Probing the symmetry energy with gravitational waves

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December 5, 2024
NTSE2024, Pusan

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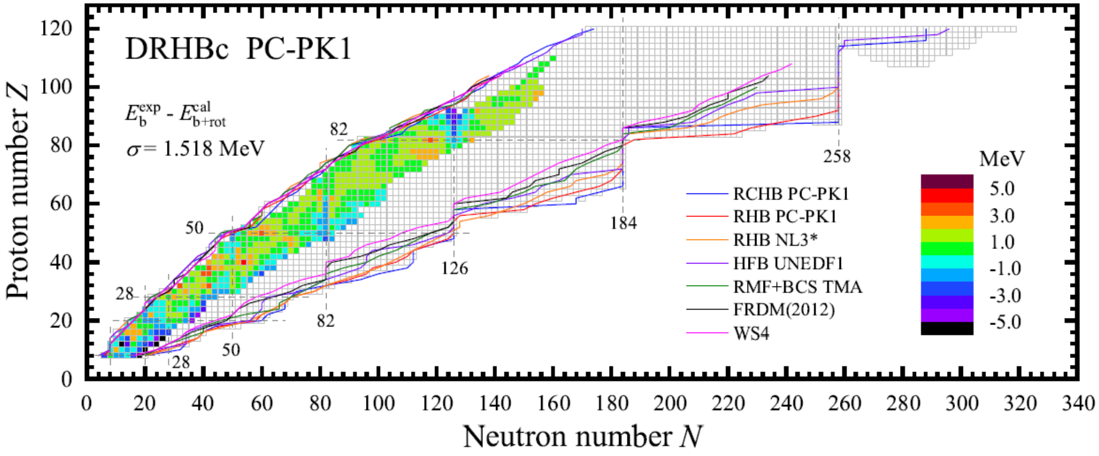
Nuclear theory near the neutron drip line

- Heavy elements are synthesized in the r-process
- r-processes happen along paths close to the neutron drip line
- Position of the neutron drip line is critical to uncover the origin of elements
- Many good mass models reproduce measured masses (~630) with RMSD ≤ 2 MeV: Fascinating
- How they predict the neutron drip line



- Apart from having a good mass model, it is necessary to have a systematic way to control, reduce and quantify the uncertainty

- **Symmetry energy**

$$\mathcal{E}(\rho,\,\delta) \;=\; \mathcal{T}(\rho,\,\delta)+E(\rho)+S(\rho)\delta^2+O(\delta^4);\;\rho=\rho_n+\rho_p,\;\delta=\frac{\rho_n-\rho_p}{\rho}$$

- Most important to understand neutron-rich systems, e.g. properties of isotopes along the neutron drip line, neutron stars

- Definition

$$\begin{aligned} E(\rho) &= E_\mathrm{B}+\frac{1}{2}K_0x^2+\cdots;\; x=\frac{\rho-\rho_0}{3\rho_0} \\ S(\rho) &= J+Lx+\frac{1}{2}K_\mathrm{sym}x^2+\cdots \end{aligned}$$

- Dependence on the symmetry energy

- My goal for uncertainty

±3% for J, ±5% for L, ±10% for K_{sym}; e.g.

J=30 ±1 MeV, L= 50 ±3 MeV,

K_{sym}=-100 ±10 MeV

H. Gil, N. Hinohara, CHH, K. Yoshida, PRC108(2023)044316

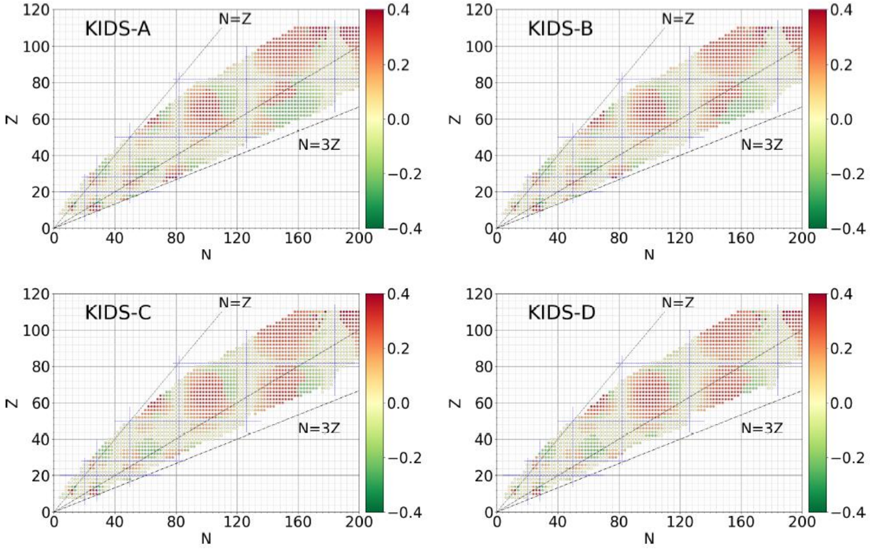


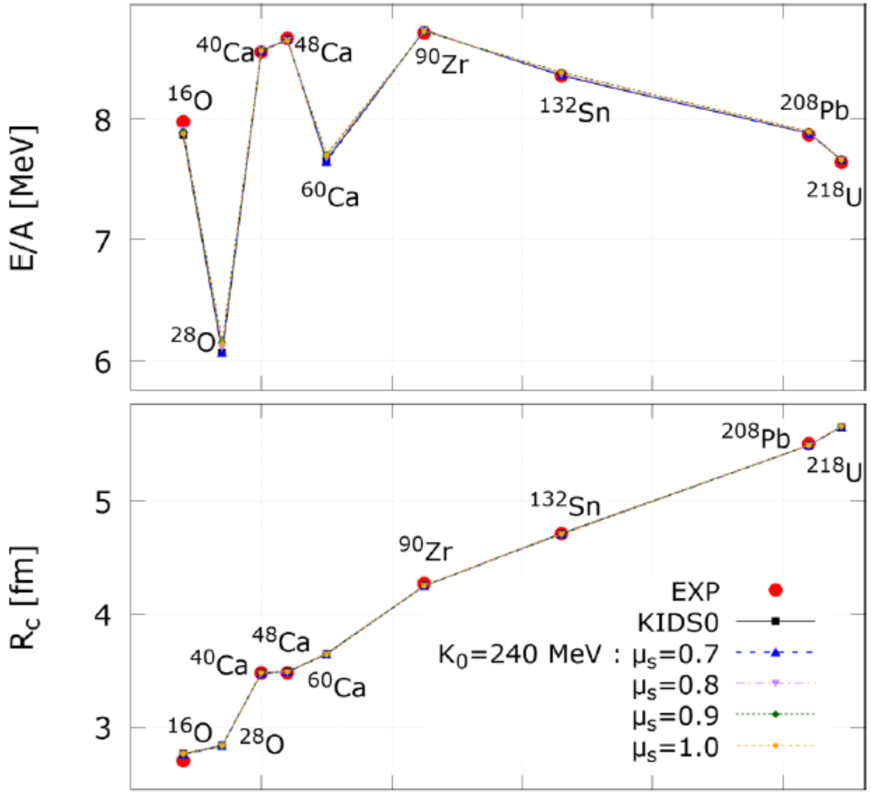
FIG. 3. Calculated quadrupole deformation $\beta_{2,p}$ for bound nuclei obtained by employing the KIDS-A–D models.

KIDS (Korea-IBS-Daegu-SKKU) formalism

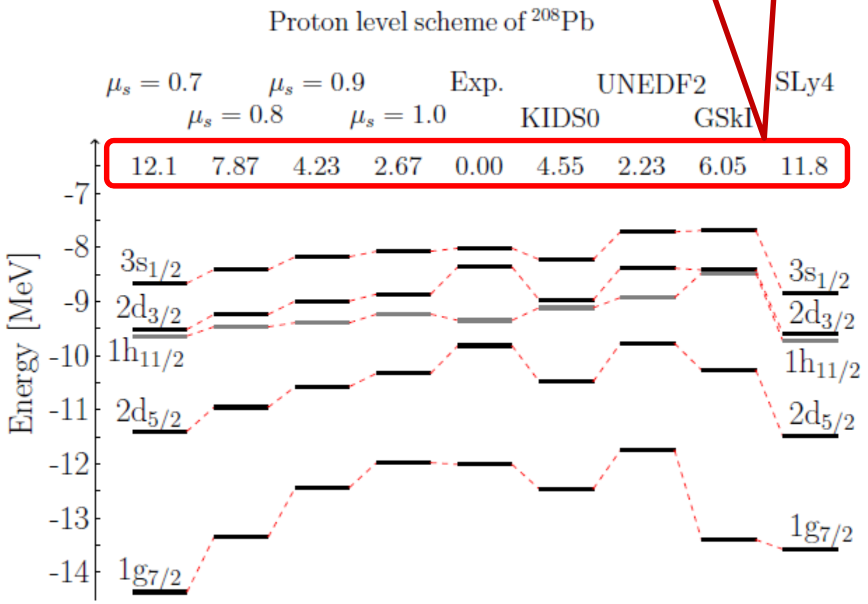
- **Expansion rule**
- Energy density of many-nucleon system expanded in the power of the Fermi momentum
- **Fitting rule**
- Coefficients in the energy density must be constrained from experiment
- Find the optimal number of terms to describe infinite nuclear matter
- Step1: Determine the coefficients to reproduce the nuclear matter EoS
- Step2: Fit the additional coefficients to reproduce nuclear properties
- Step3: Terms can be added to satisfy specific properties or conditions without changing properties determined in prior steps

- **Advantage: Nuclear matter EoS is determined independent of nuclear properties**

H. Gil, P. Papakonstantinou, CHH, Y. Oh, PRC99(2019)064319



Mean deviation from exp.



$\mu_s = M^*_s/M$

Symmetry energy with KIDS EDF

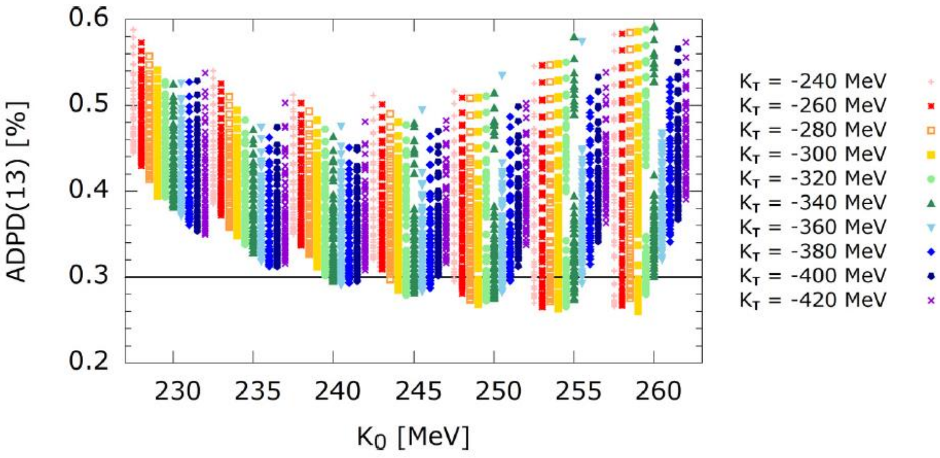
- **Nuclear matter: 5670 EoSs**

- Ko = 220, 225, … 260, J = 30, 30.5, … 34
- L = 40, 45, … 70, Kt = -420, -400, … -240 (Ksym = -300 ~ 96)

- **Nuclear data: 5670 Skyrme force models**

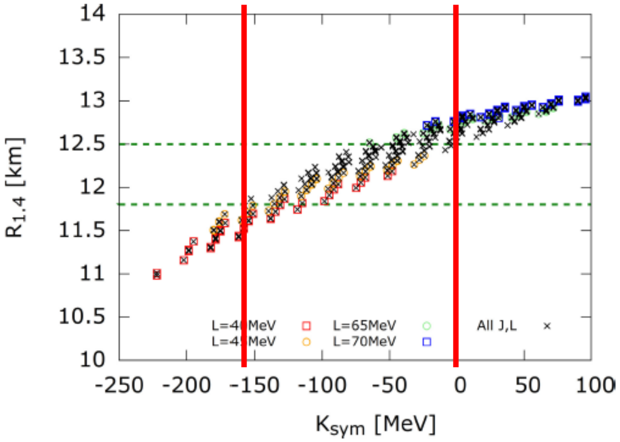
- 16O, 40Ca, 48Ca, 90Zr, 132Sn, 208Pb, 218U (13 data)
- Accuracy

$$\mathrm{ADPD}(N) \equiv \frac{1}{N}\sum_{i=1}^{N}\left|\frac{O_i^{\mathrm{exp}} - O_i^{\mathrm{cal}}}{O_i^{\mathrm{exp}}}\right|$$



358 models below the horizontal line

ADPD<0.3%



Among the 358 models, 158 models are

between $R_{1.4}$=11.8-12.5 km

Mean and standard deviation

	ADPD03		R14	
	mean	s.d.	mean	s.d.
K_0	252.0	5.6	251.1	5.3
J	30.9	0.7	30.7	0.6
L	54.7	9.7	49.8	5.2
K_{sym}	-52.4	72.0	-82.4	33.7

- Neutron star data reduce the uncertainty of L and K_{sym} substantially
- At the 2σ confidence level

- J=30.7±1.2 MeV: Satisfactory
- L=49.8±10.4: Not satisfactory

- Mass-radius data are not enough to achieve the goal
- Other constraints are demanded

Non-radial oscillation of the neutron star

Work with Debashree Sen and Atanu Guha, to appear in arXiv soon

- Quakes inside the neutron star can happen in the merger
- Oscillation modes: fundamental (f), pressure (p), gravity (g), rotational (r), space-time (w)
- Different restoring forces give rise to mode classification
- Magnitude of the frequency is clearly distinguished (e.g. f: 1-3 kHz, p > 5 kHz, g < 1 kHz)
- Forthcoming detectors like Einstein Telescope, Cosmic Explorer, LIGO O4 run are expected to measure the signals for the oscillation

- **Metric for the spherically symmetric stars**

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2.$$

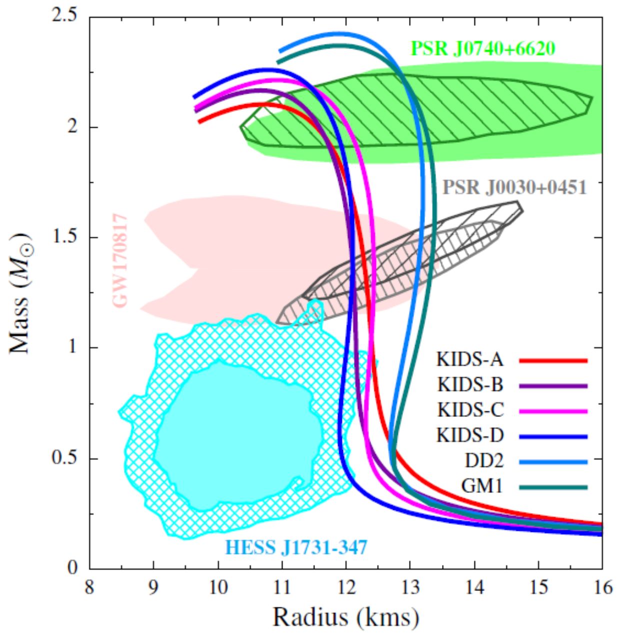
- **TOV equations**

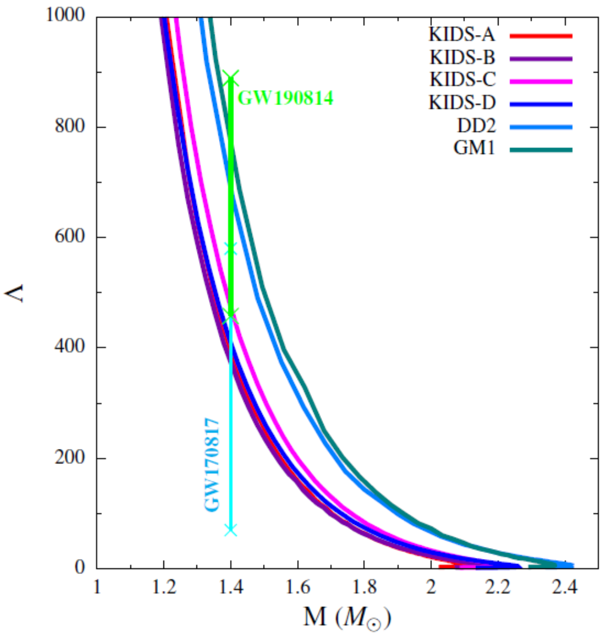
$$\begin{aligned}\frac{dP(r)}{dr} &= -\Big(\varepsilon(r) + P(r)\Big)\frac{d\Phi(r)}{dr},\\ \frac{d\Phi(r)}{dr} &= \frac{M(r) + 4\pi r^3 P(r)}{r\Big(r - 2M(r)\Big)},\\ \frac{dM(r)}{dr} &= 4\pi r^2 \varepsilon(r).\end{aligned}$$

- **Oscillation mode**

$$\begin{aligned}\frac{dW(r)}{dr} &= \frac{d\varepsilon(r)}{dP(r)}\left[\omega^2 r^2 e^{\lambda(r)-2\Phi(r)}V(r) + \frac{d\Phi(r)}{dr}W(r)\right] - l(l+1)e^{\lambda(r)}V(r),\\ \frac{dV(r)}{dr} &= 2\frac{d\Phi(r)}{dr}V(r) - e^{\lambda(r)}\frac{W(r)}{r^2}.\end{aligned}$$

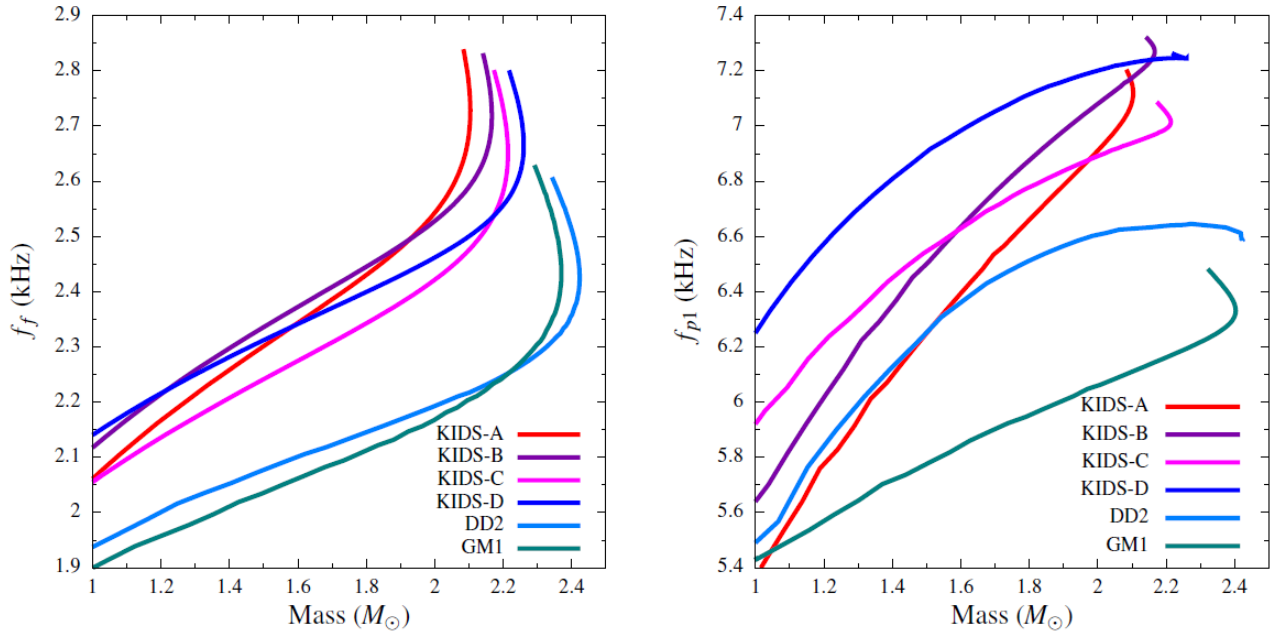
Result1: Hadronic neutron stars





- Masses are fine in KIDS and RFM
- R1.4: 12.0-12.5 km for KIDS, Above 13 km for RMF
- RMF EoSs are stiffer than KIDS
- Λ1.4: KIDS are consistent with GW170817, RMF are above the upper limit of GW170817

Result1: Hadronic neutron stars

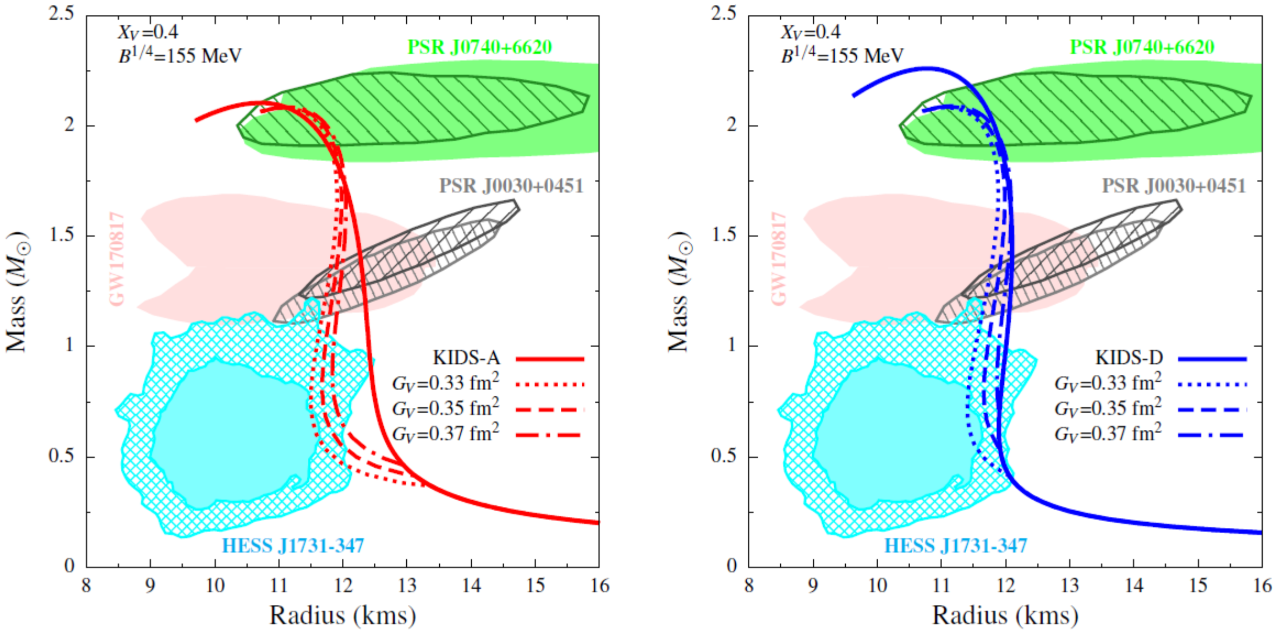


- f mode
- Stiff EoS gives small frequency
- Order of $f_{f1.4}$ is exactly in the reverse order of R1.4
- Rangees are 2.2-2.3 kHz for KIDS, and ~2.0 kHz for RMF
- p mode
- Ordering of $f_{p1.4}$ is exactly in the reverse of L

GM1	KIDS-A	DD2	KIDS-B,C	KIDS-D
94	66	55	58	47

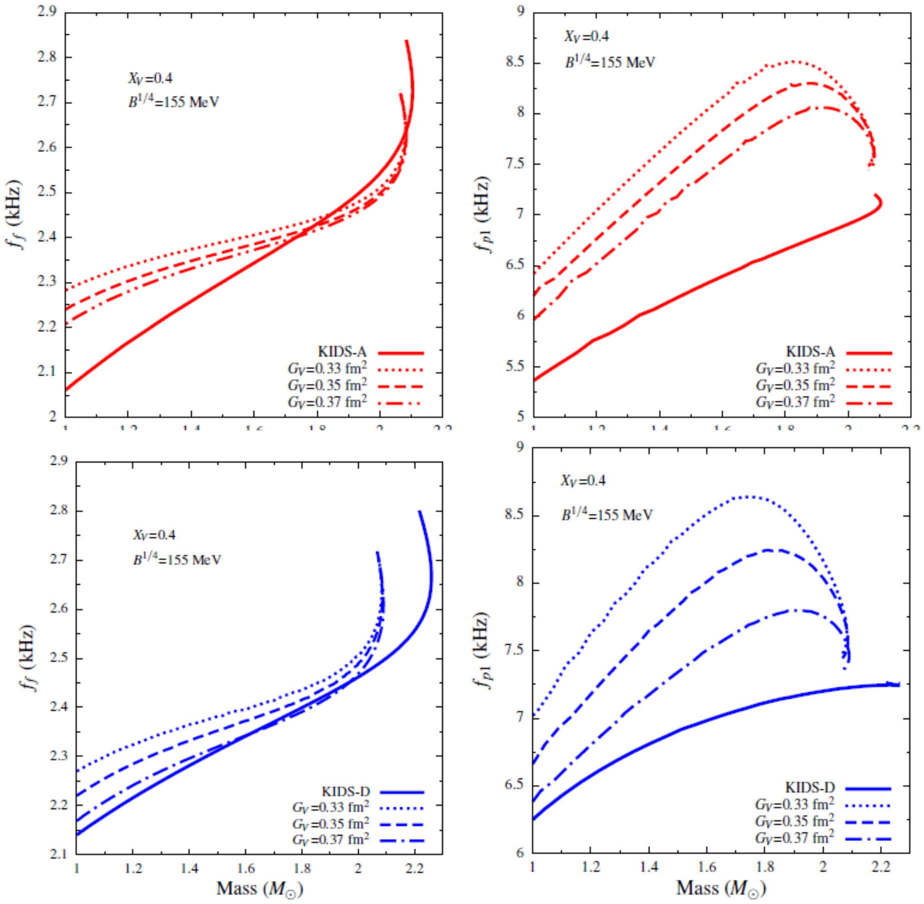
Result2: Hybrid stars

- Hadron-quark phase transition
- Hadronic EoS: KIDS-A, D
- Quark EoS: MIT bag with vector repulsion



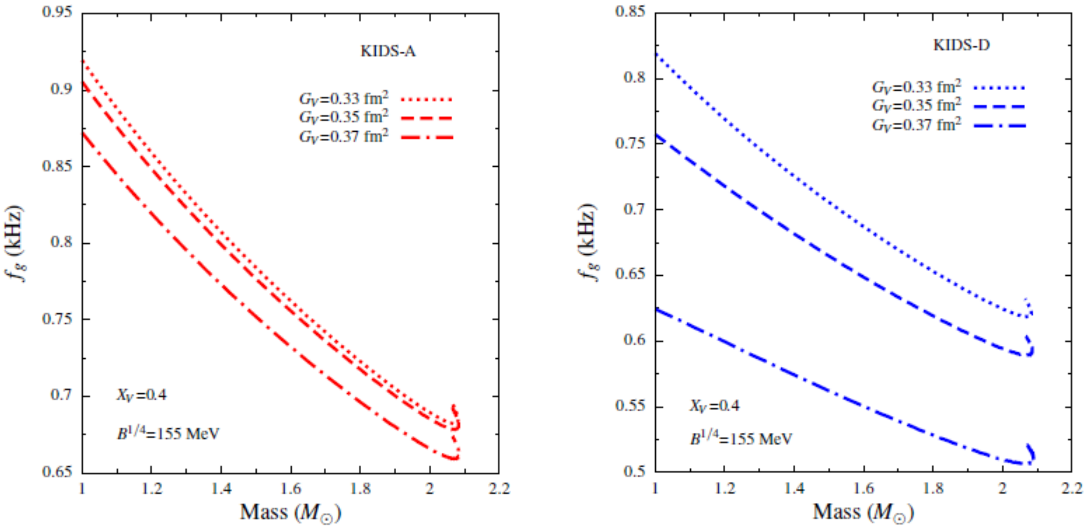
- Mass and radius are consistent with all the existing mass-radius data
- Mass-radius relations of hybrid stars do not depend strongly on the symmetry energy and the vector coupling

Result2: Hybrid stars



- Ranges of f mode are similar with the hadronic stars
- Ranges of p mode are well above the hadronic stars: useful to distinguish hadronic and hybrid stars

Result2: Hybrid stars



- **Ranges of g mode**

- **Larger than 0.77 kHz for KIDS-A, Smaller than 0.73 kHz for KIDS-D**
- **g mode distinguishes symmetry energy clear**
- **Strong dependence on the vector coupling in the KIDS-D**
- **Measurement is informative to both symmetry energy and vector coupling**

Summary

- KIDS formalism offers a systematic way to explore and control uncertainties
- Symmetry energy can be constrained significantly with mass-radius data, but it is not satisfactory
- Detection of the non-radial frequency of the compact objects opens a new way to constrain the symmetry energy
- f mode can distinguish mild and stiff EoS
- p mode can constrain the slope parameter L
- g mode, if detected, is an evidence for the 1st order phase transition
- g mode can distinguish the stiffness of symmetry energy critically