Beyond Standard Model

SM
- quarks & leptons — u, d, ν, e — $p = uud$, $n = udd$
- gauge bosons $\begin{cases} \text{gluons} \\ W^\pm, Z \\ \text{photon } A/\gamma \end{cases}$
- Higgs boson — $H_e = p p \bar{e} \bar{e}$

~~SM~~
- ν mass $\rightarrow$ oscillation.
- Dark Matter 24% of the energy density of the universe
- Baryon Asymmetry of Universe
- New particles $\begin{cases} \text{gauge bosons} \\ \text{fermions} \\ \text{Higgses} \end{cases}$

$m_\nu \lesssim 0.1\,\text{eV} \ll m_e \ll m_t$ flavor problem

CP violation in strong/weak $\Rightarrow$ Axion.

Standard Model

Gauge Symmetry

Forces: strong — $SU(3)_c$ $G^{1,\dots,8}$

weak } ElectroWeak — $SU(2)_L \times U(1)_Y$
e.m. }

$W^{1,2,3}$ B

$T_{1,2,3}$ Y

$= \frac{1}{2}\sigma_{1,2,3}$.

$Q = T_3 + Y$.

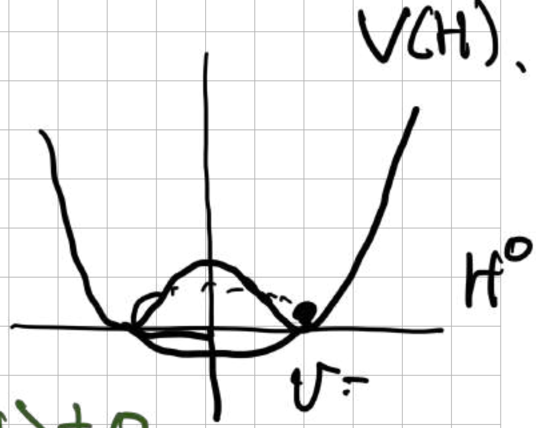
Higgs boson:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad T_3 = \begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$Y = \frac{1}{2}.$$

EW symmetry breaking

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$



$T_3 \langle H \rangle = -\frac{1}{2}\langle H \rangle$, $T_{1,2}\langle H \rangle \neq 0$

$Y \langle H \rangle = +\frac{1}{2}\langle H \rangle$

$Q \langle H \rangle = 0$.

↑ unbroken generator

$$SU(2)_L \times U(1)_Y \downarrow U(1)_Q .\ \text{em.}$$

$$V(H) = -\mu^2 |H|^2 + \frac{\lambda}{2}|H|^4$$
$$= \frac{\lambda}{2}\left(|H|^2 - \frac{\mu^2}{\lambda}\right)^2 - c.$$

$$H = \begin{pmatrix} G^+ \\ \frac{v+h+iG^0}{\sqrt{2}} \end{pmatrix} \qquad \begin{matrix} \to W^+ \\ \to Z^0 \end{matrix}$$

$$V(H) = \frac{1}{2}m_h^2 h^2 + \frac{1}{2}\frac{m_h^2}{v}h^3 + \frac{1}{8}\frac{m_h^2}{v^2}h^4$$

$$v^2 = \frac{2\mu^2}{\lambda} = (246.22\,\text{GeV})^2$$
$$m_h^2 = \lambda v^2 = (125\,\text{GeV})^2$$

EW gauge bosons

$$\text{gauge couplings}\begin{cases} g_2 & SU(2)_L \\ g_1 & U(1)_Y \end{cases}$$
$$\to e \quad U(1)_Q$$

weak mixing angle θ_W:

$$\tan\theta_W = \frac{g_1}{g_2}$$

$$\left[g_2 \left(W_\mu^1 \frac{\sigma_1}{2} + W_\mu^2 \frac{\sigma_2}{2} + W_\mu^3 \frac{\sigma_3}{2} \right) + g_1 B_\mu Y \right] \cdot H$$

$$2 \qquad \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \left[\frac{g_2}{2} \begin{pmatrix} 0 & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} g_2 W_\mu^3 + g_1 B_\mu & 0 \\ 0 & -g_2 W_\mu^3 + g_1 B_\mu \end{pmatrix} \right] H$$

$$= \left[\frac{g_2}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} + \begin{pmatrix} e A_\mu + \frac{g_Z}{2}(1 - 2 s_W^2) Z_\mu & 0 \\ 0 & -\frac{g_Z}{2} Z_\mu \end{pmatrix} \right] H$$

$$= \frac{v+h}{\sqrt{2}} \begin{pmatrix} \frac{g_2}{\sqrt{2}} W_\mu^+ \\ -\frac{g_Z}{2} Z_\mu \end{pmatrix} \quad \leftarrow H = \frac{v+h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|*|^2 = \left(1 + \frac{2h}{v} + \frac{h^2}{v^2} \right) \left(m_W^2 W_\mu^+ W^{\mu -} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right)$$

$$A_\mu = B_\mu c_W + W_\mu^3 s_W \qquad m_A = 0$$

$$Z_\mu = -B_\mu s_W + W_\mu^3 c_W \qquad m_Z = \frac{1}{2} g_Z v$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right) \qquad m_W = \frac{1}{2} g_2 v$$

$$g_Z = \sqrt{g_1^2 + g_2^2} = \frac{e}{s_W c_W}, \quad g_1 = \frac{e}{c_W}, \quad g_2 = \frac{e}{s_W}$$

$$\mathcal{L}_{hVV} = h\left(2\frac{m_W^2}{v} W_\mu^+ W^{\mu -} + \frac{m_Z^2}{v} Z_\mu Z^\mu\right)$$

Fermion content of SM

three generation.

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad u_R \quad d_R \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad e_R \quad (\nu_R)$$

$$\frac{1}{6} \qquad \frac{2}{3} \qquad -\frac{1}{3} \qquad -\frac{1}{2} \qquad -1 \qquad 0$$

Yukawa couplings

$$\mathcal{L}_{Yuk} = \bar{q}_L \tilde{H} Y_u u_R + \bar{q}_L H Y_d d_R + \bar{l}_L H Y_e e_R + h.c$$

3X3 complex.

$$\left(+ \bar{l}_L \tilde{H} Y_\nu \nu_R\right)$$

EWSB $\quad H = \frac{v+h}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \tilde{H} = \frac{v+h}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$= \frac{v}{\sqrt{2}}\left(1+\frac{h}{v}\right)\left(\bar{u}_L Y_u u_R + \bar{d}_L Y_d d_R + \bar{e}_L Y_e e_R + \left(\bar{\nu}_L Y_\nu \nu_R\right) + h.c\right)$$

Yukawa diagonalization

$$f_L \to L_f f_L \qquad f = u, d, e, \nu$$

$$f_R \to R_f f_R \qquad L_f, R_f : \text{unitary}.$$

$$Y_f \to L_f^\dagger Y_f R_f = \mathrm{Diag}[y_f].$$

$$y_f \frac{v}{\sqrt{2}} = m_f$$

$$\mathcal{L}_{Yuk} \Rightarrow \left(1 + \frac{h}{v}\right) \sum_f \left(m_f \bar{f}_L f_R + \text{h.c.}\right)$$

Vff couplings

$$\mathcal{L}_{Wff} = -\frac{g_2}{\sqrt{2}} W_\mu^+ \left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L\right) + \text{h.c.}$$

$$\Rightarrow -\frac{g_2}{\sqrt{2}} W_\mu^+ \Big(\bar{u}_L \gamma^\mu \underbrace{L_u^\dagger L_d}_{V_{CKM}} d_L + \bar{\nu}_L \gamma^\mu \underbrace{L_\nu^\dagger L_e}_{(V_{MNS})} e_L$$

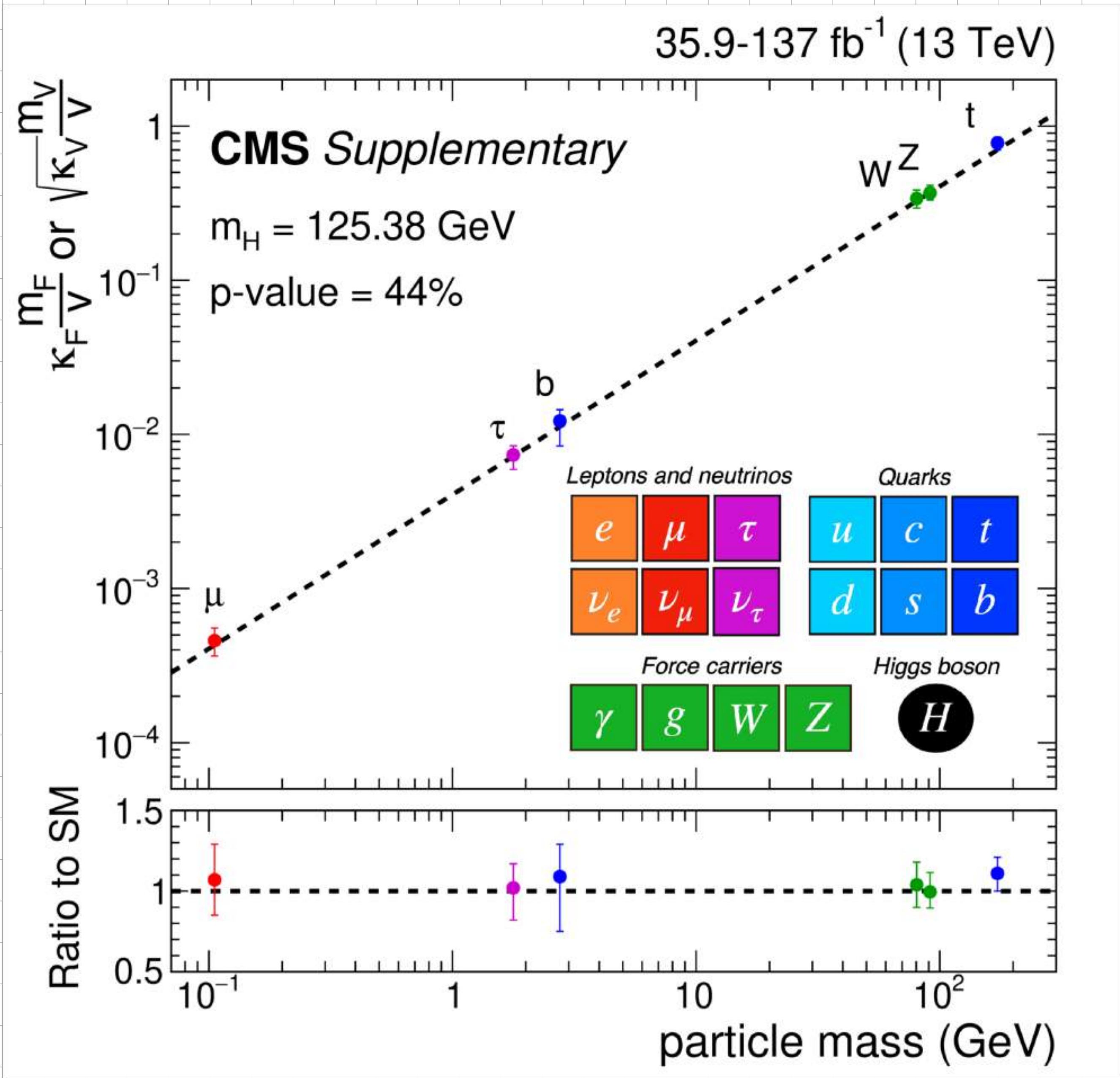
$$+ \text{h.c.}\Big)$$

$$\mathcal{L}_{Zff} = -\frac{g_Z}{2} Z_\mu \sum_f \bar{f} \gamma^\mu (g_V^f - \gamma_5 g_A^f) f$$

$\Rightarrow$ "same" "No flavor change"

$$g_V^f = T_3^f - 2 s_W^2 Q^f, \quad g_A^f = T_3^f.$$

LHC probes Higgs couplings



Fermion Masses & mixing

m_ν	m_e	$m_{u,d}$	$\cdots\cdots$	m_t
$0.1\,\mathrm{eV}$	$0.5\,\mathrm{MeV}$	$\sim 3\,\mathrm{MeV}$		$173\,\mathrm{GeV}$

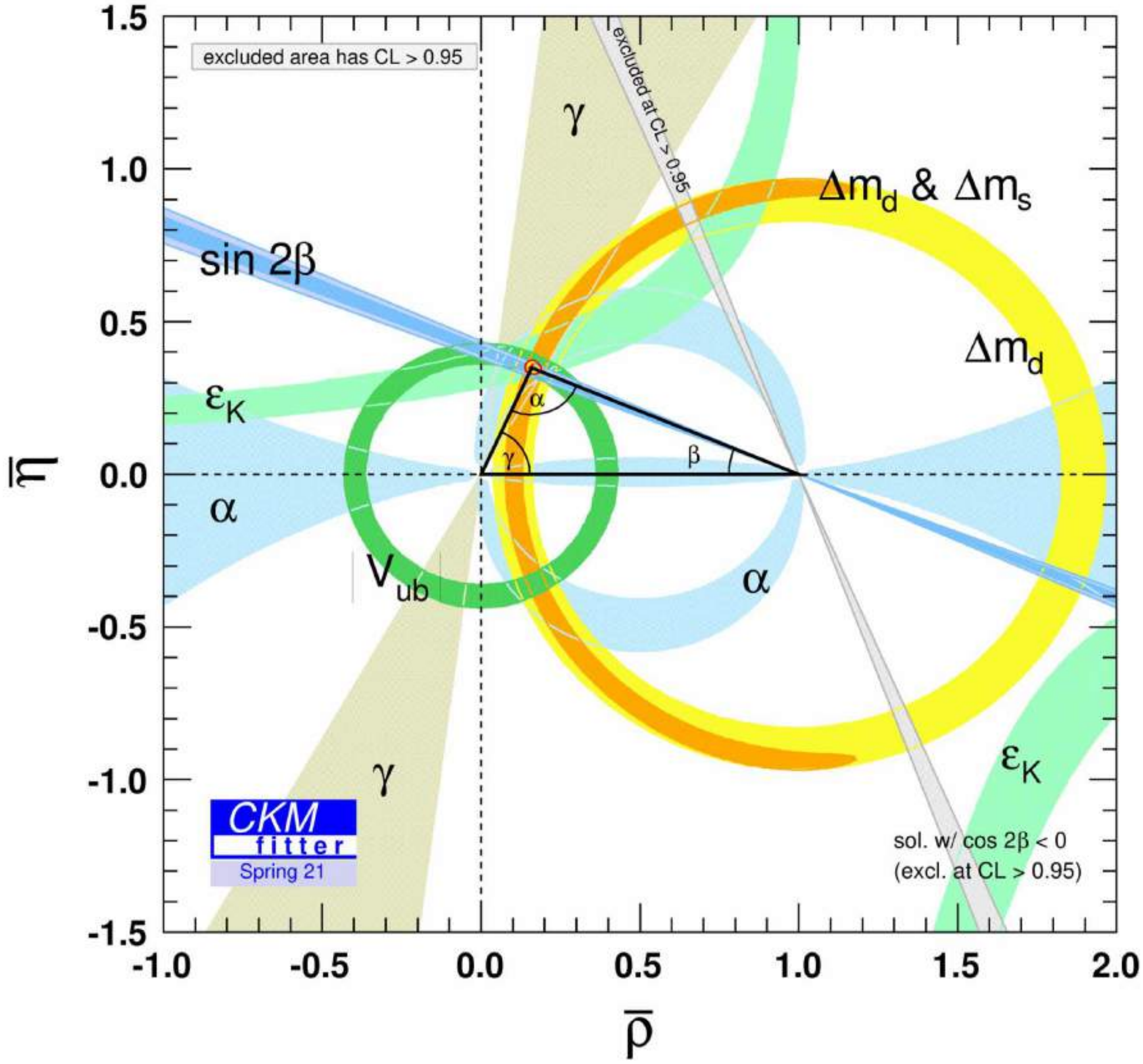
10^6 $\qquad\qquad$ 10^6

$$V_{CKM} \simeq \begin{array}{c c} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & \begin{bmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} \end{array}$$

$\lambda = 0.2257$, $A = 0.814$, $\rho = 0.135$,
$\eta = 0.349$

$$V_{CKM}^{\dagger} V_{CKM} = 1.$$

CKMology



No loopholes!

MNSology

$V_{MNS} =$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

↑ atmospheric neutrino oscillation ↑ reactor ↑ solar ↑ [Majorana phase]

Neutrino Mixing

The following values are obtained through data analyses based on the 3-neutrino mixing scheme described in the review "Neutrino Masses, Mixing, and Oscillations."

$\sin^2(\theta_{12}) = 0.307 \pm 0.013$
$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2$
$\sin^2(\theta_{23}) = 0.534^{+0.021}_{-0.024}$ (Inverted order)
$\sin^2(\theta_{23}) = 0.547^{+0.018}_{-0.024}$ (Normal order)
$\Delta m^2_{32} = (-2.519 \pm 0.033) \times 10^{-3}\ \mathrm{eV}^2$ (Inverted order)
$\Delta m^2_{32} = (2.437 \pm 0.033) \times 10^{-3}\ \mathrm{eV}^2$ (Normal order)
$\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$
δ, CP violating phase $= 1.23 \pm 0.21\ \pi$ rad (S = 1.3)
$\langle \Delta m^2_{21} - \Delta \overline{m}^2_{21} \rangle < 1.1 \times 10^{-4}\ \mathrm{eV}^2$, CL = 99.7%
$\langle \Delta m^2_{32} - \Delta \overline{m}^2_{32} \rangle = (-0.12 \pm 0.25) \times 10^{-3}\ \mathrm{eV}^2$

$\Delta m^2_{ij} = m_i^2 - m_j^2$

NO: $m_3 > m_2 > m_1$

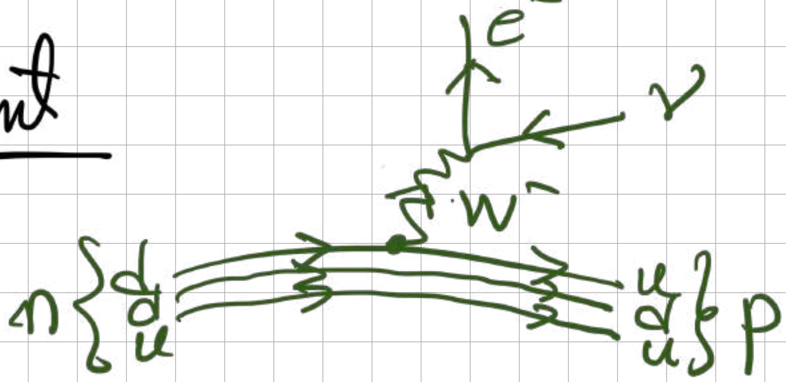
IO: $m_2 \sim m_1 > m_3$

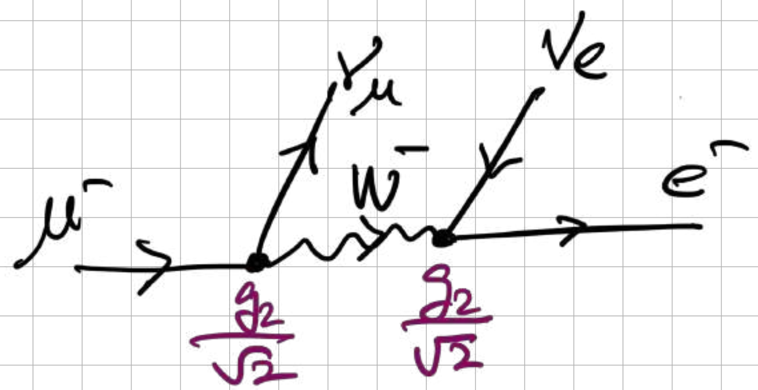
Electroweak Precision Test

Fermi theory of Weak Interaction 1933

$\sim G_F \bar{n} p \bar{e} \nu \qquad n \to p e^- \bar{\nu}$

Fermi constant





$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8 m_{W^\pm}^2} = \frac{1}{2v^2}$$

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$\boxed{G_F = 1.1663788(6) \times 10^{-5}\ \mathrm{GeV}^{-2}}$$

$v = 246.22$ GeV.

Electromagnetic coupling

$$\alpha = \frac{e^2}{4\pi} = 1/137.035999180(10)$$

$$\alpha(m_Z) = 1/127.951(9)$$

Z, W boson mass

$$m_Z = 91.1876 \pm 0.0021 \text{ GeV}$$

$$m_W = 80.377 \pm 0.012 \text{ GeV (world average)}$$

(*) CDF

Weak mixing angle

$$s_W^2(\ell) = \begin{cases} 0.2324 \pm 0.0012 \\ 0.23129 \pm 0.00033 \end{cases}$$

EW parameters

$$v, g_1, g_2 \leftrightarrow G_F, \alpha, m_Z$$

$$\begin{cases} e = 0.3184 \\ g_1 = 0.3580 \\ g_2 = 0.6485 \end{cases}$$

$$v = (\sqrt{2}G_F)^{-\frac{1}{2}} \approx 246 \text{ GeV}$$

$$e = (4\pi\alpha)^{1/2}, \quad g_1 = \frac{e}{c_W}, \quad g_2 = \frac{e}{s_W}.$$

$$m_Z = \frac{ev}{2s_W c_W} \Rightarrow s_W^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F m_Z^2}}\right)$$

SM prediction

Quantum correction

$$m_W = \frac{1}{2}g_2 v = 79.83 \Rightarrow 80.36$$

$$s_W^2 = 0.2336 \Rightarrow 0.23155$$

• ρ parameter

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} = 1.00039 \pm 0.00019$$

Higgs multiplets (T, Y)

Doublet $(\frac{1}{2}, \frac{1}{2})$ $(H^+, \underset{\sim}{H^0})$

Triplet $(1, 0)$ $(\Sigma^+, \underset{\sim}{\Sigma^0}, \Sigma^-)$ real

$(1, 1)$ $(\Delta^{++}, \Delta^+, \underset{\sim}{\Delta^0})$ complex.

Septuplet $(3, 2)$ $(5+, 4+, 3+, 2+, +, \underset{\sim}{0}, -)$

$$\rho = \frac{\sum_i c_i \left(T_i(T_i+1) - Y_i^2\right) v_i^2}{2 \sum_i Y_i^2 v_i^2}$$

$c_i = 1 \left(\frac{1}{2}\right)$ for complex (real).

1) $(\frac{1}{2}, \frac{1}{2})$ $$\rho = \frac{\sum_i \left(\frac{3}{4} - \frac{1}{4}\right) v_i^2}{2 \sum_i \frac{1}{4} v_i^2} = 1.$$

2) $(\frac{1}{2}, \frac{1}{2}) + (1, 1)$

$$\rho = \frac{\frac{1}{2} v_H^2 + v_\Delta^2}{\frac{1}{2} v_H^2 + 2 v_\Delta^2} \approx 1 - 2 \frac{v_\Delta^2}{v_H^2}$$

$$\Rightarrow v_\Delta \lesssim 10^{-2} v_H \sim \text{GeV}$$

3) $(\frac{1}{2},\frac{1}{2}) + (1,1) + (1,0)$ Georgi-Machacek

$$\rho = \frac{\frac{1}{2}v_H^2 + v_\Delta^2 + v_\Sigma^2}{\frac{1}{2}v_H^2 + 2v_\Delta^2} = 1$$

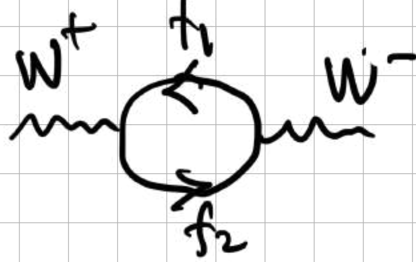
$$\uparrow$$

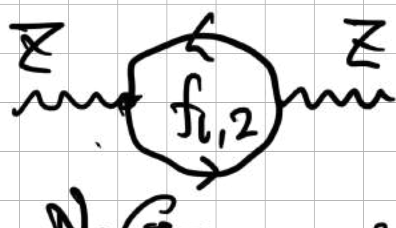
$$v_\Delta = v_\Sigma .$$

4) $(3,2)$ $\rho = \frac{(3\cdot 4 - 4)\, v_{(3,2)}^2}{2\cdot 4\, v_{(3,2)}^2} = 1 .$

Quantum correction to ρ.

ex) A doublet $\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$ * Any fermionic or bosonic multiplets.

$\delta m_W^2 =$ 

$\delta m_Z^2 =$ 

$$\Rightarrow \delta\rho = \frac{N_c G_F}{8\sqrt{2}\pi^2} F(m_1^2, m_2^2), \quad F = m_1^2 + m_2^2 - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln\frac{m_1^2}{m_2^2}$$

$\downarrow$ if $m_1 = m_2$

0

$$\geq (m_1 - m_2)^2$$

$$(16\,\mathrm{GeV})^2 \lesssim \sum_i \frac{N_c^i}{3} \Delta m_i^2 \lesssim (48\,\mathrm{GeV})^2$$

(숙제) $$\delta\rho_{SM} = \delta\rho_t + \delta\rho_{\phi} + \delta\rho_{W,Z}.$$

$$\delta\rho_t = \frac{3 G_F m_t^2}{8\sqrt{2}\pi^2} \approx 0.0084 \left(\frac{m_t}{174\,\mathrm{GeV}}\right)^2$$

$$\left(m_t = 173\,\mathrm{GeV} \gg m_b \approx 4\,\mathrm{GeV}\right)$$

Custodial Symmetry

EW gauge symmetry $\underbrace{SU(2)_L}_{\Updownarrow} \times U(1)_Y \to U(1)_Q$

global $SU(2)$ invariance.

Gauge sector

W^1, W^2, W^3, B

$$M^2 = \frac{v^2}{4}\begin{pmatrix} g_2^2 & 0 & 0 & 0 \\ 0 & g_2^2 & 0 & 0 \\ 0 & 0 & g_2^2 & -g_1 g_2 \\ 0 & 0 & -g_1 g_2 & g_1^2 \end{pmatrix}$$

$g_1 \to 0$, $s_W^2 \to 0$

$Z_\mu = W_\mu^3$

$m_{W^\pm}^2 = m_Z^2$

$SU(2)$-invariance $\begin{pmatrix} W_\mu^+ \\ Z_\mu \\ W_\mu^- \end{pmatrix}$ $T_3 = \begin{pmatrix} +1 \\ 0 \\ -1 \end{pmatrix}$

Higgs sector

$$\Phi \equiv \begin{pmatrix} H^{0*} & H^+ \\ -H^- & H^0 \end{pmatrix}$$

$$\frac{1}{2}\mathrm{Tr}(\Phi^\dagger \Phi) = |H|^2.$$

$$V(H) = -\frac{\mu^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\lambda}{8}\left[\mathrm{Tr}(\Phi^\dagger\Phi)\right]^2.$$

$SU(2)_L \times SU(2)_R$ invariant.

$$\Phi \to L \Phi R^\dagger$$

$$\langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$ invariant under $L = R$.

$SU(2)_L \times SU(2)_R \to \underline{SU(2)_V}$.

Yukawa sector

$$\mathcal{L}_F = y_u (\bar{u}_L, \bar{d}_L) \tilde{H} u_R + y_d (\bar{u}_L, \bar{d}_L) H d_R + \text{h.c.}$$

$$= y (\bar{u}_L, \bar{d}_L) \Phi \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \text{h.c.}$$

$y_u = y_d = y$ $SU(2)$ invariance

$$m_u = m_d = \frac{y v}{\sqrt{2}}.$$

$$\Downarrow$$

$$\delta \rho_{u,d} = 0.$$

Custodial symmetry is broken by g_1 and Yukawa in SM. but $\delta\rho_{SM}$ is well controlled.

Chirality vs. Helicity

Dirac spinor in chiral representation

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$P_{L,R} \equiv \frac{1 \mp \gamma_5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\not{p} = E\gamma^0 - \vec{p}\cdot\vec{\gamma} = \begin{pmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix}$$

u-spinor :
$$u = \begin{pmatrix} \sqrt{E - \vec{p}\cdot\vec{\sigma}}\,\xi \\ \sqrt{E + \vec{p}\cdot\vec{\sigma}}\,\xi \end{pmatrix} \qquad \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

v-spinor :
$$v = \begin{pmatrix} \sqrt{E - \vec{p}\cdot\vec{\sigma}}\,\eta \\ -\sqrt{E + \vec{p}\cdot\vec{\sigma}}\,\eta \end{pmatrix}$$

(숙제) $(\not{p} - m)u = 0$

$(\not{p} + m)v = 0$

※ $(E \mp \vec{p}\cdot\vec{\sigma})(E \pm \vec{p}\cdot\vec{\sigma}) = m^2$.

$$u_{\substack{L\\R}} = P_{\substack{L\\R}} u = \begin{pmatrix} \sqrt{E - \vec{p}\cdot\vec{\sigma}}\,\xi \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \sqrt{E + \vec{p}\cdot\vec{\sigma}}\,\xi \end{pmatrix}.$$

chirality: left/right-handed

Helicity operator $\mathcal{H} = \hat{p}\cdot\vec{S} = \hat{p}\cdot\frac{\vec{\sigma}}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

In the relativistic limit $E \approx p \gg m$,

take $\hat{p} = \hat{z}$, $\xi = \begin{pmatrix} \xi_\uparrow \\ \xi_\downarrow \end{pmatrix}$ $\quad \xi_{\substack{\uparrow\\\downarrow}} \quad S_z = \substack{\frac{1}{2}\\-\frac{1}{2}}$

Then $\mathcal{H} = \frac{\sigma_3}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\qquad E - \vec{p}\cdot\vec{\sigma} \approx E(1 - \sigma_3) \approx \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$u_L = \sqrt{2E}\begin{pmatrix} 0 \\ \xi_\downarrow \\ 0 \\ 0 \end{pmatrix}, \qquad u_R = \sqrt{2E}\begin{pmatrix} \xi_\uparrow \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathcal{H}\, u_L = \underset{\sim}{-\frac{1}{2}}\, u_L \qquad \mathcal{H}\, u_R = \underset{\sim}{+\frac{1}{2}}\, u_R$$

Thus, $u_{L,R}$ is the Helicity eigenstate with $\mathcal{H} = \mp\frac{1}{2} \leftarrow$ left-handed / right

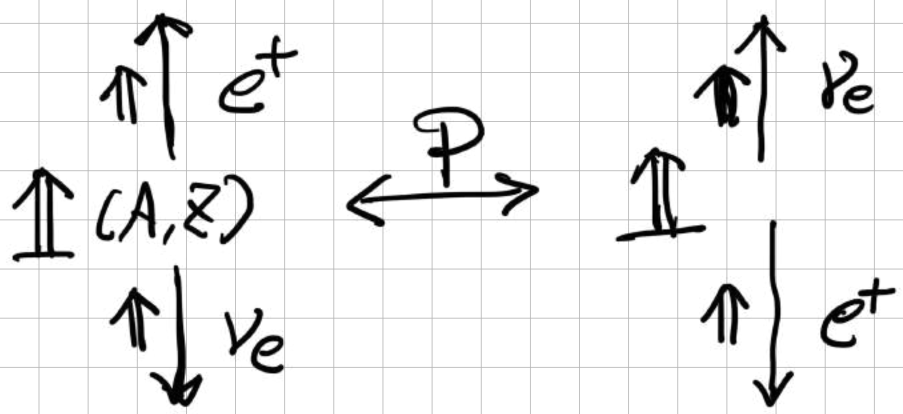
Massless neutrino or $m_\nu \ll E$

ν_L is $h = -\frac{1}{2}$ left-handed.

In β^+ decay

$$(A, Z) \to (A, Z-1) + e^+ + \nu_e$$

$$S_z = 1 \qquad 0 \qquad +\tfrac{1}{2} \qquad +\tfrac{1}{2}$$



Parity violation "Wu".

Charge conjugation

$$\psi \to \psi^c = i\gamma^2 \psi^*$$

$$i\gamma^2 = \begin{pmatrix} 0 & \varepsilon \\ -\varepsilon & 0 \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\varepsilon^2 = -1.$$

$$v = u^c = \begin{pmatrix} 0 & \varepsilon \\ -\varepsilon & 0 \end{pmatrix} \begin{pmatrix} \sqrt{E - \vec{p}\cdot\vec{\sigma}}\, \xi \\ \sqrt{E + \vec{p}\cdot\vec{\sigma}}\, \xi \end{pmatrix}^*$$

$$(*) \quad \sqrt{E \mp p\sigma_3} = \begin{pmatrix} \sqrt{E \mp p} & 0 \\ 0 & \sqrt{E \pm p} \end{pmatrix}$$

$$\varepsilon \sqrt{E \mp p\sigma_3}\, \varepsilon = -\sqrt{E \pm p\sigma_3}$$

$$u^c = \begin{pmatrix} \varepsilon \sqrt{E + p\sigma_3}\, \xi^* \\ -\varepsilon \sqrt{E - p\sigma_3}\, \xi^* \end{pmatrix} = \begin{pmatrix} \sqrt{E - p\sigma_3}\, \epsilon \xi^* \\ -\sqrt{E + p\sigma_3}\, \epsilon \xi^* \end{pmatrix}$$

$$= v \quad \text{with} \quad \eta = \epsilon \xi^* = \begin{pmatrix} \xi_\downarrow^* \\ -\xi_\uparrow^* \end{pmatrix}$$

Majorana fermion

Dirac mass: $\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$

$\psi = \begin{pmatrix} \tilde{\psi}_L \\ \tilde{\psi}_R \end{pmatrix}$, $\psi_L = \begin{pmatrix} \tilde{\psi}_L \\ 0 \end{pmatrix}$, $\psi_R = \begin{pmatrix} 0 \\ \tilde{\psi}_R \end{pmatrix}$ Two-component chiral/Weyl

$$\bar{\psi}_L \psi_R = (\tilde{\psi}_L^\dagger, 0) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \tilde{\psi}_R \end{pmatrix} = \tilde{\psi}_L^\dagger \tilde{\psi}_R .$$

Dirac mass: $\tilde{\psi}_L^\dagger \tilde{\psi}_R + \tilde{\psi}_R^\dagger \tilde{\psi}_L$

Majorana mass: $\overline{\psi_R^c} \psi_R + \bar{\psi}_R \psi_R^c$

or $\bar{\psi}_L \psi_L^c + \overline{\psi_L^c} \psi_L$.

$$\psi^c = \begin{pmatrix} 0 & \varepsilon \\ -\varepsilon & 0 \end{pmatrix} \begin{pmatrix} \tilde{\psi}_L^* \\ \tilde{\psi}_R^* \end{pmatrix} = \begin{pmatrix} \varepsilon \tilde{\psi}_R^* \\ -\varepsilon \tilde{\psi}_L^* \end{pmatrix}$$

$$\psi_R^c = (\psi_R)^c = \begin{pmatrix} \varepsilon \tilde{\psi}_R^* \\ 0 \end{pmatrix}, \quad \psi_L^c = (\psi_L)^c = \begin{pmatrix} 0 \\ -\varepsilon \tilde{\psi}_L^* \end{pmatrix}$$

$$\overline{\psi_R^c} \psi_R + \bar{\psi}_R \psi_R^c = \tilde{\psi}_R^T \varepsilon^T \psi_R + \tilde{\psi}_R^{*T} \varepsilon \psi_R^*$$

$$\bar{\psi}_L \psi_L^c + \overline{\psi_L^c} \psi_L = \tilde{\psi}_L^{*T} \varepsilon^T \tilde{\psi}_L^* + \underline{\tilde{\psi}_L \varepsilon \psi_L}$$

$\uparrow\downarrow - \downarrow\uparrow$

Define $\hat{\psi}_{L,R} \equiv \varepsilon \tilde{\psi}^*_{L,R}$, $\tilde{\psi}^*_{L,R} = -\varepsilon \hat{\psi}_{L,R}$.

Majorana mass: $\tilde{\psi}_R^T \varepsilon^T \hat{\psi}_R + \hat{\psi}_R^T \varepsilon \tilde{\psi}_R$

$\hat{\psi}_L^T \varepsilon^T \hat{\psi}_L + \hat{\psi}_L \varepsilon \tilde{\psi}_L$

Dirac fermion $\psi^c \neq \psi$

Majorana fermion $\psi^c = \psi$

$$\psi = \begin{pmatrix} \tilde{\psi}_L \\ -\varepsilon \tilde{\psi}_L \end{pmatrix} \text{ or } \psi = \begin{pmatrix} \varepsilon \tilde{\psi}_R \\ \tilde{\psi}_R \end{pmatrix}$$

Can electron get Majorana mass?

$U(1)_Q$ $\quad \psi \to e^{i\alpha Q} \psi$

$\psi^c \to e^{-i\alpha Q} \psi^c$

$\bar{\psi} \psi^c \to e^{-2i\alpha Q} \bar{\psi}\psi^c$

$\overline{\psi^c} \psi \to e^{2i\alpha Q} \overline{\psi^c}\psi$

breaks $U(1)_Q$.

"<u>Neutral</u> fermion can get <u>Majorana</u> mass"

Right Handed Neutrino ν_R

Dirac neutrino

$$m_\nu(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) \qquad m_\nu = y_\nu v/\sqrt{2}$$

$$y_\nu \sim \frac{m_\nu}{v} \sim \underset{\sim}{10^{-12}}!!$$

$$y_t \sim \frac{m_t}{v} \sim \underset{\sim}{1}$$

Majorana neutrino

Let us introduce a heavy mass for ν_R.
Majorana

$$\frac{1}{2}M_R(\overline{\nu_R^c}\nu_R + \overline{\nu_R}\nu_R^c)$$

$$= \frac{M_R}{2}(\underline{\tilde{\nu}_R^T(-\varepsilon)\tilde{\nu}_R} + \hat{\nu}_R^T\varepsilon\hat{\nu}_R)$$

Dirac mass terms (숙제)

$$m_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$$

$$= \frac{1}{2}m_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L + \overline{\nu_R^c}\nu_L^c + \overline{\nu_L^c}\nu_R^c)$$

$$\boxed{\tilde{\nu}^*_{L,R} \equiv \pm\varepsilon\hat{\nu}_{L,R}}$$

$$= \frac{1}{2} m_D \Big(\underline{-\hat{\nu}_L^T \varepsilon \tilde{\nu}_R} + \hat{\nu}_R^T \varepsilon \tilde{\nu}_L$$
$$\underline{-\tilde{\nu}_R^T \varepsilon \hat{\nu}_L} + \tilde{\nu}_L^T \varepsilon \hat{\nu}_R \Big)$$

Seesaw mass matrix

$$\begin{array}{c|cc} & \hat{\nu}_L & \tilde{\nu}_R \\ \hline \hat{\nu}_L^T & 0 & m_D \\ \tilde{\nu}_R^T & m_D & M_R \end{array} \xrightarrow{m_D \ll M} \begin{pmatrix} -\frac{m_D^2}{M_R} & 0 \\ 0 & M_R \end{pmatrix}$$

$$\hat{\nu}_L \to \hat{\nu}_L + \theta \tilde{\nu}_R, \quad \theta \approx \frac{m_D}{M_R} \ll 1.$$
$$\tilde{\nu}_R \to \tilde{\nu}_R - \theta \hat{\nu}_L$$

In the diagonalized mass basis,

$$-\frac{1}{2} m_\nu \hat{\nu}_L^T \varepsilon \hat{\nu}_L - \frac{M}{2} \tilde{\nu}_R^T \varepsilon \tilde{\nu}_R$$

$$m_\nu = -\frac{m_D^2}{M}.$$ Seesaw Mechanism.

$m_D \sim 10^2$ GeV, $M \sim 10^{14}$ GeV

$m_D \sim 0.5$ MeV, $M \sim 10^3$ GeV

"Who will order ν_R?"

Symmetries of SM

$SU(3)_c \times SU(2)_L \times U(1)_Y$

H $\quad q_L,\ u_R,\ d_R.\ l_L,\ e_R,\ \nu_R$ three copies of 6 fields

$\frac{1}{2}$ $\quad Y = \frac{1}{6} \quad \frac{2}{3} \quad -\frac{1}{3} \quad -\frac{1}{2} \quad -1 \quad 0$

Gauge invariant couplings. 4 relations

$\bar{q}_L \tilde{H} u_R,\ \bar{q}_L H d_R,\ \bar{l}_L H e_R,\ \bar{l}_L \tilde{H} \nu_R \qquad 6-4=\underline{\underline{2}}.$

Out of 6 arbitrary phase rotations $f \to e^{i\alpha_f} f$,

2 leave $\mathcal{L}_{SM}$ invariant

$f \to e^{i\alpha_1} f$ for $f = q_L, u_R, d_R$ quark #

$e^{i\alpha_2}$ $\qquad l_L, e_R, \nu_R$ lepton #.

2 conserved charges: quark number
lepton number

($p = uud$, $n = udd$, Baryon number conserved)

$q_L,\ u_R,\ d_R,\ l_L,\ e_R,\ \nu_R$ Accidental symmetries

$B = \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad 0 \quad 0 \quad 0$

$L = 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1$ $\qquad U(1)_B \times U(1)_L$

Anomalies

$U(1)_B$ current $\quad J^B_\mu = \bar{q}_L \gamma_\mu q_L + \bar{u}_R \gamma_\mu u_R + \bar{d}_R \gamma_\mu d_R$

$U(1)_L$ $\quad J^L_\mu = \bar{l}_L \gamma_\mu l_L + \bar{e}_R \gamma_\mu e_R + \bar{\nu}_R \gamma_\mu \nu_R .$

"Charge conservation $\partial^\mu J^{B,L}_\mu = 0$.

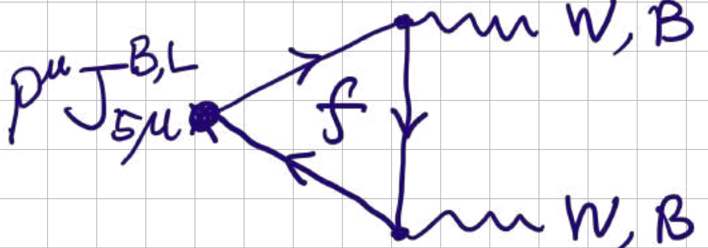
may not be guaranteed at quantum level."

ABJ anomaly.

Axial vector current is anomalous

$$p^\mu \langle \bar{f} \gamma_\mu \gamma_5 f \rangle_{Reg} = A \frac{g_a^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F^a_{\alpha\beta} F^a_{\mu\nu} .$$

(숙제)



$$J^B_{5\mu} = -\frac{1}{2} \bar{q}_L \gamma_\mu \gamma_5 q_L + \frac{1}{2} \bar{u}_R \gamma_\mu \gamma_5 u_R + \frac{1}{2} \bar{d}_R \gamma_\mu \gamma_5 d_R$$

$$J^L_{5\mu} = -\frac{1}{2} \bar{l}_L \gamma_\mu \gamma_5 l_L + \frac{1}{2} \bar{e}_R \gamma_\mu \gamma_5 e_R + \frac{1}{2} \bar{\nu}_R \gamma_\mu \gamma_5 \nu_R$$

$$p^\mu \langle J^{B,L}_{5\mu} \rangle = A_{B,L} \frac{g_a^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F^a_{\alpha\beta} F^a_{\mu\nu} .$$

$$A_B(SU(2)) = \frac{1}{3} \times N_c \times N_g = 3.$$

$$A_L(SU(2)) = 1 \times N_g = 3.$$

$$A_B(U(1)_Y) = \frac{1}{3} \times N_c \times N_g \times \underbrace{\left[\left(\frac{1}{6}\right)^2 \times 2 - \left(\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)^2\right]}_{-\frac{1}{2}}$$

$$A_L(U(1)_Y) = N_g \times \underbrace{\left[\left(\frac{1}{2}\right)^2 \times 2 - (-1)^2 - (0)^2\right]}_{-\frac{1}{2}}$$

$$\Rightarrow \begin{cases} A_{B-L} = 0 & \text{B-L is anomaly free} \\ A_{B+L} \neq 0 & \text{B+L is anomalous.} \end{cases}$$

B-L can be a gauge symmetry.

$$\begin{aligned} A_{B-L}(U(1)_{B-L}) &= N_c \times N_g \times \left[\underset{q_L}{\left(\frac{1}{3}\right)^3 \times 2} - \underset{u_R,\, d_R}{\left(\frac{1}{3}\right)^3 \times 2}\right] \\ &\quad + N_g \times \left[\underset{\ell_L}{(-1)^3 \times 2} - \underset{e_R,\, \nu_R}{(-1)^3 \times 2}\right] \\ &= 0! \end{aligned}$$

A few models beyond SM

* Origin of neutrino masses. ← related?
** Identity of DM particle(s). ←

• U(1)$_{B-L}$ gauge symmetry

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \quad \subset SO(10)$$

$$\underset{3\times 2}{q_L}, \underset{+3}{u_R}, \underset{+3}{d_R}, \underset{+2}{l_L}, \underset{+1}{e_R}, \underset{+1}{\nu_R} = \underline{\underline{16}}$$

spinor rep. of SO(10)

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{B-L}$$

$$\mathcal{L}_{B-L} = y_D \bar{l}_L \tilde{H} \nu_R + h.c. + \frac{y_N}{2} \Phi \left(\overline{\nu_R^c} \nu_R + \bar{\nu}_R \nu_R^c \right)$$

$$B-L = +1 \quad -1 \qquad\qquad +2 \quad -1 \; -1$$

Majorana mass of RHN

$$M_R = y_N \langle \Phi \rangle$$

- <u>Left-Right Symmetric model.</u>

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\qquad\qquad T_L^a \qquad\quad T_R^a \qquad B-L$$

$\ell = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad (\tfrac{1}{2}, 0)$ Weyl spinor notation.

$\ell^c = \begin{pmatrix} \nu_R^c \\ e_R^c \end{pmatrix} \quad (0, \tfrac{1}{2})$ $\qquad Q = T_L^3 + \underbrace{T_R^3 + \frac{B-L}{2}}_{Y}$

$$\Phi = \begin{pmatrix} H_2^0 & H_1^+ \\ H_2^- & H_1^0 \end{pmatrix} \updownarrow SU(2)_L \quad (\tfrac{1}{2}, \tfrac{1}{2}) \text{ Bi-doublet.}$$

$$\overset{\longleftrightarrow}{SU(2)_R}$$

$$\mathcal{L}_{Yuk} = y_D \underbrace{\ell \epsilon \Phi \epsilon \ell^c}_{\parallel}$$

$$(-e_L, \nu_L) \begin{pmatrix} H_2^0 & H_1^+ \\ H_2^- & H_1^0 \end{pmatrix} \begin{pmatrix} e_R^c \\ -\nu_R^c \end{pmatrix}$$

$$= -e_L H_2^0 e_R^c + \nu_L H_2^- e_R^c$$

$$+ e_L H_1^+ \nu_R^c - \nu_L H_1^0 \nu_R^c$$

$$\Rightarrow \quad m_e = y_D \langle H_2^0 \rangle$$

$$m_D = y_D \langle H_1^0 \rangle \longrightarrow \text{Seesaw ?}$$

Introduce $SU(2)_{L,R}$ triplet bosons

$$\Delta_L = (\Delta_L^{++}, \Delta_L^{+}, \Delta_L^{0}), \qquad \Delta_R = (\Delta_R^{0}, \Delta_R^{-}, \Delta_R^{--})$$

$$T_L^3 = 1,\ 0,\ -1 \qquad T_R^3 = 1,\ 0,\ -1$$

$$B-L = 2 \qquad\qquad -2$$

$$l\cdot\Delta_L\cdot l = \nu_L\nu_L\Delta_L^0 + (\nu_L e_L + e_L\nu_L)\Delta_L^+ + e_L e_L \Delta_L^{++}$$

$$l^c\cdot\Delta_R\cdot l^c = \nu_R^c\nu_R^c\Delta_R^0 + (\nu_R^c e_R^c + e_R^c\nu_R^c)\Delta_R^- + e_R^c e_R^c\Delta_R^{--}$$

$$\Longrightarrow \quad m_\nu = y_L\langle\Delta_L^0\rangle$$

$$M_R = y_R\langle\Delta_R^0\rangle$$

$$\text{"}m_\nu\text{"} = y_L\langle\Delta_L^0\rangle + \frac{y_D^2\langle H_b^0\rangle^2}{y_R\langle\Delta_R^0\rangle}$$

(*) Type II seesaw : SM + Δ_L

$$\mathcal{L}_{\Delta_L} = y_\nu\, l\cdot\Delta_L\cdot l + \mu_H \tilde{H}\cdot\Delta_L\cdot\tilde{H} - m_\Delta^2|\Delta_L|^2$$

$$\langle\Delta_L^0\rangle = \frac{\mu_H v^2}{2m_\Delta^2}$$

(**) ρ-parameter bound :

$$\langle\Delta_L^0\rangle \lesssim \text{GeV}$$

$$|m^2_{\Delta_L^+} - m^2_{\Delta_L^0}| \simeq |m^2_{\Delta_L^{++}} - m^2_{\Delta_L^+}| \lesssim (40\,\text{GeV})^2$$

- Two Higgs Doublet Model for Dark Matter

2HDM : H_1, H_2

H_1 ↑ SM Higgs

H_2 ↑ inert Higgs
$$\begin{cases} \langle H_2^0 \rangle = 0 \\ \text{No Yukawa} \end{cases}$$

$$H_1 = \frac{v+h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H^+ \\ \frac{H+iA}{\sqrt{2}} \end{pmatrix}$$

DM stability imposed by Z_2

H_2 : Z_2-odd, others : Z_2-even.

$$\mathcal{L}_{2HDM} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \cancel{\left(m_{12}^2 H_1^\dagger H_2 + h.c.\right)}$$
$$+ \frac{1}{2}\lambda_1 |H_1|^4 + \frac{1}{2}\lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^\dagger H_2|^2$$
$$+ \left\{ \frac{\lambda_5}{2} (H_1^\dagger H_2)^2 + \cancel{\left[\lambda_6 |H_1|^2 + \lambda_7 |H_2|^2\right](H_1^\dagger H_2)} + h.c \right\}$$

[0207010]

$$H_1^\dagger H_2 \Rightarrow \frac{v}{\sqrt{2}} \frac{H+iA}{\sqrt{2}}$$

$$\lambda_4 |H_1^\dagger H_2|^2 \Rightarrow \frac{\lambda_4}{4} v^2 (H^2 + A^2)$$

$$\frac{\lambda_5}{2} (H_1^\dagger H_2)^2 + h.c \Rightarrow \frac{\lambda_5}{4} v^2 (H^2 - A^2) \quad (\mathrm{Im}(\lambda_5) = 0)$$

$$m_H^2 = m_{H^\pm}^2 + \frac{\lambda_4+\lambda_5}{2} v^2$$

$$m_A^2 = m_{H^\pm}^2 + \frac{\lambda_4-\lambda_5}{2} v^2 .$$

(*) custodial symmetric limit:

(i) $m_H^2 = m_A^2 = m_{H^\pm}^2$.

(ii) $m_H^2 = m_{H^\pm}^2$, or $m_A^2 = m_{H^\pm}^2$. (숙제)

(H or A can be much lighter)

(*) ρ-parameter bound.

$$m_{H^\pm}^2 - m_H^2 \lesssim 40\,\text{GeV}.$$

or $m_{H^\pm}^2 - m_A^2$.

(*) vacuum stability condition

$$\lambda_1 > 0,\quad \lambda_2 > 0,\quad \lambda_3 > -(\lambda_1\lambda_2)^{\frac{1}{2}}$$

$$\lambda_3 + \lambda_4 - |\lambda_5| > -(\lambda_1\lambda_2)^{\frac{1}{2}}$$

<1903.03616>

<u>Archetype of WIMP DM.</u> <Lectures by Prof. J.C.Park>

A weakly interacting particle that is neutral and stable is a good DM candidate.

Its typical annihilation cross-section

$$\langle \sigma v \rangle \sim \frac{\lambda^2}{4\pi m_{DM}^2}$$

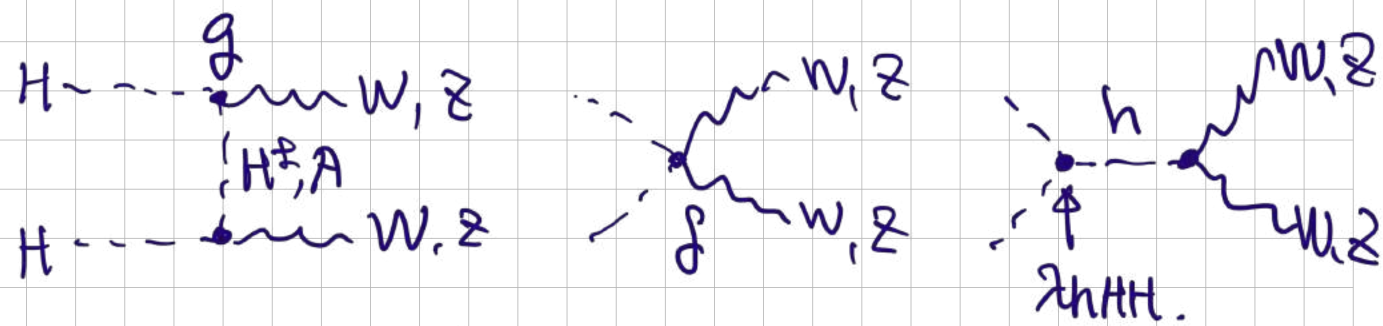
can be naturally in the range of

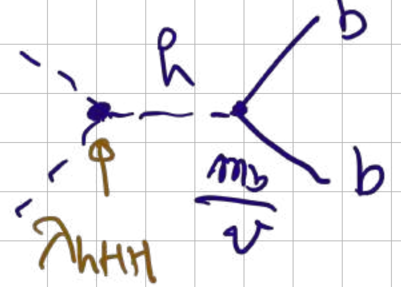
$$\langle \sigma v \rangle \approx 10^{-9}\,\mathrm{GeV}^{-2} \quad \begin{cases} m_{DM} \sim 1\ \mathrm{TeV} \\ \lambda \sim 0.1 \end{cases}$$

which produces the observed DM relic density through __freeze-out__ mechanism

$$DM = H \quad (m_H \le m_A \approx m_{H^\pm})$$
$$\phantom{DM = {}} A \quad (m_A \le m_H \approx m_{H^\pm})$$

Annihilation channels





and more.

viable mass ranges

i) $m_H \gtrsim 500\,\mathrm{GeV}$

ii) $m_H \simeq m_W \sim 2 m_W$

iii) $m_H \sim m_h/2$

• viable mass ranges

i) $m_H \gtrsim 500$ GeV $\quad HH \to VV$

ii) $m_H = 50 \sim 80$ GeV

Higgs resonance $HH \to bb$.

$$\sigma v \sim \underbrace{\lambda_{hHH}^2 \left(\frac{m_b}{v}\right)^2}_{\text{small}} \underbrace{\left(\frac{1}{s - m_h^2}\right)^2}_{\text{resonance enhancement.}}, \quad s \simeq (2m_H)^2 \sim m_h^2.$$

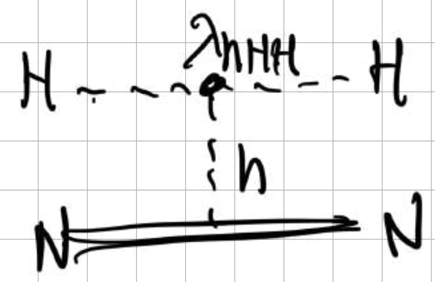
• Experimental probes

1) Indirect detection:

$HH \to VV, bb \to$ cosmic rays

$p\bar{p}, n\bar{n}, e^+e^-, \gamma\gamma, \ldots$

2) Direct detection:



$\frac{\lambda_{hHH}}{v} \lesssim 10^{-3}$ for low m_H.

3) Direct production at colliders.

- Invisible Higgs decay $h \to HH$
- $p\bar{p} \to Z^0, W^\pm \to HA, HH^\pm$.