

Dark Matter



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2023 CTPU-PTC Summer School on Cosmology & Particle Physics

Lecturer?

- 박종철 (Park, Jong-Chul)
충남대 (Chungnam National University)
- Particle Physics – Phenomenology:
Dark Matter, Neutrino, Cosmic Rays,
Early Universe Cosmology, ...
- log1079@gmail.com or jcpark@cnu.ac.kr
- <https://inspirehep.net/literature?sort=mostrecent&size=25&page=1&q=a%20j.%20c.%20park%20and%20d%20%3E%202002&ui-citation-summary=true>

References: Reviews/Lecture Notes

1. <https://arxiv.org/abs/1605.04909> → History
2. <https://arxiv.org/abs/1703.07364> → General w/ Models
3. <https://arxiv.org/abs/1705.01987> → General
4. <https://arxiv.org/abs/1910.05610> → General
5. <https://arxiv.org/abs/1903.03026> → Direct Detection
6. <https://arxiv.org/abs/1904.07915> → Direct Detection
7. <https://arxiv.org/abs/1710.05137> → Indirect Detection
8. <https://arxiv.org/abs/1812.02029> → Indirect Detection
9. <https://arxiv.org/abs/2109.02696> → Indirect Detection (Extension of 7)
10. <https://arxiv.org/abs/1912.04727> → Cosmology
11. ...

References: Books

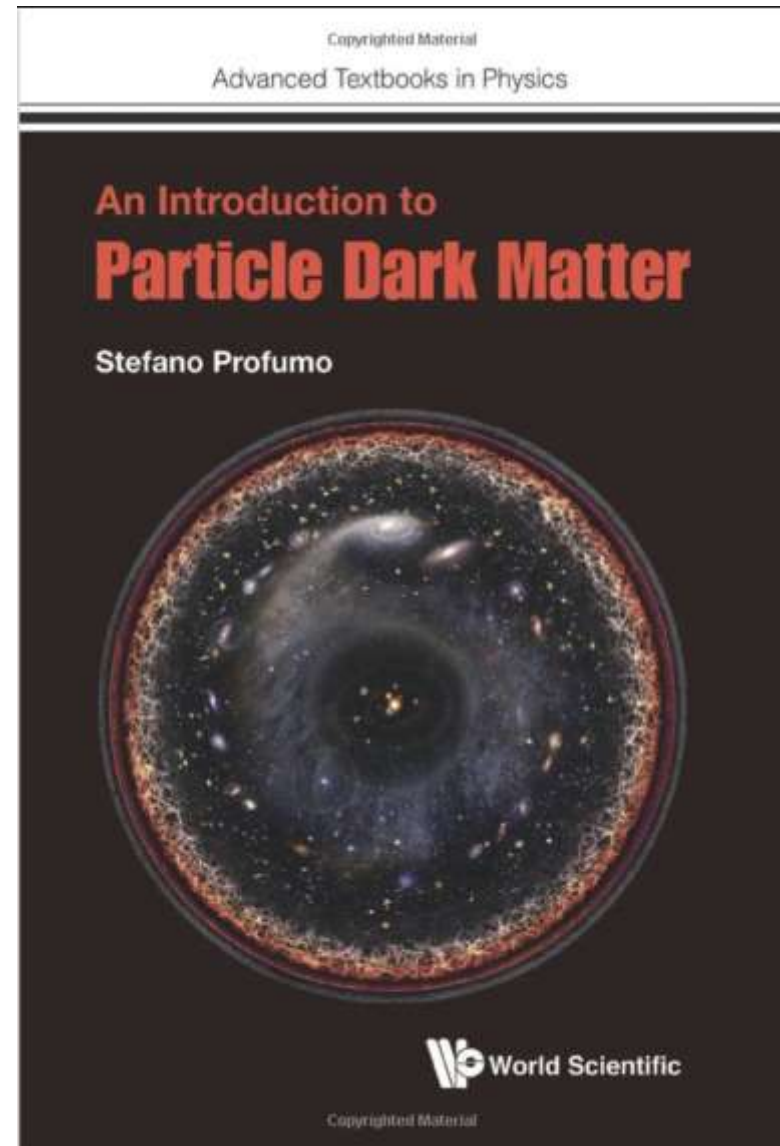


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References: Simulation Tools

1. WimPyDD (<https://wimpydd.hepforge.org/>)
2. PPC 4 DM ID (<http://www.marcocirelli.net/PPPC4DMID.html>)
3. MicrOMEGAs (<https://lapth.cnrs.fr/micromegas/>)
4. MadDM (<https://launchpad.net/maddm>)
5. ...

Things You Should Have Known ...

- Classical Mechanics, Thermal Physics
- Quantum Mechanics, Special Relativity
- General Relativity ???
- Quantum Field Theory... ????



Outline

0. Very Short Summary of the Standard Model
1. Observational Evidence of Dark Matter
2. Relic Abundance of Dark Matter
3. Direct Detection of DM – Target particle recoil
4. Indirect Detection of DM – Cosmic rays
5. Direct production of DM – Collider
6. New Approaches

**o. Very Short Summary of
the Standard Model**



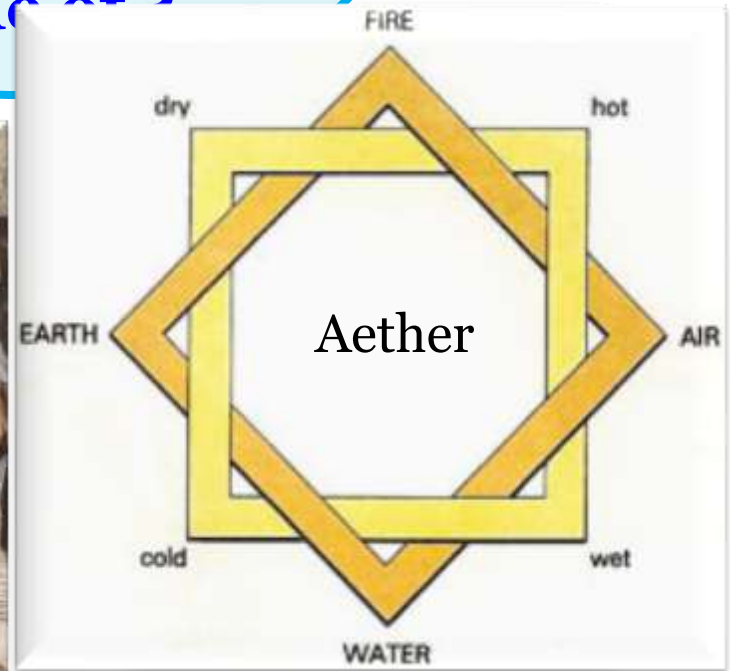
Eternal Questions

**What is the
Universe
made of?**



Eternal Questions

What is the Universe made of? ❖ Ancient Greek: 4 basic elements



19th c: Periodic Table

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18																																																																															
1 H Hydrogen 1.00794	2 He Helium 4.002602	<table border="0"> <tr> <td>Atomic #</td><td>Symbol</td><td>C Solid</td><td colspan="2">Metalloids</td><td colspan="4">Nonmetals</td><td colspan="2">Other nonmetals</td><td>Halogens</td><td>Noble gases</td><td colspan="2">273</td><td></td></tr> <tr> <td>Name</td><td></td><td>Hg Liquid</td><td colspan="2"></td><td colspan="4"></td><td colspan="2"></td><td></td><td></td><td colspan="2"></td><td></td></tr> <tr> <td>Atomic Weight</td><td></td><td>H Gas</td><td colspan="2"></td><td colspan="4"></td><td colspan="2"></td><td></td><td></td><td colspan="2"></td><td></td></tr> <tr> <td></td><td></td><td>Rf Unknown</td><td colspan="2"></td><td colspan="4"></td><td colspan="2"></td><td></td><td></td><td colspan="2"></td><td></td></tr> <tr> <td></td><td></td><td></td><td>Alkali metals</td><td>Alkaline earth metals</td><td>Lanthanoids</td><td>Actinoids</td><td>Transition metals</td><td>Post-transition metals</td><td colspan="8"></td></tr> </table>														Atomic #	Symbol	C Solid	Metalloids		Nonmetals				Other nonmetals		Halogens	Noble gases	273			Name		Hg Liquid														Atomic Weight		H Gas																Rf Unknown																	Alkali metals	Alkaline earth metals	Lanthanoids	Actinoids	Transition metals	Post-transition metals								
Atomic #	Symbol	C Solid	Metalloids		Nonmetals				Other nonmetals		Halogens	Noble gases	273																																																																																			
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			Alkali metals	Alkaline earth metals	Lanthanoids	Actinoids	Transition metals	Post-transition metals																																																																																								
3 Li Lithium 6.941	4 Be Beryllium 9.012182	5 B Boron 10.811	6 C Carbon 12.0107	7 N Nitrogen 14.0067	8 O Oxygen 15.9994	9 F Fluorine 18.9984032	10 Ne Neon 20.1797	11 Na Sodium 22.98976928	12 Mg Magnesium 24.305	13 Al Aluminum 26.9815386	14 Si Silicon 28.0855	15 P Phosphorus 30.973762	16 S Sulfur 32.06	17 Cl Chlorine 35.453	18 Ar Argon 39.948																																																																																	
19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.955912	22 Ti Titanium 47.867	23 V Vanadium 50.9415	24 Cr Chromium 51.9961	25 Mn Manganese 54.938045	26 Fe Iron 55.845	27 Co Cobalt 58.933195	28 Ni Nickel 58.6934	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.63	33 As Arsenic 74.9216	34 Se Selenium 78.96	35 Br Bromine 79.904	36 Kr Krypton 83.796																																																																															
37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.90585	40 Zr Zirconium 91.224	41 Nb Niobium 92.90638	42 Mo Molybdenum 95.94	43 Tc Technetium (98)	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.9055	46 Pd Palladium 106.42	47 Ag Silver 107.8682	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.71	51 Sb Antimony 121.75	52 Te Tellurium 127.6	53 I Iodine 126.90447	54 Xe Xenon 131.29																																																																															
55 Cs Cesium 132.90545	56 Ba Barium 137.327	57-71 Lanthanoids	72 Hf Hafnium 178.49	73 Ta Tantalum 180.94788	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.222	78 Pt Platinum 195.084	79 Au Gold 196.966569	80 Hg Mercury 200.59	81 Tl Thallium 204.3847	82 Pb Lead 207.2	83 Bi Bismuth 208.9804	84 Po Polonium (209)	85 At Astatine (210)	86 Rn Radon (222)																																																																															
87 Fr Francium (223)	88 Ra Radium (226)	89-103 Actinoids	104 Rf Rutherfordium (261)	105 Db Dubnium (268)	106 Sg Seaborgium (271)	107 Bh Bohrium (272)	108 Hs Hassium (277)	109 Mt Meitnerium (276)	110 Ds Darmstadtium (281)	111 Rg Roentgenium (280)	112 Cn Copernicium (285)	113 Nh Nihonium (284)	114 Fl Flerovium (289)	115 Uup Ununpentium (288)	116 Uuq Ununquadium (289)	117 Uuh Ununheptium (284)	118 Uuo Ununoctium (289)																																																																															

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

Periodic Table Design & Interface Copyright © 1997 Michael Dayah, Ptable.com Last updated Apr 10, 2011

57 La Lanthanum 138.90547	58 Ce Cerium 140.118	59 Pr Praseodymium 140.90765	60 Nd Neodymium 144.242	61 Pm Promethium (145)	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.92535	66 Dy Dysprosium 162.5	67 Ho Holmium 164.93032	68 Er Erbium 167.259	69 Tm Thulium 168.93421	70 Yb Ytterbium 173.054	71 Lu Lutetium 174.967
89 Ac Actinium (227)	90 Th Thorium 232.03806	91 Pa Protactinium 231.03888	92 U Uranium 238.02891	93 Np Neptunium (237)	94 Pu Plutonium (244)	95 Am Americium (243)	96 Cm Curium (247)	97 Bk Berkelium (247)	98 Cf Californium (251)	99 Es Einsteinium (252)	100 Fm Fermium (257)	101 Md Mendelevium (258)	102 No Nobelium (259)	103 Lr Lawrencium (262)



Dmitri Mendeleev
(1869)

20th c: Standard Model (SM)

mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.07 \text{ GeV}/c^2$	0
charge →	$2/3$	$2/3$	$2/3$	0
spin →	$1/2$	$1/2$	$1/2$	1
	u up	c charm	t top	g gluon
	d down	s strange	b bottom	γ photon
	e electron	μ muon	τ tau	Z Z boson
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson

QUARKS

LEPTONS

GAUGE BOSONS



Higgs (1964)!

VOLUME 13, NUMBER 16

PHYSICAL REVIEW LETTERS

19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland

(Received 31 August 1964)



Chicago (USA)



Geneva (Europe)

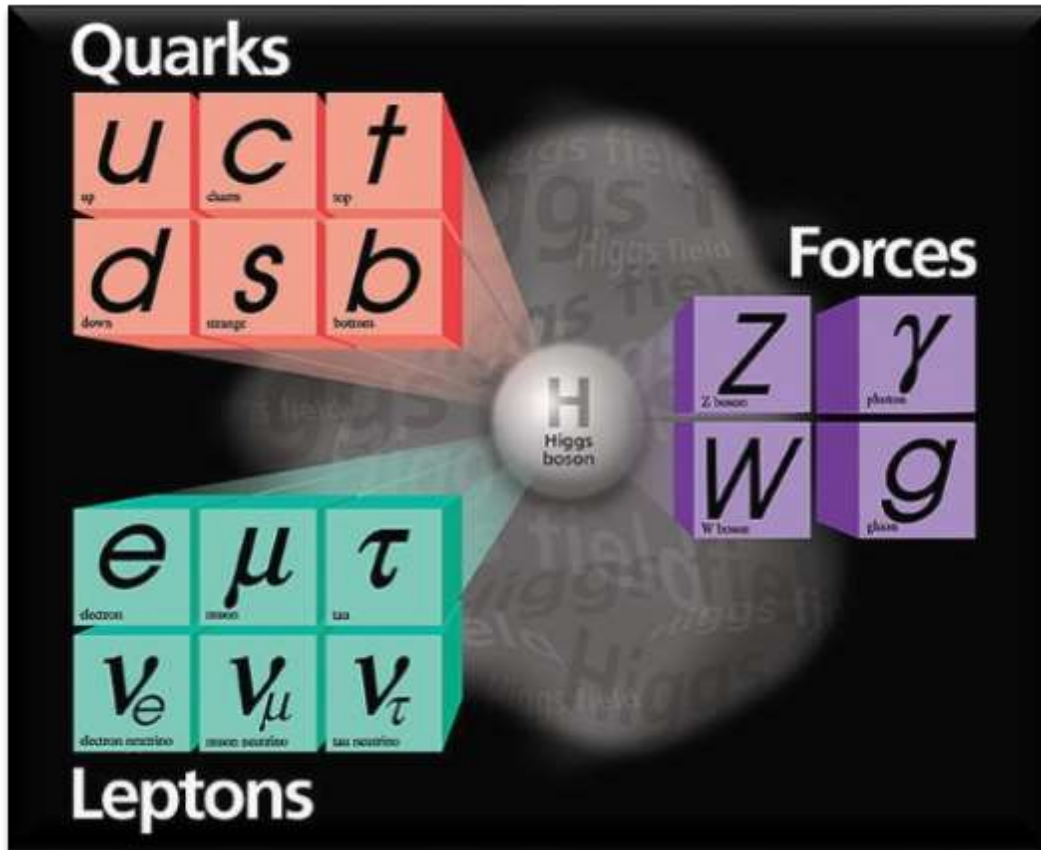
Higgs (2012)!



Higgs is discovered!



Now: Standard Model (SM)



A white mug with a black border, featuring the Standard Model Lagrangian written in black ink:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + i\bar{\Psi}\not{D}\Psi + \text{h.c.} \\ & + \Psi_i\gamma_{ij}\Psi_j\Phi + \text{h.c.} \\ & + |D_\mu\Phi|^2 - V(\Phi) \end{aligned}$$

Now: Standard Model (SM)

$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^0 \partial_\nu g_\mu^0 - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_a^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2\alpha_w} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - \\
 & Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+)) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\nu^+) - ig_s w (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\nu^+)) - \\
 & \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^- - Z_\mu^0 Z_\nu^0 W_\nu^+ W_\mu^-) + \\
 & g^2 s_w^2 (A_\mu W_\nu^+ A_\nu W_\mu^- - A_\mu A_\nu W_\nu^+ W_\mu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - \\
 & 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
 & \beta_h \left(\frac{2M^2}{g} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - g \alpha_h M (H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-) - \\
 & \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - g M W_\mu^+ W_\mu^- H - \\
 & \frac{1}{2}g \frac{M}{c_w} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\nu \phi^- - \phi^- \partial_\nu \phi^0) - W_\mu^- (\phi^0 \partial_\nu \phi^+ - \phi^+ \partial_\nu \phi^0)) + \\
 & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + \\
 & M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \\
 & W_\mu^- \phi^+) - i \\
 & \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 \\
 & \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A \\
 & \frac{1}{2}ig_s \lambda_{ij}^0 (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a - \\
 & ig_s w A_\mu (-(\bar{e}^\lambda \gamma^\mu e^\lambda \\
 & 1 - \gamma^5) \\
 & \frac{ig}{2\sqrt{2}} W \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep}{}_{\kappa\lambda}^\dagger \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^c C_{\kappa\lambda}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\lambda (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_e^\kappa (\bar{e}^\lambda U^{lep}{}_{\lambda\kappa}^\dagger (1 + \gamma^5) \nu^\kappa) - m_e^\kappa (\bar{e}^\lambda U^{lep}{}_{\lambda\kappa}^\dagger (1 - \gamma^5) \nu^\kappa) - \frac{g}{2} \frac{m_\nu^2}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
 & \frac{g}{2} \frac{m_\nu^2}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\nu^2}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\nu^2}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \bar{\nu}_\kappa - \\
 & \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \bar{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^c (\bar{u}_j^c C_{\lambda\kappa} (1 - \gamma^5) d_j^c) + m_u^c (\bar{u}_j^c C_{\lambda\kappa} (1 + \gamma^5) d_j^c)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_d^c (\bar{d}_j^c C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^c) - m_u^c (\bar{d}_j^c C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^c) - \frac{g}{2} \frac{m_\nu^2}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2} \frac{m_\nu^2}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \\
 & \frac{ig}{2} \frac{m_\nu^2}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\nu^2}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda)
 \end{aligned}$$



Probably NOT by Prof. Eung Jin Chun

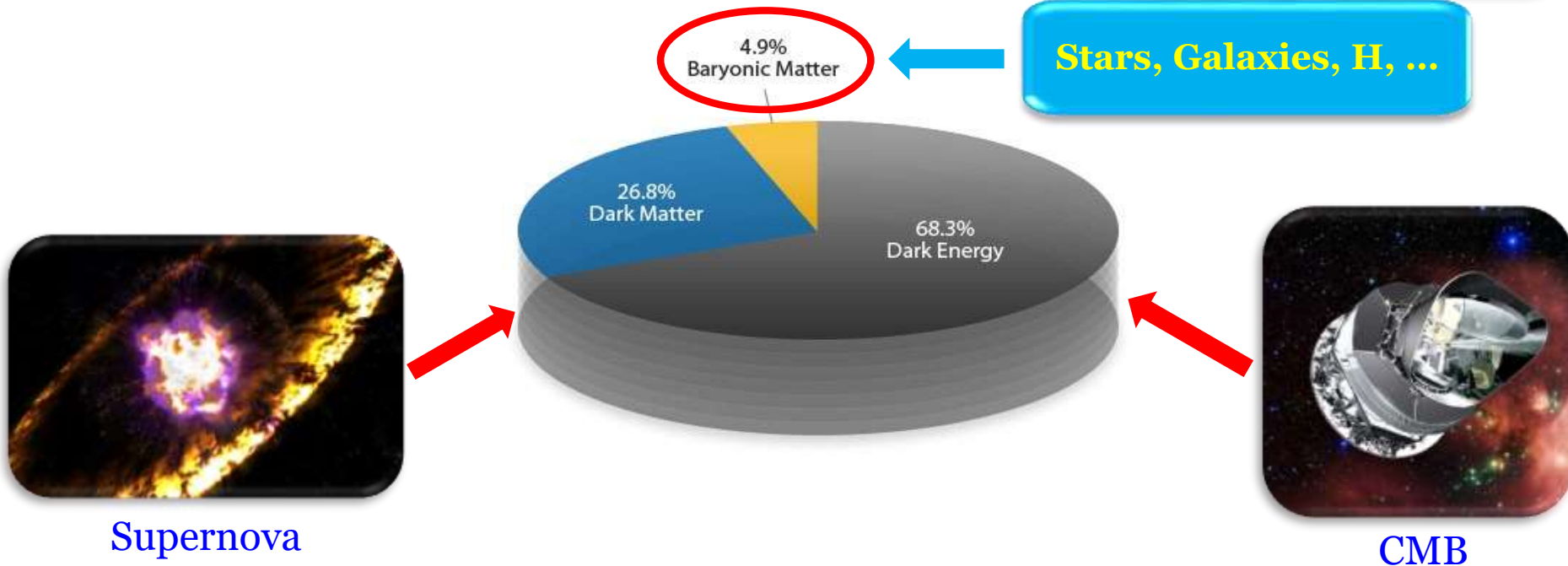


Message from Cosmology

❖ Modern cosmology → Cosmic pie



Stars, Galaxies, H, ...



❖ The **Standard Model** explains **only ~5%** of the total E of the Universe.

Question in the 20th Century!!



What's
the matter?



What's
Dark Matter?

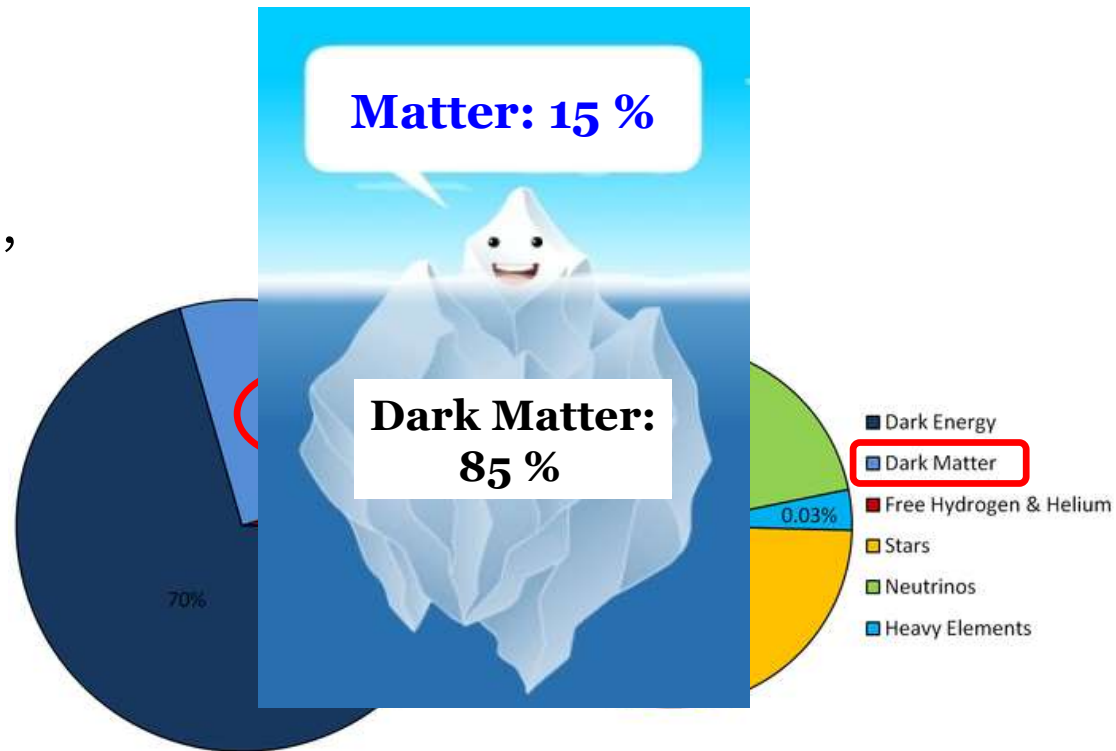
Dark Matter (DM)

- ❖ **Postulated** by **Fritz Zwicky** in early 1930's
- ❖ **Rediscovered** by **Vera Rubin** in 1970



- ❖ **Compelling paradigm:**

- ✓ massive,
- ✓ non-relativistic ($\rightarrow v \ll c$),
- ✓ non-luminous
(no/tiny EM interaction),
- ✓ stable particles
- ✓ $\sim 1/4$ of the Universe



**Existence of
Dark Matter?**



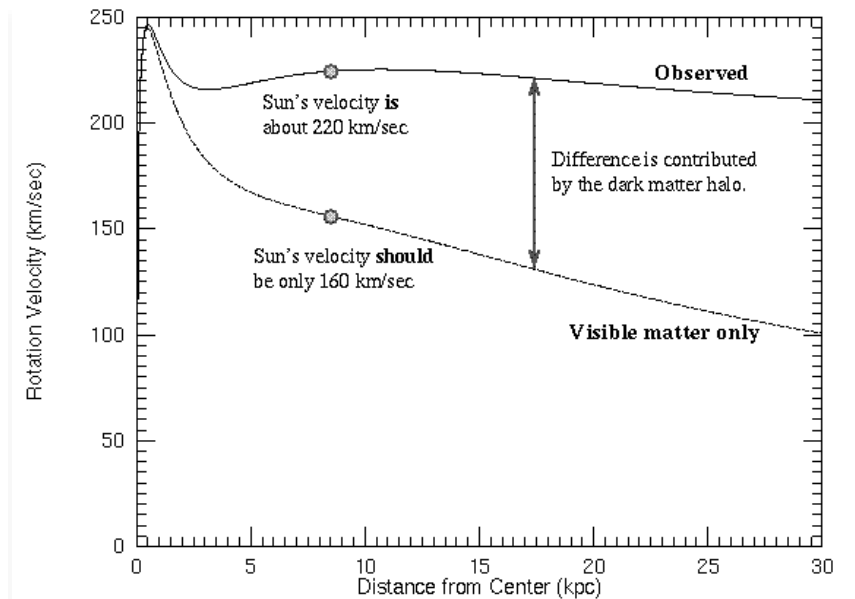
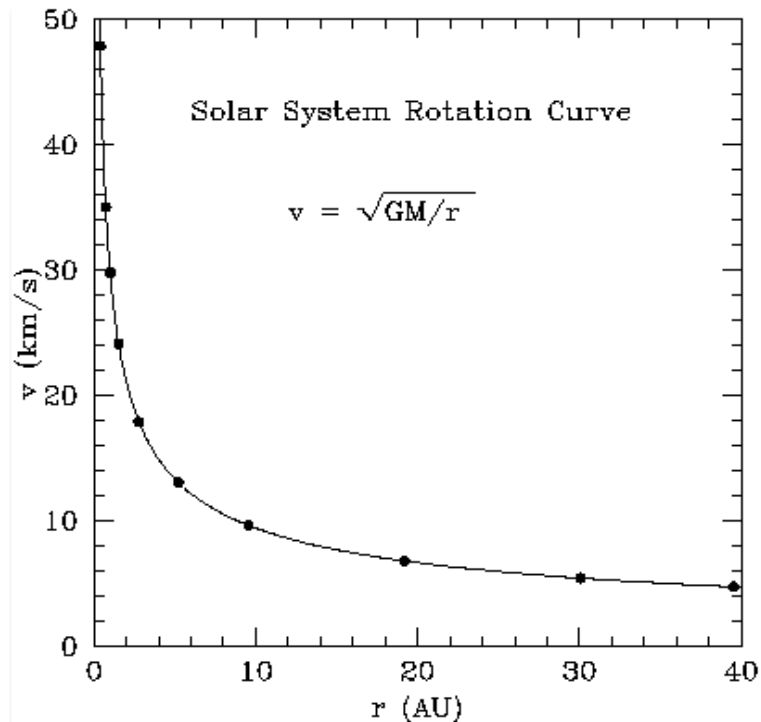
1. Observational Evidence of Dark Matter



Observational Evidence of DM

- ✓ Galaxy rotation curve
- ✓ Coma cluster
- ✓ Gravitational lensing
- ✓ Bullet cluster
- ✓ Structure formation
- ✓ Cosmic microwave background radiation (CMBR)
- ✓ Sky surveys
- ✓ Type Ia supervovae
- ✓ Baryonic acoustic oscillation (BAO)
- ✓ ...

Galaxy Rotation Curve

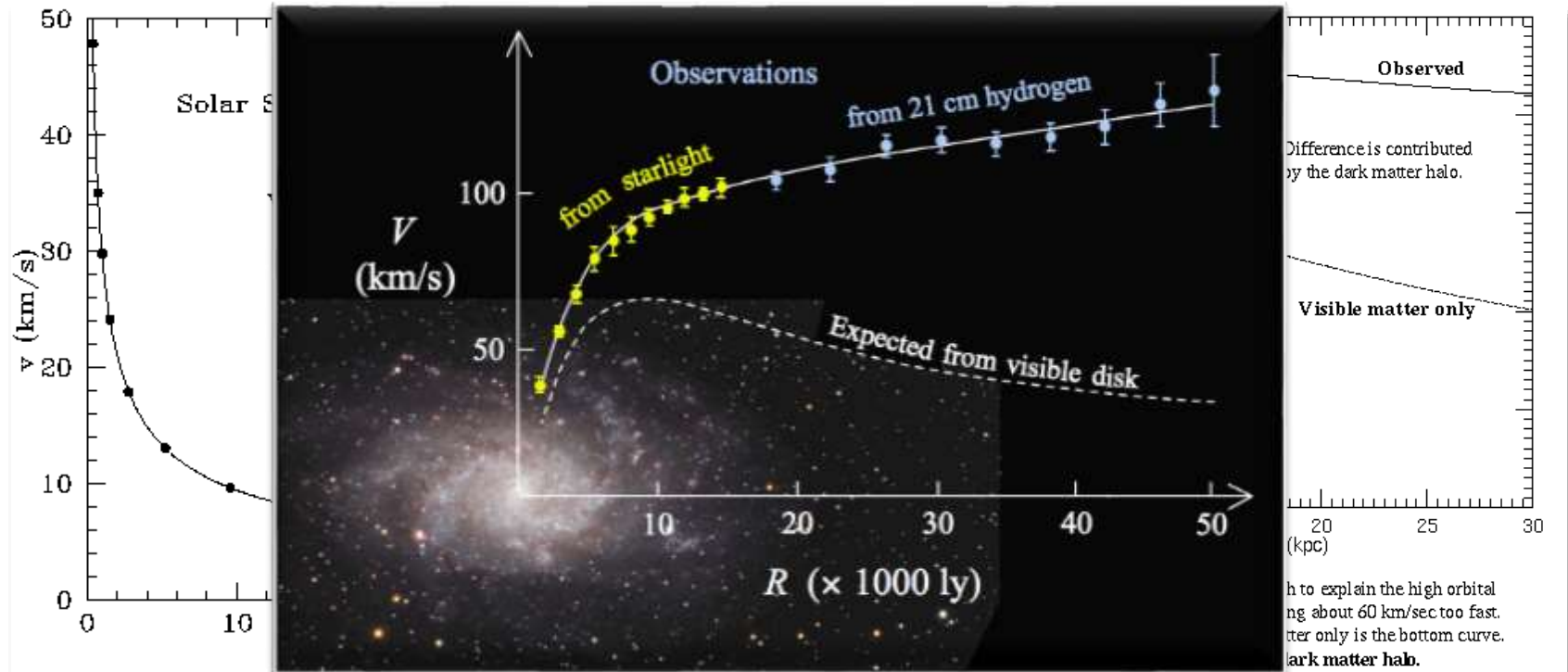


The gravity of the visible matter in the Galaxy is not enough to explain the high orbital speeds of stars in the Galaxy. For example, the Sun is moving about 60 km/sec too fast. The part of the rotation curve contributed by the visible matter only is the bottom curve. The discrepancy between the two curves is evidence for a **dark matter halo**.



Vera Rubin

Galaxy Rotation Curve



Vera Rubin

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \rightarrow v \propto \sqrt{\frac{GM}{r}}$$

$$v \sim \text{constant} \rightarrow M(r) \propto r$$

❖ Much more galaxies

(Lower luminosity galaxies)

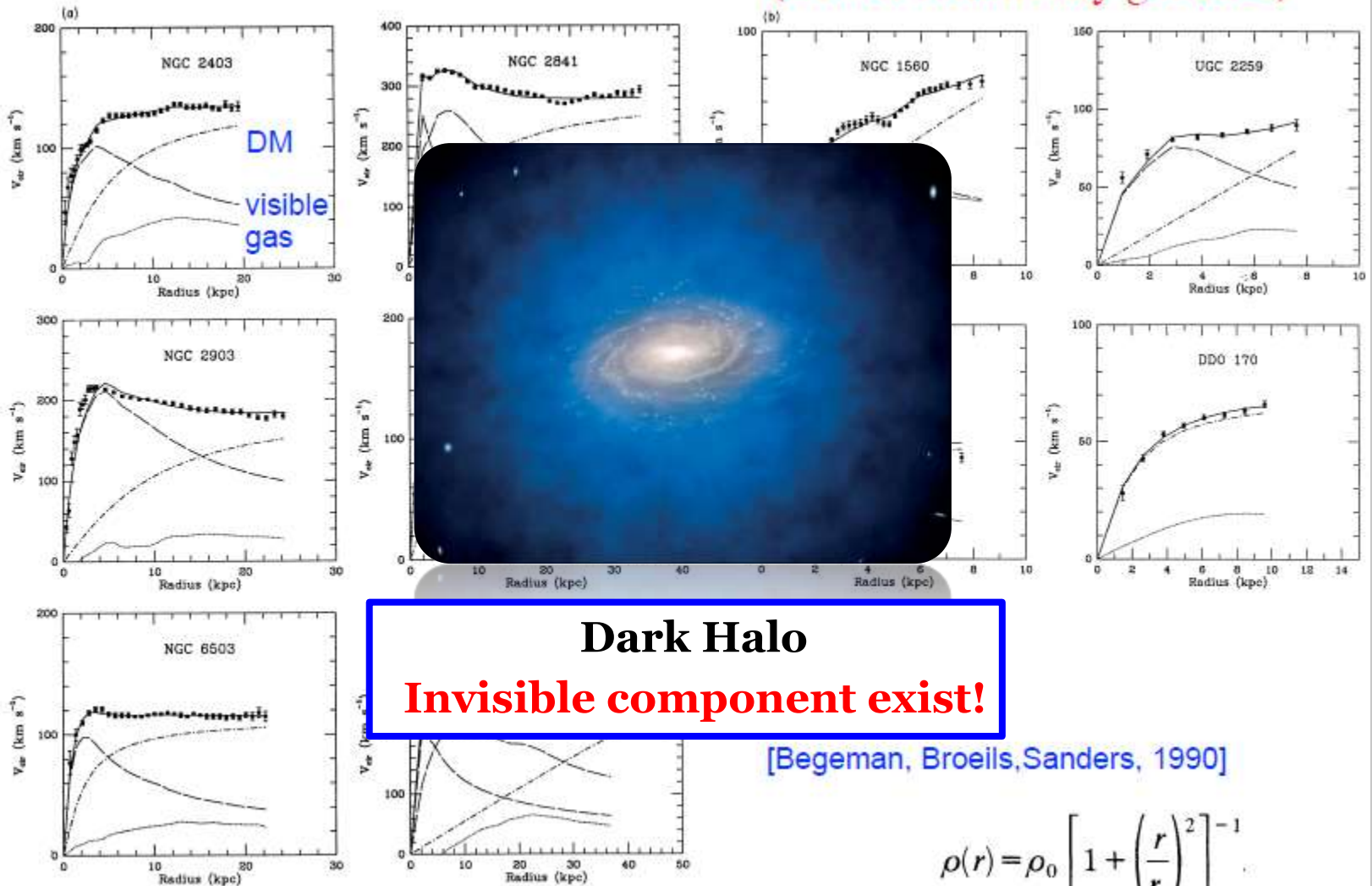
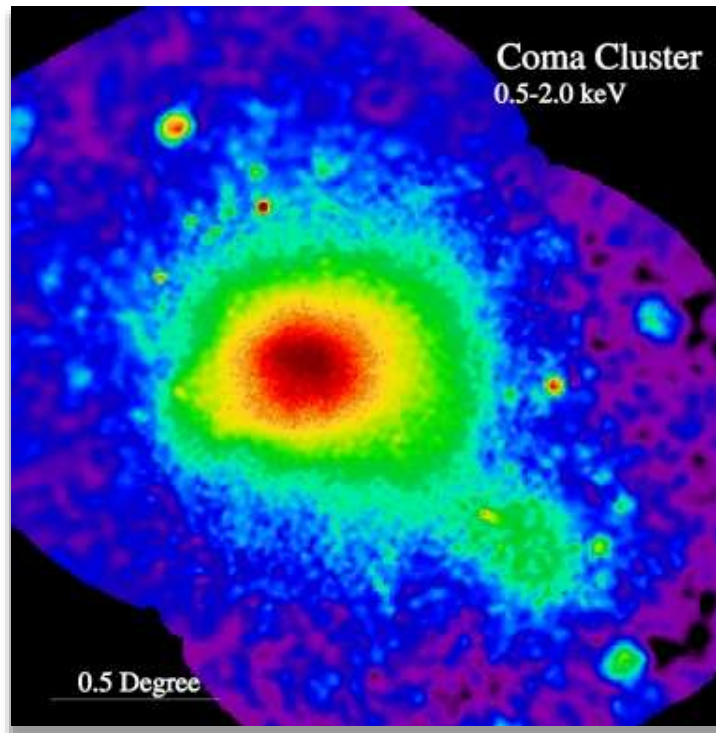


Figure 1. Three-parameter dark-halo fits (solid curves) to the rotation curves of sample galaxies. The rotation curves of the individual components are also shown: the dashed curves are for the visible components, the dotted curves for the gas, and the dash-dot curves for the dark halo. The fitting parameters are the mass-to-light ratio of the disc (M/L), the halo core radius (r_c), and the halo asymptotic circular velocity (V_s). The galaxies from the sample of Begeman are shown in (a) and the lower luminosity galaxies in (b). Best-fit values for the free parameters are given in columns 2, 3 and 4 of Table 2.

$$\rho(r) = \rho_0 \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-1}$$

Coma Cluster

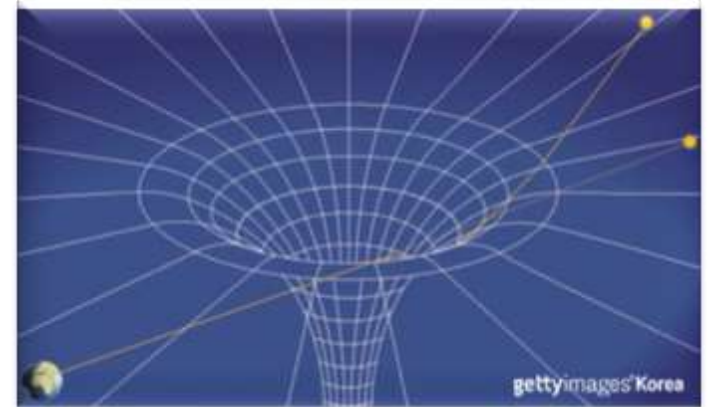
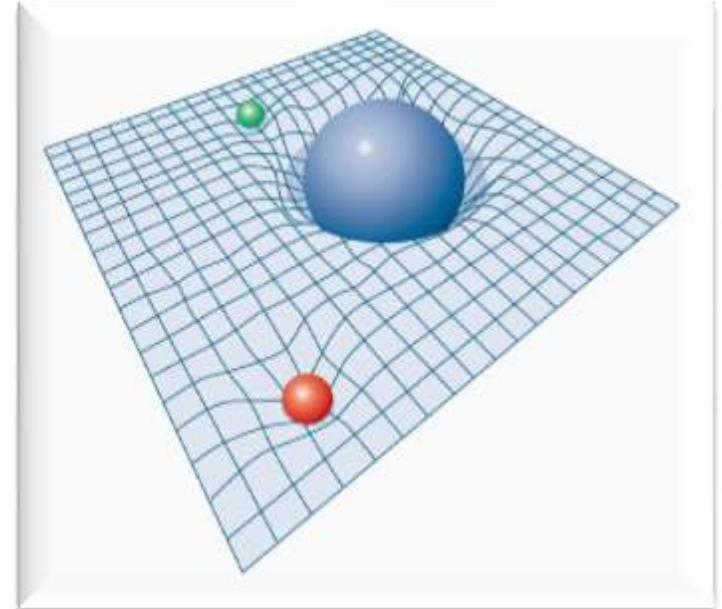
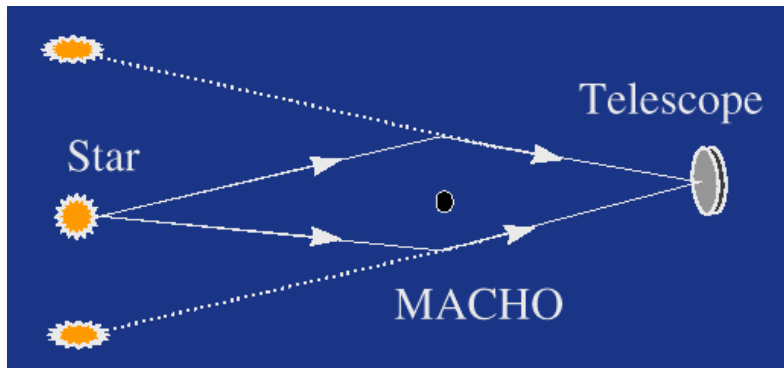
- ❖ The gravity of the cluster: **too weak** to contain the **hot gas**.
→ **It would evaporate!**: $T \propto v^2 \Leftrightarrow v^2 \propto GM/r$



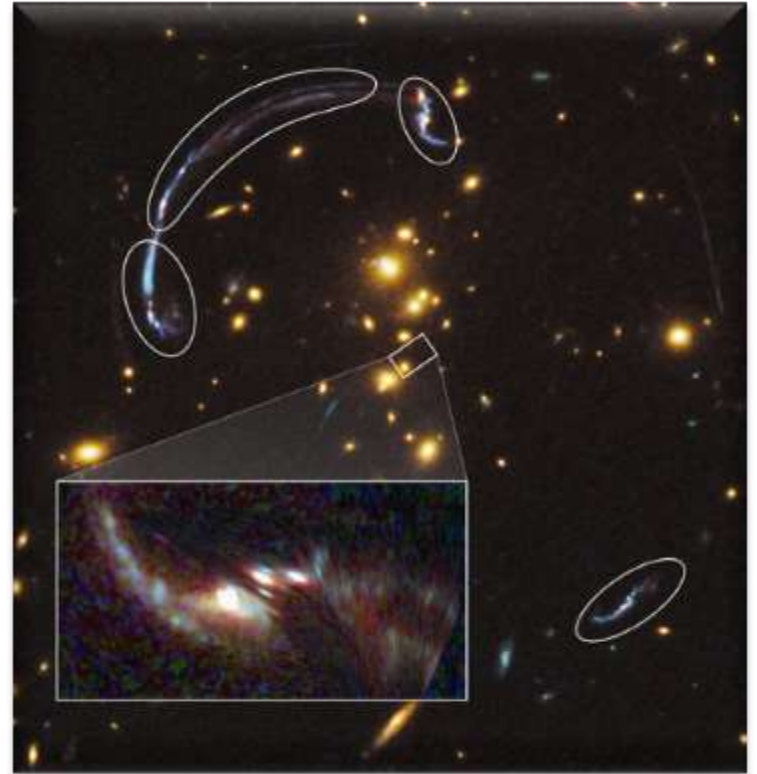
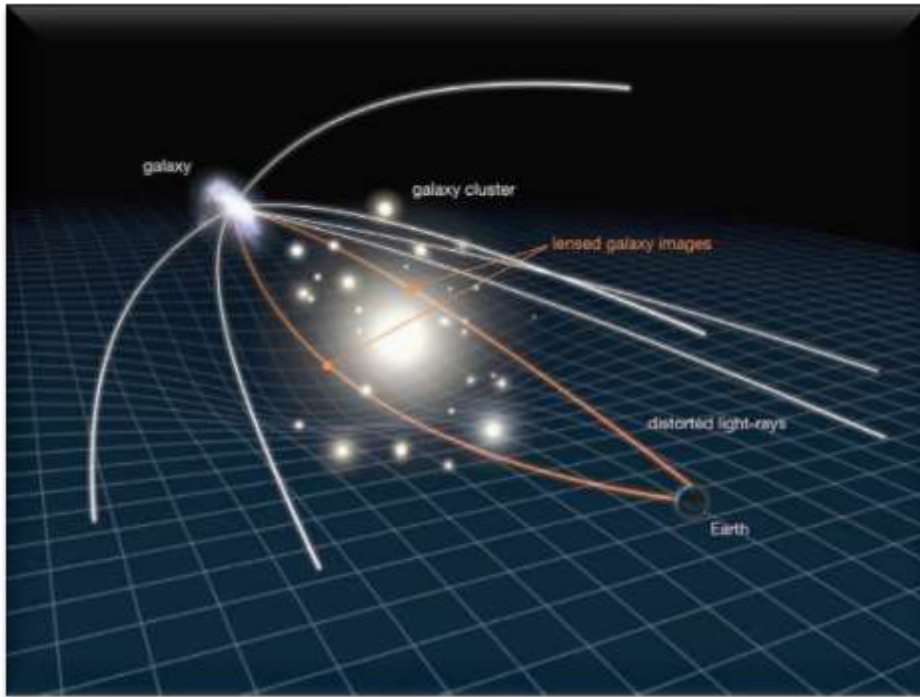
x-ray image from the ROSAT satellite

Gravitational Lensing

- General relativity: M distorts space-time
 - ➔ When light passes around a massive object, it is bent!



Gravitational Lensing



➤ Stars and hot gas:

too small to bend the light from the background galaxies so much

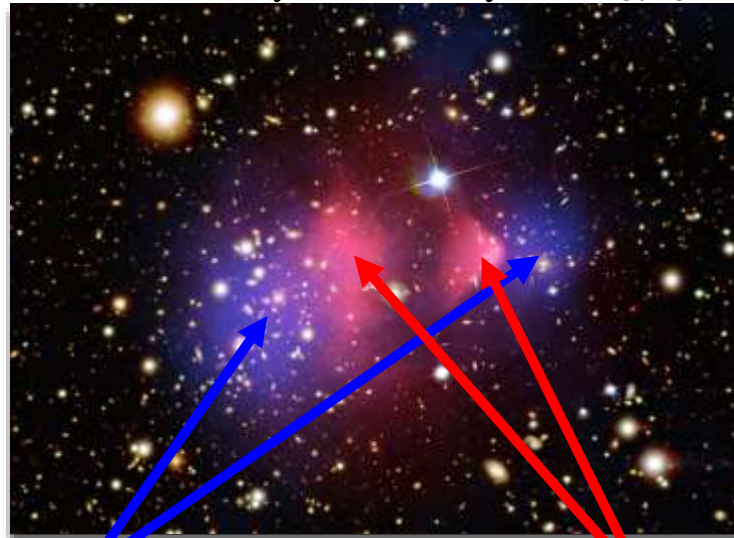
→ **Great concentration of invisible matter: Dark Matter !!**

Bullet Cluster

❖ Two colliding galaxy clusters

→ **significant displacement** between their **center of visible matter** & **gravitational potential**

Chandra X-Ray Observatory: 1E 0657-56

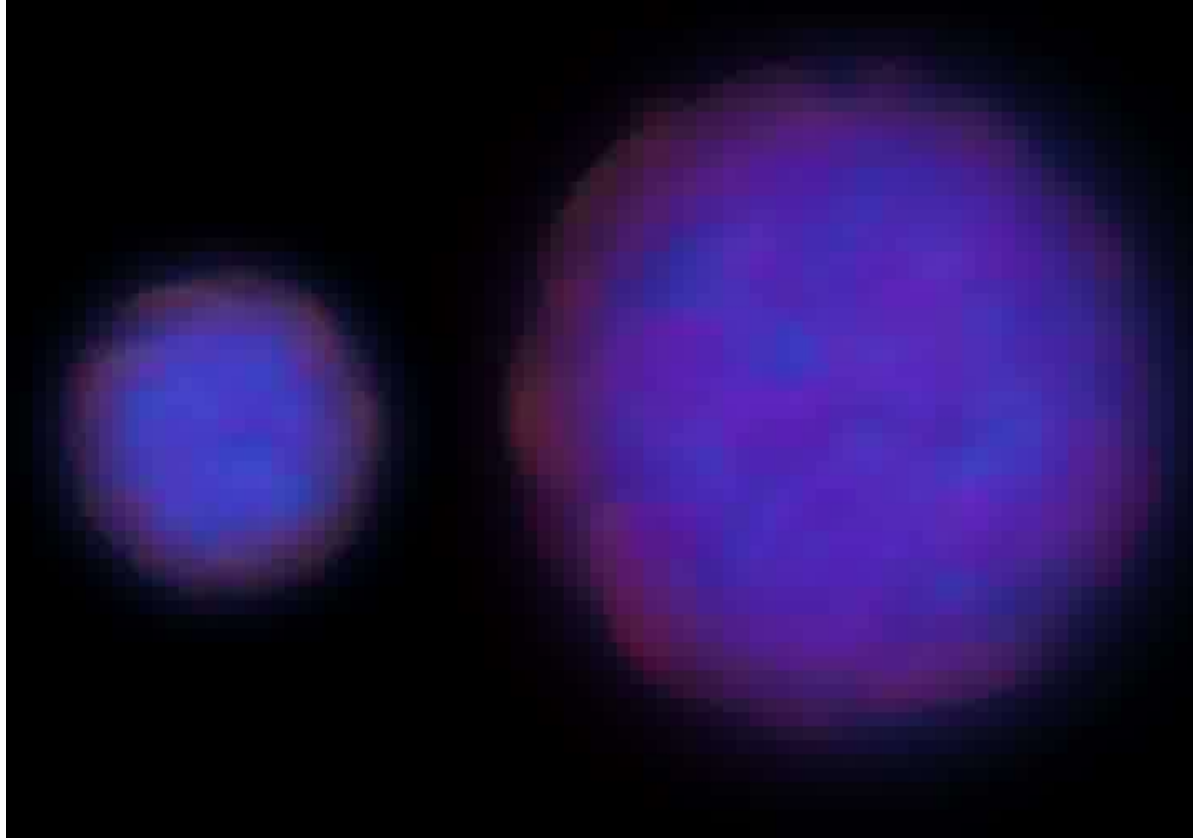


Gravitational potential
(lensing)

Ordinary matter
(X-ray)

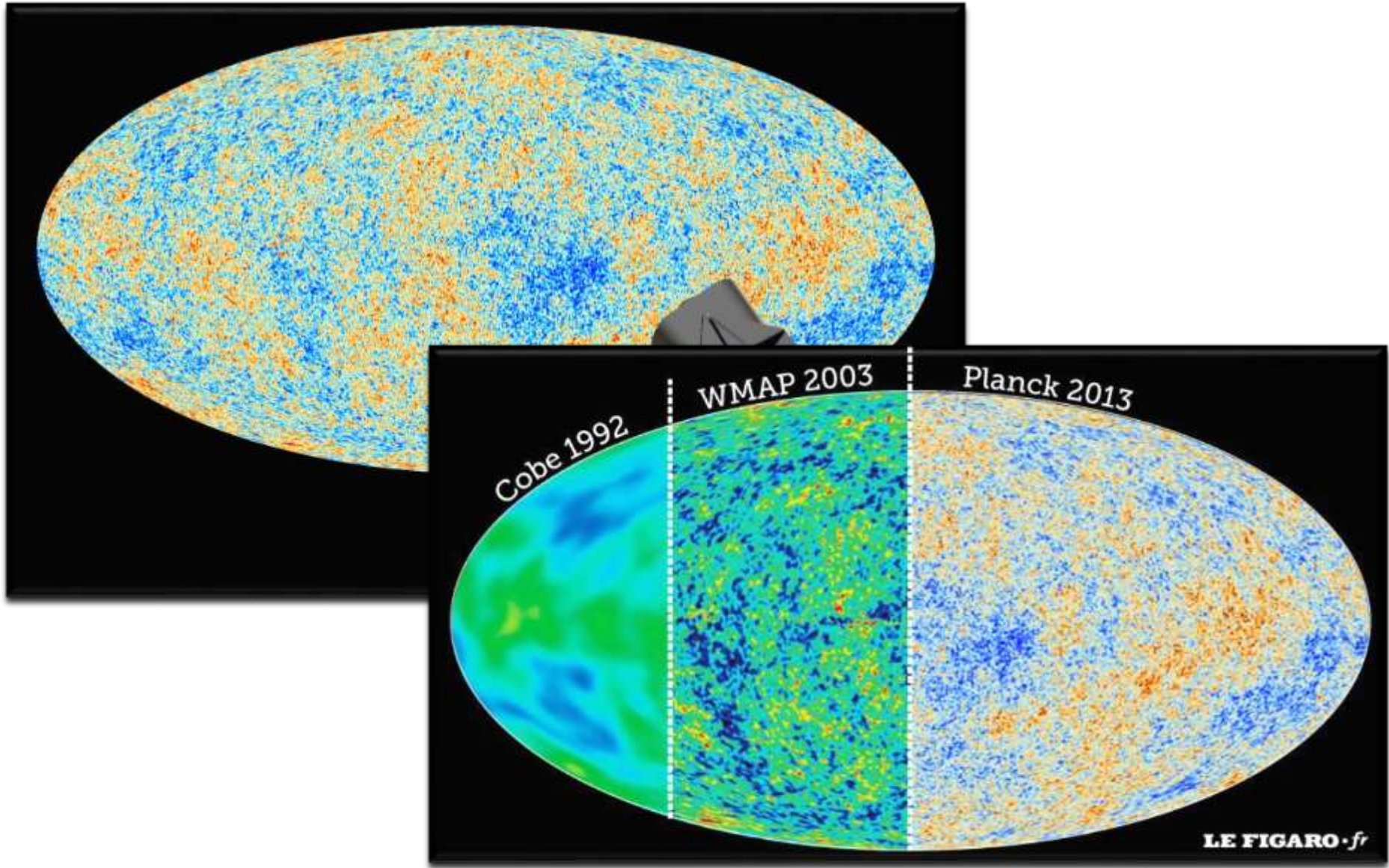
Bullet Cluster

- ❖ Simulation of **two colliding galaxy clusters**

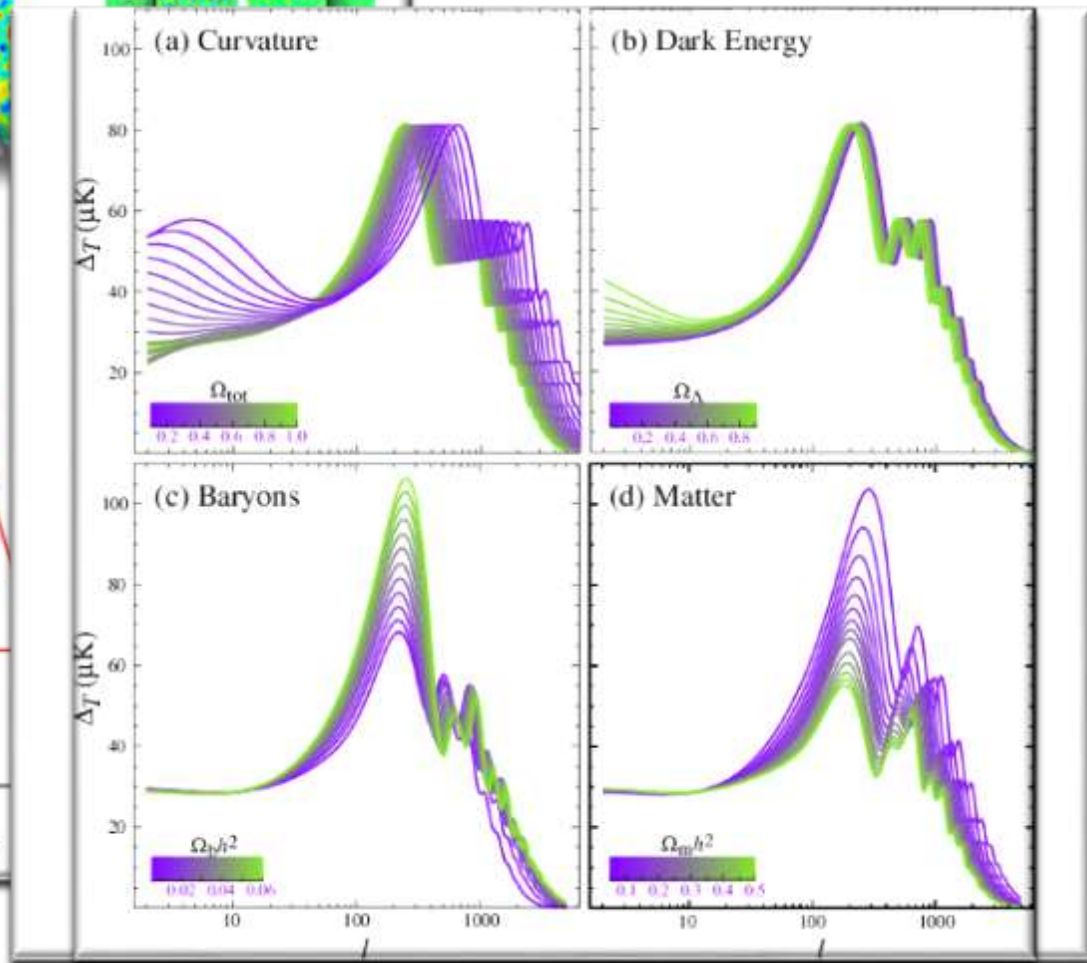
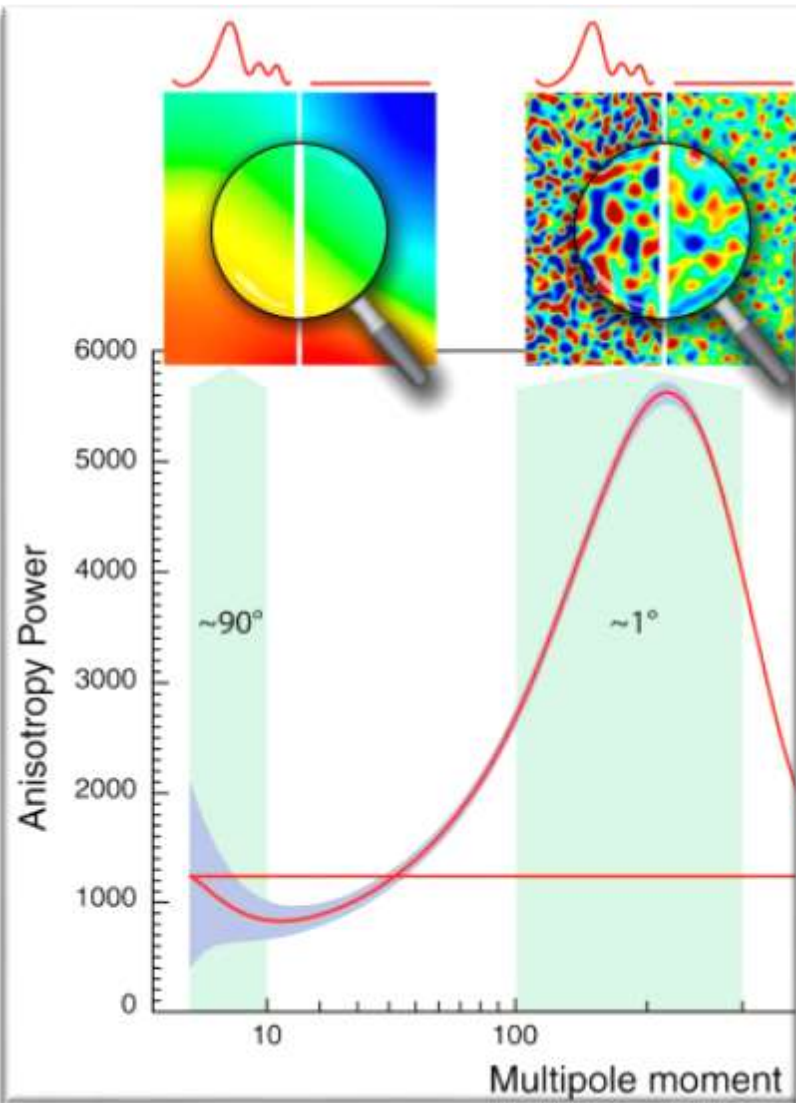


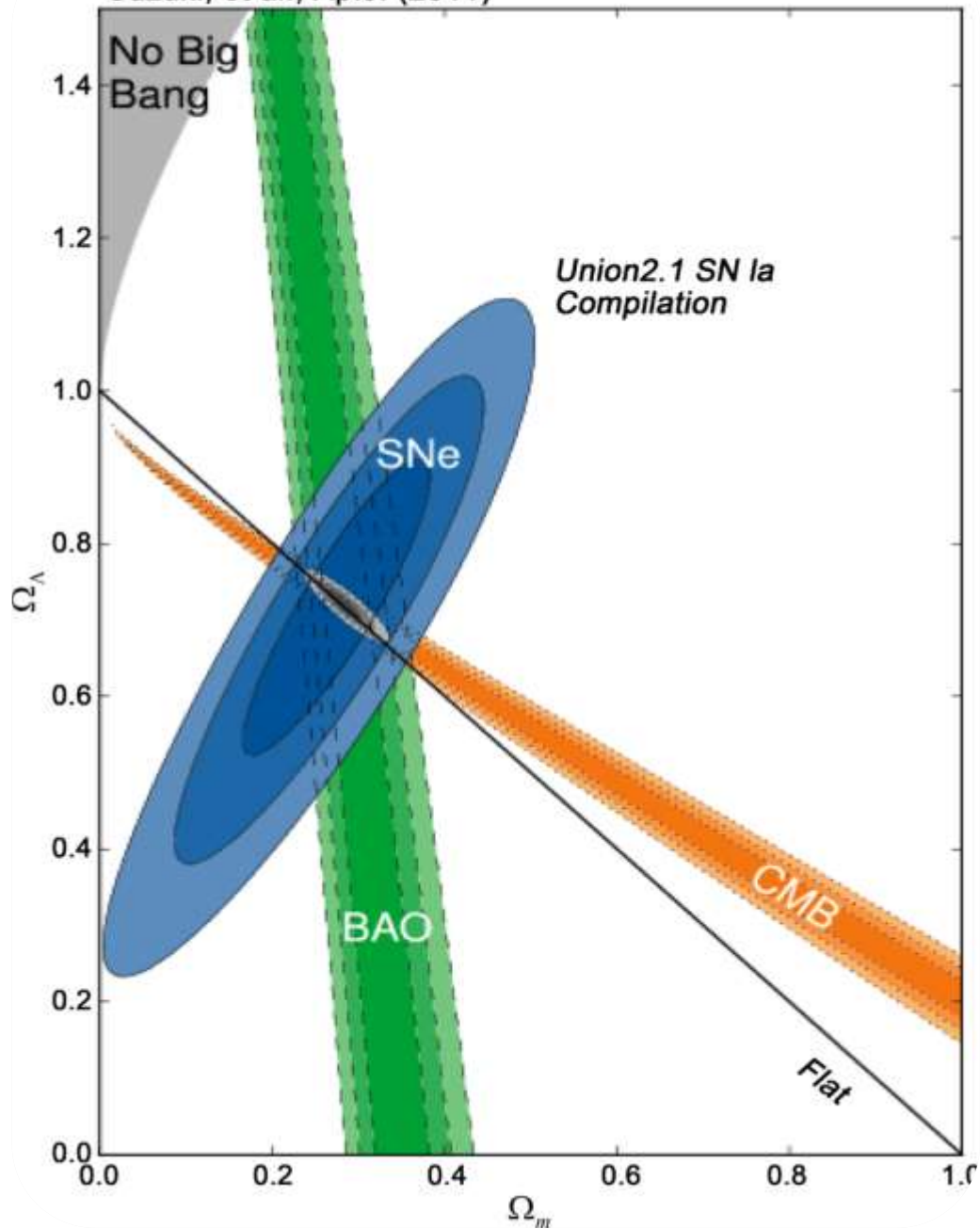
http://chandra.harvard.edu/photo/2006/1e0657/1e0657_bullett_anim_lg.mpg

CMB



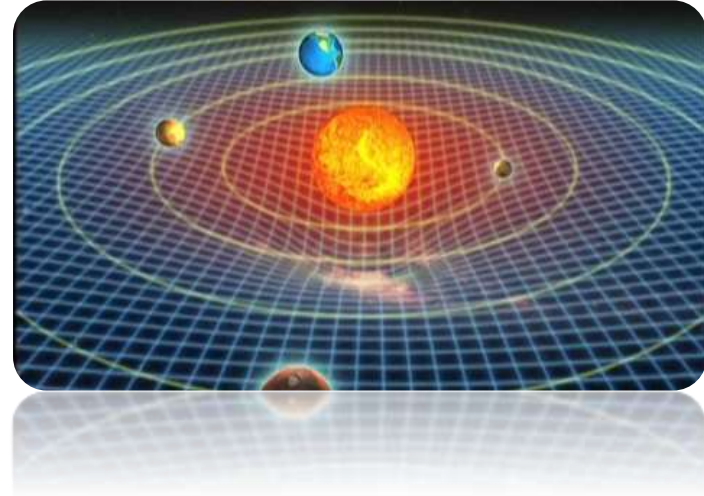
CMB





Observational Evidence of DM

- ✓ Galaxy rotation curve
- ✓ Coma cluster
- ✓ Gravitational lensing
- ✓ Bullet cluster
- ✓ Structure formation
- ✓ Cosmic microwave background radiation (CMBR)
- ✓ Sky surveys
- ✓ Type Ia supervovae
- ✓ Baryonic acoustic oscillation (BAO)
- ✓ ...



Exercises

1. DM vs Modified gravity/MOND (MODified Newtonian Dynamics):

(a) What are Modified gravity and MOND?

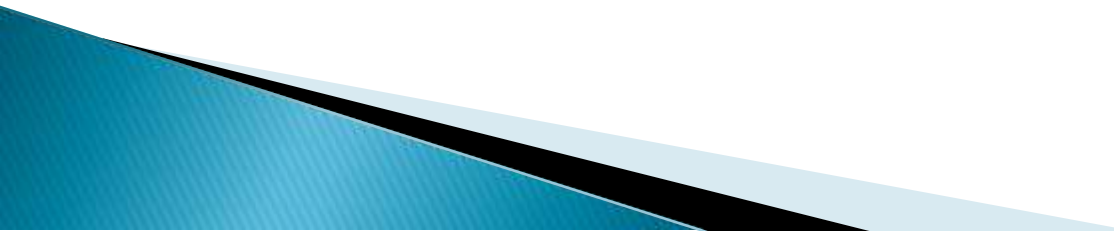
(b) Compare DM & Modified gravity/MOND: pros and cons

2. DM candidates among SM particles:

(a) Which ones?

(b) Why not? (If possible, Relic density of neutrino)

2. Relic Abundance of Dark Matter



Thermal Freeze-out

Cosmological Lower Bound on Heavy-Neutrino Masses

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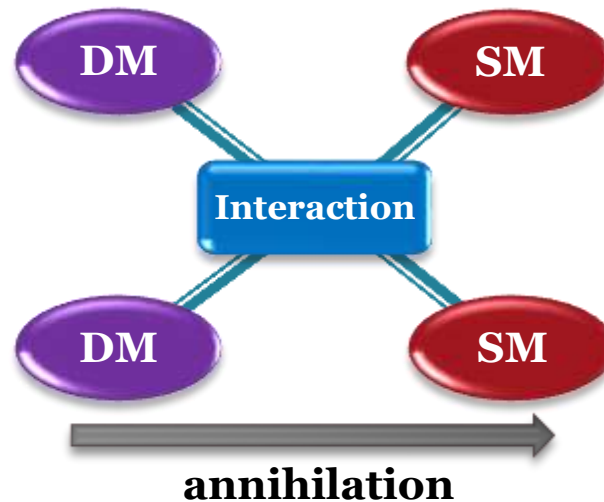
and

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(Received 13 May 1977)

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of $2 \times 10^{-29} \text{ g/cm}^3$, the lepton mass would have to be *greater* than a lower bound of the order of 2 GeV.



Basics of Freeze-out

- ❖ Boltzmann equation: the statistical behavior of a thermodynamic system not in a state of equilibrium → time evolution of # density:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_A v\rangle [(n_\chi)^2 - (n_\chi^{\text{eq}})^2]$$

- ❖ Decoupled when $\Gamma_{\text{int}} = \langle\sigma v\rangle n_\chi < H$
- ❖ Comoving number density → Scale out the expansion effect:

$$\begin{aligned}(n_\chi/s)_0 &= (n_\chi/s)_f \simeq 100/(m_\chi m_{\text{P}1} g_*^{1/2} \langle\sigma_A v\rangle) \\ &\simeq 10^{-8}/[(m_\chi/\text{GeV})(\langle\sigma_A v\rangle/10^{-27} \text{ cm}^3 \text{ s}^{-1})]\end{aligned}$$

- ❖ Relic density in units of the critical density:

$$\Omega_\chi h^2 = m_\chi n_\chi / \rho_c \simeq (3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} / \langle\sigma_A v\rangle)$$

For more details, please see e.g., Ch. 5.1 & 5.2 of “The Early Universe”