Expressing non-Abelian gauge-field dynamics in the quantum age

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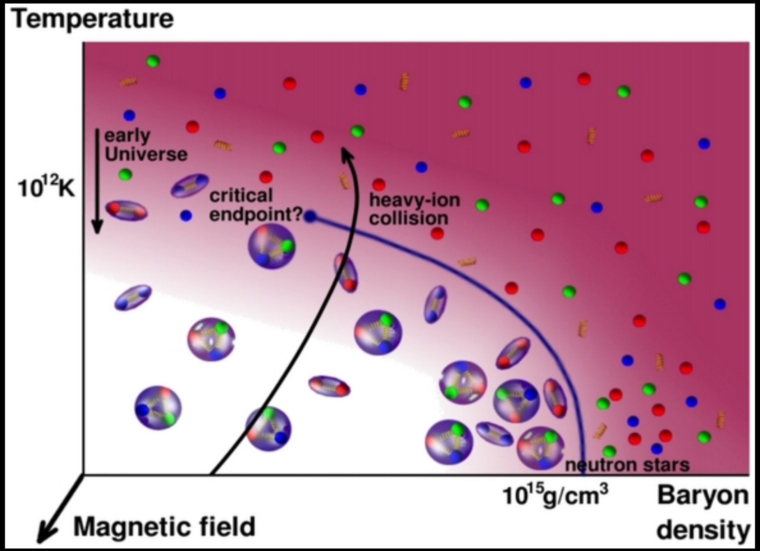
Motivation

Physics targets:
- Simulation of quantum chromodynamics
  - Hadronization
  - Microscopic understanding of scattering events
- Complete phase diagram of QCD
- Post-collision thermalization
- Roles of entanglement in HEP
- + more

Bauer, Davoudi et al. (2021) Snowmass report

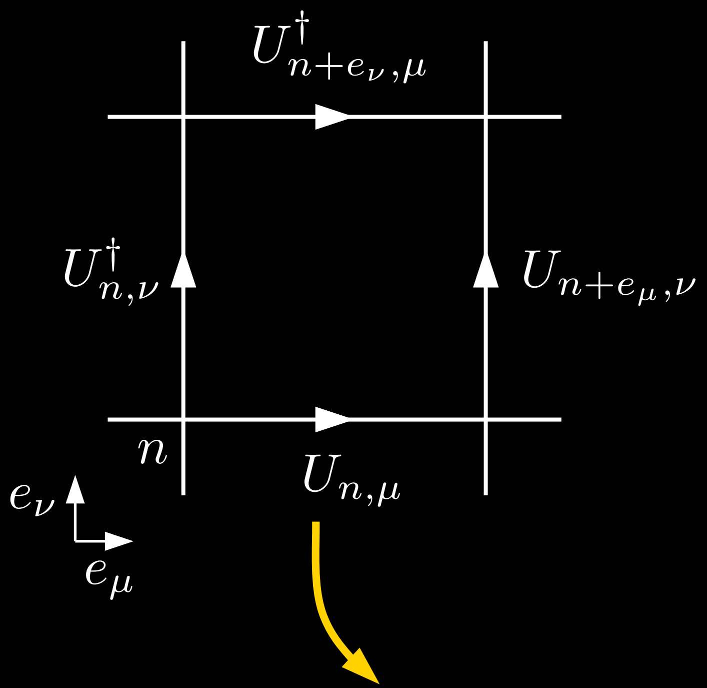
How to make these predictions?
- Nonperturbative problems
  - ➔ **Numerically simulate** QCD degrees of freedom



Conjectured phase diagram credit: G. Endrödi J.Phys.Conf.Ser. 503 (2014) 012009

Traditional lattice field theory

$x^\mu \to a n^\mu$

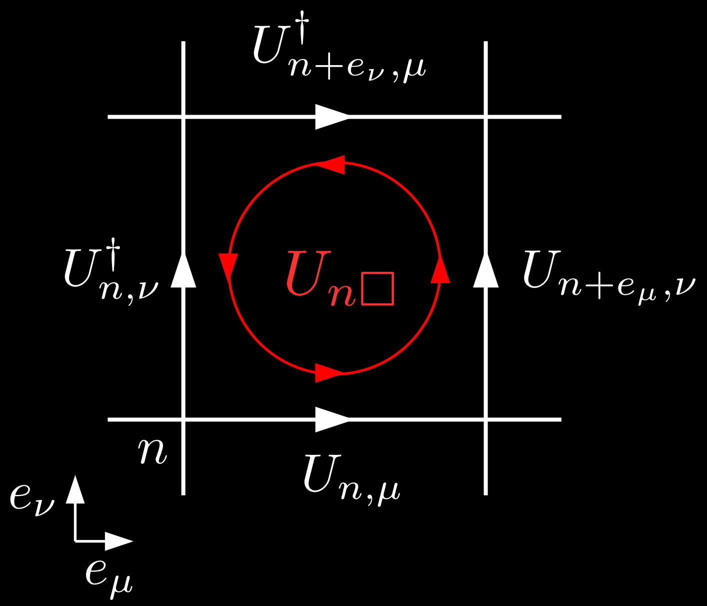


$$\begin{pmatrix} -0.7485 & -0.2744 - 0.6037i \\ 0.2744 - 0.6037i & -0.7485 \end{pmatrix}$$

- Defines a field theory nonperturbatively
- Spacetime discretized with a lattice (e.g. square, cubic, hypercubic)
- Matter particles such as quarks “live” on the sites
- Gauge bosons live on oriented links joining sites
- Gauge fields belonging to some (Lie) group–the “gauge group” G

Traditional lattice field theory

$x^\mu \to a n^\mu$



Wilson's gauge action, S_W

"link operator" matrices in gauge group G

$$S_W = -\beta \sum_{n,\mu} \mathrm{tr}(\underbrace{U_{n,\mu} U_{n+e_\mu,\nu} U^\dagger_{n+e_\nu,\mu} U^\dagger_{n,\nu}}_{U_\square} + U^\dagger_\square)$$

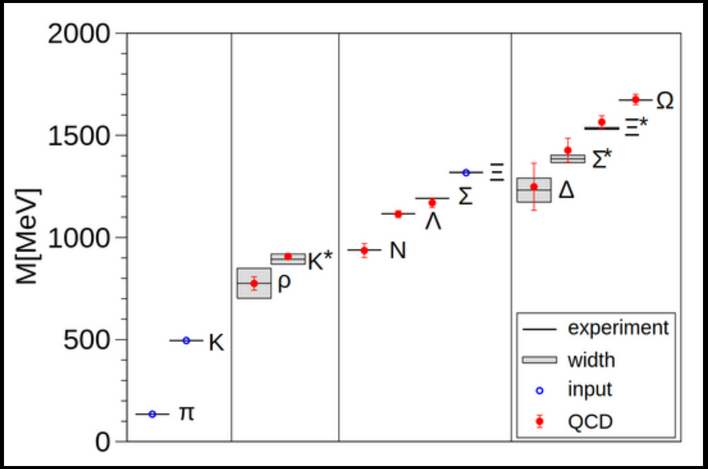
$U_\square$ "plaquette" operator

for non-Abelian

- In classical simulations, exp(-S_W) acts like a probably weight for the configuration – Monte Carlo integration

Traditional lattice field theory

- Successes
  - (light) hadron spectrum
  - *some* scattering amplitudes (Luscher formalism + generalizations)
  - muon g-2: hadronic vacuum polarization
- Drawbacks
  - dynamical fermions dramatically raise cost
  - (best for) static properties / thermal equilibrium



BMW collab., 2009

Traditional lattice field theory

- Limitations to Monte Carlo
  - Wick rotation to imaginary time underlies Euclidean path integral

$$\langle \hat{A} \rangle = \frac{\int [D\phi] e^{iS_M[\phi]} A[\phi]}{\int [D\phi] e^{iS_M[\phi]}} \rightarrow \frac{\int [D\phi] e^{-S_E[\phi]} A[\phi]}{\int [D\phi] e^{-S_E[\phi]}}$$

$$\int_{-\infty}^{\infty} dt \rightarrow \int_{-i\infty}^{i\infty} dt$$

  - $e^{iS_M[\phi]}$: violently oscillatory,
    $e^{-S_E[\phi]}$: dominated by saddle points
  - $i\,S_M \rightarrow -S_E$ may still be complex (theta term, chemical potential)
  - Finite Minkowskian time intervals preclude simply rotating the time-integration contour. Stuck with $i\,S_M$!

Classical problems; quantum solutions?

Digital quantum computers:

$$|\psi_i\rangle \left\{ \quad \boxed{\hat{U}} \quad \right\} |\psi_f\rangle$$

- Unitary gates: $e^{-it\hat{H}}$ with Hamiltonian of interest
- Want to simulate nonperturbative gauge theory
  - Gauge theory on the lattice
  - Hamiltonian lattice gauge theory
- Has no apparent sign problems

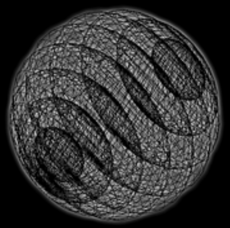
General problem:

How to map a Hilbert space $\mathcal{H}$, and $\hat{H}$, on to qubits & quantum gates?

Hamiltonian lattice gauge theory

- Temporal gauge, continuous-time limit → Kogut-Susskind Hamiltonian formulation
- Gauge fields on spatial links with on-link Hilbert spaces
- E.g., SU(2)

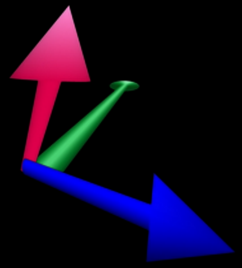
Phys. Rev. D 11, 395 (1975)



$$\langle g | j, m, m' \rangle = \sqrt{\frac{d_j}{|G|}} D^{(j)}_{m,m'}(g)$$

group-element basis — irrep basis

Left and right electric fields each have color-charge components, in addition to spatial components



$$[\hat{E}^{\alpha}_{L/R}, \hat{E}^{\beta}_{L/R}] = i f^{\alpha\beta\gamma} \hat{E}^{\gamma}_{L/R}$$
$$[\hat{E}^{\alpha}_{R}, \hat{U}_{mm'}] = (\hat{U} T^{\alpha})_{mm'}$$
$$[\hat{E}^{\alpha}_{L}, \hat{U}_{mm'}] = -(T^{\alpha} \hat{U})_{mm'}$$

canonical commutation relations for a link

Gauge transformations: $\hat{U}_{n,i} \rightarrow \Omega_n \hat{U}_{n,i} \Omega^{\dagger}_{n+e_i}$

- Rotations from the left (Ω_n) and right (Ω_{n+ei}) are generated by “left” and “right” electric fields

Hamiltonian lattice gauge theory

Lattice gauge theory Hilbert space structure
- Non-Abelian group, e.g. SU(2)

U adds representations

$$U_{m,m'} |j, M, M'\rangle = C_+(j, m, m', M, M')\times \times |j + 1/2, M + m, M' + m'\rangle + C_-(j, m, m', M, M')\times \times |j - 1/2, M + m, M' + m'\rangle$$

SU(2) example for the
2x2 link operator

Non-Abelian Hamiltonian

$$\hat{H}_E = \frac{g^2}{2} \sum_{n,i} \hat{E}^{\alpha}_{n,i} \hat{E}^{\alpha}_{n,i}$$

$$\hat{H}_B = -\sum_{n} \frac{1}{2g^2} \mathrm{tr}(\hat{U}_{n,\square} + \hat{U}^{\dagger}_{n,\square})$$

Hamiltonian lattice gauge theory

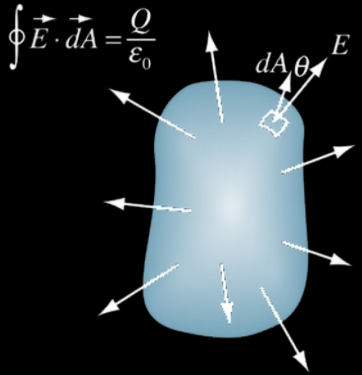
Plus Gauss law constraints

U(1)

$$\nabla \cdot \mathbf{E} - \rho = 0$$

$\hat{\mathcal{G}}_n$ ↗

$$\rho = \psi^\dagger \psi$$



SU(N)

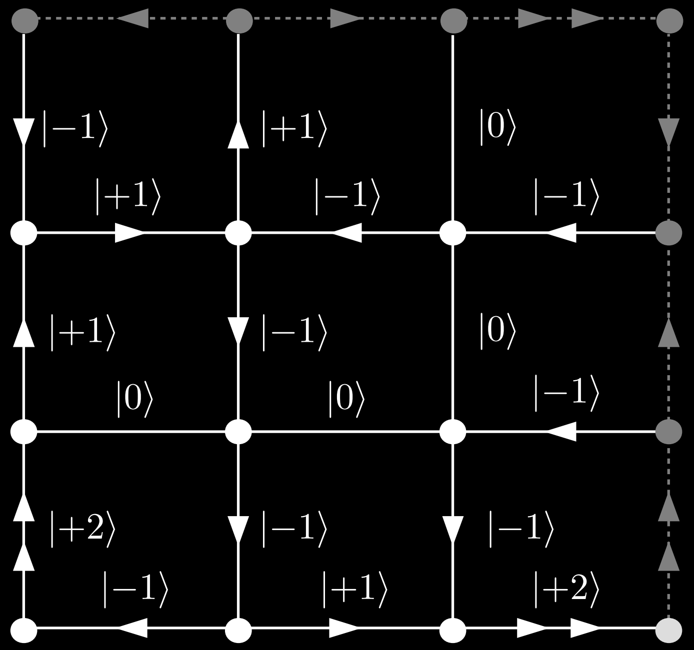
$$\mathbf{D} \cdot \mathbf{E}^a - \rho^a = 0$$

$\hat{\mathcal{G}}_n^a$ ↗

$$\rho^a = \psi^\dagger T^a \psi$$

Gauss's law ⟷ gauge invariance

charge conservation



compact U(1)
electric eigenbasis

Outline

- Formulations & bases
- SU(2) Schwinger boson formulation
- SU(2) Loop-string-hadron formulation
- Applications within quantum computing

* my working definition of formulation

Formulations & bases

- Thanks to redundancy, Hamiltonian lattice gauge theories seem to enjoy lots of different formulations
- Hamiltonian "formulation" meaning… *
  - set of degrees of freedom - often local
  - set of fields used to construct Hamiltonian/observables
  - algebraic (commutation) relations
  - constraints
  - (optional truncation scheme)

* my working definition of formulation

Formulations & bases

- Formulation ≠ basis !
  - But: Formulations are often associated with, or defined in terms of, a particular basis
  - Colloquially, different bases are at times called different “formulations” too…
- A formulation isn’t intrinsically tied to a particular Hamiltonian either – different choices are possible!
  - In practice, there usually is an implicit or explicit choice
  - We need at least one choice of Hamiltonian in order to do anything with the formulation. Constraints descend from the Hamiltonian.
- Basis choice is generally either <u>electric</u> or <u>magnetic</u>

Formulations & bases: Examples

- Kogut-Susskind formulation
  - Irrep/"angular momentum" basis
  Byrnes, Yamamoto, Zohar, Burrello, et al.
  - Group-element basis *Zohar, NuQS collab., et al.*
- Gauge magnets/quantum link models
  Wiese, Chandrasekharan, et al.
- Tensor lattice field theory
  Meurice, Sakai, Unmuth-Yockey, et al.
- Dual/rotor formulations *Kaplan, **JRS**, Haase, Dellantonio, et al., Bauer, Grabowska, et al.*
- Casimir variables / "local-multiplet basis"
  *Klco, Savage, **JRS**, Ciavarella*
- Purely fermionic formulations (1+1D & OBC)
  Muschik, Atas, Zhang, IQuS@UW group, Powell, et al.
- Prepotential/Schwinger boson formulations
  Mathur, Anishetty, Raychowdhury, et al.
- Loop-string-hadron formulation
  *Raychowdhury, **JRS**, Davoudi, Shaw, Dasgupta, Kadam*
- Light-front formulation
  Kreshchuk, Kirby, Love, Yao, et al.
- Qubit models *Chandrasekharan, Singh, et al.*
- *q*-deformed Kogut-Susskind
  Zache, González-Cuadra, Zoller
- Scalar field theory…
  - Harmonic oscillator basis
    Klco & Savage
  - Single-particle basis
    Barata, Mueller, Tarasov, Venugopalan
  - Future gauge-field generalizations??

Choice of basis

Most common basis choice: **Electric/irrep**

Electric-basis pros

- States naturally discretized (for compact Lie groups)
- Gauss's law a function of electric fields
- Natural "UV" truncation scheme

Electric-basis cons

- Better-suited to strong coupling (opposite of continuum QCD)
- Many off-diagonal operators in 3+1 Hamiltonian

Electric truncation

- Lie group Hilbert spaces are locally infinite-dimensional
- Digital quantum simulation requires truncations
  - Common choices: Finite subgroups, electric cutoff on irreps

Provably accurate simulation of gauge theories and bosonic systems

Yu Tong[1,2], Victor V. Albert[3], Jarrod R. McClean[1], John Preskill[4,5], and Yuan Su[1,4]
April 4th, 2022

- Tong et al., ‘22:
  - formal analysis on error in time evolution operator
  - U(1) and SU(2) LGTs considered in electric bases
  - Find: For fixed error ε and lattice parameters, required electric cutoff grows at worst linearly in time T and polylog($1/\varepsilon$)

Choice of basis

Group-element basis pros

- Link operators are diagonalized
- No Clebsch-Gordon coefficients
- Naively good for weak-coupling limit

A detail of Spinoza monument in Amsterdam. © Dmitry Feichtner-Kozlov

Group-element basis cons

- Limited number of regular subgroups for SU(N)
  - Limited "resolution" with subgroups
  - 120 elements for SU(2)
  - 1080 for SU(3) [NuQS collab.]
- Sub*sets* generally do not preserve gauge symmetry
- Electric fields become tricky

Choice of basis

- $E^a E^a$ is Laplace-Beltrami differential operator on the group manifold
- How to define derivatives on a subgroup or discrete subset? How to preserve gauge invariance?
- Only recently has this question been taken up by some groups in the context of quantum simulation

Jakobs, Garofalo, et al.
2304.02322
Mariani, Pradhan, and Ercolessi.
[2301.12224]
Ji, Lamm, and Ju.
Phys. Rev. D 102, 114513 (2020)

Fig. 1 Fibonacci lattices on S_2 with 20 (blue), 100 (orange) and 500 (green) vertices

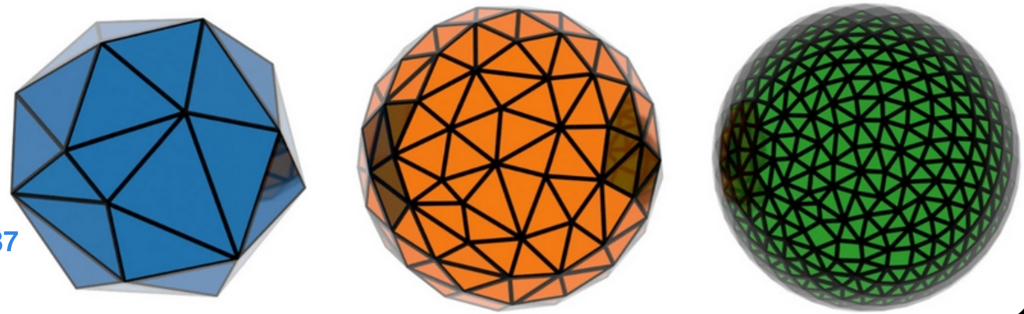
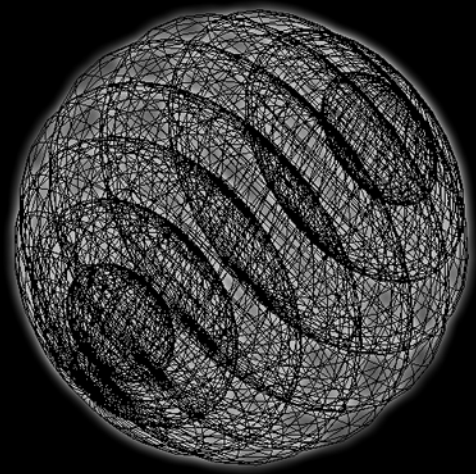


Figure by Hartung, Jakobs, Jansen, Ostmeyer, and Urbach.
Eur. Phys. J. C (2022) 82:237

SU(2) example

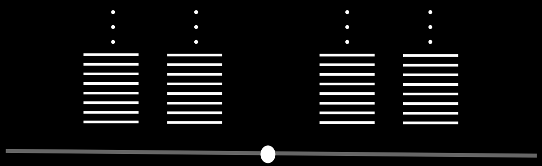
- Prototype non-Abelian gauge theory: SU(2), 1+1
- Matter: fundamental ‘quarks’
- Goal: Examine Schwinger-boson and its derivative loop-string-hadron formulation as simulation candidates



1D: Schwinger bosons + N=2 ‘quarks’

Step 1: Start with **Schwinger boson (“prepotential”) formulation.**

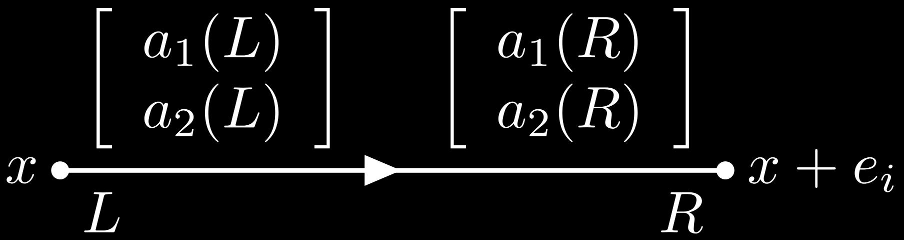
- Represents gauge field operators using many simple harmonic oscillators *



$$|n_{L,1}, n_{L,2}, n_{R,1}, n_{R,2}\rangle$$

$$n_{L,1} + n_{L,2} = 2j_L$$

$$n_{R,1} + n_{R,2} = 2j_R$$



- One bosonic doublet per end, per link
  - ➔ Four total oscillators per link
- Gauge Hilbert space → tensor product of SHOs

Gauge transformations:

$$a(L) \to \Omega(x) a(L)$$

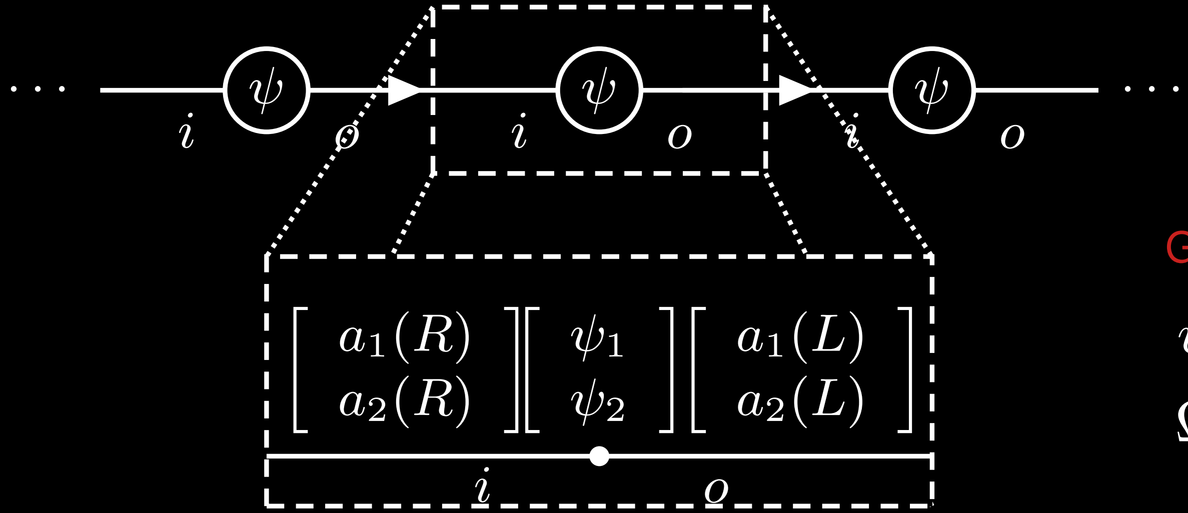
$$a(R) \to \Omega(x + e_i) a(R)$$

$$\Omega(x), \Omega(x + e_i) \in \mathrm{SU}(2)$$

* Papers by Anishetty, Mathur, Raychowdhury, Sharatchandra

1D: Schwinger bosons + N=2 'quarks'

Step 2: Add staggered fermions
- Two-color doublet



- One fermionic doublet per site
- Fock space to characterize lattice

Gauge transformations:

$$\psi(x) \to \Omega(x)\psi(x)$$

$$\Omega(x) \in \mathrm{SU}(2)$$

1D: Schwinger bosons + N=2 ‘quarks’

Step 3a: Represent E, U algebra

$$E_L^\alpha \equiv \hat{a}^\dagger(L) T^\alpha \hat{a}(L)$$
$$E_R^\alpha \equiv \hat{a}^\dagger(R) T^\alpha \hat{a}(R)$$

$$\hat{U}(x,i) = \hat{U}_L(x)\hat{U}_R(x+e_i) \ ,$$
$$\hat{U}_L(x,i) = \frac{1}{\sqrt{\mathcal{N}_L + 1}} \begin{pmatrix} \hat{a}_2^\dagger(L) & \hat{a}_1(L) \\ -\hat{a}_1^\dagger(L) & \hat{a}_2(L) \end{pmatrix}\Bigg|_{x,i}$$
$$\hat{U}_R(x,i) = \begin{pmatrix} \hat{a}_1^\dagger(R) & \hat{a}_2^\dagger(R) \\ -\hat{a}_2(R) & \hat{a}_1(R) \end{pmatrix} \frac{1}{\sqrt{\mathcal{N}_R + 1}}\Bigg|_{x,i}$$

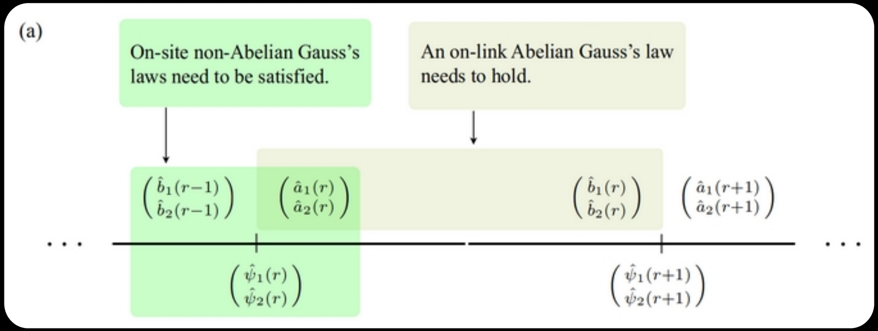
3b: Impose “Abelian Gauss law”

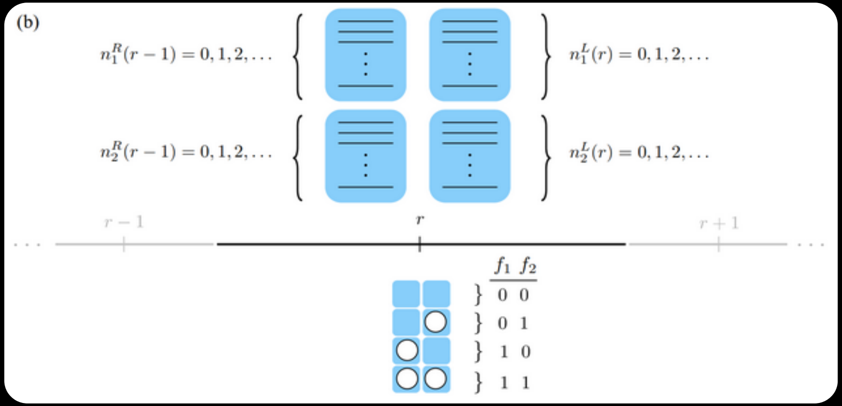
$$\mathcal{N}_{L/R} = \hat{a}^\dagger(L/R) \cdot \hat{a}(L/R)$$
$$\mathcal{N}_L(x,i)\,|\mathrm{phys}\rangle = \mathcal{N}_R(x+e_i,i)\,|\mathrm{phys}\rangle$$

Supplementary constraint from introducing extra dof’s

1D: Schwinger bosons + N=2 ‘quarks’

Pictorial summary of Schwinger boson DOFs





$$E_L^\alpha = a^\dagger \cdot \frac{\sigma^\alpha}{2} \cdot a \qquad E_R^\alpha = b^\dagger \cdot \frac{\sigma^\alpha}{2} \cdot b$$

$$U = \frac{1}{\sqrt{a^\dagger \cdot a + 1}} \begin{pmatrix} -a_1 b_2 + a_2^\dagger b_1^\dagger & a_1 b_1 + a_2^\dagger b_2^\dagger \\ -a_2 b_2 - a_1^\dagger b_1^\dagger & a_2 b_1 - a_1^\dagger b_2^\dagger \end{pmatrix} \frac{1}{\sqrt{a^\dagger \cdot a + 1}}$$

Loop-string-hadron formulation, SU(2)

Going further: Exploit doublets to make SU(2) singlets

Notice:

$$f \to \Omega \cdot f$$
$$(\epsilon f^*) \to \Omega \cdot (\epsilon f^*)$$
$$f = a(L), a(R), \psi$$

So use the doublets and their duals from a site to make *manifestly* Ω-*invariant* bilinears (have $\Omega^\dagger$ and Ω cancel)

Examples:

$$a(L/R)^\dagger \cdot a(L/R) = a_1^\dagger a_1 + a_2^\dagger a_2$$
$$\psi^\dagger \cdot \psi = \psi_1^\dagger \psi_1 + \psi_2^\dagger \psi_2$$
$$(\epsilon a(L)^*)^\dagger \cdot a(R) = a_1(R) a_2(L) - a_2(R) a_1(L)$$
$$(\epsilon \psi^*)^\dagger \cdot \psi = -(\psi_1 \psi_2 - \psi_2 \psi_1) = 2\psi_2 \psi_1$$

In this way we can form 17 bilinears that are exactly invariant under Ω

These special operators do not "know" a way to violate color charge conservation

I. Raychowdhury & JRS
PRD 101, 114502 (2020)
PRResearch 2, 033039 (2020)

Loop-string-hadron formulation, SU(2)

$$\mathcal{L}^{++} = a(R)^\dagger_\alpha a(L)^\dagger_\beta \epsilon_{\alpha\beta}$$

$$\mathcal{L}^{--} = a(R)_\alpha a(L)_\beta \epsilon_{\alpha\beta} = (\mathcal{L}^{++})^\dagger$$

$$\mathcal{L}^{+-} = a(R)^\dagger_\alpha a(L)_\beta \delta_{\alpha\beta}$$

$$\mathcal{L}^{-+} = a(R)_\alpha a(L)^\dagger_\beta \delta_{\alpha\beta} = (\mathcal{L}^{+-})^\dagger$$

$$\mathcal{S}^{++}_{\text{in}} = a(R)^\dagger_\alpha \psi^\dagger_\beta \epsilon_{\alpha\beta}$$

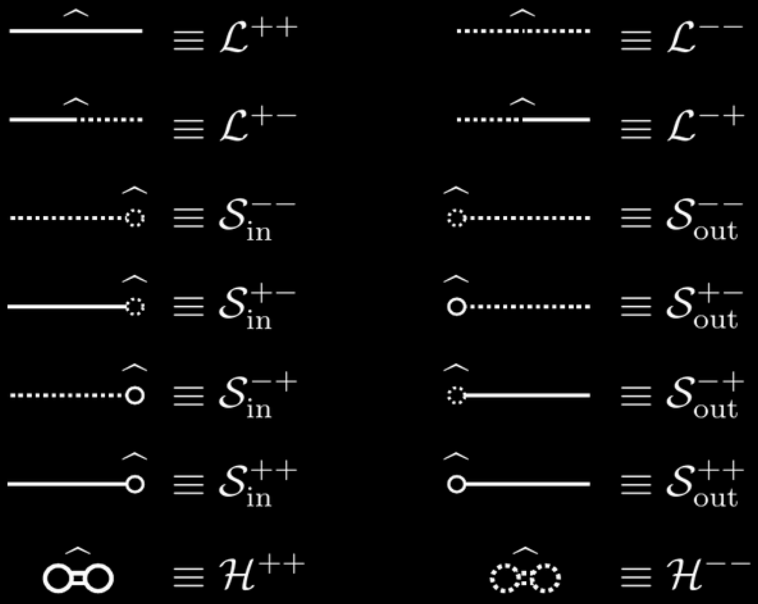
$$\mathcal{S}^{--}_{\text{in}} = a(R)_\alpha \psi_\beta \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{in}})^\dagger$$

$$\mathcal{S}^{+-}_{\text{in}} = a(R)^\dagger_\alpha \psi_\beta \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{in}} = a(R)_\alpha \psi^\dagger_\beta \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{in}})^\dagger$$

$$\mathcal{H}^{++} = -\frac{1}{2!}\psi^\dagger_\alpha \psi^\dagger_\beta \epsilon_{\alpha\beta}$$

$$\mathcal{H}^{--} = \frac{1}{2!}\psi_\alpha \psi_\beta \epsilon_{\alpha\beta} = (\mathcal{H}^{++})^\dagger$$



4 ‘**loop**’ + 4 ‘in **string**’ + 4 ‘out **string**’ + 2 ‘**hadron**’ operators (+ 3 number operators) → “LSH”

In this way we can form 17 bilinears that are exactly invariant under Ω

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LSH formulation, SU(2), 1+1

Physical, SU(2)-invariant interpretations

$\mathcal{L}^{++}(x) \equiv$	Create unit of gauge flux.
$\mathcal{L}^{--}(x) \equiv$	Destroy unit of gauge flux.
$\mathcal{L}^{+-}(x) \equiv$	Change matter-sourced flux direction. $(d > 1)$
$\mathcal{L}^{-+}(x) \equiv$	Change matter-sourced flux direction. $(d > 1)$
$\mathcal{H}^{++}(x) \equiv$	Create a hadron.
$\mathcal{H}^{--}(x) \equiv$	Destroy a hadron.

This *is* the physical intuition for interacting SU(2) excitations

SU(2) = pseudoreal
flux = unoriented

vs

U(1) = complex
flux = oriented

… string (S) operators are more involved …

LSH formulation, SU(2), 1+1

Identified the loop-string-hadron (LSH) operators

- Manifestly SU(2)-invariant
- Transparent physical interpretations
- **Can construct Hamiltonian in terms of them**

"Hard" terms

"Easy" terms

$$\hat{H}_I \to \sum_x \frac{1}{\sqrt{\mathcal{N}_L(x)+1}} \left[\sum_{\sigma=\pm} \mathcal{S}_{\text{out}}^{+,\sigma}(x) \mathcal{S}_{\text{in}}^{\sigma,-}(x+1) \right] \times \frac{1}{\sqrt{\mathcal{N}_R(x+1)+1}} + \text{H.c.}$$

$$\hat{H}_M \to m_0 \sum_x (-)^x \mathcal{N}_\psi(x)$$

$$\hat{H}_E \to \frac{g_0^2}{4} \sum_x \left[\frac{1}{2}\mathcal{N}_R(x)\left(\frac{1}{2}\mathcal{N}_R(x)+1\right) + \frac{1}{2}\mathcal{N}_L(x)\left(\frac{1}{2}\mathcal{N}_L(x)+1\right) \right]$$

$$\hat{\psi}^\dagger(x)\hat{U}_L(x) = \frac{1}{\sqrt{\mathcal{N}_L(x)+1}} \left(\mathcal{S}_{\text{out}}^{++}(x), \quad \mathcal{S}_{\text{out}}^{+-}(x) \right),$$

$$\hat{U}_R(x)\hat{\psi}(x) = \begin{pmatrix} \mathcal{S}_{\text{in}}^{+-}(x) \\ \mathcal{S}_{\text{in}}^{--}(x) \end{pmatrix} \frac{1}{\sqrt{\mathcal{N}_R(x)+1}}.$$

LSH formulation, SU(2), 1+1

LSH operators also define an SU(2)-singlet basis
- Take a reference state, e.g., 0 flux & 0 fermions
- Act locally with any product of LSH operators
- Result is SU(2)-invariant

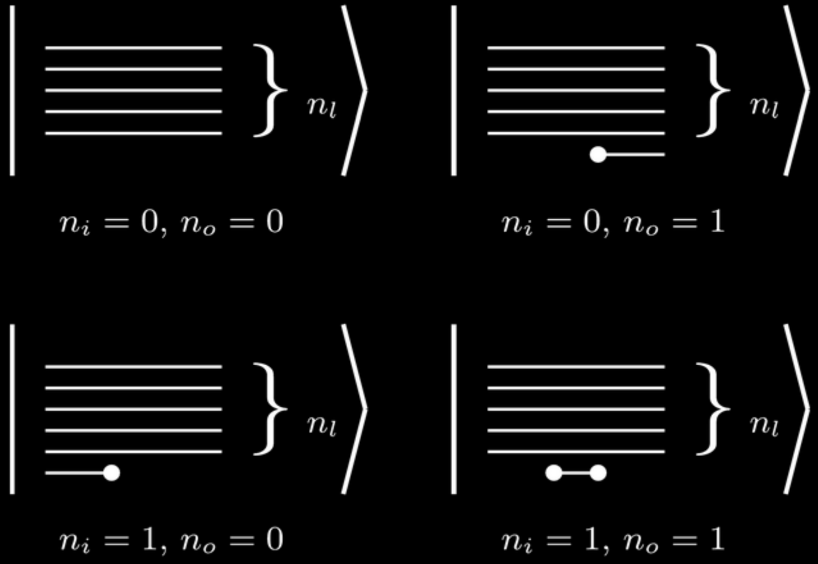
The “catch” of this framework is non-automatic flux conservation *along links.*

$$||n_l, n_i = 0, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}|0\rangle$$

$$||n_l, n_i = 0, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}_{\text{out}}^{++}|0\rangle$$

$$||n_l, n_i = 1, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}_{\text{in}}^{++}|0\rangle$$

$$||n_l, n_i = 1, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{H}^{++}|0\rangle$$



$$\mathcal{N}_\psi = \mathcal{N}_i + \mathcal{N}_o$$

$$\mathcal{N}_L = \mathcal{N}_l + \mathcal{N}_o(1 - \mathcal{N}_i)$$

$$\mathcal{N}_R = \mathcal{N}_l + \mathcal{N}_i(1 - \mathcal{N}_o)$$

This is an electric basis

LSH formulation, SU(2), 1+1

Can compute LSH-operator matrix elements using the orthonormal basis

- All operators 'factorized' into diagonal matrices and 'normalized ladder operators' (one-sparse, binary matrices)

Loop-string-hadron operator factorizations
$\mathcal{L}^{++} = \Lambda^{+}\sqrt{(\mathcal{N}_l+1)(\mathcal{N}_l+2+(\mathcal{N}_i \oplus \mathcal{N}_o))}$
$\mathcal{L}^{--} = \Lambda^{-}\sqrt{\mathcal{N}_l(\mathcal{N}_l+1+(\mathcal{N}_i \oplus \mathcal{N}_o))}$
$\mathcal{L}^{+-} = -\chi_i^\dagger \chi_o$
$\mathcal{L}^{-+} = \chi_i \chi_o^\dagger$
$\mathcal{S}_{\text{in}}^{++} = \chi_i^\dagger (\Lambda^+)^{\mathcal{N}_o}\sqrt{\mathcal{N}_l+2-\mathcal{N}_o}$
$\mathcal{S}_{\text{in}}^{--} = \chi_i (\Lambda^-)^{\mathcal{N}_o}\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_o)}$
$\mathcal{S}_{\text{out}}^{++} = \chi_o^\dagger (\Lambda^+)^{\mathcal{N}_i}\sqrt{\mathcal{N}_l+2-\mathcal{N}_i}$
$\mathcal{S}_{\text{out}}^{--} = \chi_o (\Lambda^-)^{\mathcal{N}_i}\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_i)}$
$\mathcal{S}_{\text{in}}^{-+} = \chi_o^\dagger (\Lambda^-)^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+2\mathcal{N}_i}$
$\mathcal{S}_{\text{in}}^{+-} = \chi_o (\Lambda^+)^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+1+\mathcal{N}_i}$
$\mathcal{S}_{\text{out}}^{+-} = \chi_i^\dagger (\Lambda^-)^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+2\mathcal{N}_o}$
$\mathcal{S}_{\text{out}}^{-+} = \chi_i (\Lambda^+)^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+1+\mathcal{N}_o}$
$\mathcal{H}^{++} = \chi_i^\dagger \chi_o^\dagger$
$\mathcal{H}^{--} = -\chi_i \chi_o$

$$\langle n_l', n_i', n_o' | \Lambda^{\pm} | n_l, n_i, n_o \rangle = \delta_{n_l', n_l \pm 1}\delta_{n_i', n_i}\delta_{n_o', n_o}$$

$$\{\chi_{q'}, \chi_q\} = \{\chi_{q'}^\dagger, \chi_q^\dagger\} = 0$$

$$\{\chi_{q'}, \chi_q^\dagger\} = \delta_{q'q} \qquad (q = i, o)$$

SU(2) LSH & quantum computation

Hamiltonian in operator-factorized form is the input for developing simulation algorithms

Advantages

- All constraints are *Abelian*
  - → Simultaneously diagonalizable
  - → LSH basis states are individually definitely allowed or definitely unallowed, unlike other formulations
- Hilbert space is structure is far simpler than |jmm’> states
- Hamiltonian structure looks more similar to U(1)
- Clebsch-Gordons recast as SHO scaling factors
- First SU(2) physicality quantum circuits constructed (Raychowdhury & JS 2020)

SU(2) LSH & quantum computation

- Circuits for LSH constraints, in any number of dimensions, are worked out in detail
- Speedups likely needed to make possible in NISQ era

LSH potential drawbacks:

- H_B in d>1 has many terms
- Can cost more qubits in D>1+1

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Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

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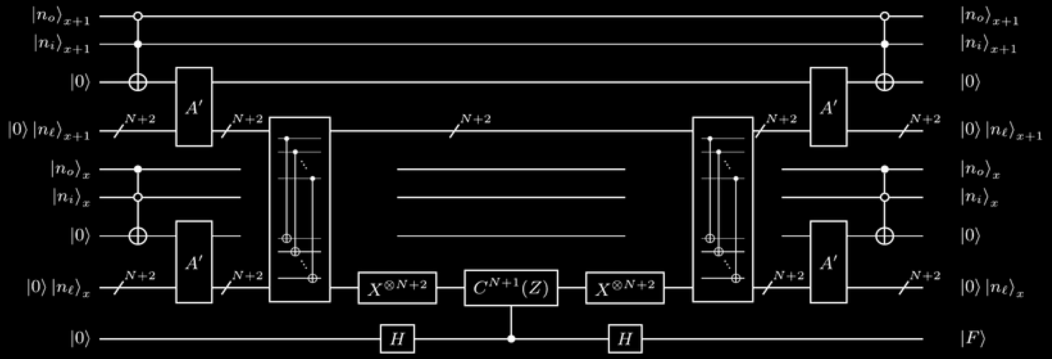
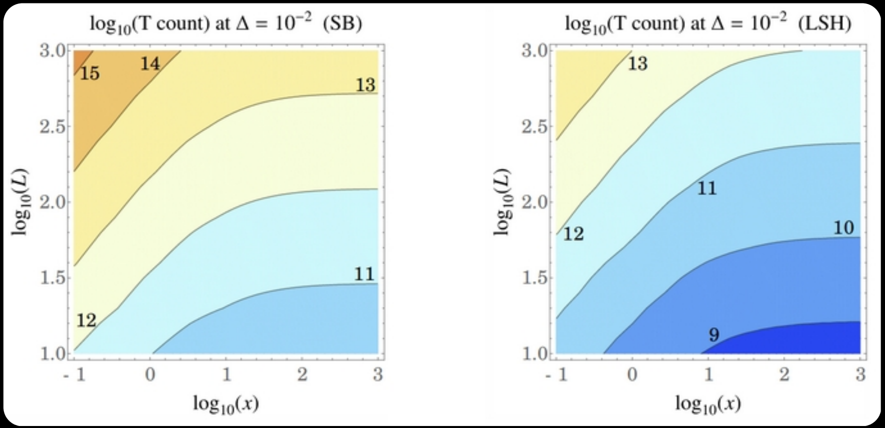
We show that using the loop-string-hadron (LSH) formulation of SU(2) lattice gauge theory (I. Raychowdhury and J. R. Stryker, Phys. Rev. D **101**, 114502 (2020)) as a basis for digital quantum computation easily solves an important problem of fundamental interest: implementing gauge invariance (or Gauss's law) exactly. We first



SU(2) LSH vs Schwinger bosons

Z. Davoudi, A.F. Shaw, & JRS, arXiv:2212.14030 (accepted to Quantum)
- Complete Trotterized time-evolution circuits for Schwinger boson and LSH formulations.
- Far-term- and near-term-inspired circuits

							Schwinger bosons		LSH	
x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Qubits	T gates	Qubits	T gates
1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}	1319	3.91817×10^{10}
1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}	1397	1.5172×10^{11}
1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}	1397	1.51643×10^{12}
1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}	1475	5.76229×10^{12}
1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{13}	6797	1.53099×10^{12}
1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}	6875	5.81562×10^{12}
1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}	6875	5.81468×10^{13}
1	4	1000	10	0.001	90%	9%	19086	4.48657×10^{15}	6979	2.29217×10^{14}
1	8	100	1	0.01	90%	9%	4398	5.79224×10^{12}	1807	2.72735×10^{11}
1	8	100	1	0.001	90%	9%	4476	2.1482×10^{13}	1885	1.03709×10^{12}
1	8	100	10	0.01	90%	9%	4476	2.14816×10^{14}	1885	1.03705×10^{13}
1	8	100	10	0.001	90%	9%	4580	8.22615×10^{14}	1963	3.87886×10^{13}
1	8	1000	1	0.01	90%	9%	35076	2.16773×10^{14}	10885	1.04652×10^{13}
1	8	1000	1	0.001	90%	9%	35180	8.30098×10^{14}	10963	3.91414×10^{13}
1	8	1000	10	0.01	90%	9%	35180	8.30094×10^{15}	10963	3.91412×10^{14}
1	8	1000	10	0.001	90%	9%	35258	2.99214×10^{16}	11067	1.5154×10^{15}



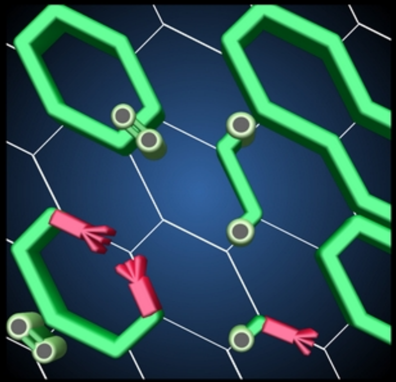
T-gate costs at fixed m/g=1. Other simulation parameters not explicitly shown are η = 8, t/a_s = 1, $\alpha_{\text{Trot.}}$ = 90%, $\alpha_{\text{Newt.}}$ = 9%, and $\alpha_{\text{synth.}}$ = 1%.

~20x T gate reduction with LSH

Conclusion

- Still in early days of quantum-assisted lattice QFT/QCD
- "What would you do today with a perfect quantum computer, gates, and lots of qubits?"
  - Until recently, we had no real answers relevant for lattice QCD
- Much theoretical development remains to be done
  - LSH recently generalized to SU(3) in 1+1D (Kadam, Raychowdhury, JRS 2022)
- Much to learn about pros and cons of different formulations

This field is vibrant and rapidly expanding!



Thank you for your attention!

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Extra slides

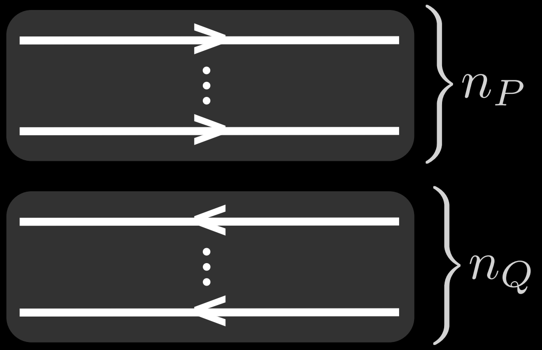
SU(3) LSH

1+1D Hilbert space construction

S. Kadam, JS, & I. Raychowdhury, Phys. Rev. D 107, 094513 (2023)

$$[A^\dagger(\underline{1}) \cdot B^\dagger(1)]^{n_P} \to$$

$$[B^\dagger(\underline{1}) \cdot A^\dagger(1)]^{n_Q} \to$$



$$|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 0, 0\rangle \to$$

$$\psi^\dagger \cdot B^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 0, 1\rangle \to$$

$$\psi^\dagger \cdot B^\dagger(\underline{1})\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 0, 0\rangle \to$$

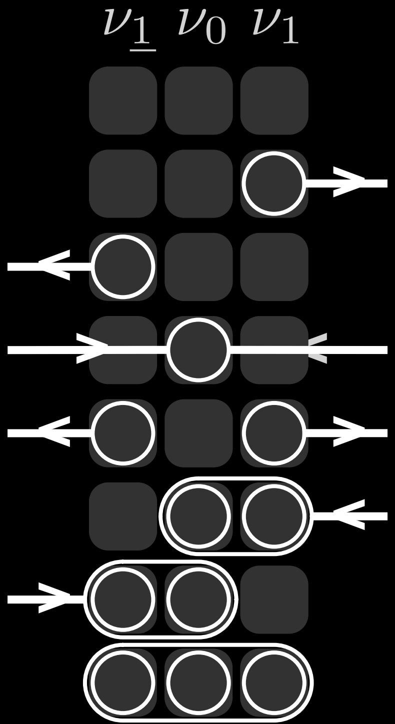
$$\psi^\dagger \cdot A^\dagger(\underline{1}) \wedge A^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 1, 0\rangle \to$$

$$\psi^\dagger \cdot B^\dagger(\underline{1})\psi^\dagger \cdot B^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 0, 1\rangle \to$$

$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 1, 1\rangle \to$$

$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1})\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 1, 0\rangle \to$$

$$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 1, 1\rangle \to$$



D>1+1 in progress!