Modelling of Massive Stars

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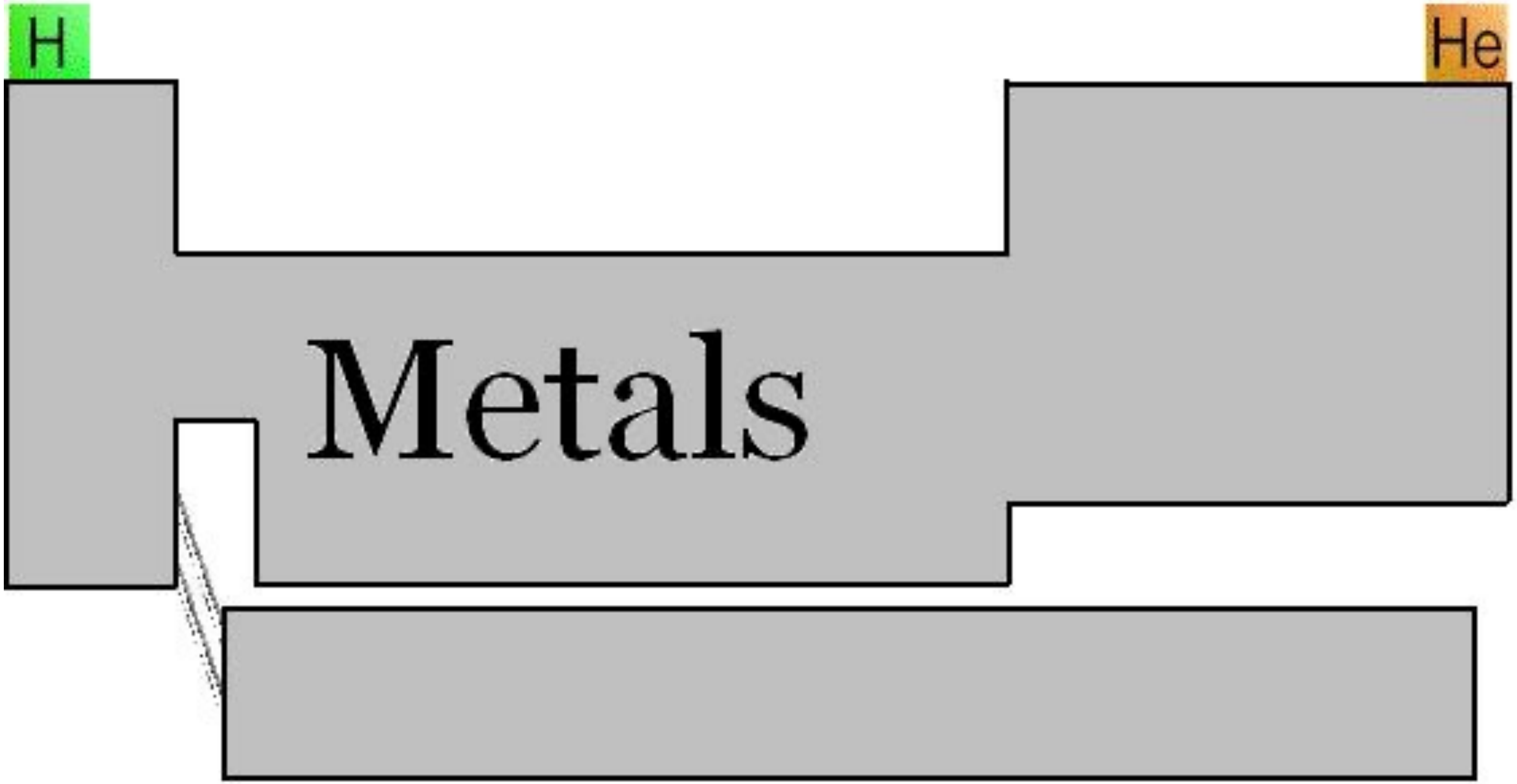
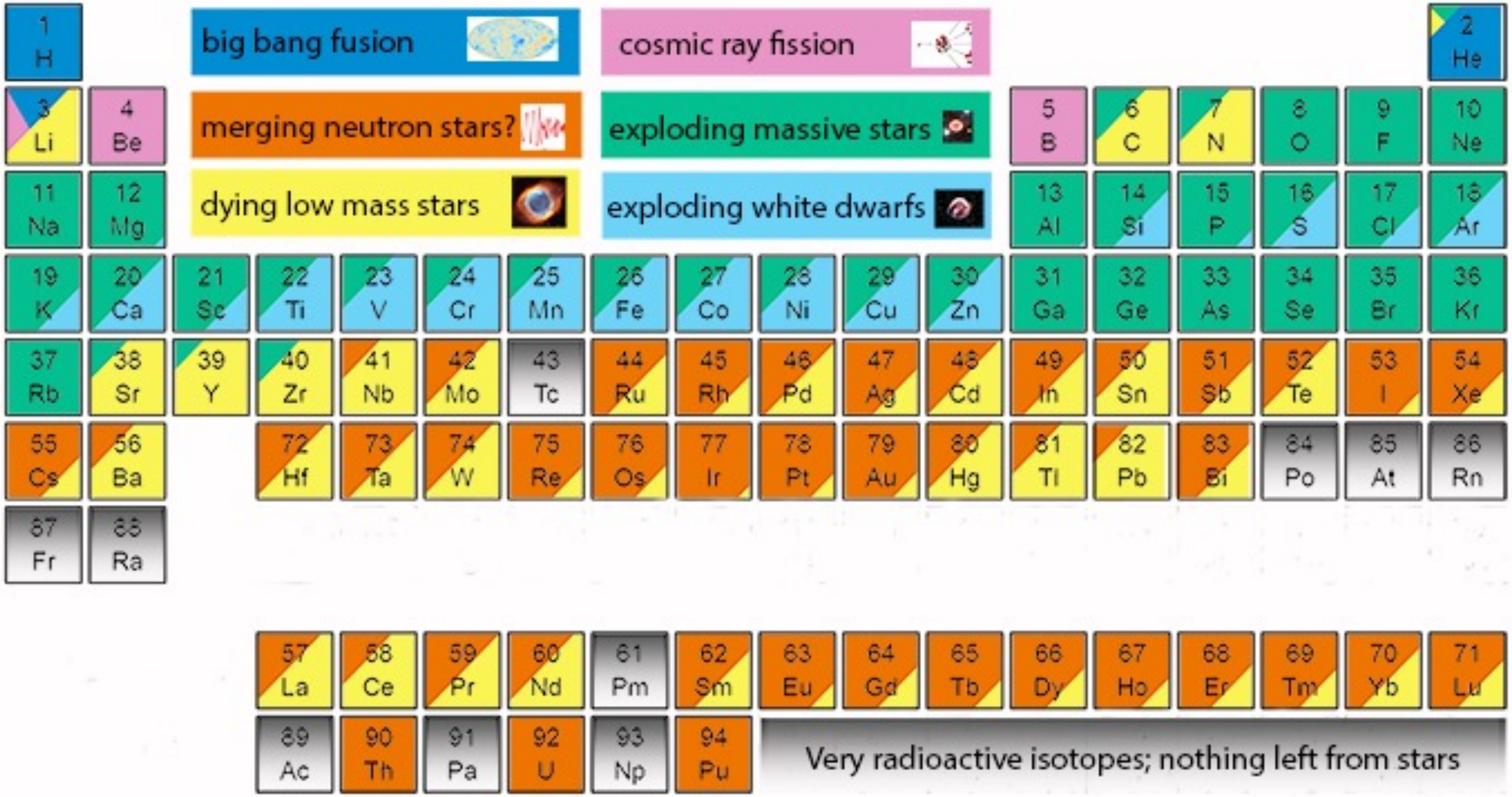
https://kaiserscience.wordpress.com/chemistry/the-periodic-table/alternative-periodic-tables/



The Origin of the Solar System Elements



Graphic created by Jennifer Johnson
http://www.astronomy.ohio-state.edu/~jaj/nucleo/

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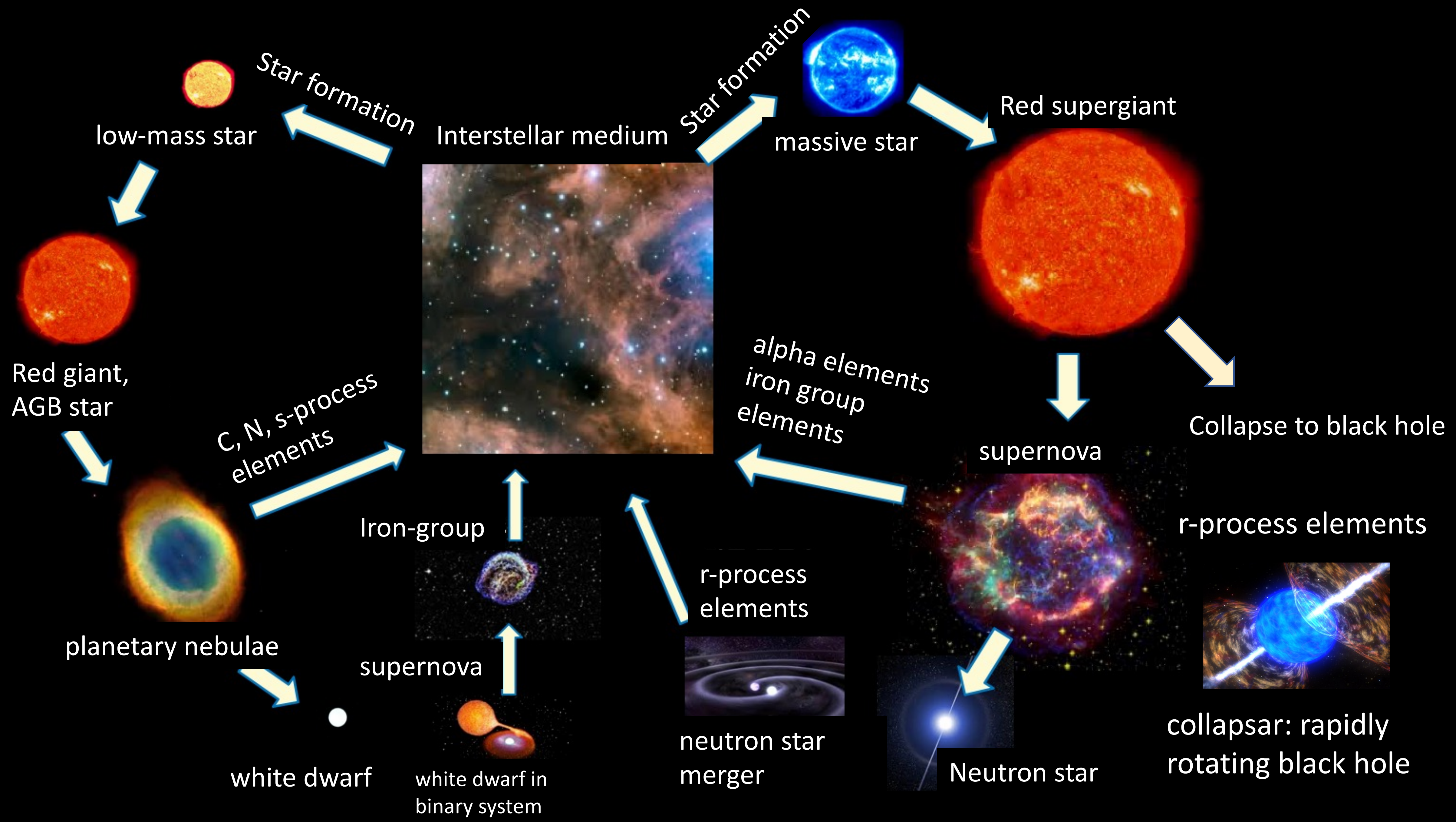
Star formation
low-mass star
Interstellar medium
Star formation
massive star
Red supergiant
Red giant,
AGB star
C, N, s-process
elements
alpha elements
iron group
elements
Collapse to black hole
supernova
Iron-group
r-process elements
r-process
elements
planetary nebulae
supernova
white dwarf
white dwarf in
binary system
neutron star
merger
Neutron star
collapsar: rapidly
rotating black hole

Massive stars

- Progenitors of supernovae
- Progenitors of neutron stars and black hole
- Source of ionizing flux in the universe
- Life time is short ($< \sim 10^7$ yr) : important role for the chemical evolution
- Main source of alpha elements (O, Ne, Mg, Si, S, Ca, etc.)
- Iron-group elements: about 30% in the universe
- r-process elements (either by neutron star mergers or collapsar)
- s-process elements (56 < A < 90, weak components)

Recommended Reference

REVIEWS OF MODERN PHYSICS, VOLUME 74, OCTOBER 2002

The evolution and explosion of massive stars

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(Published 7 November 2002)

Like all true stars, massive stars are gravitationally confined thermonuclear reactors whose composition evolves as energy is lost to radiation and neutrinos. Unlike lower-mass stars ($M \lesssim 8 M_\odot$), however, no point is ever reached at which a massive star can be fully supported by electron degeneracy. Instead, the center evolves to ever higher temperatures, fusing ever heavier elements until a core of iron is produced. The collapse of this iron core to a neutron star releases an enormous amount of energy, a tiny fraction of which is sufficient to explode the star as a supernova. The authors examine our current understanding of the lives and deaths of massive stars, with special attention to the relevant nuclear and stellar physics. Emphasis is placed upon their post-helium-burning evolution. Current views regarding the supernova explosion mechanism are reviewed, and the hydrodynamics of supernova shock propagation and "fallback" is discussed. The calculated neutron star masses, supernova light curves, and spectra from these model stars are shown to be consistent with observations. During all phases, particular attention is paid to the nucleosynthesis of heavy elements. Such stars are capable of producing, with few exceptions, the isotopes between mass 16 and 88 as well as a large fraction of still heavier elements made by the r and p processes.

Outline

- Basics of stellar evolution
- Evolution of massive stars
- Massive star modelling

Basics of stellar evolution

Stellar luminosity with black-body approximation

- If you integrate the Planck function over the whole frequency range, you get the Stefan-Boltzmann law as the following:

$$F = \sigma T^4 \quad \text{at stellar surface}$$

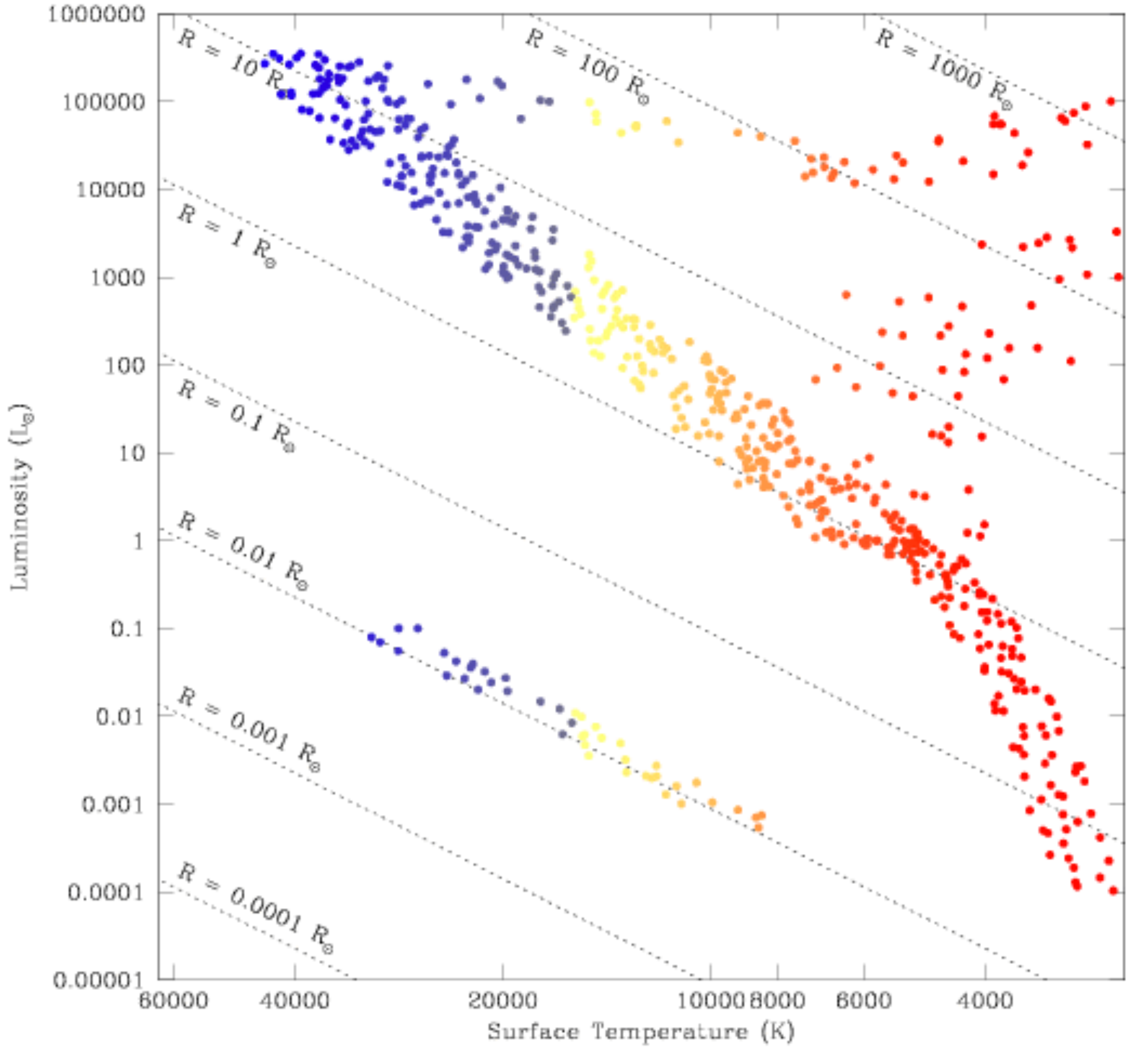
- Here F = flux (energy per unit area per unit time)

- The stellar luminosity (total energy per unit time) is then given by:

$$L = 4\pi R^2 F = 4\pi R^2 \sigma T^4$$

- This means: stars are bright because they are hot (regardless of their energy source).

Hertzsprung-Russell Diagram

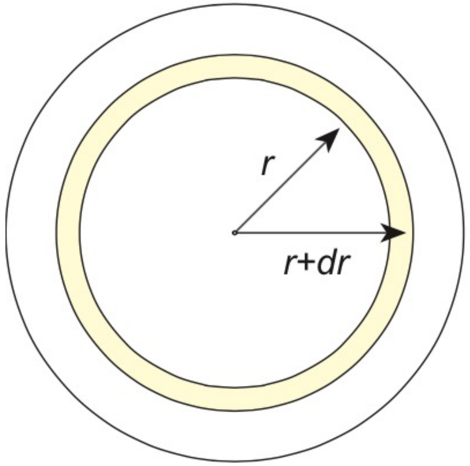


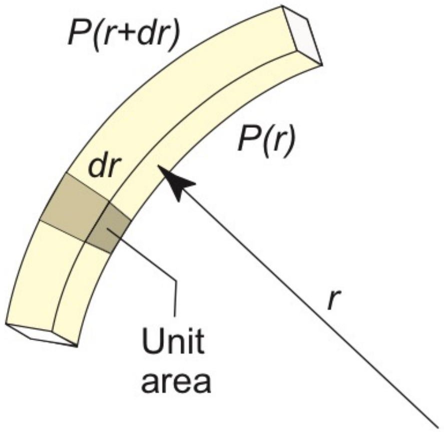
Three principles that govern the evolution of stars

- **Stars are in hydrostatic equilibrium**
- **Stars have a negative heat capacity**
- **Stars lose energy by radiation**

1. Stars are in hydrostatic equilibrium.

Mass Continuity and Mass Coordinate





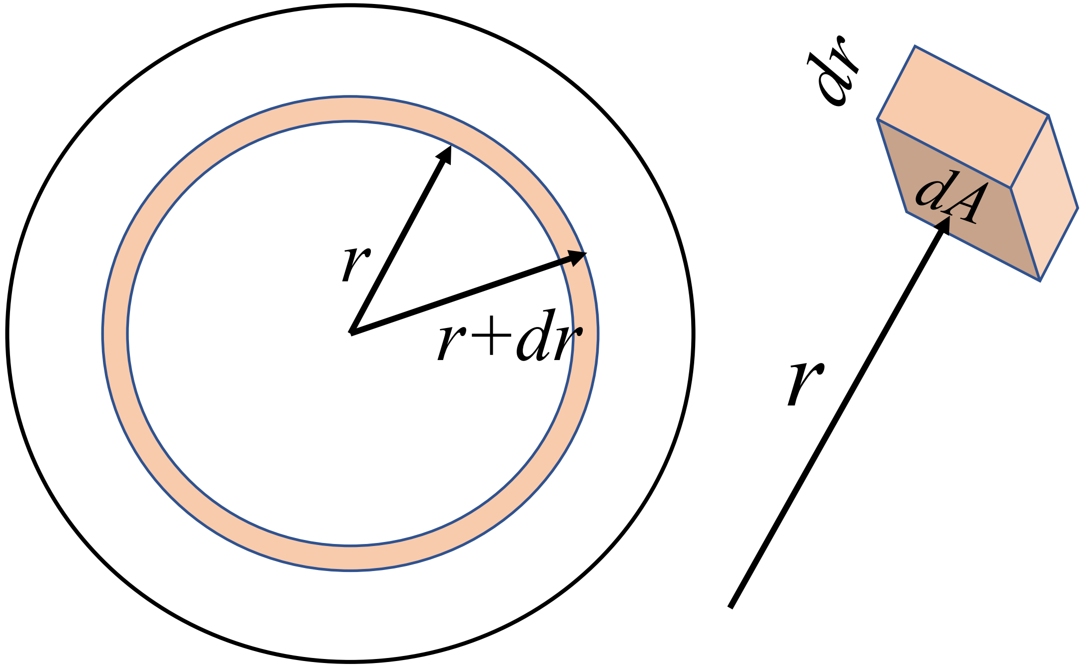
$dM_r = 4\pi r^2 \rho(r) dr$:mass contained in a spherical shell between r and $r + dr$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho(r) \quad \text{:Mass Continuity Equation}$$

M_r : mass coordinate

$$M_r = \int_0^r 4\pi r'^2 \rho(r') dr' \quad \text{Total mass contained within the radius r}$$

Hydrostatic Equilibrium Equation



$$dm = \rho dr dA$$

$$F_{\text{grav}} = -\frac{GMdm}{r^2} = -\frac{GM\rho dr dA}{r^2}$$

$$F_{\text{pressure}} = dA[P(r) - P(r+dr)] = dAdP$$

By equating the two: $F_{\text{grav}} = F_{\text{pressure}},$

We get the equation that describes the hydrostatic equilibrium in stars:

$$\boxed{\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho = -g\rho}$$

Force per unit volume

Equation of State for a non-degenerate gas

$$P = \underbrace{nkT}_{\text{Ideal Gas pressure}} + \underbrace{\frac{1}{3}aT^4}_{\text{Radiation pressure}}$$

n = number density

$k = \text{Boltzmann constant}$

$a = \text{radiation constant}$

Central pressure and temperature from the hydrodynamic equilibrium

using $\rho \sim \dfrac{M}{R^3}$ and $\dfrac{dP}{dr} \sim \dfrac{P_c}{R}$, we get

$$P_c \sim \frac{GM^2}{R^4} = 1.1 \times 10^{16} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R}{R_\odot}\right)^{-4} \text{ dyn cm}^{-2}$$

and

$$T_c = \frac{\mu_c m_u P_c}{k \rho_c} \sim \frac{G \mu_c m_u M}{kR} = 1.9 \times 10^7 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{R_\odot}\right)^{-1} \left(\frac{\mu_c}{0.85}\right) \text{ K}$$

If the system is not in hydrostatic equilibrium -

If a self-gravitating spherical system is not in hydrostatic equilibrium: the force (inward force by gravity and outward force by pressure) must be balanced by the inertia term (F = ma):

$$\rho \frac{\partial^2 r}{\partial t^2} = -\frac{\partial P}{\partial r} - \frac{G M_r}{r^2}\rho$$

- If there is no gravity, the matter will undergo **free expansion.**
- If there is no pressure, the matter will undergo **free fall.**

In hydrodynamic equilibrium state, we have $\partial^2 r/\partial t^2 = 0$

A star responds to any perturbation that causes $\partial^2 r/\partial t^2 \neq 0$ on a **dynamical timescale.**

For example, if the star is dynamically stable, the hydrostatic equilibrium will be restored on a dynamical timescale if there's any perturbation. If the star is dynamically unstable, the star will either collapse or disrupted roughly on a dynamical timescale.

Dynamical timescale

- **Free fall timescale**

$$\rho \frac{\partial^2 r}{\partial t^2} = -\frac{GM_r}{r^2}\rho \longrightarrow \frac{R}{t_{\rm ff}^2} \approx \frac{GM}{R^2} \approx \frac{GR^3\bar{\rho}}{R^2} \rightarrow \boxed{t_{\rm ff} \approx \sqrt{\frac{1}{G\bar{\rho}}}}$$

- **Free expansion timescale**

$$\rho \frac{\partial^2 r}{\partial t^2} = -\frac{\partial P}{\partial r} \longrightarrow \frac{R}{t_{\rm exp}^2} \approx \frac{\bar{P}}{\bar{\rho}R} \approx \frac{c_s^2}{R} \rightarrow \boxed{t_{\rm exp} \approx \frac{R}{c_s}}$$

$t_{\rm exp}$ is roughly the time for a sound wave to travel from the center to the surface of a star.

$$c_{\rm s} = \sqrt{\gamma \frac{P}{\rho}}$$

c_s: Speed of Sound

$\gamma =$ adiabatic index

Dynamical timescale

$$t_{\mathrm{ff}} << t_{\mathrm{exp}} \rightarrow \quad \text{Gravitational Collapse}$$
$$t_{\mathrm{ff}} >> t_{\mathrm{exp}} \rightarrow \quad \text{Disruption of the star by expansion (or explosion)}$$

- In hydrostatic equilibrium, the free-fall time, and the free expansion time must be comparable. Hence, we can define the dynamical timescale for a star in hydrostatic equilibrium as:

$$t_{\mathrm{dyn}} \approx t_{\mathrm{ff}} \approx t_{\mathrm{exp}} \approx \frac{1}{\sqrt{G\bar{\rho}}}$$

- The dynamical timescale for a star in hydrostatic equilibrium means the timescale on which the condition for hydrostatic equilibrium can be restored in response to a perturbation.

Dynamical timescale

- For the Sun: $\rho = 1.4$ g cm^{-3}

$$\longrightarrow \quad t_{\mathrm{dyn}} \approx 55 \text{ min.}$$

- This is very very very short compared to the evolutionary time of the Sun. ***Therefore, we can safely assume that the Sun is in hydrostatic equilibrium at each evolutionary stage, even if the stellar structure continuously changes as the star evolves*** with the nuclear burning and the resulting change in the chemical composition.

2. Stars have a negative heat capacity.

Virial Theorem

VIRIAL :
German, from Latin *vires*, plural, strength, power + German *-ial*; akin to Latin *vis* strength, force, violence (from Merriam-Webster)

Hydrostatic equilibrium equation: $$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho = -g\rho$$

$$\frac{4}{3}\pi r^3 dP = -\frac{4}{3}\pi r^3 \frac{GM_r}{r^2}\rho dr = -\frac{GM_r}{3r}4\pi r^2 \rho dr = -\frac{GM_r}{3r}dM_r$$

If we integrate the above equation, we get the following relation.

$$\boxed{2E_{\text{int}} = -E_{\text{grav}}}$$: Virial Theorem

where we assumed the ideal gas equation of state.

The Virial Theorem and the ρ-T relation in stars

Virial Theorem: $$2E_{\rm int} = -E_{\rm grav}$$

Therefore: $$\frac{3Mk\bar{T}}{\mu m_u} = \alpha\frac{GM^2}{R}$$

And: $$\bar{T} = \frac{\alpha\mu m_u G}{3k}\frac{M}{R} = \frac{\alpha\mu m_u G}{3k}\left(\frac{4\pi}{3}\right)^{1/3} M^{2/3}\bar{\rho}^{1/3}$$

$\bar{T} =$ average temperature of the star

$\alpha =$ some constant that depends on the density profile

$$\boxed{T \propto M^{2/3}\rho^{1/3}}$$

- For a given temperature, density is lower for a more massive star.
- As the star contracts, temperature increases.

Virial Theorem and Heat Capacity of a Star

- Total Energy: $$E_{\text{tot}} = E_{\text{grav}} + E_{\text{int}}$$

- Virial Theorem: $$2E_{\text{int}} = -E_{\text{grav}}$$

$$E_{\text{tot}} = -E_{\text{init}}$$

$$\Rightarrow \quad \boxed{\Delta E_{\text{tot}} = -\Delta E_{\text{init}}}$$

Stars have a negative heat capacity!

3. Stars lose energy by radiation.

Stars radiate light.

Luminosity in hydrostatic equilibrium ≈ Energy loss rate by radiation. (Energy loss by neutrino emission is not important for ordinary stars)

$$L = -\frac{dE_{\rm tot}}{dt}$$

If we ignore any internal energy source, from the virial theorem,

$$E_{\rm tot} = -E_{\rm int} = \frac{1}{2}E_{\rm grav}$$

we get:

$$\boxed{L = -\frac{dE_{\rm tot}}{dt} = \frac{dE_{\rm init}}{dt} = -\frac{1}{2}\frac{dE_{\rm grav}}{dt}}$$

This means: Stars in hydrostatic equilibrium would become hotter and more compact as they lose energy by radiation, if there is no internal energy source.

Kelvin-Helmholtz Timescale

If a star does not have any internal energy source (i.e., nuclear burning), the star would contract as they lose energy by radiation to maintain hydrostatic equilibrium.

The timescale for this ***thermal contraction*** *while maintaining hydrostatic equilibrium (this is NOT dynamical free fall)* is called "Kelvin-Helmholtz" time scale (or thermal timescale).

$$\boxed{\tau_{\mathrm{KH}} = \frac{E_{\mathrm{tot}}}{L} = \frac{GM^2}{2RL}}$$

$$\tau_{\mathrm{KH}} = 1.5 \times 10^7 [\mathrm{yr}] \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R_\odot}{R}\right) \left(\frac{L_\odot}{L}\right)$$

KH time is much longer than the dynamical time (i.e., sound crossing time or free fall time)

Stars with nuclear burning

$\boldsymbol{L_{\mathrm{nuc}}}$ = energy generation rate by nuclear burning

$\boldsymbol{L}$ = energy loss rate by radiation

The virial theorem tells us:

$$L - L_{\mathrm{nuc}} = -\frac{dE_{\mathrm{tot}}}{dt} = -\frac{dE_{\mathrm{int}}}{dt} - \frac{dE_{\mathrm{grav}}}{dt} = \frac{dE_{\mathrm{int}}}{dt} = -\frac{1}{2}\frac{dE_{\mathrm{grav}}}{dt}$$

Thermal equilibrium : the energy loss by radiation occurs at the same rate with the energy generation by an internal source (e.g., nuclear reactions).

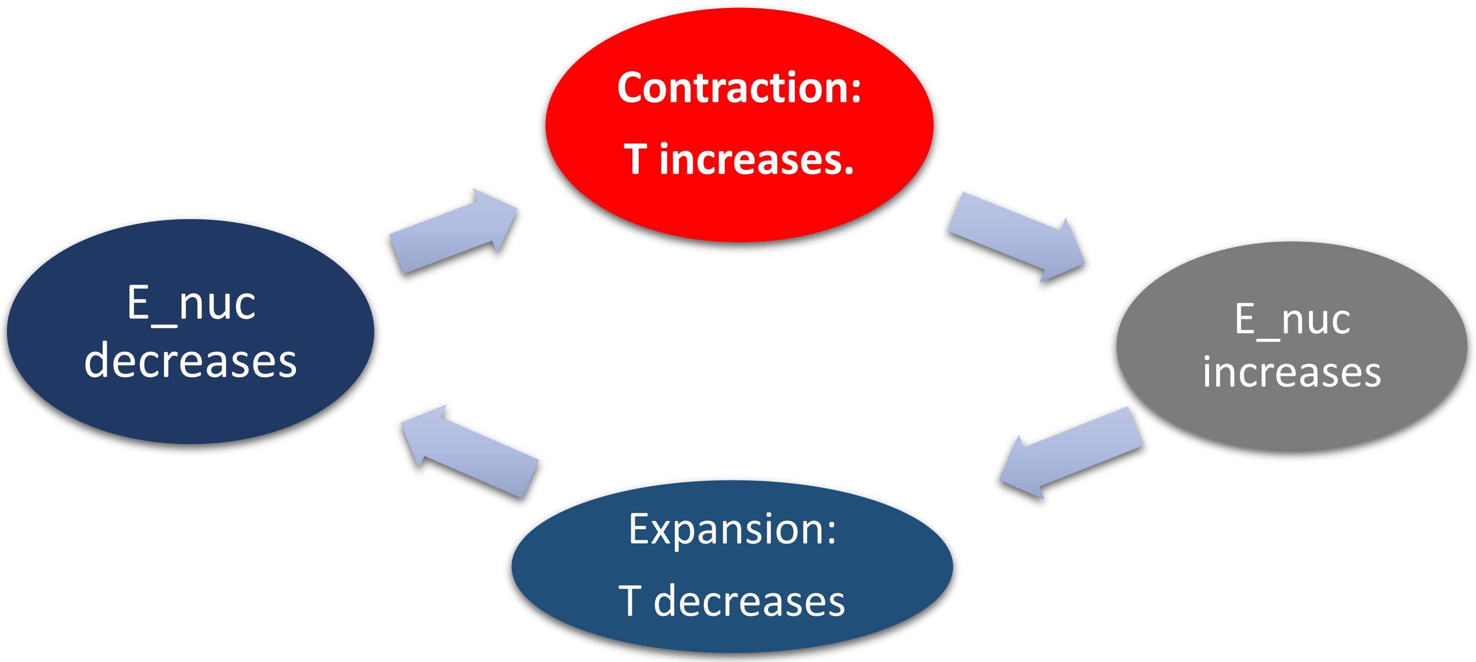
$$L = L_{\mathrm{nuc}}$$

in thermal equilibrium for main sequence star.

If there's a perturbation in terms of energy (i.e., instant extra-energy input or loss), thermal equilibrium can be restored on **the thermal timescale**.

Stars with nuclear burning

$$L - L_{\rm nuc} = -\frac{dE_{\rm tot}}{dt} = \frac{dE_{\rm int}}{dt} = -\frac{1}{2}\frac{dE_{\rm grav}}{dt}$$



- $L - L_{\rm nuc} > 0$: the star loses more energy than nuclear energy → contraction
- $L - L_{\rm nuc} < 0$: Stars gain more energy due to nuclear burning than it loses → expansion

Nuclear Burning Timescale

$$\tau_{\mathrm{nuc}} = \phi f_{\mathrm{nuc}} \frac{Mc^2}{L} \approx 10^{10} \frac{M}{\mathrm{M}_\odot} \frac{\mathrm{L}_\odot}{L} \ \mathrm{yr}$$

$\phi =$ the fraction of the rest mass of the nuclei that is converted into energy
≈ 0.007 for hydrogen burning

$f_{\mathrm{nuc}} =$ the fraction of the stellar mass that serve as nuclear fuel

$$\tau_{\mathrm{nuc}} >> \tau_{\mathrm{KH}} >> \tau_{\mathrm{dyn}}$$

For the Sun : 10^{10} yr $>>$ 10^7 yr $>>$ 1.0 hr

Stars can find thermal equilibrium on a very short timescale compared to the nuclear burning timescale.

Energy transport by radiation

$$F_r = -\frac{1}{3} c\lambda \frac{du}{dr}$$

F = photon flux (energy per unit area per unit time)
u = radiation energy density
c = speed of light
λ = mean free path of a photon

$$\left.\begin{aligned} u &= aT^4 \\ \lambda &= \frac{1}{\rho\kappa} \end{aligned}\right\}$$

$\rho = \text{mass density}$

$\kappa = \text{opacity}$

$$F_r = -\frac{4acT^3}{3\rho\kappa}\frac{dT}{dr}$$ (energy per unit area per unit time)

$$L_r = 4\pi r^2 F_r = -\frac{16\pi acr^2T^3}{3\rho\kappa}\frac{dT}{dr}$$

(energy per unit time)

Dimensional analysis to derive the M-L relation

$$\frac{dP}{dr} = -\rho \frac{GM_r}{r^2}$$

$$L_r = -\frac{16\pi a c r^2 T^3}{3\kappa\rho}\frac{dT}{dr}$$

$$P = nkT = \frac{\rho k T}{\mu m_{\mathrm{u}}}$$

$$\frac{\rho T}{\mu R} \sim \frac{\rho M}{R^2}$$

$$\frac{L}{R^2} \sim \frac{R^3 T^4}{\kappa M R}$$

$$TR \sim M\mu$$

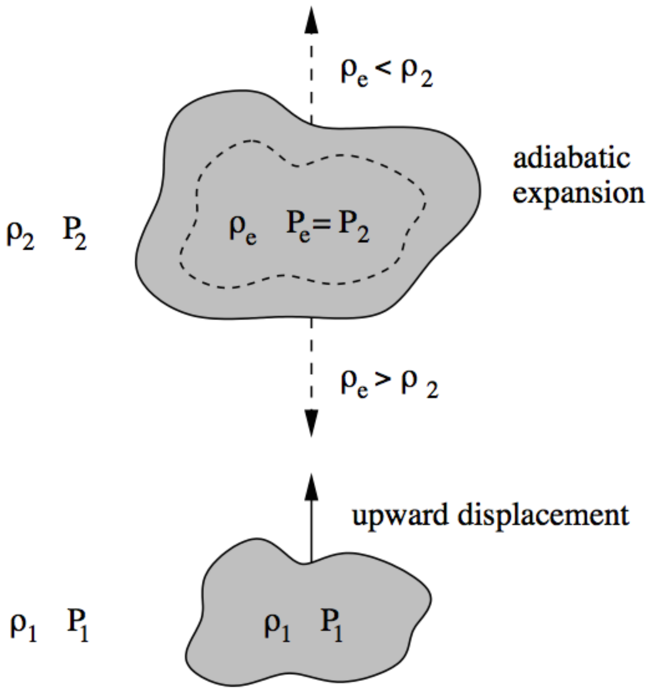
$$L \sim \frac{(RT)^4}{\kappa M}$$

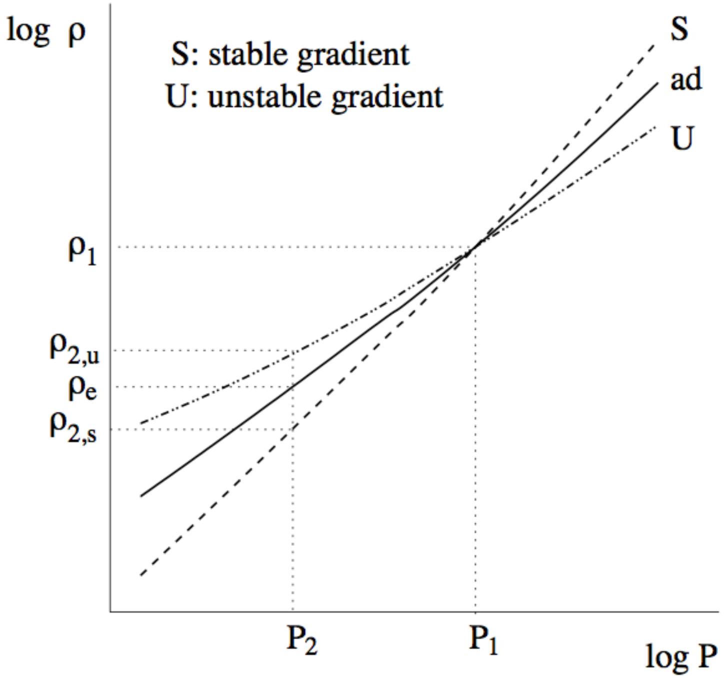
$$\boxed{L \sim \frac{\mu^4 M^3}{\kappa}}$$

for an ideal gas where the gas pressure is dominant.

Energy and matter transport by convection

Condition for convection





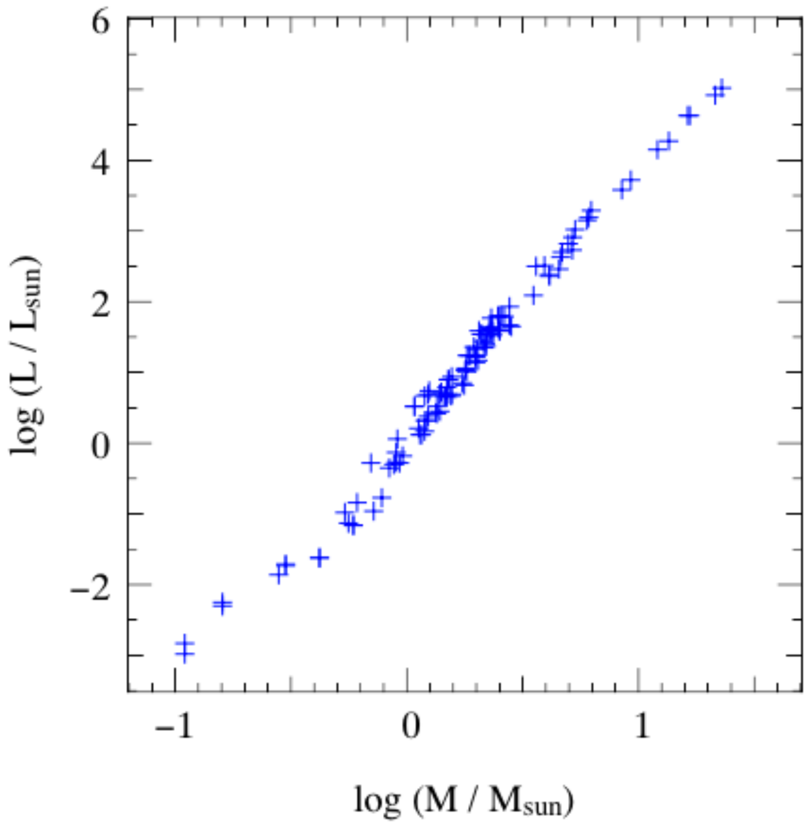
Convection occurs if:

$$\left|\frac{dT}{dr}\right| > \left|\frac{dT}{dr}\right|_{\rm ad} \qquad \text{or} \qquad \left|\frac{d\rho}{dr}\right| < \left|\frac{d\rho}{dr}\right|_{\rm ad}$$

Here, “ad” means “adiabatic change”: any physical change that occurs much faster than thermal diffusion.

Mas-Luminosity Relation and Nuclear Burning Time

$$L \propto M^{3.8}$$



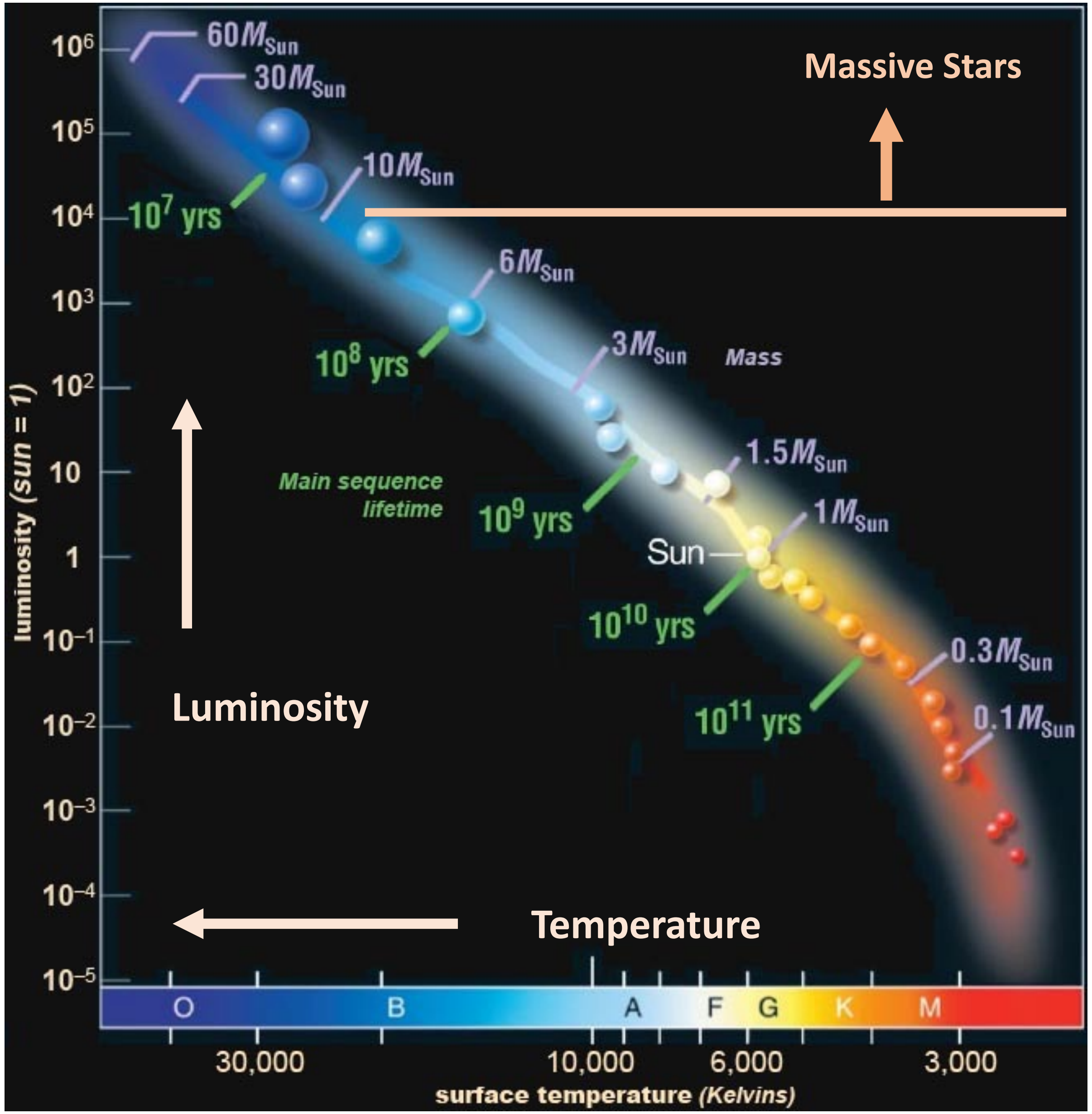
$$\tau_{\rm nuc} \propto \frac{M}{L} \propto M^{-2.8}$$

More massive stars have a shorter lifetime.

e.g.

- 1 Msun star : 10 Gyr
- 20 Msun star: 10 Myr

Lifetime of stars

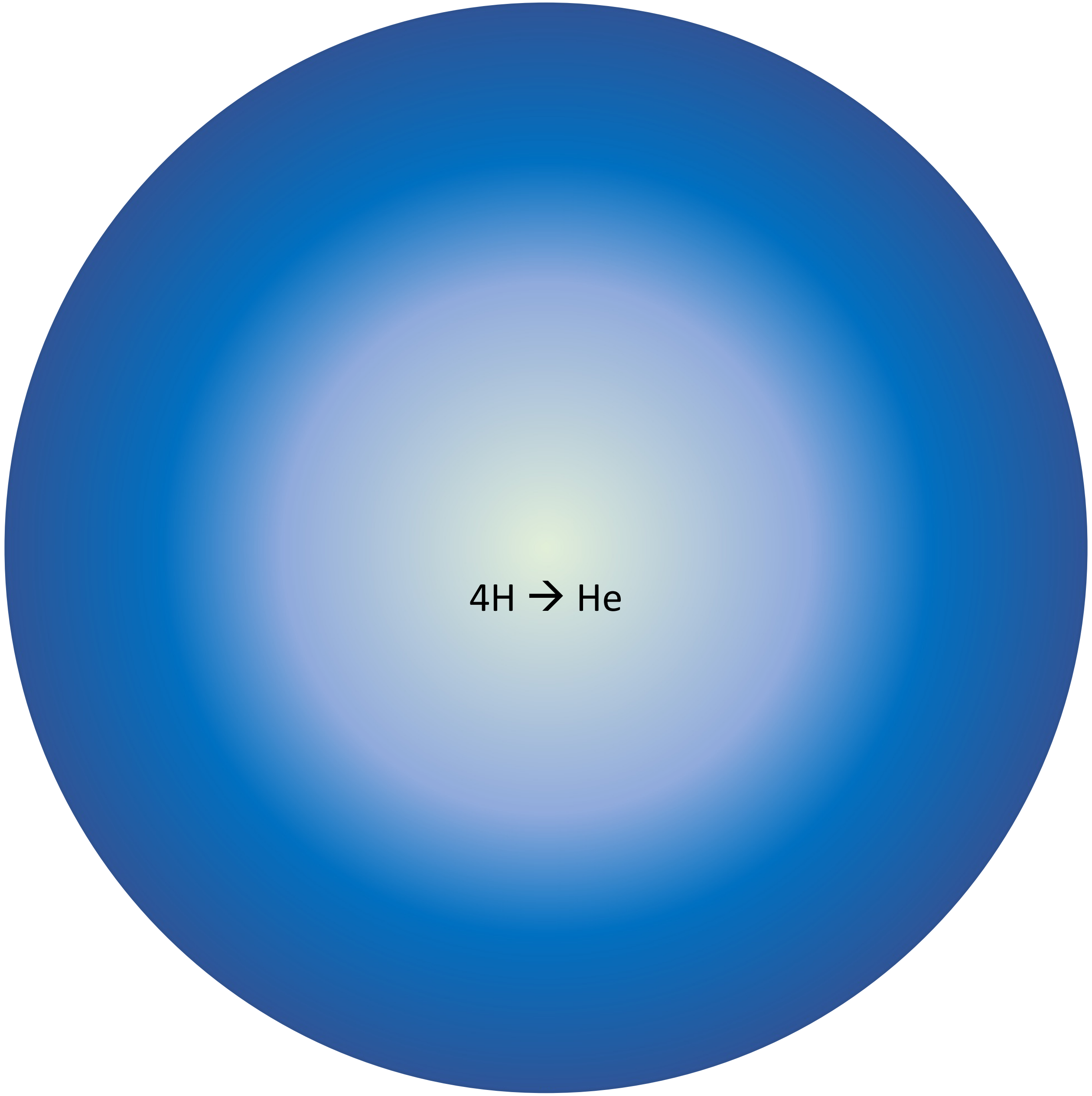


https://www.handprint.com/ASTRO/index.html

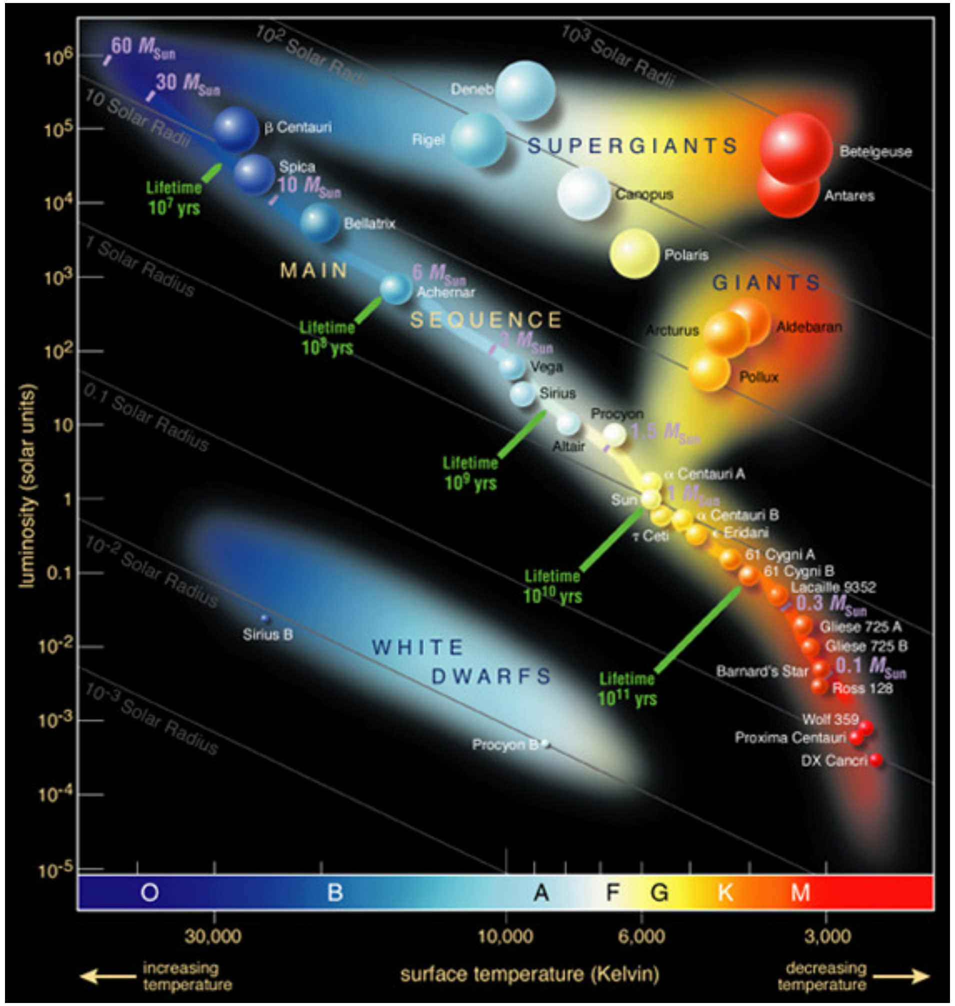
Evolution of massive stars

Beginning of evolution: hydrogen burning

- Main Sequence: hydrogen burning stage (CNO cycle)
- The core is convective.
- The envelope is radiative.
- About 90% of the stellar life time (~ 10 M yr for a 20 Msun star.



Main Sequence

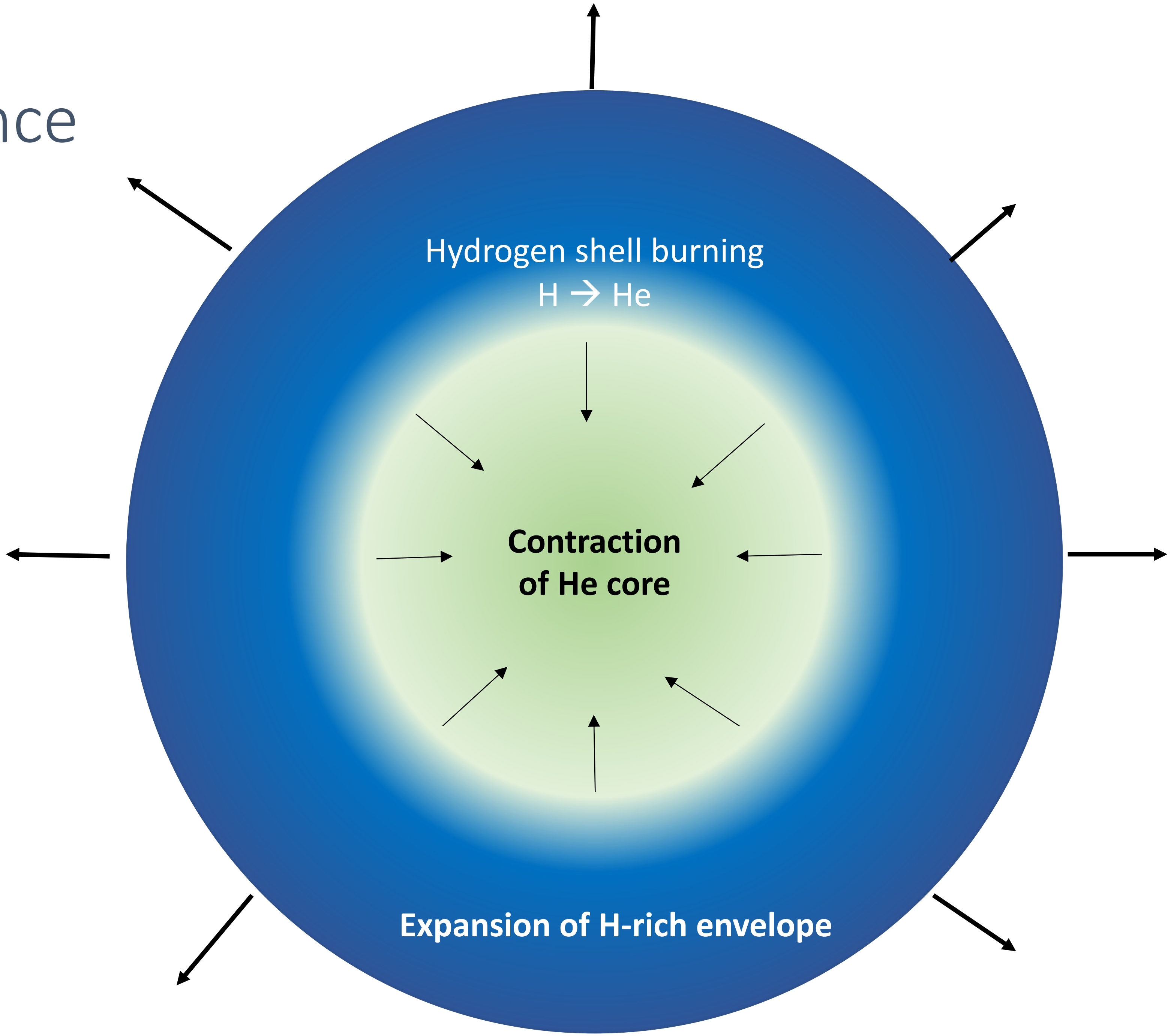


Stars on the main sequence is in thermal equilibrium.

$$L \approx L_{\text{nuc}}$$

on the main sequence

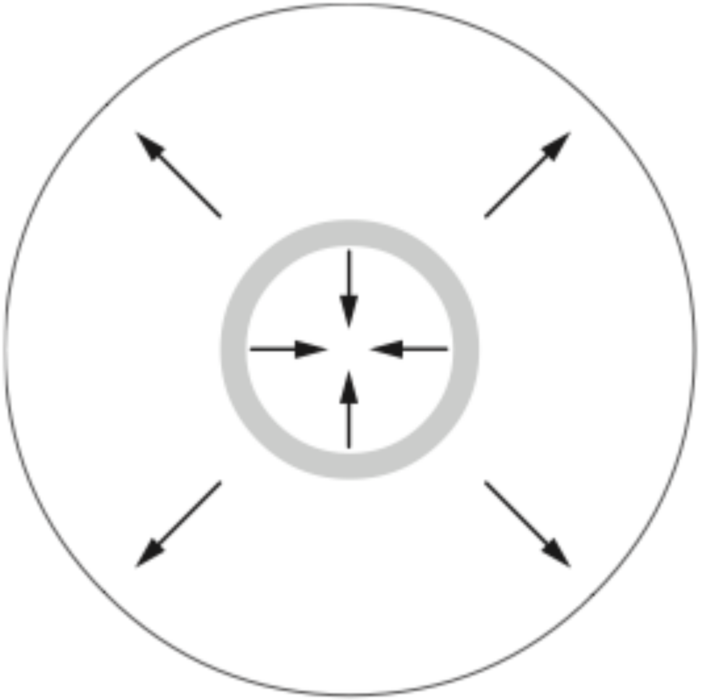
Post-main sequence



Mirror principle

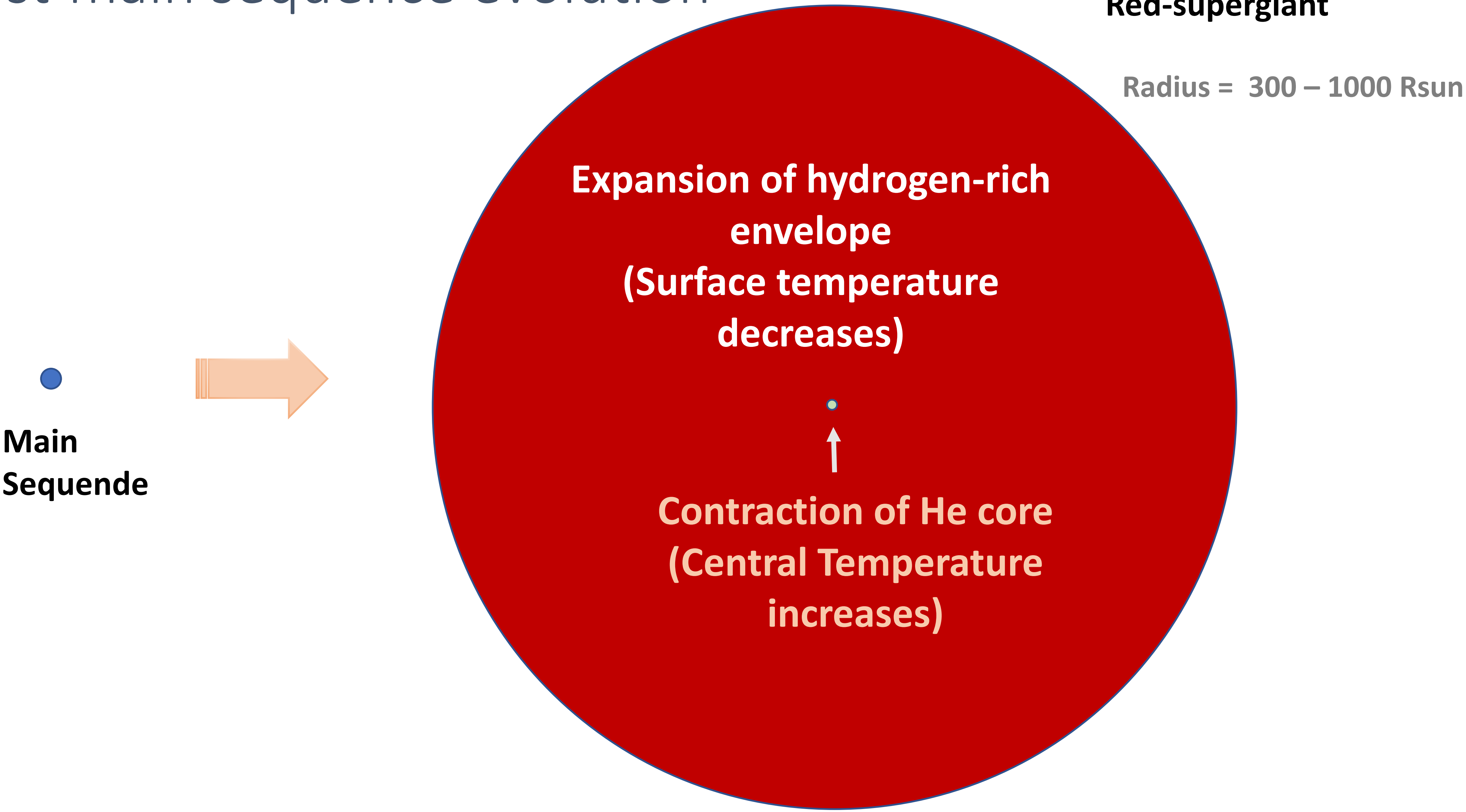
If a star has an active shell burning source, the burning shell acts as a mirror between the core and the envelope:

core contraction → envelope expansion
core expansion → envelope contraction

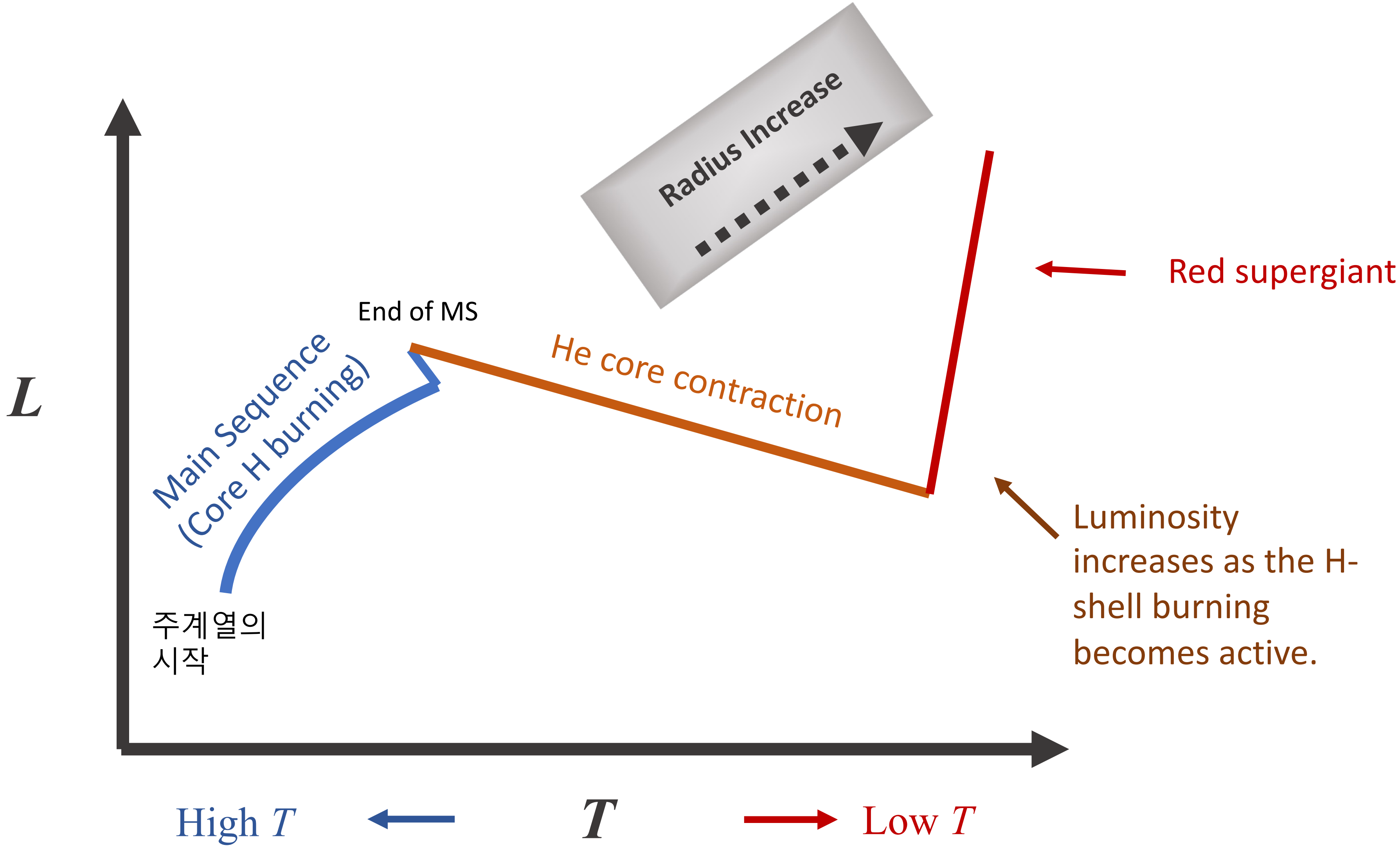


Numerical simulations show that, as the core contracts, the envelope expands, if there is a nuclear burning shell: I.e., the star becomes a giant star as the core contracts.

Post-main sequence evolution



Evolution on the HR diagram



Post-main sequence evolution

Core Helium burning stage

$T_c \approx 2\times10^8$ K

$$3\alpha \rightarrow {}^{12}\mathrm{C} + \gamma$$

- Further reactions:
  - ^{12}C(a, g)^{16}O : very important reaction, but the rate of this reaction is very uncertain.
  - ^{16}O(a, g)^{20}Ne : significant only at very high T.

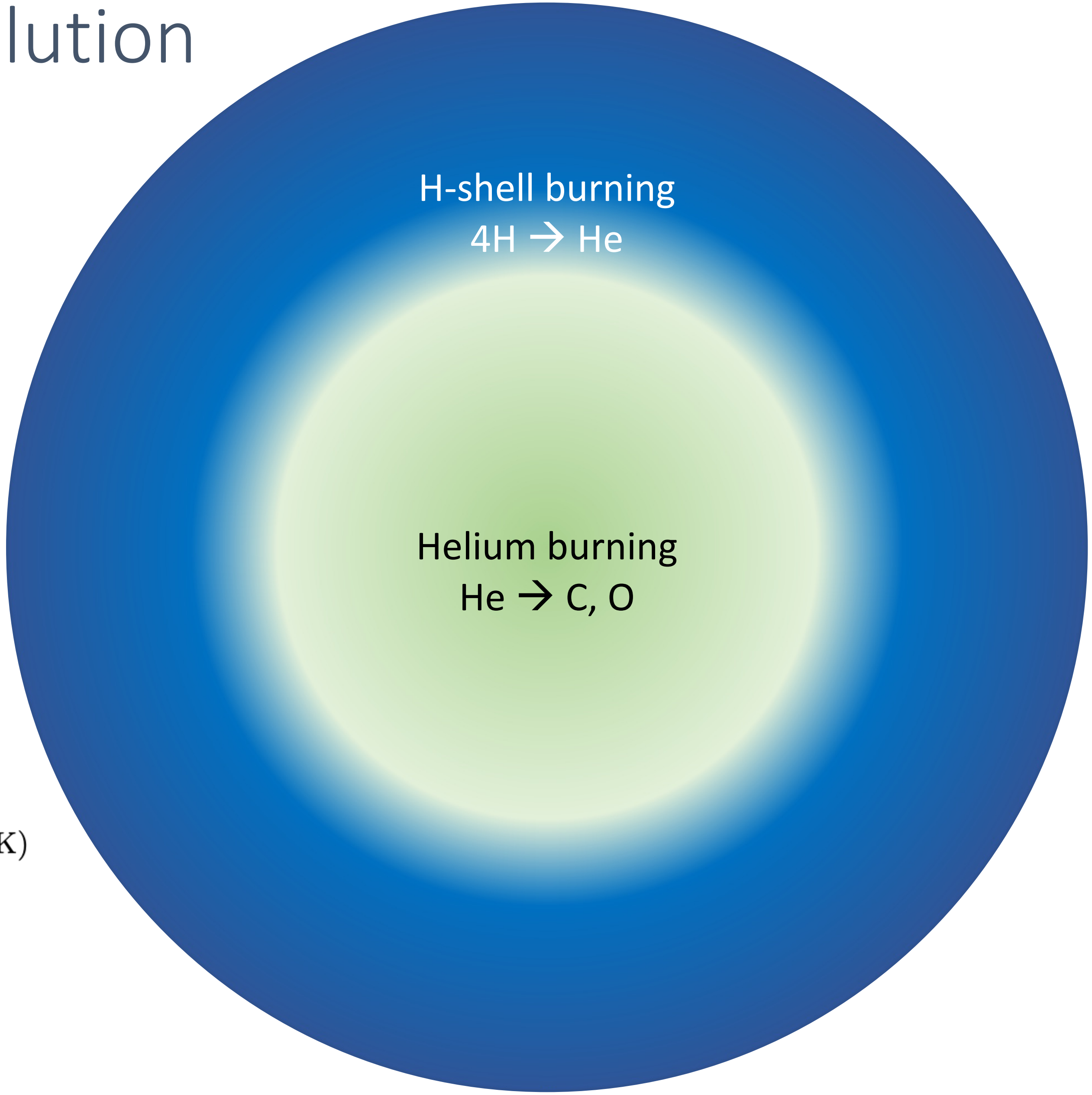
- Secondary nucleosynthesis

  - production of ^{18}O and ^{22}Ne:

$$^{14}\mathrm{N}(\alpha,\gamma)^{18}\mathrm{F}(\beta^+)^{18}\mathrm{O}(\alpha,\gamma)^{22}\mathrm{Ne} \quad (\text{already at } T \simeq 10^8\,\mathrm{K})$$

  -production of free neutrons: important for s-process in massive stars:

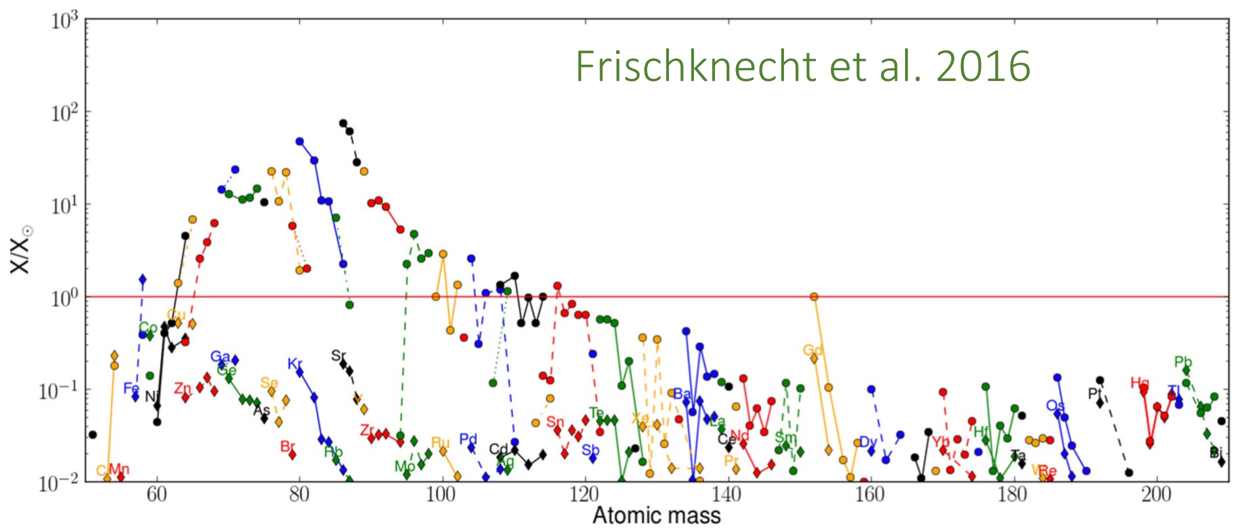
$$^{22}\mathrm{Ne}(\alpha,\mathrm{n})^{25}\mathrm{Mg} \quad (\text{for } T \geq 3\cdot10^8\,\mathrm{K}$$

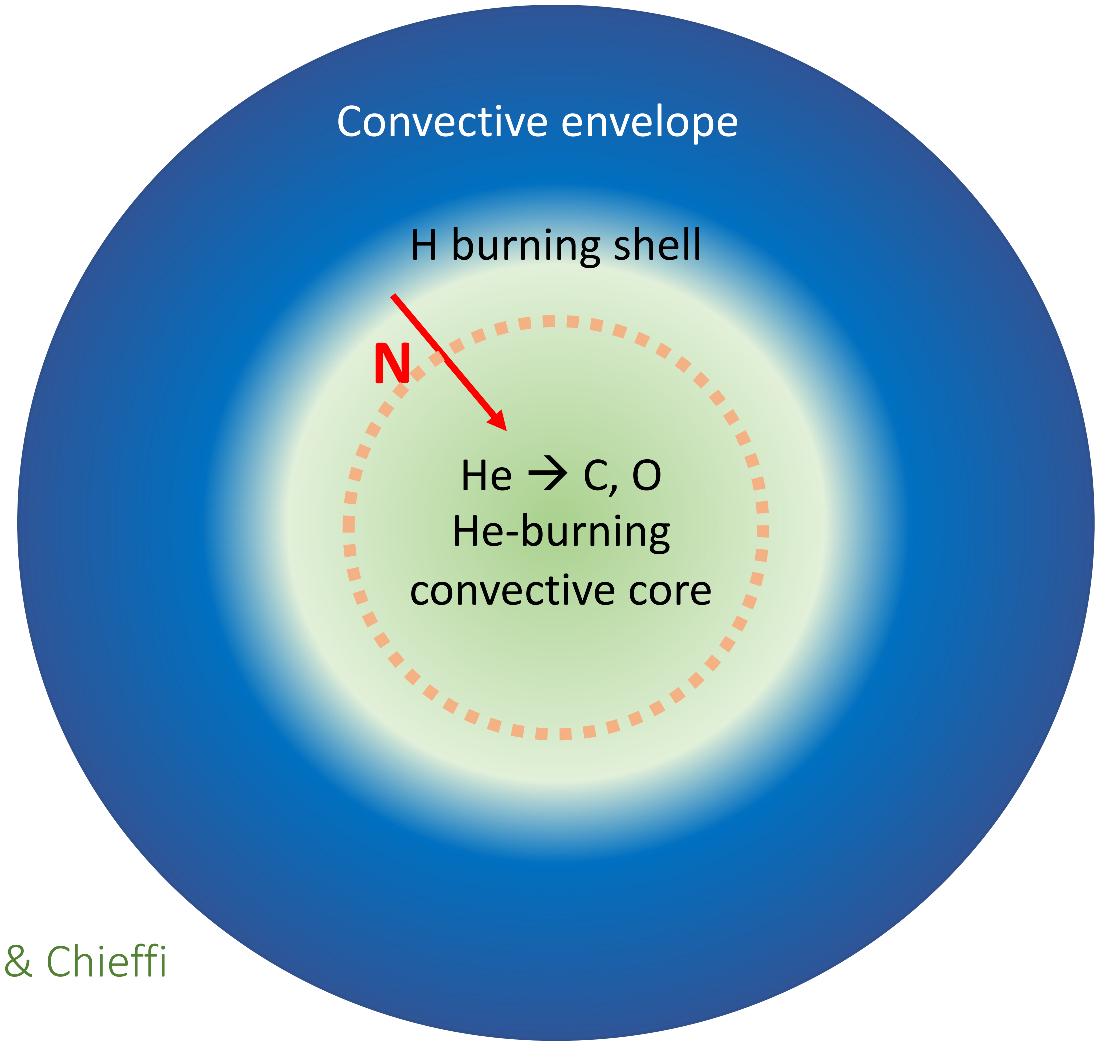


S-process in the He-burning core in massive stars

$$^{14}\mathrm{N}(\alpha,\gamma)^{18}\mathrm{F}(\beta^+,\nu_e)^{18}\mathrm{O}(\alpha,\gamma)^{22}\mathrm{Ne}(\alpha,n)$$

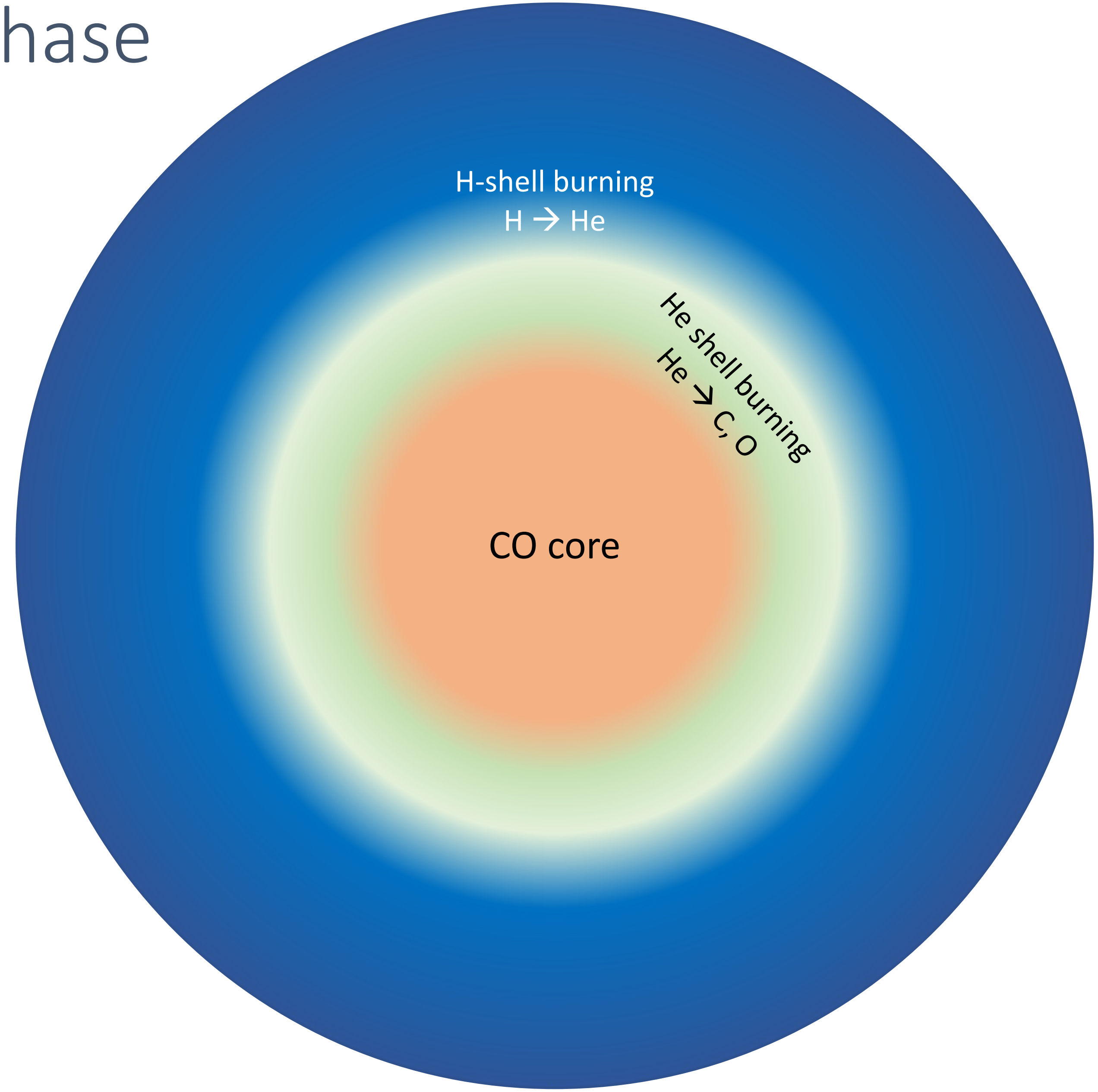
N produced by CNO cycle can be mixed into the He-burning core by rotationally induced mixing
→ More s-process in the He burning core.





see also: Banerjee et al. 2019, 2018, Choplin et al. 2018, Limongi & Chieffi 2018, Pranzos et al. 2018

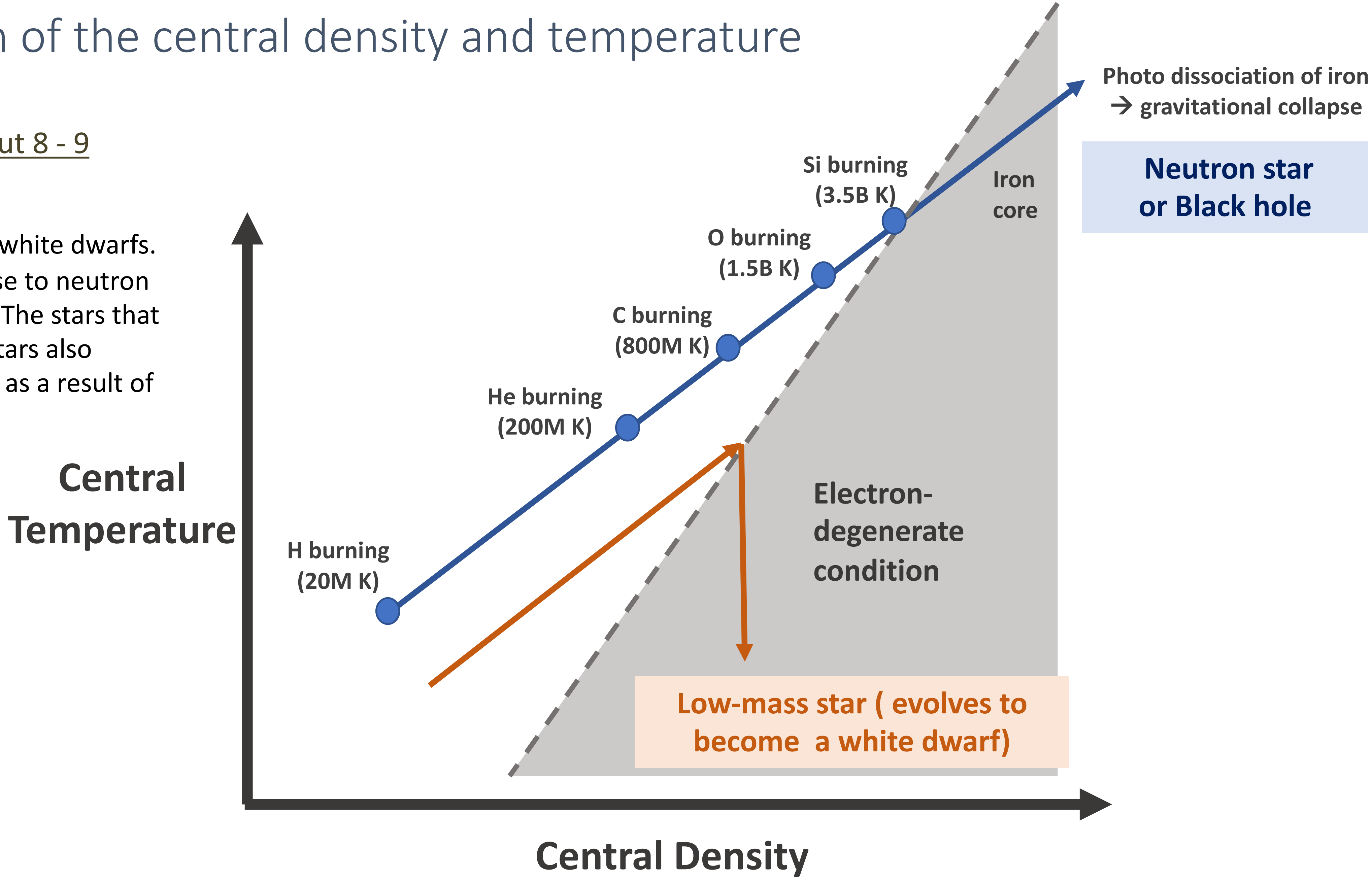
After Core He burning phase



Evolution of the central density and temperature

<u>Stars with M > about 8 - 9 Msun</u>

They do not become white dwarfs. The iron cores collapse to neutron stars or black holes. The stars that collapse to neutron stars also produce supernovae, as a result of the core-collapse.



Carbon burning stage

For the case of 20 Msun star:

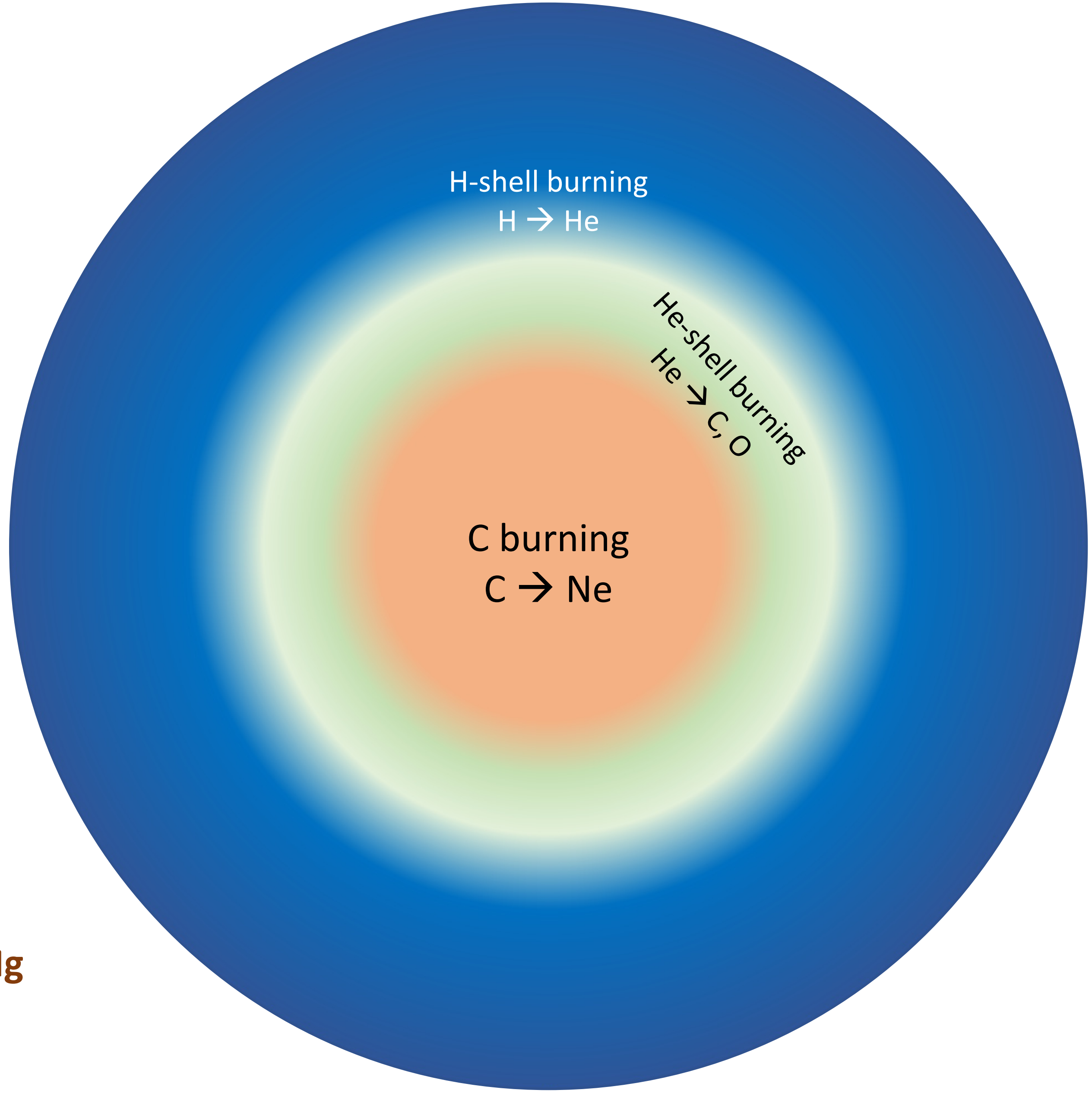
$T_c \approx 8{\times}10^8$ K

Duration: 1000 yr

$$
\begin{aligned}
{}^{12}\mathrm{C}+{}^{12}\mathrm{C}\rightarrow {}^{24}\mathrm{Mg}^{*} &\rightarrow {}^{23}\mathrm{Mg}+n-2.62\ \mathrm{MeV}\\
&\rightarrow {}^{20}\mathrm{Ne}+\alpha+4.62\ \mathrm{MeV}\\
&\rightarrow {}^{23}\mathrm{Na}+p+2.24\ \mathrm{MeV}.
\end{aligned}
$$

$$
{}^{23}\mathrm{Na}(\mathrm{p},\alpha)^{20}\mathrm{Ne},\quad {}^{20}\mathrm{Ne}(\alpha,\gamma)^{24}\mathrm{Mg}
$$

Main products of carbon burning: ^{16}O, ^{20}Ne, ^{24}Mg



Neon burning stage

For the case of 20 Msun star:

$T_c \approx 1.5 \times 10^9$ **K**

Duration: 3 yr

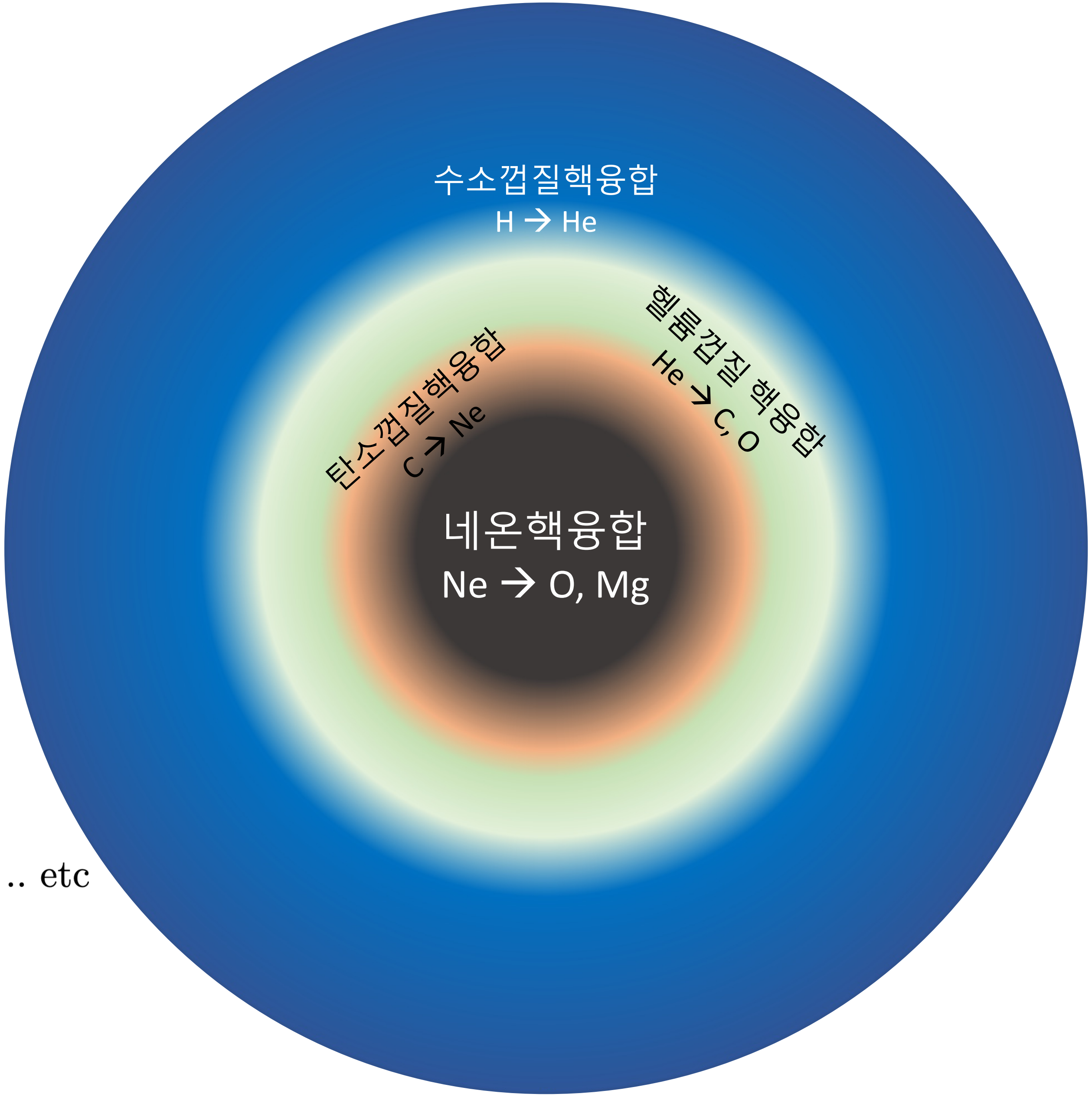
$$^{20}\mathrm{Ne} + \gamma \longleftrightarrow {}^{16}\mathrm{O} + \alpha$$

$$^{20}\mathrm{Ne} + \alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$$

Secondary reactions:

$$^{24}\mathrm{Mg}(\alpha,\gamma)^{28}\mathrm{Si}, {}^{25}\mathrm{Mg}(\alpha,\gamma)^{29}\mathrm{Si}, {}^{26}\mathrm{Mg}(\alpha,\gamma)^{26}\mathrm{Al}, \ldots \text{ etc}$$

Main products: ^{16}O, ^{24}Mg



Oxygen burning stage

For the case of 20 Msun star:

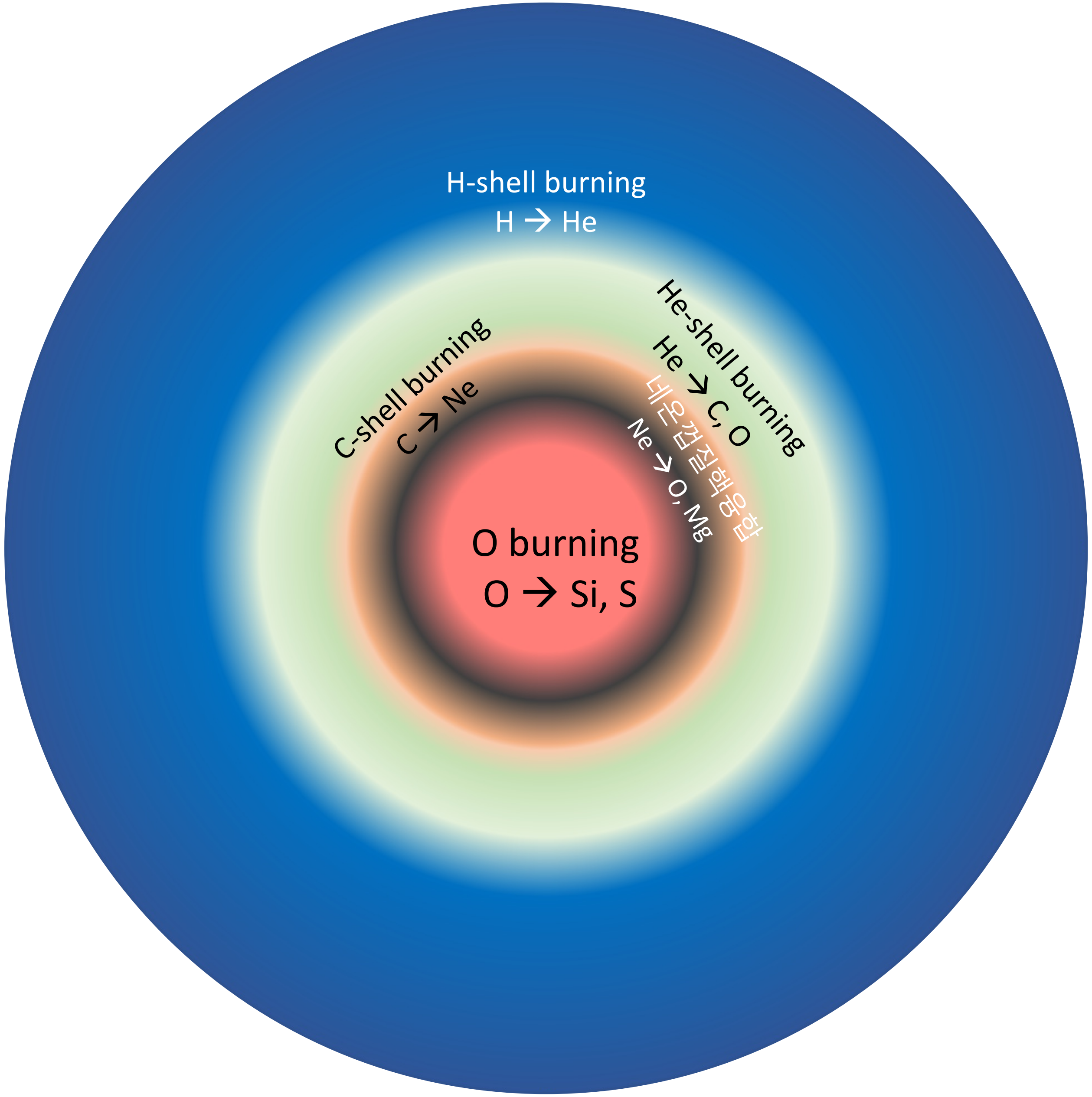
$T_c \approx 2.0 \times 10^9$ K

Duration: 10 months

$$
\begin{aligned}
{}^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}^* &\rightarrow {}^{31}\mathrm{S} + n + 1.45\ \mathrm{MeV} && 5\% \\
&\rightarrow {}^{31}\mathrm{P} + p + 7.68\ \mathrm{MeV} && 56\ \% \\
&\rightarrow {}^{30}\mathrm{P} + d - 2.41\ \mathrm{MeV} && 5\% \\
&\rightarrow {}^{28}\mathrm{Si} + \alpha + 9.59\ \mathrm{MeV.} && 34\%
\end{aligned}
$$

Side reactions: ${}^{31}\mathrm{P}(\mathrm{p}, \alpha)^{28}\mathrm{Si}$, ${}^{28}\mathrm{Si}(\alpha, \gamma)^{32}\mathrm{S}$ …

Main products: ^{28}Si, ^{32}S



Si-burning stage

For the case of 20 Msun star:

$T_c \approx 3.5\times10^9$ K

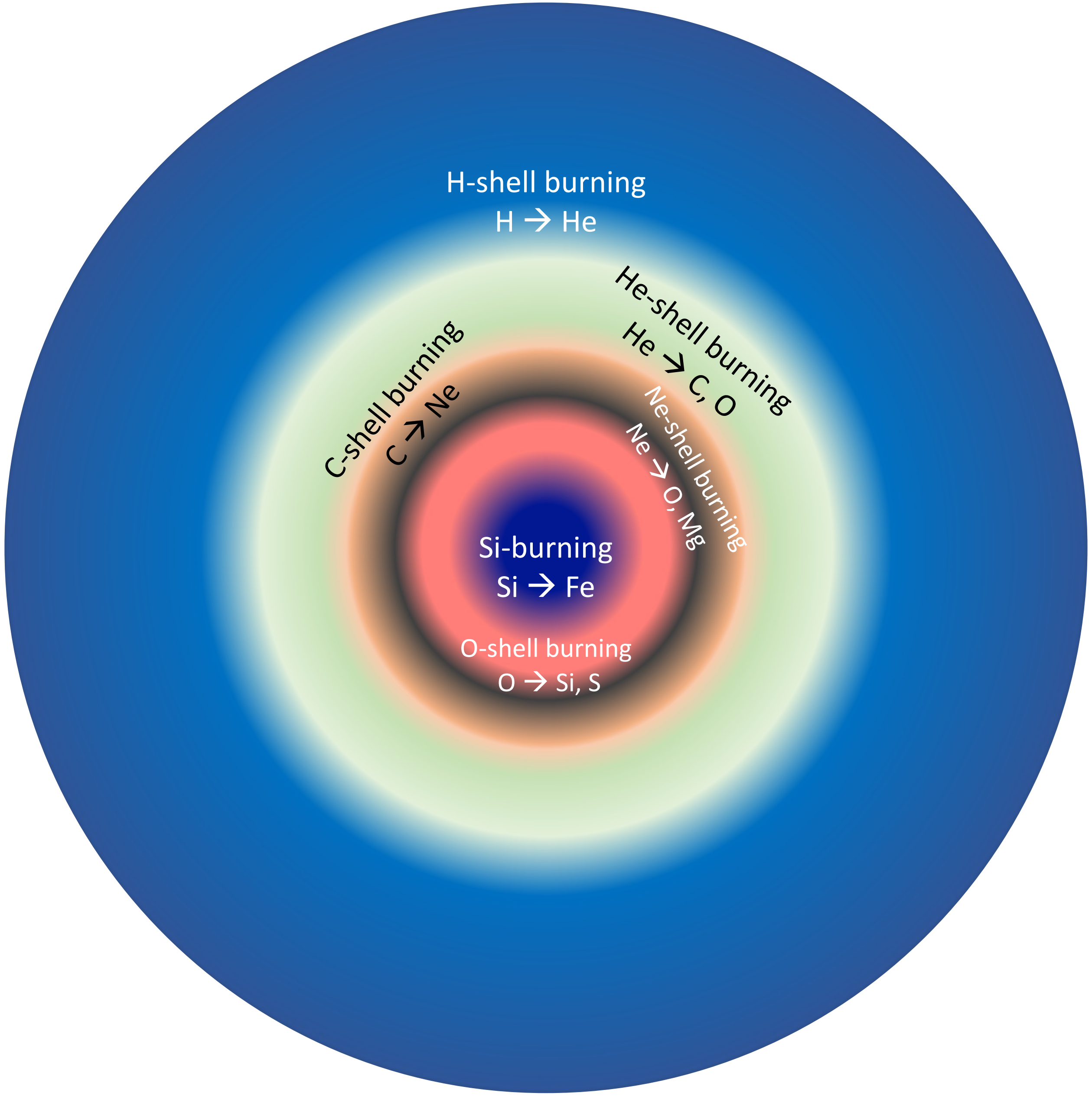
Duration: 1.0 week

$$^{28}\mathrm{Si}\,(\gamma,\alpha)\,^{24}\mathrm{Mg}\,(\gamma,\alpha)\,^{20}\mathrm{Ne}\,(\gamma,\alpha)\,^{16}\mathrm{O}\,(\gamma,\alpha)\,^{12}\mathrm{C}\,(\gamma,\alpha)\,2\alpha$$

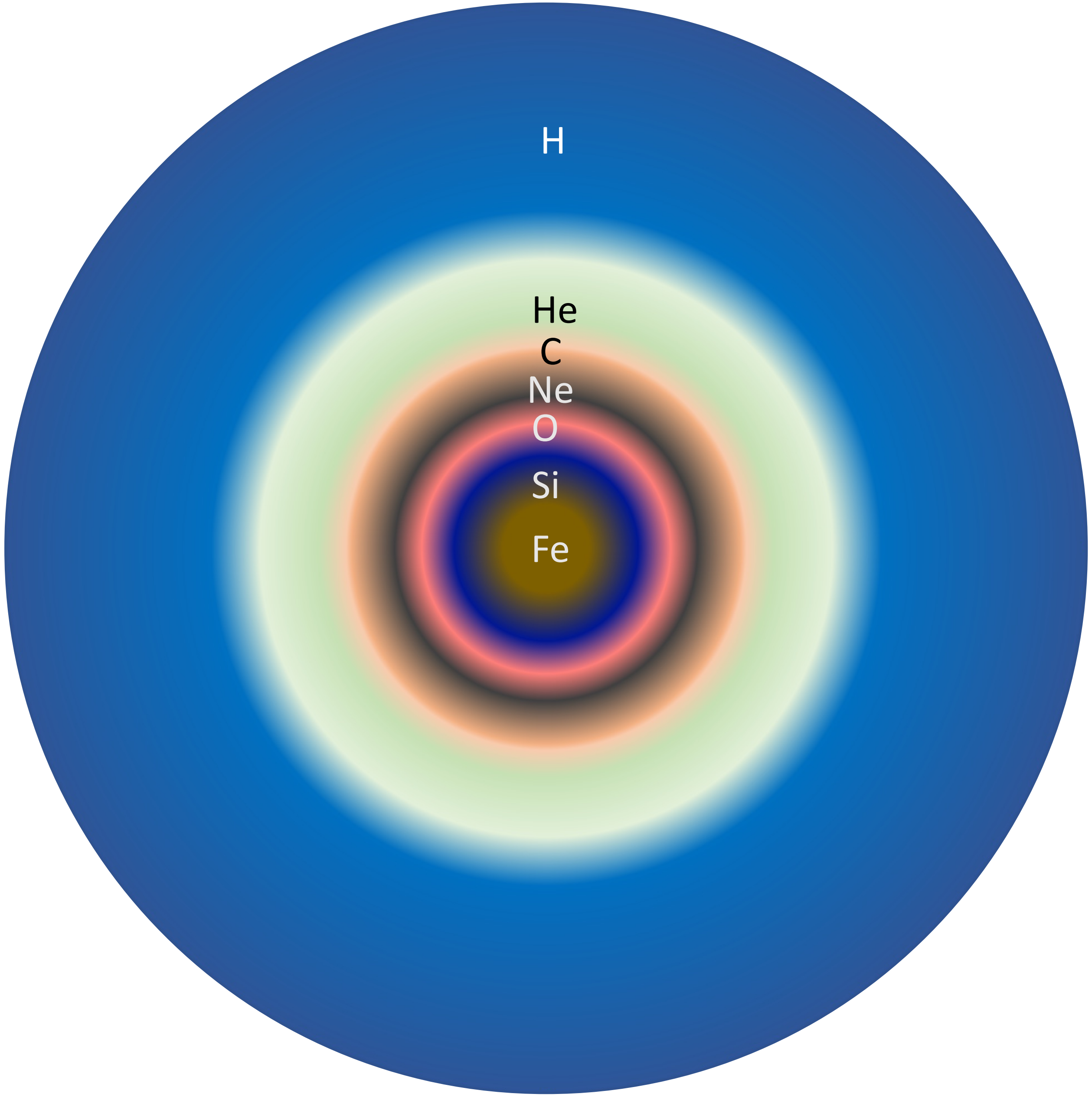
$$^{28}\mathrm{Si}\,(\alpha,\gamma)\,^{32}\mathrm{S}\,(\alpha,\gamma)\,^{36}\mathrm{Ar}\,(\alpha,\gamma)\,^{40}\mathrm{Ca}\,(\alpha,\gamma)\,^{44}\mathrm{Ti}\,(\alpha,\gamma)\,\ldots\,^{56}\mathrm{Ni}$$

In fact, ^{28}Si not only releases alpha particles but also protons and neutrons:

$$^{28}\mathrm{Si} + \alpha + p + n \;\rightarrow\; \text{iron peak elements.}$$



Final structure



Final Structure

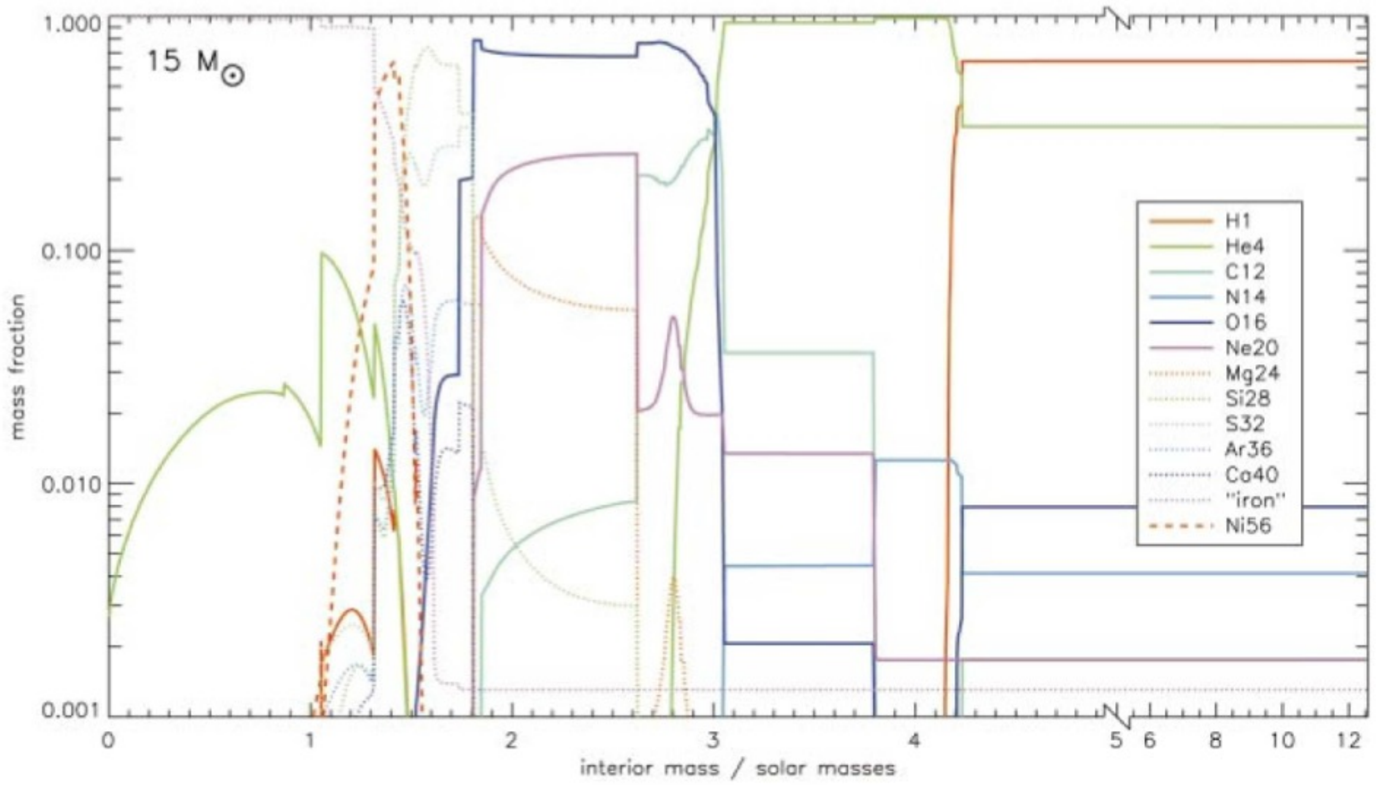
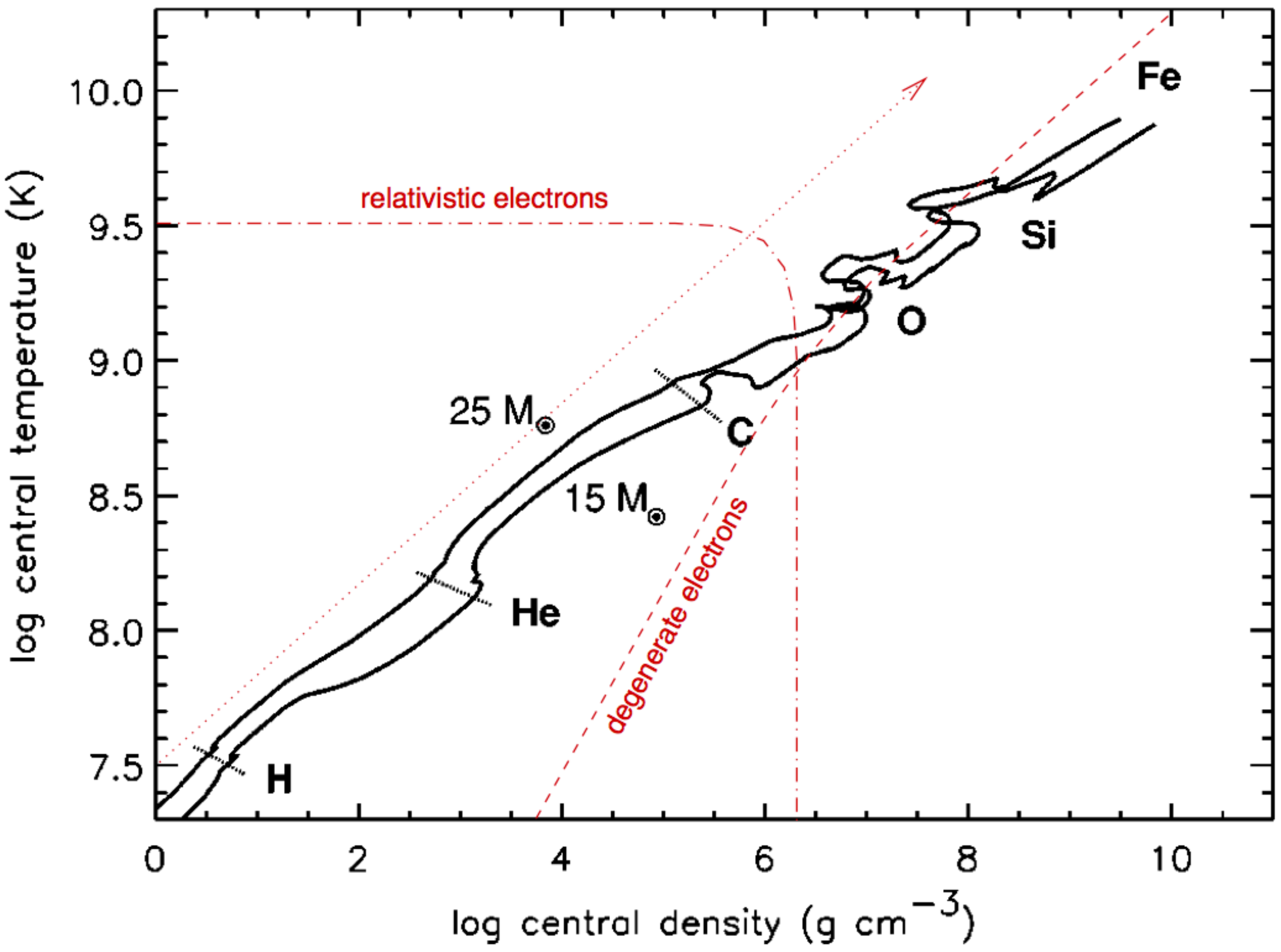


Figure 12.9. Final composition profiles of a 15 $M_\odot$ star (see Fig. 12.7), just before core collapse. "Iron" refers to the sum of neutron-rich nuclei of the iron group, especially ^{56}Fe. Figure from Woosley, Heger & Weaver (2002).

Evolution of central density and temperature



Advanced Nuclear Burning Stages

(e.g., 20 solar masses)

Fuel	Main Product	Secondary Products	Temp (10^9 K)	Time (yr)
H	He	^{14}N	0.02	10^7
He	C,O	^{18}O, ^{22}Ne s- process	0.2	10^6
C	Ne, Mg	Na	0.8	10^3
Ne	O, Mg	Al, P	1.5	3
O	Si, S	Cl, Ar K, Ca	2.0	0.8
Si	Fe	Ti, V, Cr Mn, Co, Ni	3.5	1 week

Courtesy: S. Woosly

Massive Star Modelling

The stellar structure & evolution equations

mass conservation:

$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho}$$

momentum conservation:

$$\frac{\partial P}{\partial M_r} = -\frac{G M_r}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

dynamical timescale

energy conservation:

$$\frac{\partial L_r}{\partial M_r} = \epsilon_{\rm nuc} - \epsilon_\nu - T\frac{\partial s}{\partial t}$$

thermal timescale

heat transfer:

$$\frac{\partial T}{\partial M_r} = -\frac{G M_r}{4\pi r^4}\frac{T}{P}\nabla \quad \text{where} \quad \nabla = \frac{\partial \ln T}{\partial \ln P}$$

chemical composition change:

$$\frac{\partial X_i}{\partial t} = \frac{A_i m_{\rm u}}{\rho}\left(-\sum_j (1+\delta_{ij}) r_{ij} + \sum_{k,l} r_{kl,i}\right) + (\text{chemical mixing})$$

nuclear timescale

mixing timescale (e.g., convection, rotation, etc)

Boundary Conditions : Center

$$M_r = 0 \quad ; r = 0 \quad ; L_r = 0$$

cf. Physical properties at the center:

Near the center, for a very small value of m, we have $r \simeq \left(\frac{3}{4\pi\rho_c}\right)^{1/3} m^{1/3}$.

The hydrostatic equation becomes

$$\frac{\partial P}{\partial m} \simeq -\frac{G}{4\pi}\left(\frac{4\pi\rho_c}{3}\right)^{4/3} m^{-1/3},$$

and finally we get

$$P - P_c \simeq -\frac{3G}{8\pi}\left(\frac{4\pi}{3}\rho_c\right)^{4/3} m^{2/3}$$

Boundary Conditions : Surface

Simplest Conditions :
Zero boundary condition

$$T = 0 \quad ; \quad P = 0$$

Photospheric Boundary Conditions :

$$\tau = 2/3$$

: Optical depth

$$L = 4\pi R^2 \sigma T^4$$

$$P \simeq \frac{2}{3}\frac{GM}{\kappa R^2}$$

There are several different options for the surface boundary condition.

Various physical factors that determine massive star evolution

In modelling of massive stars, we should consider uncertainties due to:

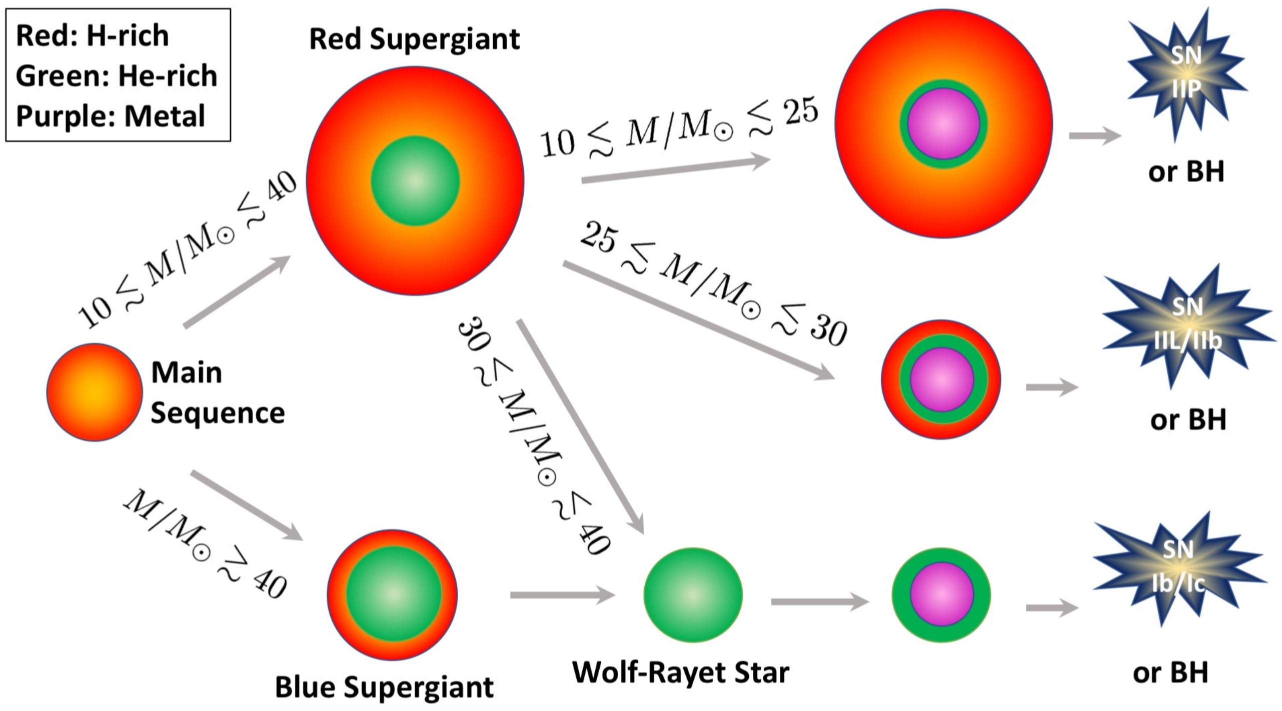
- Stellar winds and mass-loss
- Convection
- Metallicity
- Detailed nucleosynthesis
- Rotation
- Binary interactions

Stellar winds, mass loss and massive star evolution

Causes of mass-loss:

- Radiation-driven winds
- Episodic mass eruptions
- Binary interactions

Standard scenario of massive star evolution



$$\dot{M}_{\rm wind} \propto L^{1.5} Z^{0.7}$$

Stellar winds and massive star evolution

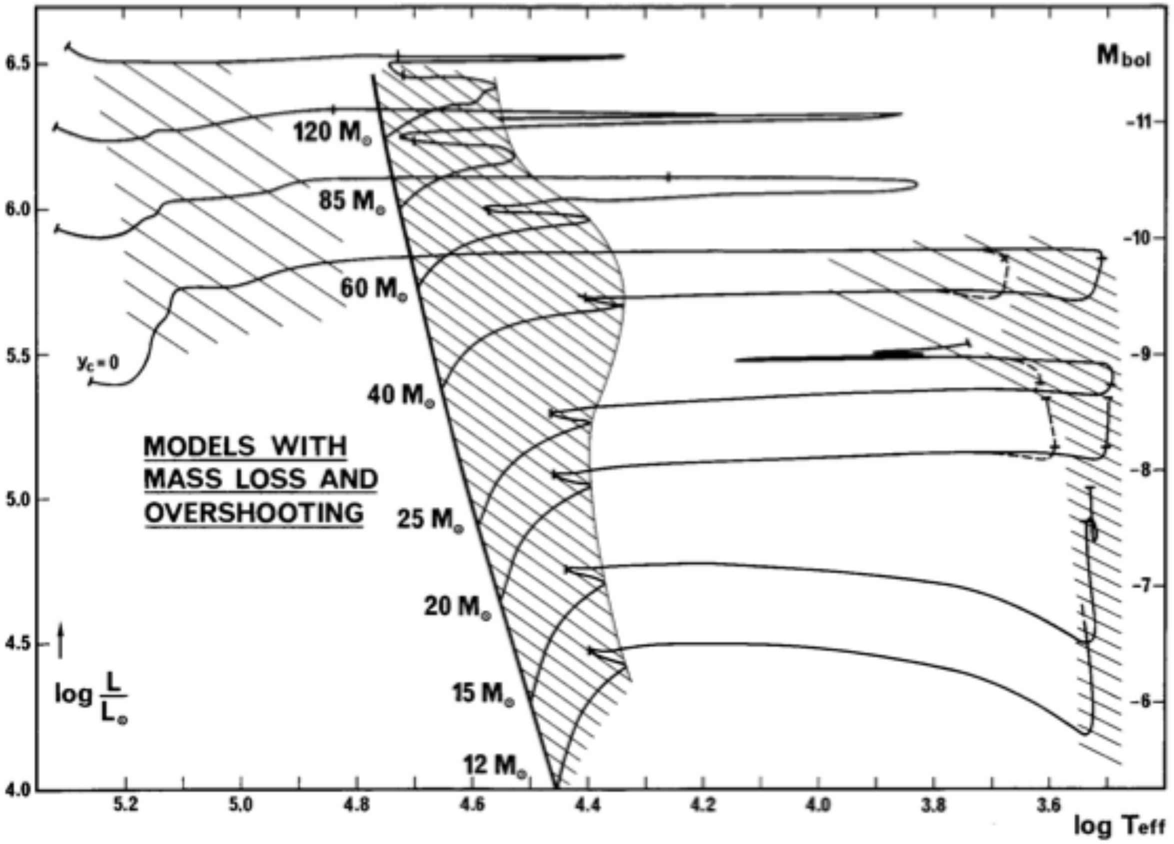


Figure 12.3. Evolution tracks of massive stars ($12 - 120\ M_\odot$) calculated with mass loss and a moderate amount of convective overshooting ($0.25\ H_P$). The shaded regions correspond to long-lived evolution phases on the main sequence, and during core He burning as a RSG (at $\log T_{\rm eff} < 4.0$) or as a WR star (at $\log T_{\rm eff} > 4.8$). Stars with initial mass $M > 40\ M_\odot$ are assumed to lose their entire envelope due to LBV episodes and never become RSGs. Figure from Maeder & Meynet (1987, A&A 182, 243).

The Humphreys-Davidson limit

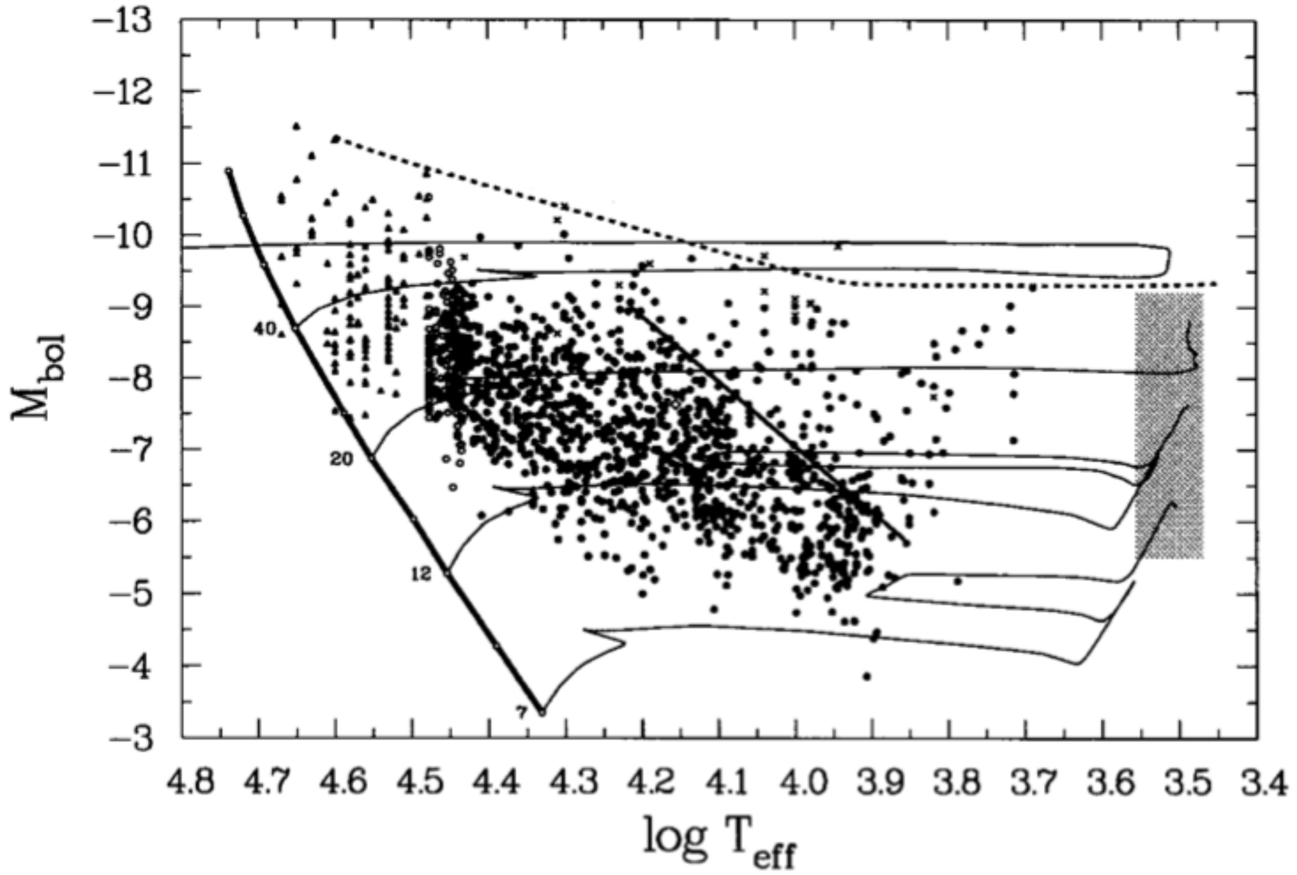
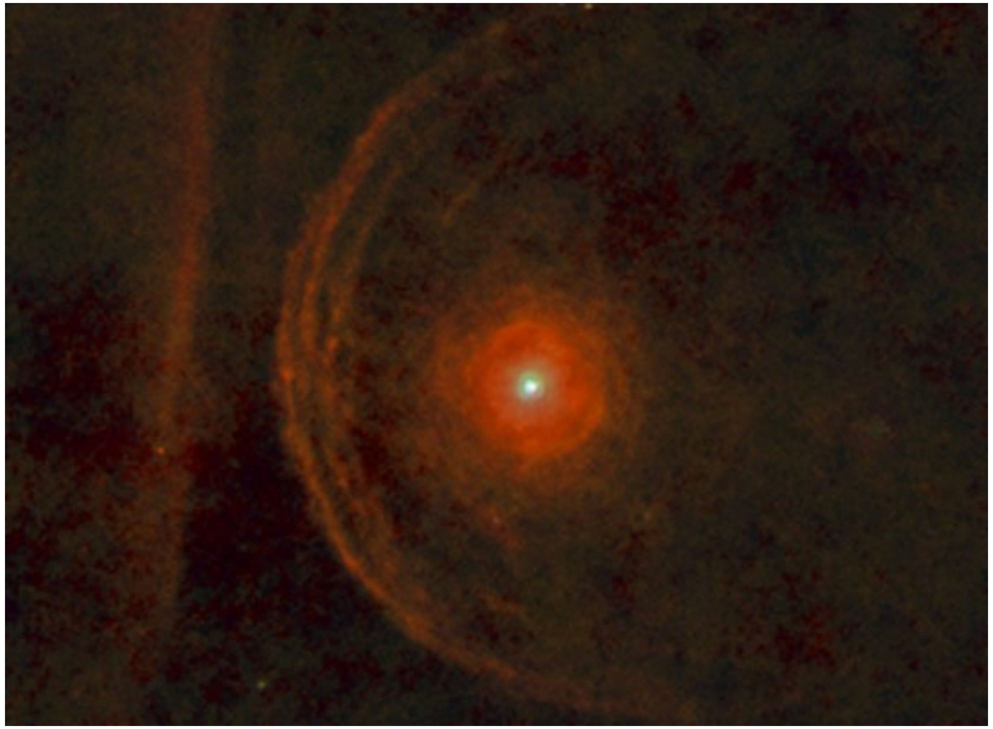


Figure 12.2. The HRD of the brightest supergiants in the LMC. The shaded region contains several hundred red supergiants that are not individually shown. The upper envelope of observed stars traced by the dotted line is known as the Humphreys-Davidson limit (the lower envelope is simply an observational cut-off). Figure adapted from Fitzpatrick & Garmany (1990, ApJ 363, 119).

Red-supergiant star: Betelgeuse

This is the closest RSG from the earth (about 640 light year away)

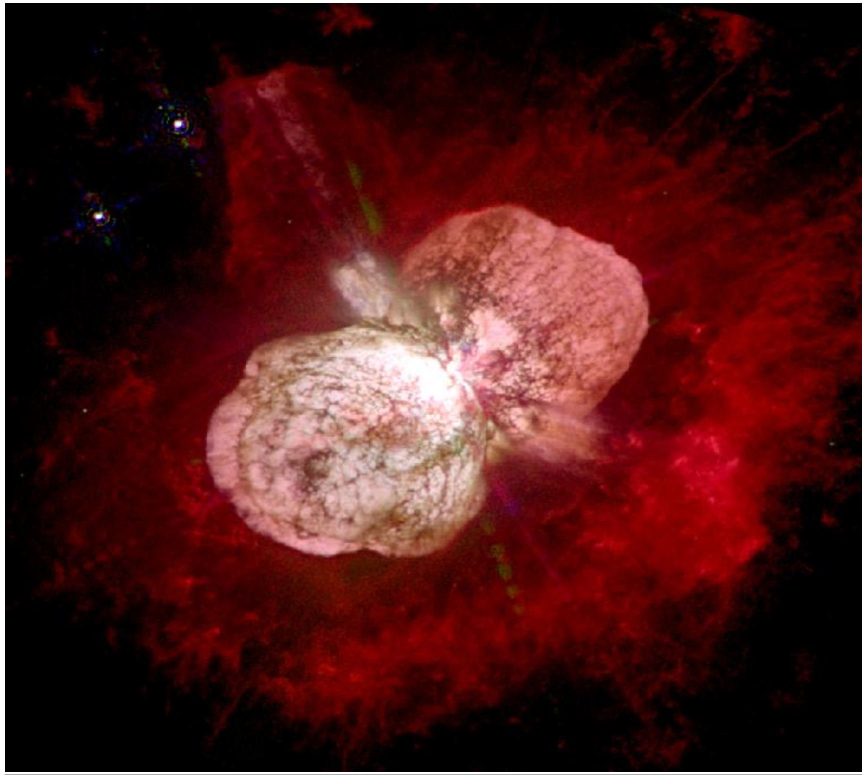


IR image by the Herschel space telescope.

Optical Image

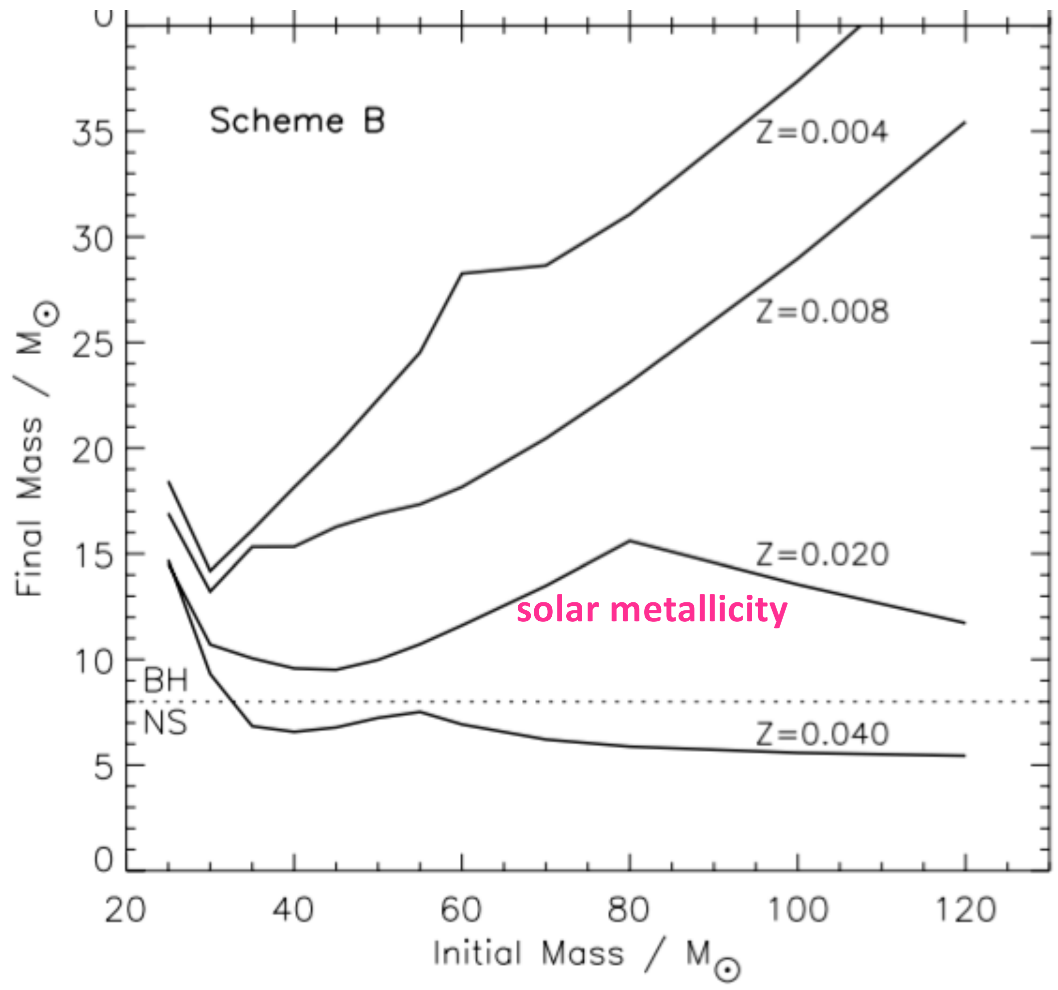
V838 Mon: Another example of mass-loss from red-supergiant star.

A ***Wolf-Rayet Star*** (WR star). It is a naked helium star that has lost its hydrogen envelope. A star of this kind has usually very strong winds.



Eta Carinae nebula: The star at the center has a mass of about 100 Msun. This star underwent a great eruption in the 1840s, ejecting about 10 Msun.

Final mass of single stars as a function of metallicity



Eldridge & Vink 2006

Role of nucleosynthesis for the final structure

Chandrasekhar mass:

$$M_{\mathrm{Ch,eff}} = M_{\mathrm{Ch}}\left[1+\left(\frac{\pi^2 k^2 T^2}{\epsilon_{\mathrm{F}}^2}\right)\right]$$

$$M_{\mathrm{Ch}} = 5.83 Y_e^2$$

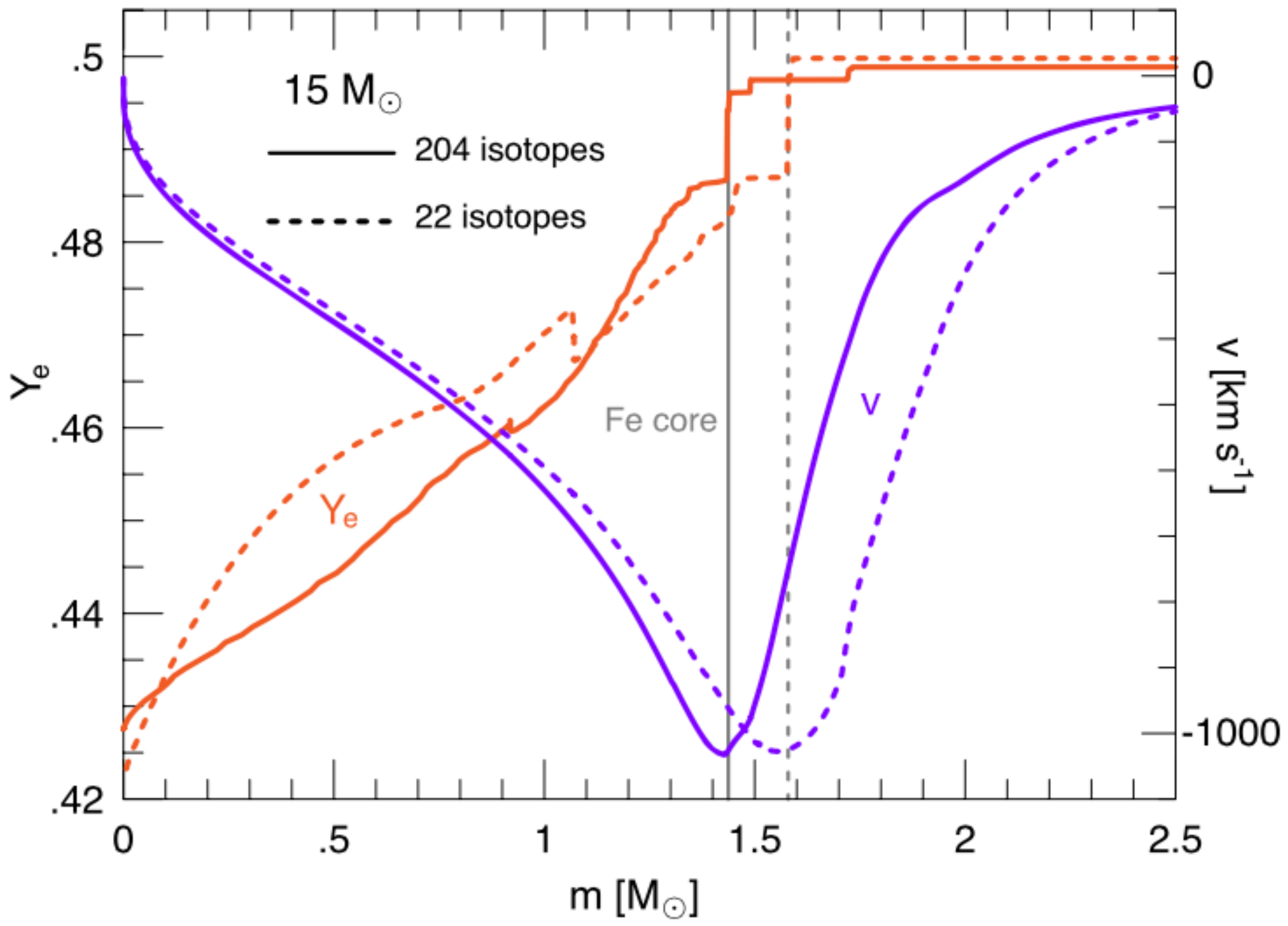


Figure from Paxton et al. 2015

Role of convection

The stellar evolution at the final stages is highly non-linear with complicated convection history, and the final stellar structure is not easy to predict robustly.
→
Different assumptions lead to different predictions for nucleosynthesis!

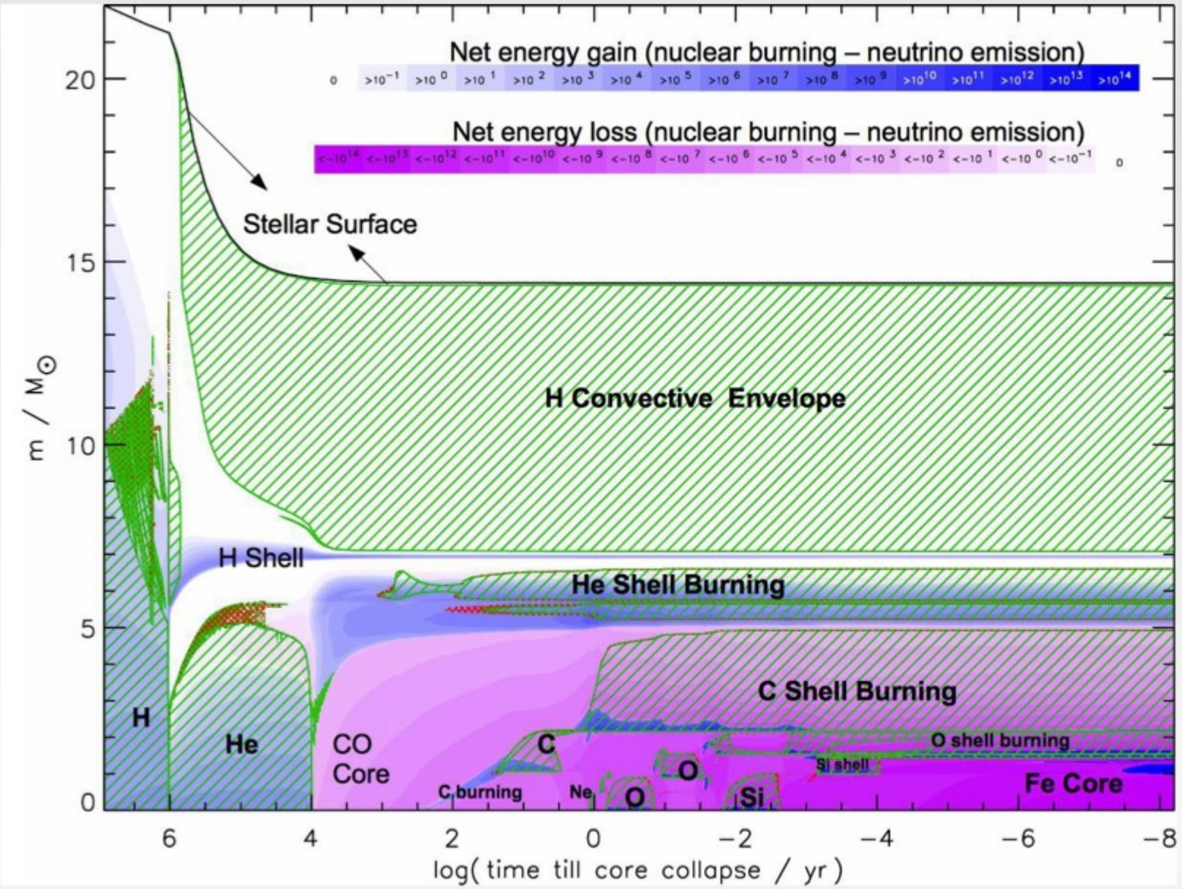
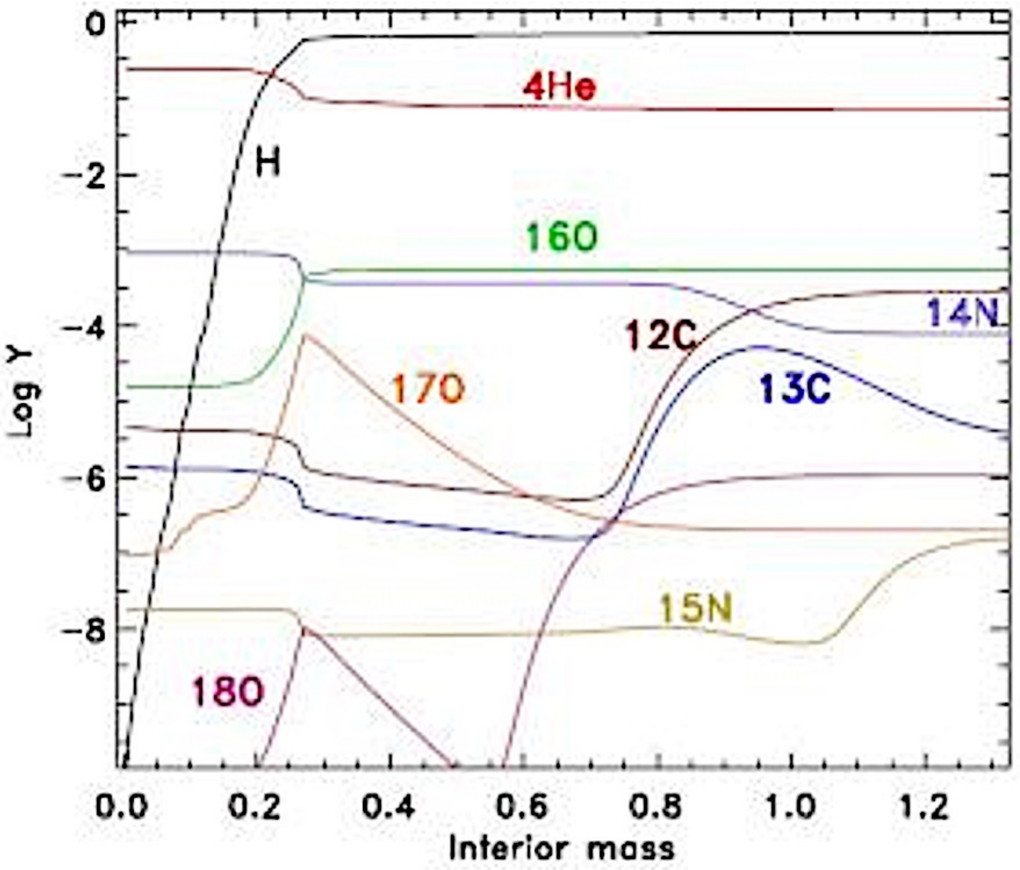


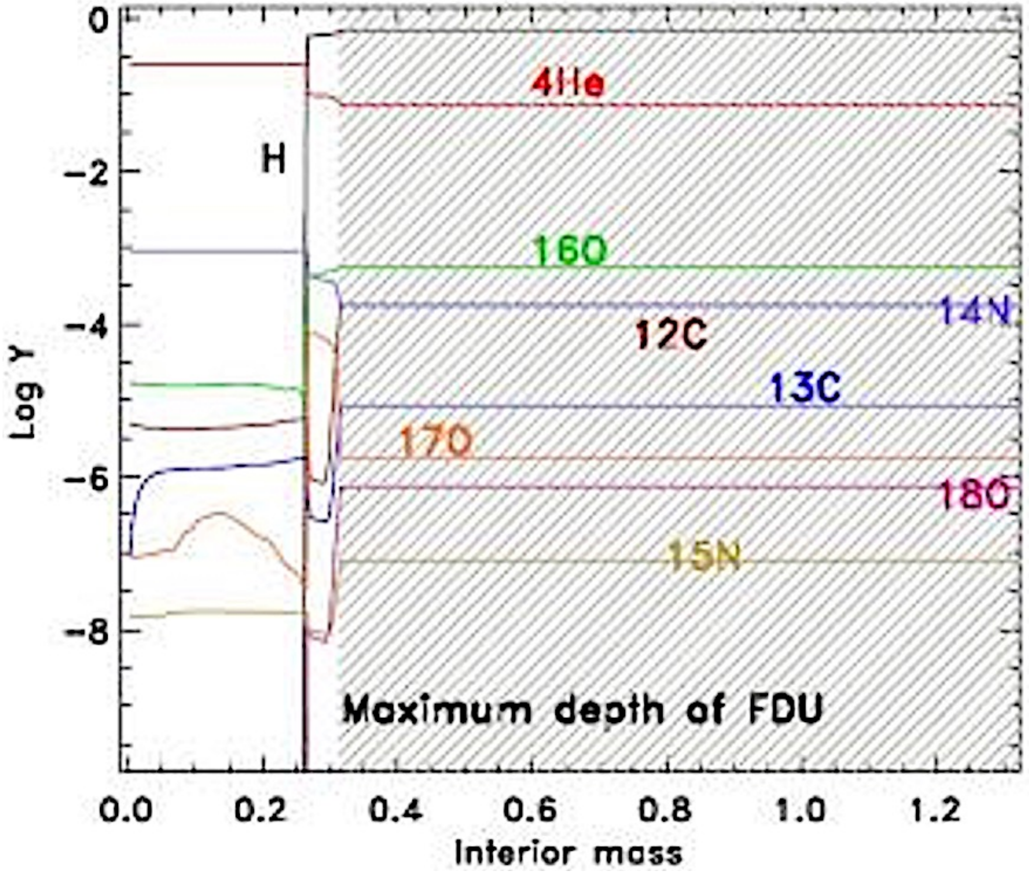
Figure from A. Heger

Convective Mixing

Immediately after the main sequence

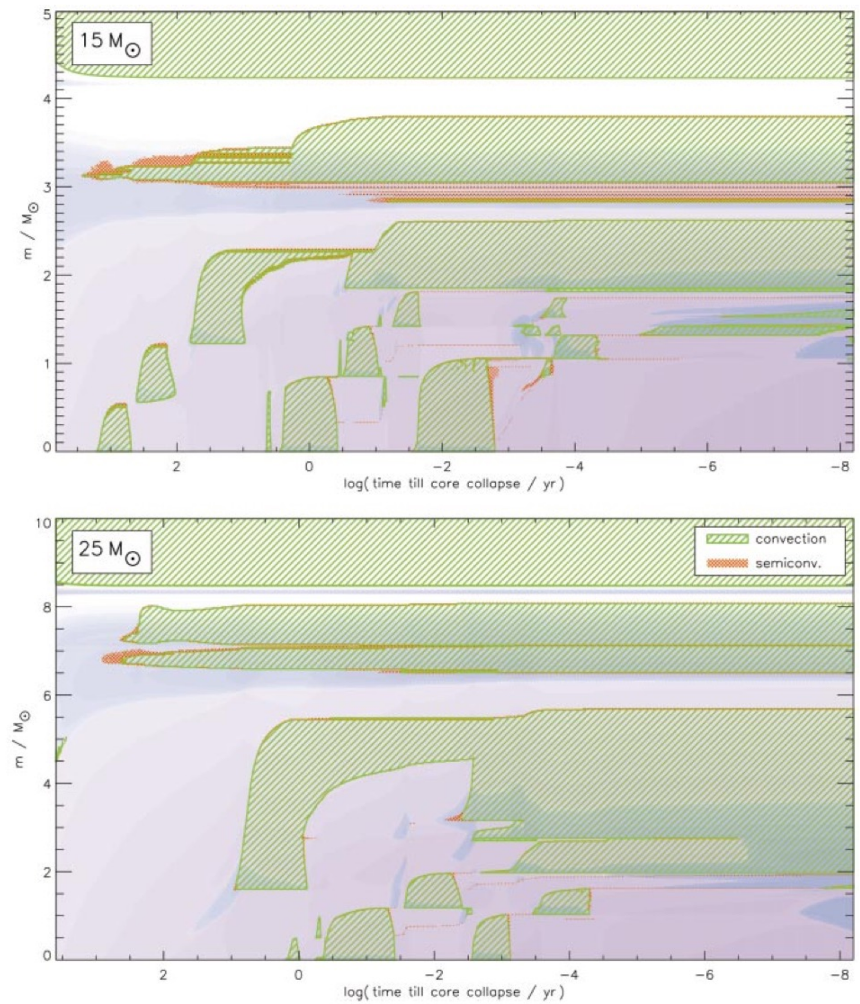


Convective envelope develops as the star becomes a red giant.



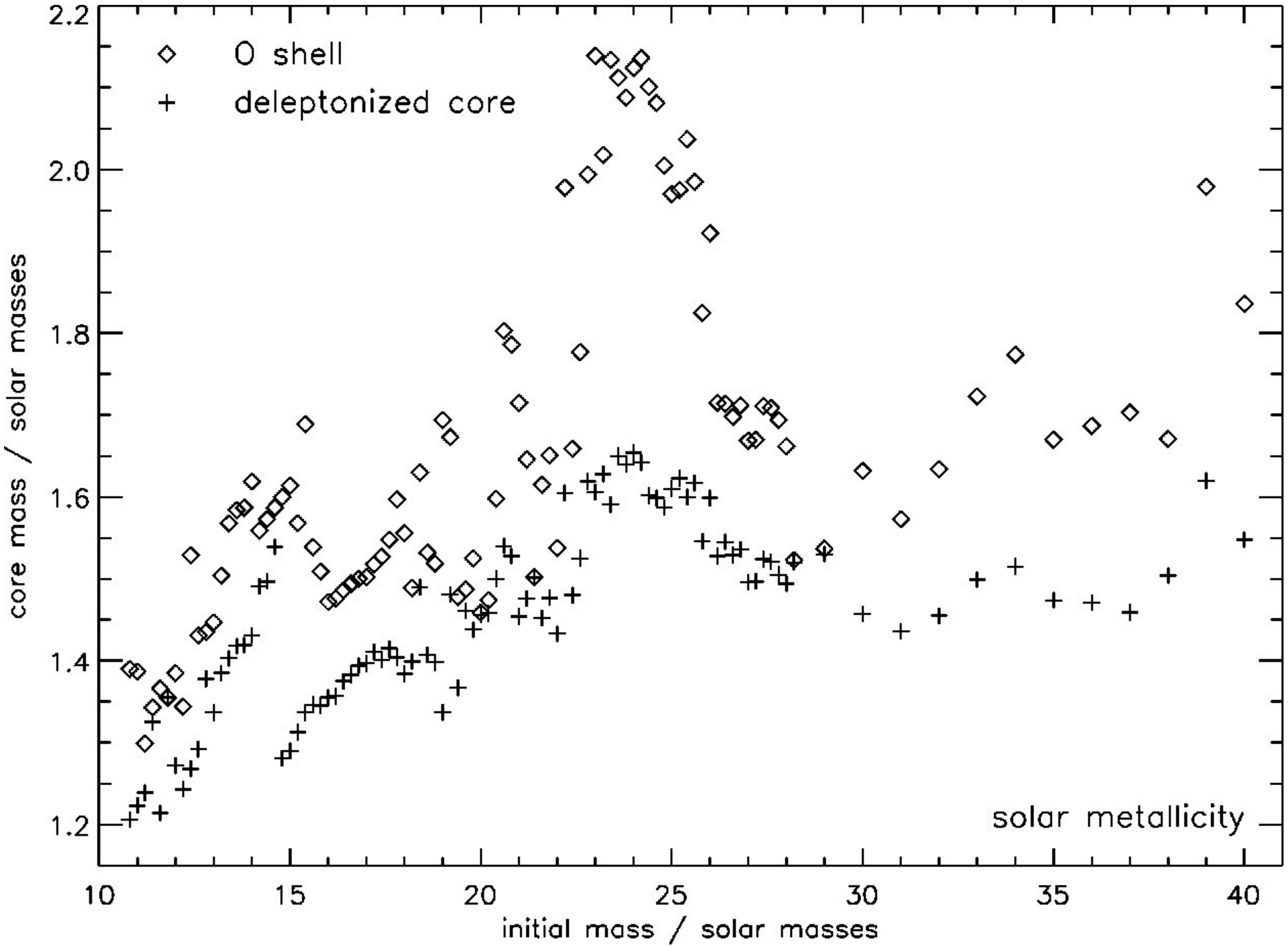
http://ned.ipac.caltech.edu/level5/March15/Karakas/Karakas2.html

Pre-SN Evolution



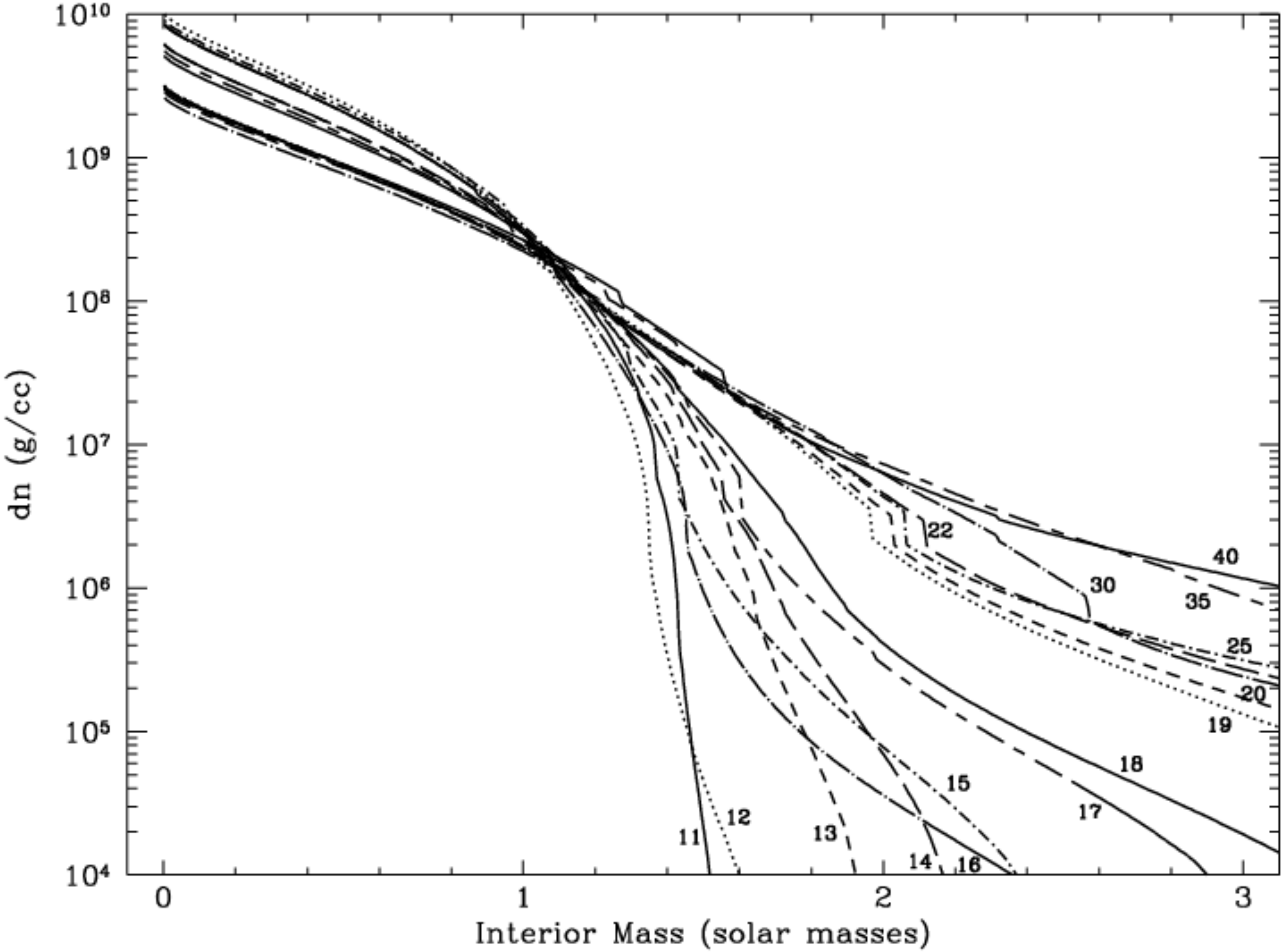
Woosley & Heger 2002

Due to the complex history with convection and shell burning, the iron core size is not a monotonic function of the intial mass.



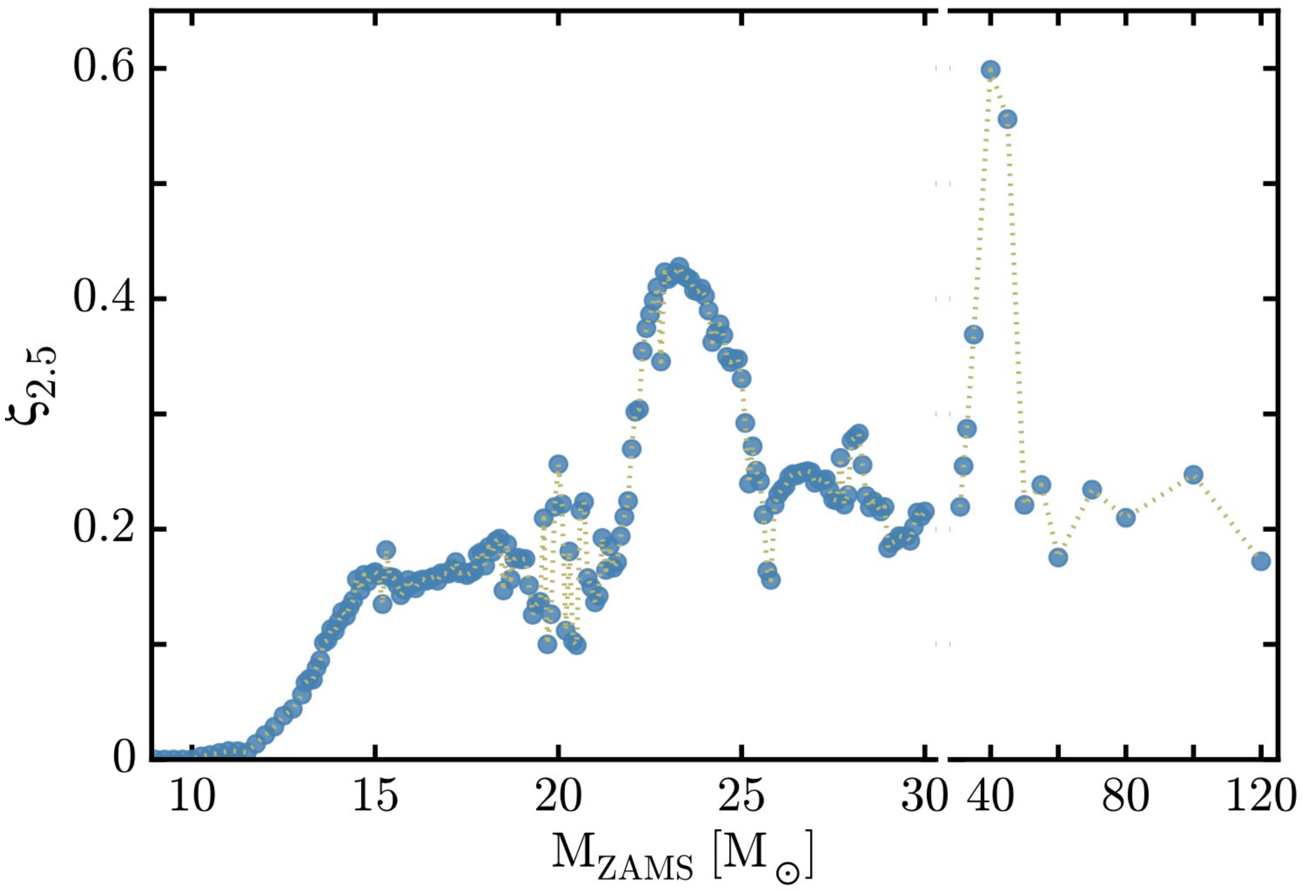
Woosley et al. 02

binding energy becomes higher for a higher initial mass.

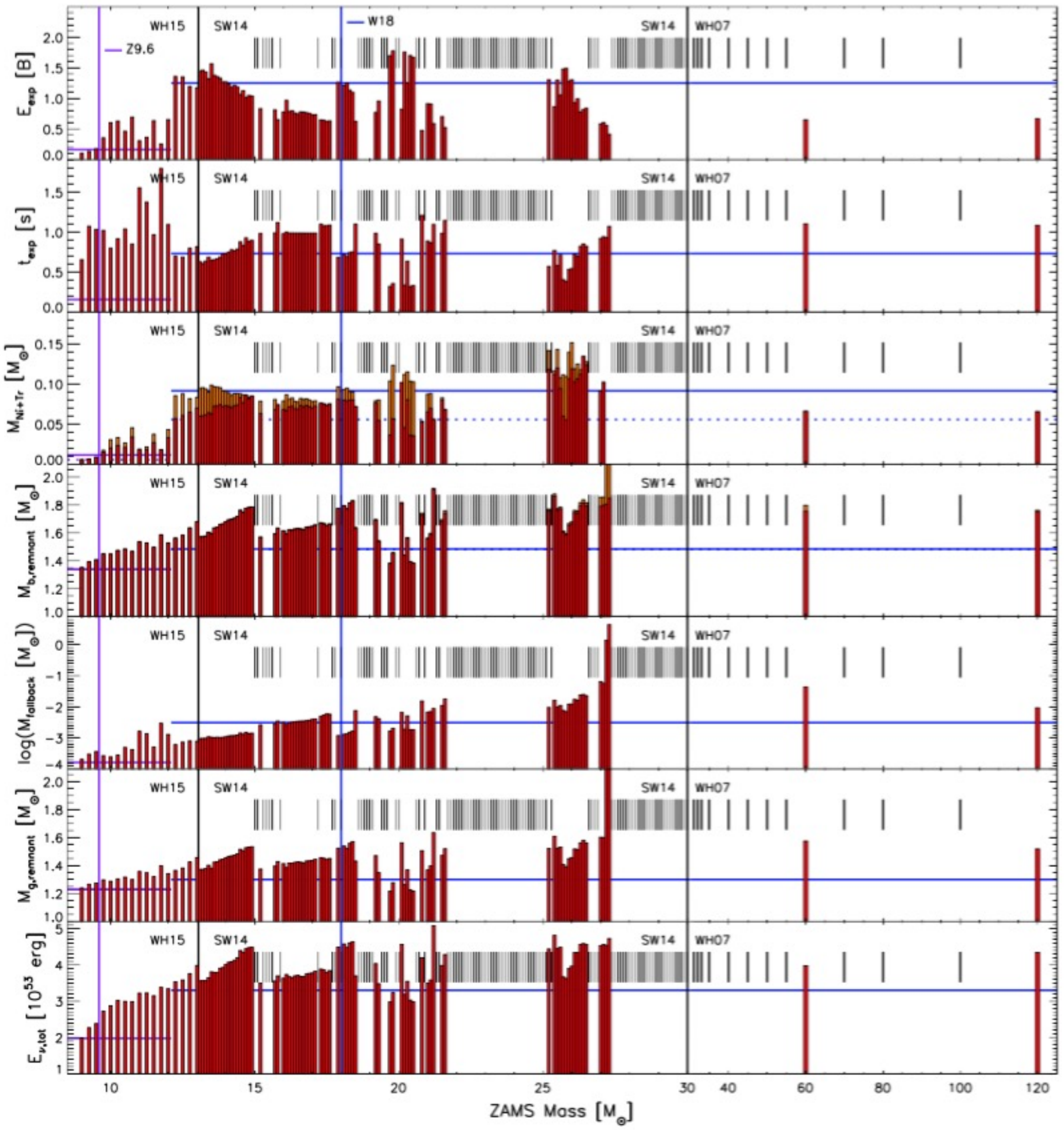


Compactness parameter

$$\xi_M = \frac{M/\mathrm{M}_\odot}{R(M)/1000\,\mathrm{km}}\Big|_{t_\mathrm{bounce}}$$



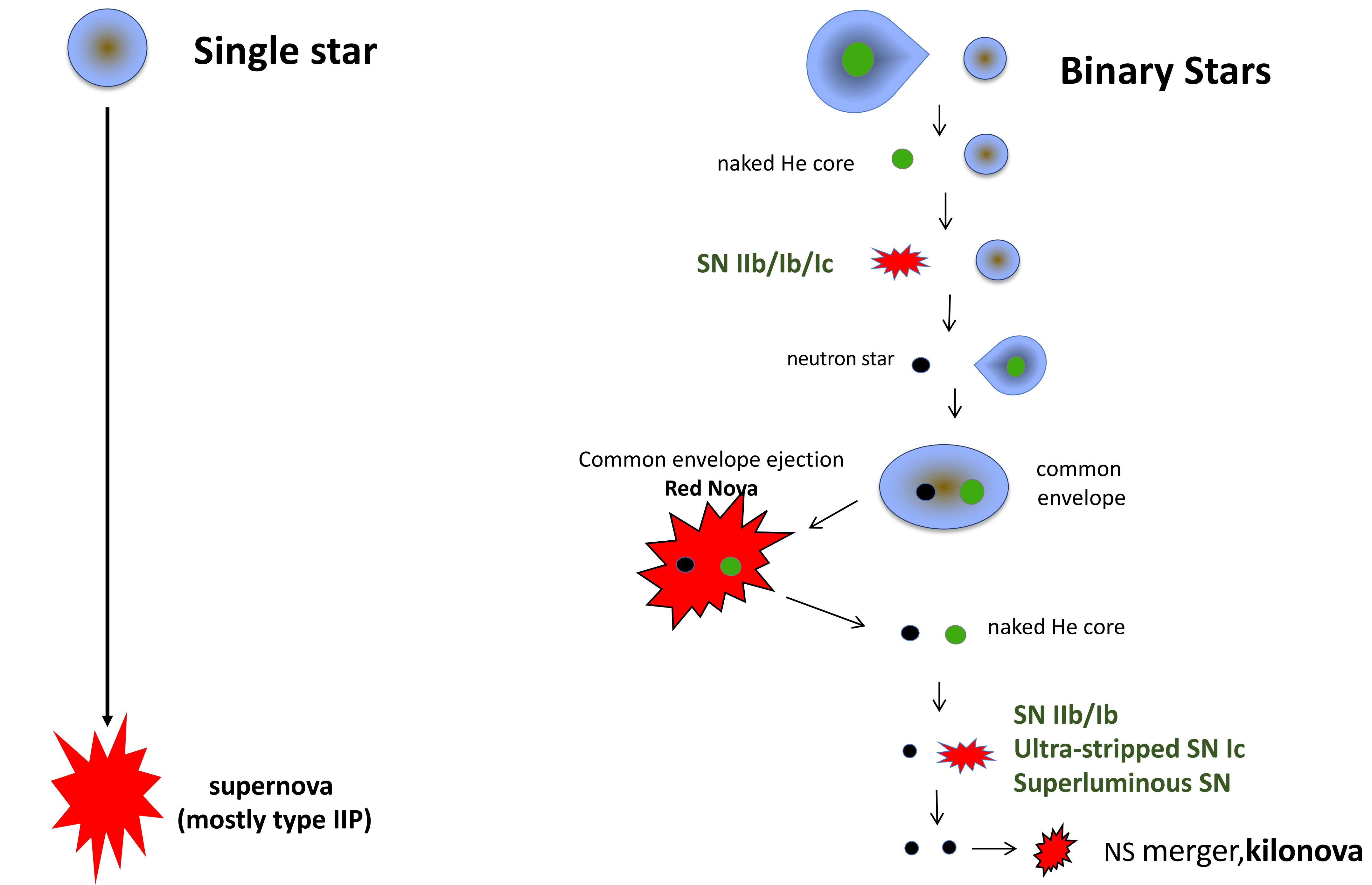
Sukhbold et al. 2016



Sukhbold et al. 2016

Binary interactions

- Almost all massive stars are born in binary systems.
- They are progenitors for double black hole and double neutron star systems.
- Origin of supernova diversity



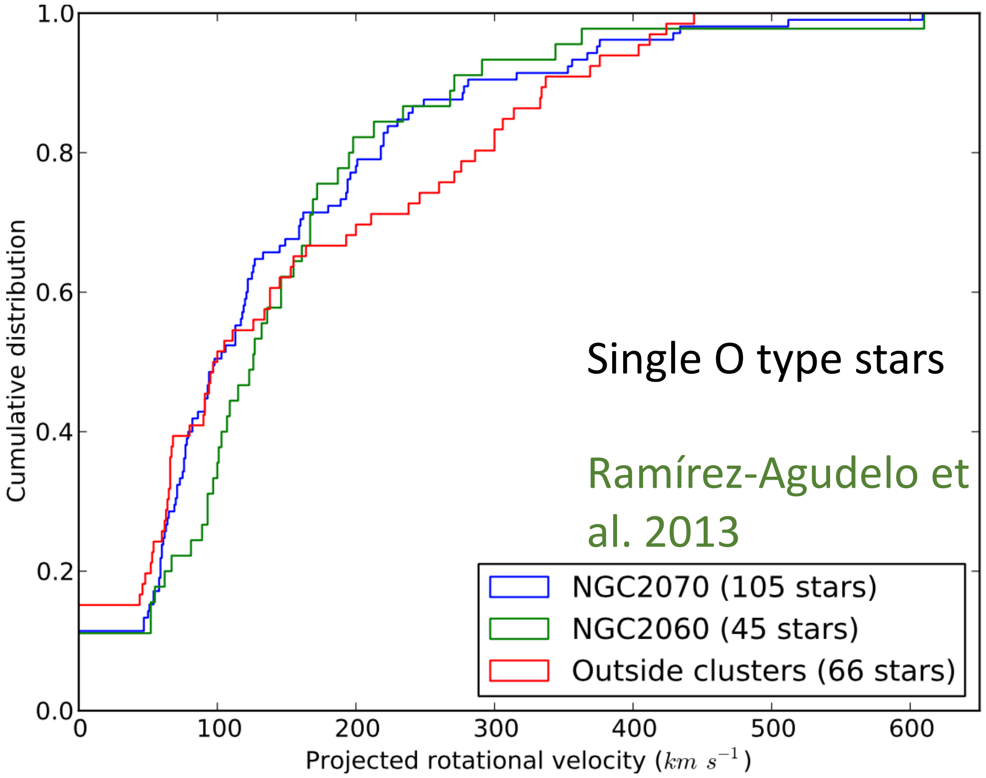
Single star
supernova
(mostly type IIP)
Binary Stars
naked He core
SN IIb/Ib/Ic
neutron star
Common envelope ejection
Red Nova
common
envelope
naked He core
SN IIb/Ib
Ultra-stripped SN Ic
Superluminous SN
NS merger,kilonova

Rotation

A significant fraction of massive stars are rapid rotators.

<u>Effects of rotation:</u>

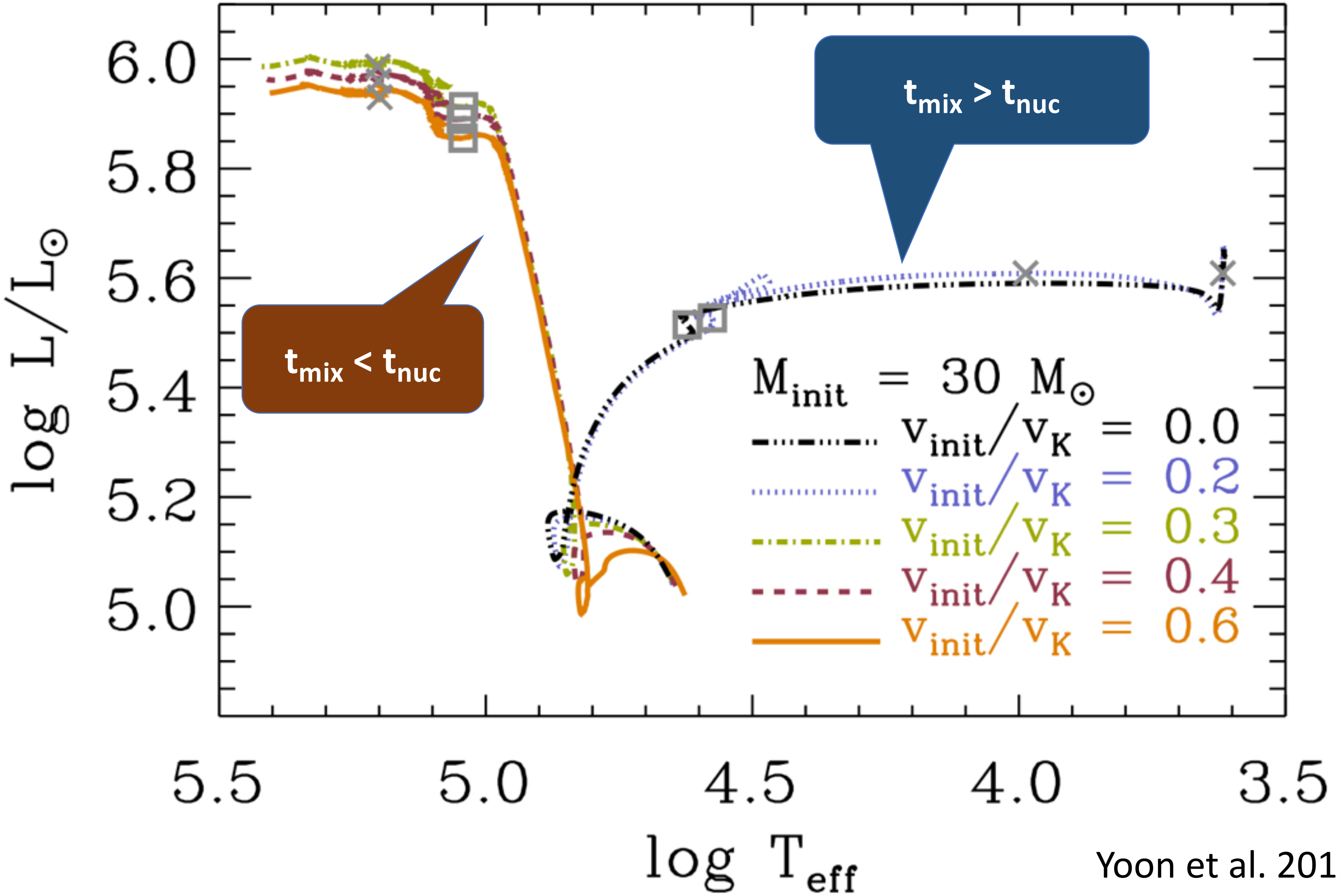
- change of stellar structure
- rotationally induced chemical mixing
- enhancement of mass loss



Ramírez-Agudelo et al. 2013

e.g.,
Mokiem et al. 2006
Bouret et al. 2013, 2021
Dufton et al. 2019
Ramachandran et al. 2019

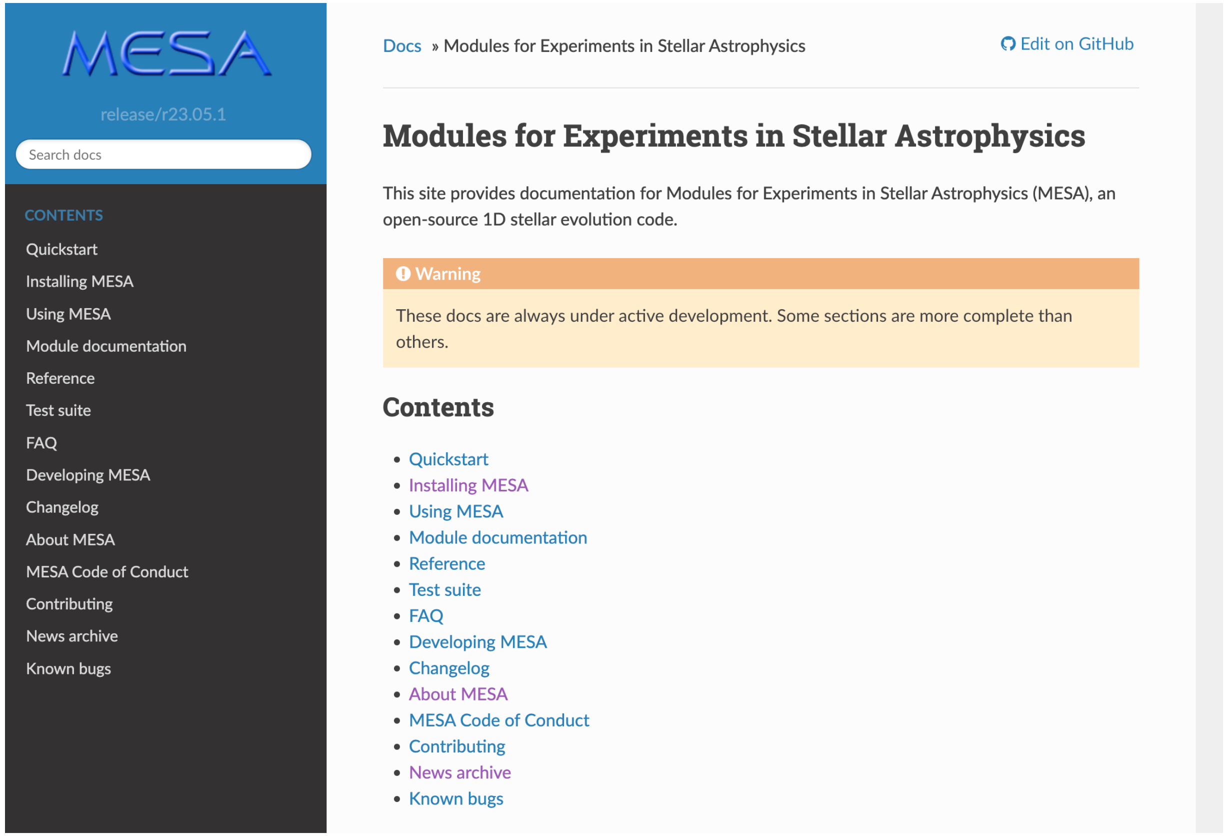
An Example of the effects of rotation: Chemically Homogeneous Evolution (CHE)



Yoon et al. 2012

MESA code: https://docs.mesastar.org/

- You can try various nuclear network options for different numbers of elements (up to 3335 isotopes).
- Many physical factors are included:
    - Effects of rotation
    - Binary interactions
    - Atomic diffusion
    - Explosive nucleosynthesis due to supernova explosions
    - Stellar oscillations



Massive star models publicly available

https://2sn.org/

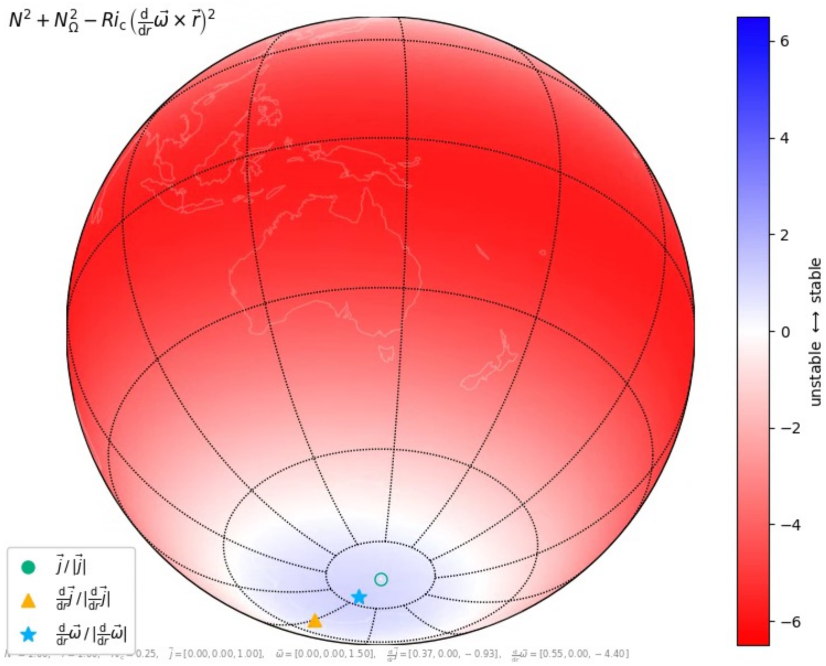


Figure: Stability of layer oblique differentially rotating star.

Dr.

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Professor

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Massive star models publicly available

http://orfeo.iaps.inaf.it/

Massive star models publicly available

https://www.unige.ch/sciences/astro/evolution/en/database/

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STELLAR EVOLUTION

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Database

Welcome on the database for stellar models.

You'll find below the links to the stellar evolution tracks computed by our group.

These tracks come from two distinct codes:

- The Geneva Code
- STAREVOL

Visit our interactive tools page

Massive star models publicly available

https://bpass.auckland.ac.nz/

BPASS

Binary Population and Spectral Synthesis

THE UNIVERSITY OF AUCKLAND

NEW ZEALAND

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- BPASSv2.2
- Hoki
- BPASS Results
- CURVEPOPS
- Nebular Emission
- Other Results
- FAQ & Outreach
- Older BPASS versions

Have questions not answered here? CONTACT US: j.eldridge [at] auckland.ac.nz and e.r.stanway [at] warwick.ac.uk

The Binary Population and Spectral Synthesis code (BPASS) is the result of combining my stellar evolution models with libraries of synthetic atmosphere spectra to create a unique tool to model many details of stellar populations. While similar codes (such as starburst99) exist BPASS has important features, each of which set it apart from other codes and in combination make it the cutting edge. First, and most important, is the inclusion of binary evolution in modelling the stellar populations. The general effect of binaries is to cause a population of stars to look bluer at an older age than predicted by single-star models. Secondly, detailed stellar evolution models are used rather than an approximate rapid population synthesis method. Thirdly, only theoretical model spectra are used in the syntheses with as few empirical inputs as possible to create completely synthetic models to compare with observations.

On this site we make available standard outputs from our code for single and binary star populations. Select the data you require from the menu above. If you require data that is not here please email us.

The current version of the code is Version 2.3. (Released March 2022)