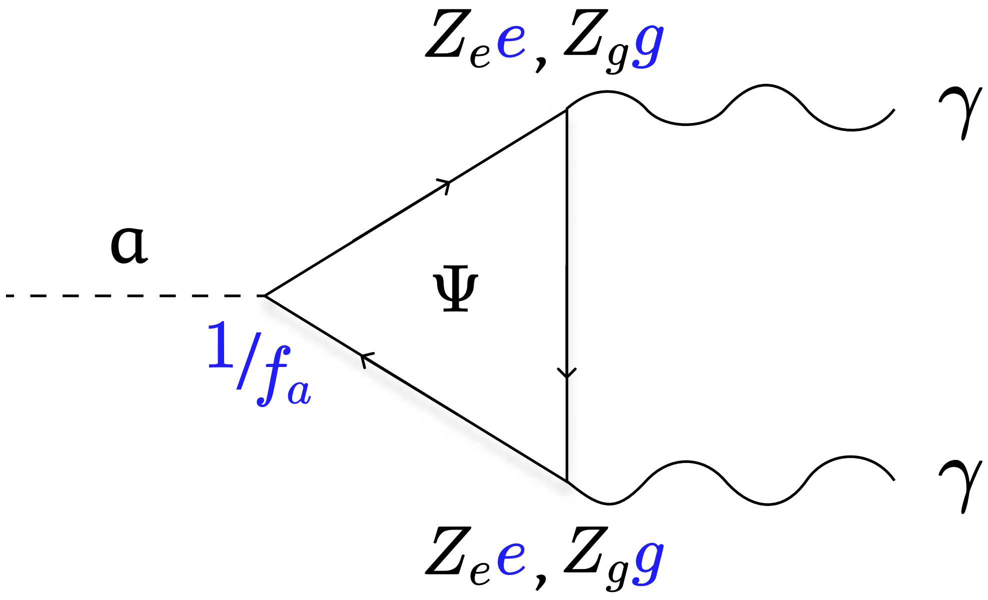

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  Ψ is a new heavy particle
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- $g = \frac{2\pi n}{e} \gg e$

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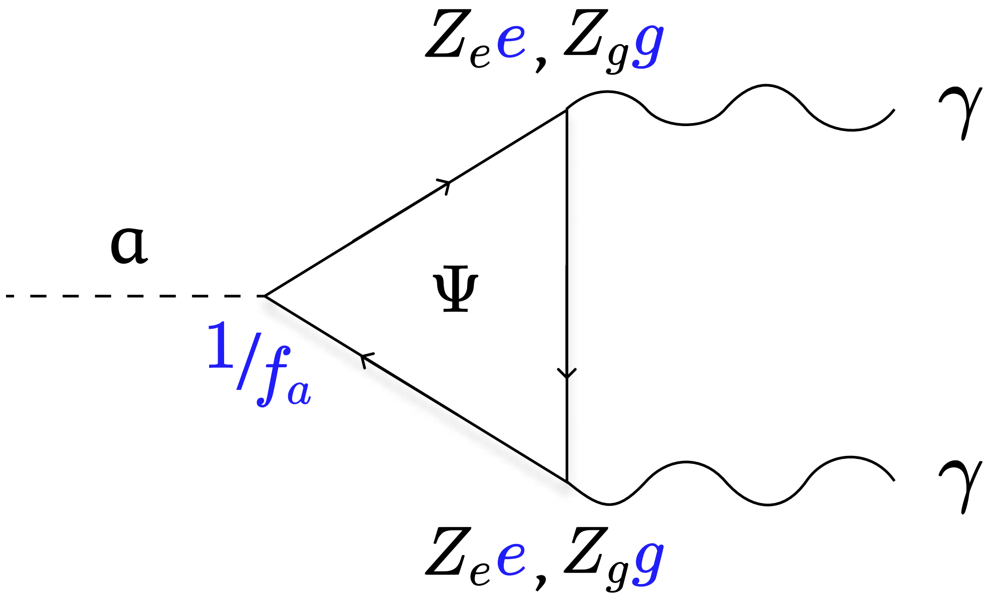

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Adler-Bardeen theorem ⇒ no higher order corrections in g

MAGNETIC ANOMALY COEFFICIENTS

- Magnetic couplings dominate low energy physics of hadronic axion

$$\mathcal{L}_{a\gamma\gamma} = -\tilde{g}_{a\gamma\gamma} a\vec{E}\vec{H}$$

$$\tilde{g}_{a\gamma\gamma} = \frac{M}{N} \cdot \frac{g^2}{8\pi^2 f_a}\,, \quad M = \sum_{\psi} M_\psi = \sum_{\psi} Z_g^2(\psi) \cdot d(C_\psi)$$

- M_ψ – magnetic anomaly coefficients
- $d(C_\psi)$ – dimension of the color representation of ψ

COMPARISON WITH KSVZ MODELS

- Consider a simple conventional hadronic model

  with one new heavy quark having $Z_e = 1/3$:

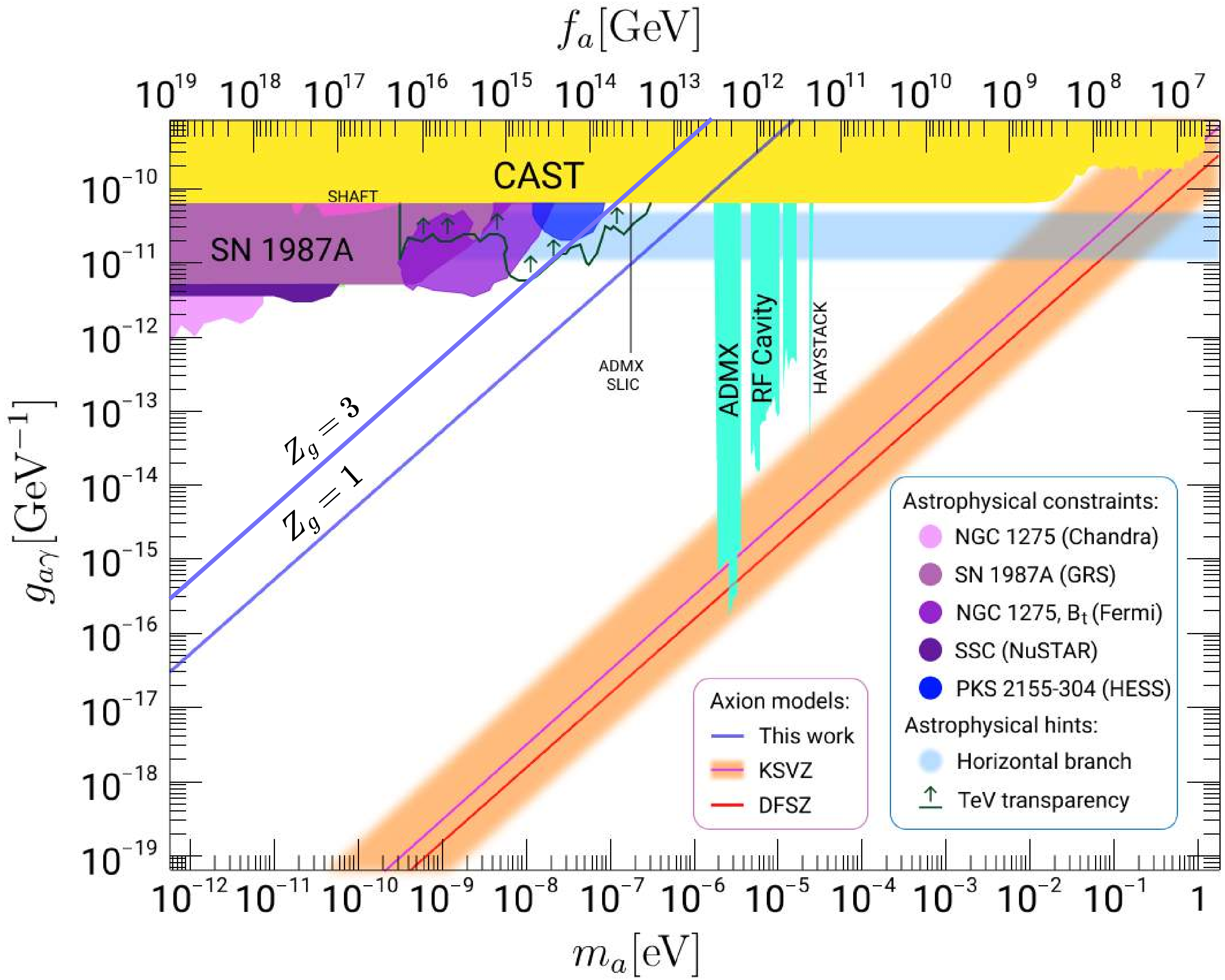
$$g_{a\gamma\gamma} = \frac{e^2}{8\pi^2 f_a} \cdot \left(\frac{E}{N} - 1.92 \right) = -\frac{e^2}{8\pi^2 f_a} \cdot 1.26$$

- Compare with the result of the general hadronic model:

$$\tilde{g}_{a\gamma\gamma} = \frac{g^2}{8\pi^2 f_a} \cdot \frac{M}{N} = -g_{a\gamma\gamma} \cdot \frac{g^2}{e^2} \cdot \frac{M/N}{1.26} = -g_{a\gamma\gamma} \cdot 2 \cdot 10^5 Z_g^2$$

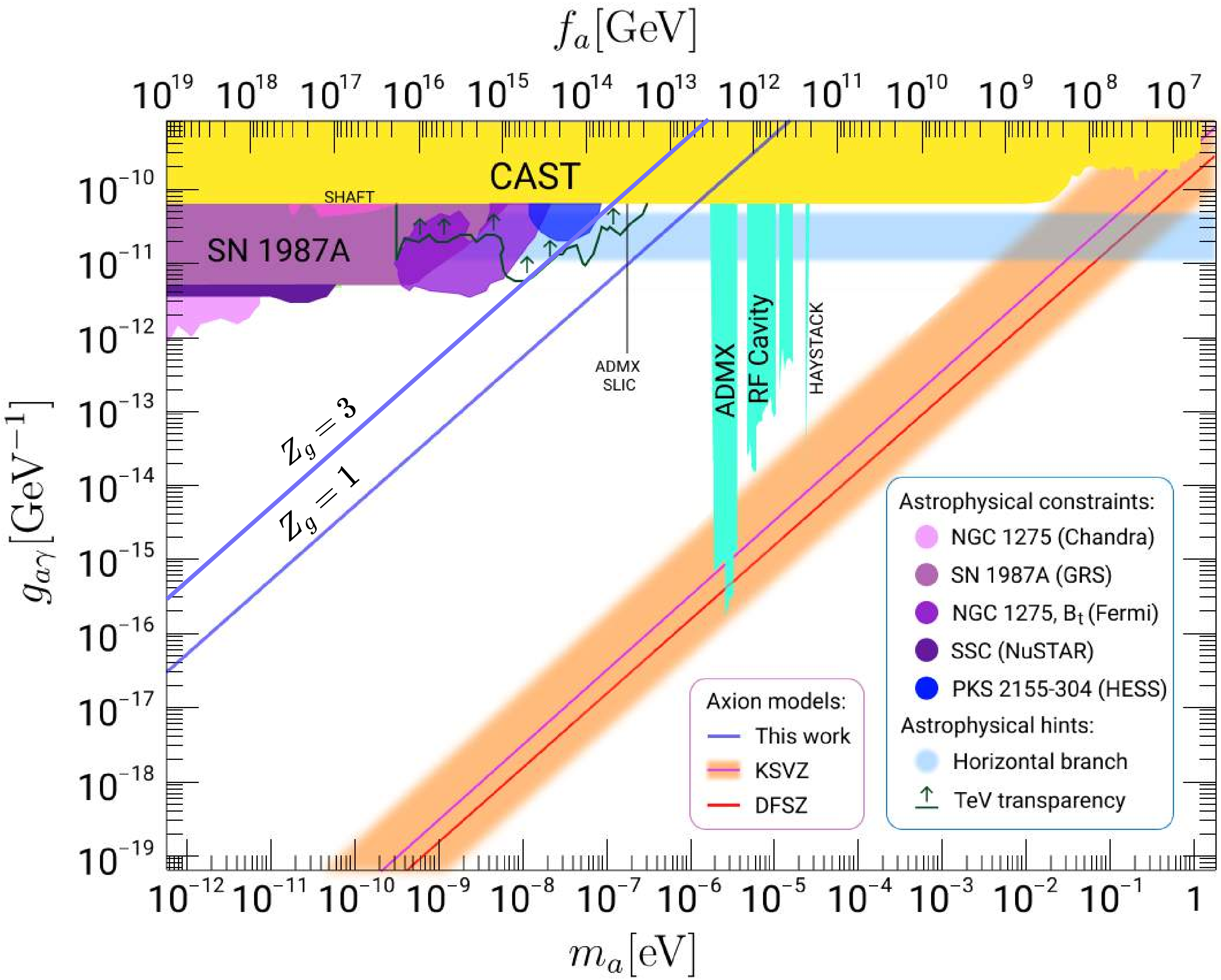
where we took into account $g = 6\pi/e$

COMPARISON PLOT



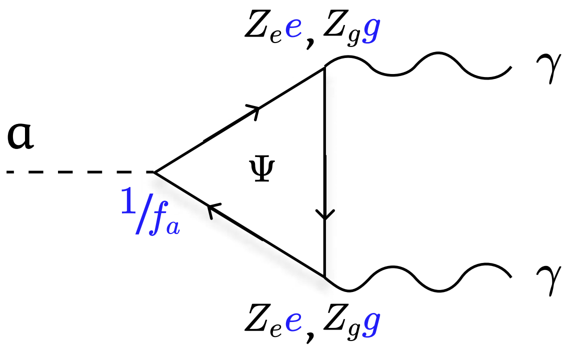
- Axion-photon coupling is hugely enhanced

COMPARISON PLOT



- Axion-photon coupling is hugely enhanced
- In the strong sector, the model is analogous to KSVZ ⇒
  - same CDM abundance
  - same EDM coupling

MIXED ELECTRIC-MAGNETIC ANOMALY COEFFICIENT



$$g_{aAB} = \frac{D}{N} \cdot \frac{eg}{8\pi^2 f_a} \,,$$

$$D = \sum_\psi D_\psi = \sum_\psi Z_e(\psi) Z_g(\psi) \cdot d(C_\psi) \,.$$

- D_ψ — magnetic anomaly coefficients
- $d(C_\psi)$ — dimension of the color representation of ψ

- CP violation is transfered from heavy dyons to axion-photon interactions

APPARENT CONTRADICTION?

- Previously in the literature, there has been a claim that the main contribution to $g_{a\gamma\gamma}$ is necessarily quantized in units $\propto e^2$.

[Agrawal et al. '17]

- The argument proceeds as follows:

1) $\mathcal{L} \supset \frac{a g_s^2}{32\pi^2 f_a} G^{a\,\mu\nu} G^{d\,a}_{\mu\nu}$ inv. wrt $a \to a + 2\pi n v_a\,,\ n \in \mathbb{Z} \quad \Rightarrow \quad f_a = v_a / N_{\rm DW}\,,\ N_{\rm DW} \in \mathbb{Z}$

2) $\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma}\, a\, F^{\mu\nu} F^{d}_{\mu\nu}$ inv. wrt $a \to a + 2\pi n v_a\,,\ n \in \mathbb{Z} \quad \Rightarrow \quad g_{a\gamma\gamma} = \frac{E}{N_{\rm DW}} \frac{e^2}{4\pi^2 f_a}\,,\ E \in \mathbb{Z}$

(note that $N_{\rm DW} = 2N$)

- The second line takes advantage of the Witten effect:

$\theta_{\rm em} \cdot \frac{e^2}{16\pi^2} \int d^4x\, F^{\mu\nu} F^{d}_{\mu\nu}$ is physically relevant due to monopoles/topology

NO CONTRADICTION!

- In the framework of QED, we have no choice but to identify the two structures

$$\theta_{\text{em}} \cdot \frac{e^2}{16\pi^2} \int d^4x \, \underline{F^{\mu\nu} F^d_{\mu\nu}} \qquad vs \qquad -\frac{1}{4}\, g_{a\gamma\gamma}\, a\, \underline{F^{\mu\nu} F^d_{\mu\nu}}$$

- Such formalism is justified if the magnetic monopoles are treated semi-classically
- A more general formalism of QEMD allows us to see the dynamical effects of monopoles and thus leads to new couplings which are not subject to the arguments from a semi-classical treatment
- In QEMD, the Witten-effect induced coupling is still quantized:

$$\mathcal{L} \supset -\left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a}\, j_m^\phi\right) \cdot (A - \partial\phi) \;, \qquad j_m^\phi \propto n_m g \,, \; n_m \in \mathbb{Z}$$

GENERIC AXION-PHOTON EFT

$(\partial \wedge A)_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $(G)^d = \tilde{G}$

All dimension-five operators consistent with the symmetries:

$\mathcal{L} = \mathcal{L}_{\text{kin}}(A, B, n)$	Kinetic part
$-\frac{1}{4} g_{aAA}\, a\, \mathrm{tr}\left\{(\partial \wedge A)(\partial \wedge A)^d\right\} - \frac{1}{4} g_{aBB}\, a\, \mathrm{tr}\left\{(\partial \wedge B)(\partial \wedge B)^d\right\}$	Anomalous axion-photon interactions, CP-conserving
$-\frac{1}{2} g_{aAB}\, a\, \mathrm{tr}\left\{(\partial \wedge A)(\partial \wedge B)^d\right\}$	Anomalous axion-photon interaction, CP-violating
$-\left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a} j_m^\phi\right) \cdot (A - \partial\phi) - j_m \cdot B$	Witten effect induced axion-photon interaction, includes j_m^ϕ - current of 't Hooft-Polyakov monopoles

This Effective Field Theory is valid for any axion or axion-like particle.

In each particular UV model, one can calculate the coefficients g_{aAA}, g_{aBB} and g_{aAB}.

General feature due to the quantization condition: $g_{aBB} \gg |g_{aAB}| \gg g_{aAA}$.

AXION MAXWELL EQUATIONS

Classical equations of motion corresponding to our Lagrangian are the axion Maxwell equations:

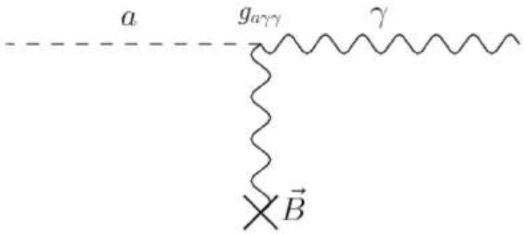
$$\begin{aligned} \partial_\mu F^{\mu\nu} - g_{a\mathrm{AA}}\, \partial_\mu a\, F^{d\,\mu\nu} + \underline{g_{a\mathrm{AB}}}\, \partial_\mu a\, F^{\mu\nu} - \frac{e^2 a}{4\pi^2 v_a} j_m^{\phi\,\nu} &= \bar{j}_e^{\,\nu} \; , \\ \partial_\mu F^{d\,\mu\nu} + \underline{\underline{g_{a\mathrm{BB}}}}\, \partial_\mu a\, F^{\mu\nu} - \underline{g_{a\mathrm{AB}}}\, \partial_\mu a\, F^{d\,\mu\nu} &= j_m^{\nu} \; , \\ (\partial^2 - m_a^2)\, a &= -\frac{1}{4}\, (g_{a\mathrm{AA}} + \underline{\underline{g_{a\mathrm{BB}}}})\, F_{\mu\nu} F^{d\,\mu\nu} - \frac{1}{2}\, \underline{g_{a\mathrm{AB}}} F_{\mu\nu} F^{\mu\nu} \end{aligned}$$

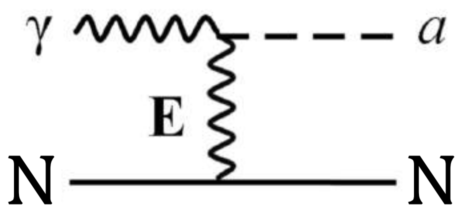
In the experimentally relevant case, in terms of electric and magnetic fields:

$$\begin{aligned} &\nabla \times \mathbf{B}_a - \dot{\mathbf{E}}_a = g_{a\mathrm{AA}}\, (\mathbf{E}_0 \times \nabla a - \dot{a}\mathbf{B}_0) + \underline{g_{a\mathrm{AB}}}\, (\mathbf{B}_0 \times \nabla a + \dot{a}\mathbf{E}_0) \; , \\ &\nabla \times \mathbf{E}_a + \dot{\mathbf{B}}_a = -\underline{\underline{g_{a\mathrm{BB}}}}\, (\mathbf{B}_0 \times \nabla a + \dot{a}\mathbf{E}_0) - \underline{g_{a\mathrm{AB}}}\, (\mathbf{E}_0 \times \nabla a - \dot{a}\mathbf{B}_0) \; , \\ &\nabla \cdot \mathbf{B}_a = -\underline{\underline{g_{a\mathrm{BB}}}}\, \mathbf{E}_0 \cdot \nabla a + \underline{g_{a\mathrm{AB}}}\, \mathbf{B}_0 \cdot \nabla a \, , \\ &\nabla \cdot \mathbf{E}_a = g_{a\mathrm{AA}}\, \mathbf{B}_0 \cdot \nabla a - \underline{g_{a\mathrm{AB}}}\, \mathbf{E}_0 \cdot \nabla a \, , \\ &(\partial^2 - m_a^2)\, a = (g_{a\mathrm{AA}} + \underline{\underline{g_{a\mathrm{BB}}}})\, \mathbf{E}_0 \cdot \mathbf{B}_0 + \underline{g_{a\mathrm{AB}}}\, (\mathbf{E}_0^2 - \mathbf{B}_0^2) \; , \end{aligned}$$

where we separated external fields sustained in the detector and axion-induced fields.

AXION-PHOTON CONVERSION IN EXTERNAL FIELD





Axion-photon conversion can be calculated using the axion equation of motion:

$$(\partial^2 - m_a^2)a = (g_{aAA} + g_{aBB})\,\mathbf{E}\cdot\mathbf{B} + g_{aAB}\,(\mathbf{E}^2 - \mathbf{B}^2)$$

- The two couplings g_{aAA} , g_{aBB} enter the equation symmetrically, which means that the conversion probability has the usual form, but $g_{aAA} \longrightarrow g_{aBB}$

HALOSCOPE EXPERIMENTS FOR LOW-MASS AXION DM

For axion DM detection, leaving only the dominant terms on the right-hand side, we obtain:

$$\nabla \times \mathbf{B}_a - \dot{\mathbf{E}}_a = 0\,,$$

$$\nabla \times \mathbf{E}_a + \dot{\mathbf{B}}_a = -g_{a\mathrm{BB}}\left(\mathbf{B}_0 \times \nabla a + \dot{a}\mathbf{E}_0\right) + g_{a\mathrm{AB}}\dot{a}\mathbf{B}_0\,,$$

$$\nabla \cdot \mathbf{B}_a = 0\,,$$

$$\nabla \cdot \mathbf{E}_a = 0\,,$$

$$\xrightarrow{m_a L \ll 1} \quad E_a \gg B_a$$

m_a - axion mass

L - detector size

This is to be contrasted with the conventional axion Maxwell equations used for axion DM detection:

$$\nabla \times \mathbf{B}_a - \dot{\mathbf{E}}_a = -g_{a\mathrm{AA}}\dot{a}\mathbf{B}_0,$$

$$\nabla \times \mathbf{E}_a + \dot{\mathbf{B}}_a = 0\,,$$

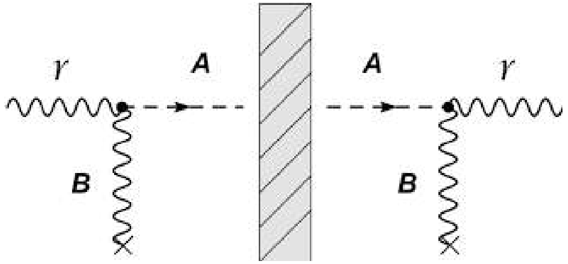
$$\nabla \cdot \mathbf{B}_a = 0\,,$$

$$\nabla \cdot \mathbf{E}_a = 0\,,$$

$$\xrightarrow{m_a L \ll 1} \quad B_a \gg E_a$$

The models with and without super heavy monopoles have completely different low energy phenomenology! One should aim to measure both electric and magnetic axion-induced fields.

LSW EXPERIMENTS



For LSW experiments, the effect can be calculated using the axion equation of motion:

$$(\partial^2 - m_a^2)a \;=\; (g_{aAA} + g_{aBB})\,\mathbf{E}\cdot\mathbf{B} + g_{aAB}\,(\mathbf{E}^2 - \mathbf{B}^2)$$

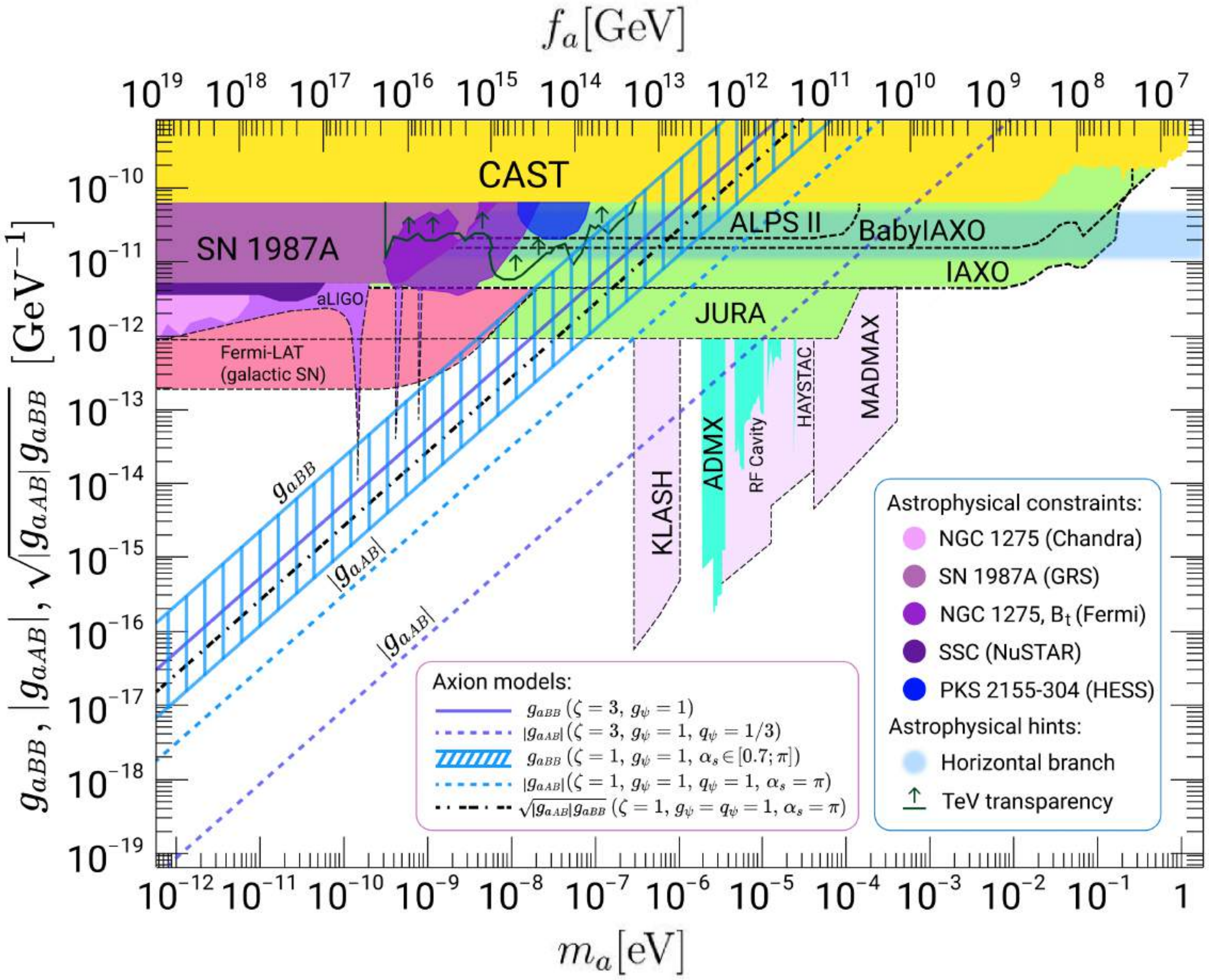
The effect depends on the polarization of the incoming light:

$$P(\gamma_{\parallel} \to a \to \gamma) \simeq 16\,\frac{(g_{a\mathrm{BB}}\omega B_0)^4}{m_a^8}\,\sin^4\!\left(\frac{m_a^2\,L_{B_0}}{4\omega}\right),$$

$$P(\gamma_{\perp} \to a \to \gamma) \simeq 16\,\frac{(g_{a\mathrm{AB}}\omega B_0)^2(g_{a\mathrm{BB}}\omega B_0)^2}{m_a^8}\,\sin^4\!\left(\frac{m_a^2\,L_{B_0}}{4\omega}\right)$$

This means that in the case of a signal detected in both channels, one can compare the theoretically derived ratio of CP-violating and CP-conserving couplings in a given model with the experiment.

PHENOMENOLOGY OF THE NEW COUPLINGS



- Axion DM could be easier to detect
- Haloscopes with electric sensors would be useful
- ALPS II is sensitive to the QCD axion

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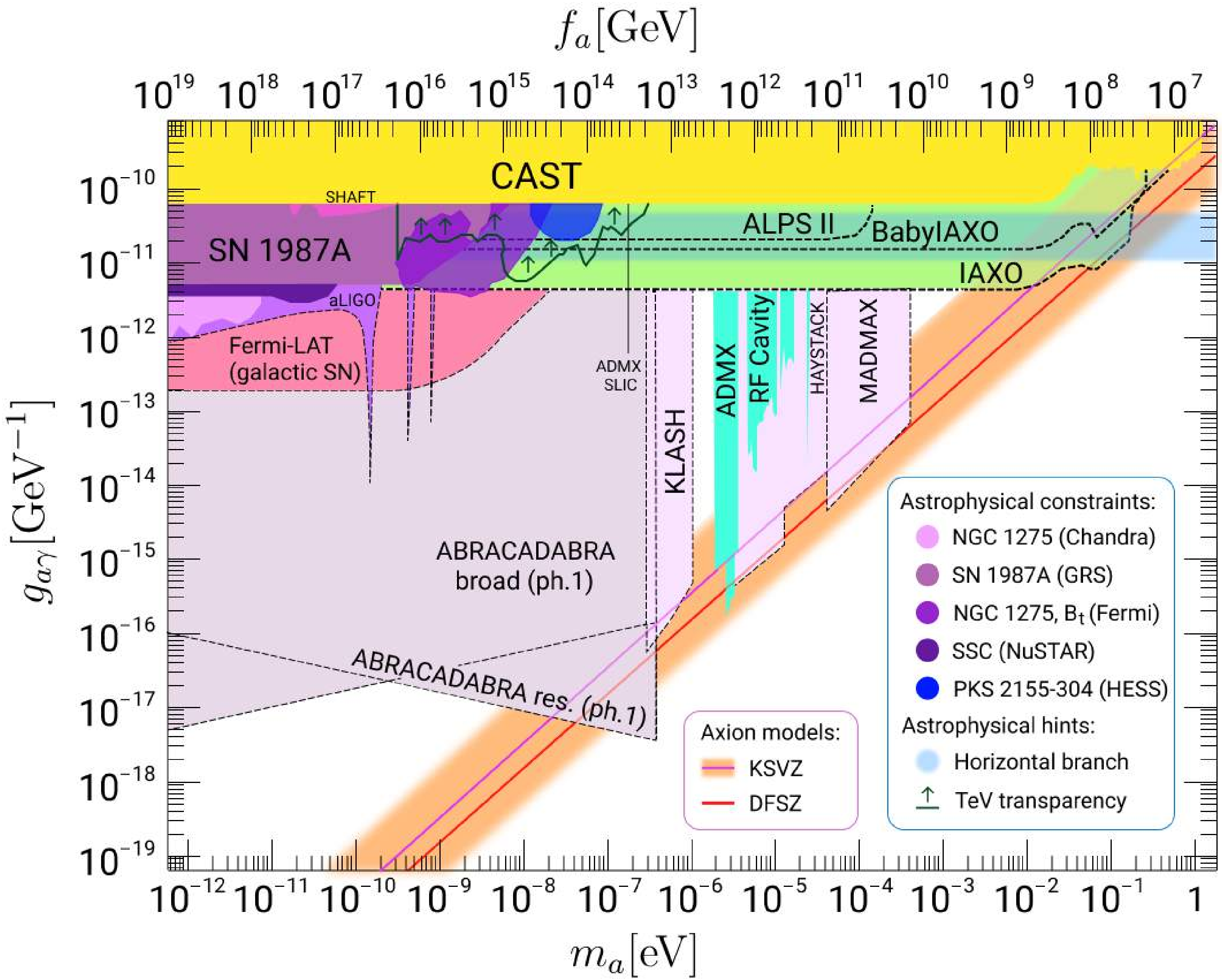

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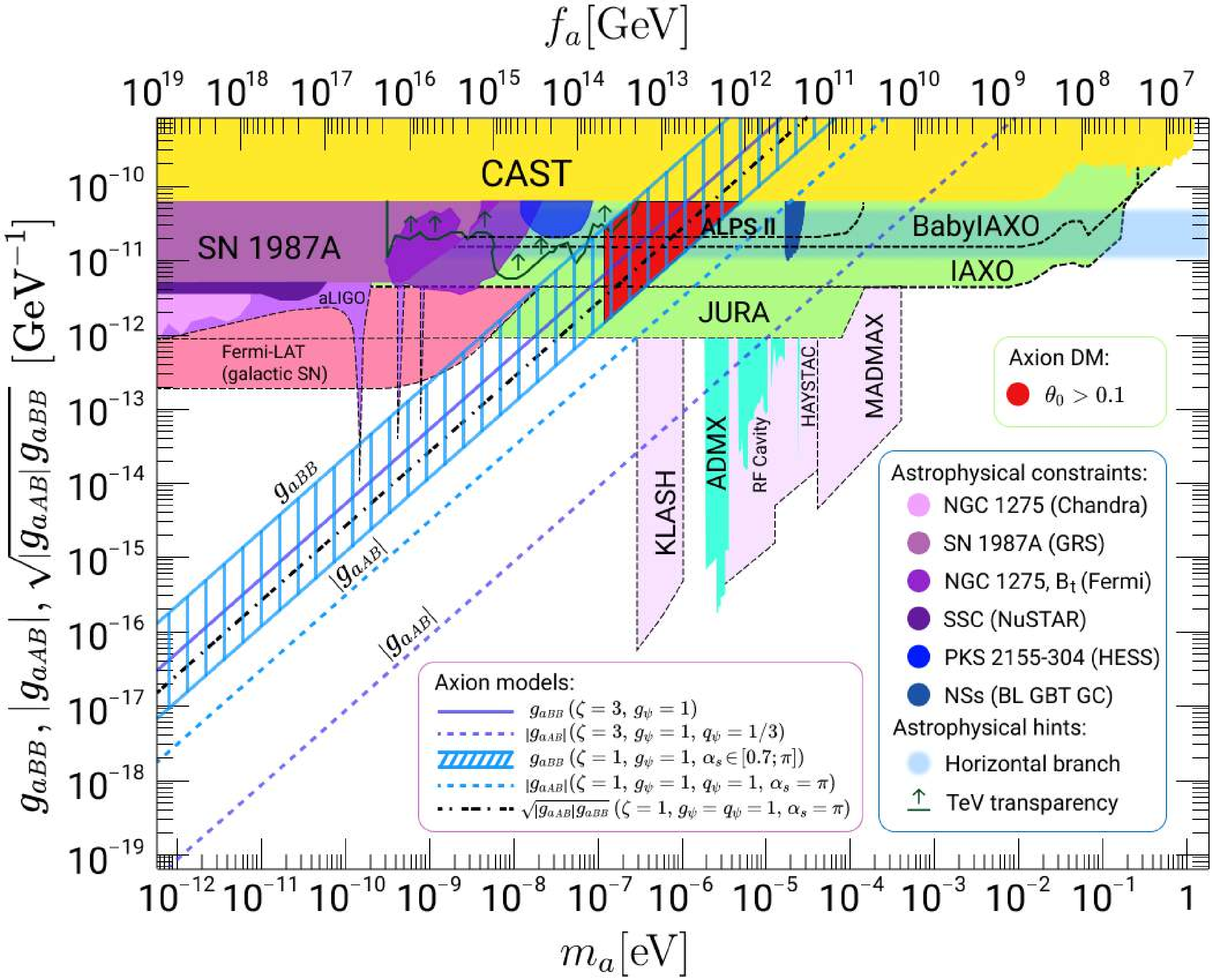

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AXION MODEL WITH NON-ABELIAN MONOPOLE

- $[U_M(1) \times SU_M(3)]$

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$\exp(4\pi i \beta_i T_i) = 1 \quad \Rightarrow \quad \beta_i$ lie in the weight lattice of G^V

magnetic charges

Cartan generators of G

Laglands dual of G

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GNO conjecture:

$$G_M = (G_E)^V$$

$$g_m = 2\pi/g$$

$(U(1))^V = U(1)$	$(SU(3)/\mathbb{Z}_3)^V = SU(3)$

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Due to the GNO conjecture we introduce: $C_\mu = g B_\mu + g_m B^a_\mu t^a$

Lagrangian of the PQ field Φ and fermion ψ is standard:

$$\mathcal{L} \supset i\bar{\psi}\gamma^\mu\partial_\mu\psi + \bar{\psi}\gamma^\mu C_\mu\psi + y\left(\Phi\,\bar{\psi}_L\psi_R + \mathrm{h.c.}\right) - \lambda_\Phi\left(|\Phi|^2 - \frac{v_a^2}{2}\right)^2$$

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Since monopole ψ has color-magnetic charge, the quantization condition allows for the minimal Dirac magnetic charge value:

$\min\{g\} = 2\pi/e$

EFFECTIVE LOW ENERGY LAGRANGIAN

● $[U_M(1) \times SU_M(3)]$

Low energy physics

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{4}\, g^{0}_{aBB}\, a\, F^{B}_{\mu\nu}\tilde{F}^{\mu\nu}_{B} - \frac{a g_s^2}{32\pi^2 f_a}\, G^{a}_{\mu\nu}\tilde{G}^{a\,\mu\nu} + \underbrace{\mathcal{L}_{\text{off}}}_{0 \text{ in IR}}$$

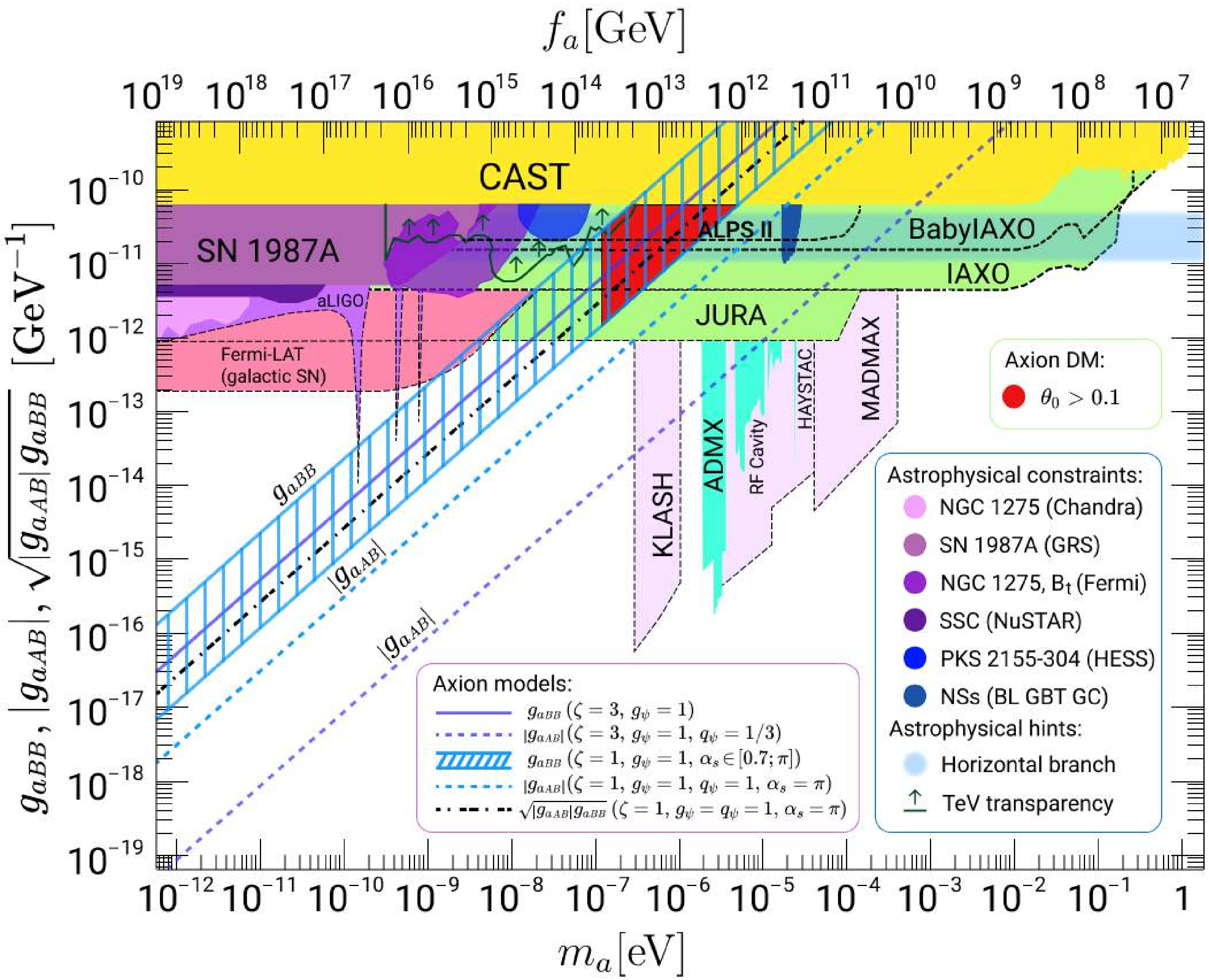
$$g^{0}_{aBB} = 3\alpha_s^2/\left(\pi\alpha f_a\right)$$

- Non-perturbative calculation
- $\mathscr{C}^{\alpha}_{\mu\nu} = \tilde{\mathscr{G}}^{\alpha}_{\mu\nu}$ for Abelian field strengths
- Abelian dominance of IR QCD

suggests that $\mathcal{L}_{\text{off}}$ is small

- Axion-photon coupling is special

PHENOMENOLOGY OF THE NEW COUPLINGS



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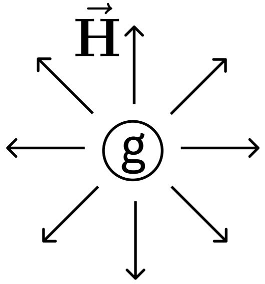
CONCLUSIONS

- Heavy magnetic monopoles can influence low energy physics through axion-photon couplings and thus can be indirectly probed in this way.
- New axion-photon couplings were found in the EFT approach and two classes of UV-complete models featuring these couplings were constructed.
- New axion-photon couplings give unique signatures in haloscopes searching for low-mass ALP dark matter and in some other experiments.
- Axion-photon interaction can violate CP.
- Low-mass axion dark matter could be detected earlier than previously thought.
- We clarified some issues within axion theory, such as axion mass from monopoles, Witten-effect induced axion interaction and quantization of the axion-photon coupling.



THANK YOU FOR YOUR ATTENTION!