Nuclear structure input to low-energy precision tests of the Standard Model via superallowed $0^+ \to 0^+$ Fermi β decay

Latsamy Xayavong[1], Nadya Smirnova[2], Yeunhwan Lim[1]

[1] Department of Physics, Yonsei University, Seoul 03722, South Korea

[2] LP2IB, CNRS/IN2P3, Université de Bordeaux, 33175 Gradignan, France

International Conference on Nuclear Theory in the Supercomputing Era

December 1-7, 2024

Outline

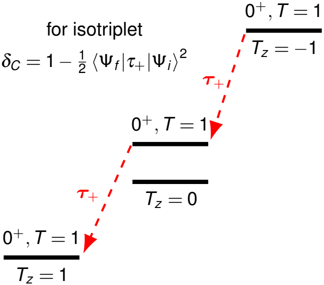
- Superallowed $0^+ \rightarrow 0^+$ Fermi β decay provides the cleanest probe of the Standard Model

$$\mathscr{F}t = ft(1+\delta_R')(1-\delta_C+\delta_{NS}) = \frac{K}{2G_F^2 V_{ud}^2(1+\Delta_R^V)},$$

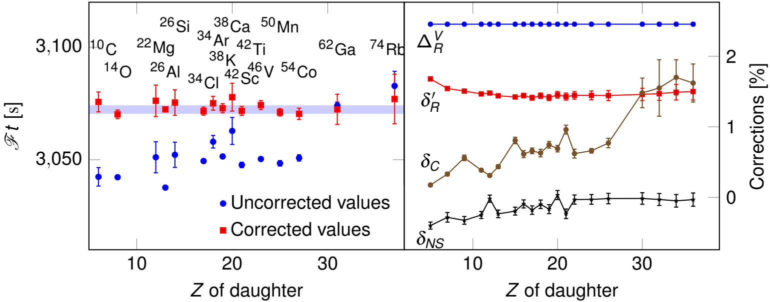
1) Inputs:
   - ft ~experimental input, sub-percent precision for 15 cases;
   - δ_C ~isospin correction, Towner-Hardy 2008;
   - δ_R' ~nucleus-dependent radiative correction, Marciano-Sirlin 1984;
   - Δ_R^V ~nucleus-independent radiative correction, Seng *et al.* 2018;
   - δ_{NS} ~structure-dependent radiative correction. Talk by Petr Navratil on Wednesday,
2) Outputs:
   - CVC implies constant $\mathscr{F}t$
   - $|V_{ud}|$ must satisfy the CKM unitarity.

for isotriplet

$$\delta_C = 1 - \frac{1}{2}\langle\Psi_f|\tau_+|\Psi_i\rangle^2$$



- Current status (using δ_C from shell model with Woods-Saxon)



  - ▶ The alignment of $\mathscr{F}t$ is mostly determined by δ_C,
  - ▶ CVC is verified

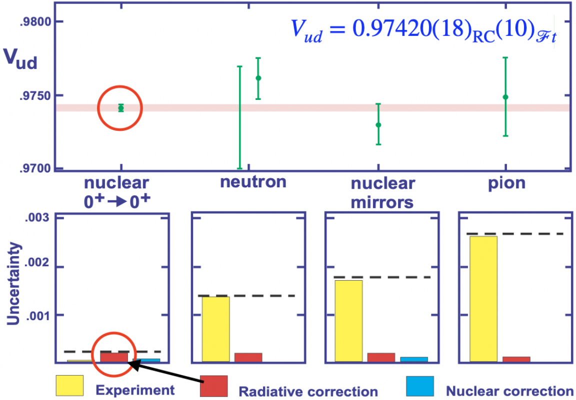
$$\overline{\mathscr{F}t} = 3072.24 \pm 0.57_{\delta_{C,exp}} \pm 0.36_{\delta'_R} \pm 1.73_{\delta_{NS}}\, s, \qquad \chi^2/\nu = 0.47$$

but uncertainties appear to be overestimated.

  - ▶ **Phenomenological shell model doesn't provide a reliable framework for uncertainty quantification.**

Hardy & Towner, PRC 102, 045501 (2020)

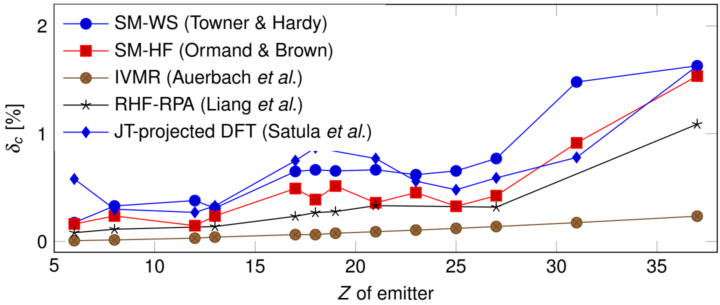
- Current status (using δ_C from shell model with Woods-Saxon)



Based on data from superallowed $0^+ \to 0^+$ Fermi β decay, **CKM top-row unitarity is found to be violated by more than** 2σ.

Hardy & Towner, PRC 102, 045501 (2020)

- Current status



  - Typical range: 0 to 1 %.
  - Among existing calculations, only the shell-model approach provides reasonable agreement with the Standard Model,
  - However, SM-WS and SM-HF yield significantly different results. **This discrepancy has been the subject of intent debate over the past decades**

Towner & Hardy, PRC 82, 065501 (2010)

Formalism

- Fermi matrix element in realistic basis

$$\begin{aligned} M_F &= \langle \Psi_f | \tau_+ | \Psi_i \rangle \\ &= \sum_{\alpha} \langle \tilde{\alpha} | \alpha \rangle \langle \Psi_f | n^\dagger_{\tilde{\alpha}} p_\alpha | \Psi_i \rangle + \sum_{\alpha\alpha'}^{n \neq n'} \langle \tilde{\alpha} | \alpha' \rangle \langle \Psi_f | n^\dagger_{\tilde{\alpha}} p_{\alpha'} | \Psi_i \rangle , \end{aligned}$$

where $\alpha = nljm$, $\alpha' = n'ljm$, and

$$\langle \tilde{\alpha} | \alpha' \rangle = \int_0^\infty R_{\tilde{\alpha},n}(r) R_{\alpha',p}(r) r^2 dr \equiv \Omega_{\tilde{\alpha}\alpha'}$$

  - Terms in red involve nodal mixing, and are unsuitable for the shell-model approach
  - Within simplified interaction, Miller & Schwenk demonstrated that non-orthogonal terms are not negligible and destructively contribute.

Miller & Schwenk, PRC 80, 064319 (2009)

Formalism

- Correction due to isospin-symmetry breaking

$$\delta_C = 1 - M_F^2/2$$

  - At LO, we can approximately decouple the radial mismatch and isospin-mixing contribution,

$$\delta_{C1} = \sqrt{2}\sum_{\alpha}\left[\langle\Psi_f|n^{\dagger}_{\tilde{\alpha}}p_{\alpha}|\Psi_i\rangle^{IC} - \langle\Psi_f|n^{\dagger}_{\tilde{\alpha}}p_{\alpha}|\Psi_i\rangle^{INC}\right], \qquad \text{LO}$$

$$\delta_{C2} = \sqrt{2}\left[\sum_{\alpha}\langle\Psi_f|n^{\dagger}_{\tilde{\alpha}}p_{\alpha}|\Psi_i\rangle^{IC}(1-\Omega_{\tilde{\alpha}\alpha}) + \sum_{\alpha\alpha'}^{n\neq n'}\langle\Psi_f|n^{\dagger}_{\tilde{\alpha}}p_{\alpha'}|\Psi_i\rangle^{IC}(1-\Omega_{\tilde{\alpha}\alpha'})\right], \qquad \text{LO}$$

$$\delta_{C3} = -\delta_{C2} + \sqrt{2}\sum_{\alpha}\langle\Psi_f|n^{\dagger}_{\tilde{\alpha}}p_{\alpha}|\Psi_i\rangle^{INC}(1-\Omega_{\tilde{\alpha}\alpha}), \qquad \text{NLO}$$

$$\delta_{C4} = -(\delta_{C1}+\delta_{C2})^2/4, \qquad \text{NLO}$$

$$\delta_{C5} = -(\delta_{C1}+\delta_{C2})\delta_{C3}/2, \qquad \text{NNLO}$$

$$\delta_{C6} = -(\delta_{C3})^2/4, \qquad \text{NNNLO}$$

where $[H_{IC}, T^2] = 0$ and $[H_{INC}, T^2] \neq 0$.

The radial excitation and higher-order terms were not considered by Towner-Hardy

Large-scale shell-model calculations

- Model space and effective interactions

Parent nuclei	model spaces	IC interactions	Refs.
^{10}C, ^{14}O	p shell	CKPOT/CKI/CKII	Cohen-Kurath, 1965
^{22}Mg	$1p_{\frac{3}{2}}\,1d_{\frac{5}{2}}\,2s_{\frac{1}{2}}$	REWIL ZBMI/ZBMII	Rehal-Wildenthal, 1973 Zuker et al., 1969
^{26}Al, ^{26}Si, ^{34}Cl ^{34}Ar	sd shell	USD USDA/USDB	Wildenthal, 1984 Brown-Richter, 2006
^{38}K, ^{38}Ca, ^{42}Sc, ^{46}V	$2s_{\frac{1}{2}}\,1d_{\frac{3}{2}}\,1f_{\frac{1}{2}}\,2p_{\frac{3}{2}}$	ZBM2	Nowacki et al., 2014
^{50}Mn, ^{56}Co	pf shell	GXPF1A KB3G FPD6	Honma et al., 2004 Poves et al., 2004 Richter et al., 1991
^{62}Ga, ^{74}Rb	$2p_{\frac{3}{2}}\,2p_{\frac{1}{2}}\,1f_{\frac{5}{2}}\,1g_{\frac{9}{2}}$	JUN45 MRG	Honma et al., 2009 Nowacki et al., 1996

  - The INC component is taken from Ormand & Brown, NPA 491, 1 (1989) 1-23.
  - Our calculations were performed in full model spaces (except ^{74}Rb) using NuShellX@MSU

- The overlap integral is evaluated with Woods-Saxon and Hartree-Fock radial wave functions

- Essentially, we expand δ_{C2} in terms of intermediate states

$$\begin{aligned}\delta_{C2} &= \sqrt{2}\Bigg[\sum_{\alpha\pi} |\langle\Psi_f|n^{\dagger}_{\tilde{\alpha}}|\pi\rangle^{IC}|^2(1-\Omega^{\pi}_{\tilde{\alpha}\alpha}) \\ &+ \sum_{\alpha\alpha'\pi}^{n\neq n'} \langle\Psi_f|n^{\dagger}_{\tilde{\alpha}}|\pi\rangle^{IC}\langle\pi|p_{\alpha'}|\Psi_i\rangle^{IC}(1-\Omega^{\pi}_{\tilde{\alpha}\alpha'})\Bigg]\end{aligned}$$

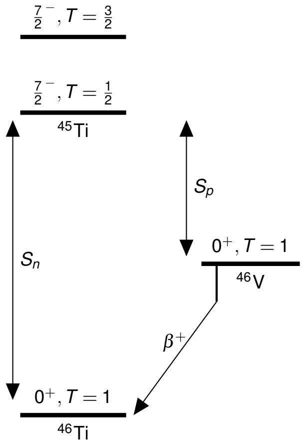
  - Spectroscopic amplitudes are calculated using eigenvectors
  - Constraints for radial wave functions

    1) Energies

$$\begin{aligned}\varepsilon_p &= -S_p - E^x_\pi \\ \varepsilon_n &= -S_n - E^x_\pi\end{aligned}$$

    2) Charge radii

$$\langle r^2\rangle = \frac{1}{Z}\sum_{\alpha\pi}|\langle\Psi_i|p^{\dagger}_{\alpha}|\pi\rangle^{IC}|^2\int_0^\infty r^4 R^{\pi}_{\alpha,p}(r)dr$$



This technique was first introduced by Towner & Hardy. PRC 77, 025501 (2008)

Incorporation of spectator states

- Completeness check for the matrix element

Parent nuclei	IC interaction	model space	# of states	M_F^0	cutoff error [%]
^{42}Sc	ZBM2	$2s_{\frac{1}{2}}\,1d_{\frac{3}{2}}\,1f_{\frac{1}{2}}\,2p_{\frac{3}{2}}$	1400	1.358	-3.975
^{46}V	ZBM2	$2s_{\frac{1}{2}}\,1d_{\frac{3}{2}}\,1f_{\frac{1}{2}}\,2p_{\frac{3}{2}}$	1400	1.345	-4.894
	GX1A	fp shell	800	1.415	0.056
	KB3G	fp shell	800	1.415	0.056
	FPD6	fp shell	800	1.415	0.056
^{50}Mn	GX1A	fp shell	1200	1.429	1.046
	KB3G	fp shell	400	1.428	0.975
	FPD6	fp shell	600	1.415	0.056
^{54}Co	GX1A	fp shell	600	1.452	2.672
	KB3G	fp shell	600	1.427	0.904
	FPD6	fp shell	600	1.417	0.197
^{66}AS	JUN45	$2p_{\frac{3}{2}}\,2p_{\frac{1}{2}}\,1f_{\frac{5}{2}}\,1g_{\frac{9}{2}}$	3200	1.42	0.409
	MRG	$2p_{\frac{3}{2}}\,2p_{\frac{1}{2}}\,1f_{\frac{5}{2}}\,1g_{\frac{9}{2}}$	3200	1.417	0.197
^{70}Br	JUN45	$2p_{\frac{3}{2}}\,2p_{\frac{1}{2}}\,1f_{\frac{5}{2}}\,1g_{\frac{9}{2}}$	570	1.519	7.410

where

$$M_F^0 = \sum_{\alpha\pi} |\,\langle \Psi_f | n^{\dagger}_{\tilde{\alpha}} | \pi \rangle^{IC}\,|^2,$$

must be normalized to $\sqrt{2}$.

Incorporation of spectator states

- Convergence of δ_{C2}^{sm} (excluding radial excitation)

$$\delta_{C2}^{sm} = \sqrt{2}\sum_{\alpha\pi} |\langle\Psi_f|n_{\tilde{\alpha}}^{\dagger}|\pi\rangle^{IC}|^2(1-\Omega_{\tilde{\alpha}\alpha}^{\pi}),$$

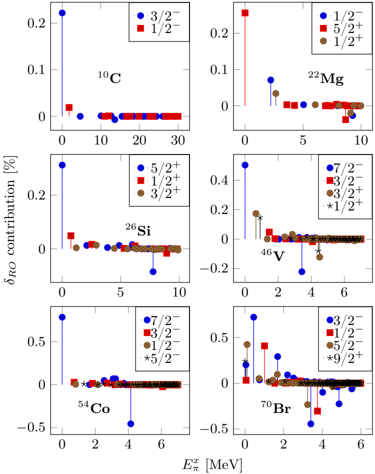
- δ_{C2}^{sm} converges faster than M_F, because $(1-\Omega_{\tilde{\alpha}\alpha}^{\pi}) \sim 0$ at high energy.

  Typically, a few hundred states are sufficient

- Furthermore,

$$|\langle\Psi_f|n_{\tilde{\alpha}}^{\dagger}|\pi\rangle^{IC}|^2 \propto |\langle\Psi_f|||n_{\tilde{\alpha}}^{\dagger}|||\pi_<\rangle^{IC}|^2 - \frac{1}{2}|\langle\Psi_f|||n_{\tilde{\alpha}}^{\dagger}|||\pi_>\rangle^{IC}|^2$$

  leading to cancellation between isospin-lesser (positive) and isospin-greater (negative) contributions. The cancellation is more pronounced when $\tilde{\alpha}$ is highly filled (FM sum rule).



Investigation of radial excitation

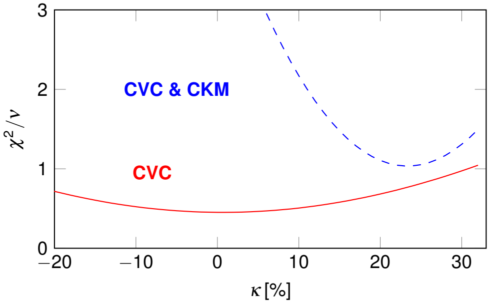
- We assume a constant proportionality between the diagonal and radial excitation terms,

$$\delta_{C2} = \delta_{C2}^{sm} + \delta_{C2}^{re} = \delta_{C2}^{sm}(1+\kappa).$$

so that the corrected $\mathscr{F}t$ is modified as

$$\mathscr{F}t = \mathscr{F}t^{sm} - ft(1+\delta'_R)\kappa\delta_{C2}^{sm}.$$

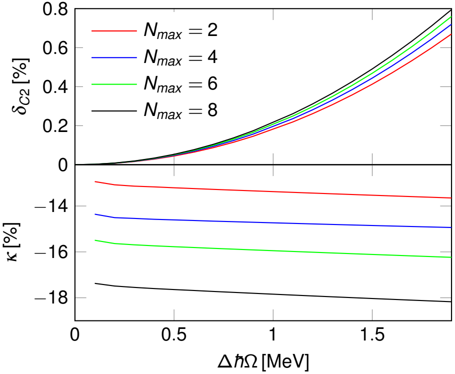
With existing data for $\{\mathscr{F}t^{sm}, ft, \delta'_R, \delta_{C2}^{sm}, V_{us}\}$, we investigate the Standard-Model consistency as a function of κ



- CVC test suggests $\kappa \approx 0$
- However, to maintain both CVC & CKM unitarity, $\kappa \sim 23\,\%$ is required
- **Contrary, Miller & Schwenk have predicted a significantly negative value**.

Investigation of radial excitation

- Exact NCSM calculations for ^{3}H$\rightarrow^3$He (mirror β decay),



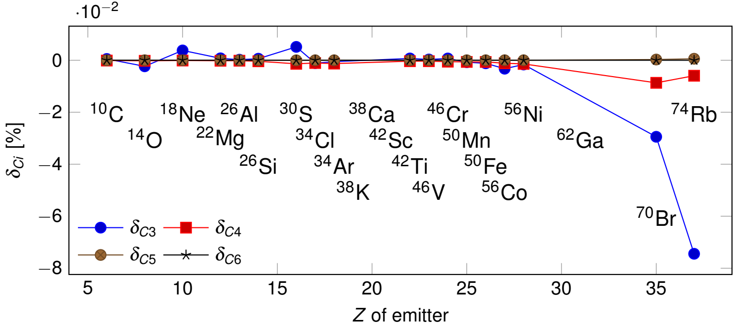
- Interaction: NN, EMN500, N4LO, SRG ($\lambda = 2\,\text{fm}^{-1}$)
- Nonorthogonal HO basis ($\Delta\hbar\Omega = \hbar\Omega_i - \hbar\Omega_f$)
- We don't have constraint for fixing κ at small N_{max}. However, κ appears to be lower than -10 % (negative sign, agreeing with Miller & Schwenk !)
- Keeping this for $0^+ \rightarrow 0^+$, the CKM sum rule would be further underestimated.

This property must be verified within the shell-model framework through effective τ_+ operator

These calculations were carried using the Bigstick and NuHamil codes

Higher-order terms (δ_{C3}, δ_{C4}, δ_{C5}, δ_{C6})

- Generally, higher-order terms are negligibly small. Only δ_{C3} is significant for ^{74}Rb (δ_{C3} is destructive).

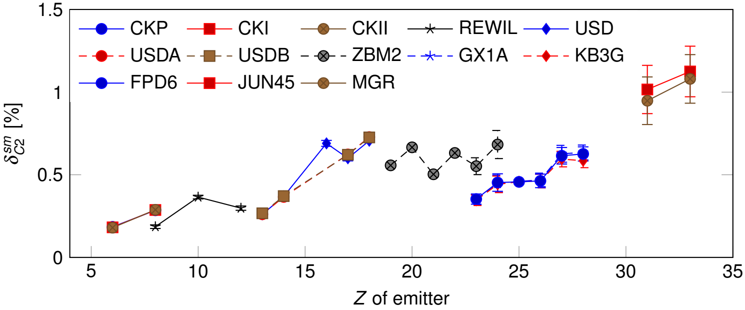


  - These terms are more pronounced in Fermi transitions of higher isospin multiplets, and Gamow-Teller transitions

Xayavong & Smirnova, PRC 109, 014317 (2024)

Radial mismatch correction (δ_{C2}^{sm})

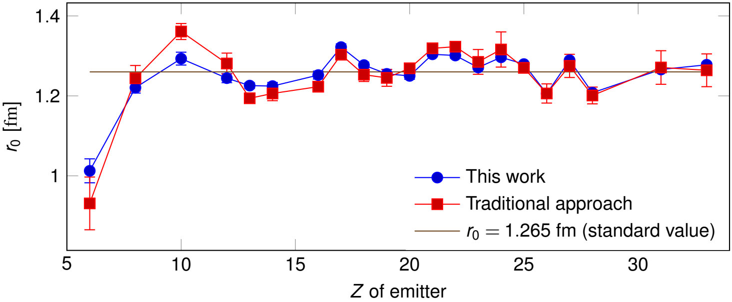
- Shell model with Woods-Saxon radial wave functions



  - For a given model space, interaction dependence is insignificant
  - Uncertainty is mainly due to the variations in fitting of radial wave functions.

Radial mismatch correction (δ_{C2}^{sm})

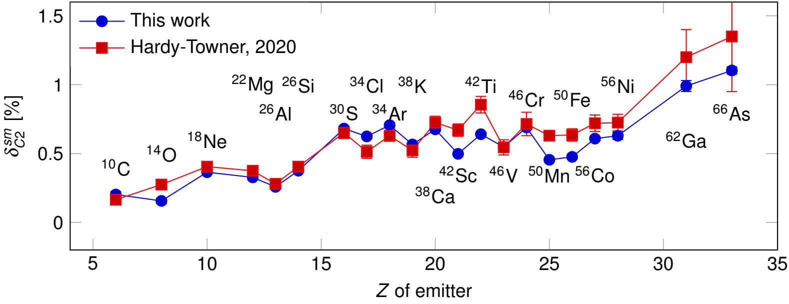
- Shell model with Woods-Saxon radial wave functions



  - The incorporation of intermediate states remarkably influences charge radii
  - r_0 suddenly decreases at $A \leq 10$. Sophisticated CoM correction is required (we currently adopt the harmonic oscillator formula).

Radial mismatch correction (δ_{C2}^{sm})

- Shell model with Woods-Saxon radial wave functions

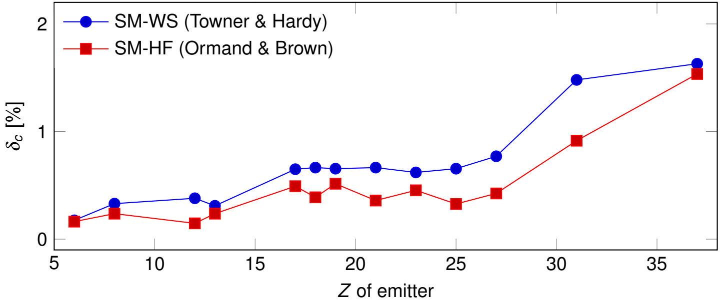


- In addition to the charge-radius generalization, our calculations are performed in full model spaces. HT used truncated model spaces
- We conservatively recommend to use the averaged values.

Xayavong & Smirnova; PRC 97, 024324 (2018)

Radial mismatch correction (δ_{C2}^{sm})

- Discrepancy between SM-WS and SM-HF



  - Ormand-Brown suggested self-consistent Hartree-Fock radial wave functions.
  - To account for excitation energy of intermediate states, they scaled the central field after variation.
  - Their calculations were based the conventional Skyrme interaction, which is isospin-invariant.

Ormand & Brown, NPA 491, 1 (1989) 1-23; Towner & Hardy, PRC 77, 025501 (2008).

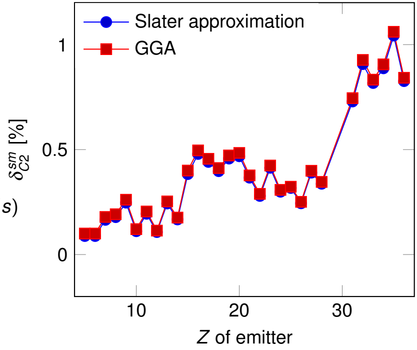
Radial mismatch correction (δ_{C2}^{sm})

- Shell model with Hartree-Fock radial wave functions

  - The Slater approximation doesn't produce correct asymptotics.
  - As an improvement, we treat the Coulomb-exchange term using GGA

$$V_{coul}^{ex}(r) = V_{Sl}^{ex}(r)\left\{F(s) - \left[s + \frac{3}{4k_F r}\right]F'(s) + \left[s^2 - \frac{3\rho_{ch}''(r)}{8\rho_{ch}(r)k_F^2}\right]F''(s)\right\}$$

where s denotes the density gradient



The GGA values are 2-14 % larger than those obtained with the Slater approximation. This tends to support SM-WS !

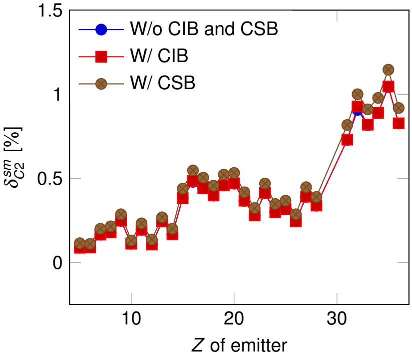
Xayavong & Smirnova, PRC 105, 044308 (2022)

Radial mismatch correction (δ_{C2}^{sm})

- Shell model with Hartree-Fock radial wave functions
  - We also include the CIB and CSB forces from Suzuki *et al.*, PRL 112, 102502 (1995),

$$
\begin{aligned}
V_{CIB} &= 2t_{iz}t_{jz}\delta[u_0(1-P_\sigma) \\
&+\frac{u_1}{2}(1-P_\sigma)\left(\boldsymbol{k}^2+\boldsymbol{k}'^2\right) \\
&+u_2(1-P_\sigma)\boldsymbol{k}'\cdot\boldsymbol{k}], \\
V_{CSB} &= (t_{iz}+t_{jz})\delta[s_0(1-P_\sigma) \\
&+\frac{s_1}{2}(1-P_\sigma)\left(\boldsymbol{k}^2+\boldsymbol{k}'^2\right) \\
&+s_2(1-P_\sigma)\boldsymbol{k}'\cdot\boldsymbol{k}]
\end{aligned}
$$

where u_i and s_i are adjustable constant.

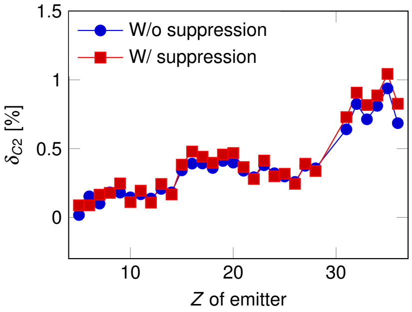


The CIB effect is completely negligible, whereas the CSB contributes 10 to 30 %. This tends to support SM-WS !

Xayavong & Smirnova, PRC 105, 044308 (2022)

Radial mismatch correction (δ_{C2}^{sm})

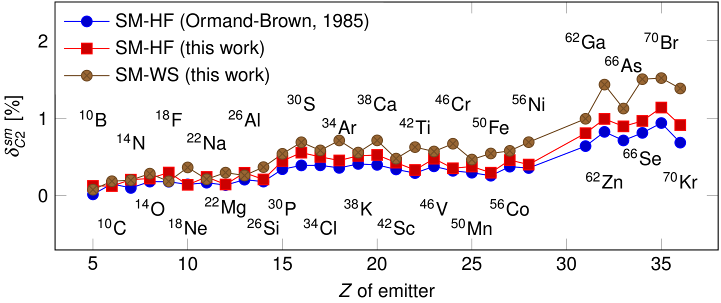
- Shell model with Hartree-Fock radial wave functions
  - Self-consistent approach causes spurious isospin mixing
  - This issue is not resolvable without going beyond HF
  - To approximate this issue, we perform HF calculations for the $N = Z$ nucleus of a given mass multiplet, and then construct the potential for the actual nucleus (with addition of charge-dependent forces) after variation.



The suppression of spurious isospin mixing also tends to increase δ_{C2}^{sm}, supporting SM-WS !

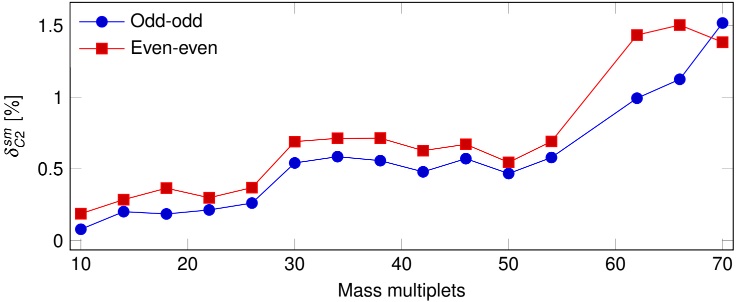
Xayavong & Smirnova, PRC 105, 044308 (2022)

- Shell model with Hartree-Fock radial wave functions
  - We have also considered other higher-order effects, including finite size, Coulomb spin-orbit, vacuum polarization, and two-body CoM. These contributions were found to be negligible



Despite significant improvements, a considerable gap remains between SM-WS and SM-HF. Although HF has a solid foundation, it is unsuitable as a basis for the shell model. For instance, spurious isospin-mixing is fundamentally unresovable .

- With WS radial wave functions, we obtained larger δ_{C2}^{sm} values for even-even emitters

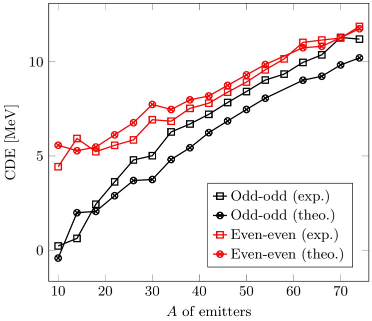


  - This odd-even staggering is highly sensitive to the mirror *ft* ratio
  - Without this key property the agreement with CVC would be lost.

- This local property is strongly correlated with Coulomb displacement energy (CDE). For isotriplets:

$$CDE = S_n^f - S_p^i$$

where i and f indicate initial and final nuclei.



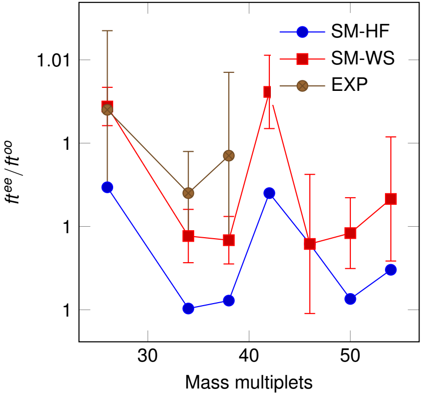
- With WS, we adjust depth or additional surface-peaked term such that CDE is exactly reproduced. This refinement improves δ_{C2}^{sm} and agreement with the Standard Model.
- Unfortunately, this desired local property of δ_{C2}^{sm} disappears when using HF radial wave functions.

HF field is determined through a well-defined EDF. A direct scaling is fundamentally illegitimate.

- Mirror asymmetry of Fermi transitions
  - Mirror *ft* ratio is mostly determined by the odd-even staggering discussed in the previous slides,

$$\frac{ft^{ee}}{ft^{oo}} \approx 1 + (\delta_C^{ee} - \delta_C^{oo})$$

  - Unluckily, only the data for $A = 26, 34, 38$ are precise enough for this test to be meaningful,
  - For most cases, the SM-HF fails to reproduce the experimental/WS data.



Summary

- The discrepancy between WS and HF basis is participially resolved.
- Principal features of δ_{C2} are well-understood.
- Radial excitation contribution appears to be significant. It should be investigated through construction of effective Fermi operator.
- While several experimental constraints have been introduced, numerious free parameters remain. Ideally, the shell-model approach can be tested in two different ways:
  1) Using light nuclei where it intersects with ab-initio methods, particularly ^{6}He, ^{10}C, and ^{14}O
  2) Verifying consistency of the theoretical approach in cases where δ_C is expected to be extremely substantial. This include Fermi transitions of higher isospin multiplets, and Gamow-Teller transitions.