# Recent Developments in Supersymmetric Dualities

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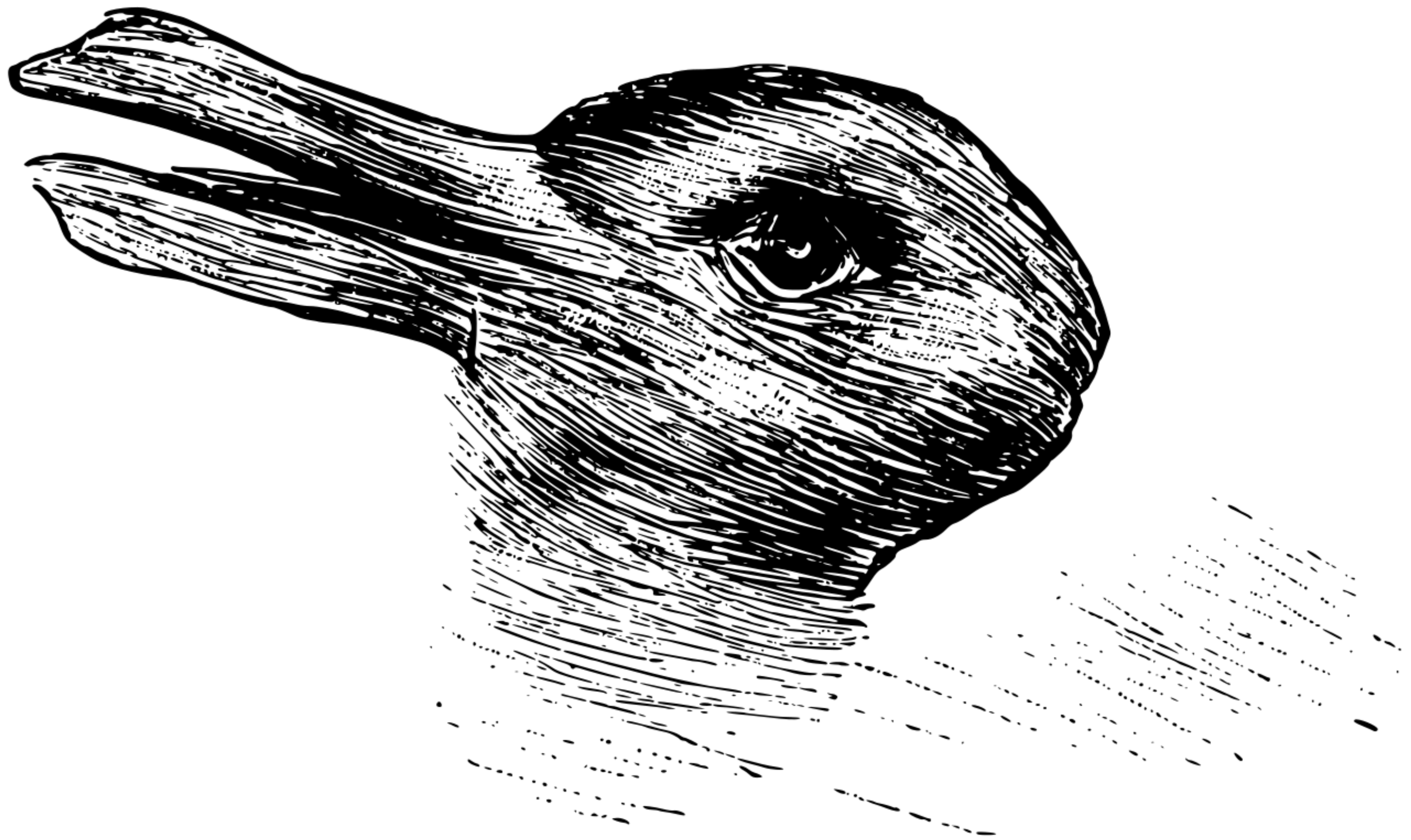
CTPU workshop on Particle Physics and Cosmology, Oct. 23, 2023

# Rabbit or Duck?

- Duality—the same phenomenon, multiple descriptions
- Essential roles in various fields, such as high energy and condensed matter physics
- Many different kinds known—today: IR duality of QFT

Welche Thiere gleichen ein-
ander am meisten?



Kaninchen und Ente.

“Rabbit and Duck,” Fliegende Blätter (23 Oct 1892)

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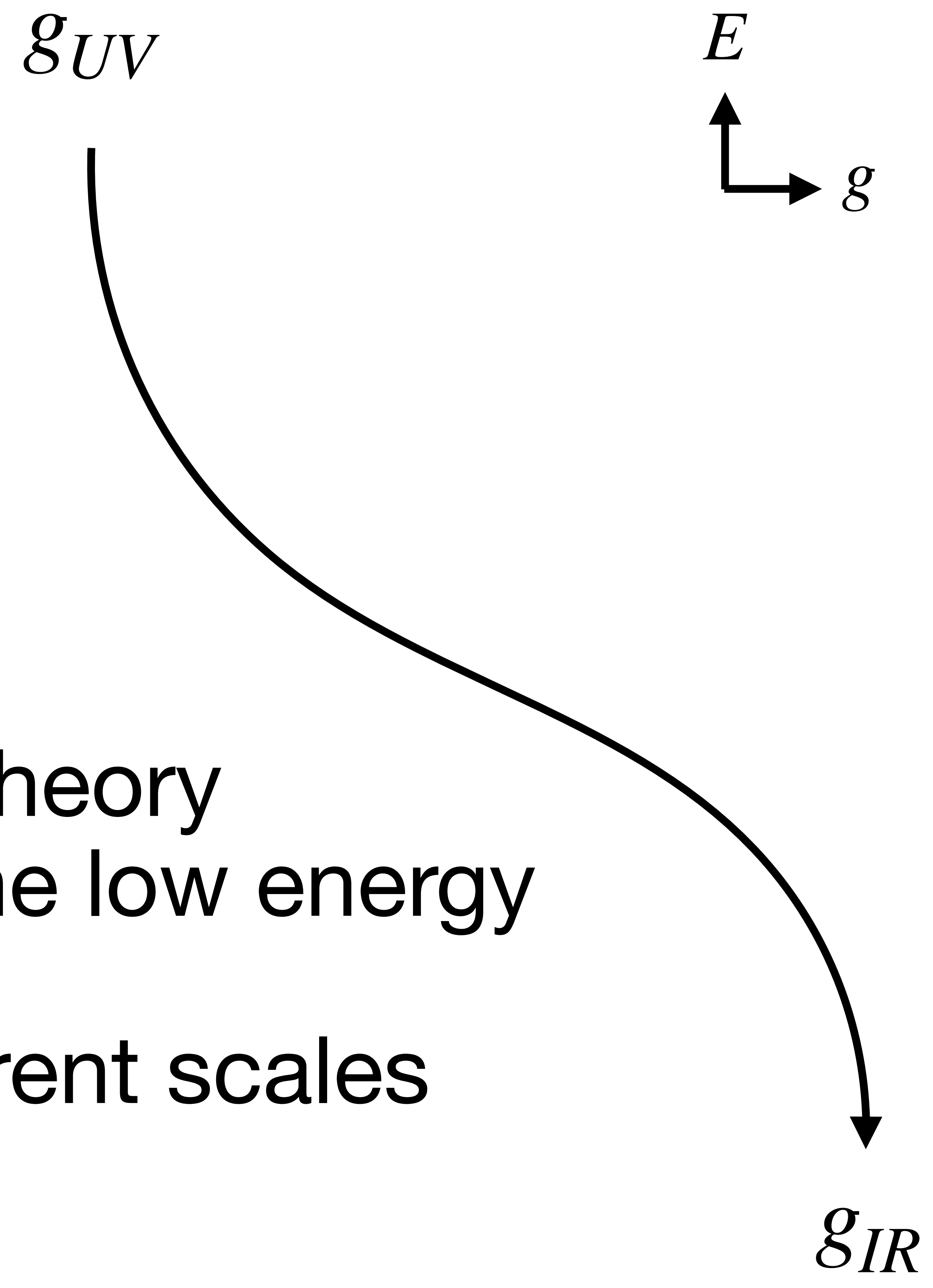
# Outline

- Introduction
  - Rabbit or duck?
  - Renormalization
  - Web of dualities—Example I, II, III
- What are the most fundamental dualities?
- Example
- Conclusion

# Renormalization

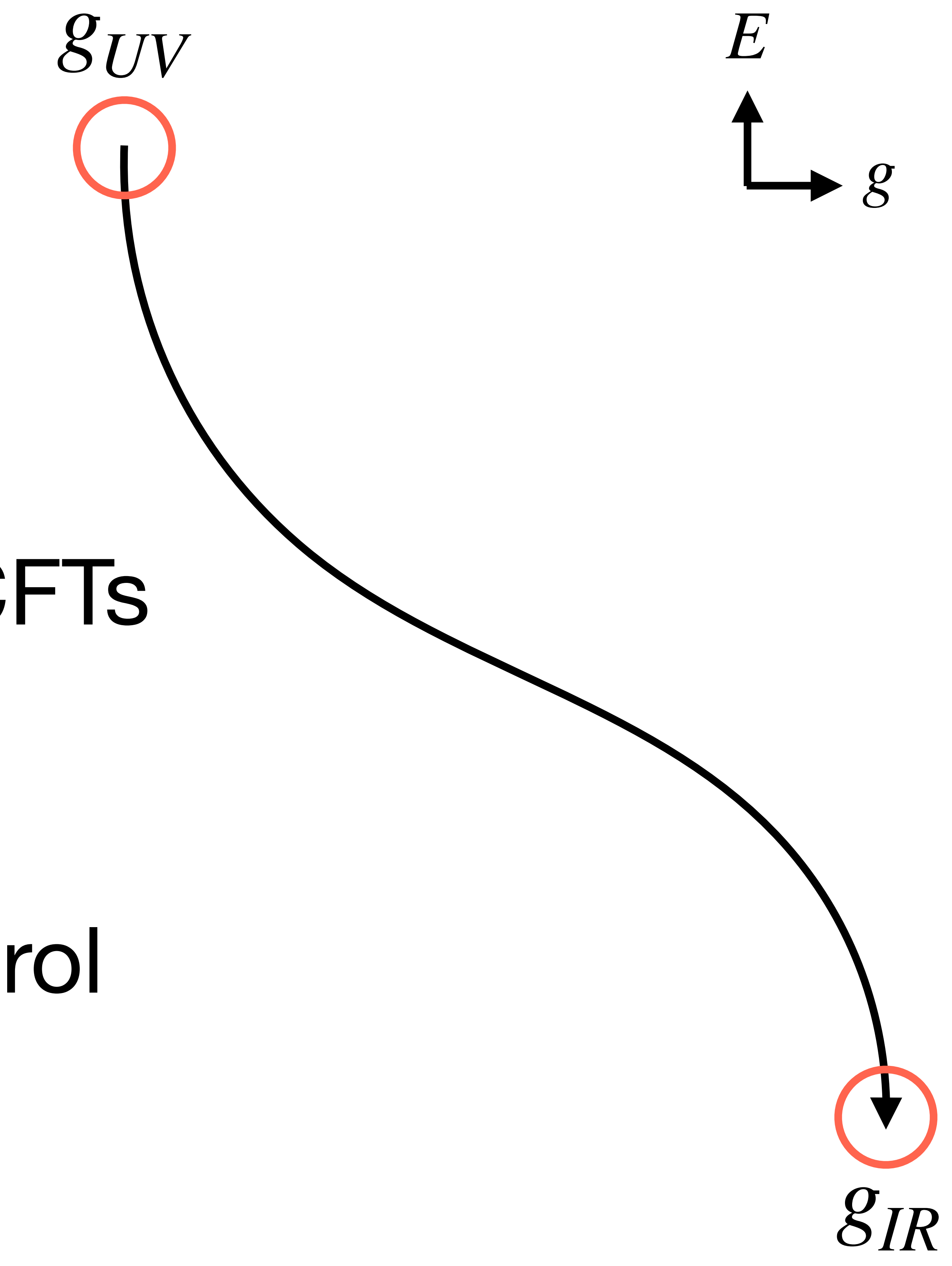
# Renormalization in QFT

- **Couplings in QFT** varying along the scale
- Different dynamics at different scales
- Rich low-energy dynamics from an almost free theory
- “QCD” in the Standard Model—an asymptotically-free theory (i.e., free in the high energy) exhibiting confinement in the low energy
- Understanding a QFT -> understanding physics at different scales



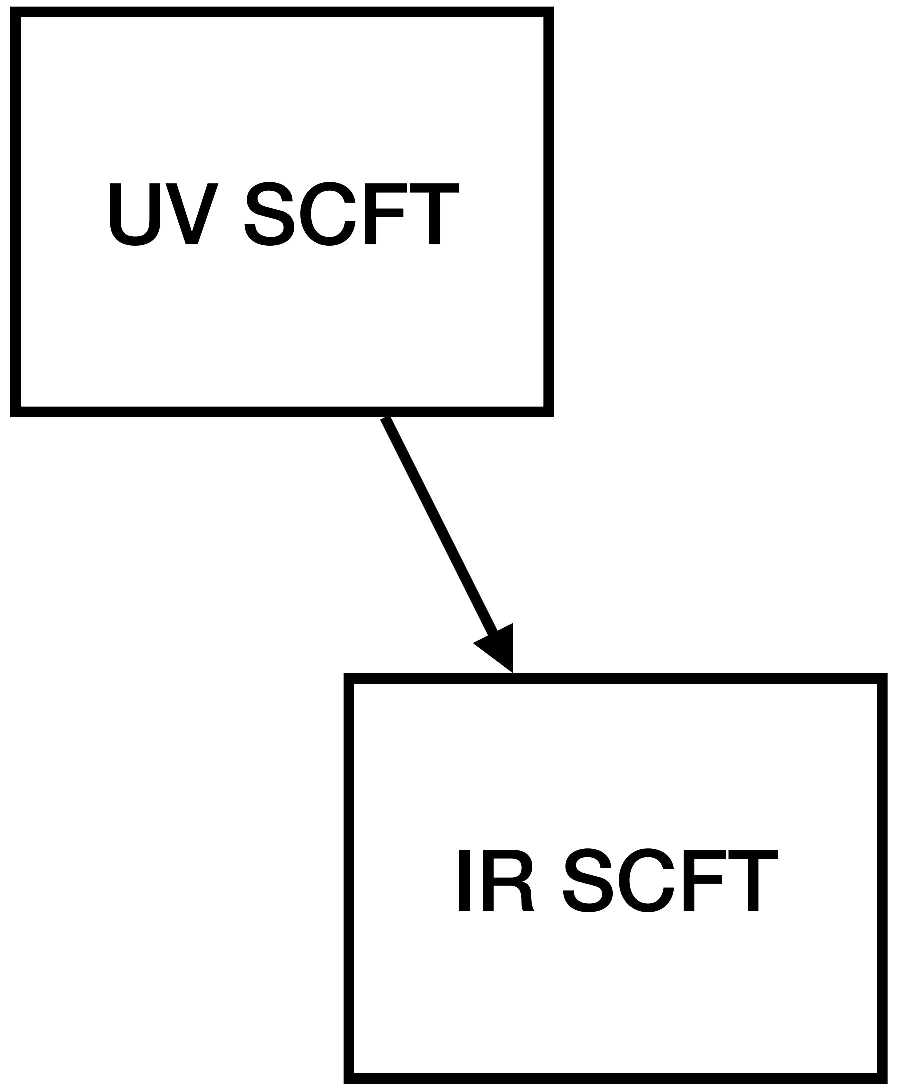
# Renormalization in QFT

- A **conformal field theory** at each end of the RG-flow
- Invariant under the scale transformation
  (e.g., a free theory)
- Interesting emergent phenomena in strongly coupled CFTs
- E.g., *symmetry enhancement, duality, gravity, …*
- Still difficult to solve; nevertheless, more universal control
- Especially with *supersymmetry*



# SCFTs and Dualities

- SCFTs—QFTs preserving a **superconformal** symmetry
- Interacting SCFTs: deformation of free theories (Lagrangian constructions), string theory constructions (strongly coupled), or
- SCFTs from SCFTs
  - Adding interactions
  - Gauging symmetries
  - Compactification
- Fixed points of RG-flows
- An Infra-Red (IR) duality—the same IR SCFT from multiple UV SCFTs



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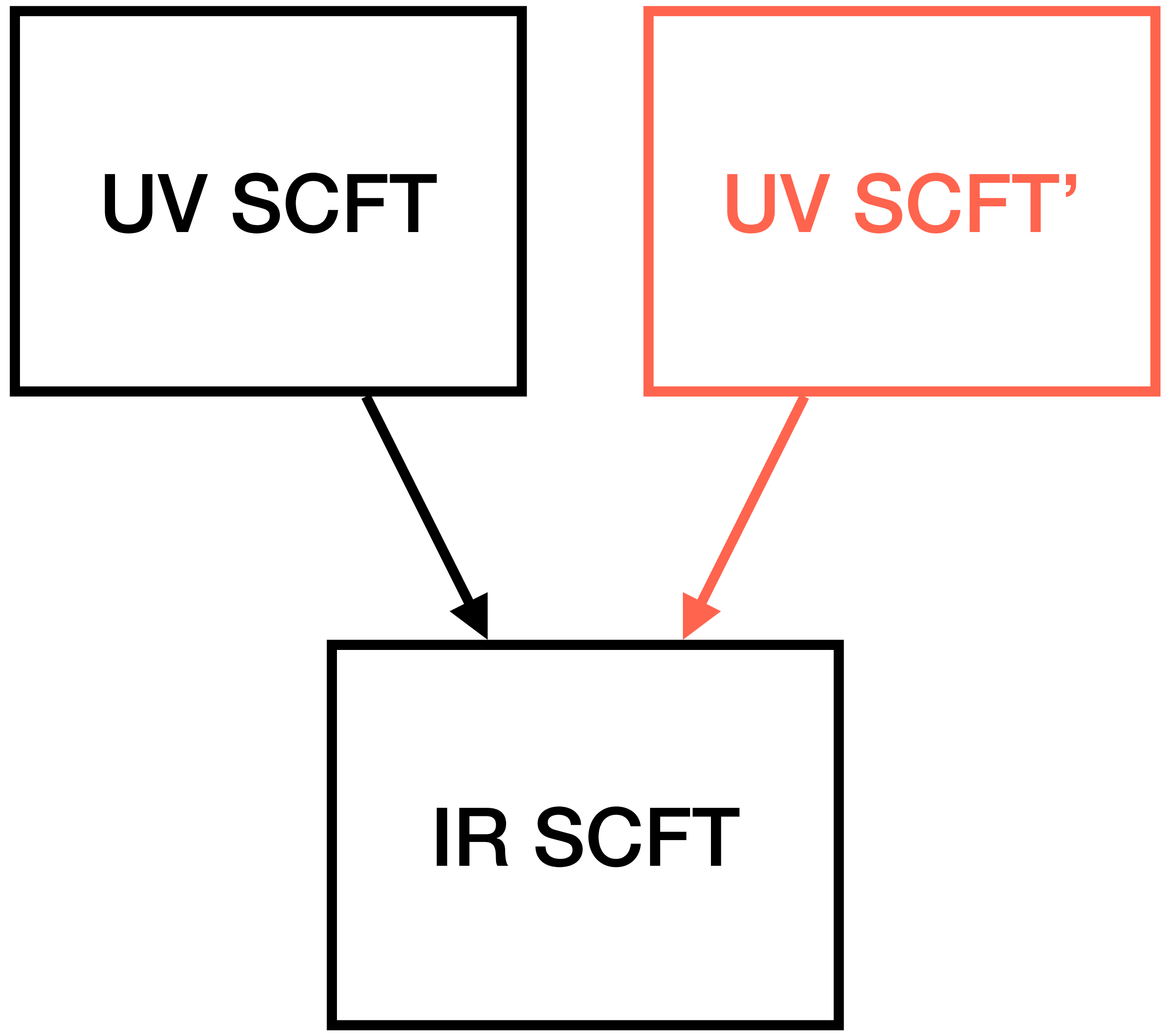

# Web of dualities

# Example I: 3D Gauge-Scalar Duality

- **The 3d Maxwell theory**

$$Z_{Maxwell} = \int \mathscr{D}A_\mu \exp\left[\int d^3x \, \frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu}\right]$$

- The field-strength satisfies the Bianchi identity:

$$\epsilon^{\mu\nu\rho} \, \partial_\mu F_{\nu\rho} = 0$$

- Integrating over $F_{\mu\nu}$, $Z_{Maxwell}$ can be written as follows:

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$$= \int \mathscr{D}\gamma \, \exp\left[\int d^3x \, \frac{e^2}{8\pi^2} \, \partial_\mu\gamma \, \partial^\mu\gamma\right]$$

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- A simple duality example between ***the 3d Maxwell theory*** and ***a scalar theory***, both of which are free.

# Example II: 3D Vortex-Particle Duality

- Generalization to interacting theories: **the 3d bosonization**

$$i\bar{\chi}\gamma_\mu D_a^\mu \chi - \frac{1}{2\pi}Bda - \frac{1}{4\pi}BdB \quad \longleftrightarrow \quad D_B\phi^{2} - \phi^{4}$$

- A number of new dualities derived; e.g., **the vortex-particle duality**

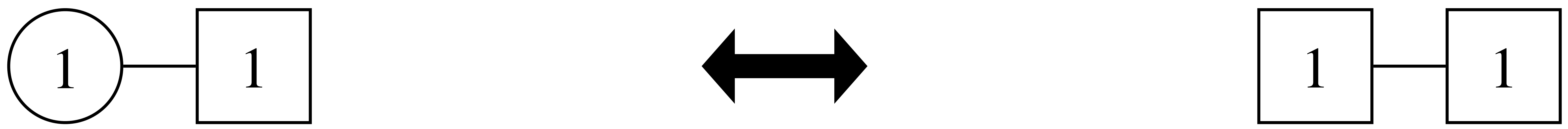
$$D_b\phi^{2} - \phi^{4} + \frac{1}{2\pi}bdC \quad \longleftrightarrow \quad i\bar{\chi}\gamma_\mu D_a^\mu \chi - \frac{1}{2\pi}bda - \frac{1}{4\pi}bdb + \frac{1}{2\pi}bdC \quad \longleftrightarrow \quad D_C\hat{\phi}^{2} - \hat{\phi}^{4}$$

- Adding $\frac{1}{2\pi}BdC$ + gauging $B \to b$ + the (time-reversed) bosonization

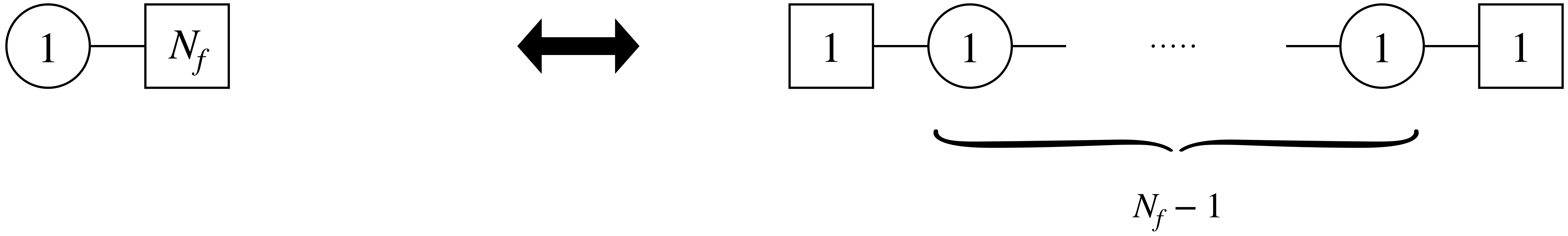
- The bosonization -> the vortex-particle duality, but not the other way around; namely, the bosonization is more fundamental than the vortex-particle duality.

# Example III: 3D Mirror Symmetry

- Generalization to **supersymmetric** theories
- The simplest case:
  - 3d $\mathcal{N} = 4$ supersymmetric $U(1)$ gauge theory with a fundamental hypermultiplet
  - A free hypermultiplet




- Multiple flavors



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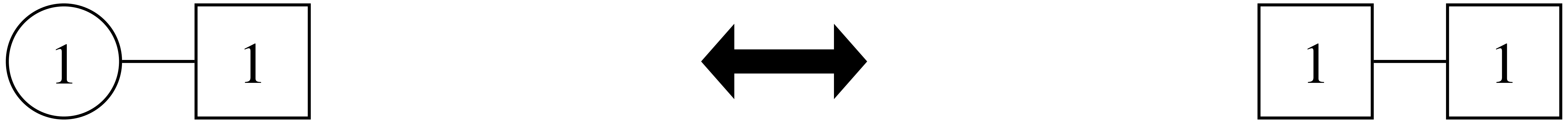


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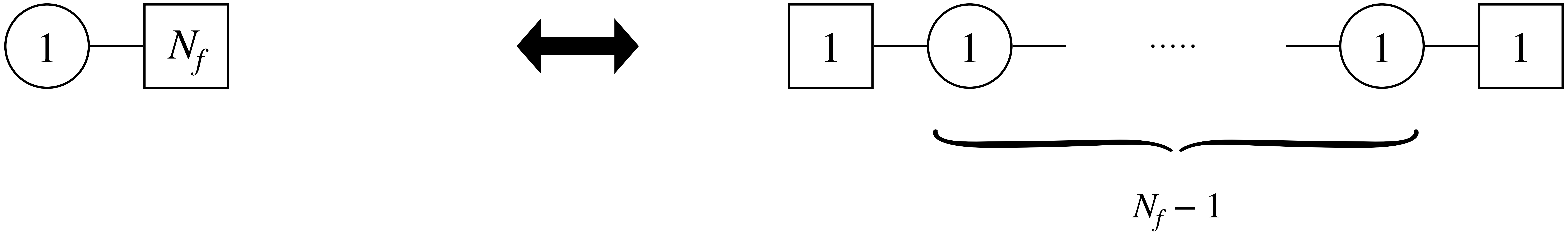

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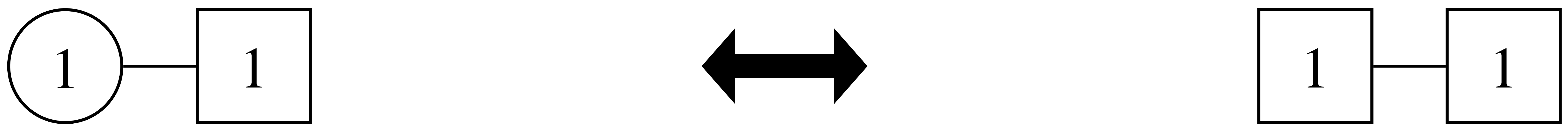
  **<-> Topological $U(1)$** **<-> Flavor $U(1)$**

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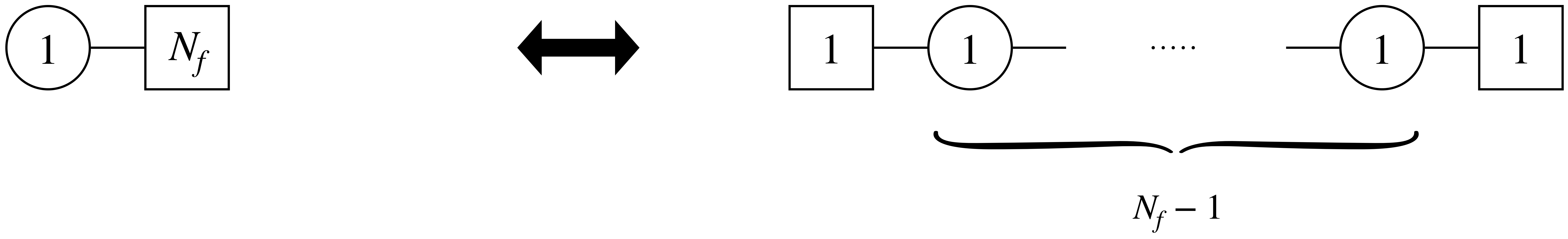

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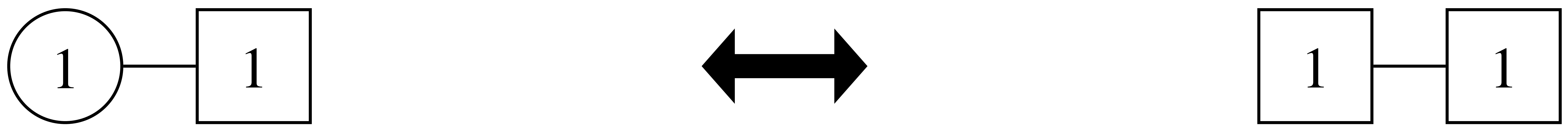


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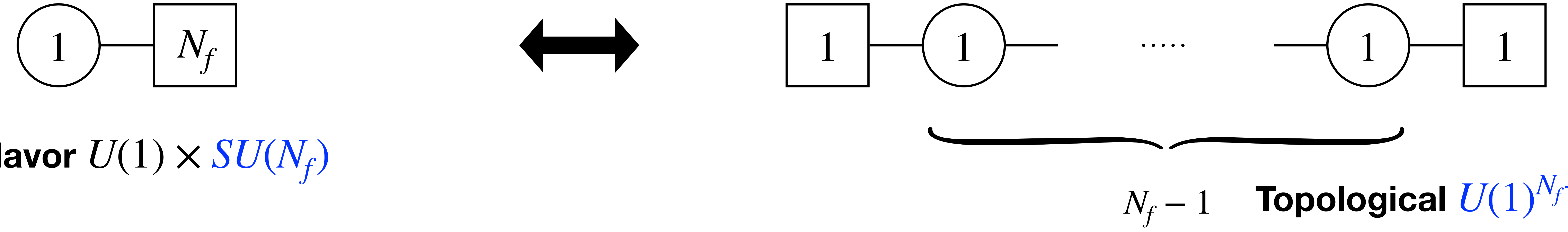



**Flavor** $U(1) \times SU(N_f)$

**Topological** $U(1)^{N_f - 1}$

# Web of IR Dualities

Seiberg duality
Intriligator-Pouliot duality
Intriligator-Leigh-Strassler duality
Kutasov-Schwimmer-Seiberg duality
Brodie duality
Kutasov-Lin duality
...

Hwang-Pasquetti-Sacchi duality
Comi-Hwang-Marino-Pasquetti-Sacchi duality

4d

Aharony duality
Giveon-Kutasov duality
Niarchos duality
Kim-Park duality
Hwang-Kim-Park duality
...
and their monopole deformations

Intriligator-Seiberg duality
Dorey-Tong duality
Gaiotto-Witten duality of $T_\rho^\sigma[G]$
...

3d bosonization
Particle-vortex duality
Karch-Robinson-Tong duality
...

3d

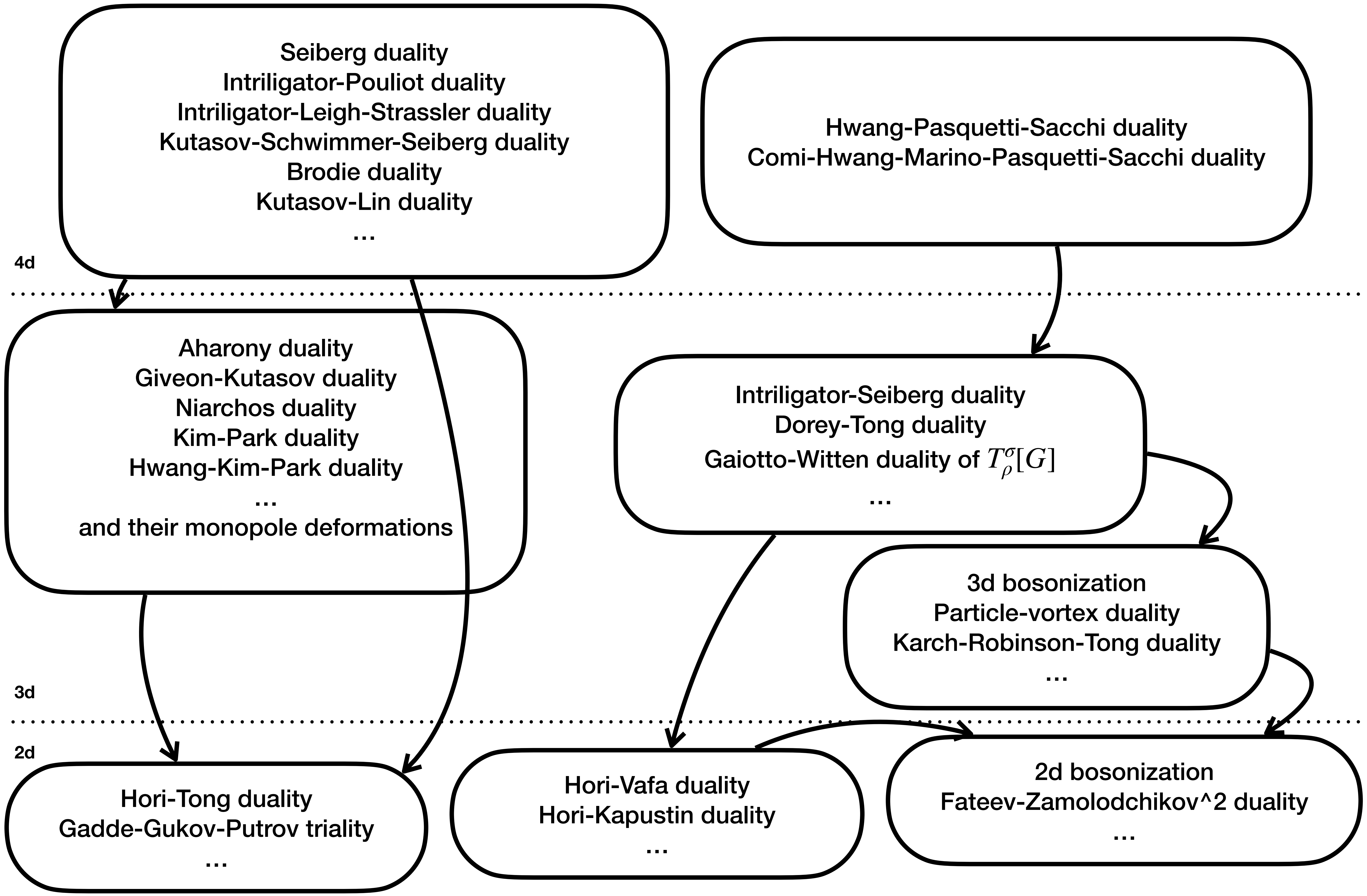
2d

Hori-Tong duality
Gadde-Gukov-Putrov triality
...

Hori-Vafa duality
Hori-Kapustin duality
...

2d bosonization
Fateev-Zamolodchikov^2 duality
...

# Web of IR Dualities



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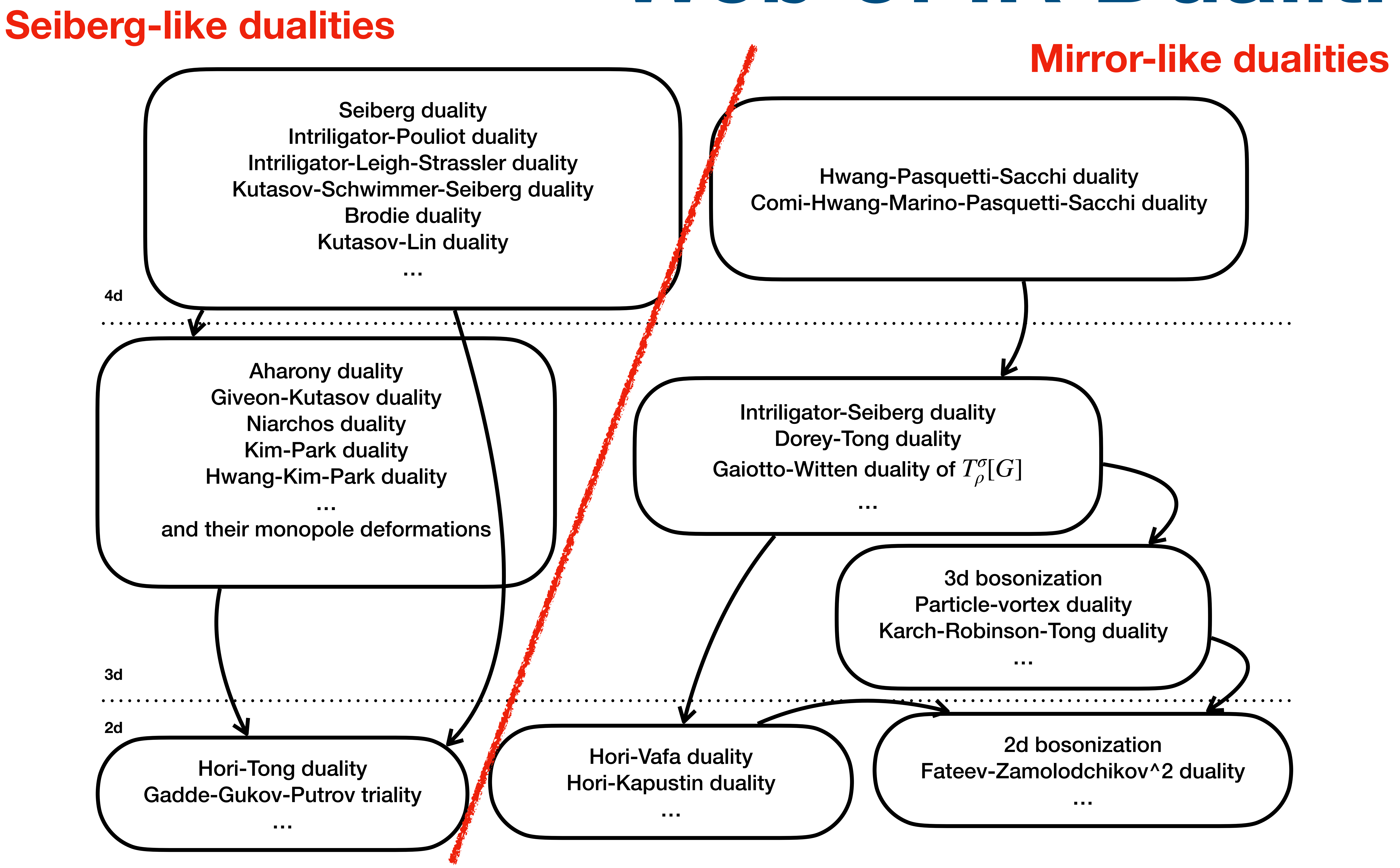


# What are the most fundamental dualities?

# Most Fundamental Dualities?

- Why do those dualities work?
- No single (interacting) duality has been proven. (Not surprising since we don't know how to handle strongly coupled QFTs in general.)
- Are they really different?—Own microscopic mechanism each or any universal mechanism?
- If the latter is the case, ***what are the most fundamental dualities?***

# Duality of Theories vs Duality of Fields

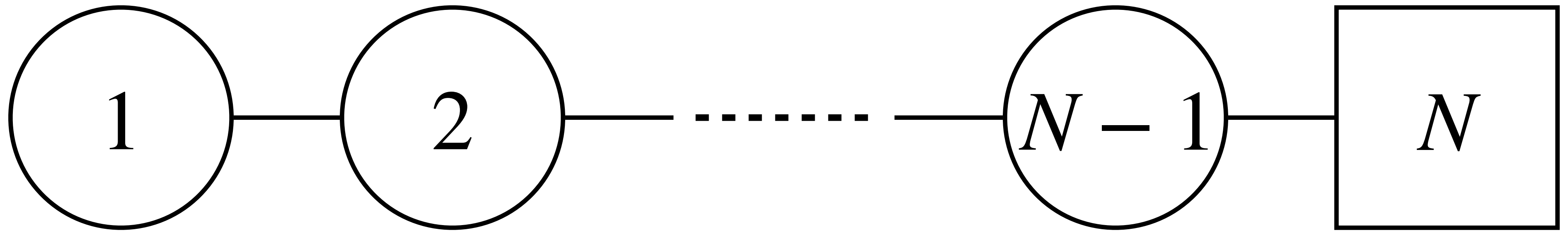
- A QFT duality relates a “theory” to another “theory”.
- Many of them are motivated by string theory, but no systematic construction in the QFT language.
- **Proposal:** dualize a “field” rather than a theory!
- The duality of a “theory” can be obtained by gauging symmetries (and adding interactions if necessary) as in the previous examples.

- Today let's focus on mirror symmetry of 3d $\mathcal{N} = 4$ *linear quiver* gauge theories:
    - Consisting of (bi-)fundamental matter fields
    - Interactions fixed by the $\mathcal{N} = 4$ supersymmetry


- Based on
  - R. Comi, CH, F. Marino, S. Pasquetti, M. Sacchi, "The SL(2, $\mathbb{Z}$) dualization algorithm at work," JHEP 06 (2023) 119, [arXiv:2212.10571].
  - CH, S. Pasquetti, M. Sacchi, "Rethinking mirror symmetry as a local duality on fields," Phys.Rev.D 106 (2022) 10, 105014, [arXiv:2110.11362].
  - L. E. Bottini, CH, S. Pasquetti, M. Sacchi, "4d S-duality wall and SL(2, $\mathbb{Z}$) relations," JHEP 03 (2022) 035, [arXiv:2110.08001].

# Basic Ingredients: S-Wall

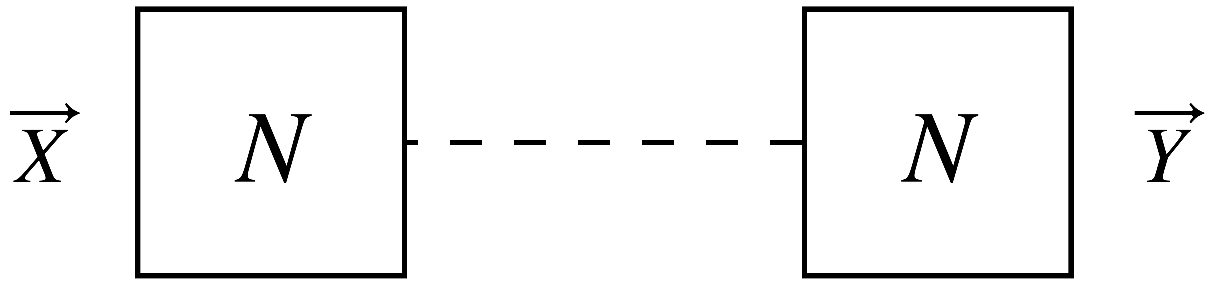
- The S-wall theory: $T[U(N)]$




- The S-duality domain-wall theory of the 4d $\mathcal{N} = 4$ SYM (Giaotto-Witten 08)

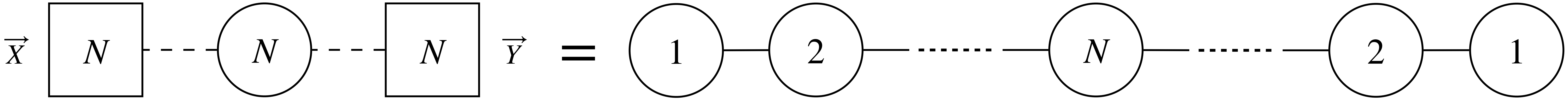
- $U(1)^{N-1}$ topological symmetry + background $U(1)$ coupled via mixed CS + $U(N)_Y$ flavor symmetry

- Enhanced $U(N)_X \times U(N)_Y$ symmetry in the IR



# Basic Ingredients: I-Wall

- The identity-wall theory: two S-walls glued by gauging common $U(N)$




- The partition function proportional to the delta function

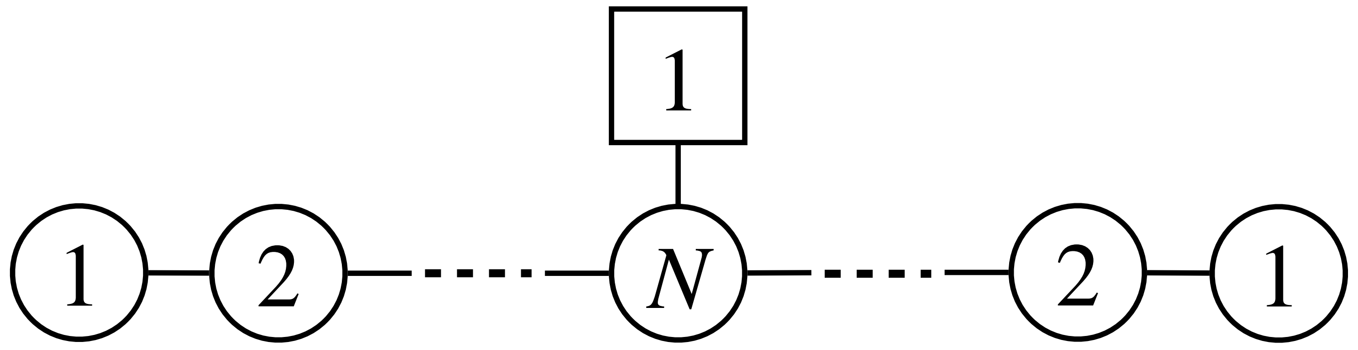
$$\sim \sum_{\sigma \in S_N} \prod_{j=1}^{N} \delta\left(X_j - Y_{\sigma(j)}\right)$$

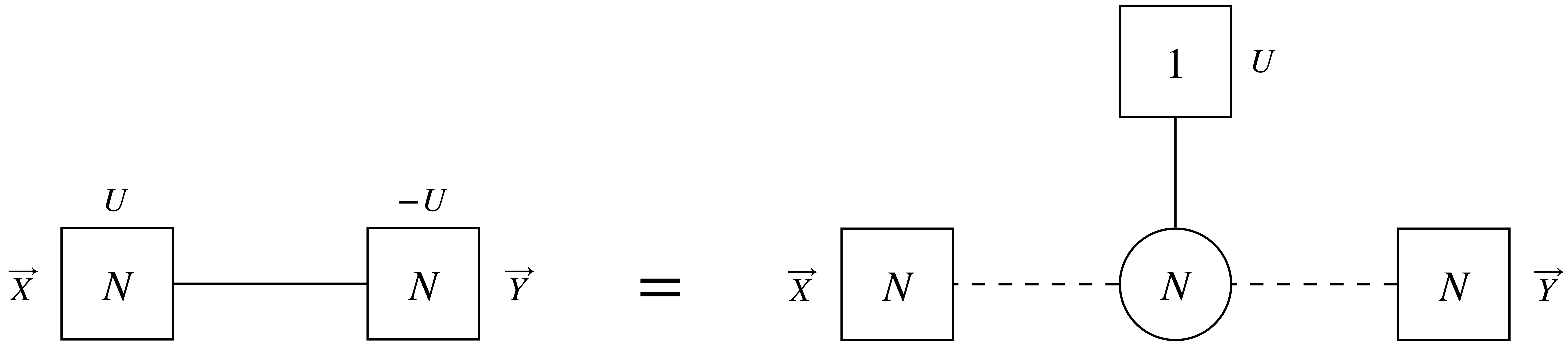
- $\langle \mathfrak{M} \rangle \neq 0$, where $\mathfrak{M}$ is a (monopole) operator in the $U(N)_X \times U(N)_Y$ bifund. rep., breaking

$$U(N)_X \times U(N)_Y \to U(N)_D$$

# Building Blocks of 3D Mirror Symmetry

- Dualization of a $U(N) \times U(N)$ **bifundamental** hypermultiplet

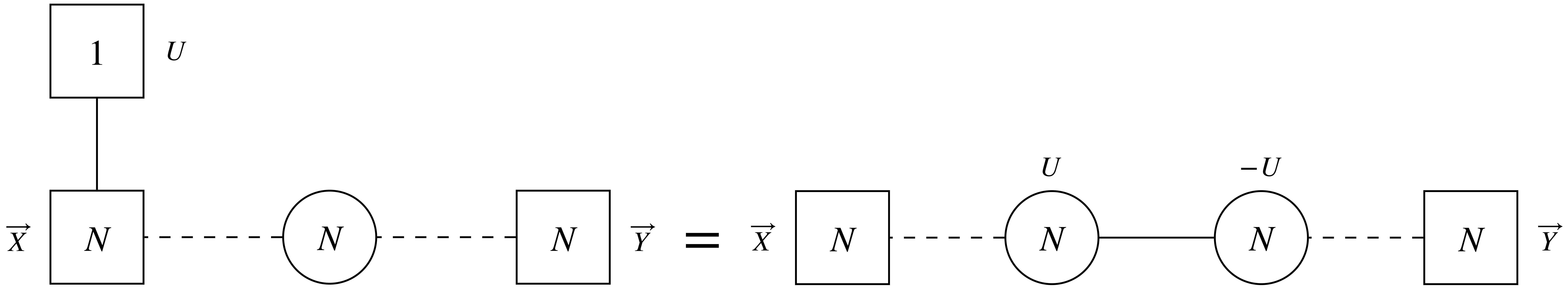



- A bifundamental hypermultiplet -> a fundamental (twisted) hypermultiplet
- Topological $U(1)$ -> flavor $U(1)$
- A QFT version of the S-transformation in IIB string theory, exchanging NS5 and D5

# Building Blocks of 3D Mirror Symmetry

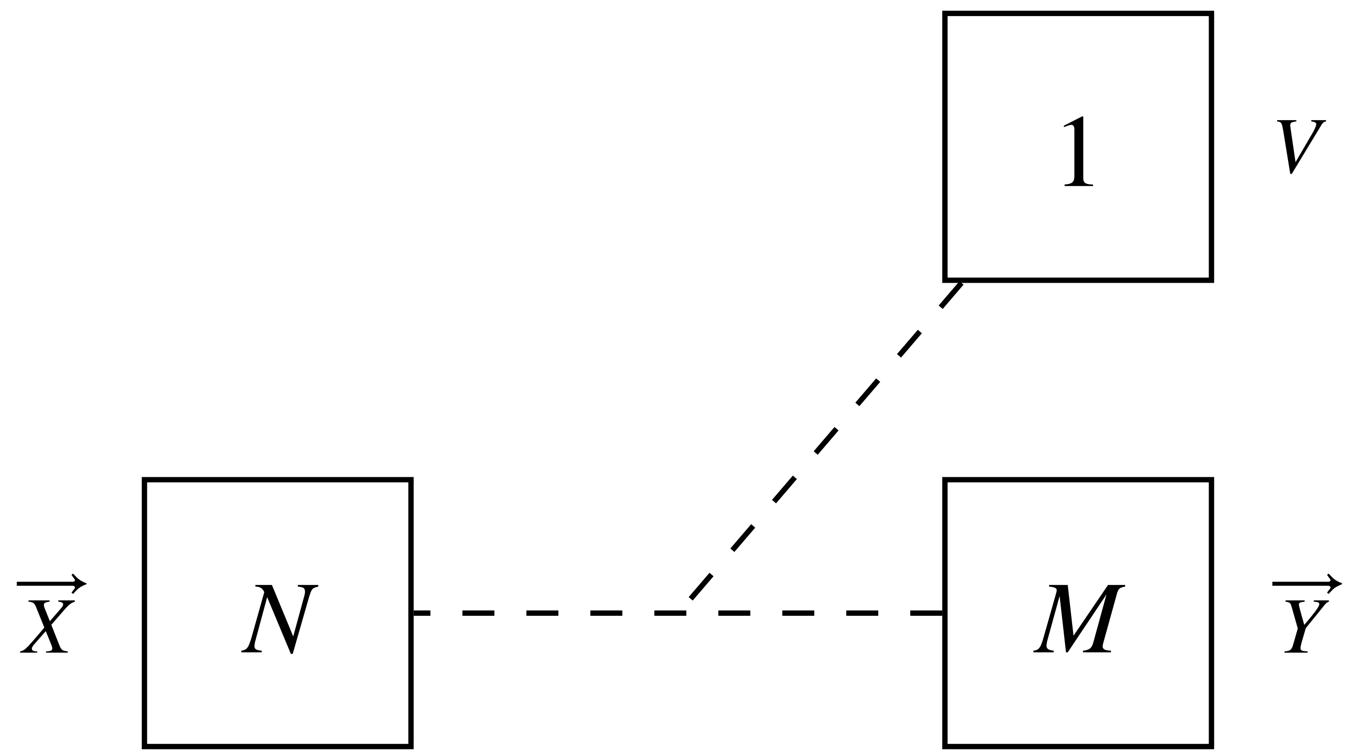
- Dualization of a **fundamental** hypermultiplet (+ an identity wall)




- Obtained by using the I-wall property
- A fundamental hypermultiplet -> a bifundamental (twisted) hypermultiplet
- Flavor U(1) -> topological U(1)

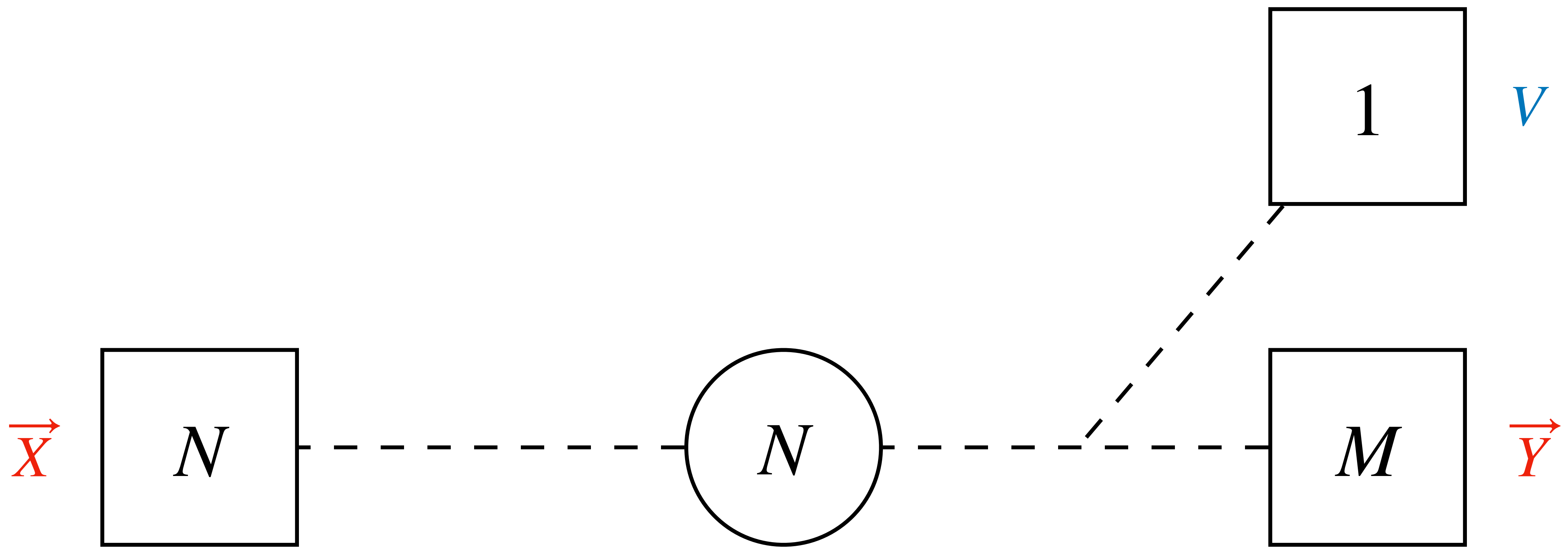
# Generalization: Mass-Deformed S- & I-Walls

- Mass-deformed S-wall: S-wall + mass terms breaking $U(N)_Y \to U(M) \times U(1)$




$$Y_{M+j} = V + \frac{N-M+1-2j}{2}(iQ - 2m_A)\,, \qquad j = 1,\ldots,N-M$$

- Mass-deformed I-wall: S-wall + mass-deformed S-wall



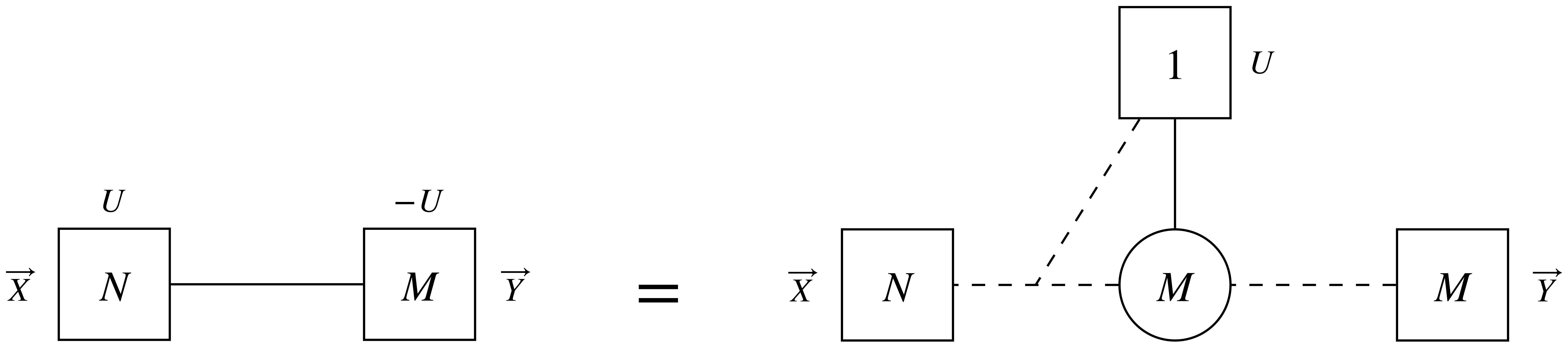

$U(N)_X \times U(M)_Y \times U(1)$
$\to U(M) \times U(1)$

$$\begin{aligned}
X_1 &\longleftrightarrow Y_1 \\
&\vdots \\
X_M &\longleftrightarrow Y_M \\
X_{M+1} &\longleftrightarrow V + (N-M-1)(iQ-2m_A)/2 \\
&\vdots \\
X_N &\longleftrightarrow V - (N-M-1)(iQ-2m_A)/2
\end{aligned}$$

**+ permutations**

# (Generalized) Building Blocks of 3D Mirror

- Dualization of a $U(N) \times U(M)$ **bifundamental** hypermultiplet

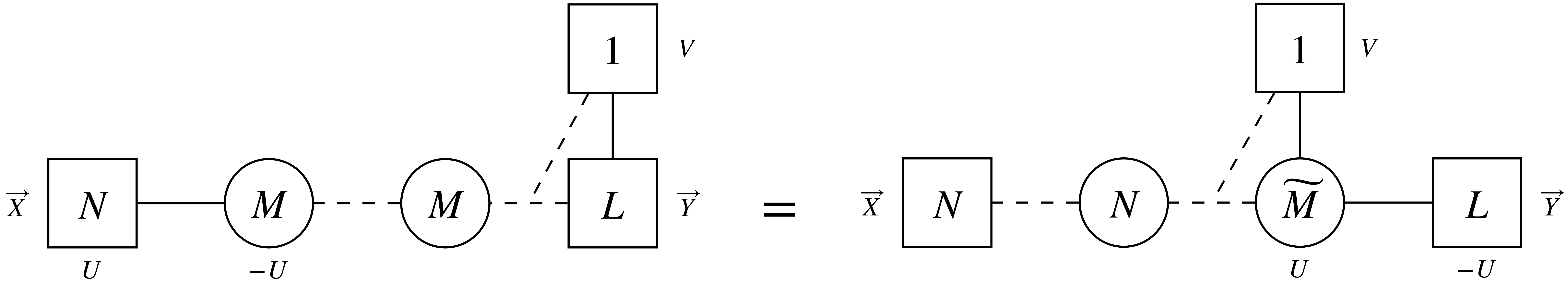



- A bifundamental hypermultiplet between different ranks = a fundamental (twisted) hypermultiplet dualized by the ***mass-deformed S-wall***

# Swap Fundamental and Bifundamental

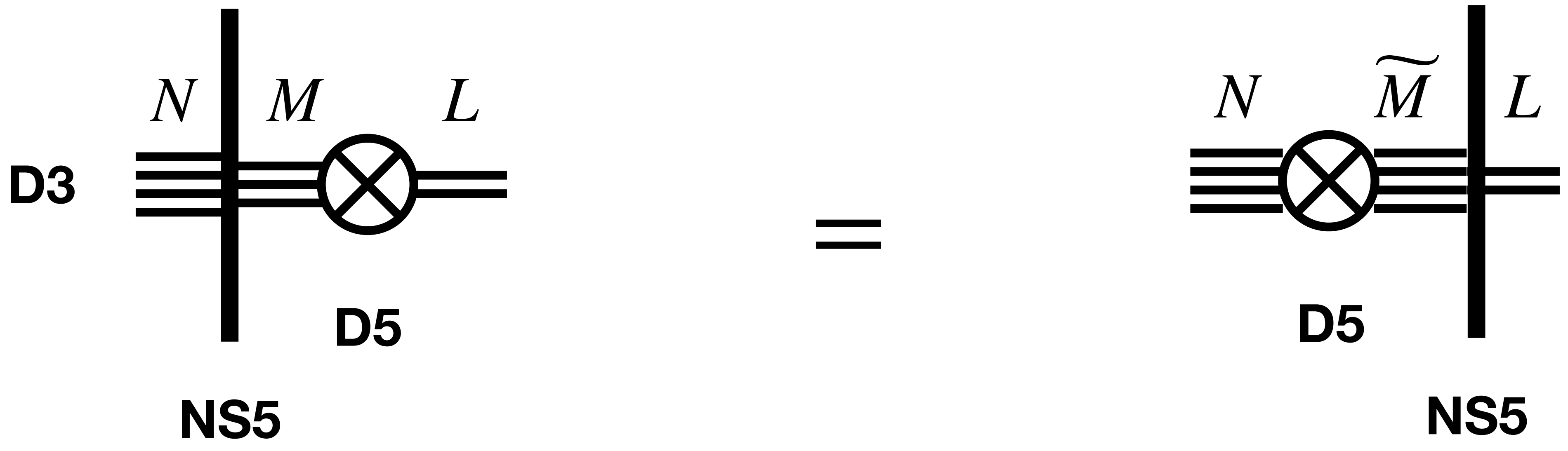
- The mass-deformed S- & I-walls satisfy an interesting property resembling the *Hanany-Witten brane move* in IIB string theory.

- Nothing but Higgs mechanism




$$\widetilde{M} = N + L - M + 1$$

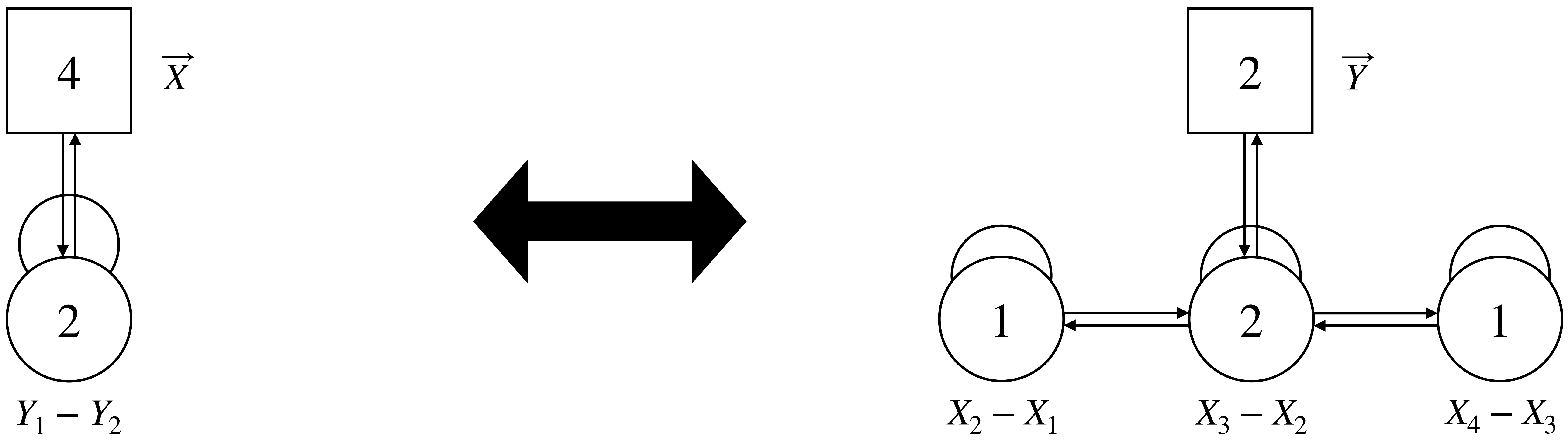
**IIB String Theory**



# Example

# Mirror symmetry of 3d SQCD

- **Mirror symmetry of the 3d $\mathcal{N} = 4$ $U(2)$ theory with 4 flavors**

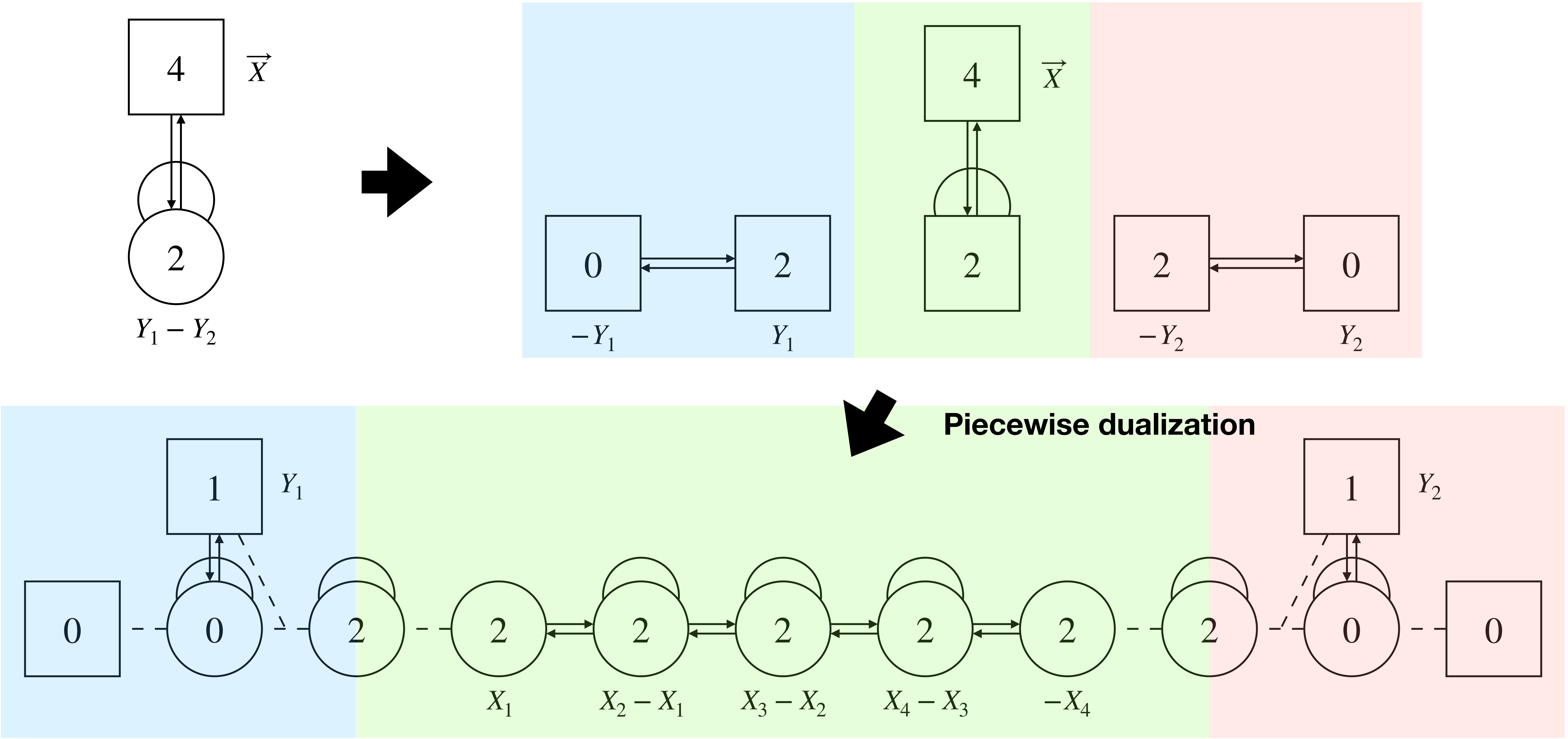



- $U(1) \times SU(4)$ vs $SU(2) \times U(1)^3$
- The global symmetry enhanced to $SU(2) \times SU(4)$ in the IR
- Monopoles and mesons exchanged under the duality

# Mirror symmetry of 3d SQCD

- Dualization Algorithm
  - Chop the quiver into the basic blocks.
  - Dualize each block and glue them back.
  - Carry out the (QFT version of) Hanany-Witten move until I-walls disappear.

# Mirror symmetry of 3d SQCD



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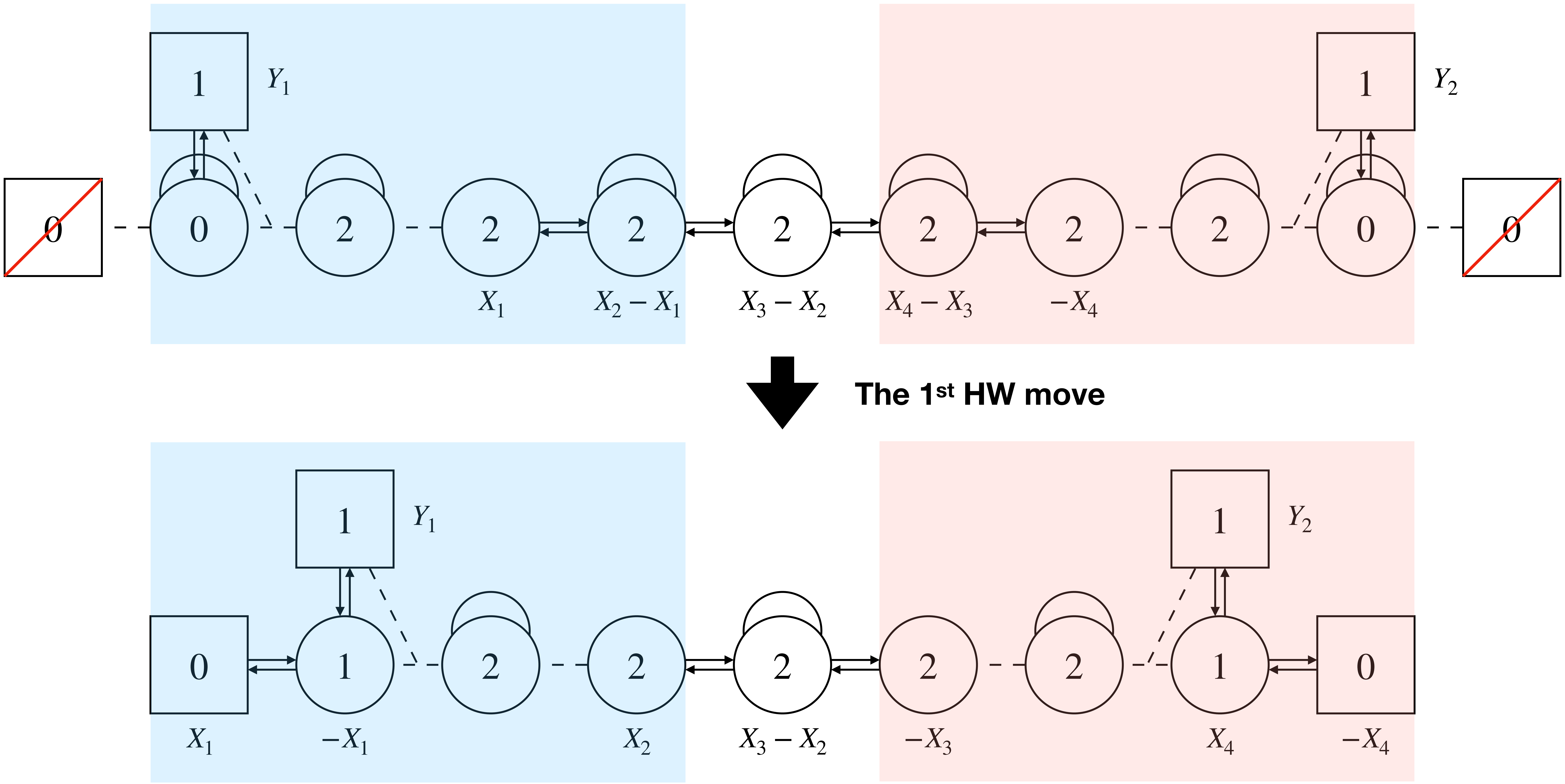

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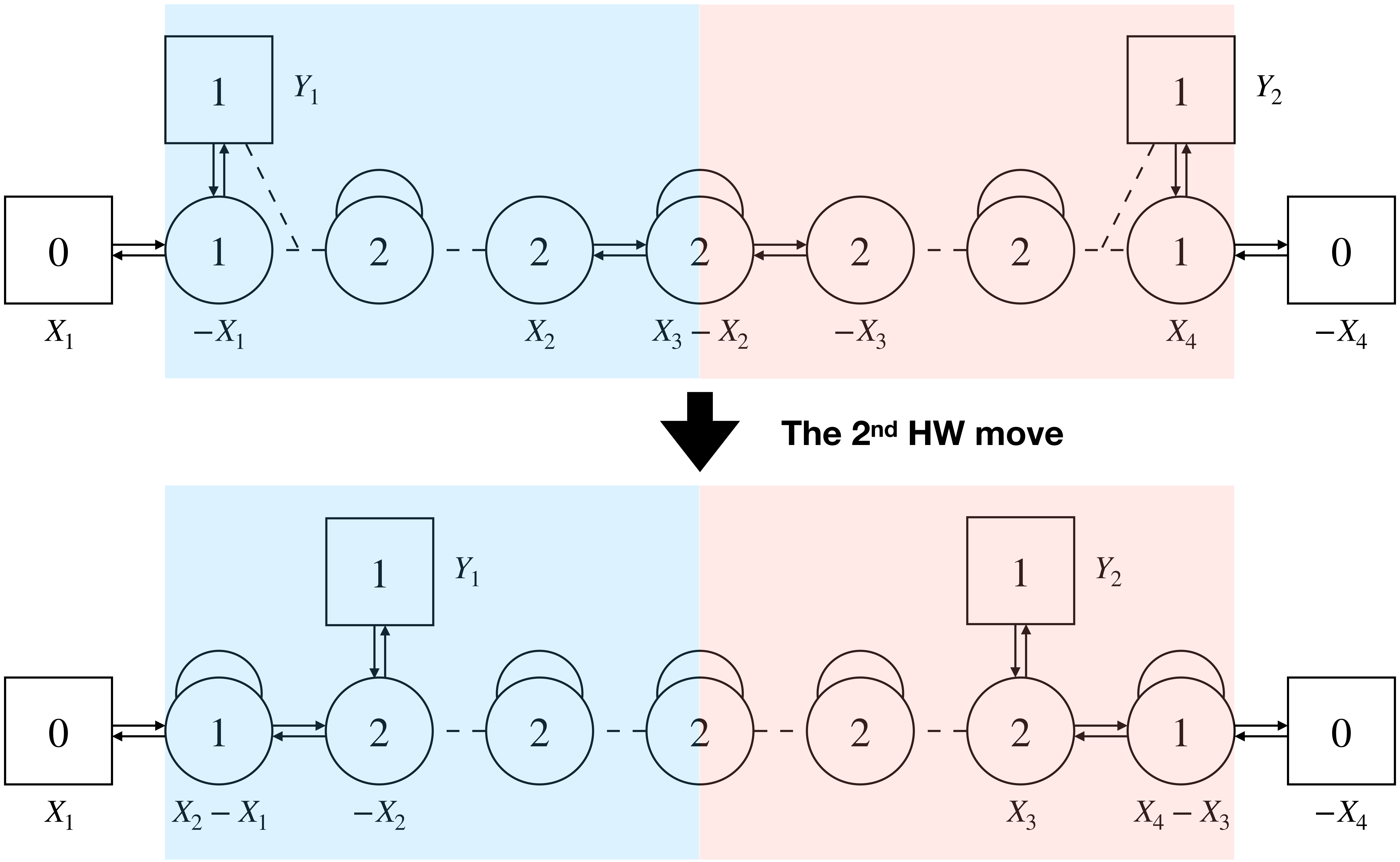

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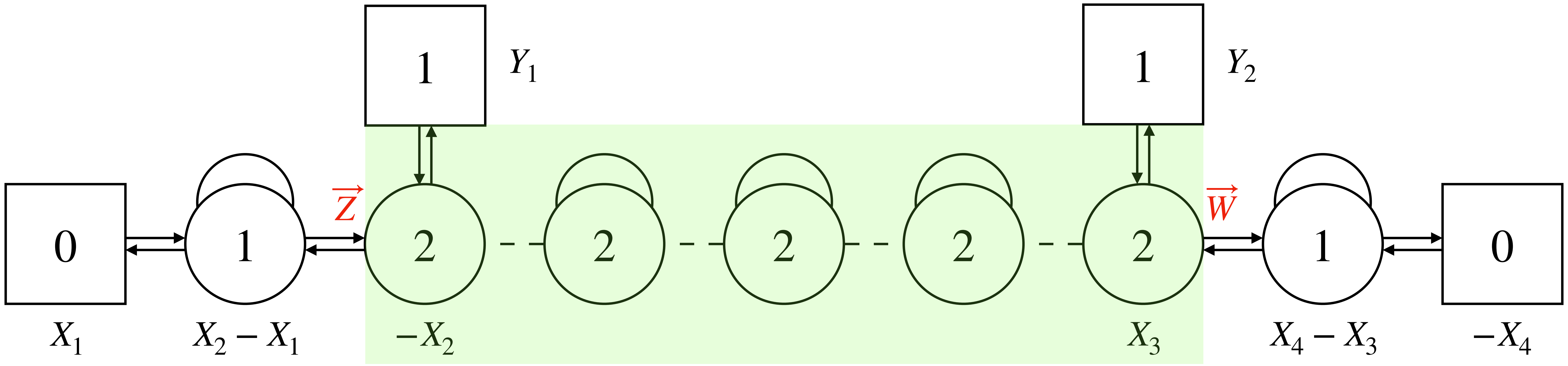
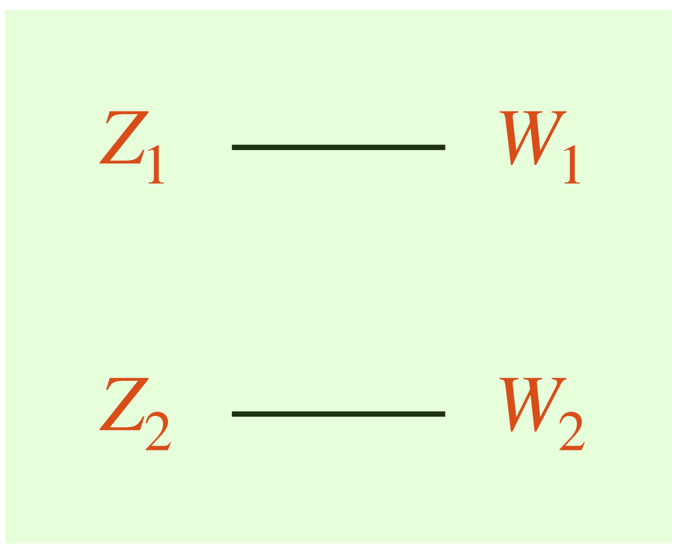
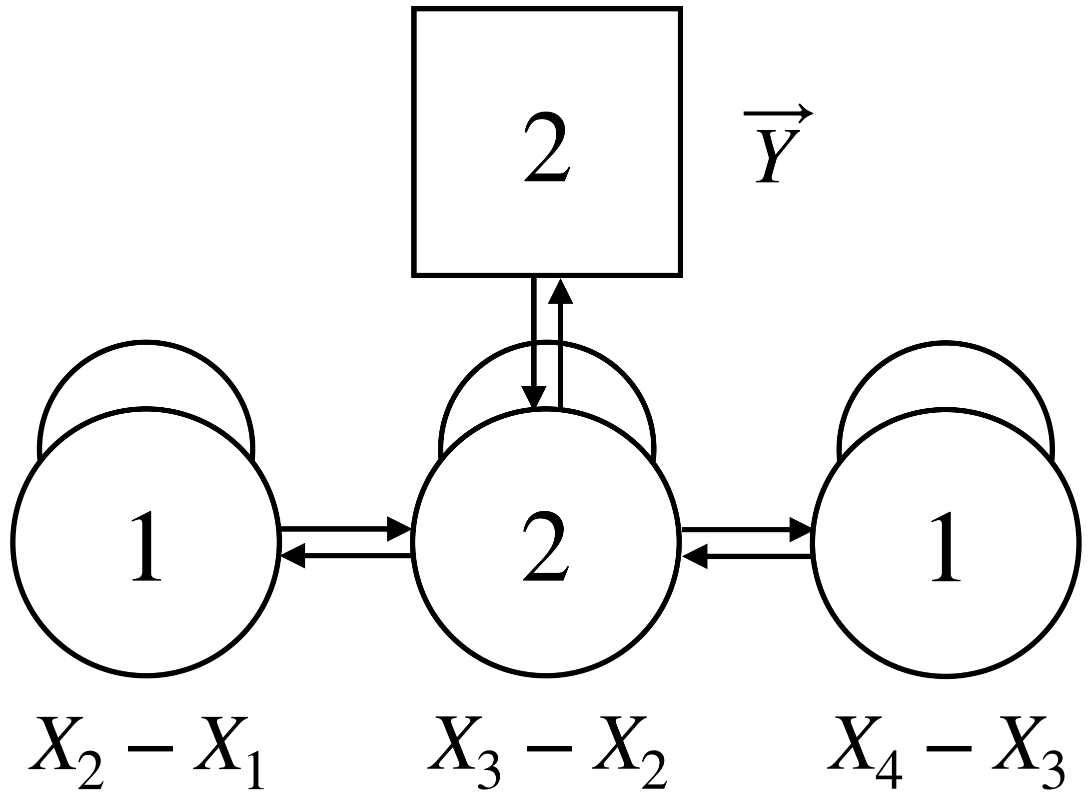



**Collapsing I-walls**




**The permutations belong to the Weyl of U(2)**



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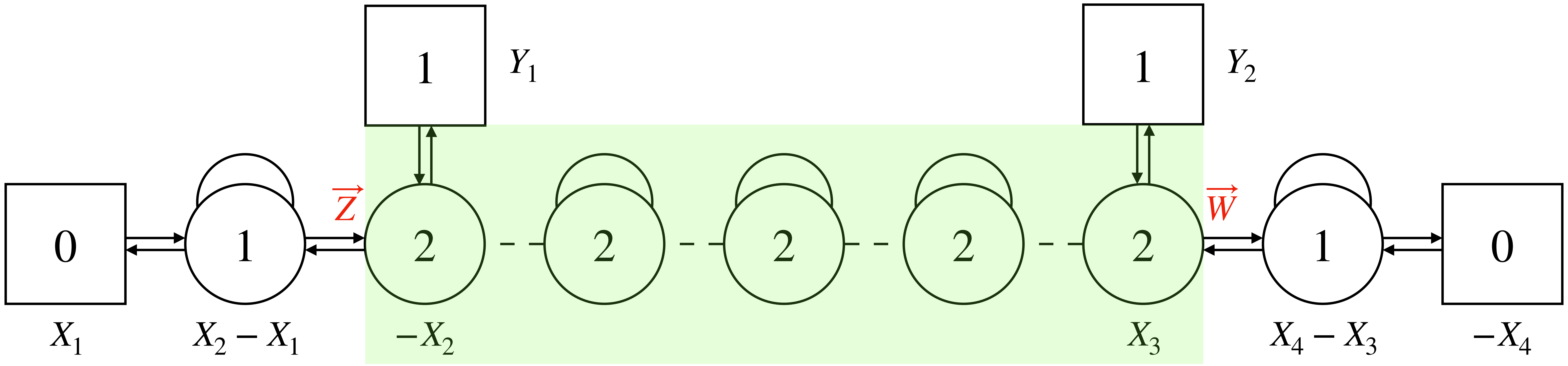


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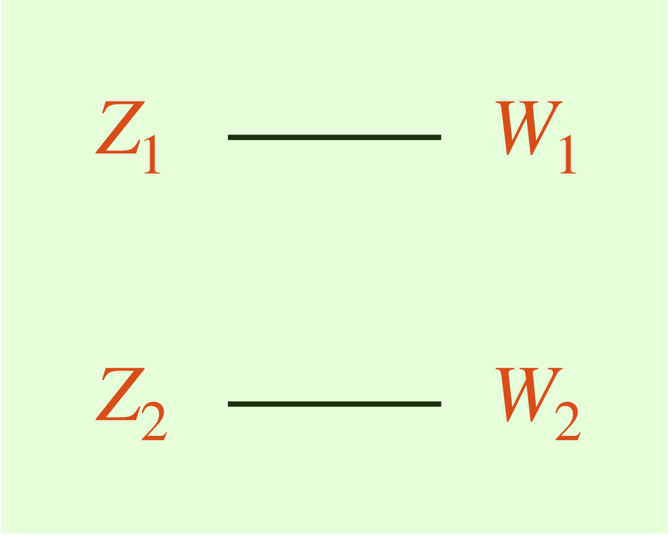
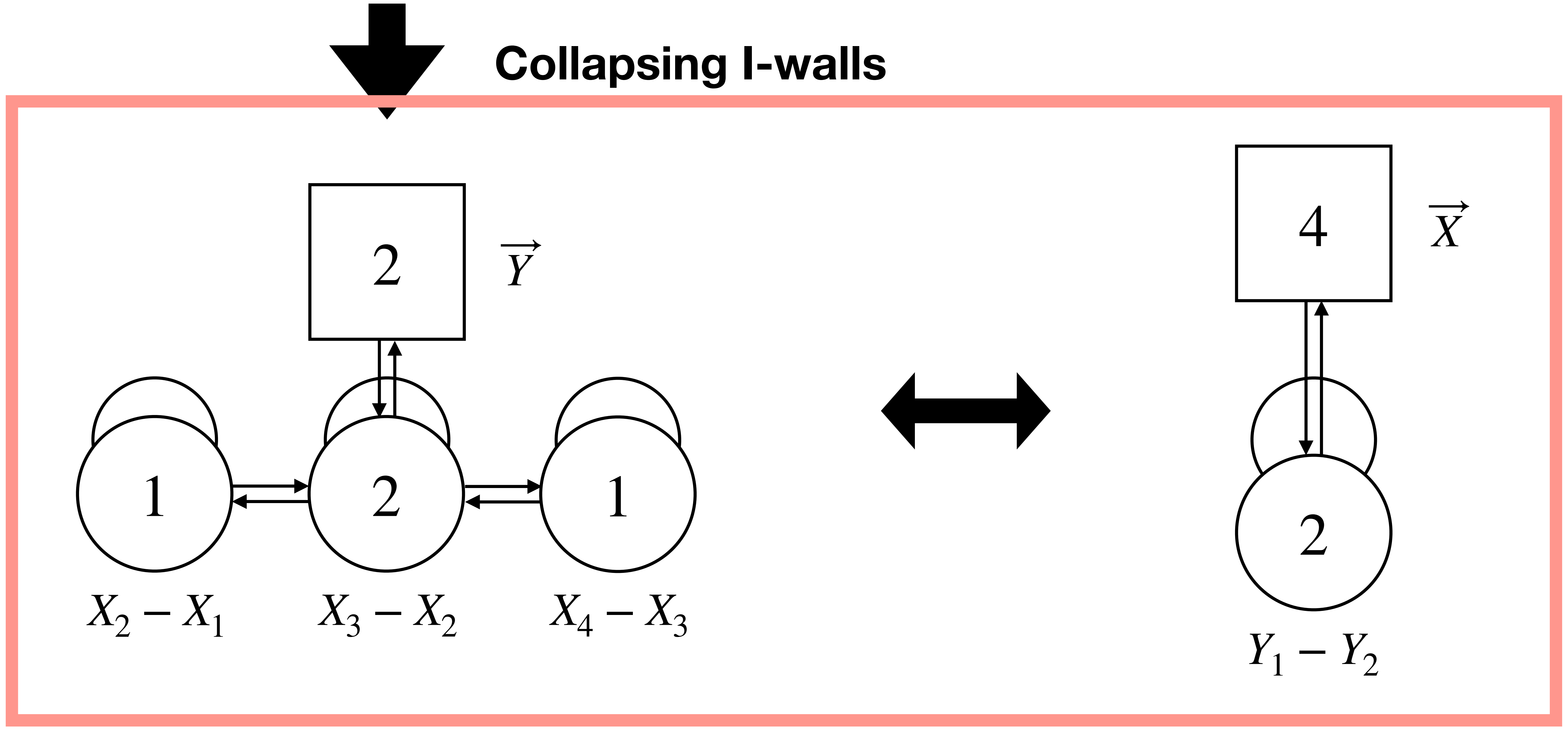



**The permutations belong to the Weyl of U(2)**

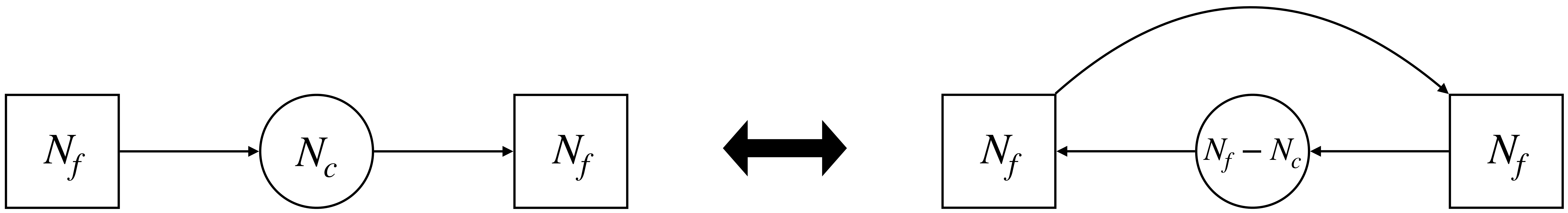



One more thing…

# Where do these building blocks come from?

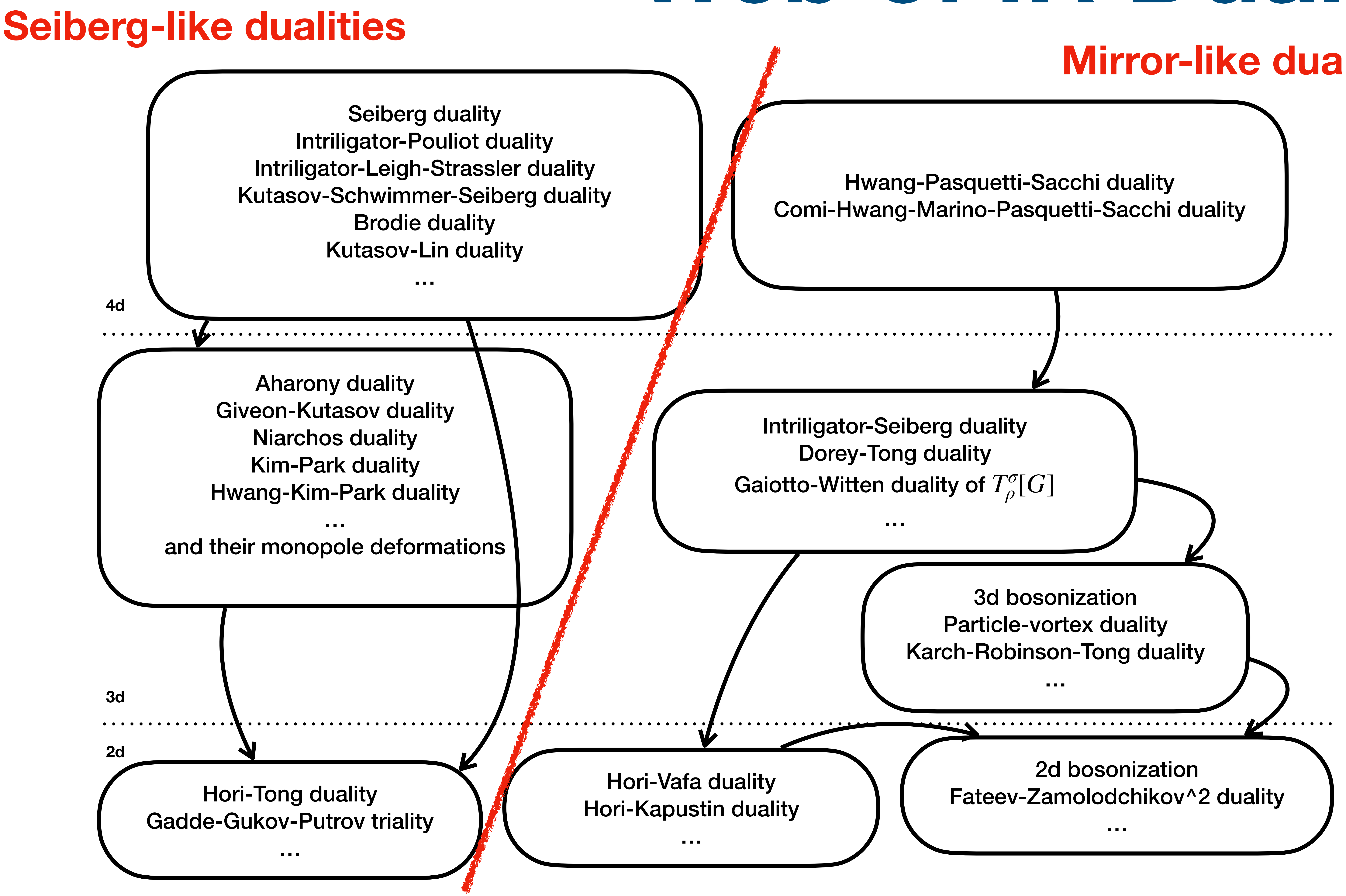
# Where do these building blocks come from?

- The 3d mirror building blocks are derived from the Aharony duality!
- The Aharony duality:

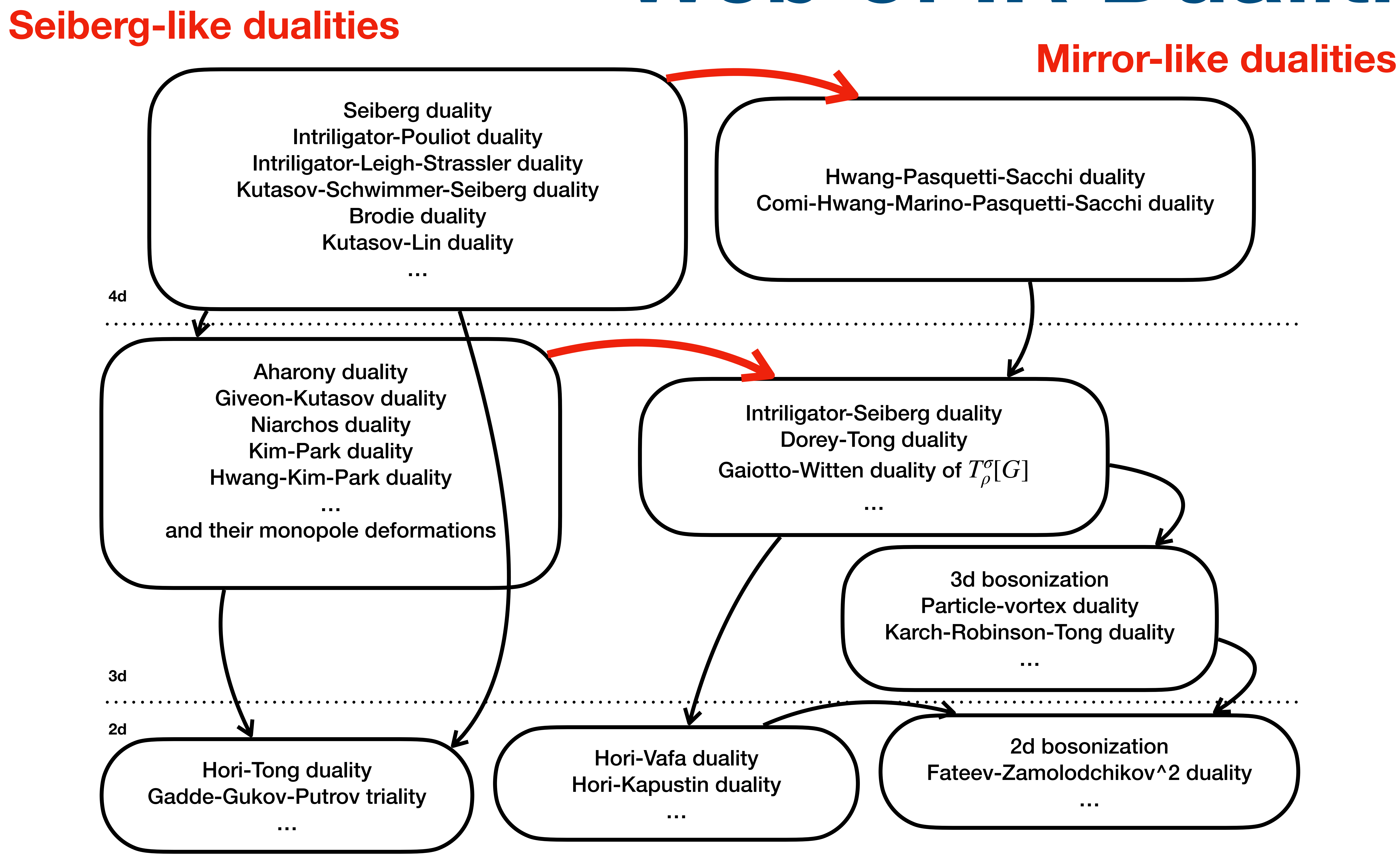



- ***A new bridge between mirror-like dualities and Seiberg-like dualities***
- The same relation in 4d

# Web of IR Dualities



# Web of IR Dualities



# Conclusion

- We have found a new properties of T[U(N)], which can be used to find building blocks of 3d mirror symmetry.
- Generalization
  - 3d $SL(2,\mathbb{Z})$ dualities
  - 4d mirror and $PSL(2,\mathbb{Z})$ dualities
  - Some Seiberg-like dualities
- These building blocks can be derived from the Aharony duality.
- Different looking dualities can be derived from common fundamental dualities.
  -> ***Microscopic mechanism of these dualities would be universal.***
- *Deeper understanding of the duality building blocks from other perspectives? E.g., connection to higher dimensional SCFTs? Holography?*

# Thank you

# Congratulation CTPU!