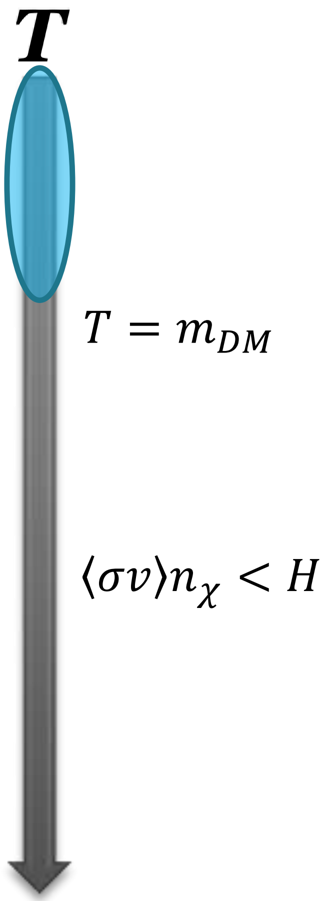
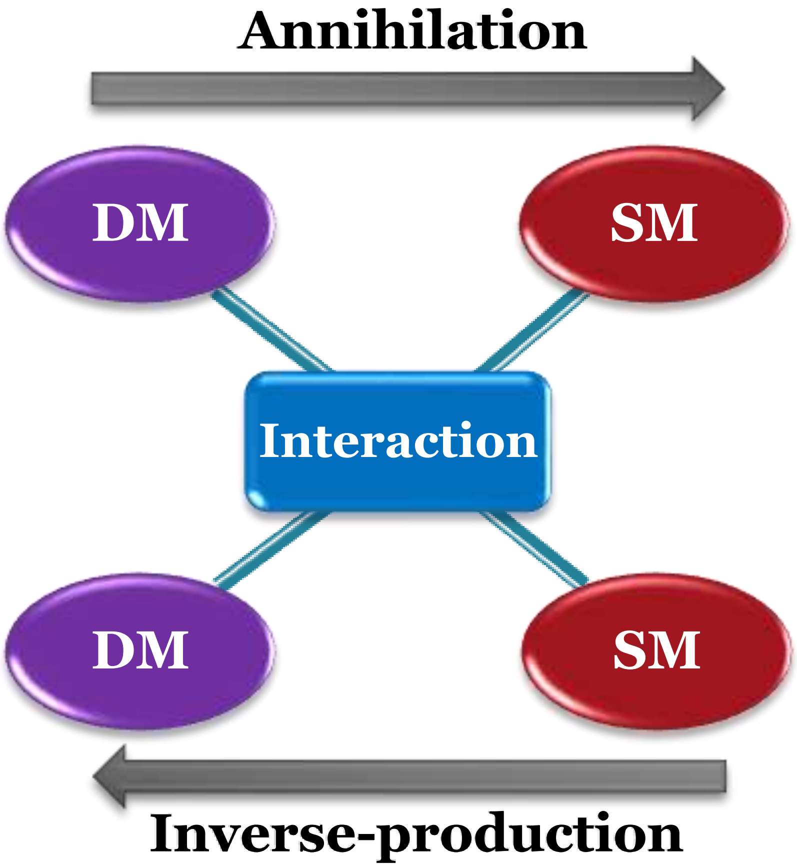
Thermal Freeze-out

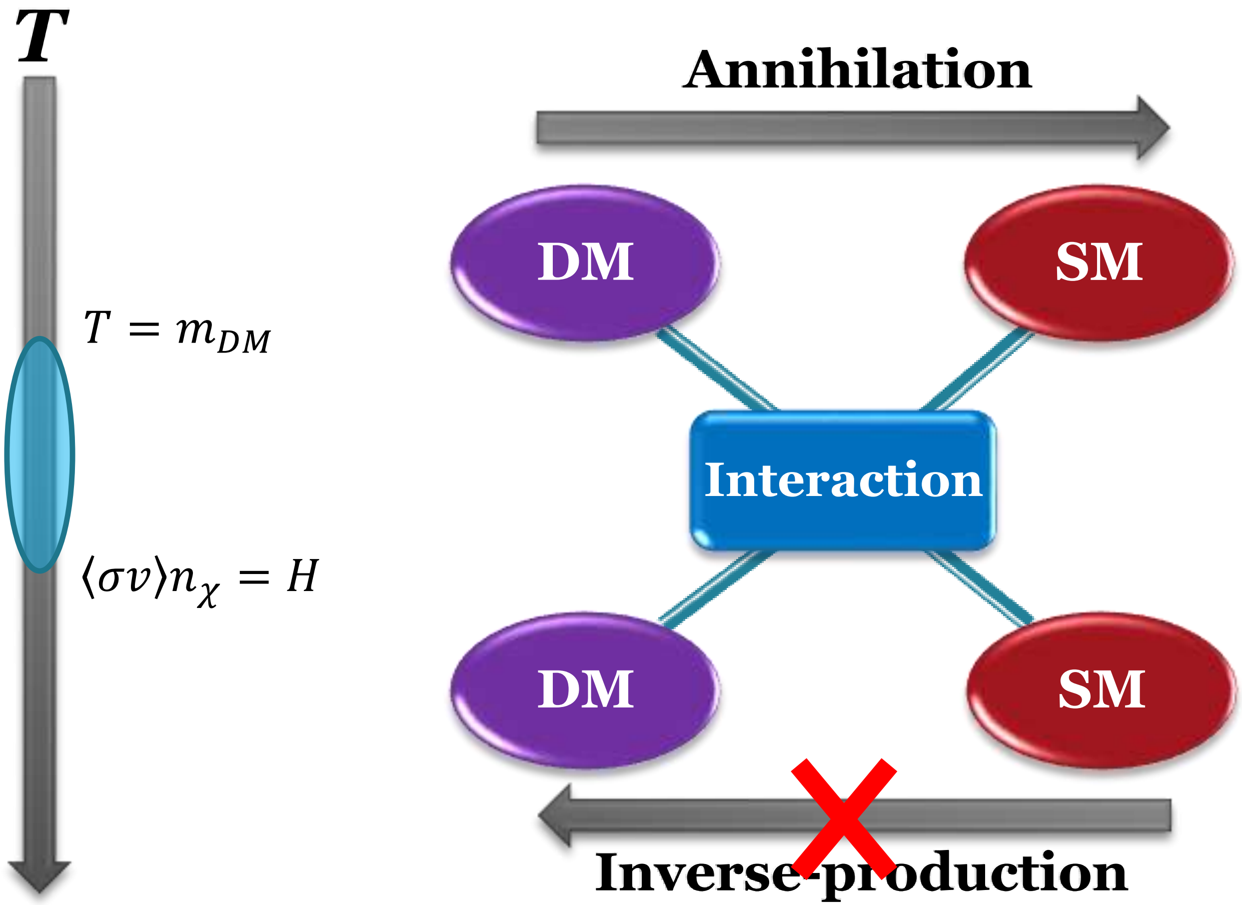


$T = m_{DM}$

$\langle \sigma v \rangle n_{\chi} < H$



Thermal Freeze-out



T

$T = m_{DM}$

$\langle \sigma v \rangle n_{\chi} = H$

Annihilation

DM

SM

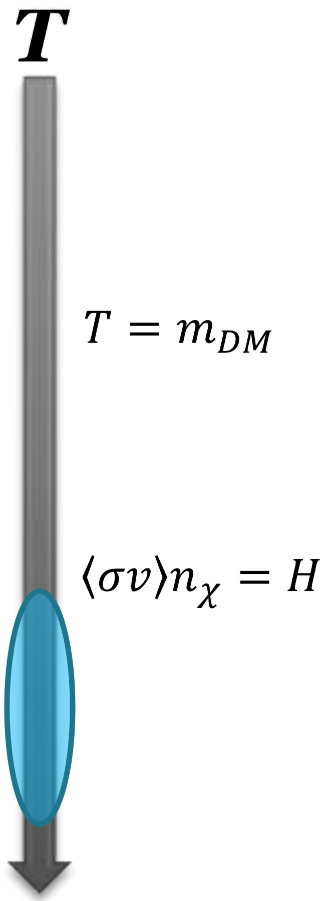
Interaction

DM

SM

Inverse-production

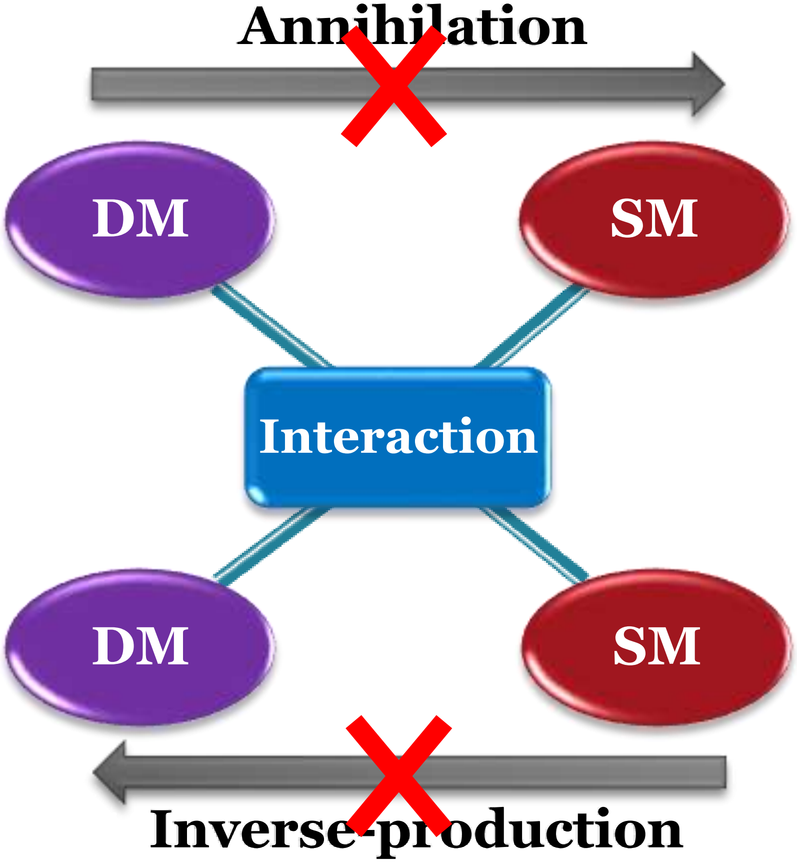
Thermal Freeze-out



T

$T = m_{DM}$

$\langle \sigma v \rangle n_{\chi} = H$



Annihilation

DM

SM

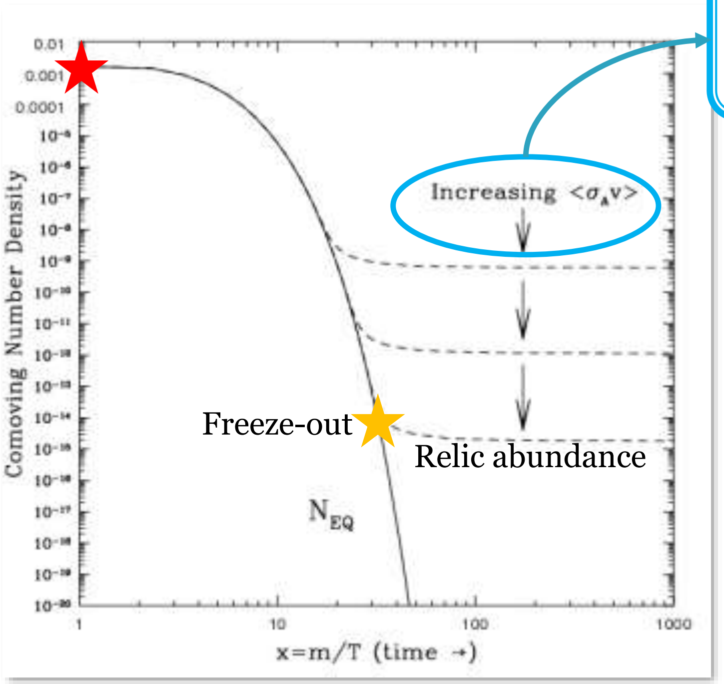
Interaction

DM

SM

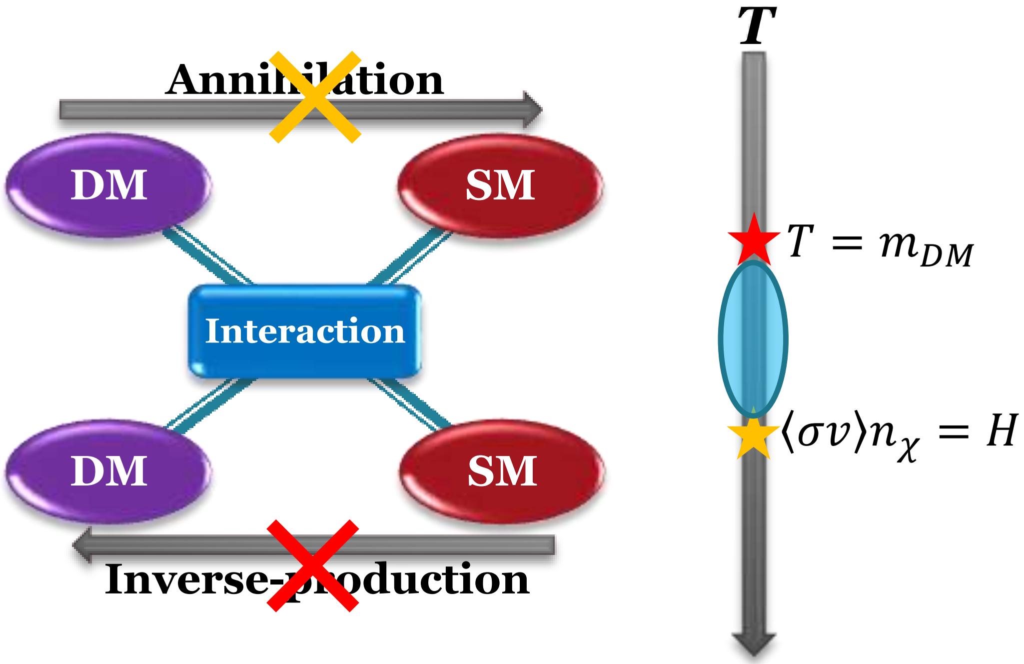
Inverse-production

Summary of Conventional Thermal FO



Stay in equilibrium longer

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_A v\rangle[(n_\chi)^2 - (n_\chi^{eq})^2]$$



- Correct thermal relic abundance:

$$\Omega h^2 \sim \frac{0.1\,pb}{\langle\sigma v\rangle} \sim \frac{3\times10^{-27}\,\text{cm}^3/\text{s}}{\langle\sigma v\rangle} \text{ with } \langle\sigma v\rangle \sim \frac{\alpha_\chi^2 m_\chi^2}{M^4} \;(M\text{: dark scale/mediator}) \text{ vs } \Omega h^2_{\text{obs}} \sim 0.1$$

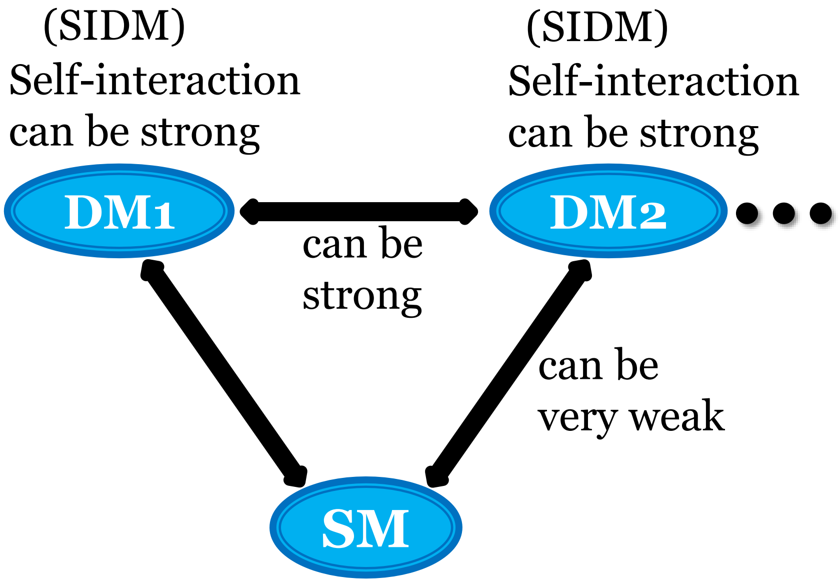
- Weak coupling ➔ Naturally Weak scale mass ➔ WIMP miracle!

~O(GeV – TeV) mass range favored ➔ weak scale (new) physics

New Ideas for DM Relic Abundance

❖ Alternative mechanisms for DM relic determination:

- ✓ Assisted freeze-out [1112.4491]
- ✓ Asymmetric dark matter [0901.4117]
- ✓ Cannibal dark matter [1602.04219, 1607.03108]
- ✓ Co-annihilation [PRD43 (1991) 3191]
- ✓ Co-decaying dark matter [1105.1652, 1607.03110]
- ✓ Continuum dark matter [2105.07035]
- ✓ Co-scattering mechanism [1705.08450]
- ✓ Dynamical dark matter [1106.4546]
- ✓ Dark freeze-out cogenesis [2112.10784]
- ✓ ELastically DEcoupling Relic (ELDER) [1512.04545]
- ✓ Freeze-in [0911.1120]
- ✓ Forbidden channels [PRD43 (1991) 3191 , 1505.07107]
- ✓ Inverse decay dark matter [2111.14857]
- ✓ Pandemic dark matter [2103.16572]
- ✓ Semi-annihilation [0811.0172, 1003.5912]
- ✓ Strongly Interacting Massive Particle (SIMP) [1402.5143, 1702.07860]
- ✓ ...



(SIDM)
Self-interaction
can be strong

(SIDM)
Self-interaction
can be strong

DM1

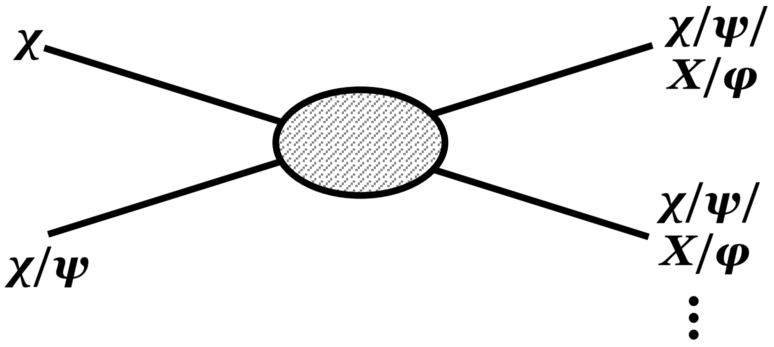
DM2

• • •

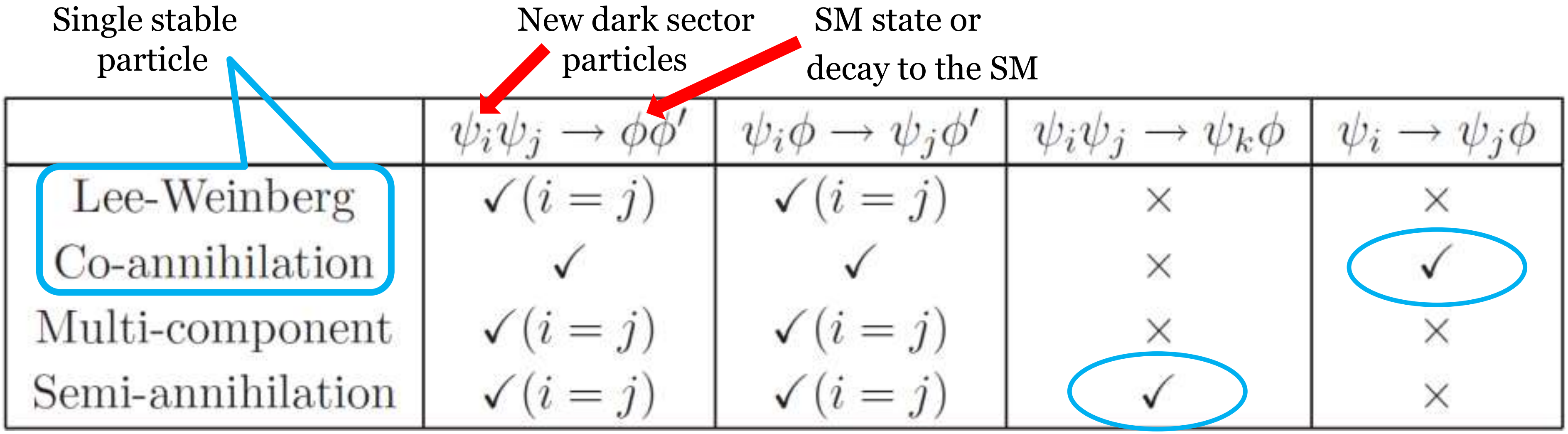
can be
strong

can be
very weak

SM



Freeze-out Scenarios



Single stable particle

New dark sector particles

SM state or decay to the SM

	$\psi_i\psi_j \to \phi\phi'$	$\psi_i\phi \to \psi_j\phi'$	$\psi_i\psi_j \to \psi_k\phi$	$\psi_i \to \psi_j\phi$
Lee-Weinberg	$\checkmark (i=j)$	$\checkmark (i=j)$	$\times$	$\times$
Co-annihilation	$\checkmark$	$\checkmark$	$\times$	$\checkmark$
Multi-component	$\checkmark (i=j)$	$\checkmark (i=j)$	$\times$	$\times$
Semi-annihilation	$\checkmark (i=j)$	$\checkmark (i=j)$	$\checkmark$	$\times$

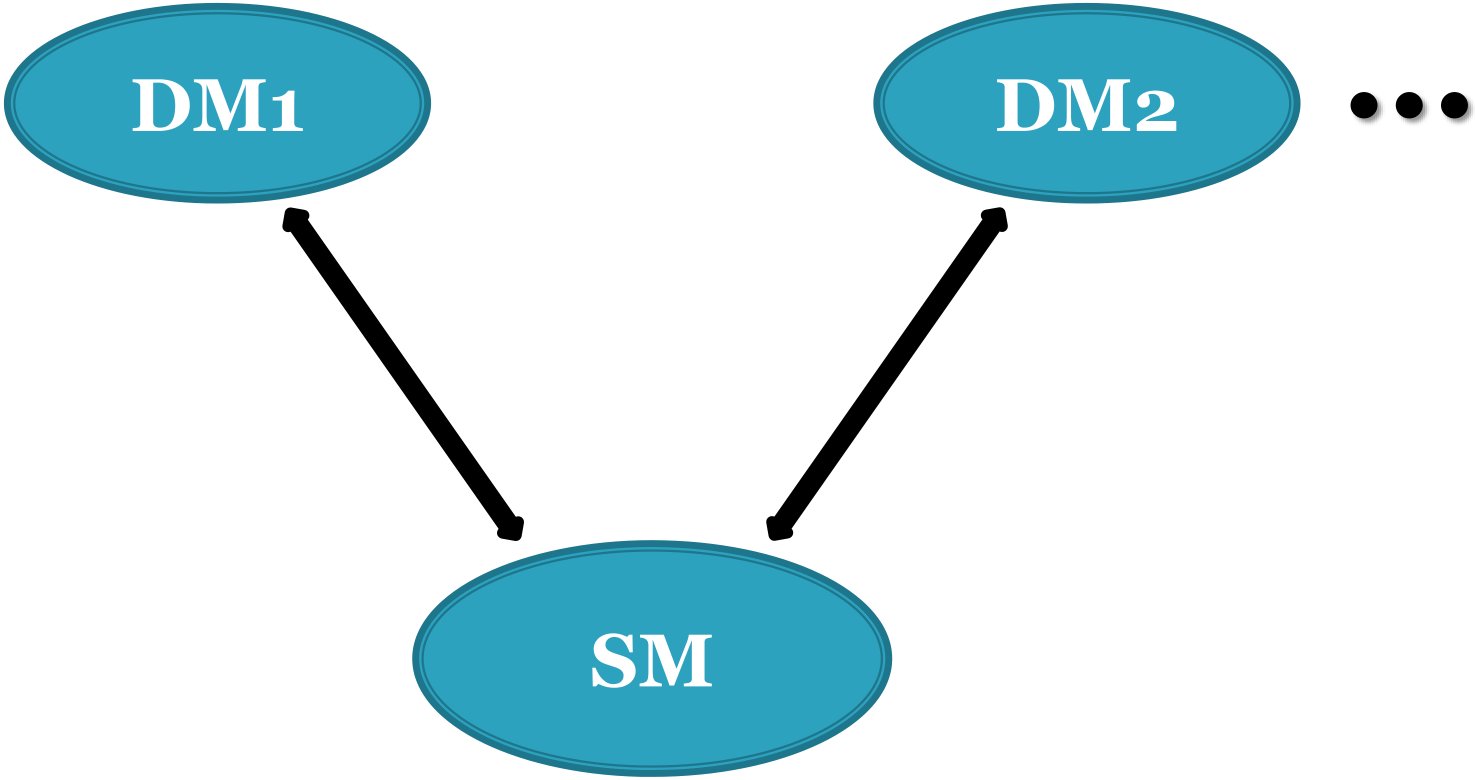
- $\psi_i\psi_j \to \psi_k\psi_m$: always present

- Lee-Weinberg: simplest & usual case. Lee & Weinberg PRL (1977)
- Co-annihilation: multi-particles but only one stable particle. Griest & Seckel PRD (1991)
- Multi-component: multi decoupled stable particles.
- Semi-annihilation: reactions among > 2 stable particles are important in determining DM relic density. D'Eramo & Thaler, JHEP (2010)

Standard Multi-component

- Standard approach: to assume that each particle is thermalized independently.

- Total DM density is $\Omega_{\rm DM} = \sum_i \Omega_i$.



Co-annihilation

- Involves *N* coupled Boltzmann equations:

Griest & Seckel, PRD43 (1991)

$$\begin{aligned}\frac{dn_i}{dt} &= -3Hn_i - \sum_{j=1}^{N}\langle\sigma_{ij}v_{ij}\rangle\left(n_in_j - n_i^{\rm eq}n_j^{\rm eq}\right)\\ &\quad-\sum_{j\neq i}\left[\langle\sigma'_{Xij}v_{ij}\rangle\left(n_in_X - n_i^{\rm eq}n_X^{\rm eq}\right) - \langle\sigma'_{Xji}v_{ij}\rangle\left(n_jn_X - n_j^{\rm eq}n_X^{\rm eq}\right)\right]\\ &\quad-\sum_{j\neq i}\left[\Gamma_{ij}\left(n_i - n_i^{\rm eq}\right) - \Gamma_{ji}\left(n_j - n_j^{\rm eq}\right)\right].\end{aligned}$$

- But possible to compute the relic density via standard methods:

$$\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^{N}\langle\sigma_{ij}v_{ij}\rangle\left(n_in_j - n_i^{\rm eq}n_j^{\rm eq}\right) \qquad n = \sum_{i=1}^{N} n_i$$

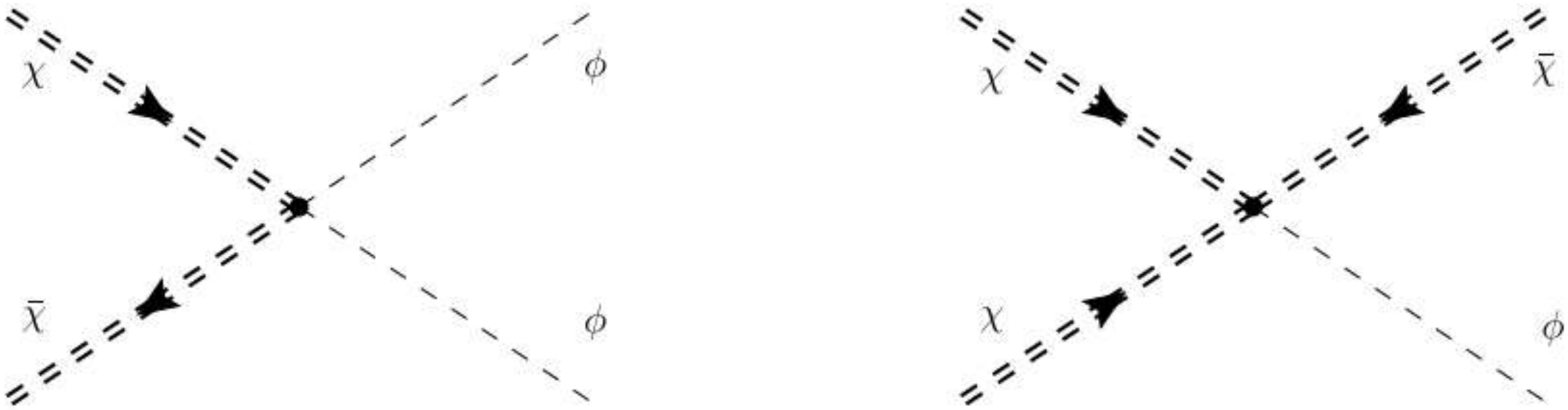
$$\frac{dn}{dt} = -3Hn - \langle\sigma_{\rm eff}v\rangle\left(n^2 - n_{\rm eq}^2\right) \qquad \langle\sigma_{\rm eff}v\rangle = \sum_{ij}\langle\sigma_{ij}v_{ij}\rangle\frac{n_i^{\rm eq}}{n^{\rm eq}}\frac{n_j^{\rm eq}}{n^{\rm eq}}$$

Semi-annihilation

D'Eramo& Thaler, JHEP1006 (2010)

- One has to solve a system of coupled Boltzmann equations.

$$\frac{dn_i}{dt} + 3Hn_i = -\langle\sigma_{ii}v_{\mathrm{rel}}\rangle\left(n_i^2 - n_i^{\mathrm{eq}\,2}\right) - \sum_{j,k}\langle\sigma_{ijk}v_{\mathrm{rel}}\rangle\left(n_i n_j - \frac{n_k}{n_k^{\mathrm{eq}}}n_i^{\mathrm{eq}}n_j^{\mathrm{eq}}\right)$$

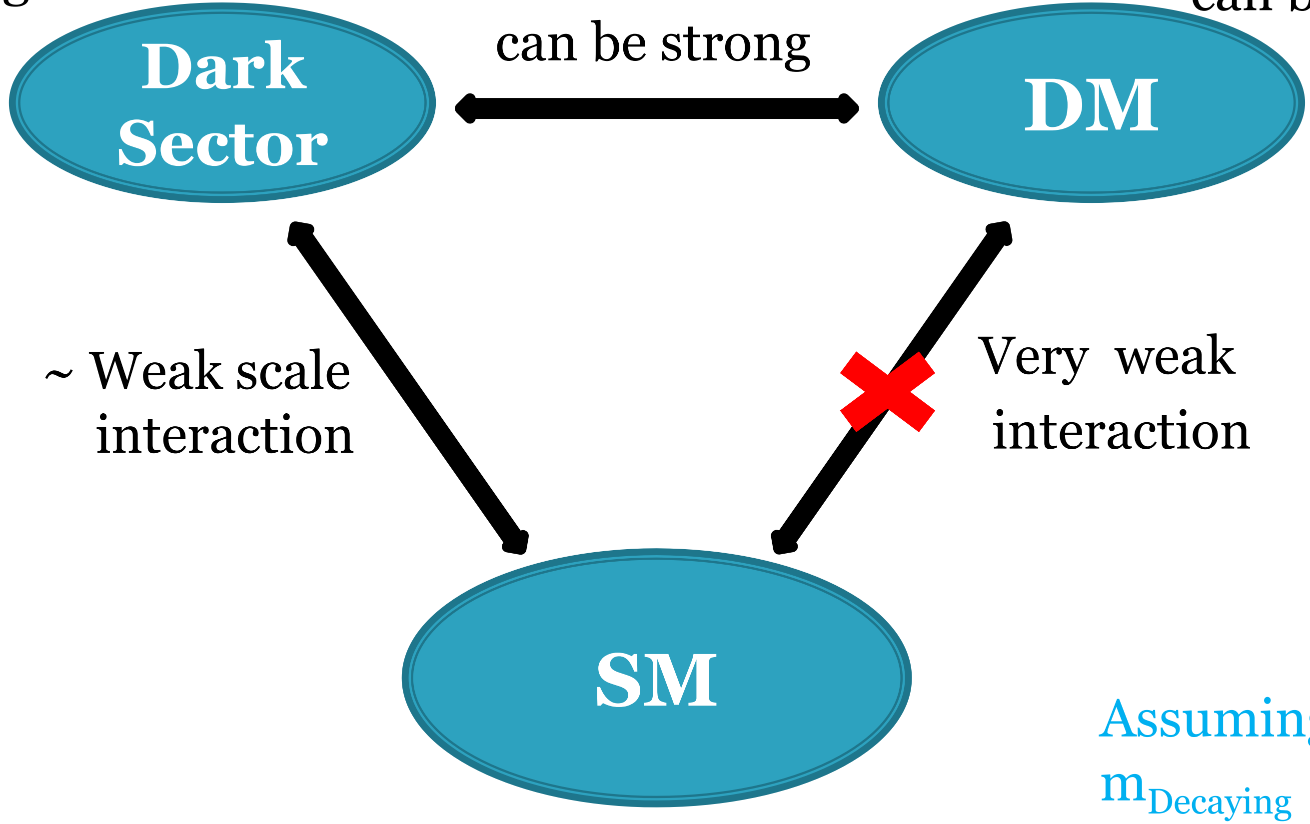


Co-decaying: Basic Set-up

[P. Bandyopadhyay, E. J. Chun & J.-C. Park, 1105.1652]

Dark sector particle
But decaying to SM

Self-interaction
can be strong



Dark Sector

can be strong

DM

~ Weak scale interaction

Very weak interaction

SM

Assuming
$m_{\text{Decaying}} < m_{\text{DM}}$

Co-decaying: Timeline

[arXiv:1607.03110]

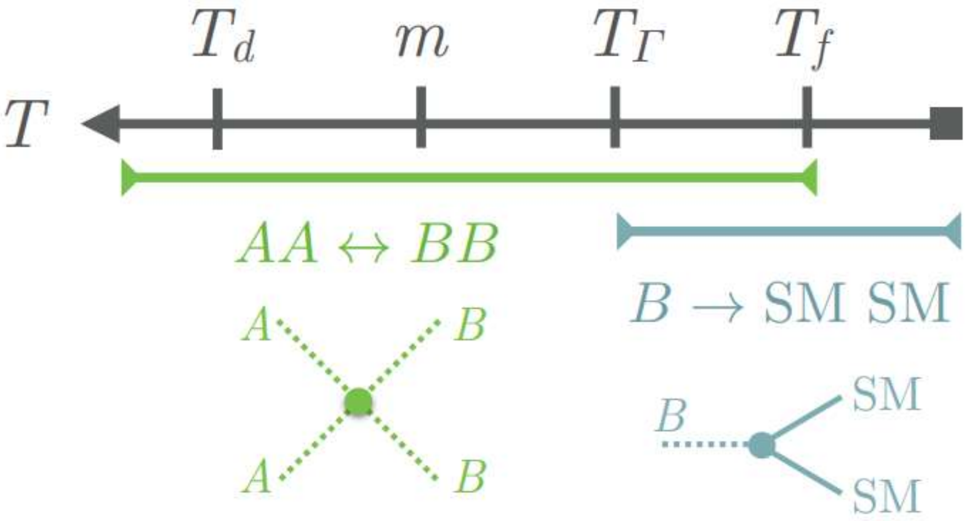
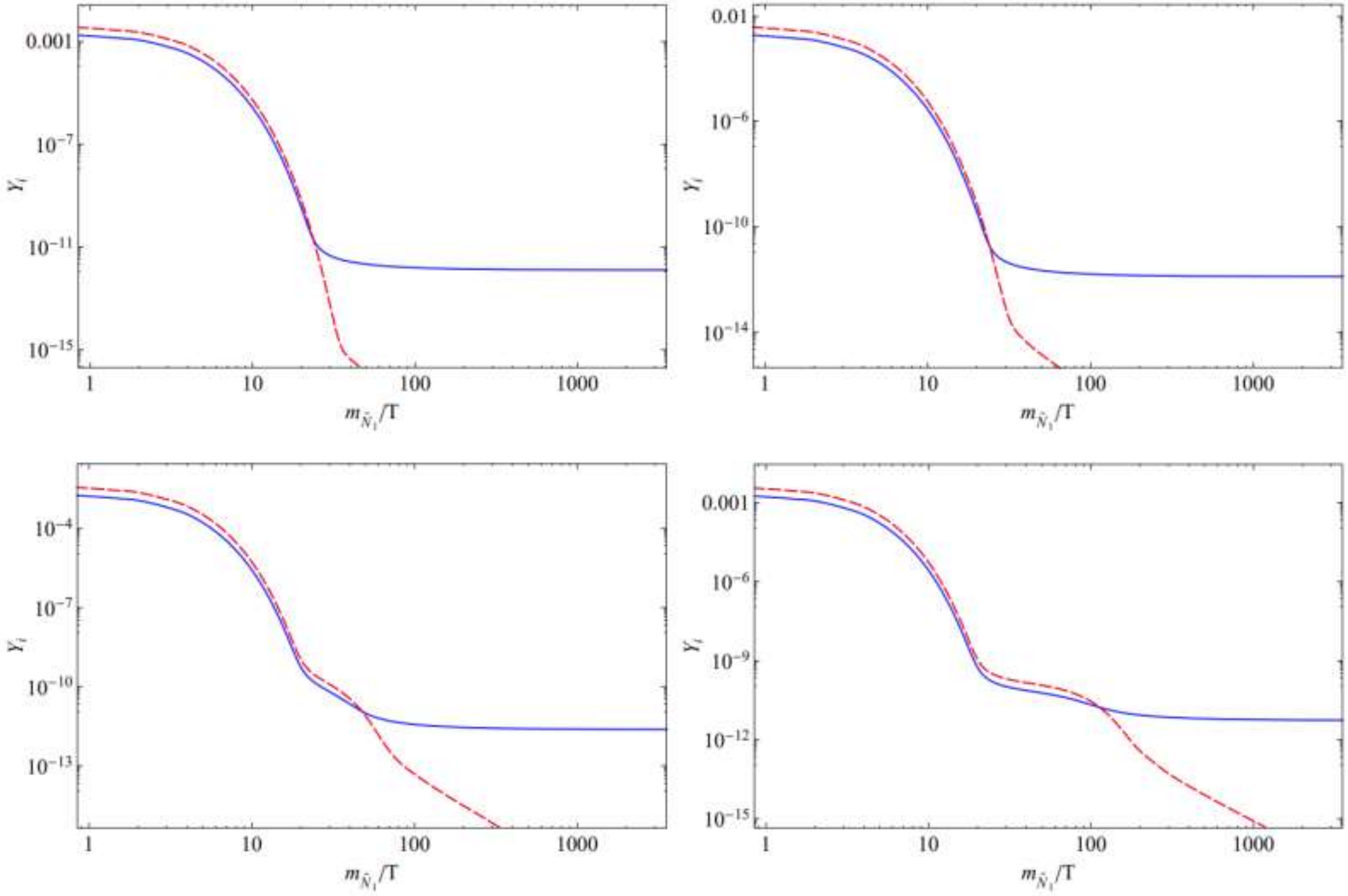


FIG. 1: Co-decay dark matter timeline. At T_d the SM and dark sector decouple; at T_Γ the decay of B's begin to deplete the dark sector density; and at T_f the $AA \leftrightarrow BB$ process freezes out, resulting in a relic abundance for the A particles.

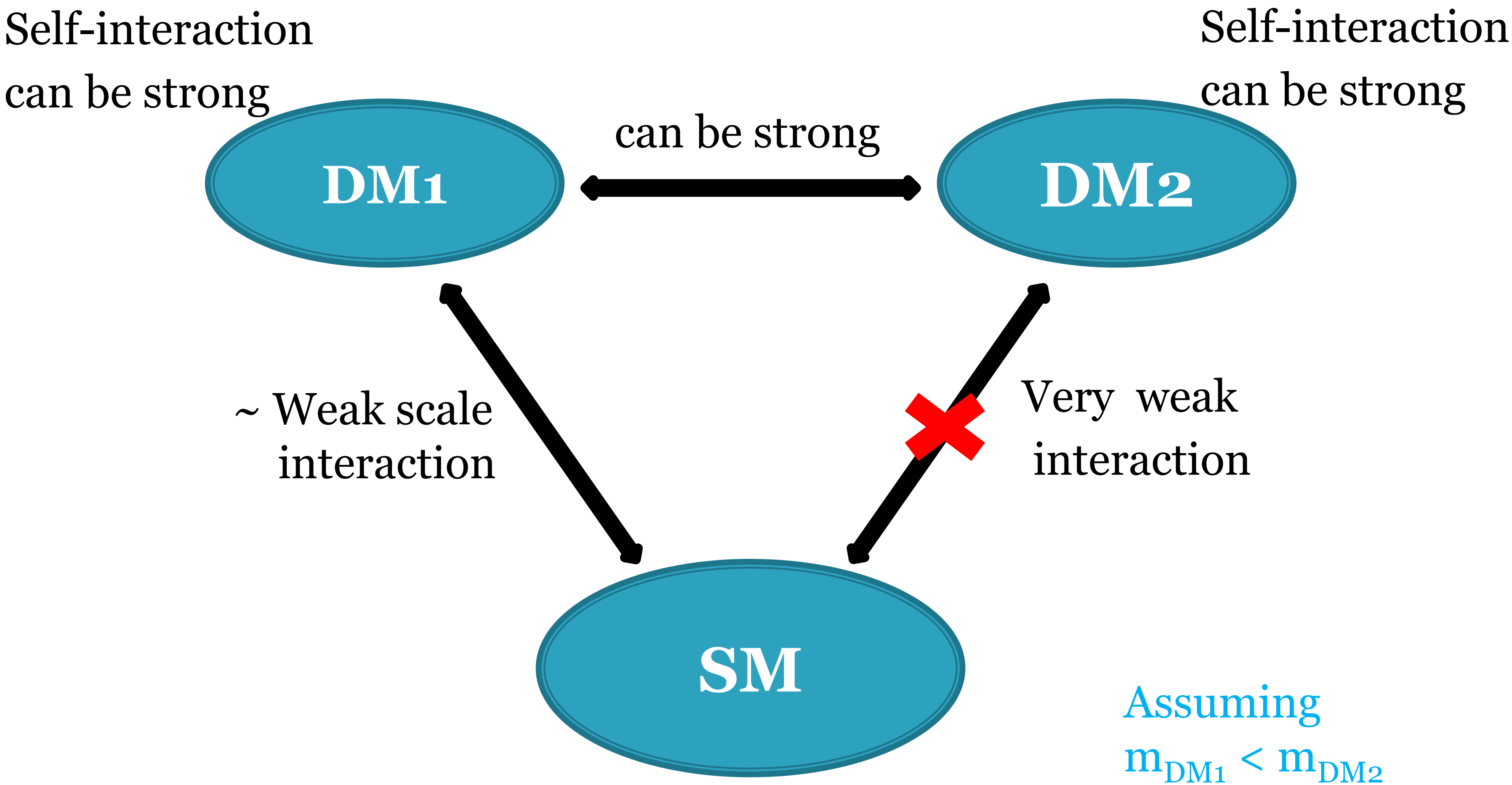
Co-decaying: Abundance Evolution

[P. Bandyopadhyay, E. J. Chun & J.-C. Park, 1105.1652]



Assisted FO: Basic Set-up

[G. Belanger & J.-C. Park, 1112.4491]



Self-interaction
can be strong

Self-interaction
can be strong

DM1

can be strong

DM2

~ Weak scale
interaction

Very weak
interaction

SM

Assuming
$m_{DM1} < m_{DM2}$

Assisted FO: Boltzmann Equations

[G. Belanger & J.-C. Park, 1112.4491]

- To find the relic abundances of DM 1&2, we should solve a set of coupled Boltzmann equations.

$$\frac{dn_2}{dt} + 3Hn_2 = -\langle\sigma v\rangle_{22\to 11}\left[(n_2)^2 - \frac{(n_2^{\rm eq})^2}{(n_1^{\rm eq})^2}(n_1)^2\right] ,$$

$$\frac{dn_1}{dt} + 3Hn_1 = -\langle\sigma v\rangle_{11\to XX}\left[(n_1)^2 - (n_1^{\rm eq})^2\right] - \langle\sigma v\rangle_{11\to 22}\left[(n_1)^2 - \frac{(n_1^{\rm eq})^2}{(n_2^{\rm eq})^2}(n_2)^2\right]$$

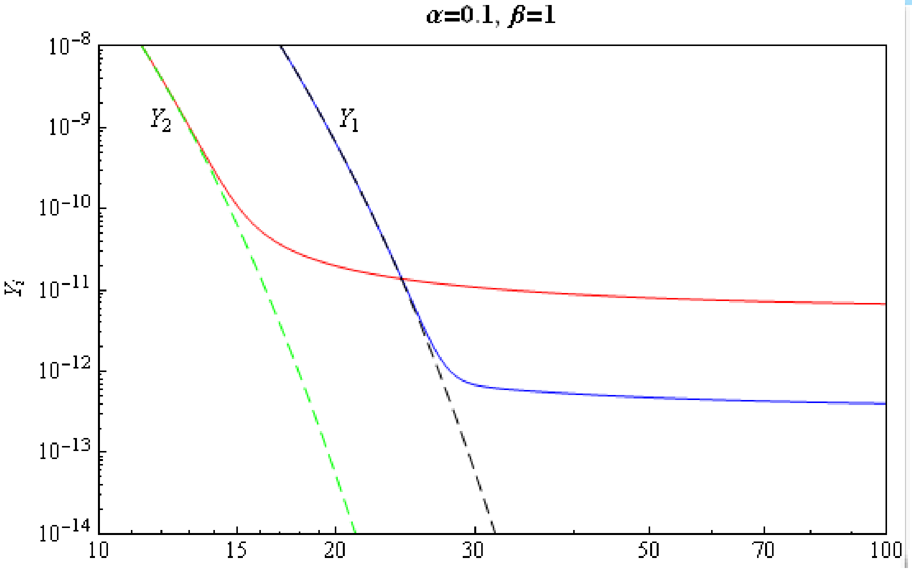
X: SM particles

- If we limit our analysis to s-wave annihilation, we can simply express the relevant matrix elements:

$$\alpha \equiv \mathcal{M}_{22\to 11} = \mathcal{M}_{11\to 22} , \quad \beta \equiv \mathcal{M}_{11\to XX}$$

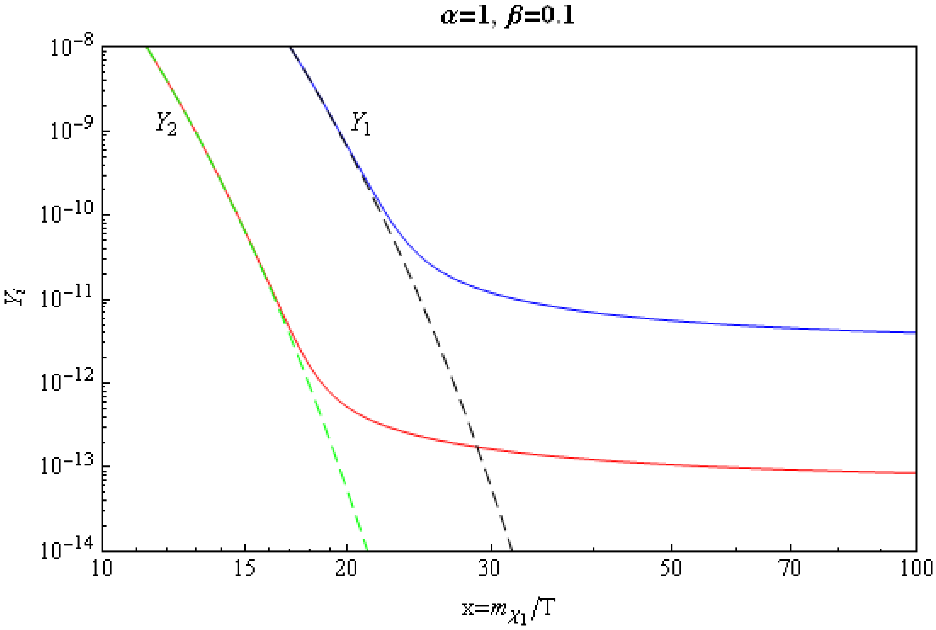
Assisted FO: Abundance Evolution

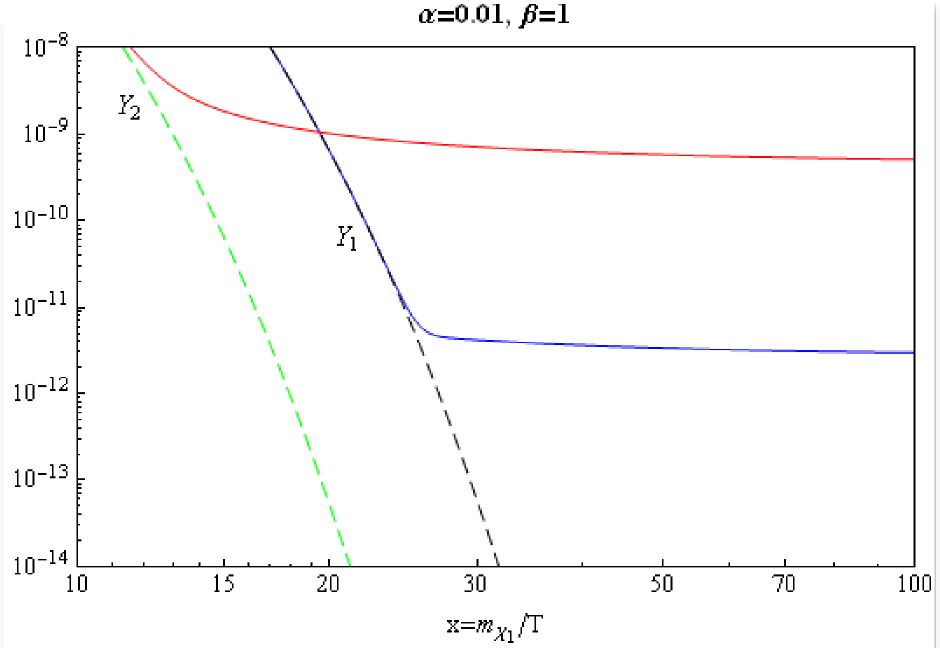
[G. Belanger & J.-C. Park, 1112.4491]



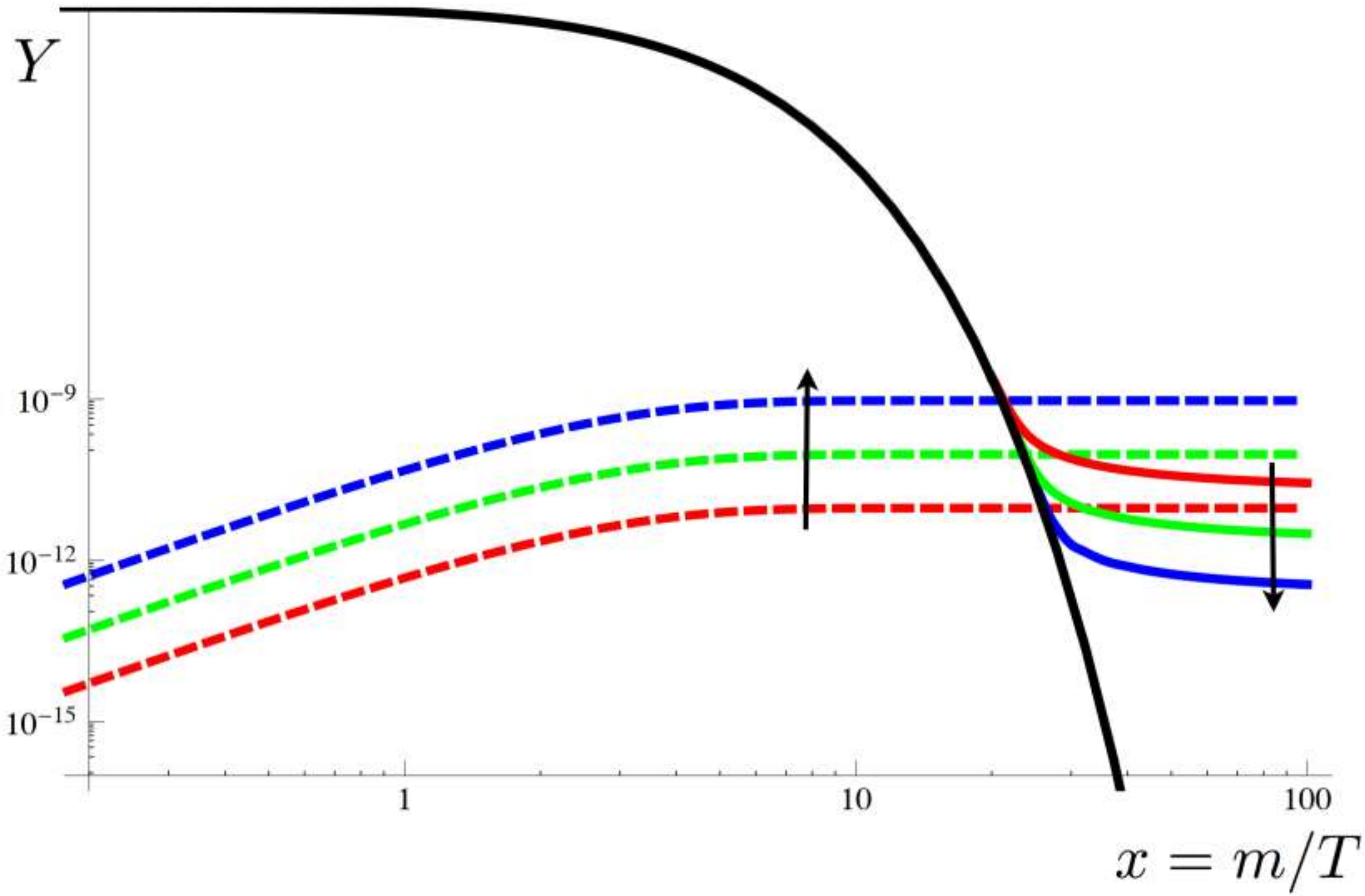
$Y_i=n_i/s$, $x=m_1/T$,
$m_1=100$ GeV, $m_2=150$ GeV

$$\alpha \equiv \mathcal{M}_{22\to 11} = \mathcal{M}_{11\to 22}\ , \quad \beta \equiv \mathcal{M}_{11\to XX}$$

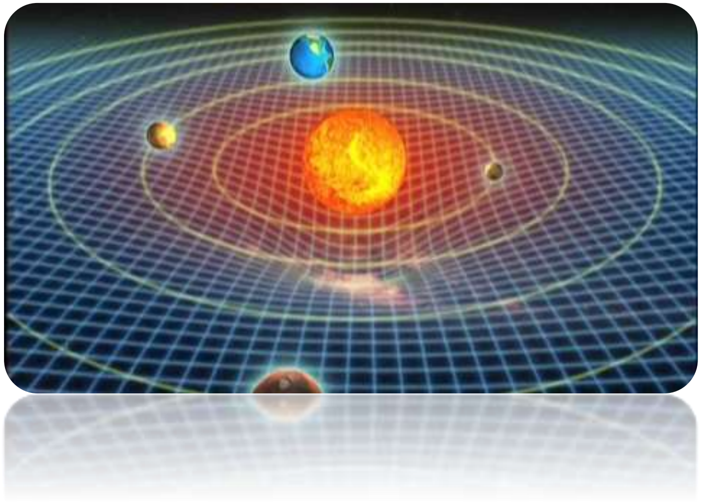




Freeze-in: Abundance Evolution



Observational Evidence of DM



- ✓ Galaxy rotation curve
- ✓ Coma cluster
- ✓ Gravitational lensing
- ✓ Bullet cluster
- ✓ Structure formation
- ✓ Cosmic microwave background radiation (CMBR)
- ✓ Sky surveys
- ✓ Type Ia supervovae
- ✓ Baryonic acoustic oscillation (BAO)
- ✓ …

Nature of **DM**?

Irene ⊂ SM

but DM ✖ SM

Exercises

3. Nuclear recoil spectrum in DM direct detection:

(a) $v_{\min} = \sqrt{E_R m_N / 2\mu_{\chi N}^2} = \frac{m_\chi + m_N}{m_\chi} \sqrt{E_R / 2 m_N}$

(b) Shape of nuclear recoil spectrum (dependence on m_χ)

4. Fluxes of DM annihilation products, e.g., $e^{\pm}, \bar{p}$:

(a) Annihilation cross section dependence

(b) m_χ dependence