Lectures on Axion Physics

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Outline

01 Overview of axion physics

- Motivation: Strong CP problem
- KSVZ & DFSZ & Axion-like particles
- Effective couplings to SM ($g_{a\gamma}$)
- Current status

02 Experimental searches

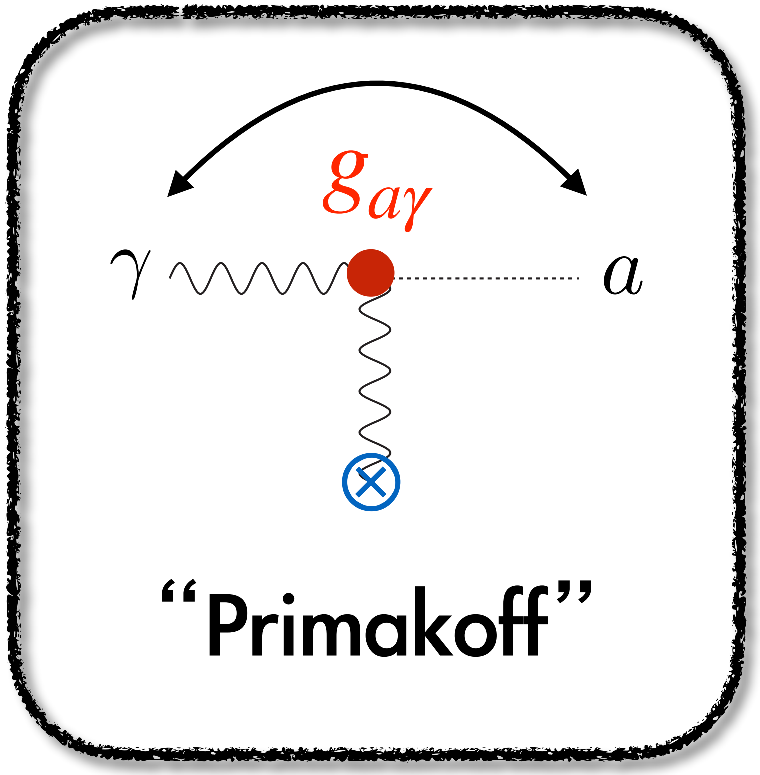
- Light-Shining-Through-Walls
- Helioscope
- Microwave cavity experiments

03 Astrophysical searches

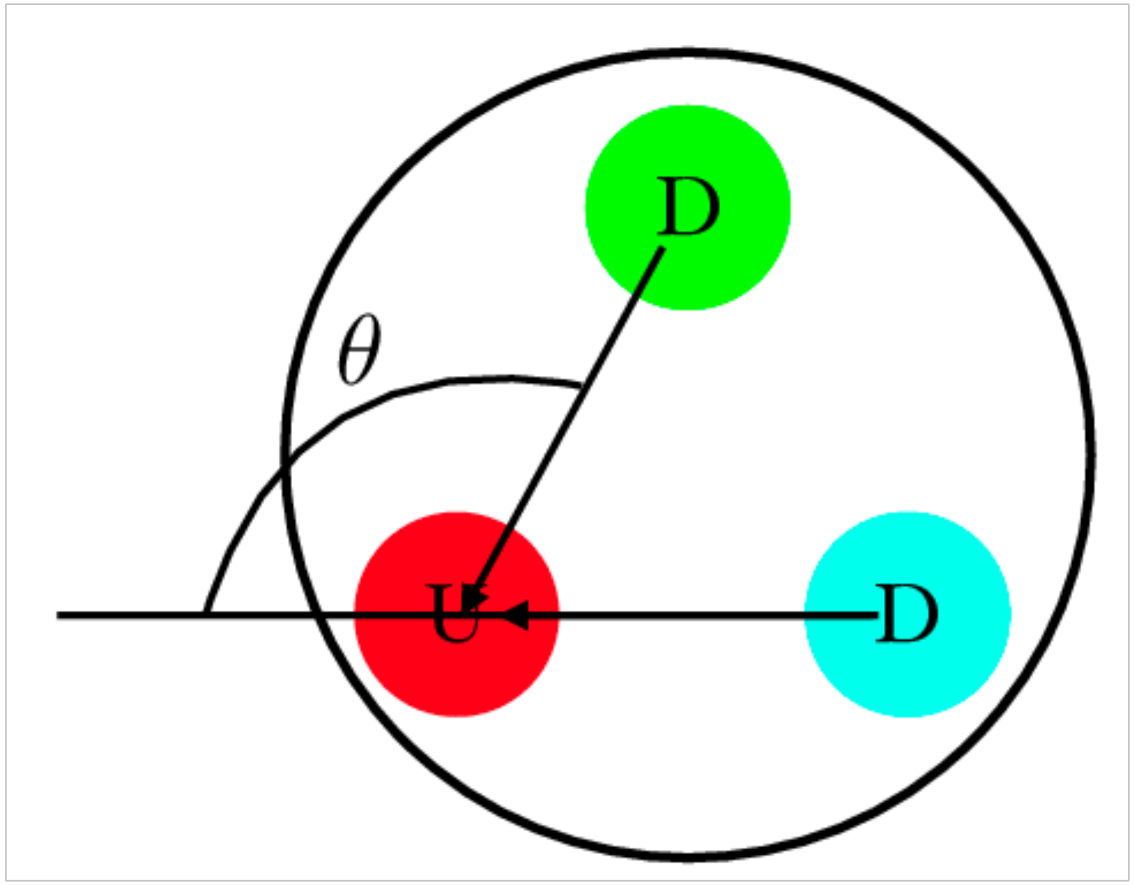
- Stellar argument (Sun & Horizontal branch & SN)

04 Cosmological searches

- Dark radiation



Strong CP problem **@ classical level**



[A. Hook, TASI 2018]

- $\vec{d} = \sum q\vec{r}$
- Classical estimate with a size of neutron $\sim m_\pi^{-1}$: $|d_n| \approx 10^{-13}\sqrt{1-\cos\theta}\, e\,\mathrm{cm}$

Low energy QCD

θ vacua

[H. Forkel, hep-ph/0009136]

[S. Coleman, "*Aspects of Symmetry*"]

- The QCD Lagrangian around the confinement scale

$$\mathscr{L}_{\rm QCD} = -\frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu} - i\bar{q}\gamma^{\mu} D_{\mu} q - \frac{g_s^2}{32\pi^2} \theta_{\rm QCD} G^{a\mu\nu} \tilde{G}^{a}_{\mu\nu} - \left(\bar{q}_L M_q q_R + {\rm h.c.} \right)$$

$$q_{L,R} = P_{L,R} \left(u, d, s\right)^{\rm T}, \; M_q = {\rm diag}\left[m_u, m_d, m_s\right]$$

- CP violation in the strong interactions

$$\bar{\theta} = \theta_{\rm QCD} + \arg {\rm Det}\, M_q$$

Anomalous symmetries

[M. Peskin and D. V. Schroeder, "An Introduction to Quantum Field Theory" Sec. 19]

[M. Srednicki, "*Quantum Field Theory*" Sec. 77]

- Global symmetries $U(3)_L \times U(3)_R$

$$q_L \to L q_L, \;\; q_R \to R q_R, \;\; M_q \to L M_q R^{\dagger}, \;\; \bar{\theta} \to \bar{\theta} + \arg {\rm Det}\left[L R^{\dagger}\right]$$

Effective Chiral Lagrangian

- Quark-antiquark condensation as the chiral breaking

$$\begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} + \frac{2}{\sqrt{6}}\eta'$$

$\left\langle \bar{q}_L q_R \right\rangle = \Lambda_{\mathrm{QCD}}^3 \Sigma$ where $U = e^{i\Phi}$ with Φ the meson-field matrix (octet+singlet)

- Effective theory of mesons

$$M_q \to L M_q R^\dagger, \quad \bar{\theta} \to \bar{\theta} + \arg \mathrm{Det} \left[L R^\dagger \right], \quad \Sigma \to R \Sigma L^\dagger$$

- Meson potential at the leading order

$$-V = \left(\Lambda^4 e^{i\bar{\theta}} \, \mathrm{Det}\, \Sigma + \mathrm{h.c.} \right) + \left(\frac{m_\pi^2 f_\pi^2}{2(m_u + m_d)} \, \mathrm{tr} \left[M_q e^{i\Phi} \right] + \mathrm{h.c.} \right)$$

$$U(1)_A \Rightarrow \eta'$$

The vacuum in the meson space

- Meson vacuum in the presence of $\bar{\theta} \neq 0$, i.e., $\left.\dfrac{\partial V}{\partial \Phi}\right|_{\bar{\theta}} = 0$

$$\langle \Phi \rangle = \mathrm{diag}\ \left[\phi_u, \phi_d, \phi_s\right] \quad \Rightarrow \quad -V = \Lambda^4 \cos\left(\sum \phi_q + \bar{\theta}\right) + \frac{m_\pi^2 f_\pi^2}{m_u + m_d} \sum m_q \cos \phi_q$$

- In the limit of $m_{u,d} \ll m_s \ll \Lambda_{\mathrm{QCD}}$,

i) $\bar{\theta} + \sum \phi_q \approx 0,$ ii) $m_u \sin \phi_u = m_d \sin \phi_d = m_s \sin \phi_s$

$$\boxed{\sin \phi_u \simeq - \frac{m_d \sin \bar{\theta}}{\sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos \bar{\theta}}}, \quad \sin \phi_d \simeq - \frac{m_u \sin \bar{\theta}}{\sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos \bar{\theta}}}, \quad \sin \phi_s \simeq - \frac{m_u m_d \sin \bar{\theta}}{m_s \sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos \bar{\theta}}}}$$

if $\bar{\theta} \ll 1$ ☞

$$\langle \Phi \rangle = - \frac{M_q^{-1}}{\mathrm{tr}\, M_q^{-1}} \bar{\theta}$$

Strong CP problem **@ quantum level**

Chiral perturbation theory

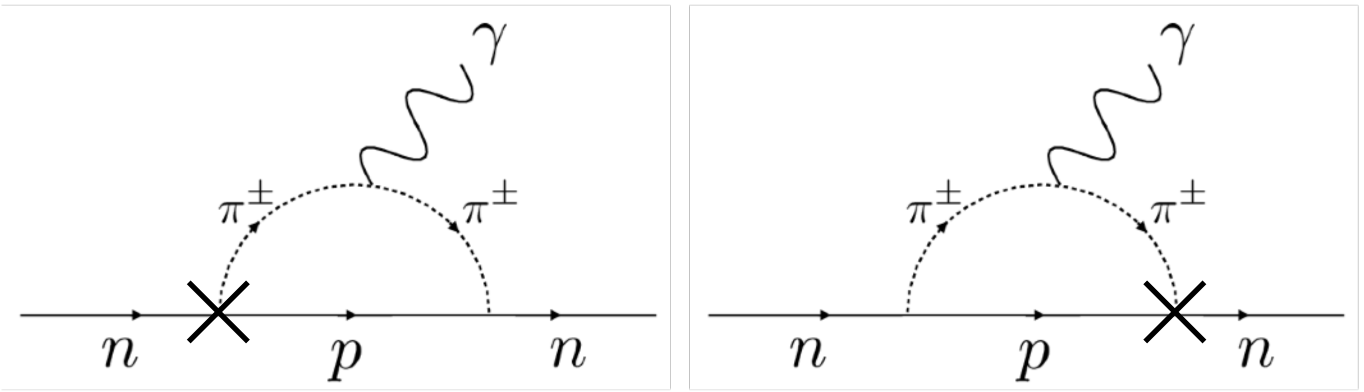
[M. Srednicki, "*Quantum Field Theory*" Sec. 83 & 94]

- P & T violating pion-nucleon coupling for $\bar{\theta} \ll 1$

$$\mathscr{L}_{\bar{\theta}\pi N} = -\bar{\theta}\frac{c_+\mu}{f_\pi}\pi^a N\sigma^a N, \quad \text{where } \mu = \frac{m_u m_d}{m_u + m_d} \ \& \ c_+ \approx 1.7$$

baryon mass spliting

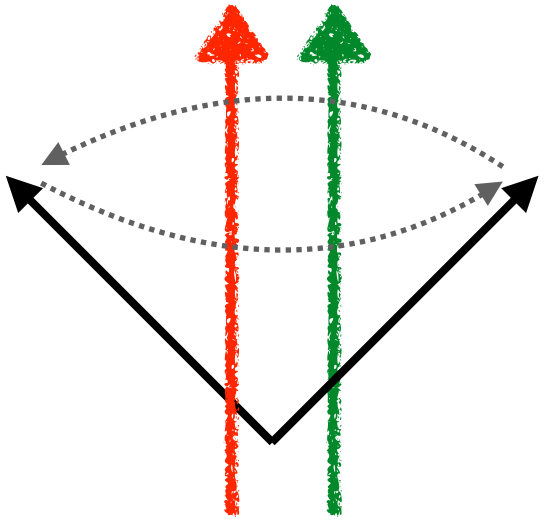
- Electric dipole moment of the neutron



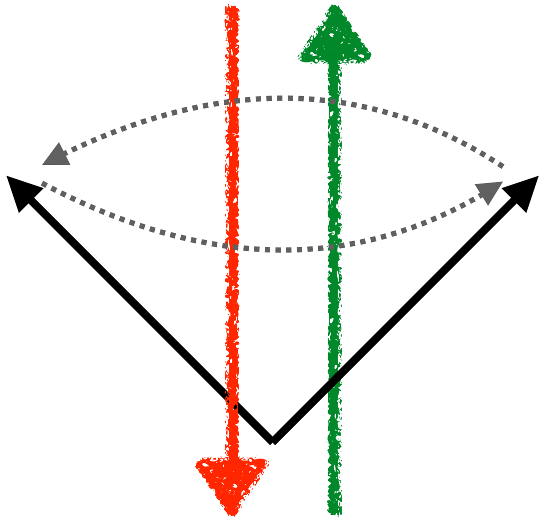
$$\boxed{d_n F_{\mu\nu}\bar{n}\gamma^{\mu\nu}i\gamma^5 n \quad \textbf{with} \quad d_n \sim 3\times 10^{-16}\,\bar{\theta}\,e\,\mathrm{cm}}$$

Experimental bound on d_n

- Conceptual way to measure d_n via a (Larrmor) precession in $\vec{E}$ & $\vec{B}$



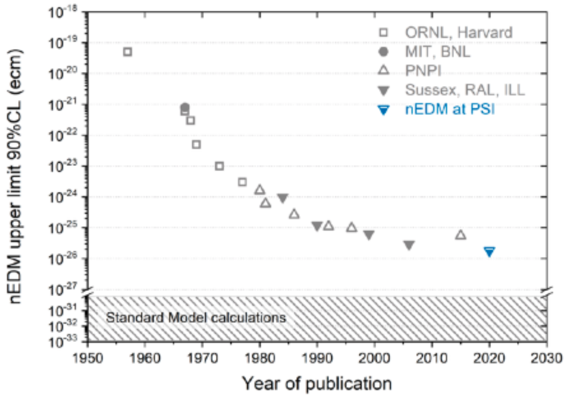
$$\nu = 2\mu B + 2dB$$



$$\nu = 2\mu B - 2dB$$

- Bound on electric dipole moment of the neutron

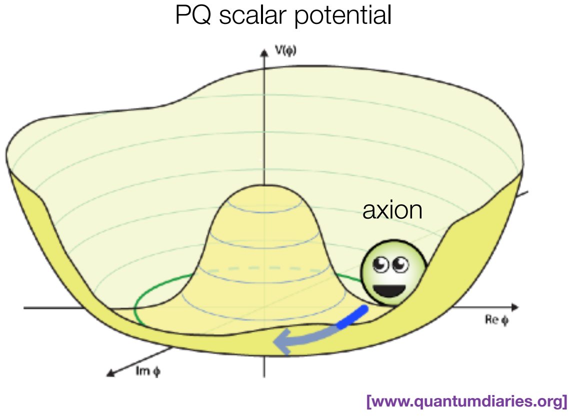
$$\boxed{d_n \sim 3 \times 10^{-16}\,\bar{\theta}\,e\,\mathrm{cm} \leq 10^{-26} e\,\mathrm{cm} \quad ☞ \quad \bar{\theta} < 10^{-10}}$$



Axion

- Solution for the strong CP problem
- Global Peccei-Quinn symmetry anomalous to the SM - **PQ fermions**

$$\Psi \to e^{i\alpha\gamma^5/2}\Psi \quad ☞ \quad \frac{g_s^2}{32\pi^2}\alpha G_{\mu\nu}\tilde{G}^{\mu\nu}$$



- Spontaneous breaking of the PQ symmetry - **PQ scalar field**

$$V_\varphi = \lambda_\varphi\left(|\varphi|^2 - f_a^2/2\right) \quad ☞ \quad \varphi \to \left(f_a/\sqrt{2}\right)e^{ia/f_a}$$

- NG boson a, the so-called '*axion*'

$$\mathscr{L}_{\rm PQ} \supset -y_\Psi \varphi \bar{\Psi}_L \Psi_R + \mathrm{h.c.} \quad ☞ \quad \bar{\theta} \to \bar{\theta} + a/f_a$$

Axion potential

$$-V = \Lambda^4 \cos\left(\sum \phi_q + \bar{\theta}\right) + \frac{m_\pi^2 f_\pi^2}{m_u + m_d} \sum m_q \cos\phi_q$$

$$\sin\phi_u \simeq -\frac{m_d \sin\bar{\theta}}{\sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos\bar{\theta}}}, \quad \sin\phi_d \simeq -\frac{m_u \sin\bar{\theta}}{\sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos\bar{\theta}}}, \quad \sin\phi_s \simeq -\frac{m_u m_d \sin\bar{\theta}}{m_s\sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos\bar{\theta}}}$$

$$\Rightarrow \quad \boxed{V_{\bar{\theta}} = -f_\pi^2 m_\pi^2 \sqrt{\frac{m_u^2 + m_d^2 + 2m_u m_d \cos\bar{\theta}}{\left(m_u + m_d\right)^2}}}$$

- With the minimum of $V_{\bar{\theta}}$ at $\bar{\theta} = 0$ & $\bar{\theta} \to \bar{\theta} + a/f_a$,

the strong CP problem is solved dynamically

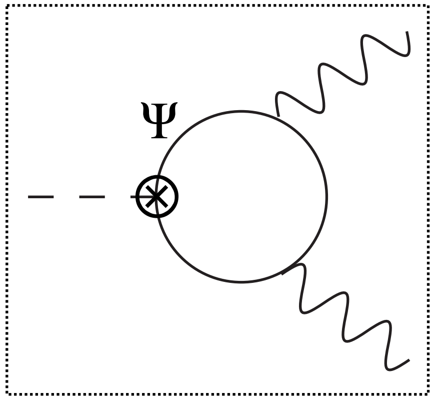
KSVZ axion scenario

- Ingredients: heavy colored PQ fermion (Ψ) & PQ singlet scalar ($\varphi \to f_a e^{ia/f_a}/\sqrt{2}$)

$$-y_\Psi \varphi \bar{\Psi}_L \Psi_R + \mathrm{h.c.}$$

- Triangle diagram with Ψ-loop leads to

$$-\frac{g_s^2}{32\pi^2}\frac{a}{f_a}G_{\mu\nu}\tilde{G}^{\mu\nu}$$



if $\bar{\theta} \ll 1$

$$\langle \Phi \rangle = -\frac{M_q^{-1}}{\mathrm{tr}\, M_q^{-1}}\bar{\theta} \quad \Rightarrow \quad \boxed{L = \exp\left[-i\frac{1}{2}\frac{M_q^{-1}}{\mathrm{Tr}\, M_q^{-1}}\bar{\theta}\right] \quad \& \quad R = \exp\left[i\frac{1}{2}\frac{M_q^{-1}}{\mathrm{Tr}\, M_q^{-1}}\bar{\theta}\right]}$$

$$\boxed{L_{\bar{\theta}\gamma} = \frac{e^2}{8\pi^2}N_c \mathrm{Tr}\left[\frac{1}{2}\frac{M_q^{-1}}{\mathrm{Tr}\, M_q^{-1}}\bar{\theta}\cdot Q_E^2\right]F_{\mu\nu}\tilde{F}^{\mu\nu} \approx \frac{e^2}{48\pi^2}\left(\frac{4m_d + m_u}{m_u + m_d}\right)\bar{\theta}F_{\mu\nu}\tilde{F}^{\mu\nu}}$$

[M. Dine, W. Fischler and M. Srednicki, 1980] [A.R. Zhitnitsky, 1980]

DFSZ axion scenario

- Ingredients: extension of Higgs sector & PQ singlet scalar ($\varphi \to v_\varphi e^{ia/v_\varphi}/\sqrt{2}$)

$$-B\left(\frac{\varphi}{v_\varphi/\sqrt{2}}\right)^n H_u^{\mathrm{T}} i\sigma^2 H_d + \mathrm{h.c.} \quad ☞ \quad -Be^{in\frac{a}{v_\varphi}} H_u^{\mathrm{T}} i\sigma^2 H_d + \mathrm{h.c.}$$

✓ $Y_{H_{u,d}} = \pm\frac{1}{2}$

$$\mathscr{L}_{\mathrm{Yukawa}} = -\bar{Q}_L \tilde{H}_u y_u u_R + \bar{Q}_L \tilde{H}_d y_d d_R + \bar{L}_L \tilde{H}_d y_e e_R + \mathrm{h.c.}$$

- Axionic rotation $H_i \to e^{-i\frac{n}{2}\frac{a}{v_\varphi}} H_i$ & $\psi_{\mathrm{SM}} \to e^{iq_\psi \frac{a}{v_\varphi}} \psi_{\mathrm{SM}}$,

$$-\frac{a}{v_\varphi}\left(3n\frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + 8n\frac{e^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}\right) - \frac{\partial_\mu a}{v_\varphi}\left(\sum_f q_f \bar{f}\gamma^\mu f - \sum_i \frac{n}{2} H_i^\dagger i\overleftrightarrow{D}^\mu H_i\right)$$

$$\downarrow$$

$$-\frac{a}{f_a}\left(\frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \frac{8}{3}\frac{e^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}\right) + \frac{\partial_\mu a}{f_a}\left(\sum_{f=u,d,e} \frac{1}{12}\bar{f}\gamma^\mu\gamma^5 f + \sum_i \frac{1}{6} H_i^\dagger i\overleftrightarrow{D}^\mu H_i\right)$$

DFSZ axion scenario

$$-\frac{a}{f_a}\left(\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}+\frac{8}{3}\frac{e^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)+\frac{\partial_\mu a}{f_a}\left(\sum_{f=u,d,e}\frac{1}{12}\bar{f}\gamma^\mu\gamma^5 f+\sum_i\frac{1}{6}H_i^\dagger i\overleftrightarrow{D}^\mu H_i\right)$$

- $D_\mu H_i^0 \supset -ig_Z Z_\mu\left(-Y_{H_i}\right)|H_i|$ ☞ a mixing of $\left(\partial_\mu a\right)Z^\mu$

✓ $Y_{H_{u,d}}=\pm\frac{1}{2}$

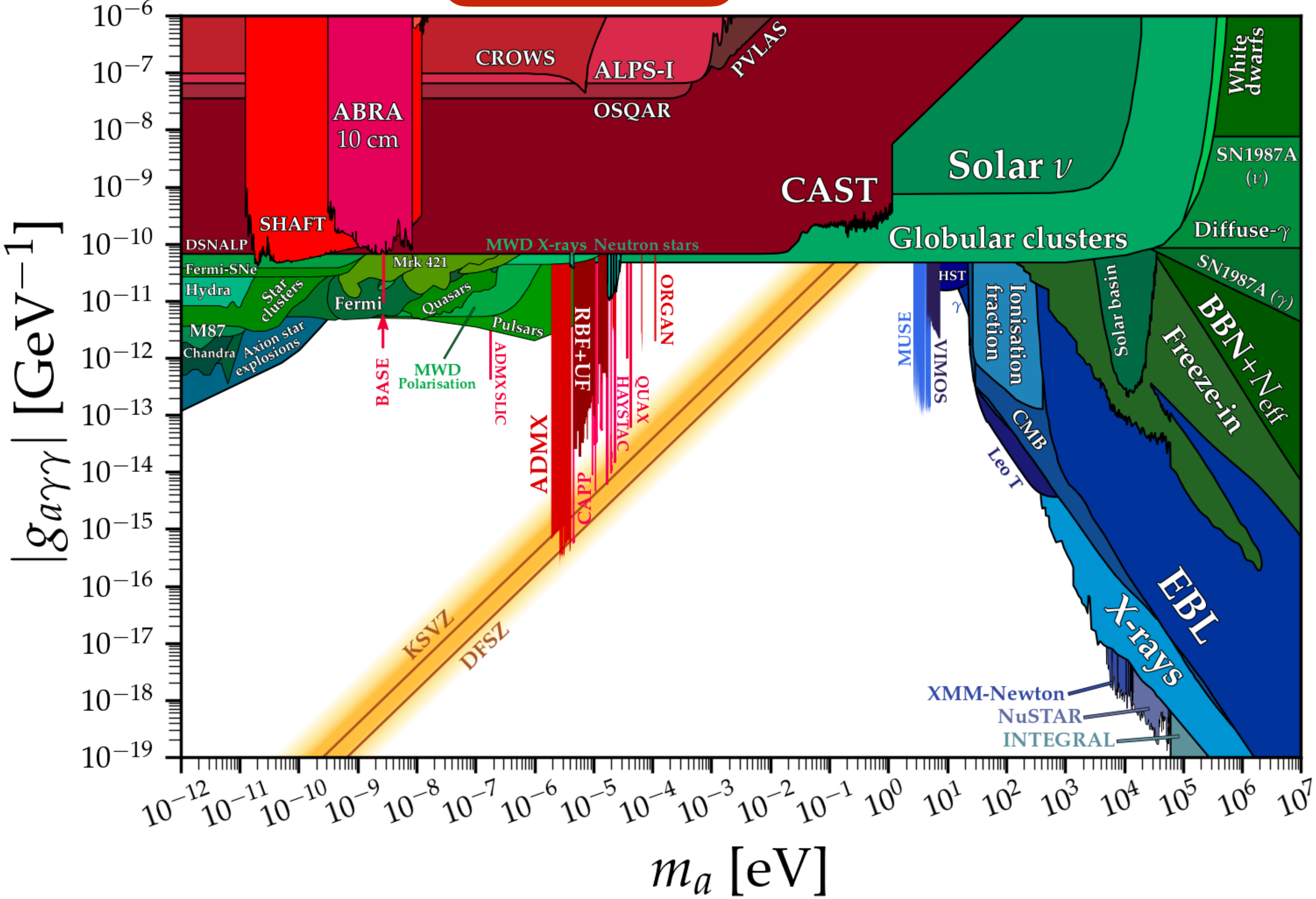
✓ $\tan\beta=\frac{|H_u|}{|H_d|}$

- Can be erased by the axionic rotation $\Psi\to e^{ic_\beta Y_\Psi\frac{a}{f_a}}\Psi$ with $c_\beta=\frac{1}{3}\left(\cos^2\beta-\sin^2\beta\right)$

$$-\frac{a}{f_a}\left(\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}+\frac{8}{3}\frac{e^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)+\frac{\partial_\mu a}{f_a}\left(\frac{\cos^2\beta}{6}\bar{u}\gamma^\mu\gamma^5 u+\frac{\sin^2\beta}{6}\bar{d}\gamma^\mu\gamma^5 d+\frac{\sin^2\beta}{6}\bar{e}\gamma^\mu\gamma^5 e\right)$$

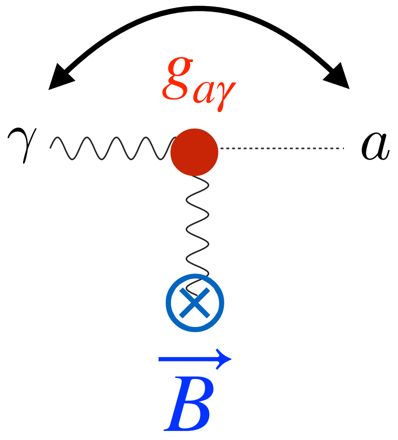
Current status on $\frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu}$

Experimental



Astrophysical

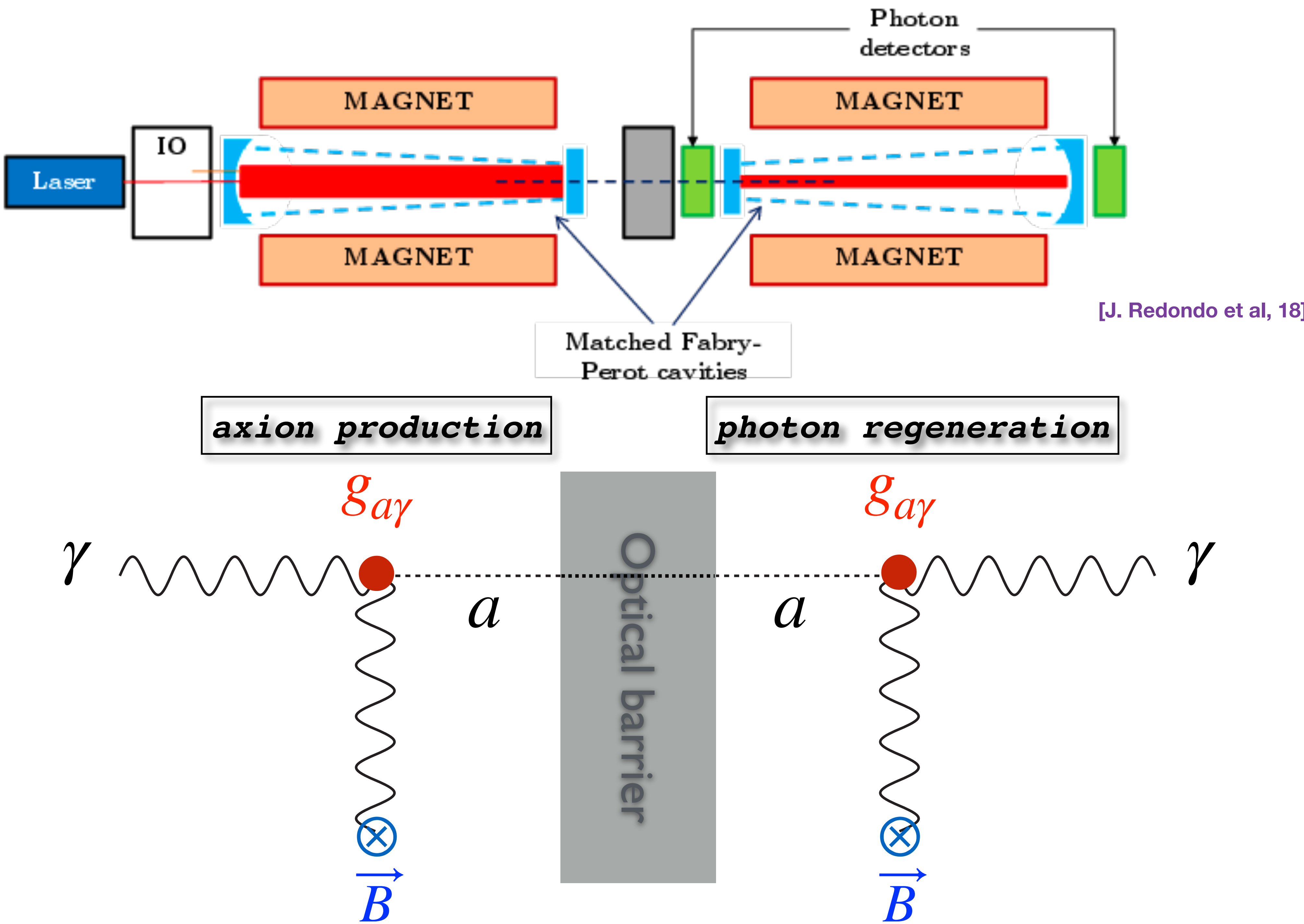
Cosmological



Experimental searches

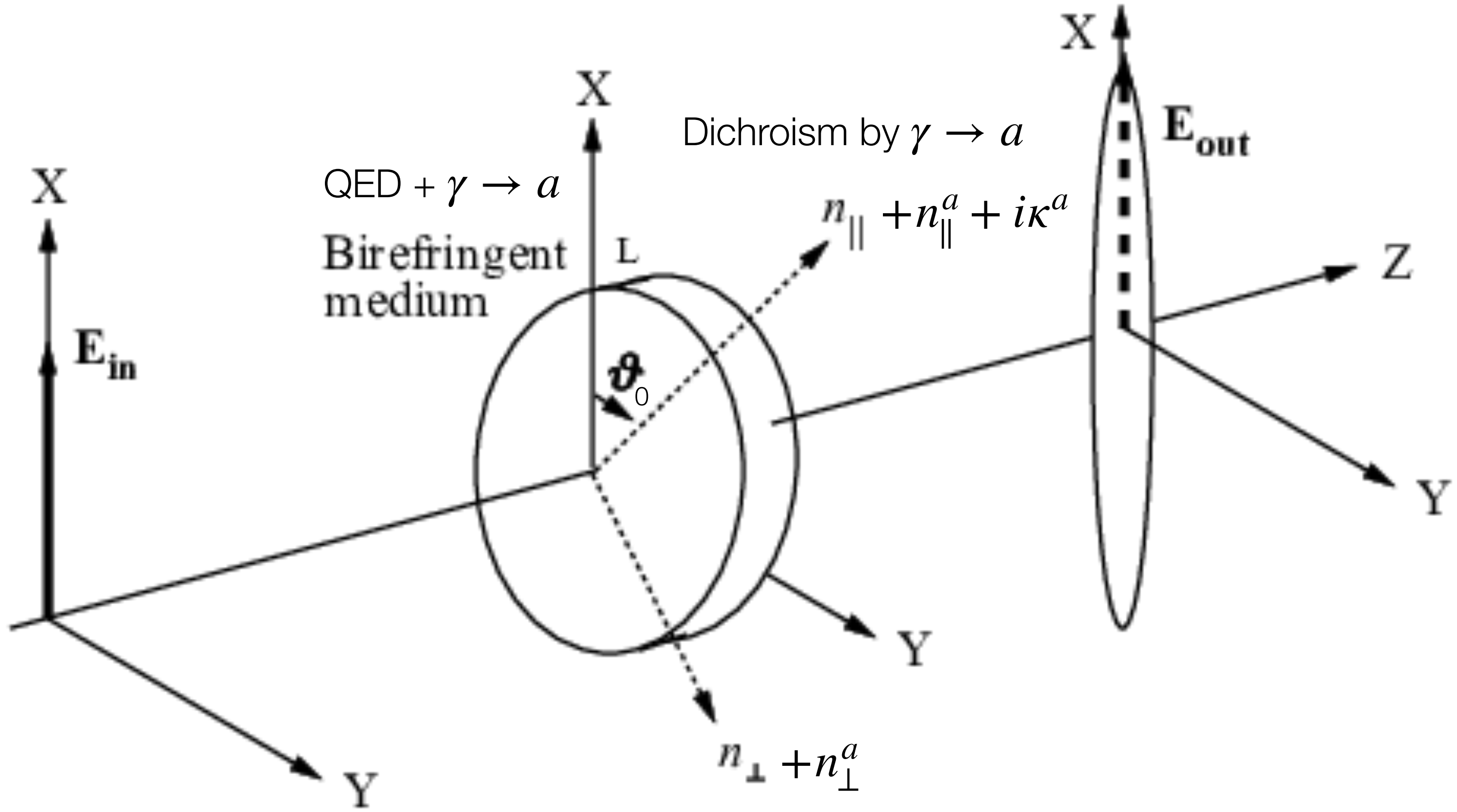
- Axion-photon conversion in a magnetic background
- Light-Shining-Through-Walls
- Photon polarizations (Dichroism & Birefringence)
- Haloscope (Microwave cavity)

Light-Shining-Through-Walls



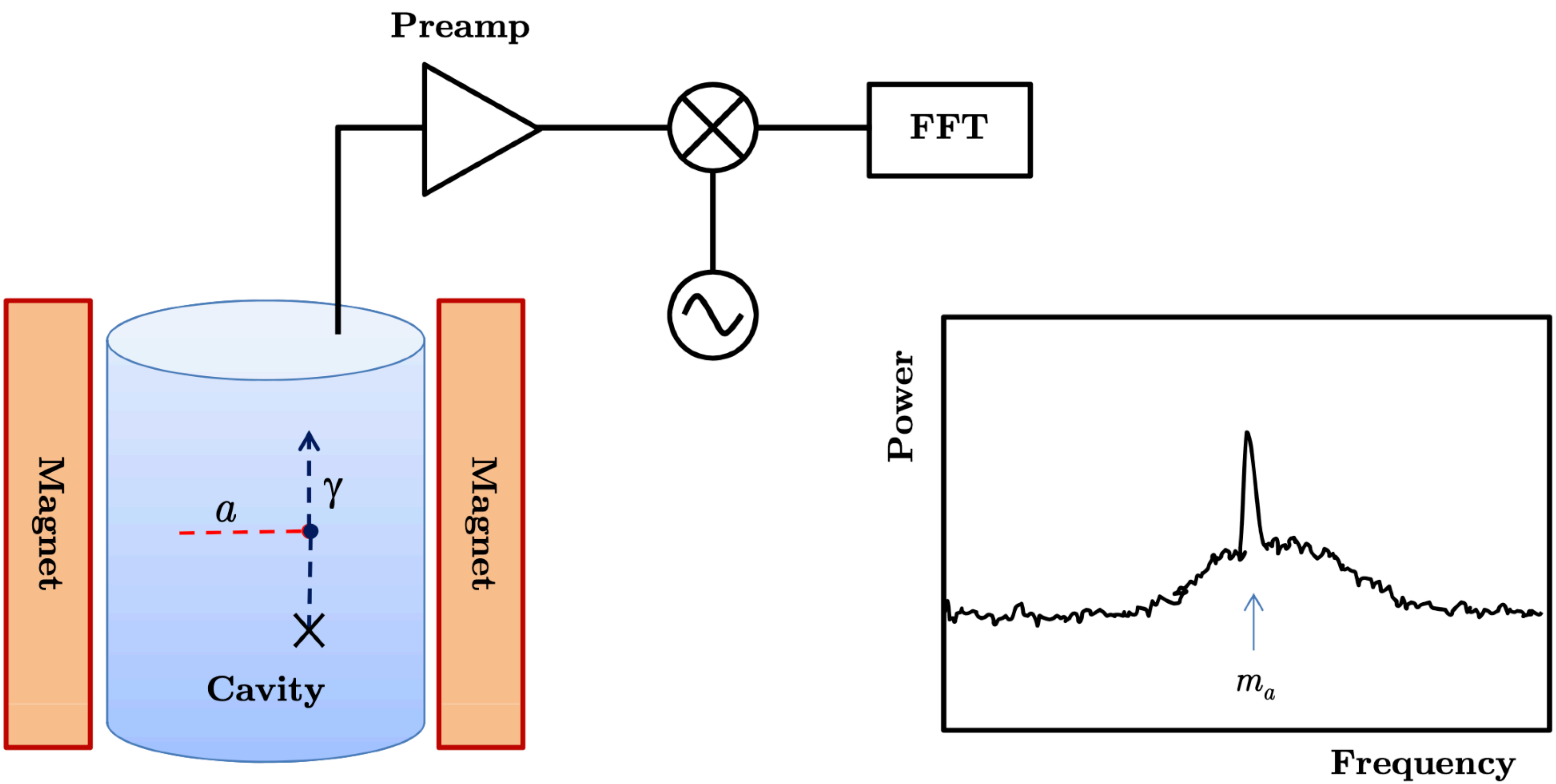
Photon polarization

[PVRAS, arXiv:2005.12913]



1. Initial linearly polarized light to the $\hat{x}$-direction & propagating to the $\hat{z}$-direction
2. Passing through a magnetic field background
   - **Birefringence: ellipticity of the γ-polarization (phase retardation by QED & $\gamma \to a$)**
   - **Dichroism: rotation of the γ-polarization (depletion by $\gamma \to a$)**

Microwave cavity



Axion-Photon oscillation

- Equation of motion in the presence of a background (transverse) magnetic field $\vec{B}$

$$\mathcal{L}_{a\,\&\,\gamma} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 \underline{- \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu}} = g_{a\gamma}\vec{E}\cdot\vec{B}a$$

$$\partial_\mu F^{\mu\nu} = g_{a\gamma}\tilde{F}^{\mu\nu}\partial_\mu a \qquad\qquad \partial_\mu\partial^\mu \vec{A} = g_{a\gamma}\vec{B}\partial_t a$$

$$\partial_\mu\partial^\mu a = -m_a^2 a - \frac{1}{4}g_{a\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad ☞ \qquad \partial_\mu\partial^\mu a = -m_a^2 a - g_{a\gamma}\vec{B}\cdot\partial_t\vec{A}$$

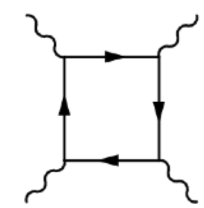
- Considering $\vec{B}$ is constant & propagation in z-direction with $\psi = e^{-i\omega t}\psi(z)$,

$$\left[\omega^2 + \partial_z^2 + 2\omega \begin{pmatrix} \omega(n_\perp - 1) & 0 & 0 \\ 0 & \omega(n_\parallel - 1) & g_{a\gamma}B/2 \\ 0 & g_{a\gamma}B/2 & -m_a^2/2\omega \end{pmatrix}\right] \begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix} = 0$$

Axion-Photon oscillation

$$\left[\omega^2+\partial_z^2+2\omega\begin{pmatrix}\omega\left(n_\perp-1\right) & 0 & 0\\ 0 & \omega\left(n_\parallel-1\right) & g_{a\gamma}B/2\\ 0 & g_{a\gamma}B/2 & -m_a^2/2\omega\end{pmatrix}\right]\begin{pmatrix}A_\perp\\ A_\parallel\\ a\end{pmatrix}=0$$

- Vacuum refractive indices leading to the QED birefringence



$$\frac{\alpha^2}{90m_e^4}\left[\left(F_{\mu\nu}F^{\mu\nu}\right)^2+\frac{7}{4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right]$$

$$n_\perp-1=4\frac{2\alpha}{45}\frac{B^2}{m_e^4}\,,\qquad n_\parallel-1=7\frac{2\alpha}{45}\frac{B^2}{m_e^4}$$

`with` $\frac{2\alpha}{45}\frac{B^2}{m_e^4}=1.32\times10^{-24}\left(\frac{B}{\mathrm{T}}\right)^2$

- For relativistic axions and photons (i.e., $\omega\gg m_a$) ☞ $\omega^2+\partial_z^2\approx2\omega\left(\omega+i\partial_z\right)$

$$\left[\omega+i\partial_z+\begin{pmatrix}\omega\left(n_\parallel-1\right) & g_{a\gamma}B/2\\ g_{a\gamma}B/2 & -m_a^2/2\omega\end{pmatrix}\right]\begin{pmatrix}A_\parallel\\ a\end{pmatrix}=0$$

Axion-Photon oscillation

not relevant

$$\left[\cancel{\omega} + i\partial_z + \begin{pmatrix} \Delta_\parallel & \Delta_{a\gamma} \\ \Delta_{a\gamma} & \Delta_a \end{pmatrix}\right] \begin{pmatrix} A_\parallel \\ a \end{pmatrix} = 0$$

$$\Delta_\parallel = \omega\left(n_\parallel - 1\right), \quad \Delta_a = -m_a^2/2\omega, \quad \Delta_{a\gamma} = g_{a\gamma}B/2$$

- Simple linear algebra!

$$\begin{pmatrix} \Delta_\parallel & \Delta_{a\gamma} \\ \Delta_{a\gamma} & \Delta_a \end{pmatrix} = \begin{pmatrix} \cos\theta_{a\gamma} & \sin\theta_{a\gamma} \\ -\sin\theta_{a\gamma} & \cos\theta_{a\gamma} \end{pmatrix} \begin{pmatrix} \frac{\Delta_\parallel + \Delta_a}{2} + \frac{\Delta_{\mathrm{osc}}}{2} & 0 \\ 0 & \frac{\Delta_\parallel + \Delta_a}{2} - \frac{\Delta_{\mathrm{osc}}}{2} \end{pmatrix} \begin{pmatrix} \cos\theta_{a\gamma} & -\sin\theta_{a\gamma} \\ \sin\theta_{a\gamma} & \cos\theta_{a\gamma} \end{pmatrix}$$

$$\tan 2\theta_{a\gamma} = \frac{2\Delta_{a\gamma}}{\Delta_a - \Delta_\parallel} \quad \text{and} \quad \Delta_{\mathrm{osc}} = \sqrt{\left(\Delta_a - \Delta_\parallel\right)^2 + 4\Delta_{a\gamma}^2}$$

Axion-Photon oscillation

$$\Delta_{\parallel} = \omega\left(n_{\parallel} - 1\right), \quad \Delta_a = -\, m_a^2/2\omega, \quad \Delta_{a\gamma} = g_{a\gamma}B/2$$

$$\tan 2\theta_{a\gamma} = \frac{2\Delta_{a\gamma}}{\Delta_a - \Delta_{\parallel}} \quad \texttt{and} \quad \Delta_{\rm osc} = \sqrt{\left(\Delta_a - \Delta_{\parallel}\right)^2 + 4\Delta_{a\gamma}^2}$$

$$\begin{pmatrix} A_{\parallel} \\ a \end{pmatrix}_z = \exp\left[i\begin{pmatrix} \Delta_{\parallel} & \Delta_{a\gamma} \\ \Delta_{a\gamma} & \Delta_a \end{pmatrix} z\right]\begin{pmatrix} A_{\parallel} \\ a \end{pmatrix}_{z=0}$$

$$\exp\left[i\begin{pmatrix} \Delta_{\parallel} & \Delta_{a\gamma} \\ \Delta_{a\gamma} & \Delta_a \end{pmatrix} z\right] = \begin{pmatrix} \cos\theta_{a\gamma} & \sin\theta_{a\gamma} \\ -\sin\theta_{a\gamma} & \cos\theta_{a\gamma} \end{pmatrix}\begin{pmatrix} e^{i\frac{\Delta_{\parallel}+\Delta_a}{2}z} e^{i\frac{\Delta_{\rm osc}}{2}z} & 0 \\ 0 & e^{i\frac{\Delta_{\parallel}+\Delta_a}{2}} e^{-i\frac{\Delta_{\rm osc}}{2}z} \end{pmatrix}\begin{pmatrix} \cos\theta_{a\gamma} & -\sin\theta_{a\gamma} \\ \sin\theta_{a\gamma} & \cos\theta_{a\gamma} \end{pmatrix}$$

$$= e^{i\frac{\Delta_{\parallel}+\Delta_a}{2}z}\begin{pmatrix} \cos\left(\frac{\Delta_{\rm osc}}{2}z\right) + i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{\rm osc}}{2}z\right) & -i\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)\sin 2\theta_{a\gamma} \\ -i\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)\sin 2\theta_{a\gamma} & \cos\left(\frac{\Delta_{\rm osc}}{2}z\right) - i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{\rm osc}}{2}z\right) \end{pmatrix}$$

Axion-Photon oscillation

$$\Delta_\parallel = \omega\left(n_\parallel - 1\right), \quad \Delta_a = -m_a^2/2\omega, \quad \Delta_{a\gamma} = g_{a\gamma}B/2$$

$$\tan 2\theta_{a\gamma} = \frac{2\Delta_{a\gamma}}{\Delta_a - \Delta_\parallel} \quad \text{and} \quad \Delta_{\rm osc} = \sqrt{\left(\Delta_a - \Delta_\parallel\right)^2 + 4\Delta_{a\gamma}^2}$$

$$\exp\left[i\begin{pmatrix}\Delta_\parallel & \Delta_{a\gamma}\\ \Delta_{a\gamma} & \Delta_a\end{pmatrix}z\right] = \begin{pmatrix}\cos\theta_{a\gamma} & \sin\theta_{a\gamma}\\ -\sin\theta_{a\gamma} & \cos\theta_{a\gamma}\end{pmatrix}\begin{pmatrix}e^{i\frac{\Delta_\parallel+\Delta_a}{2}z}e^{i\frac{\Delta_{\rm osc}}{2}z} & 0\\ 0 & e^{i\frac{\Delta_\parallel+\Delta_a}{2}}e^{-i\frac{\Delta_{\rm osc}}{2}z}\end{pmatrix}\begin{pmatrix}\cos\theta_{a\gamma} & -\sin\theta_{a\gamma}\\ \sin\theta_{a\gamma} & \cos\theta_{a\gamma}\end{pmatrix}$$

$$= e^{i\frac{\Delta_\parallel+\Delta_a}{2}z}\begin{pmatrix}\cos\left(\frac{\Delta_{\rm osc}}{2}z\right) + i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{\rm osc}}{2}z\right) & -i\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)\sin 2\theta_{a\gamma}\\ -i\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)\sin 2\theta_{a\gamma} & \cos\left(\frac{\Delta_{\rm osc}}{2}z\right) - i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)\end{pmatrix}$$

$$\boxed{P_{A_\parallel\leftrightarrow a} = \sin^2 2\theta_{a\gamma}\sin^2\left(\frac{\Delta_{\rm osc}}{2}z\right)}$$

Condition in experimental setups

$\text{Gauss} = 1.95 \times 10^{-2}\,\text{eV}^2$

$$\omega \sim 1\,\text{eV}, \quad B \sim 10\,\text{T} = 10^5\,\text{Gauss}, \quad L_B \sim (1\text{-}100)\ \text{m}$$

$$\Delta_{\parallel} = 4.7 \times 10^{-15}\,\text{m}^{-1} \left(\frac{\omega}{1\,\text{eV}}\right)\left(\frac{B}{10\,\text{T}}\right)^2$$

$$\Delta_a = -\,2.5 \times 10^{-6}\,\text{m}^{-1} \left(\frac{m_a}{\mu\text{m}}\right)^2 \left(\frac{\omega}{1\,\text{eV}}\right)^{-1}$$

$$\Delta_{a\gamma} = 4.9 \times 10^{-7}\,\text{m}^{-1} \left(\frac{g_{a\gamma}}{10^{-7}\,\text{GeV}^{-1}}\right)\left(\frac{B}{10\,\text{T}}\right)$$

$$\boxed{P_{A_{\parallel}\leftrightarrow a} = \sin^2 2\theta_{a\gamma} \sin^2\left(\frac{\Delta_{\text{osc}}}{2} L_B\right)}$$

Linear & non-linear regime

$$P_{A_\parallel \leftrightarrow a} = \sin^2 2\theta_{a\gamma} \sin^2\left(\frac{\Delta_{\rm osc}}{2} L_B\right)$$

$$\boxed{\begin{aligned} \tan 2\theta_{a\gamma} &= \frac{2\Delta_{a\gamma}}{\Delta_a} \\ \Delta_{\rm osc} &= \sqrt{\Delta_a^2 + 4\Delta_{a\gamma}^2} \end{aligned}}$$

$$\Delta_a = -\,2.5 \times 10^{-6}\,\mathrm{m}^{-1}\left(\frac{m_a}{\mu\mathrm{m}}\right)^2\left(\frac{\omega}{1\,\mathrm{eV}}\right)^{-1}, \quad \Delta_{a\gamma} = 4.9 \times 10^{-7}\,\mathrm{m}^{-1}\left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)\left(\frac{B}{10\,\mathrm{T}}\right)$$

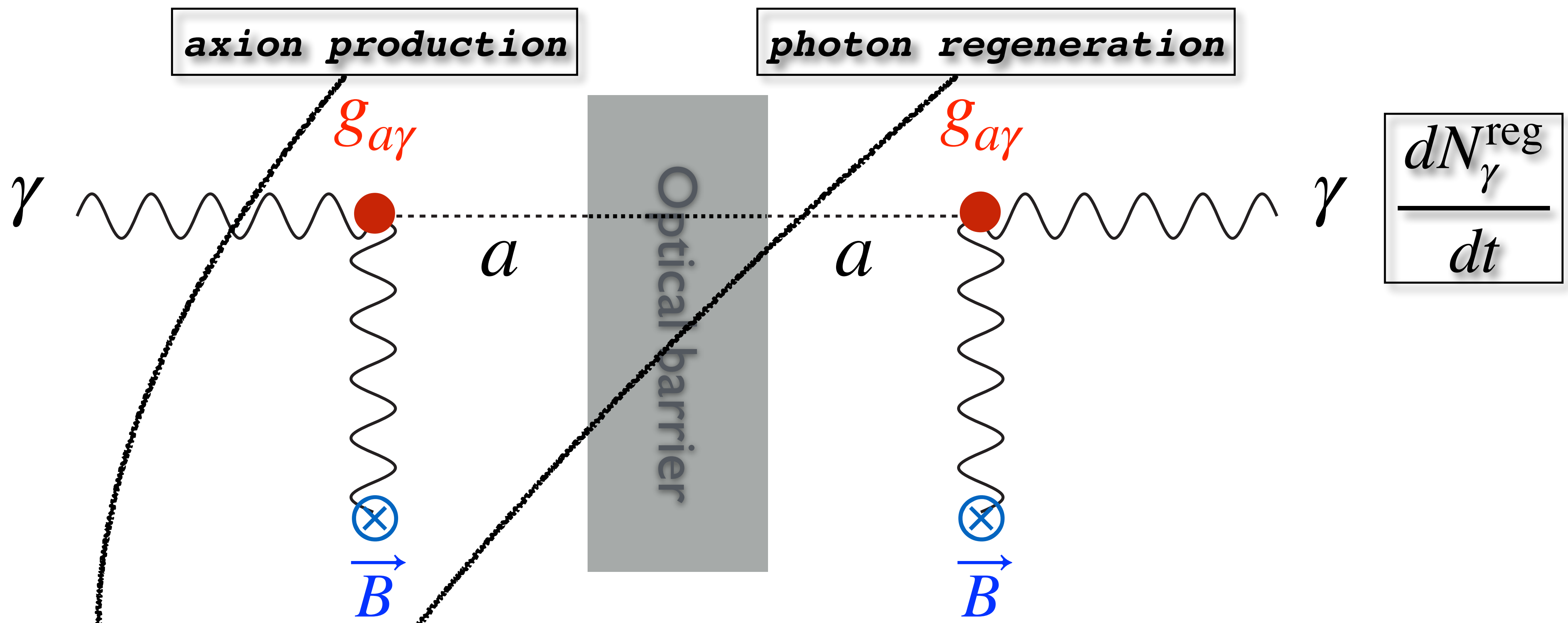
- When $\Delta_{\rm osc} L_B \ll 1$, so-called 'linear regime',

$$P_{A_\parallel \leftrightarrow a} \simeq \left(\Delta_{a\gamma} L_B\right)^2 = 2.4 \times 10^{-13}\left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{B}{10\,\mathrm{T}}\right)^2\left(\frac{L_B}{1\,\mathrm{m}}\right)^2$$

- When $\Delta_{\rm osc} L_B \gg 1$, so-called 'non-linear regime', typically $\Delta_a \gg \Delta_{a\gamma}$

$$P^{\rm max}_{A_\parallel \leftrightarrow a} \simeq \sin^2 2\theta_{a\gamma} = 1.5 \times 10^{-13}\left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{B}{10\,\mathrm{T}}\right)^2\left(\frac{\omega}{1\,\mathrm{eV}}\right)^2\left(\frac{m_a}{\mathrm{meV}}\right)^{-4}$$

Light-Shining-Through-Walls



$$\frac{dN_\gamma^{\rm reg}}{dt} = W_\gamma \times P_{A_\parallel \to a} \times P_{a \to A_\parallel}$$

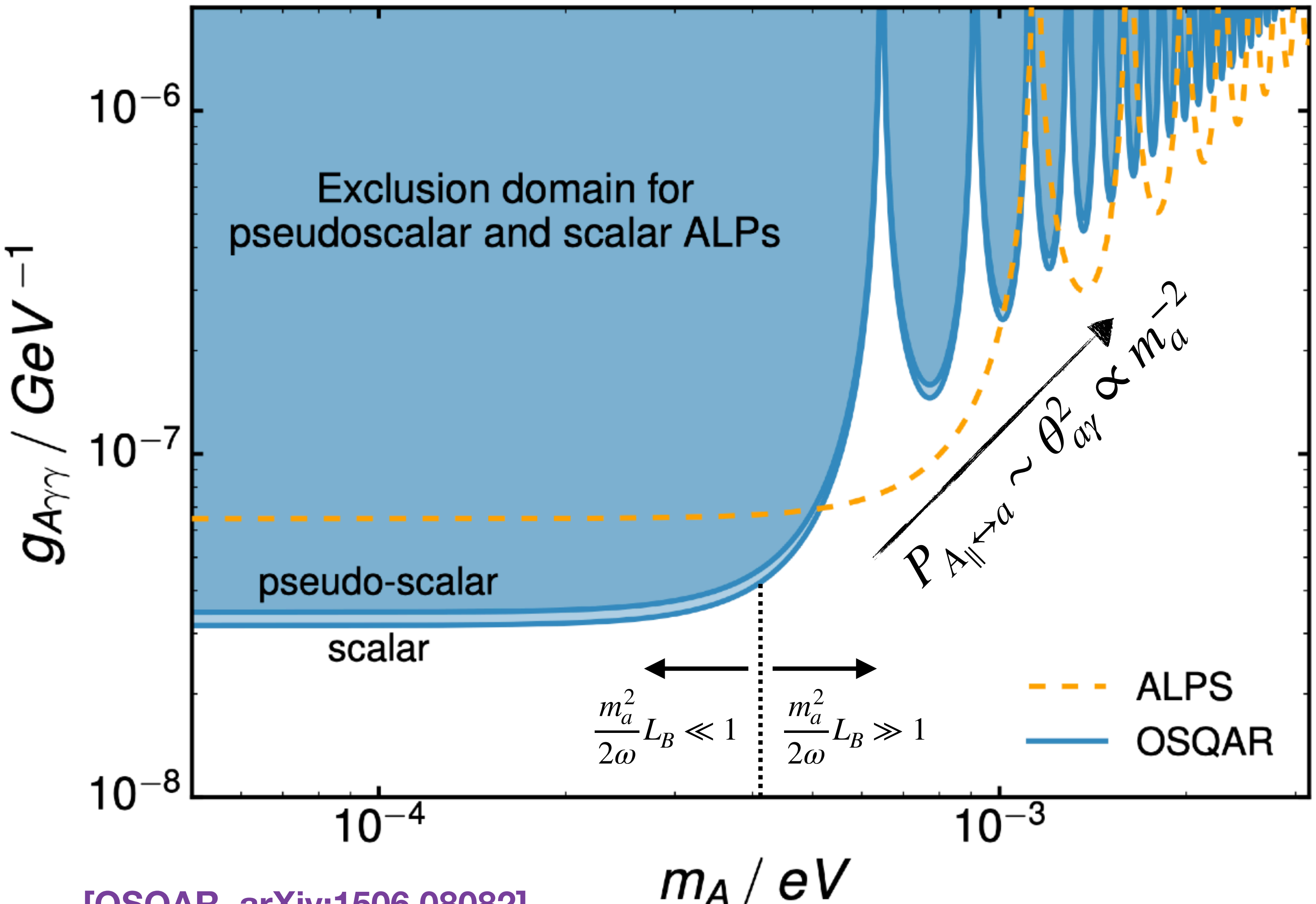
"linear"

$$= 3.6 \times 10^{-2}\,{\rm s}^{-1} \left(\frac{W_\gamma}{10\,{\rm Watt}}\right) \left(\frac{\omega}{1\,{\rm eV}}\right)^{-1} \left(\frac{g_{a\gamma}}{10^{-7}\,{\rm GeV}^{-1}}\right)^4 \left(\frac{B}{10\,{\rm T}}\right)^4 \left(\frac{L_B}{10\,{\rm m}}\right)^4$$

${\rm Watt} = 6.24 \times 10^{18}\,{\rm eV\,s}^{-1}$

Light-Shining-Through-Walls

$$\frac{dN_\gamma^{\rm reg}}{dt} = 3.6\times 10^{-2}\,{\rm s}^{-1}\left(\frac{W_\gamma}{10\,{\rm Watt}}\right)\left(\frac{\omega}{1\,{\rm eV}}\right)^{-1}\left(\frac{g_{a\gamma}}{10^{-7}\,{\rm GeV}^{-1}}\right)^4\left(\frac{B}{10\,{\rm T}}\right)^4\left(\frac{L_B}{10\,{\rm m}}\right)^4 < \frac{dN_{\rm sen}}{dt}$$



[OSQAR, arXiv:1506.08082]

$W_\gamma = 4\,{\rm Watt}\times 300,\ \omega = 2.33\,{\rm eV}$

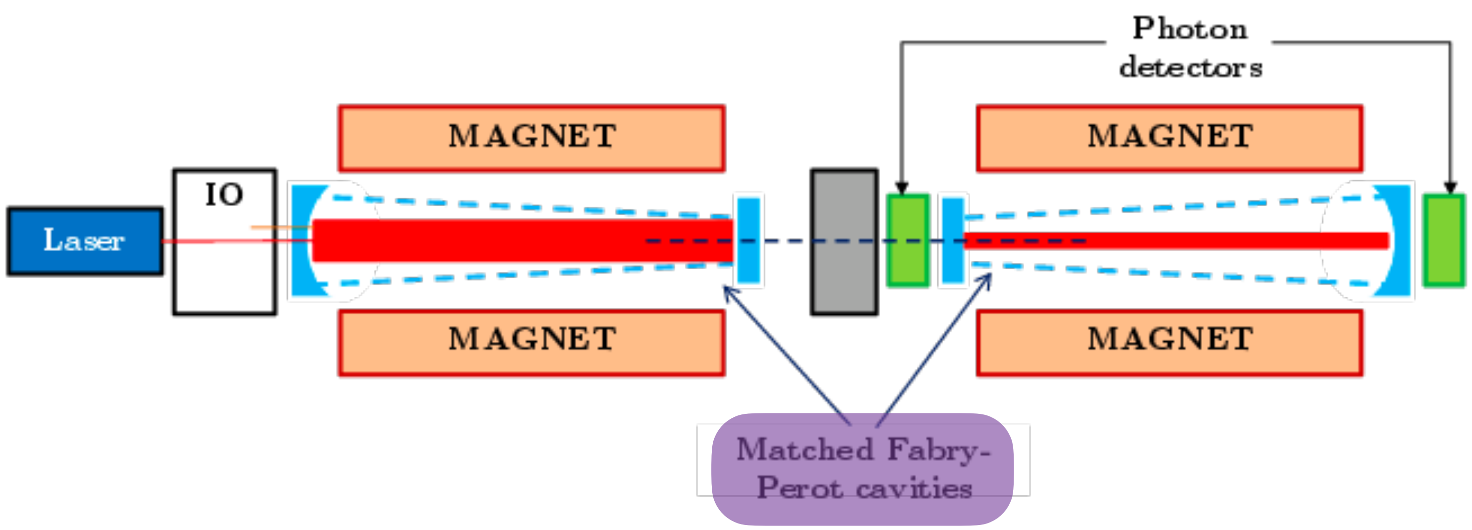
$B = 5\,{\rm T},\ L_B = 4.3\,{\rm m},\ \frac{dN_{\rm sen}}{dt} = 7.9\,{\rm mHz}$

$W_\gamma = 18.5\,{\rm Watt},\ \omega = 2.33\,{\rm eV}$

$B = 9\,{\rm T},\ L_B = 14.3\,{\rm m},\ \frac{dN_{\rm sen}}{dt} = {\rm mHz}$

Improvement on LSTW

- Exploiting cavities in both the production & regeneration regions,



axion production

- Cavity in production as a mirror
- Large reflectivity $\Rightarrow$ Amplified γ power
- $\omega L_B = n\pi$ needed to a coherently enhancement

photon regeneration

- Cavity mode by boundary $\omega_n = n\pi/L_B$
- Matching for resonance: $\omega = \omega_n$

Improvement on LSTW

axion production

- Cavity in production as a mirror
- Large reflectivity $\Rightarrow$ Amplified γ power
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photon regeneration

- Cavity mode by boundary $\omega_n = n\pi/L_B$
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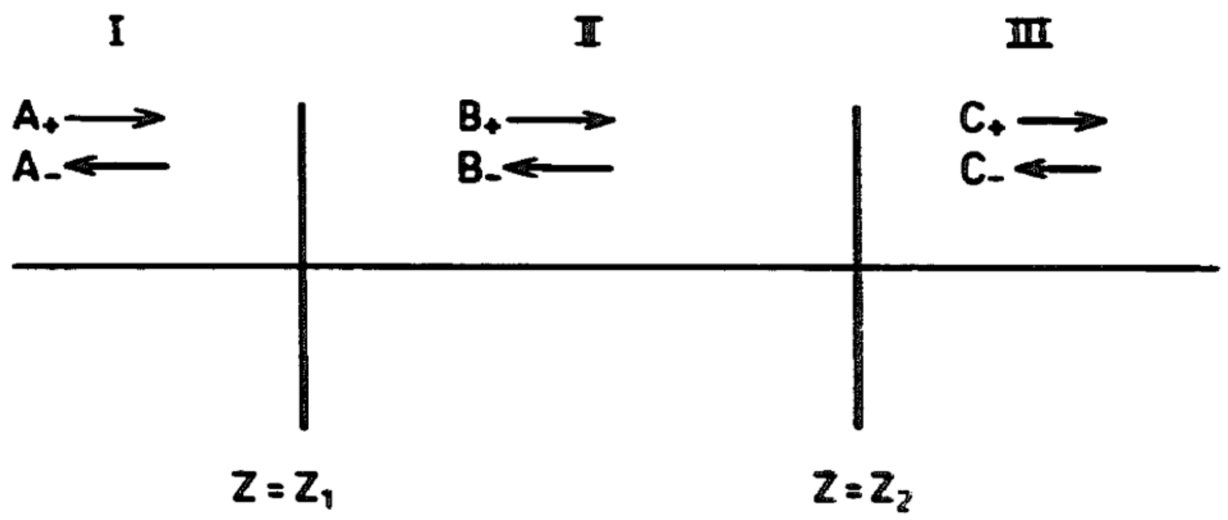


$$\begin{pmatrix} B_+ e^{-i\omega z_1} \\ A_- e^{+i\omega z_1} \end{pmatrix} = \begin{pmatrix} s_{11}^{(1)} & s_{12}^{(1)} \\ s_{21}^{(1)} & s_{22}^{(1)} \end{pmatrix} \begin{pmatrix} B_- e^{+i\omega z_1} \\ A_+ e^{-i\omega z_1} \end{pmatrix}$$

$$\begin{pmatrix} C_+ e^{-i\omega z_2} \\ B_- e^{+i\omega z_2} \end{pmatrix} = \begin{pmatrix} s_{22}^{(2)} & s_{21}^{(2)} \\ s_{12}^{(2)} & s_{11}^{(2)} \end{pmatrix} \begin{pmatrix} C_- e^{+i\omega z_2} \\ B_+ e^{-i\omega z_2} \end{pmatrix}$$

Improvement on LSTW

axion production

- Cavity in production as a mirror
- Large reflectivity $\Rightarrow$ Amplified γ power
- $\omega L_B = n\pi$ needed to a coherently enhancement

photon regeneration

- Cavity mode by boundary $\omega_n = n\pi/L_B$
- Matching for resonance: $\omega = \omega_n$

$$\begin{pmatrix} B_+ e^{-i\omega z_1} \\ A_- e^{+i\omega z_1} \end{pmatrix} = \begin{pmatrix} s_{11}^{(1)} & s_{12}^{(1)} \\ s_{21}^{(1)} & s_{22}^{(1)} \end{pmatrix} \begin{pmatrix} B_- e^{+i\omega z_1} \\ A_+ e^{-i\omega z_1} \end{pmatrix}$$

only injection from left

$$\begin{pmatrix} C_+ e^{-i\omega z_2} \\ B_- e^{+i\omega z_2} \end{pmatrix} = \begin{pmatrix} s_{22}^{(2)} & s_{21}^{(2)} \\ s_{12}^{(2)} & s_{11}^{(2)} \end{pmatrix} \begin{pmatrix} \cancel{C_-} e^{+i\omega z_2} \\ B_+ e^{-i\omega z_2} \end{pmatrix}$$

$$B_+ = \frac{s_{12}^{(1)}}{1 - s_{11}^{(1)} s_{11}^{(2)} e^{-i2\omega(z_2 - z_1)}} A_+$$

maximized at $\omega L_B = n\pi$

$$\frac{|B_+|^2}{|A_+|^2} = \boxed{\frac{1}{1-r}}$$ with $|s_{12}^{(1)}| = \sqrt{1-r}$, $|s_{11}^{(i)}| = \sqrt{r}$

r reflectivity

Improvement on LSTW

axion production

- Cavity in production as a mirror
- Large reflectivity $\Rightarrow$ Amplified γ power
- $\omega L_B = n\pi$ needed to a coherently enhancement

photon regeneration

- Cavity mode by boundary $\omega_n = n\pi/L_B$
- Matching for resonance: $\omega = \omega_n$

- Equation of motion for the EM field $\overrightarrow{A}$ with the axion source, $a = \mathscr{A}\sin\left(k_a z - \omega t\right)$

$$\partial_\mu \partial^\mu \overrightarrow{A} = g_{a\gamma}\overrightarrow{B}\partial_t a = -\, g_{a\gamma}\overrightarrow{B}\omega\mathscr{A}\cos\left(k_a z - \omega t\right)$$

- Cavity mode as $\overrightarrow{A} = A_n(t)\,\hat{B}\sin\left(\frac{n\pi}{L}z\right)$ due to $\overrightarrow{A} = 0$ at the boundary

$$\left(\frac{d^2}{dt^2} + \frac{\omega}{Q}\frac{d}{dt} + \omega_n^2\right)A_n(t) \times \sin\left(\frac{n\pi}{L}z\right) = -\, g_{a\gamma}B\omega\mathscr{A}\cos\left(k_a z - \omega t\right)$$

Improvement on LSTW

axion production

- Cavity in production as a mirror
- Large reflectivity $\Rightarrow$ Amplified γ power
- $\omega L_B = n\pi$ needed to a coherently enhancement

photon regeneration

- Cavity mode by boundary $\omega_n = n\pi / L_B$
- Matching for resonance: $\omega = \omega_n$

$$\int_0^{L_B} dz \,\sin\left(\frac{n\pi}{L}z\right)\left[\left(\frac{d^2}{dt^2} + \frac{\omega}{Q}\frac{d}{dt} + \omega_n^2\right) A_n(t) \times \sin\left(\frac{n\pi}{L}z\right) = -\, g_{a\gamma} B \omega \mathscr{A} \cos\left(k_a z - \omega t\right)\right]$$

✓ $k_a - \omega_n \ll k_a + \omega_n \approx 2\omega$

$$\boxed{\left(\frac{d^2}{dt^2} + \frac{\omega}{Q}\frac{d}{dt} + \omega_n^2\right) A_n(t) = -\frac{2}{L_B} g_{a\gamma} B \omega \mathscr{A} \frac{1}{k_a - \omega_n} \sin\left(\frac{k_a - \omega_n}{2} L_B\right) \sin\left(\omega t - \frac{k_a - \omega_n}{2} L_B\right)}$$

Improvement on LSTW

axion production

- Cavity in production as a mirror
- Large reflectivity $\Rightarrow$ Amplified γ power
- $\omega L_B = n\pi$ needed to a coherently enhancement

photon regeneration

- Cavity mode by boundary $\omega_n = n\pi / L_B$
- Matching for resonance: $\omega = \omega_n$

$$A_n(t) = \frac{1}{\cancel{\omega^2 - \omega_n^2} + i\omega^2/Q} g_{a\gamma} B\omega\mathscr{A} \left[\frac{2}{(k_a - \omega_n)\, L_B} \sin\left(\frac{k_a - \omega_n}{2} L_B \right) \right] \sin\left(\omega t - \frac{k_a - \omega_n}{2} L_B \right)$$

$$\left\langle \vec{A}^2 \right\rangle_{t,z} = \frac{1}{4} \frac{Q^2}{\omega^4} \left(g_{a\gamma} B\omega\mathscr{A} \left[\frac{2}{(k_a - \omega_n)\, L_B} \sin\left(\frac{k_a - \omega_n}{2} L_B \right) \right] \right)^2$$

Improvement on LSTW

axion production

- Cavity in production as a mirror
- Large reflectivity $\Rightarrow$ Amplified γ power
- $\omega L_B = n\pi$ needed to a coherently enhancement

photon regeneration

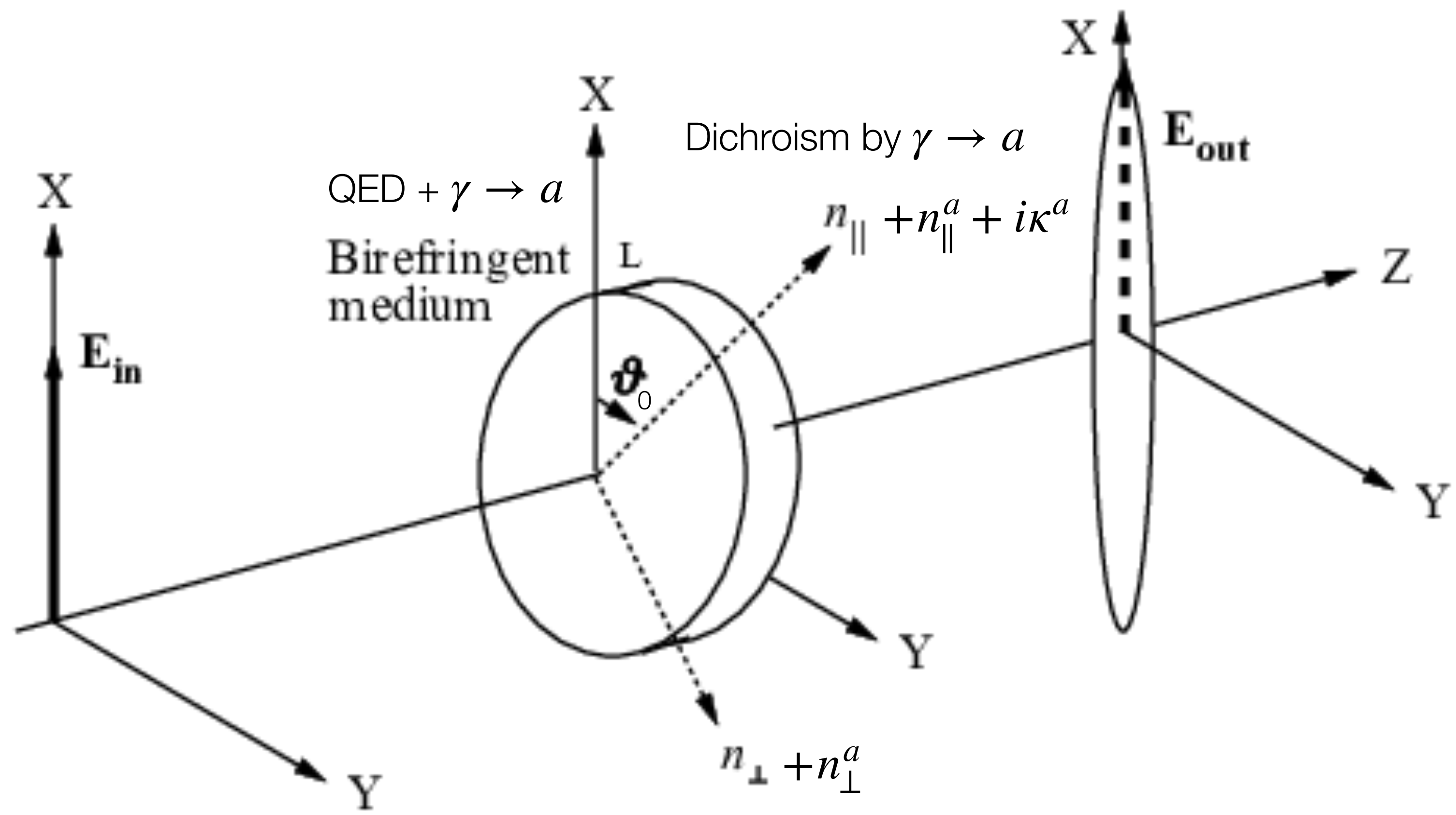
- Cavity mode by boundary $\omega_n = n\pi / L_B$
- Matching for resonance: $\omega = \omega_n$

- Power emitted by the cavity $P_{\text{cav}} = \frac{\omega}{Q} \left\langle \vec{A}^2 \right\rangle S L_B \omega^2$ vs $P_a = \frac{1}{2} S \mathscr{A}^2 \omega^2$

$$p_{\text{reg}} = \frac{P_{\text{cav}}}{P_a} = \frac{2Q}{\omega L_B} \left(\frac{g_{a\gamma} B L_B}{2} \right)^2 \left(\frac{2}{(k_a - \omega_n) L_B} \sin\left(\frac{k_a - \omega_n}{2} L_B \right) \right)^2 = \frac{2Q}{\omega L_B} p_{a \to A_\parallel}$$

Photon polarization

[PVRAS, arXiv:2005.12913]



1. Initial linearly polarized light to the $\hat{x}$-direction & propagating to the $\hat{z}$-direction
2. Passing through a magnetic field background
   - **Birefringence: ellipticity of the γ-polarization (phase retardation by QED & $\gamma \to a$)**
   - **Dichroism: rotation of the γ-polarization (depletion by $\gamma \to a$)**

Photon polarization

① Initial state

$$\begin{pmatrix} A_x \\ A_y \\ a \end{pmatrix}_{\rm in} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

② Rotating to the frame in terms of $\overrightarrow{B}$

$$\begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix}_{\rm in} = \begin{pmatrix} \sin\theta_0 & -\cos\theta_0 & 0 \\ \cos\theta_0 & \sin\theta_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ a \end{pmatrix}_{\rm in}$$

③ Propagation in $\overrightarrow{B}$ background

$$\begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix}_{\rm out} = \exp\left[i \begin{pmatrix} \Delta_\perp & 0 & 0 \\ 0 & \Delta_\parallel & \Delta_{a\gamma} \\ 0 & \Delta_{a\gamma} & \Delta_a \end{pmatrix} L_B \right] \begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix}_{\rm in}$$

④ Return to $\hat{x}$ & $\hat{y}$

$$\begin{pmatrix} A_x \\ A_y \\ a \end{pmatrix}_{\rm out} = \begin{pmatrix} \sin\theta_0 & \cos\theta_0 & 0 \\ -\cos\theta_0 & \sin\theta_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix}_{\rm out}$$

Photon polarization

- **Birefringence: ellipticity (phase retardation by QED & $\gamma \to a$)** → $\frac{|\mathrm{Im\,A_y^{out}}|}{|A_x^{out}|}$
- **Dichroism: rotation (depletion by $\gamma \to a$)** → $\frac{|\mathrm{Re\,A_y^{out}}|}{|A_x^{out}|}$

③ Propagation in $\overrightarrow{B}$ background

$$\begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix}_{\rm out} = \exp\left[i \begin{pmatrix} \Delta_\perp & 0 & 0 \\ 0 & \Delta_\parallel & \Delta_{a\gamma} \\ 0 & \Delta_{a\gamma} & \Delta_a \end{pmatrix} L_B \right] \begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix}_{\rm in}$$

$$\exp\left[i \begin{pmatrix} \Delta_\parallel & \Delta_{a\gamma} \\ \Delta_{a\gamma} & \Delta_a \end{pmatrix} z \right] = \begin{pmatrix} \cos\theta_{a\gamma} & \sin\theta_{a\gamma} \\ -\sin\theta_{a\gamma} & \cos\theta_{a\gamma} \end{pmatrix} \begin{pmatrix} e^{i\frac{\Delta_\parallel+\Delta_a}{2}z} e^{i\frac{\Delta_{\rm osc}}{2}z} & 0 \\ 0 & e^{i\frac{\Delta_\parallel+\Delta_a}{2}z} e^{-i\frac{\Delta_{\rm osc}}{2}z} \end{pmatrix} \begin{pmatrix} \cos\theta_{a\gamma} & -\sin\theta_{a\gamma} \\ \sin\theta_{a\gamma} & \cos\theta_{a\gamma} \end{pmatrix}$$

$$= e^{i\frac{\Delta_\parallel+\Delta_a}{2}z} \begin{pmatrix} \boxed{\cos\left(\frac{\Delta_{\rm osc}}{2}z\right) + i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)} & -i\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)\sin 2\theta_{a\gamma} \\ -i\sin\left(\frac{\Delta_{\rm osc}}{2}z\right)\sin 2\theta_{a\gamma} & \cos\left(\frac{\Delta_{\rm osc}}{2}z\right) - i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{\rm osc}}{2}z\right) \end{pmatrix}$$

Photon polarization

- **Birefringence: ellipticity (phase retardation by QED & $\gamma \to a$)** $\Rightarrow \dfrac{|\mathrm{Im}\, A_y^{out}|}{|A_x^{out}|}$
- **Dichroism: rotation (depletion by $\gamma \to a$)** $\Rightarrow \dfrac{|\mathrm{Re}\, A_y^{out}|}{|A_x^{out}|}$

③ Propagation in $\overrightarrow{B}$ background

$$A^{out}_{\perp,\parallel} = \left(1 - \eta_{\perp,\parallel} + i\,\phi_{\perp,\parallel}\right) A^{in}_{\perp,\parallel}$$

$$\eta_\perp = 0 \qquad \phi_\perp = \left(n_\perp - 1\right)\omega L_B$$

$$\eta_\parallel = 2\theta_{a\gamma}^2 \sin^2\left(\Delta_a z/2\right) \qquad \phi_\perp = \left(n_\parallel - 1\right)\omega L_B - \theta_{a\gamma}^2\left(\Delta_a L_B - \sin \Delta_a L_B\right)$$

$$\Delta_a = -m_a^2/2\omega$$

$$\exp\left[i\begin{pmatrix}\Delta_\parallel & \Delta_{a\gamma}\\ \Delta_{a\gamma} & \Delta_a\end{pmatrix} z\right] = \begin{pmatrix}\cos\theta_{a\gamma} & \sin\theta_{a\gamma}\\ -\sin\theta_{a\gamma} & \cos\theta_{a\gamma}\end{pmatrix}\begin{pmatrix}e^{i\frac{\Delta_\parallel+\Delta_a}{2}z}e^{i\frac{\Delta_{osc}}{2}z} & 0\\ 0 & e^{i\frac{\Delta_\parallel+\Delta_a}{2}}e^{-i\frac{\Delta_{osc}}{2}z}\end{pmatrix}\begin{pmatrix}\cos\theta_{a\gamma} & -\sin\theta_{a\gamma}\\ \sin\theta_{a\gamma} & \cos\theta_{a\gamma}\end{pmatrix}$$

$$= e^{i\frac{\Delta_\parallel+\Delta_a}{2}z}\begin{pmatrix}\boxed{\cos\left(\frac{\Delta_{osc}}{2}z\right) + i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{osc}}{2}z\right)} & -i\sin\left(\frac{\Delta_{osc}}{2}z\right)\sin 2\theta_{a\gamma}\\ -i\sin\left(\frac{\Delta_{osc}}{2}z\right)\sin 2\theta_{a\gamma} & \cos\left(\frac{\Delta_{osc}}{2}z\right) - i\cos 2\theta_{a\gamma}\sin\left(\frac{\Delta_{osc}}{2}z\right)\end{pmatrix}$$

Photon polarization

- **Birefringence: ellipticity (phase retardation by QED & $\gamma \to a$)** $\rightarrow \dfrac{|\mathrm{Im}\, A_y^{out}|}{|A_x^{out}|}$
- **Dichroism: rotation (depletion by $\gamma \to a$)** $\rightarrow \dfrac{|\mathrm{Re}\, A_y^{out}|}{|A_x^{out}|}$

③ Propagation in $\vec{B}$ background

$$A_{\perp,\parallel}^{out} = \left(1 - \eta_{\perp,\parallel} + i\,\phi_{\perp,\parallel}\right) A_{\perp,\parallel}^{in}$$

$$\eta_\perp = 0 \qquad \phi_\perp = \left(n_\perp - 1\right)\omega L_B$$

$$\eta_\parallel = 2\theta_{a\gamma}^2 \sin^2\left(\Delta_a z/2\right) \qquad \phi_\parallel = \left(n_\parallel - 1\right)\omega L_B - \theta_{a\gamma}^2\left(\Delta_a L_B - \sin \Delta_a L_B\right)$$

$$\Delta_a = -m_a^2/2\omega$$

$$\frac{|\mathrm{Im}\, A_y^{out}|}{|A_x^{out}|} = \frac{\sin 2\theta_0}{2}\left|\phi_\parallel - \phi_\perp\right| \quad ☞ \quad \Delta n\Big|_a = \frac{1}{\omega L_B}\theta_{a\gamma}^2\left(\Delta_a L_B - \sin \Delta_a L_B\right)$$

$$\frac{|\mathrm{Re}\, A_y^{out}|}{|A_x^{out}|} = \frac{\sin 2\theta_0}{2}\left|\eta_\parallel - \eta_\perp\right| \quad ☞ \quad \Delta \kappa\Big|_a = \frac{1}{\omega L_B} 2\theta_{a\gamma}^2 \sin^2\left(\Delta_a L_B/2\right)$$

Photon polarization

- Birefringence: ellipticity (phase retardation by QED & $\gamma \to a$) $\Rightarrow$ $\Delta n\big|_a$
- Dichroism: rotation (depletion by $\gamma \to a$) $\Rightarrow$ $\Delta\kappa\big|_a$

• In the linear regime, $\Delta_a L_B \ll 1$,

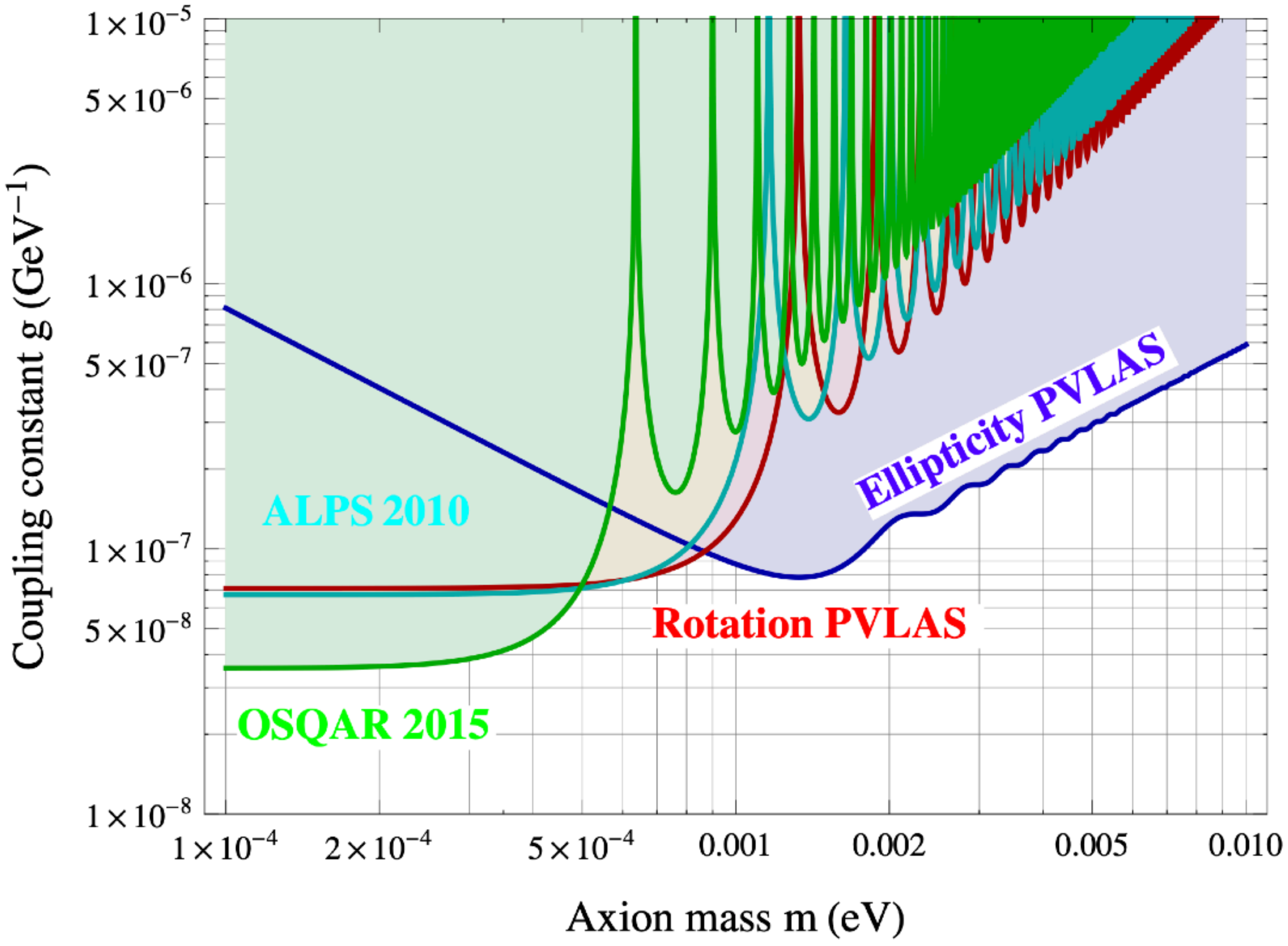
$$\Delta n\big|_a = \frac{1}{\omega L_B}\theta_{a\gamma}^2\left(\Delta_a L_B - \sin\Delta_a L_B\right) \approx \frac{1}{3}\left(\frac{g_{a\gamma}Bm_aL_B}{4\omega}\right)^2 = 2.0\times 10^{-22}\left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{B}{1\,\mathrm{T}}\right)^2\left(\frac{L_B}{1\,\mathrm{m}}\right)^2\left(\frac{m_a}{\mathrm{meV}}\right)^2\left(\frac{\omega}{1\,\mathrm{eV}}\right)^{-2}$$

$$\Delta\kappa\big|_a = \frac{1}{\omega L_B}2\theta_{a\gamma}^2\sin^2\left(\Delta_a L_B/2\right) \approx \frac{2}{\omega L_B}\left(\frac{g_{a\gamma}BL_B}{4}\right)^2 = 2.4\times 10^{-22}\left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{B}{1\,\mathrm{T}}\right)^2\left(\frac{L_B}{1\,\mathrm{m}}\right)\left(\frac{\omega}{1\,\mathrm{eV}}\right)^{-1}$$

Photon polarization

$$\Delta n\Big|_a \approx 2.0\times10^{-22}\left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{B}{1\,\mathrm{T}}\right)^2\left(\frac{L_B}{1\,\mathrm{m}}\right)^2\left(\frac{m_a}{\mathrm{meV}}\right)^2\left(\frac{\omega}{1\,\mathrm{eV}}\right)^{-2}$$

$$\Delta\kappa\Big|_a \approx 2.4\times10^{-22}\left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{B}{1\,\mathrm{T}}\right)^2\left(\frac{L_B}{1\,\mathrm{m}}\right)\left(\frac{\omega}{1\,\mathrm{eV}}\right)^{-1}$$



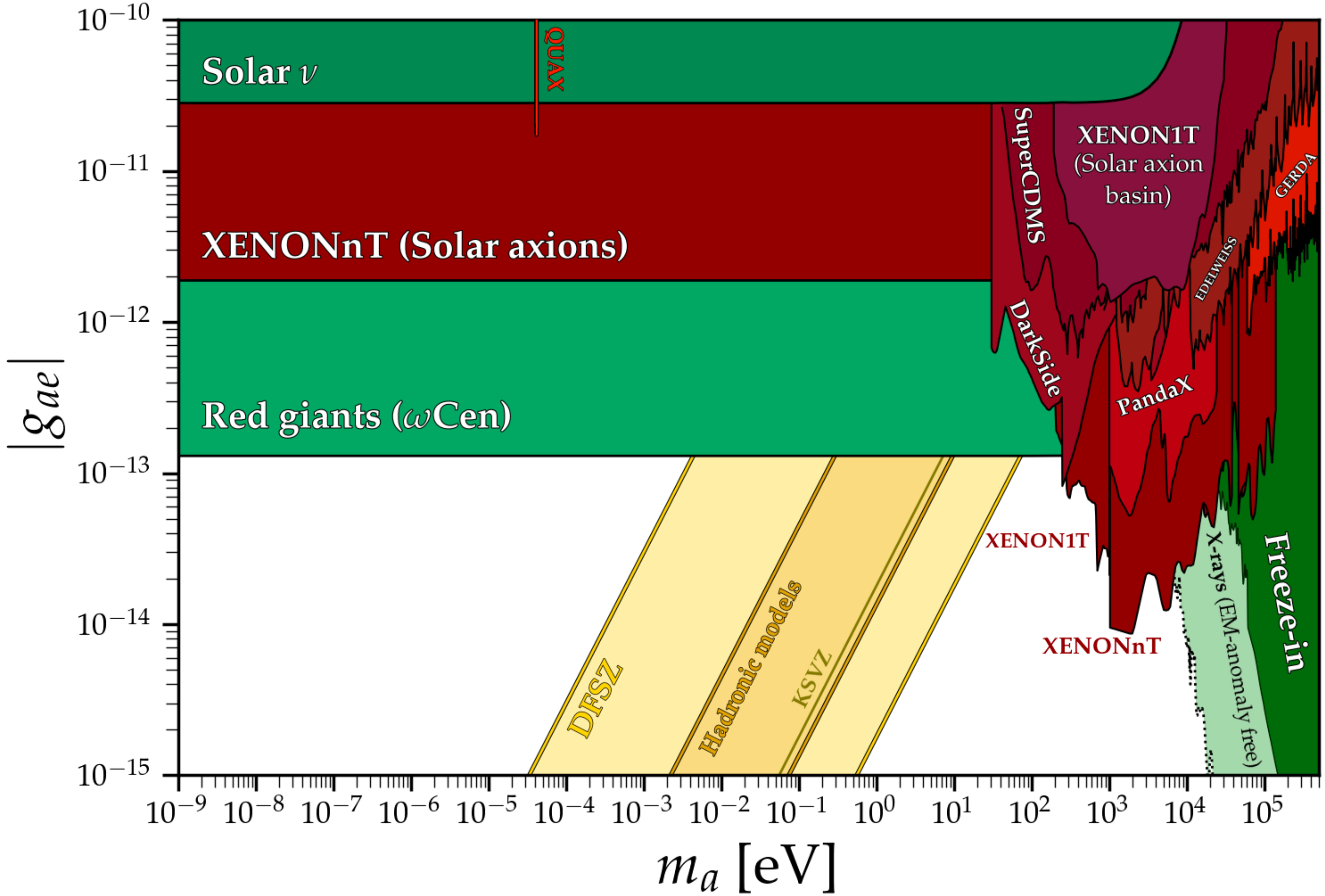
[PVRAS, arXiv:2005.12913]

$B = 2.5\,\mathrm{T}$
$L_B = 1.64\,\mathrm{m}$
$\omega = 1.2\,\mathrm{eV}$

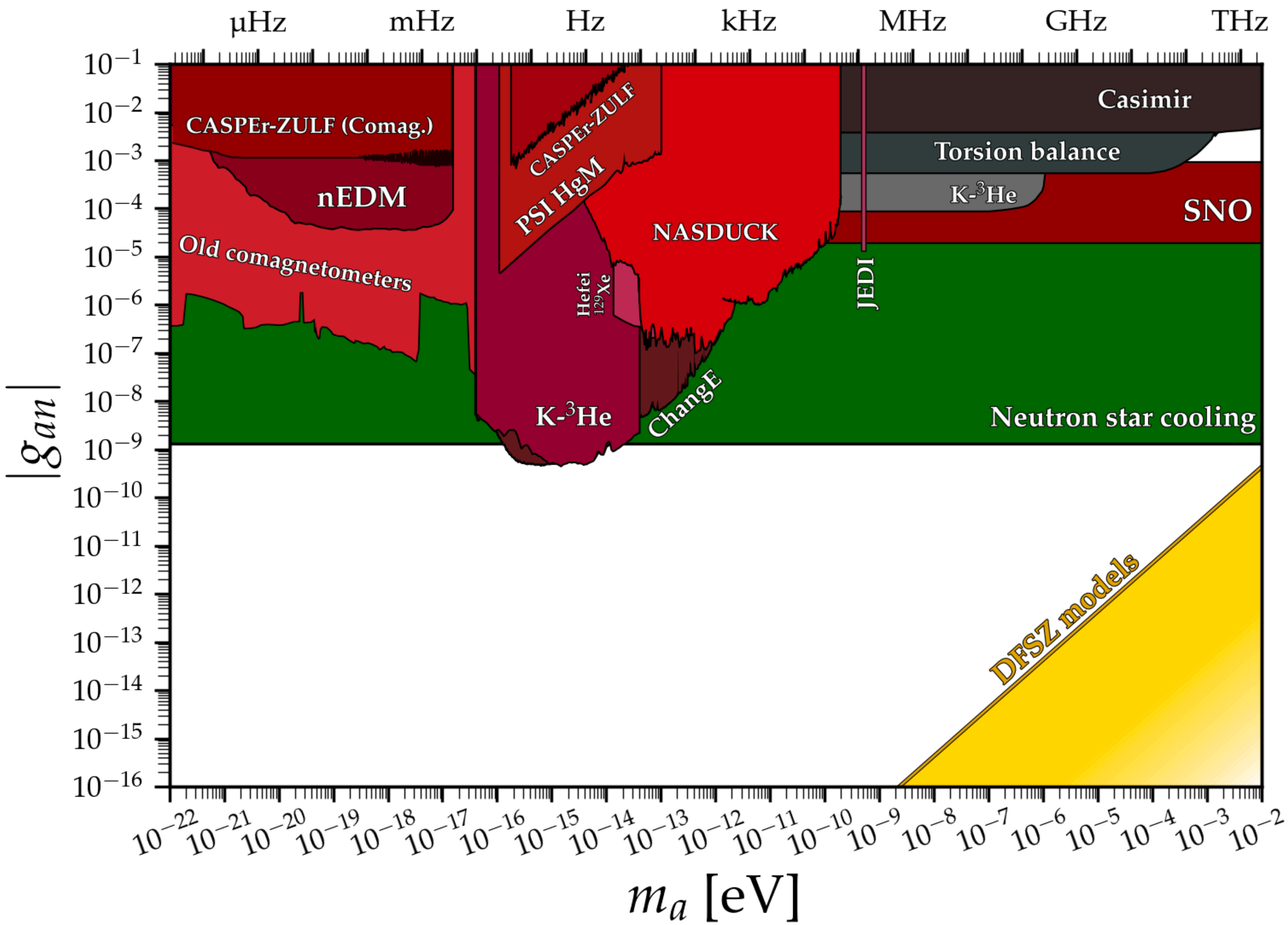
$\Delta n = 1.2\times10^{-22}$
$\Delta\kappa = 1.0\times10^{-22}$

Back up

Current status on $ig_{aee} a \bar{e} \gamma^5 e$



Current status on $ig_{ann}a\bar{n}\gamma^5 n$



Current status on $ig_{ann}a\bar{n}\gamma^5 n$