Thermodynamics with conformal Killing vector in the charged Vaidya metric [1]

in collaboration with Abbas Sherif and Seoktae Koh (Jeju National University)

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Content

Congrats 10 Years of IBS-CTPU-PTC

I joined this center as a YSF on August 1, 2021 and am happy to join this group.

Introduction

Vaidya spacetime

Vaidya spacetime in the ingoing Eddington-Finkelstein coordinate :

$$\mathrm{d}s^2 = -\left(1 - \frac{2m(v)}{r}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

where

$$v = t + r_*, \qquad r_* = \int \left(1 - \frac{2m(v)}{r}\right)\mathrm{d}r$$

- it is a dynamical spacetime, meaning that the metric coefficients are functions of time

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- a solution to the Einstein equation with energy-momentum tensor

$$T_{\alpha\beta} = \frac{1}{4\pi r^2}\frac{\mathrm{d}m(v)}{\mathrm{d}v}l_\alpha l_\beta, \qquad l_\alpha = -\partial_\alpha v$$

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- describes a black hole that absorbing a null dust (a pressureless fluid)
- can be used to model the spacetime around a black hole that is either emitting or absorbing radiation. This can help us to understand how black holes evolve over time.

Conformal Killing Equations

Conformal Killing Equation (CKE)

: defined by a vector ξ_a in a spacetime M provided Lie derivative of the spacetime metric along ξ_a

$$\mathcal{L}_\xi g_{ab} = 2\varphi g_{ab}$$

where φ is a smooth function on M

$$4\varphi = \nabla_a \xi^a$$

- If φ is zero, ξ^a is a true Killing vector (KV)
- If φ is constant, ξ^a is a homothetic conformal Killing vector (HKV)
- If φ is non-constant, ξ^a is a (proper) conformal Killing vector (CKV)

Conformal Transformation

The existence of a conformal Killing vector ξ^a implies the existence of a metric $\bar{g}_{ab}$, which is related to g_{ab} via conformal transformation

$$g_{ab} = \Omega^2 \bar{g}_{ab}, \qquad g^{ab} = \Omega^{-2} \bar{g}^{ab}, \qquad \Omega \neq 0,$$

for smooth Ω.

- A spacetime $\bar{M}$ with metric $\bar{g}_{ab}$, is static, i.e. admits a timelike KV $\bar{\xi}^a$, so that

$$\mathcal{L}_{\bar{\xi}} \bar{g}_{ab} = 0.$$

- The vectors in the static and conformal spacetimes are related as

$$\xi^a = \bar{\xi}^a, \qquad \xi_a = g_{ab}\xi^b = \Omega^2 \bar{g}_{ab}\xi^b = \Omega^2 \bar{\xi}_a$$

where $\bar{\xi}^a$ and $\bar{\xi}_a$ are Killing vectors associated with $\bar{g}_{ab}$.

Motivation

Linear Vaidya spacetime with CKV [2] [3]

$$\mathrm{d}s^2 = -\left(1 - \frac{2m(v)}{r}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

where

$$m(v) = M + \mu(v - v_0)$$

- a mass function is linear in v
- in a static limit $\mu \to 0$, Schwarzschild spacetime is recovered
- Conformal Killing Vectors (CKV)

$$\xi^a = \frac{m(v)}{M}\delta^a_v + \frac{r\mu}{M}\delta^a_r$$

- surface gravity for an observer following the trajectory of the conformal Killing vector in the dynamical Vaidya spacetime

$$\kappa = \Omega c\kappa_1 = \frac{r_0^{1/2}}{\sqrt{m^2 r_0 - 2m^3 - 2\mu m r_0^2}} \frac{2\mu\sqrt{1-16\mu}}{(1-\sqrt{1-16\mu})}$$

where κ_1 is the conformally invariant surface gravity and r_0 is the observer's location

[2] Alex B. Nielsen, Galaxies 2014, 2(1), 62-71

[3] Alex B. Nielsen and Andrey A. Shoom, Class. Quant. Grav. **35** (2018)

Charged Vaidya spacetime solution

Charged Vaidya spacetime solution[4]

$$\mathrm{d}s^2 = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$
$$A = \frac{1}{\sqrt{4\pi}}\left(\frac{q(v)}{r} - \frac{q(v)}{r_0}\right)\mathrm{d}v$$

- equations of motion

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G(T^{(EM)}_{\mu\nu} + T^{(ex)}_{\mu\nu}), \qquad \nabla^\mu F_{\mu\nu} = j_\nu,$$

- the energy-momentum tensor and the current are required to be

$$T^{(EM)\mu}{}_{\nu} = \frac{q(v)^2}{8\pi r^4}\mathrm{diag}(-1,-1,1,1),$$
$$T^{(ex)}_{vv} = \frac{2}{r^3}\left(r\dot{m}(v) - q(v)\dot{q}(v)\right), \qquad j_v = -\frac{\dot{q}(v)}{2\sqrt{\pi}r^2}.$$

- null energy condition $(T_{ab}k^a k^b \geq 0)$ requires

$$r\dot{m}(v) - q(v)\dot{q}(v) \geq 0, \qquad \left(T^{(EM)}_{\mu\nu}k^\mu k^\nu = 0\right)$$

[4]W.B. Bonnor and P.C. Vaidya, “Spherically symmetric radiation of charge in Einstein-Maxwell theory", Gen. Relativ. Grav., vol. 1, no. 2, p. 127, 1970.

Conformal Killing vectors in the charged Vaidya spacetime

CKV in the charged Vaidya spacetime

$$\mathcal{L}_\xi g_{ab} = 2\varphi g_{ab}, \qquad \mathrm{d}s^2 = -f(v,r)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

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- Ansatz for conformal Killing vector

$$\xi^a = \{h(v,r), \bar{h}(v,r), 0, 0\}, \qquad \xi_a = \{-f(v,r)h(v,r) + \bar{h}(v,r), h, 0, 0\}.$$

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- the CKE gives the following set of equations

$$0 = 2h' = 0, \qquad 0 = \frac{2\bar{h}}{r} + 2fh' + \bar{h}' + \dot{h},$$

$$0 = -\bar{h}f' - h\dot{f} + f\left(-\frac{\bar{h}}{r} + \frac{3}{2}\bar{h}' - \frac{1}{2}\dot{h}\right) + 2\dot{\bar{h}}, \qquad 0 = 2\bar{h} - r(\bar{h}' + \dot{h}),$$

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- the solution should satisfy

$$\ddot{h}(v) = 0, \qquad h(v,r) = h(v) = c_2 m(v) \text{ or } h(v) = c_3 q(v), \qquad \bar{h}(v,r) = r\dot{h}(v,r),$$

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- the solution to the CKE

$$c_2 = \frac{1}{M}, \; c_3 = \frac{1}{Q}, \; \dot{h} = rc_2\dot{\mu} = \frac{r\mu}{M}, \; \rightarrow \; \xi^a = \left\{\frac{m(v)}{M}, \frac{r\mu}{M}, 0, 0\right\}$$

Linear charged Vaidya spacetime with CKV

Charged Vaidya spacetime

$$\mathrm{d}s^2 = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

$$A = \frac{1}{\sqrt{4\pi}}\left(\frac{q(v)}{r} - \frac{q(v)}{r_0}\right)\mathrm{d}v$$

CKE requires

$$q(v) = \frac{Q}{M}m(v), \qquad m(v) = M + \mu(v - v_0)$$

CKV is

$$\xi^a = \left\{\frac{m(v)}{M}, \frac{r\mu}{M}, 0, 0\right\}$$

Homothetic conformal Killing vector

$$\mathcal{L}_\xi g_{ab} = 2\varphi g_{ab}, \qquad \varphi = \frac{1}{4}\nabla_a\xi^a = \frac{\mu}{M}$$

Conformal mapping to Static Spacetime

$$g_{ab} = \Omega(v,r)^2 \bar{g}_{ab}$$

Conformal mappling to Static Spacetime

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

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- static spacetime $\bar{M}$ associated with $\bar{g}_{ab}$

$$d\bar{s}^2 = \bar{g}_{ab}d\bar{x}^a d\bar{x}^b = \frac{1}{\Omega(v,r)^2}\left[-\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)dv^2 + 2dvdr + r^2 d\Omega^2\right]$$

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- solution of $\Omega(v,r)$

$$\Omega(v,r)^2 = \left[\frac{m(v)}{M}c_4\left(\frac{r}{m(v)}\right)\right]^2$$

Conformal mapping to Static Spacetime

$$\mathrm{d}\bar{s}^2 = \bar{g}_{ab}\mathrm{d}\bar{x}^a\mathrm{d}\bar{x}^b = \frac{1}{\Omega(v,r)^2}\Big[-f(v,r)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2\Big],$$

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where c_4 is a smooth function and not zero except $r = 0$.

1. Let us redefine the coordinate as : (v,r) to (V,y)

$$y = M\frac{r}{m(v)}, \qquad dv = \frac{m(v)}{M}dV$$

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where

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2. Let us redefine the coordinate as : (V, y) to (V, R)

$$R = \frac{y}{c_4(y)},$$

$$d\bar{s}^2 = -\frac{MF(y) - 2\mu y}{M(c_4)^2} dV^2 + \frac{2}{(c_4 - yc_4')} dR dV + R^2 d\Omega_2.$$

Conformal mapping to Static Spacetime : case 1

Remind you the dynamical spacetime that we started with

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

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[5] V.V. Kiseleve, "Quintessence and black holes", Class. Quant. Grav., vol. 20, pp. 1187-1198, 2003.

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$$c_4(y) = 1, \qquad \Omega(v,y)^2 = \left[\frac{m(v)}{M}c_4(y)\right]^2 = \left[\frac{m(v)}{M}\right]^2$$

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Then

$$\mathrm{d}\bar{s}^2 = -\tilde{F}(R)\mathrm{d}V^2 + 2\mathrm{d}R\mathrm{d}V + R^2\mathrm{d}\Omega_2, \qquad \tilde{F}(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{2\mu}{M}R.$$

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$$c_4(y) = 1, \qquad \Omega(v,y)^2 = \left[\frac{m(v)}{M}c_4(y)\right]^2 = \left[\frac{m(v)}{M}\right]^2$$

Then

$$\mathrm{d}\bar{s}^2 = -\tilde{F}(R)\mathrm{d}V^2 + 2\mathrm{d}R\mathrm{d}V + R^2\mathrm{d}\Omega_2, \qquad \tilde{F}(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{2\mu}{M}R.$$

- this spacetime is neither asymptotically flat nor asymptotically de Sitter.

[5] V.V. Kiseleve, “Quintessence and black holes", Class. Quant. Grav., vol. 20, pp. 1187-1198, 2003.

Conformal mapping to Static Spacetime : case 1

Remind you the dynamical spacetime that we started with

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

$$g_{ab} = \Omega(v,r)^2\bar{g}_{ab}$$

- redefinition of the coordinates : $(v,r) \to (V,y) \to (V,R)$

$$\mathrm{d}\bar{s}^2 = -\frac{MF(y) - 2\mu y}{M(c_4)^2}\mathrm{d}V^2 + \frac{2}{(c_4 - yc_4')}\mathrm{d}R\mathrm{d}V + R^2\mathrm{d}\Omega_2, \quad R = \frac{y}{c_4(y)}$$

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- this spacetime is neither asymptotically flat nor asymptotically de Sitter.
- one of black hole solutions surrounded by the quintessence matter [5]

[5] V.V. Kiseleve, “Quintessence and black holes", Class. Quant. Grav., vol. 20, pp. 1187-1198, 2003.

Conformal mapping to Static Spacetime : case 2

Remind you the dynamical spacetime that we started with

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

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Conformal mapping to Static Spacetime : case 2

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$$c_4(y) = \sqrt{\frac{2\mu}{M}y}, \qquad \Omega(v,y)^2 = \left[\frac{m(v)}{M}c_4(y)\right]^2 = \left[\frac{m(v)}{M}\sqrt{\frac{2\mu}{M}y}\right]^2$$

this choice makes the norm of Killing vectors to be normalized at infinity.

Conformal mapping to Static Spacetime : case 2

Remind you the dynamical spacetime that we started with

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

$$g_{ab} = \Omega(v,r)^2\bar{g}_{ab}$$

- redefinition of the coordinates : $(v,r) \to (V,y) \to (V,R)$

$$\mathrm{d}\bar{s}^2 = -\frac{MF(y) - 2\mu y}{M(c_4)^2}\mathrm{d}V^2 + \frac{2}{(c_4 - yc_4')}\mathrm{d}R\mathrm{d}V + R^2\mathrm{d}\Omega_2, \quad R = \frac{y}{c_4(y)}$$

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$$\mathrm{d}\bar{s}^2 = -\tilde{F}(R)\mathrm{d}V^2 + \frac{M}{R\mu}\mathrm{d}R\mathrm{d}V + R^2\mathrm{d}\Omega_2, \qquad \tilde{F}(R) = 1 - \frac{M^2}{4\mu^2R^2} + \frac{M^4}{4\mu^3R^4} - \frac{M^4Q^2}{16\mu^4R^6}$$

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$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

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- This metric is not asymptotically flat. This implies that the asymptotic flatness, the linear charged Vaidya spacetime with bounded mass function, is not recovered in the corresponding static spacetime under the conformal map.

Horizons

Classification : χ and $\tilde{\chi}$

σ_{AB} is pull-back of σ_{ab}

$$\sigma_{ab} = g_{ab} + l_a n_b + n_a l_b, \qquad \mathrm{d}s_{\Sigma}^2 = \sigma_{AB}\mathrm{d}x^A \mathrm{d}x^B = r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \mathrm{d}\phi^2$$

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$$\chi = l_{a;b}\sigma^{ab} \ (l_a : \text{outgoing null vector}), \quad \tilde{\chi} = n_{a;b}\sigma^{ab} \ (n_a : \text{ingoing null vector}) \tag{1}$$

- untrapped surfaces : $\chi > 0, \qquad \tilde{\chi} < 0$
- trapped surfaces : $\chi < 0, \qquad \tilde{\chi} < 0$
- Marginally Trapped Surfaces (MTS) : $\chi = 0, \qquad \tilde{\chi} < 0$

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$$C = \frac{\mathcal{L}_l \chi}{\mathcal{L}_n \chi} \tag{2}$$

- isolated horizons : $C = 0$
- dynamical horizons : $C > 0$
- timelike membrane : $C < 0$

$f(v,r)=0$ horizons

$$ds^2 = -f(v,r)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2, \qquad f(v,r) = \left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right),$$

$$f(v,r) = 0 \ \rightarrow \ r_\pm = \frac{m(v)}{M}\left(M \pm \sqrt{M^2 - Q^2}\right)$$

Outgoing: $\begin{cases} \ell^a & = (1, \frac{f}{2}, 0, 0) \\ \ell_a & = (-\frac{f}{2}, 1, 0, 0), \end{cases}$ Ingoing: $\begin{cases} n^a & = (0, -1, 0, 0) \\ n_a & = (-1, 0, 0, 0). \end{cases}$

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$f(v,r)=0$ horizons

$$ds^2 = -f(v,r)dv^2 + 2dvdr + r^2 d\Omega^2, \qquad f(v,r) = \left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right),$$

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$$\mathcal{L}_\ell \chi|_{r_\pm} = \left.\frac{\dot{f}}{r}\right|_{r_\pm} = -\frac{2\mu m(v)}{M^2 r_\pm^3}\left[\left(M^2-Q^2\right) \pm M\sqrt{M^2-Q^2}\right],$$

$$\mathcal{L}_n \chi|_{r_\pm} = -\left.\frac{f'}{r}\right|_{r_\pm} = -\frac{2m(v)^2}{M^2 r_\pm^4}\left[\left(M^2-Q^2\right) \pm M\sqrt{M^2-Q^2}\right]$$

$$C = \frac{\mu r_\pm}{m(v)} = \frac{\mu}{M}\left(M \pm \sqrt{M^2-Q^2}\right) > 0 \qquad \rightarrow \textbf{dynamical horizon}$$

conformal Killing horizons

$$\xi^\alpha \xi_\alpha = \frac{m(v)\left(-m(v)\left(q(v)^2 + r^2\right) + 2rm(v)^2 + 2\mu r^3\right)}{M^2 r^2} = 0,$$

$$r_1 = \frac{m(v)}{6\mu}\left(1 + \sqrt[3]{b} + \frac{1-12\mu}{\sqrt[3]{b}}\right),$$

$$r_2 = \frac{m(v)}{6\mu}\left(1 - \frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{b}}{2} - \frac{2(1-12\mu)}{\left(1 - i\sqrt{3}\right)\sqrt[3]{b}}\right),$$

$$r_3 = \frac{m(v)}{6\mu}\left(1 - \frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{b}}{2} - \frac{2(1-12\mu)}{\left(1 + i\sqrt{3}\right)\sqrt[3]{b}}\right),$$

$$b = 1 - 18\mu + \frac{6\mu}{M^2}\left(9\mu Q^2 + i\sqrt{3}\sqrt{(1-16\mu)M^4 + (18\mu - 1)M^2 Q^2 - 27\mu^2 Q^4}\right).$$

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In order to have distinct three roots

$$0 < \mu < \frac{M\left(-8M^3 + 9MQ^2 + \left(4M^2 - 3Q^2\right)^{3/2}\right)}{27Q^4},$$

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$$Q=0: 0<\mu<\frac{1}{16}, \qquad Q=\sqrt{\frac{4}{3}}M: 0<\mu<\frac{1}{12}, \qquad Q=M: 0<\mu<\frac{2}{27}$$

conformal Killing horizons : small μ expansion

$$r_1 \sim \frac{M}{2\mu} + \frac{1}{2}(-4M + v - v_0) + 2\mu \left(\frac{Q^2}{M} - 4M - v + v_0 \right) + \cdots ,$$

$$r_2 \sim M - \sqrt{M^2 - Q^2} + \mu \left(\frac{\left(\sqrt{M^2 - Q^2} - M\right)\left(\sqrt{M^2 - Q^2} - 3M - v + v_0\right)}{M} - \frac{Q^2}{\sqrt{M^2 - Q^2}} \right) + \cdots ,$$

$$r_3 \sim M + \sqrt{M^2 - Q^2} + \mu \left(\frac{\left(\sqrt{M^2 - Q^2} + M\right)\left(\sqrt{M^2 - Q^2} + 3M + v - v_0\right)}{M} + \frac{Q^2}{\sqrt{M^2 - Q^2}} \right) + \cdots$$

CKHs maps to $\tilde{F}(R) = 0$ horizons in static spacetime

- dynamical spacetime : linear charged Vaidya spacetime

$$\mathrm{d}s^2 = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

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- r_i and R_i are related as

$$R_1 = \frac{M}{m(v)} r_1, \qquad R_2 = \frac{M}{m(v)} r_2, \qquad R_3 = \frac{M}{m(v)} r_3$$

Surface gravity [6]

$$\nabla_a(\xi^b\xi_b) = -2\kappa_1\xi_a,$$
$$\xi^b\nabla_b\xi^a = \kappa_2\xi^a,$$
$$(\kappa_3)^2 = -\frac{1}{2}(\nabla^a\xi^b)(\nabla_{[a}\xi_{b]}).$$

These are identical for Killing vector case, but for conformal Killing vectors, they are related as

$$\kappa_1 = \kappa_2 - 2\varphi = \kappa_3 - \varphi.$$

[6] T. Jacobson and G. Kang, “Conformal invariance of black hole temperature," Class. Quant. Grav., vol. 10, pp. L201-L206, 1993

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$$\nabla_c\nabla_a\xi_b = \xi^d R_{dcab} + g_{ab}\nabla_c\phi + g_{bc}\nabla_a\phi - g_{ac}\nabla_b\phi, \;\rightarrow\; 2\kappa_3\xi^c\nabla_c\kappa_1 = 0$$
which implies that κ_1 is constant along each of the null CKV curve

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- Then κ_1 is the one that is identified to Hawking temperature in a static spacetime
$$T = \frac{\kappa_1}{2\pi}$$

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Expansion of surface gravity on small μ

$$
\begin{aligned}
\kappa_1|_{r_1} &= -\frac{1}{M}\mu + \frac{4}{M}\mu^2 + \cdots\,, \\
\kappa_1|_{r_2} &= \frac{-2M^3 + 2MQ^2 - \left(2M^2 - Q^2\right)\sqrt{M^2 - Q^2}}{Q^4} \\
&\quad + \frac{-4\sqrt{M^2 - Q^2} - M}{M\sqrt{M^2 - Q^2}}\mu - \left(\frac{2}{M} - \frac{4M^4 - 6M^2Q^2 + 3Q^4}{2M^2\left(M^2 - Q^2\right)^{3/2}}\right)\mu^2 + \cdots\,, \\
\kappa_1|_{r_3} &= \frac{-2M^3 + 2MQ^2 + \left(2M^2 - Q^2\right)\sqrt{M^2 - Q^2}}{Q^4} \\
&\quad + \frac{M - 4\sqrt{M^2 - Q^2}}{M\sqrt{M^2 - Q^2}}\mu - \left(\frac{2}{M} + \frac{4M^4 - 6M^2Q^2 + 3Q^4}{2M^2\left(M^2 - Q^2\right)^{3/2}}\right)\mu^2 + \cdots
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Hawking temperature in static spacetime

The static spacetime when $c_4 = 1$:

$$\mathrm{d}\bar{s}^2 = -\tilde{F}(R)\mathrm{d}V^2 + 2\mathrm{d}R\mathrm{d}V + R^2\mathrm{d}\Omega_2$$

$$\tilde{F}(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{2\mu}{M}R.$$

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Hawking temperatures are computed as

$$T_i = \frac{\tilde{F}'(R)}{4\pi}\bigg|_{R=R_i} = \frac{\kappa_i}{2\pi}$$

Null Energy Condition

The null energy condition requires

$$r\dot{m}(v) - q(v)\dot{q}(v) \geq 0$$

[7] A.Ori, “Charged null fluid and the weak energy condition", Class. Quantum Grav., vol. 8, no. 8, p. 1559, 1991.

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  - To resolve this problem, there have been studies to remove the unphysical region and to glue another spacetime. [7]

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The linear charge Vaidya spacetime is

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$$\kappa_1|_{r_1} = -\frac{\mu}{M} + \cdots, \qquad \kappa_1|_{r_2} = -\frac{\sqrt{2}\sqrt{\mu} + 4\mu}{M} + \cdots, \qquad \kappa_1|_{r_3} = \frac{\sqrt{2}\sqrt{\mu} - 4\mu}{M} + \cdots.$$

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- thus the degenerate horizon does not yield vanishing surface gravity in the linear charged Vaidya spacetime, and hence non-zero Hawking temperature in the static spacetime.