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Daejeon16 beyond the *p*-shell nuclei

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2024. 12. 05.

iRIS 중이온가속기연구소
Institute for Rare Isotope Science

Contents

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Daejeon16

- *Ab initio* nuclear physics

  - No-core shell model : all nucleons are treated as active degrees of freedom equally

  - *Ab initio* theory requires a **realistic** nucleon-nucleon interactions accurately describing, e.g.
    - ✓ *NN* scattering data
    - ✓ deuteron properties

  - Interaction could be suggested,
    - ✓ phenomenologically
    - ✓ from the first principles

    and also combination of two ways above

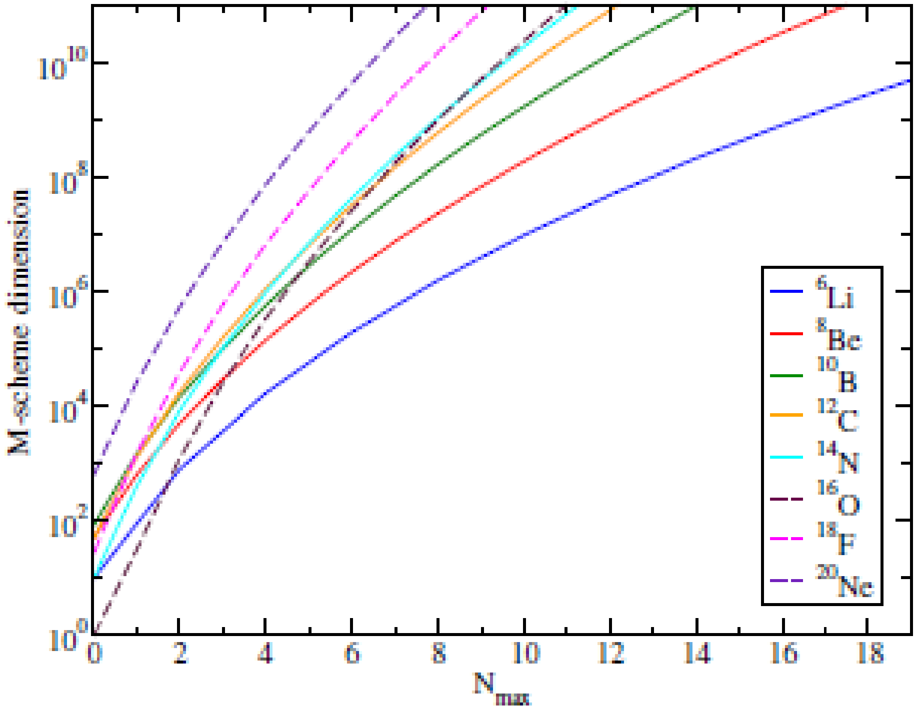
Daejeon16

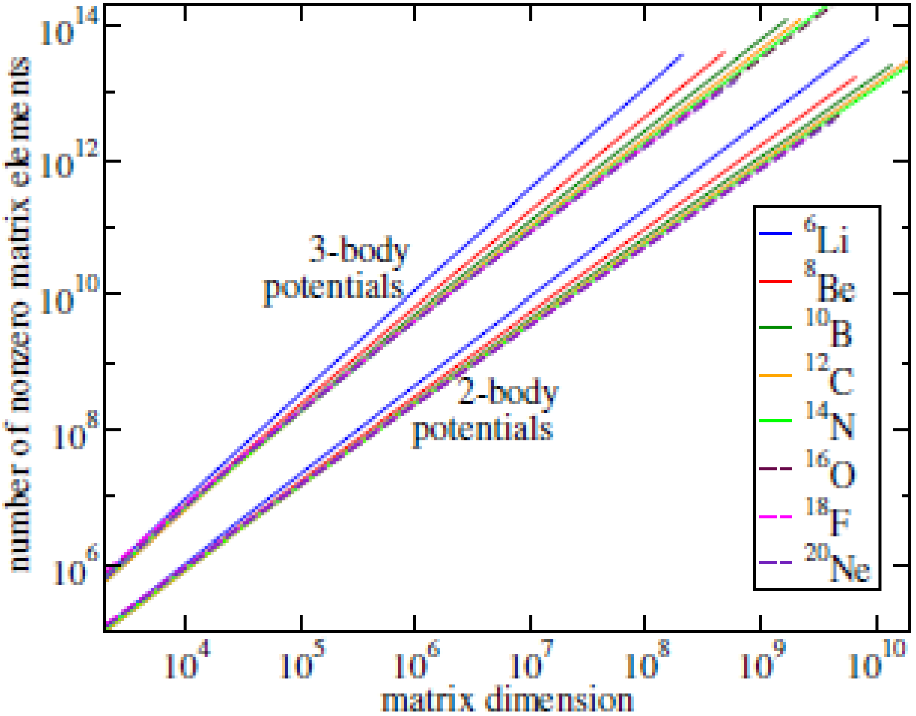
- Historical evolution
  - Fitting deuteron rms radius and quadrupole moment by PET : good ^{3}H and ^{4}He bindings
  - Fitting p waves to ^{6}Li spectrum : JISP6
  - Additional fitting p waves to ^{16}O binding : JISP16
  - More accurate fit to light nuclei : JISP16$_{2010}$
  - from SRG-evolved N3LO : Daejeon16

Almost taken from [A.M.Shirokov's talk at NTSE-2016 @ Khabarovsk

Daejeon16

- *NNN* interaction
  - *NNN* force contribution to nuclear physics is small but can be essential
  - However this requires more computing resources (especially memory)
  - If we can avoid using 3*N* forces ?





Daejeon16

- *NNN* interaction

  - W. Polyzou & W. Glöckle theorem [Few-body Syst. 9, 97 (1990))]

$$H = T + V_{ij} \Rightarrow H' = T + V'_{ij} + V_{ijk}$$

where V_{ij} and V'_{ij} are phase-equivalent, H and H' are isospectral.

Is it possible to transform as

$$H' = T + V'_{ij} + V_{ijk} \Rightarrow H = T + V_{ij}$$

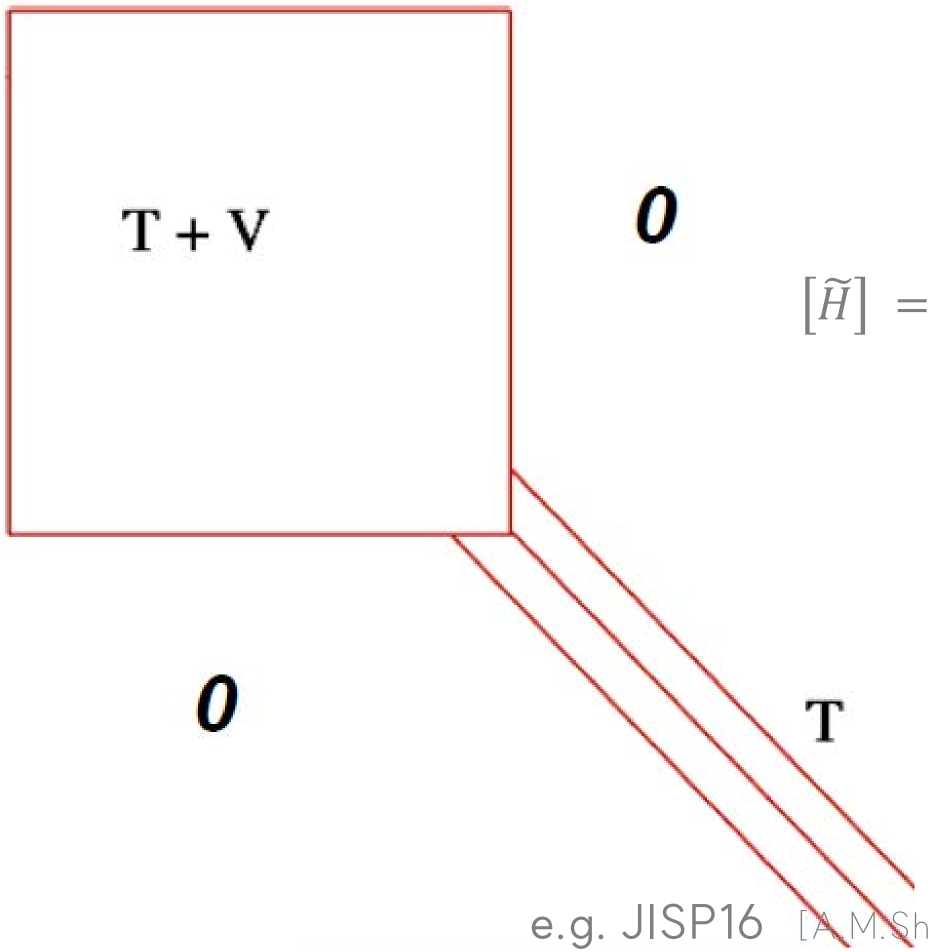
with (approximately) isospectral H and H' ?

In other words, intrinsic *NNN* interaction could be cancelled out by induced *NNN* interaction?

  - Calculation without *NNN* interaction requires so smaller computational resources that we can reach larger model spaces.

Daejeon16

- PET (phase-equivalent transformation)



- oscillator basis
- truncated potential energy matrix V
- complete infinite kinetic energy matrix T

$$[\tilde{H}] = [U^{\dagger}]\,[H]\,[U]$$

- do not change scattering phase shifts and bound state energies of two–body system
- but are supposed to modify two-body bound state observables such as the rms radii and electromagnetic moments

e.g. JISP16 [A.M.Shirokov, J.P.Vary, A.I.Mazur and T.A.Weber, Phys.Lett.B 644, 33 (2007)]

Daejeon16

N3LO interaction

↓

SRG
(similarity renormalization group)

λ=1.5 fm^{-1}

↓

SRG-evolved N3LO

↓

PET
(phase equivalent transformation)

$\hbar\Omega$=25 MeV

↓

Daejeon16

Simplest PETs with continuous parameters are used to fit the B.E. of several light nuclei in NCSM calculations.

[A.M.Shirokov, I.J.Shin, Y.Kim, M.Sosonkina, P.Maris and J.P.Vary, "N3LO NN interaction adjusted to light nuclei in *ab exitu* approach," Phys. Lett.B **761**, 87 (2016)]

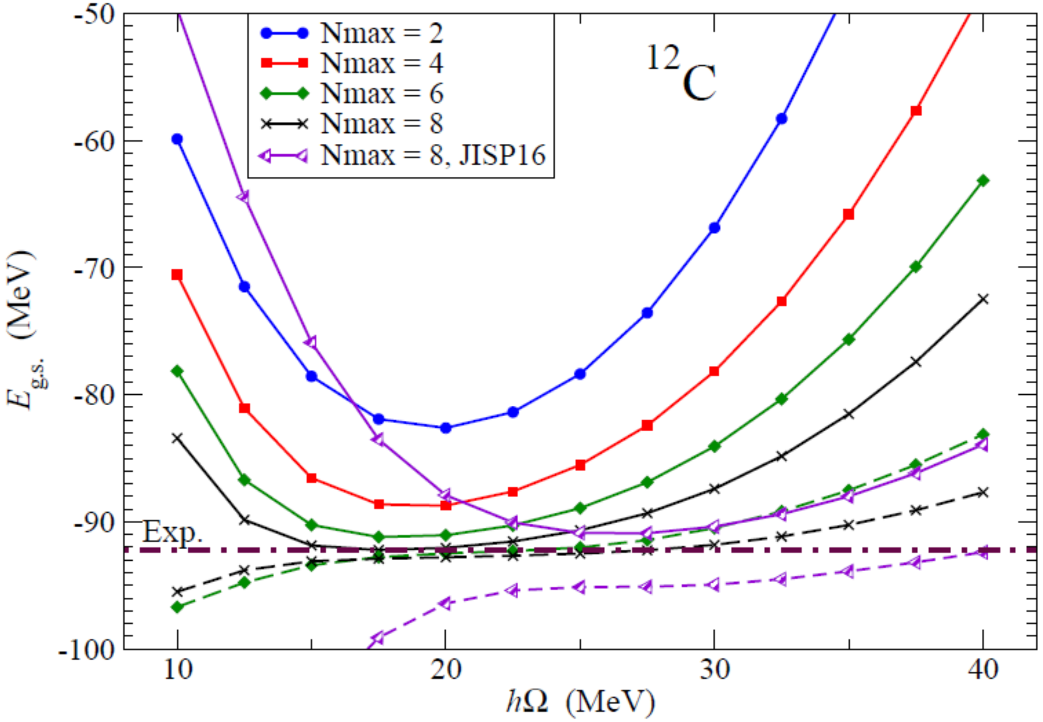
How to fit :

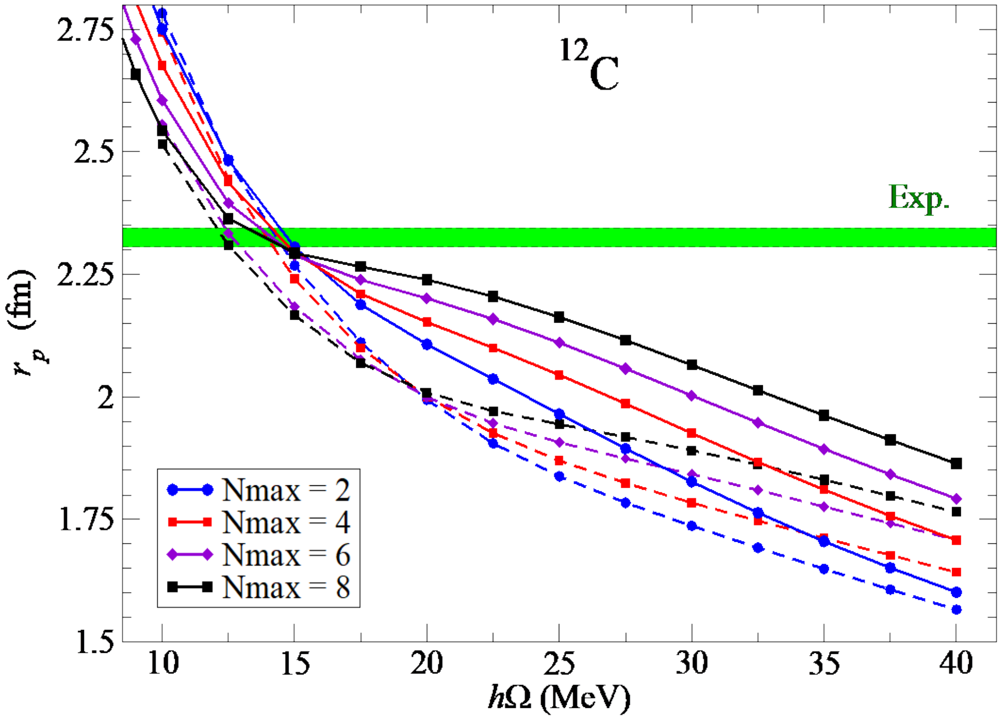
- ✓ using petsc (TAO)
- ✓ ground : ^{3}H, ^{4}He, ^{6}Li, ^{10}B, ^{12}C, ^{16}O and **^{8}He**
- ✓ excited : ^{6}Li [(3^+,0), (0^+,1)], ^{10}B [(1^+,0)], ^{12}C [(2^+,0)]
- ✓ target values are estimated from comparison between the results of NCSM calculation and experimental values

wave	$^1s_0$	$^3sd_1$	$^1p_1$	$^3p_0$	$^3p_1$	$^3pf_2$	$^3d_2$
angle	−2.997	4.461	5.507	1.785	4.299	−2.031	7.833

Daejeon16

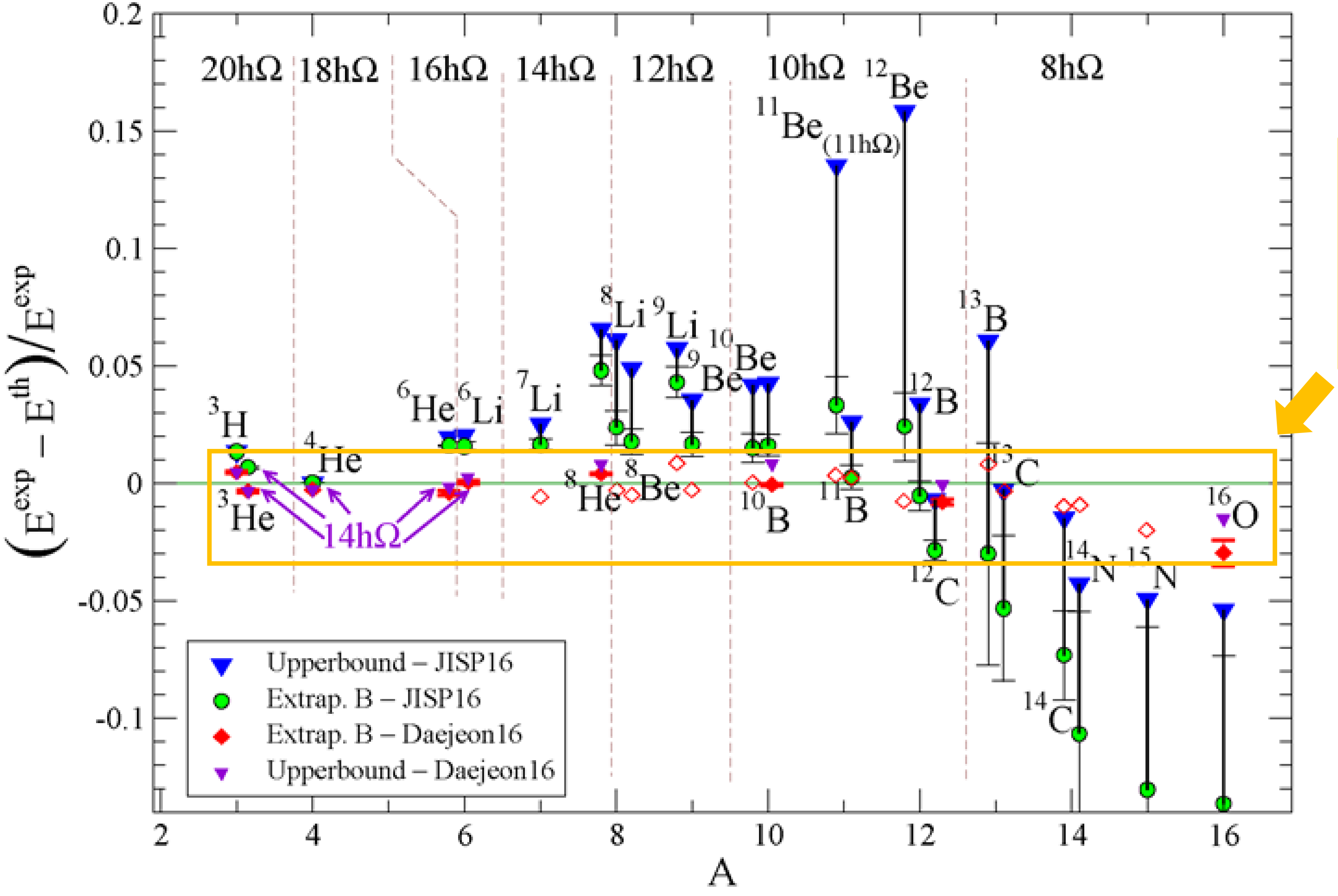
- Good convergence on NCSM calculation
- Good description of binding energies and spectra
- Improved description of other observables, e.g. rms radii





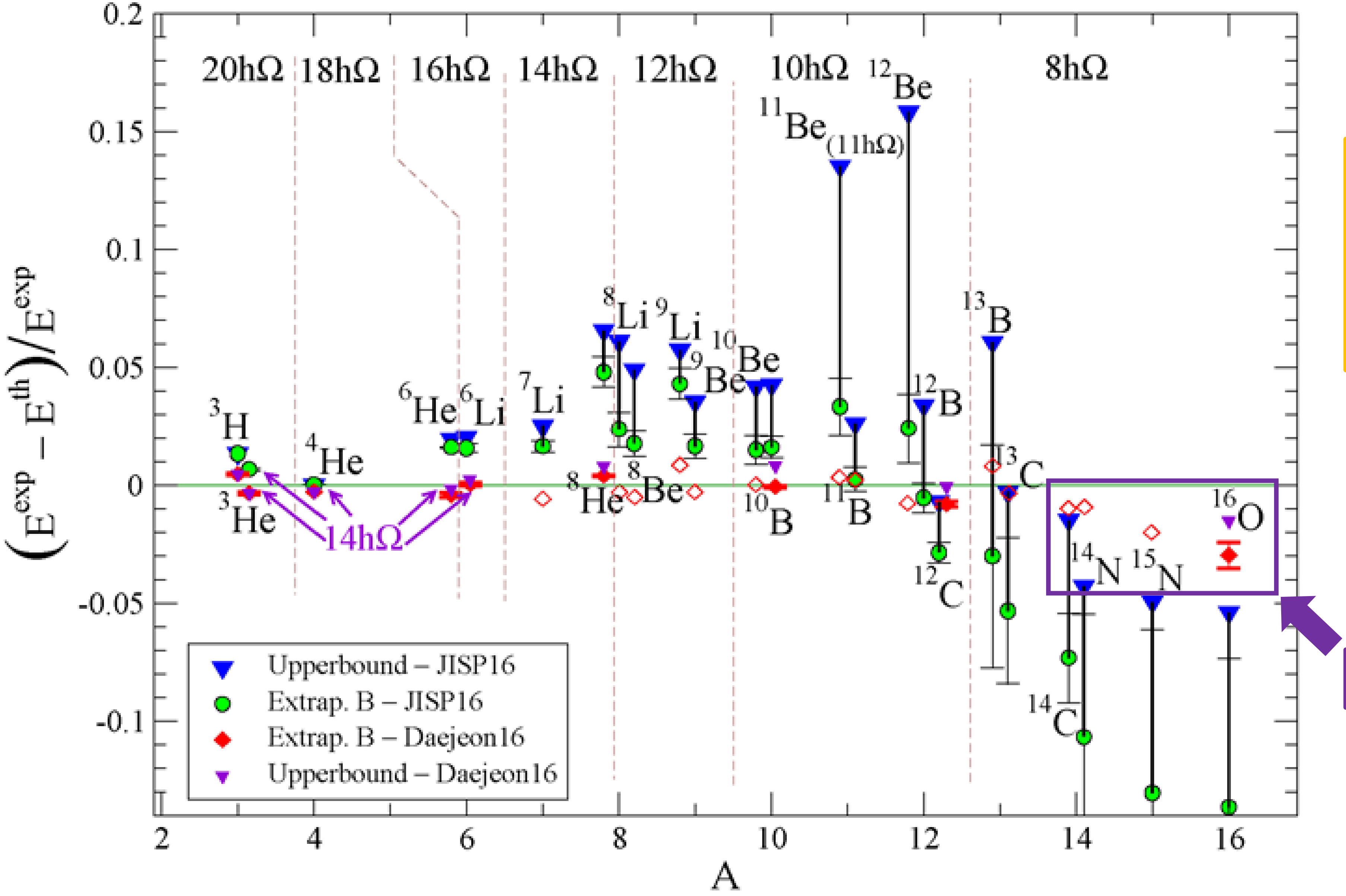
[A.M.Shirokov, *et al.*, Phys. Lett. B **761**, 87 (2016)]

Daejeon16



Using Daejeon16, the binding energies of many *p*-shell nuclei agree with experiments within error less than 3%

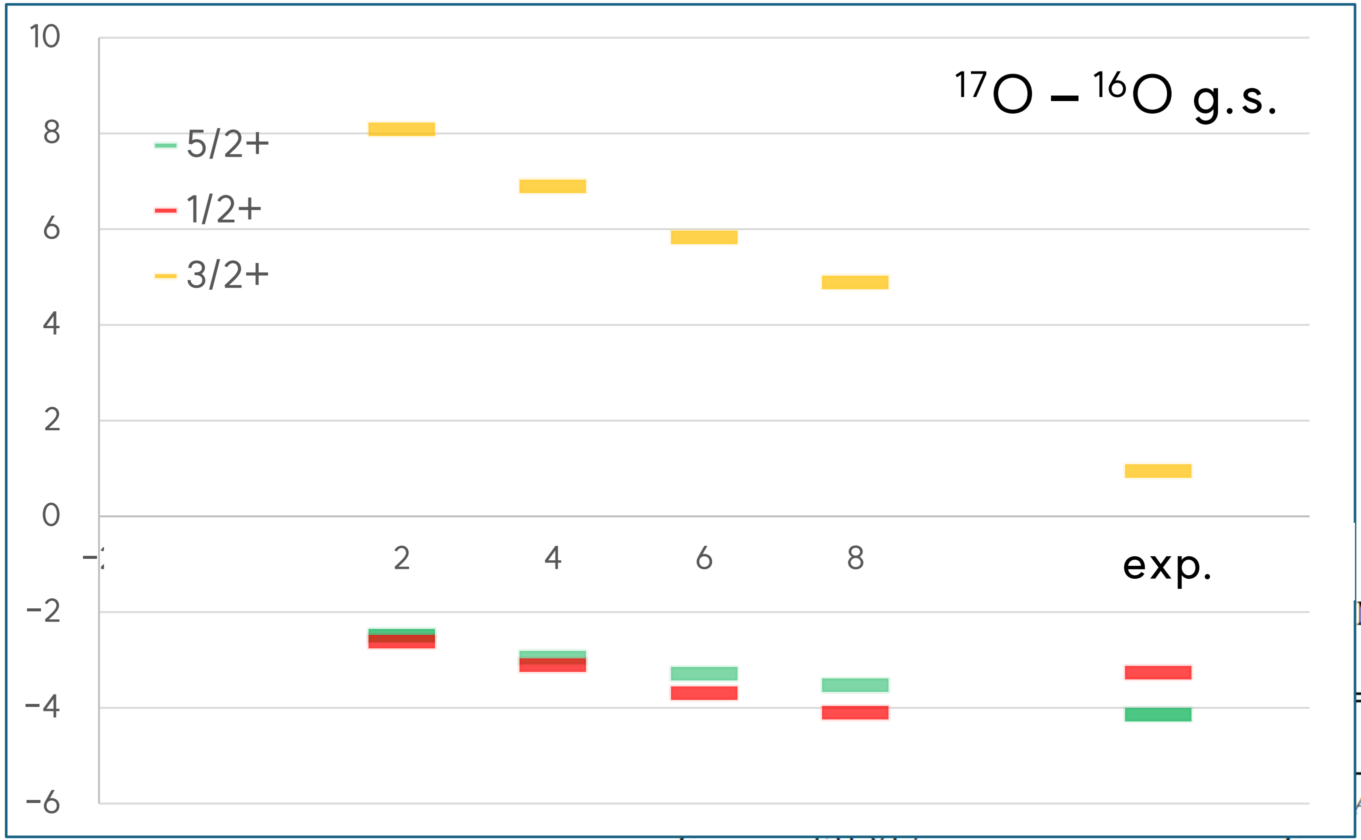
Daejeon16



Using Daejeon16, the binding energies of many *p*-shell nuclei agree with experiments within error less than 3%

over-bound around ^{16}O !

Daejeon16



MeV) obtained from the OLS transformed Daejeon16 potential (for

							Bare		
		$E_{core} = -121.817$			$A = 19$ $E_{core} = -121.783$			$A = 18$ $E_{core} = -118.307$	
(nlj)	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$
$\epsilon_\nu(nlj)$	−3.576	−3.302	6.675	−3.572	−3.299	6.677	−3.115	−2.953	6.889
$\epsilon_\pi(nlj)$	−0.077	0.291	9.974	−0.073	0.294	9.976	0.362	0.621	10.174

[N.Smirnova, *et al.*, Phys.Rev.C 100, 054329 (2019)]

Daejeon16

- $d_{5/2}$ and $s_{1/2}$ orbitals are inverted
- the spin-orbit splitting between $d_{5/2}$ and $d_{3/2}$ is larger than the empirical value

USDB single particle energies

$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$
-3.2079	-3.9257	2.1117

TABLE II. Neutron ("ν") and proton ("π") single-particle energies (in MeV) obtained from the OLS transformed Daejeon16 potential (for $A = 18$ and $A = 19$) and from bare Daejeon16.

	OLS						Bare		
	$A = 18$, $E_{\text{core}} = -121.817$			$A = 19$, $E_{\text{core}} = -121.783$			$A = 18$, $E_{\text{core}} = -118.307$		
$(nl\,j)$	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$
$\epsilon_\nu(nl\,j)$	−3.576	−3.302	6.675	−3.572	−3.299	6.677	−3.115	−2.953	6.889
$\epsilon_\pi(nl\,j)$	−0.077	0.291	9.974	−0.073	0.294	9.976	0.362	0.621	10.174

[N.Smirnova, *et al.*, Phys.Rev.C 100, 054329 (2019)]

New Daejeon16

N3LO interaction

↓

SRG
(similarity renormalization group)

↓

SRG-evolved N3LO

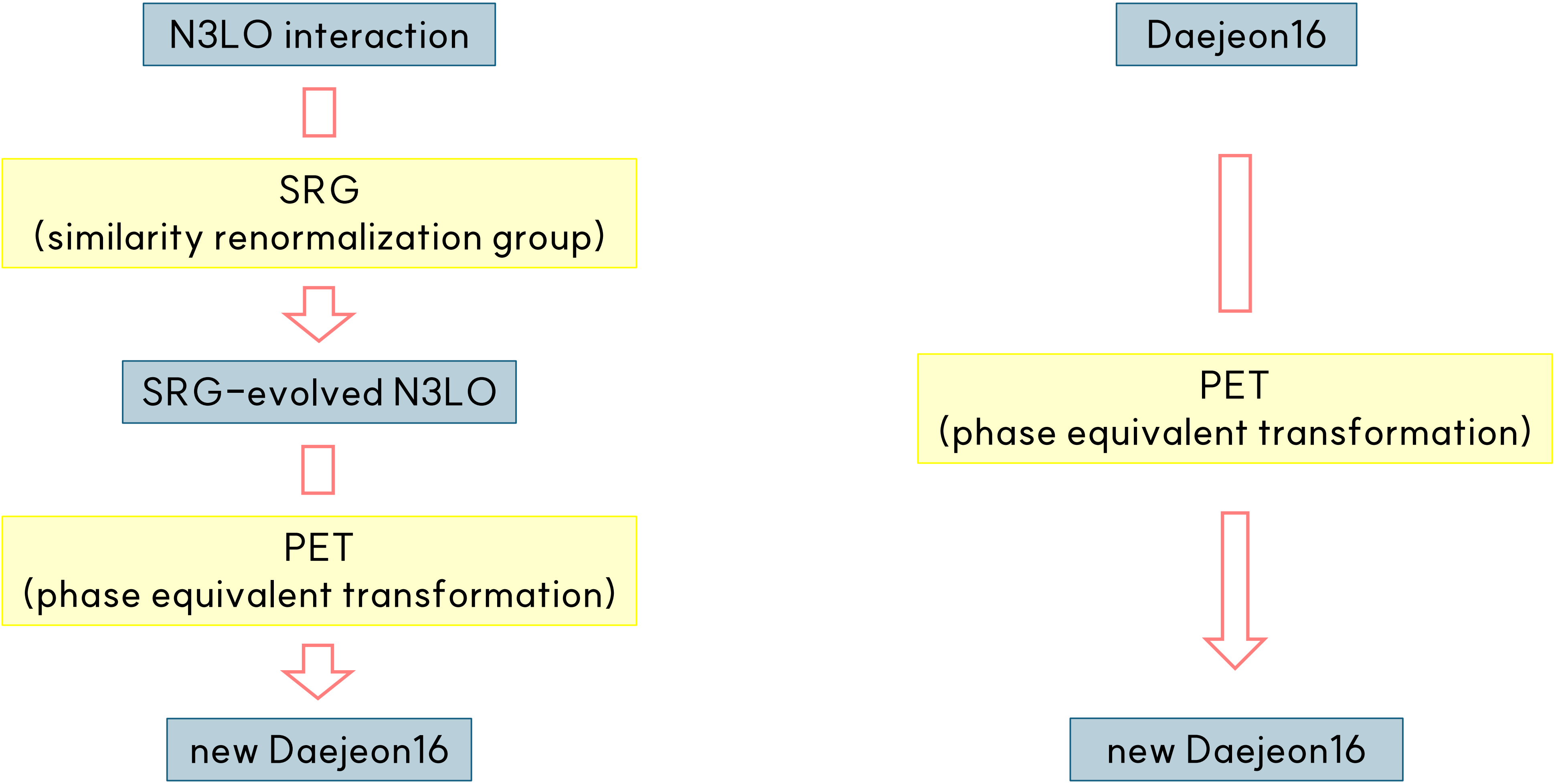
↓

PET
(phase equivalent transformation)

↓

new Daejeon16

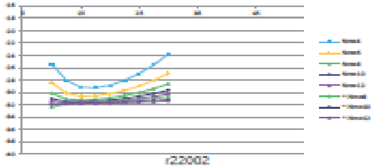
New Daejeon16

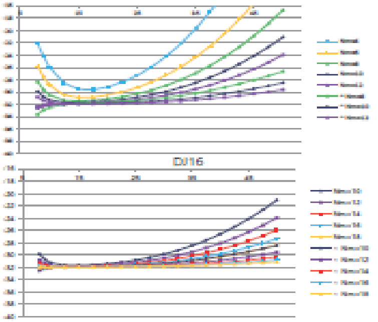


New Daejeon16

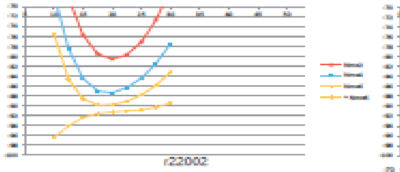
```
* 132_p1 (in case r005) ==> Daejeon16
::: pick up one parameter set from fevctlLog of case_r005
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -2.99726247452 4.46130674849 1.78535330322 4.29913281611 -2.030529
::: starting points -4.05927139467 5.5337758277 1.53321330908 3.2177908374
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.5 ex1:0.1 ex2:0.1) 8He(0.25) 10B(0.
::: res [3H(-8.434,+0.49%) 4He(-28.362,-0.27%) 6Li(-31.434,+0.25%) 8He(-29
12C(-88.629,-0.86%) 16O(-122.967,-2.74%)]


* r22002
::: final PETSc parameter set from fevctlLog of case_r22002
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -3.77744147919 5.26043333132 1.41811625717 4.58920685652 -2.750170
::: starting points -2.99726247452 4.46130674849 1.78535330322 4.299132816
Daejeon16 angle
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.3 ex1:0.1 ex2:0.1) 8He(0.2) 10B(0.4
::: res [3H(-8.445,+0.35%) 4He(-28.346,-0.21%) 6Li(-31.230,+0.89%) 8He(-29
12C(-86.990,+1.01%) 16O(-120.116,-0.35%)]
```

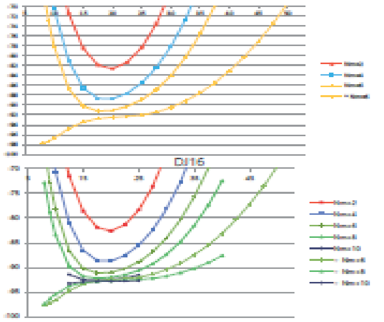
New Daejeon16



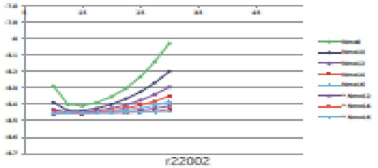


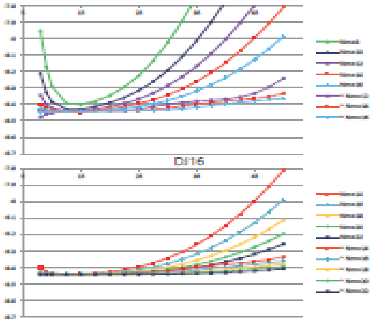
6Li	Nm=12	extrap.	Nm=18	extrap.
exp.		-31.995		-31.995
DJ16	-31.814	-31.939	-31.977	-32.007
r22002	-31.642	-31.776		



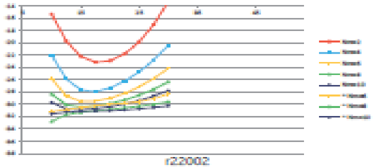


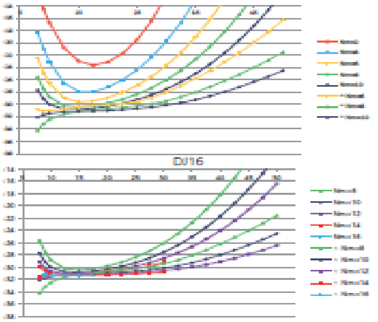
12C	Nm=6	extrap.	Nm=10	extrap.
exp.		-92.163		-92.163
DJ16	-91.048	-92.482	-92.619	-92.882
r22002	-89.600	-91.225		



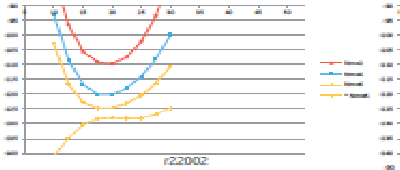


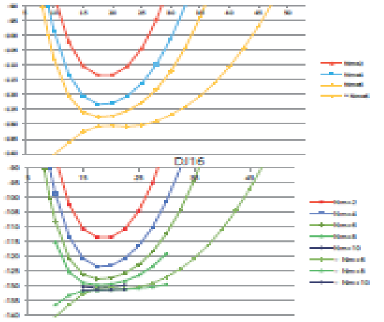
3H	Nm=16	extrap.	Nm=22	extrap.
exp.		-8.482		-8.482
DJ16	-8.442	-8.442	-8.442	-8.442
r22002	-8.452	-8.452		



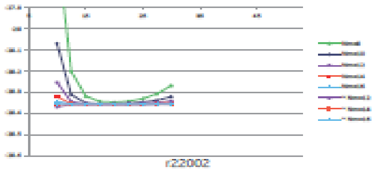


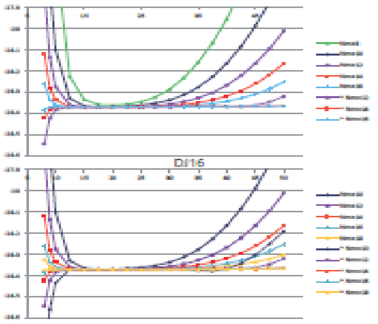
8He	Nm=10	extrap.	Nm=16	extrap.
exp.		-31.409		-31.409
DJ16	-30.787	-31.121	-31.225	-31.302
r22002	-30.833	-31.164		



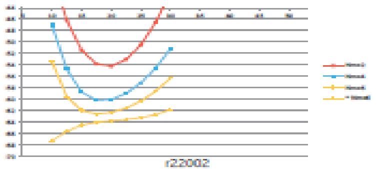


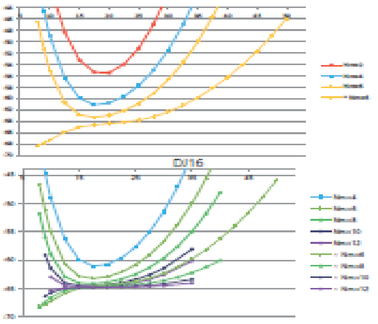
16O	Nm=6	extrap.	Nm=10	extrap.
exp.		-127.619		-127.619
DJ16	-127.523	-130.735	-130.481	-131.265
r22002	-124.693	-128.247		





4He	Nm=16	extrap.	Nm=18	extrap.
exp.		-28.296		-28.296
DJ16	-28.372	-28.372	-28.372	-28.372
r22002	-28.357	-28.357		





10B	Nm=6	extrap.	Nm=12	extrap.
exp.		-64.753		-64.753
DJ16	-63.264	-64.615	-64.713	-64.833
r22002	-62.340	-63.779		

New Daejeon16

```
* 132_p1 (in case r005) ==> Daejeon16
::: pick up one parameter set from fevctlLog of case_r005
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -2.99726247452 4.46130674849 1.78535330322 4.29913281611 -2.030529
::: starting points -4.05927139467 5.5337758277 1.53321330908 3.217790837
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.5 ex1:0.1 ex2:0.1) 8He(0.25) 10B(0.
::: res [3H(-8.434,+0.49%) 4He(-28.362,-0.27%) 6Li(-31.434,+0.25%) 8He(-29
12C(-88.629,-0.86%) 16O(-122.967,-2.74%)]

* r22002
::: final PETSc parameter set from fevctlLog of case_r22002
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -3.77744147919 5.26043333132 1.41811625717 4.58920685652 -2.75017
::: starting points -2.99726247452 4.46130674849 1.78535330322 4.29913281
Daejeon16 angle
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.3 ex1:0.1 ex2:0.1) 8He(0.2) 10B(0.4
::: res [3H(-8.445,+0.35%) 4He(-28.346,-0.21%) 6Li(-31.230,+0.89%) 8He(-29
12C(-86.990,+1.01%) 16O(-120.116,-0.35%)]
```

New Daejeon16

```
* 132_p1 (in case r005) ==> Daejeon16
::: pick up one parameter set from fevctlLog of case_r005
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -2.99726247452 4.46130674849 1.78535330322 4.2991328
::: starting points -4.05927139467 5.5337758277 1.5332133090
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.5 ex1:0.1 ex2:0.1) 8H
::: res [3H(-8.434,+0.49%) 4He(-28.362,-0.27%) 6Li(-31.434,+
12C(-88.629,-0.86%) 16O(-122.967,-2.74%)]

* r22002
::: final PETSc parameter set from fevctlLog of case_r22002
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -3.77744147919 5.26043333132 1.41811625717 4.5892068
::: starting points -2.99726247452 4.46130674849 1.785353303
Daejeon16 angle
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.3 ex1:0.1 ex2:0.1) 8H
::: res [3H(-8.445,+0.35%) 4He(-28.346,-0.21%) 6Li(-31.230,+
12C(-86.990,+1.01%) 16O(-120.116,-0.35%)]
```

```
# 6 andgles : 3p0 3p1 3pf[pp] 1d2 3d2 3dg[dd]

-fev etarget 1 0.5 0.5 -8.4762 0.03
-fev etarget 2 0 0 -28.2944 0.09
-fev etarget 3 1 0 -31.2973 0.25; 3 0 2.05 0.1; 0 1 3.56 0.
-fev etarget 4 0 2 -29.0514 0.2
-fev etarget 5 3 0 -59.8529 0.4; 1 0 2.05 0.1
-fev_etarget_6 0 0 -87.9101 0.7; 2 0 5.2 0.1
-fev_etarget_7 0 0 -118.6314 0.1
-fev_etarget_8 2.5 0.5 -119.1967 0.2; 0.5 0.5 2.0 0.03; 1.5

***** r22301 [PET @ HW=25MeV]
pointX : ['-6.13906014363', '0.968609219422', '-0.446820987
Current energy levels for nucleus #7: [((3.0, 0.0), [-92.21
[-119.203, -95.23199999999999]), ((4.0, 0.0), [-92.4849999
Current energy levels for nucleus #5: [((3.0, 0.0), [-59.03
[-57.393999999999998, -55.648000000000003]), ((2.0, 1.0), [
Current energy levels for nucleus #4: [((1.0, 2.0), [-22.05
-18.856999999999999]), ((2.0, 2.0), [-23.998999999999999, -
Current energy levels for nucleus #1: [((2.5, 0.5), [4.6479
1.0700000000000001]), ((1.5, 0.5), [2.532, 4.6429999999999
Current energy levels for nucleus #2: [((0.0, 1.0), [0.104]
[-28.495999999999999, -6.5529999999999999]), ((1.0, 1.0), [
Current energy levels for nucleus #3: [((3.0, 0.0), [-29.36
[-31.324999999999999]), ((2.0, 1.0), [-25.788]), ((2.0, 0.0
Current energy levels for nucleus #6: [((2.0, 0.0), [-81.61
[-87.626000000000005, -70.899000000000001]), ((1.0, 1.0), [
Current energy levels for nucleus #8: [((1.5, 0.5), [-104.9
-95.356999999999999]), ((0.5, 0.5), [-116.593])]

***** r23101  [PET @ HW=20MeV]
pointX : ['-2.99624757311', '0.615026311095', '0.0257810435
Current energy levels for nucleus #7: [((3.0, 0.0), [-93.86
-96.177000000000007]), ((4.0, 0.0), [-94.278000000000006])]
Current energy levels for nucleus #5: [((3.0, 0.0), [-59.20
[-57.405000000000001, -55.680999999999997]), ((2.0, 1.0), [
Current energy levels for nucleus #4: [((1.0, 2.0), [-21.74
((2.0, 2.0), [-23.901, -18.059000000000001])]
Current energy levels for nucleus #1: [((2.5, 0.5), [4.6429
0.5), [2.5249999999999999, 4.6440000000000001])]
Current energy levels for nucleus #2: [((0.0, 1.0), [0.1029
[-28.420000000000002, -6.5279999999999996]), ((1.0, 1.0), [
Current energy levels for nucleus #3: [((3.0, 0.0), [-29.15
[-31.119]), ((2.0, 1.0), [-25.599]), ((2.0, 0.0), [-25.114999999999
Current energy levels for nucleus #6: [((2.0, 0.0), [-81.72100000000
[-87.813999999999993, -71.314999999999998]), ((1.0, 1.0), [-71.84999
Current energy levels for nucleus #8: [((1.5, 0.5), [-105.315, -98.23
((0.5, 0.5), [-117.208])]
```

```
# 6 andgles : 3p0 3p1 3pf[pp] 1d2 3d2 3dg[dd]

***** r22301 [PET @ HW=25MeV]
pointX : ['-6.13906014363', '0.968609219422', '-0.44682098
Current energy levels for nucleus #7: [((3.0, 0.0), [-92.2
[-119.203, -95.23199999999999]), ((4.0, 0.0), [-92.484999
Current energy levels for nucleus #5: [((3.0, 0.0), [-59.0
[-57.393999999999998, -55.648000000000003]), ((2.0, 1.0),
Current energy levels for nucleus #4: [((1.0, 2.0), [-22.0
-18.856999999999999]), ((2.0, 2.0), [-23.998999999999999,
Current energy levels for nucleus #1: [((2.5, 0.5), [4.647
1.0700000000000001]), ((1.5, 0.5), [2.532, 4.642999999999
Current energy levels for nucleus #2: [((0.0, 1.0), [0.104
[-28.495999999999999, -6.5529999999999999]), ((1.0, 1.0),
Current energy levels for nucleus #3: [((3.0, 0.0), [-29.3
[-31.324999999999999]), ((2.0, 1.0), [-25.788]), ((2.0, 0.
Current energy levels for nucleus #6: [((2.0, 0.0), [-81.6
[-87.626000000000005, -70.899000000000001]), ((1.0, 1.0),
Current energy levels for nucleus #8: [((1.5, 0.5), [-104.
-95.356999999999999]), ((0.5, 0.5), [-116.593])]

# 7 angles : 1p1 3p0 3p1 3pf[pp] 1d2 3d2 3dg[dg]

***** r23401
-prob_starting_point 0.0 -6.13906014363 0.968609219422 -0.
-prob_lower_bound -20 -15 -10 -10 -10 0 -20
-prob upper bound 20 5 10 10 10 20 20
1:      ['0.0', '-6.13906014364', '0.96860921942', '-0.446
2 ~ 99 :
100:    ['-0.533909765247', '-9.1345431801', '1.1339210437
'8.17871400871']     1652.39301899
101:    ['-0.530722982632', '-9.13673271', '1.13393724379'
'8.18228492379']       1649.88586189
102:    ['-0.532636622368', '-9.13731999835', '1.132993751
'8.18414891898']     1650.23010976
103:    ['-0.530581890876', '-9.13674635921', '1.13413684
'8.18020496978']    1650.76243189
104:    ['-0.531283181845', '-9.13657941693', '1.13443100
'8.18330379078']     1650.25108073
```

New Daejeon16

```
* 132_p1 (in case r005) ==> Daejeon16
::: pick up one parameter set from fevctlLog of case_r005
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -2.99726247452 4.46130674849 1.78535330322 4.2991328
::: starting points -4.05927139467 5.5337758277 1.5332133090
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.5 ex1:0.1 ex2:0.1) 8H
::: res [3H(-8.434,+0.49%) 4He(-28.362,-0.27%) 6Li(-31.434,+
12C(-88.629,-0.86%) 16O(-122.967,-2.74%)]

* r22002
::: final PETSc parameter set from fevctlLog of case_r22002
::: 1s0 3sd[ss] 3p0 3p1 3pf[pp] 1p1 3d2
::: res -3.77744147919 5.26043333132 1.41811625717 4.5892068
::: starting points -2.99726247452 4.46130674849 1.785353303
Daejeon16 angle
::: 7 nuclei [3H(0.03) 4He(0.09) 6Li(0.3 ex1:0.1 ex2:0.1) 8H
::: res [3H(-8.445,+0.35%) 4He(-28.346,-0.21%) 6Li(-31.230,+
12C(-86.990,+1
```

```
# 6 andgles : 3p0 3p1 3pf[pp] 1d2 3d2 3dg[dd]

-fev etarget 1 0.5 0.5 -8.4762 0.03
-fev etarget 2 0 0 -28.2944 0.09
-fev etarget 3 1 0 -31.2973 0.25; 3 0 2.05 0.1; 0 1 3.56 0.
-fev etarget 4 0 2 -29.0514 0.2
-fev etarget 5 3 0 -59.8529 0.4; 1 0 2.05 0.1
-fev_etarget_6 0 0 -87.9101 0.7; 2 0 5.2 0.1
-fev_etarget_7 0 0 -118.6314 0.1
-fev_etarget_8 2.5 0.5 -119.1967 0.2; 0.5 0.5 2.0 0.03; 1.5

***** r22301 [PET @ HW=25MeV]
pointX : ['-6.13906014363', '0.968609219422', '-0.446820987
Current energy levels for nucleus #7: [((3.0, 0.0), [-92.21
[-119.203, -95.2319999999999]), ((4.0, 0.0), [-92.4849999
Current energy levels for nucleus #5: [((3.0, 0.0), [-59.03
[-57.3939999999998, -55.6480000000003]), ((2.0, 1.0), [
Current energy levels for nucleus #4: [((1.0, 2.0), [-22.05
-18.8569999999999]), ((2.0, 2.0), [-23.9989999999999, -
Current energy levels for nucleus #1: [((2.5, 0.5), [4.6479
1.0700000000000001]), ((1.5, 0.5), [2.532, 4.64299999999999
Current energy levels for nucleus #6: [((2.0, 0.0)nergy levels for nucleus #2: [((0.0, 1.0), [0.104]
[-87.655000000000001, -70.918000000000006]), ((1.0, 1.0),99999999999, -6.5529999999999]), ((1.0, 1.0), [
Current energy levels for nucleus #8: [((1.5, 0.5), [-104nergy levels for nucleus #3: [((3.0, 0.0), [-29.36
-95.393000000000001]), ((0.5, 0.5), [-116.619])]99999999999]), ((2.0, 1.0), [-25.788]), ((2.0, 0.0
                                                 nergy levels for nucleus #6: [((2.0, 0.0), [-81.61
***** r23304                                     00000000005, -70.899000000000001]), ((1.0, 1.0), [
-prob_starting_point 0.0 -6.13906014363 0.968609219422 -0
-prob_lower_bound -20 -15 -10 -10 -10 0 -10
-prob upper bound 20 5 10 10 10 20 10
1:      ['0.0', '-6.13906014364', '0.96860921942', '-0.44
1753.31517286
2~28
29:     ['-0.00238453698183', '-6.13957670481', '0.970827
'1.02026693348']          1753.78969219
30:     ['0.00367268707103', '-6.13944318864', '0.9659403
'1.01911557011'] 1754.63064306

Result: ['0.0', '-6.13906014364', '0.96860921942', '-0.44
```

No change of 1p1 channel is better..

```
999]), ((2.0, 0.0), [-25.4259999999998])]
105.28100000000001, -97.772000000000006]), ((2.5, 0.5), [-118.0

-0.446820987236 -1.2750766695 8.61612476985 1.01787306243
```

starting from r22301

```
.44682098724', '-1.2750766695', '8.61612476985', '1.01787306243

nergy levels for nucleus #2: [((0.0, 1.0), [0.1029
00000000002, -6.5279999999996]), ((1.0, 1.0), [
nergy levels for nucleus #3: [((3.0, 0.0), [-29.15
), ((2.0, 1.0), [-25.599]), ((2.0, 0.0), [-25.1149999999999
nergy levels for nucleus #6: [((2.0, 0.0), [-81.72100000000
99999999993, -71.3149999999998]), ((1.0, 1.0), [-71.84999
Current energy levels for nucleus #8: [((1.5, 0.5), [-105.315, -98.23
((0.5, 0.5), [-117.208])]
```

```
# 6 andgles : 3p0 3p1 3pf[pp] 1d2 3d2 3dg[dd]

***** r22301 [PET @ HW=25MeV]
pointX : ['-6.13906014363', '0.968609219422', '-0.44682098
Current energy levels for nucleus #7: [((3.0, 0.0), [-92.2
[-119.203, -95.2319999999999]), ((4.0, 0.0), [-92.484999
Current energy levels for nucleus #5: [((3.0, 0.0), [-59.0
[-57.3939999999998, -55.6480000000003]), ((2.0, 1.0),
Current energy levels for nucleus #4: [((1.0, 2.0), [-22.0
-18.8569999999999]), ((2.0, 2.0), [-23.9989999999999,
Current energy levels for nucleus #1: [((2.5, 0.5), [4.647
1.0700000000000001]), ((1.5, 0.5), [2.532, 4.6429999999999
Current energy levels for nucleus #2: [((0.0, 1.0), [0.104
[-28.4959999999999, -6.5529999999999]), ((1.0, 1.0),
Current energy levels for nucleus #3: [((3.0, 0.0), [-29.3
[-31.3249999999999]), ((2.0, 1.0), [-25.788]), ((2.0, 0.
Current energy levels for nucleus #6: [((2.0, 0.0), [-81.6
[-87.626000000000005, -70.899000000000001]), ((1.0, 1.0),
Current energy levels for nucleus #8: [((1.5, 0.5), [-104.
-95.3569999999999]), ((0.5, 0.5), [-116.593])]

# 7 angles : 1p1 3p0 3p1 3pf[pp] 1d2 3d2 3dg[dg]

***** r23401
-prob_starting_point 0.0 -6.13906014363 0.968609219422 -0.
bound -20 -15 -10 -10 -10 0 -20
bound 20 5 10 10 10 20 20
.0', '-6.13906014364', '0.96860921942', '-0.446
).533909765247', '-9.1345431801', '1.1339210437
371']       1652.39301899
).530722982632', '-9.13673271', '1.13393724379'
379']       1649.88586189
).532636622368', '-9.13731999835', '1.132993751
398']       1650.23010976
).530581890876', '-9.13674635921', '1.13413684C
978']       1650.76243189
).531283181845', '-9.13657941693', '1.13443100E
'8.18330379078']      1650.25108073
```

New Daejeon16

Daejeon16 : from SRG evolved N3LO with λ=1.5 fm^{-1} at ħΩ=25 MeV

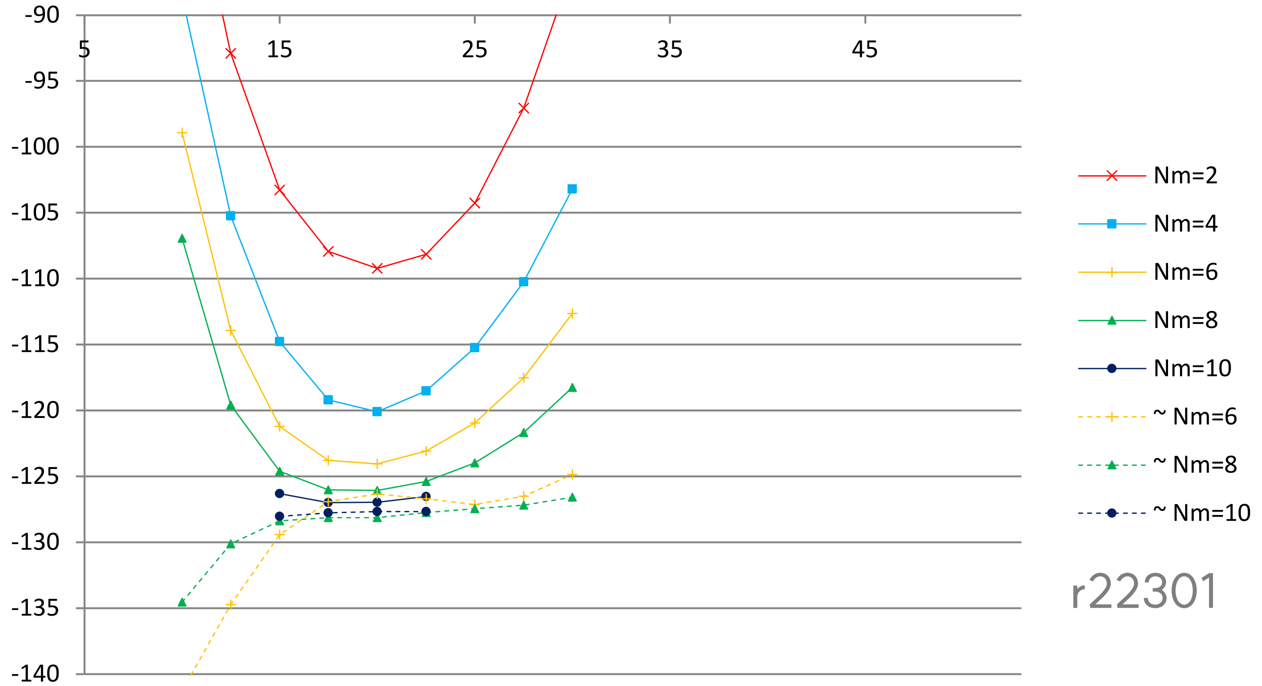
wave	$^1s_0$	$^3sd_1$	$^1p_1$	$^3p_0$	$^3p_1$	$^3pf_2$	$^1d_2$	$^3d_2$	$^3dg_3$
Daejeon16	−2.997	4.461	5.507	1.785	4.299	−2.031	–	7.833	–

New Daejeon16 : from Daejeon16 at ħΩ=25 MeV

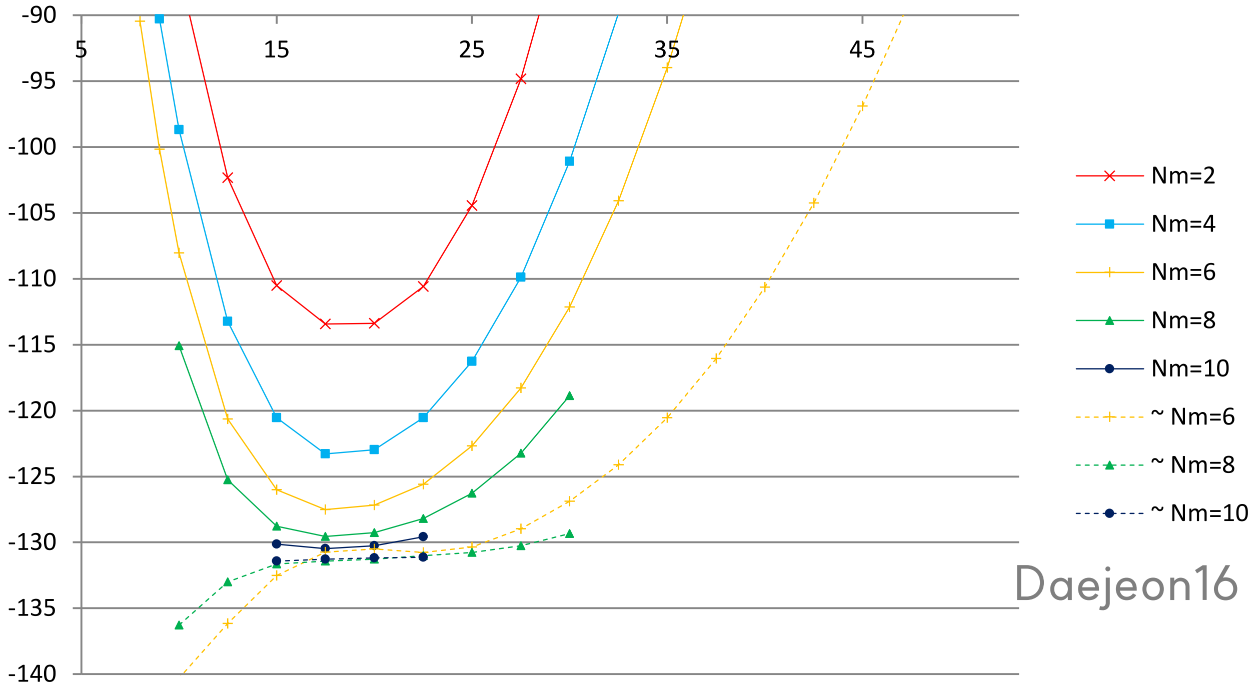
wave	$^1s_0$	$^3sd_1$	$^1p_1$	$^3p_0$	$^3p_1$	$^3pf_2$	$^1d_2$	$^3d_2$	$^3dg_3$
r22301	–	–	–	−6.139	0.969	−0.446	−1.275	8.616	1.018
r23701	–	1.605	–	−2.124	1.324	2.721	−3.644	−0.502	−11.538

Results

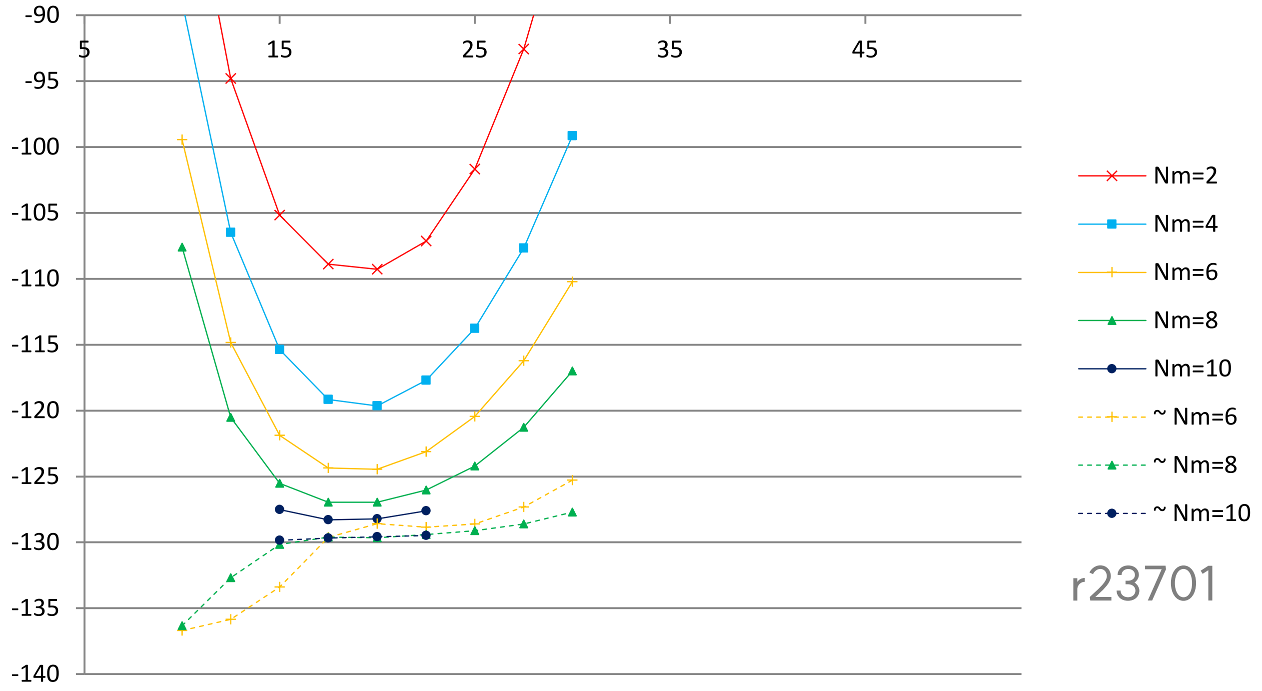
^{16}O	Nmax=8	extrapol.	Nmax=10	extrapol.
exp.		−127.619		−127.619
Daejeon16	−129.553	−131.411	−130.481	−131.265
r22301	−126.077	−128.144	−126.971	−127.687
r23701	−126.959	−129.597	−128.223	−129.565



r22301



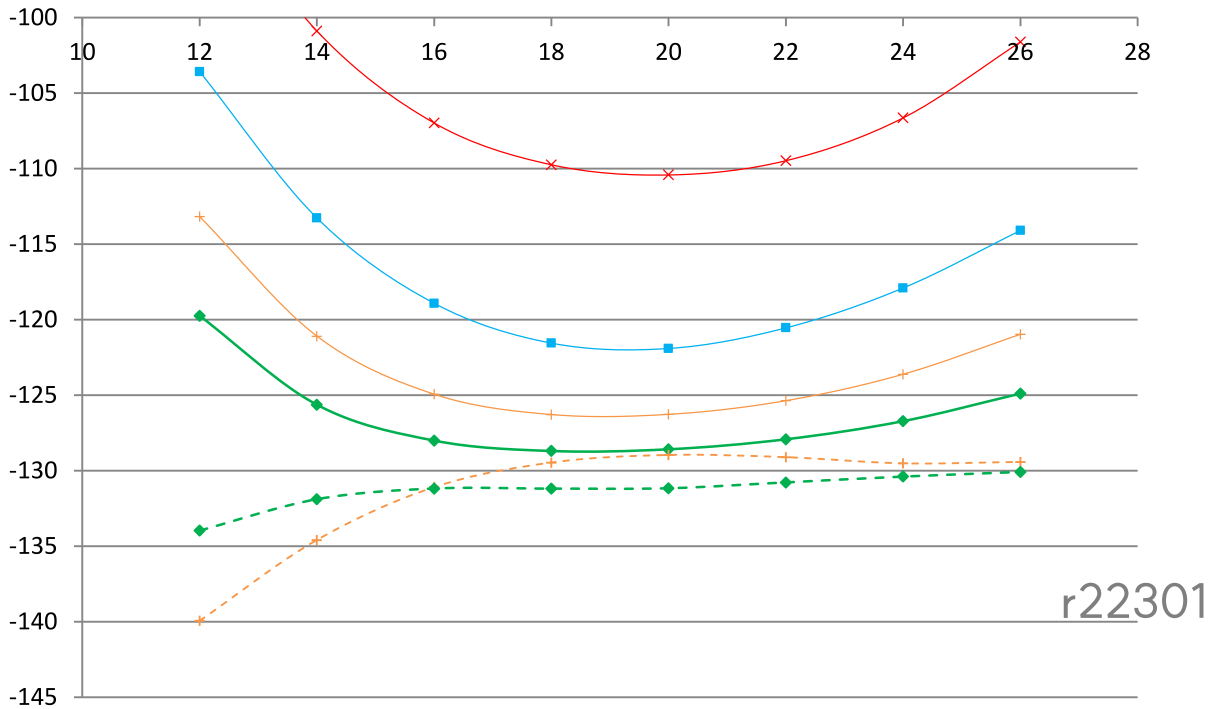
Daejeon16

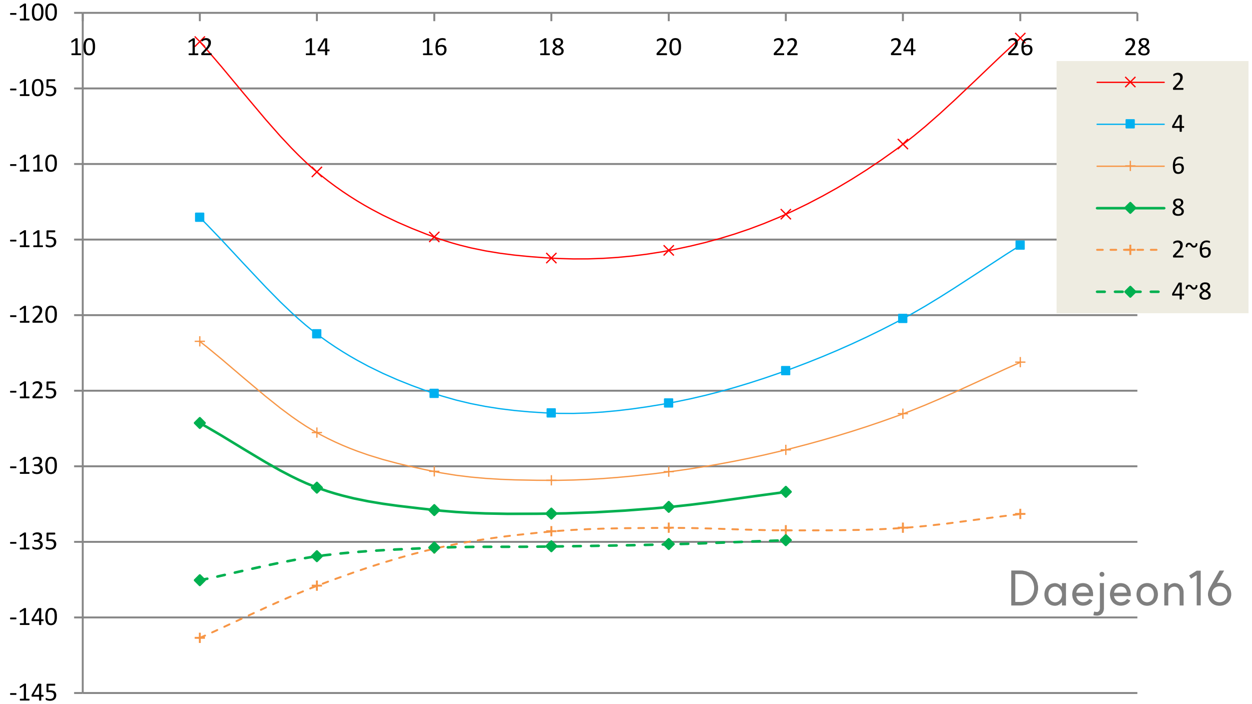


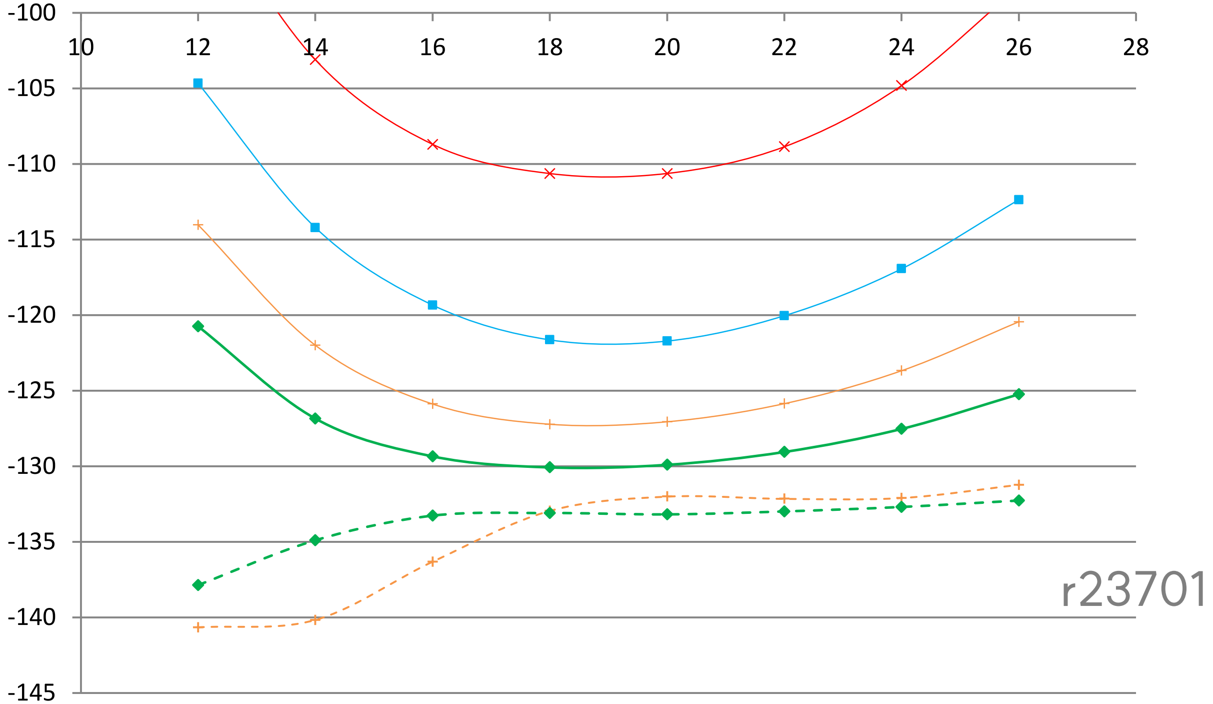
r23701

Results

^{17}O	Nmax=6	extrapol.	Nmax=8	extrapol.
exp.		−131.765		−131.765
Daejeon16	−130.347	−134.311	−133.133	−135.302
r22301	−126.280	−129.454	−128.689	−131.179
r23701	−127.215	−132.947	−130.075	−133.085

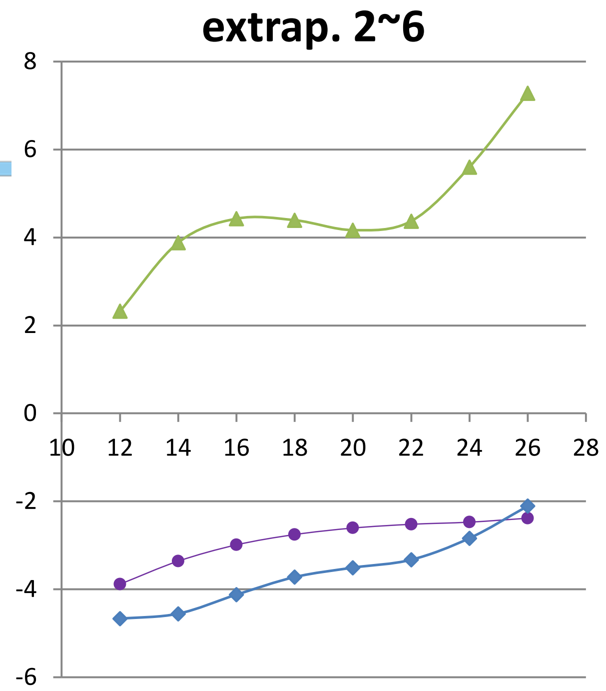




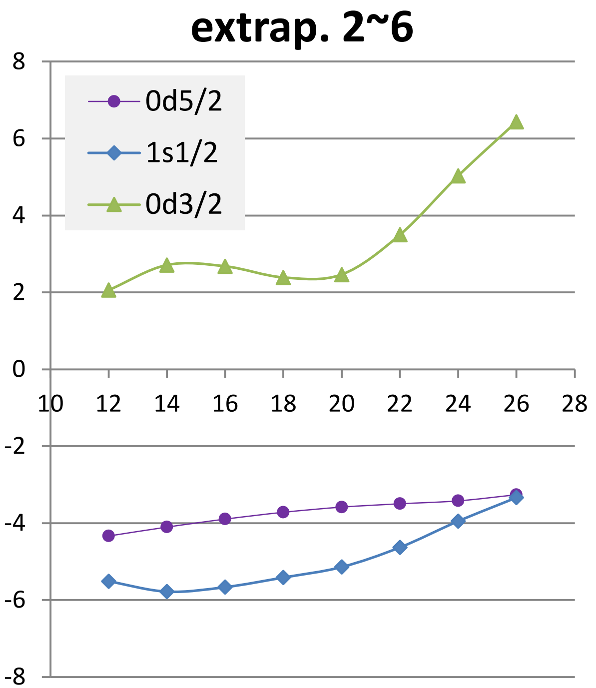


Results

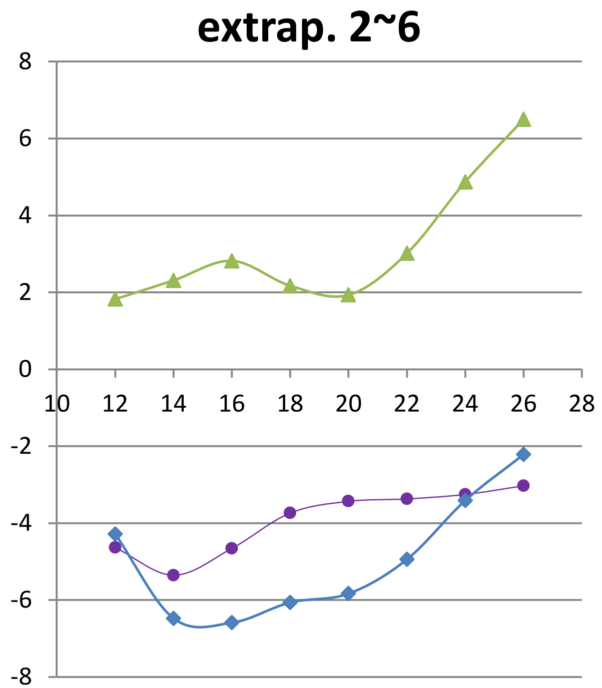
1. To take differences between ^{17}O spectra and ^{16}O g.s.
2. To adopt extrapolation



r22301



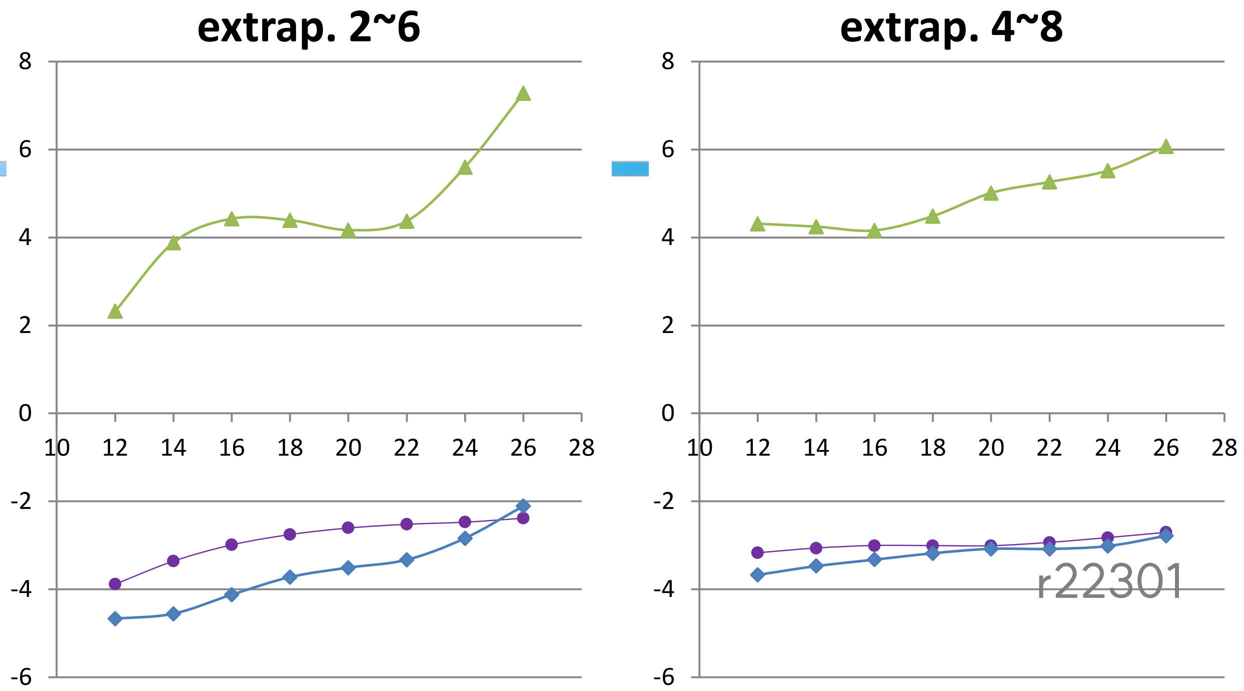
Daejeon16

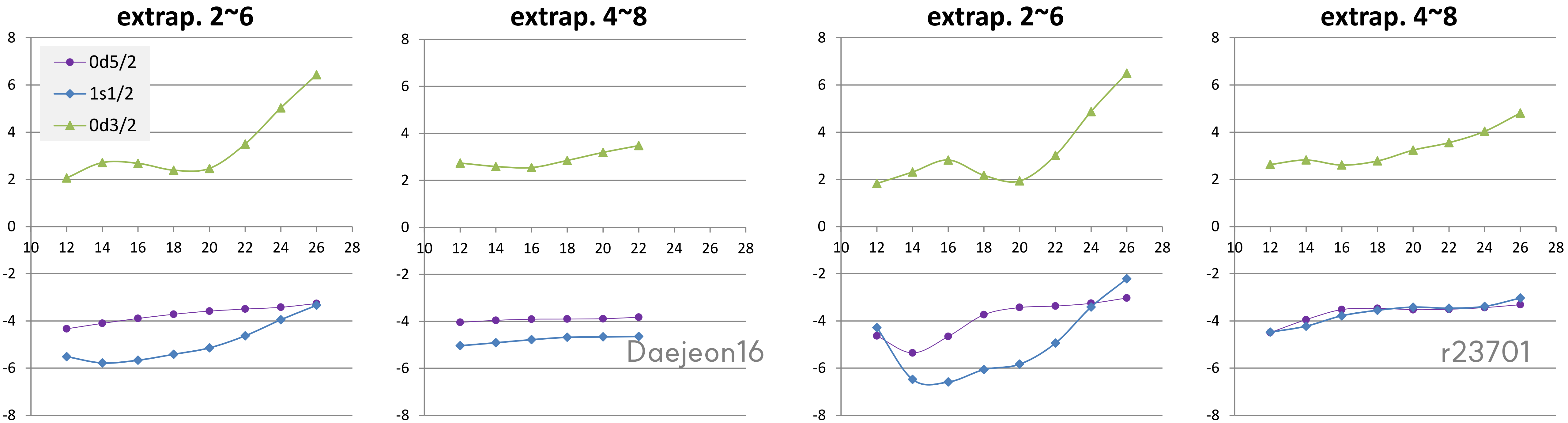


r23701

Results

1. To take differences between ^{17}O spectra and ^{16}O g.s.
2. To adopt extrapolation

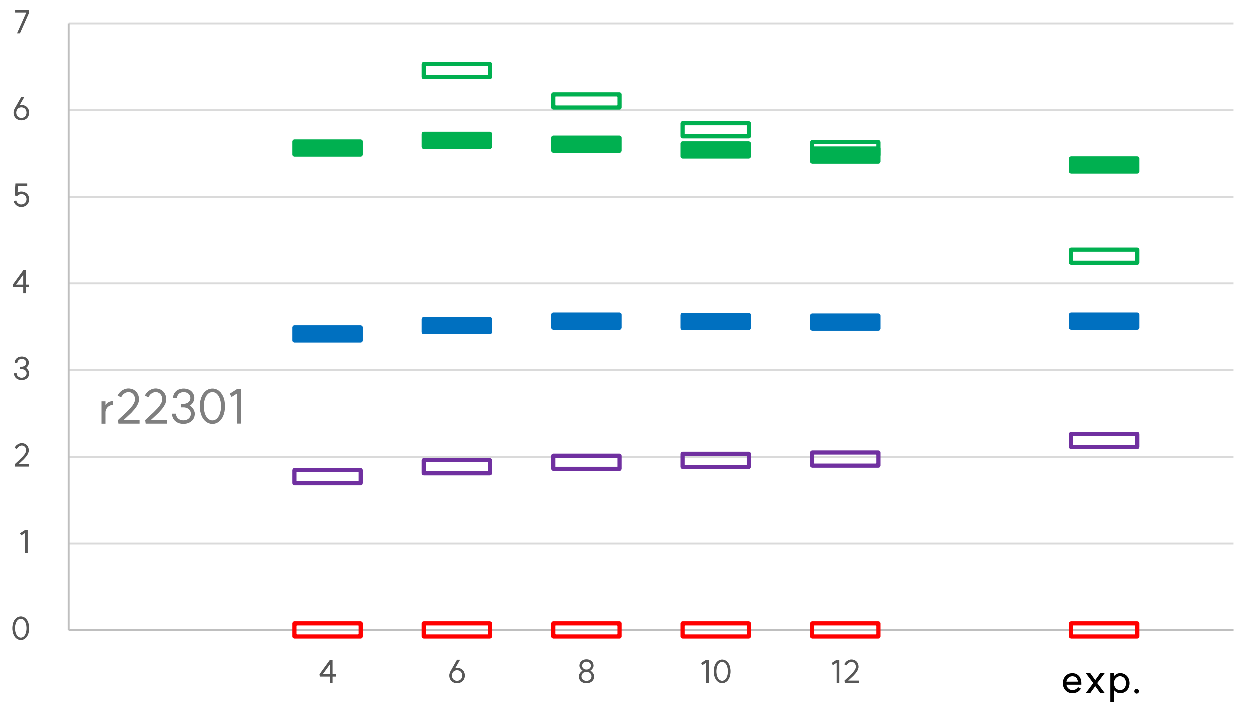


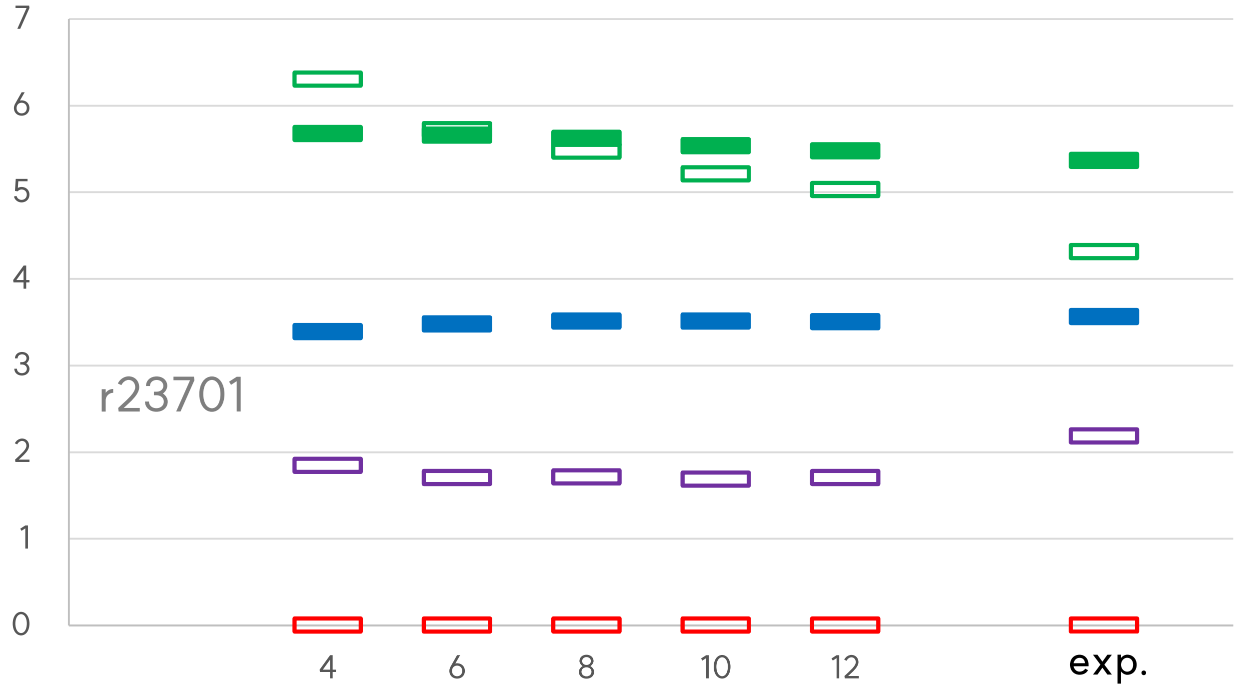


Results

^6Li @ Nmax=12 & HW=15.0 MeV

^6Li	exp.	Daejeon16	r22301	r23701
3+,0	2.186	1.885	1.971	1.705
0+,1	3.563	3.538	3.552	3.505
2+,0	4.312	5.062	5.555	5.028
2+,1	5.366	5.603	5.480	5.474

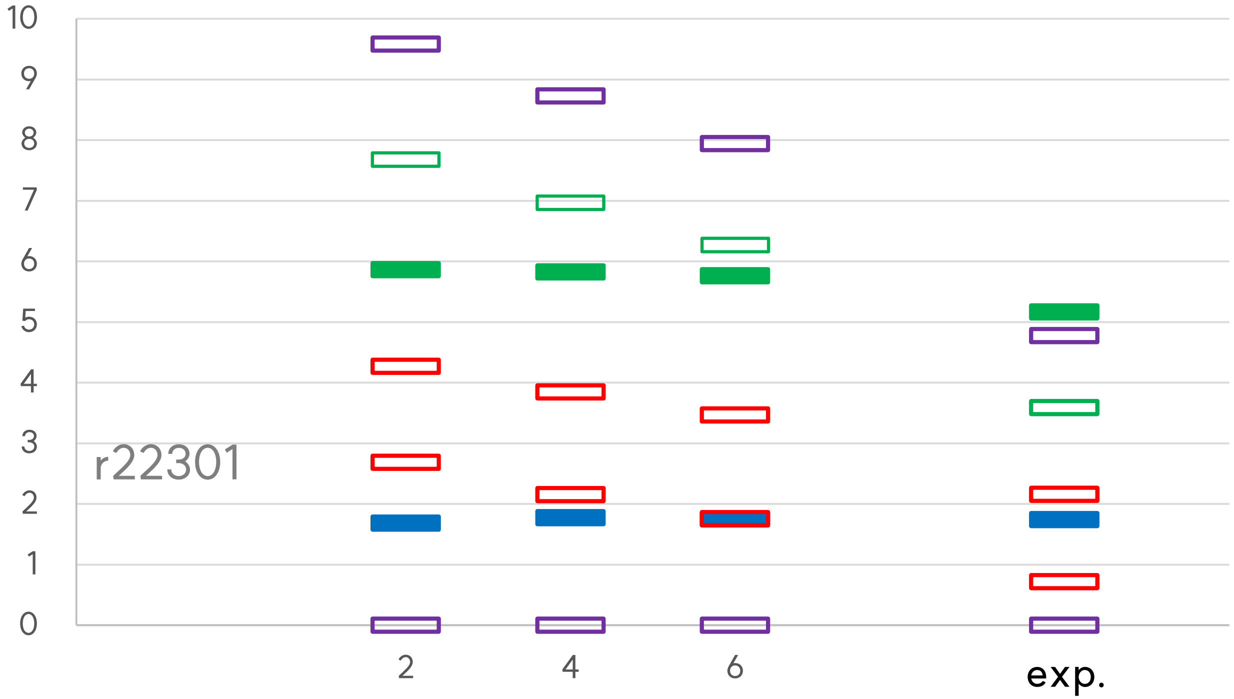


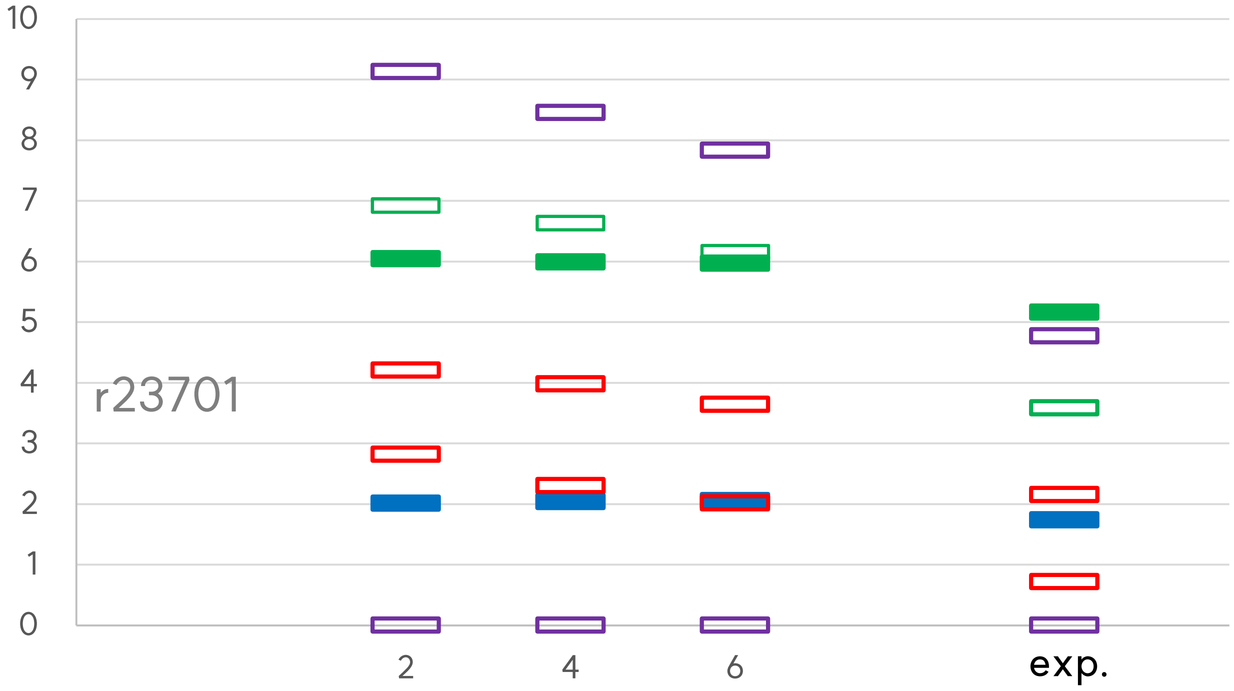


Results

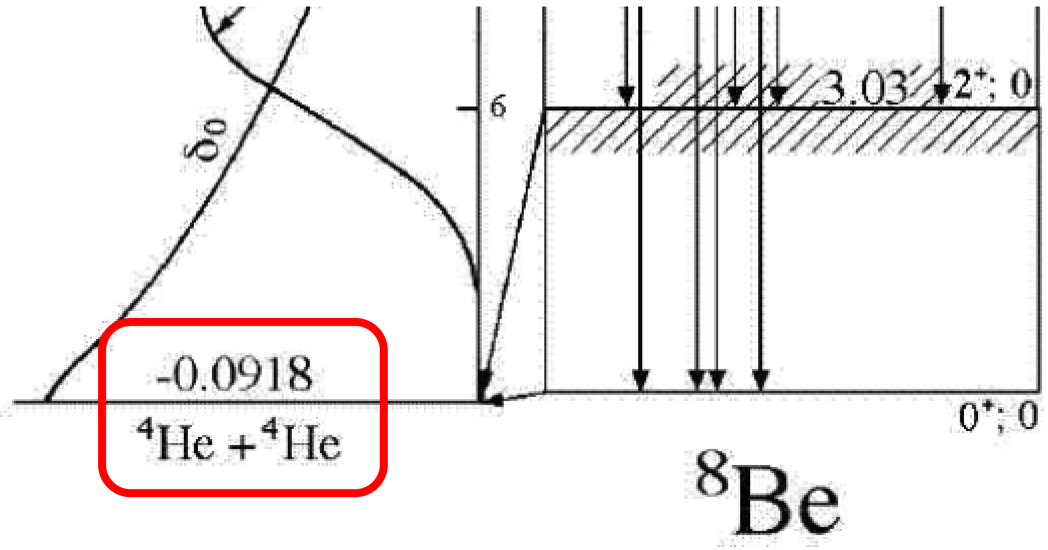
^{10}B @ Nmax=6 & HW=17.5 MeV

^{10}B	exp.	Daejeon16	r22301	r23701
1+,0	0.718	1.693	1.756	2.019
0+,1	1.740	1.960	1.750	2.058
1+,0	2.154	3.401	3.467	3.644
2+,0	3.587	5.771	6.268	6.150
3+,0	4.774	7.457	7.941	7.832
2+,1	5.164	5.879	5.763	5.968





Results



^{4}He	Nm=16	extrapol.	Nm=18	extrapol.
exp.		−28.296		−28.296
Daejeon16	−28.372	−28.372	−28.372	−28.372
r22301	−28.498	−28.498		
r23701	−28.356	−28.356		

^{4}He+^{4}He		extrapol.	Nm=18	extrapol.
exp.		−56.592		−56.592
Daejeon16		−56.744		−56.744
r22301		−56.996		
r23701		−56.712		

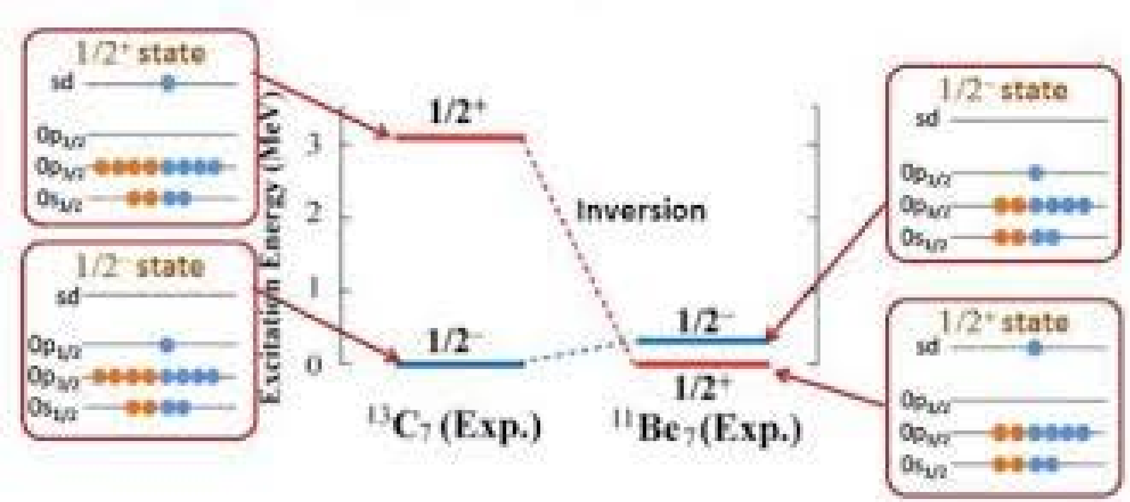
^{8}Be – (^{4}He+^{4}He)	~ Nm=12	~ Nm=14
exp.	+0.092	
Daejeon16	−0.031	−0.103
r22301	+0.777	?
r23701	+0.277	?

^{8}Be	Nm=12	extrapol.	Nm=14	extrapol.
exp.		−56.500		−56.500
Daejeon16	−56.470	−56.775	−56.629	−56.847
r22301	−55.940	−56.219		
r23701	−56.044	−56.435		

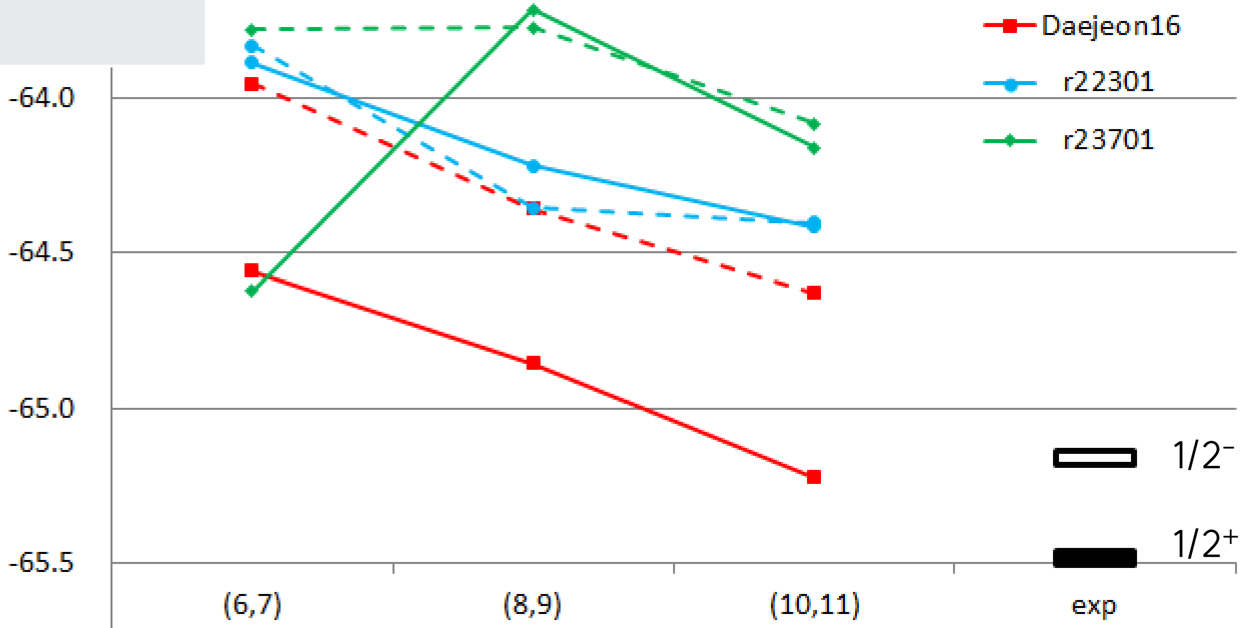
Results

^{11}Be	Nmax	Daejeon16	r22301	r23701	exp.
$1/2^+$	3~7	−64.6	−63.9	−64.6	−65.483
	5~9	−64.9(3)	−64.2(3)	−63.7(9)	
	7~11	−65.22(7)	−64.4(2)	−64.2(4)	
$1/2^-$	2~6	−64.0	−63.8	−63.8	−65.163
	4~8	−64.4(4)	−64.4(5)	−63.78(1)	
	6~10	−64.62(2)	−64.40(5)	−64.1(3)	

- $1/2^+$ ground state of ^{11}Be
  - ✓ loosely bound by 0.5 MeV with respect to the ^{10}Be+n threshold
  - ✓ slightly separated by only 0.3 MeV from excited $1/2^-$ state



[From Alessia Di Pietro, EURISOL-DF 2017]



Summary and Outlook

- **beyond Daejeon16**

  - We have adopted additional PET to Daejeon16 in order to extend its application to the heavier region

  - Two candidates for the new Daejeon16 interaction show the better description the $p\,/\,sd$−shell nuclei at least around ^{16}O

  - Further study
    - ✓ More study various nuclei using the new candidate interactions
    - ✓ To find appropriate partial waves and optimized angles

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1, Gukjegwahak-ro, Yuseong-gu, Daejeon, Korea

THANK YOU !

01 Backup

- Backup

- PET (phase-equivalent transformation)

Hamiltonian can be expressed as an infinite dimensional matrix $[H]$ in the oscillator basis $\{|n\rangle\}$.

$$\text{matrix elements of } [H] \ : H_{nm} = \langle \mathrm{n}|H|m\rangle$$

PET is based on the unitary transformation as

$$[\widetilde{H}] = [U^{\dagger}]\,[H]\,[U]$$

with the help of the unitary matrix $[U]$ which is supposed to be of the form

$$[U] = [U_0] \oplus [I] = \begin{pmatrix} [U_0] & 0 \\ 0 & [I] \end{pmatrix}$$

where $[I]$ is the infinite dimensional unit matrix.

- PET (phase-equivalent transformation)

Clearly the spectra of Hamiltonians H and $\widetilde{H}$ are identical. Corresponding eigenfunction can be written as

$$[\widetilde{\Psi}] = [U^{\dagger}]\ [\Psi]$$

in the oscillator basis $\{|n\rangle\}$. Then the difference is

$$\Delta[\Psi] = [\widetilde{\Psi}] - [\Psi] = \left([U^{\dagger}] - [I] \right) [\Psi] = \begin{pmatrix} [U_0] & 0 \\ 0 & 0 \end{pmatrix} [\Psi] \,.$$

That is, the only difference is shown as a superposition of a finite number of functions. If we consider a simple 2×2 matrix $[U_0]$ as

$$[U_0] = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} ,$$

then only two oscillator basis $|0\rangle$ and $|1\rangle$ are related.

- PET (phase-equivalent transformation)

The superposition of a finite number of $\mathcal{L}^2$ functions cannot affect the asymptotics of scattering wave functions.

$$\widetilde{\Psi} = \Psi + \sum_i^N c_i \, |i\rangle \quad \Rightarrow \quad \lim_{r\to\infty} \widetilde{\Psi}(r) = \lim_{r\to\infty} \Psi(r) \qquad \text{when} \quad |i\rangle \in \mathcal{L}^2$$

Since the scattering phase shifts and the $S-$ matrix are defined through the asymptotic behavior of the wave functions, the phase shifts δ associated with the Ψ and $\widetilde{\Psi}$ are identical.

So the Hamiltonians H and $\widetilde{H}$ are phase-equivalent.

- PET (phase-equivalent transformation)

Practically,

$$
\begin{aligned}
[\tilde{V}] &= [V] + [\Delta V] \\
&= [V] + \left([\tilde{H}] - [H] \right) \\
&= [\tilde{H}] - \left([H] - [V] \right) \\
&= [U^{\dagger}] \left([V] + [T] \right) [U] - [T]
\end{aligned}
$$

1. Add kinetic term to original potential in order to construct Hamiltonian
2. Take unitary transformation
3. Subtract kinetic term to obtain potential part

New Daejeon16

- Binding energies

Nucleus	Nature	r22301		r23701		Daejeon16			JISP16		
		Theory	hw	Theory	hw	Theory	hw	Nmax	Theory	hw	Nmax
^{3}H	8.482	8.472	12.5	8.439	12.5	8.442	12.5	16	8.370(3)	15	20
^{3}He	7.718	7.773	12.5	7.742	12.5	7.744	12.5	16	7.667(5)	17.5	20
^{4}He	28.296	28.498(0)	15	28.356(0)	17.5	28.372(0)	17.5	16	28.299(0)	22.5	18
^{6}He	29.269	29.45(3)	12.5	29.183(1)	15	29.39(3)	12.5	14	28.80(5)	17.5	16
^{8}He	31.409	31.75(2)	12.5	30.958(8)	15	31.28(1)	12.5	14	29.9(2)	20	14
^{6}Li	31.995	32.04(1)	12.5	31.744(3)	15	31.98(2)	12.5	14	31.48(3)	20	16
^{10}B	64.751	64.28(0)	17.5	64.6(1)	17.5	64.79(3)	17.5	10	63.9(1)	22.5	10
^{12}C	92.162	91.0(3)	20	92.28(9)	20	92.9(1)	17.5	8	94.8(3)	27.5	10
^{16}O	127.619	127.7(5)	20	129.57(9)	20	131.3(1)	17.5	10	145(8)	35	8

New Daejeon16

- Excitation energies

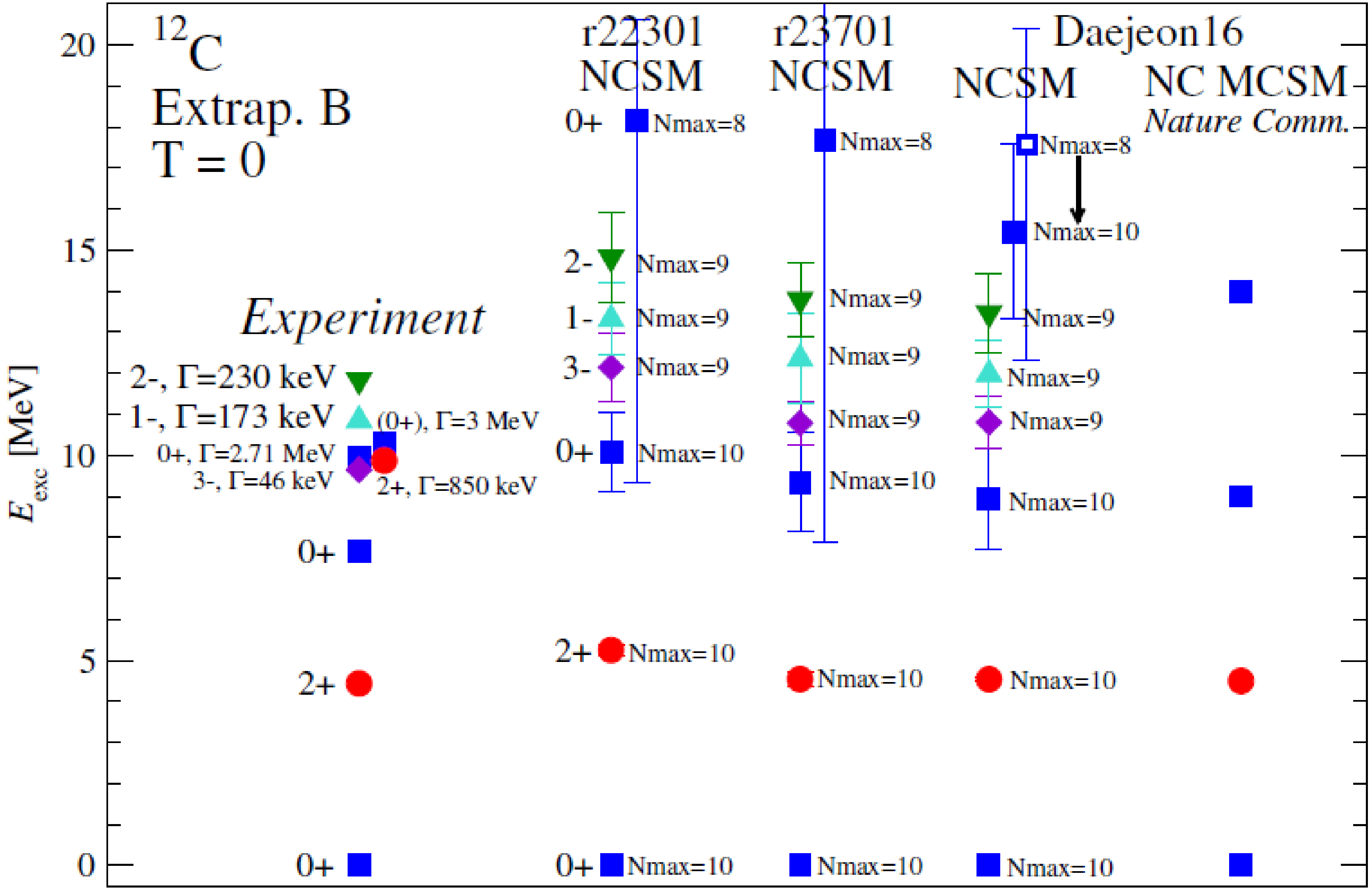
Nucleus, level	Nature	r22301		r23701		Daejeon16			JISP16		
		Theory	hw	Theory	hw	Theory	hw	Nmax	Theory	hw	Nmax
^{6}He											
(0+,1)	0	0		0		0			0		
(2+,1)	1.797	1.81	12.5	1.78	12.5	1.91(5)	12.5	14	2.3(1)	17.5	16
^{6}Li											
(1+,0)	0	0		0		0			0		
(3+,0)	2.186	1.98	12.5	1.72	15	1.91(1)	12.5	14	2.55(7)	20	16
(0+,1)	3.563	3.50	12.5	3.47	15	3.50(4)	12.5	14	3.65(6)	17.5	16
(2+,0)	4.312	4.81	12.5	4.44	15	4.4(3)	12.5	14	4.5(2)	20	16
(2+,1)	5.366	5.38	15	5.21	12.5	5.36(7)	12.5	14	5.9(1)	17.5	16
(1+,0)	5.65					5.0(4)	12.5	14	5.4(2)	17.5	16

New Daejeon16

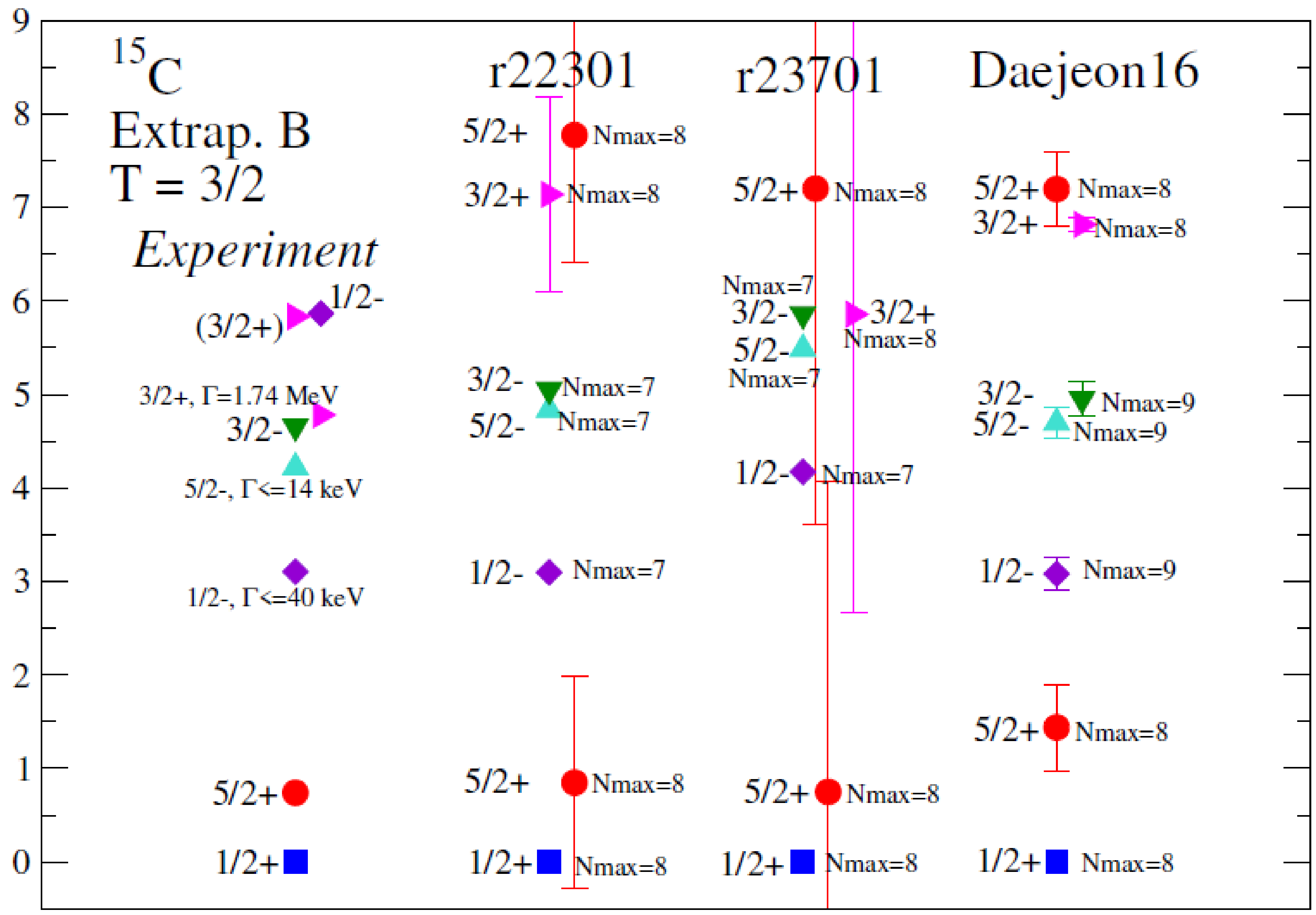
- Excitation energies (continued)

Nucleus, level	Nature	r22301		r23701		Daejeon16			JISP16		
		Theory	hw	Theory	hw	Theory	hw	Nmax	Theory	hw	Nmax
^{10}B											
(3+,0)	0	0		0		0			0		
(1+,0)	0.718	0.82	17.5	1.01	17.5	0.5(1)	15	10	0.9(2.4)	22.5	10
(0+,1)	1.740	1.58	17.5	1.87	17.5	1.74(7)	17.5	10	1.8(1.4)	25	8
(1+,0)	2.154	2.88	17.5	2.95	17.5	2.8(2)	17.5	10	4.1(1.7)	30	10
(2+,0)	3.587	4.90	17.5	4.83	17.5	4.3(2)	15	10	3.8(2)	27.5	10
(3+,0)	4.774	5.72	17.5	5.71	17.5	5.1(7)	17.5	10	5.6(3)	22.5	10
(2+,1)	5.164	5.47	17.5	5.58	17.5	5.49(9)	17.5	10	4.6(3)	22.5	10
^{12}C											
(0+,0)	0	0		0		0			0		
(2+,0)	4.439	5.16	17.5	4.61	20	4.57(15)	17.5	8	3.9(4)	27.5	10

New Daejeon16

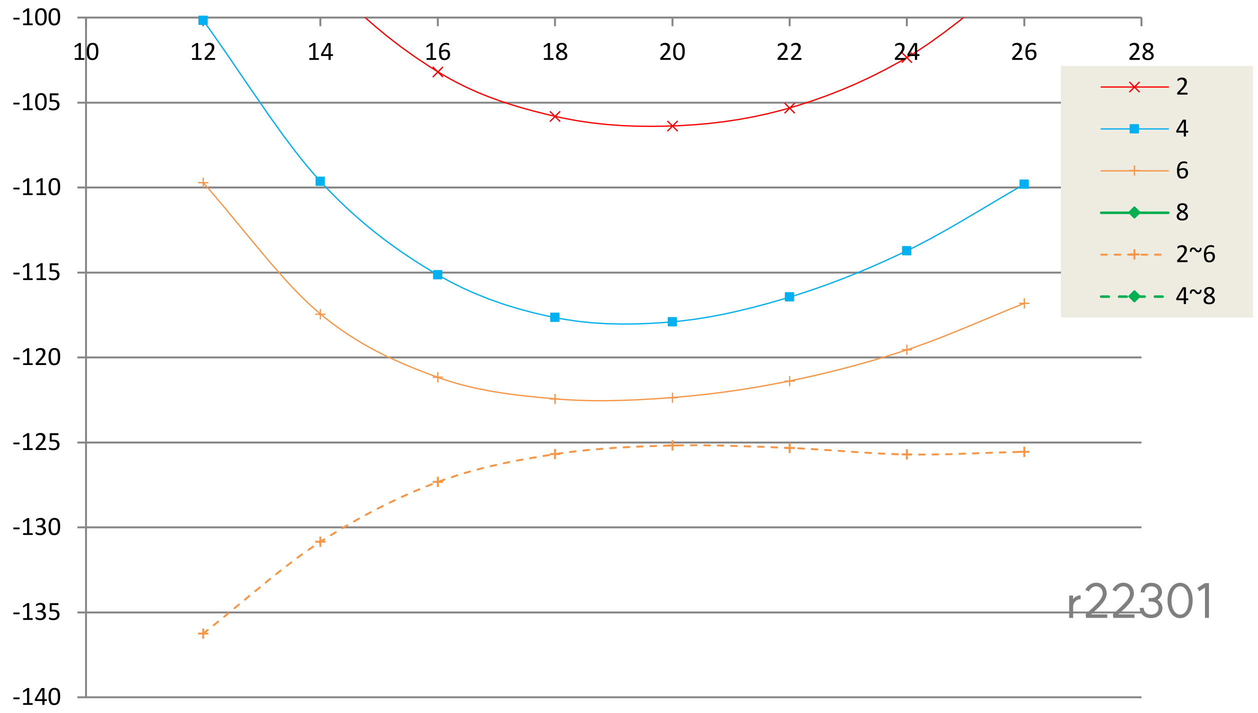


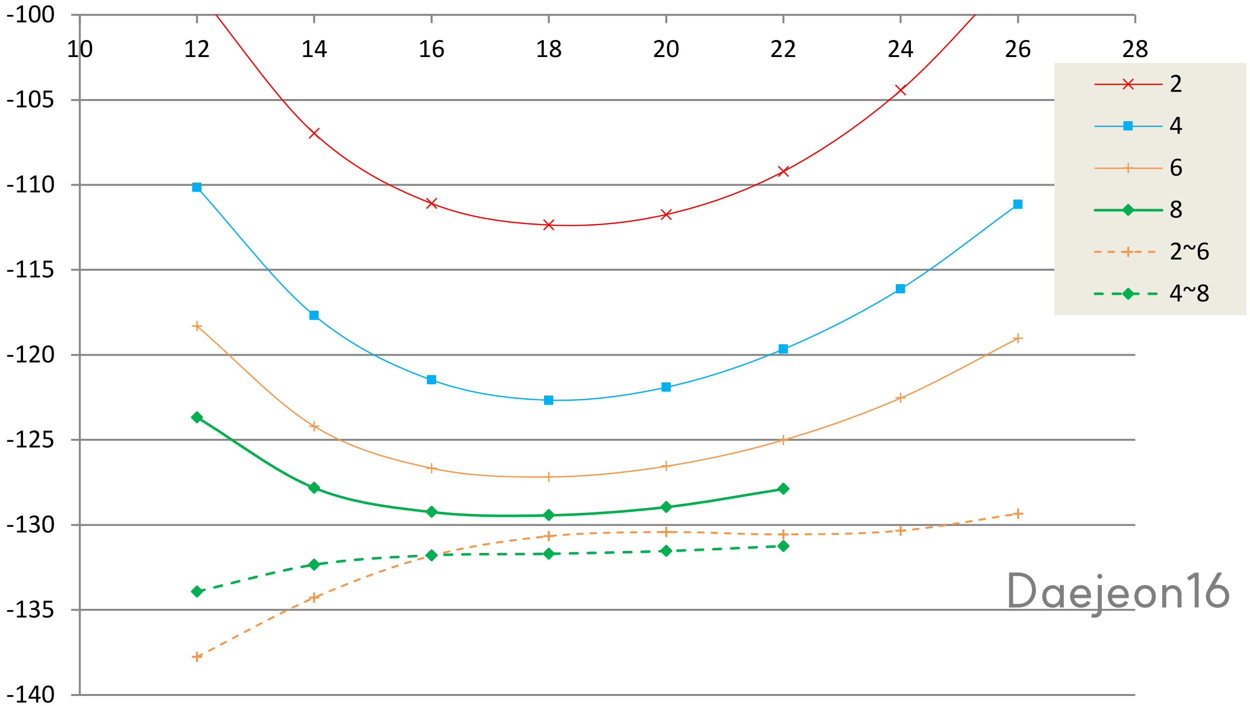
New Daejeon16

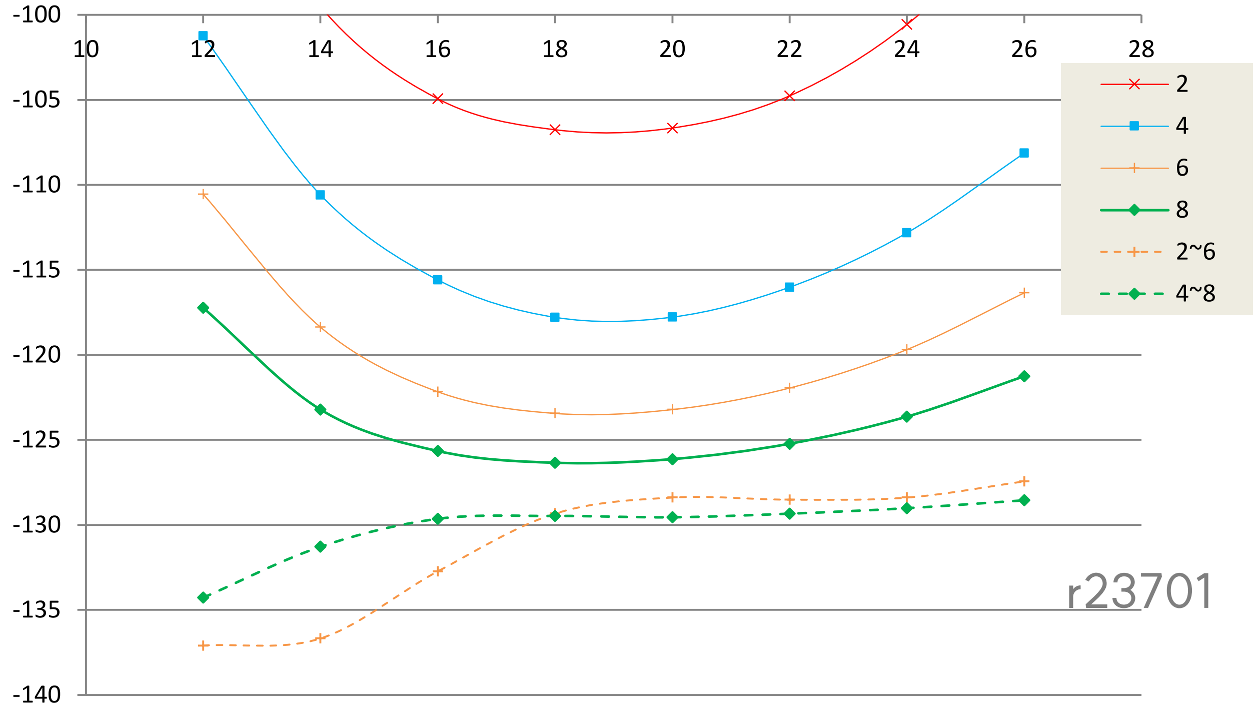


New Daejeon16

^{17}F	Nmax=6	extrapol.	Nmax=8	extrapol.
exp.		−128.222		−128.222
Daejeon16	−127.170	−130.664	−129.426	−131.692
r22301	−122.436	−125.685		
r23701	−123.434	−129.328	−126.350	−129.470



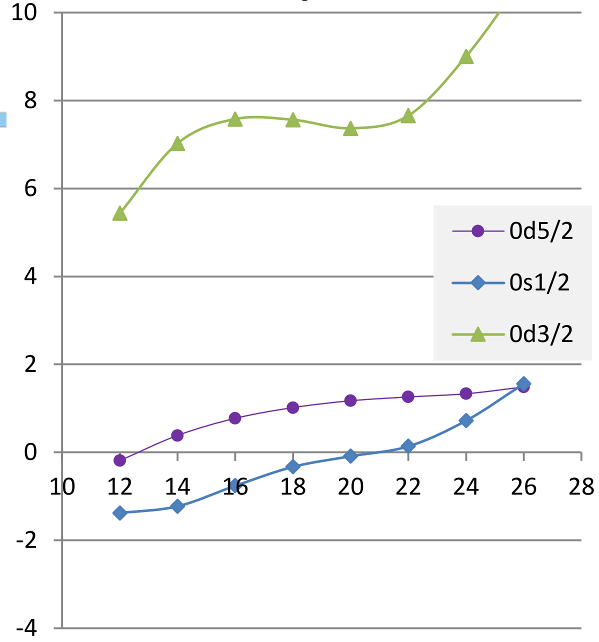




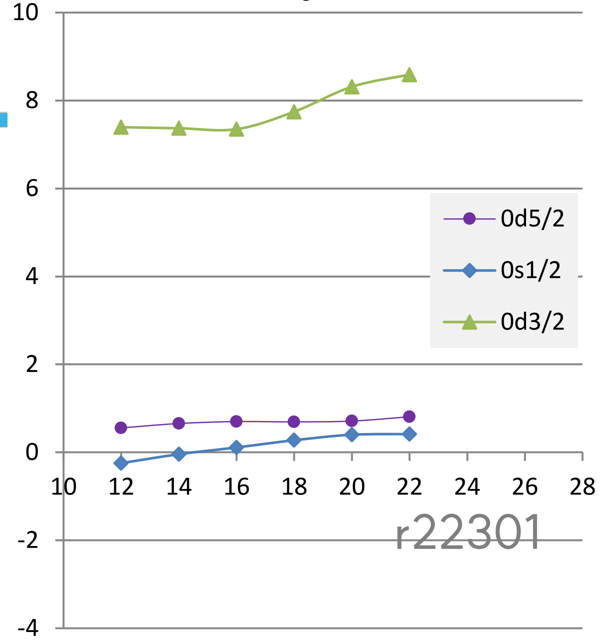
New Daejeon16

To take differences between ^{17}F spectra and ^{16}O g.s., then to adopt extrapolation

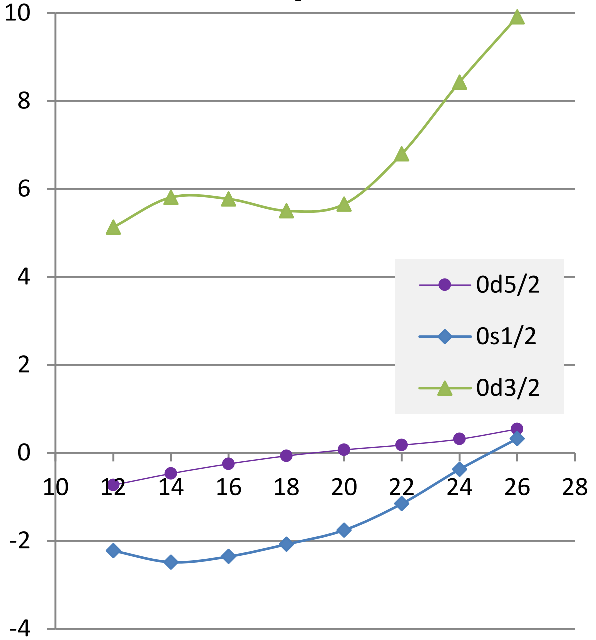
extrap. 2~6



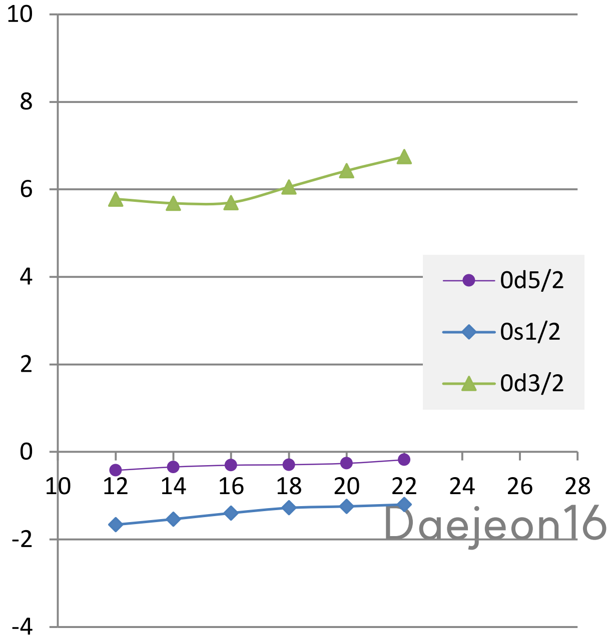
extrap. 4~8



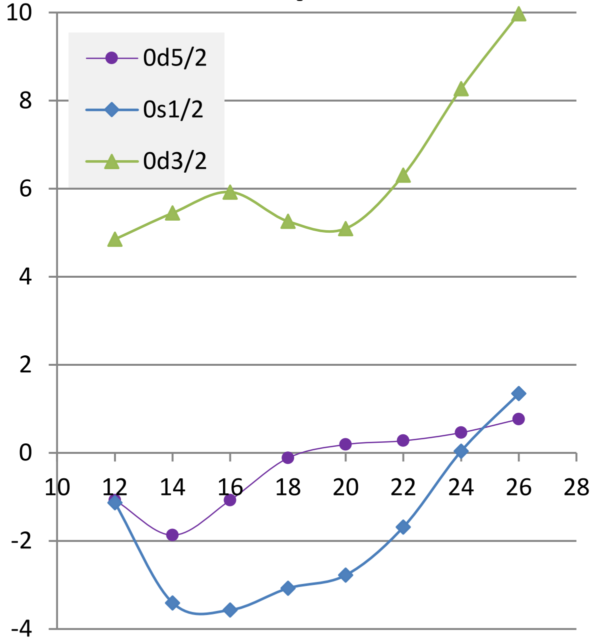
extrap. 2~6



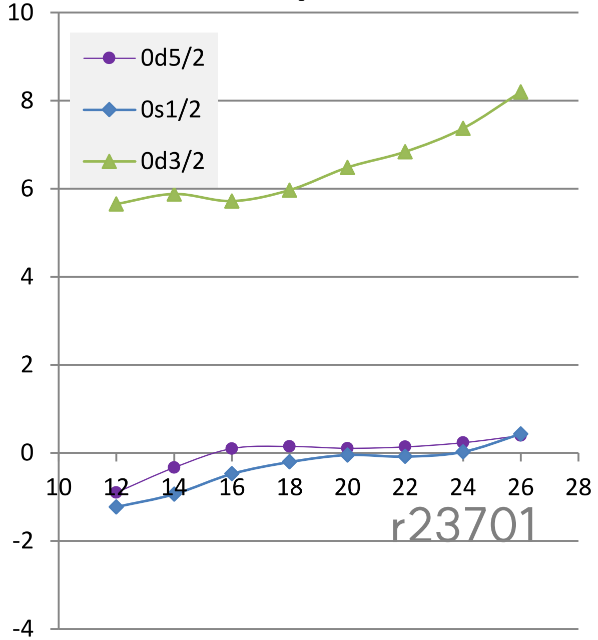
extrap. 4~8



extrap. 2~6



extrap. 4~8



New Daejeon16

- ^{18}F spectrum

state	r22301		r23701		Daejeon16		exp.
	Nm=6	Nm=8	Nm=6	Nm=8	Nm=6	Nm=8	
1+,0	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3+,0	−0.115	0.175	−0.300	-0.311	0.589	0.595	0.937
0+,1	1.310	1.001	1.108	1.131	1.744	1.229	1.042
5+,0	1.639	1.455	1.520	1.385	2.839	2.158	1.121

Optimal HO basis (for all states)

r22301 & r23701 : 20 MeV (Nm=6) / 18 MeV (Nm=8)

Daejeon16 : 18 MeV (Nm=6, 8)