Nucleon structure in Minkowski space

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Dedicated to the Memory of Prof. Ruprecht Machleidt

Int. Conference on Nuclear Theory in the Supercomputing Era (NTSE-2024)

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NUCLEON

- How do we see?

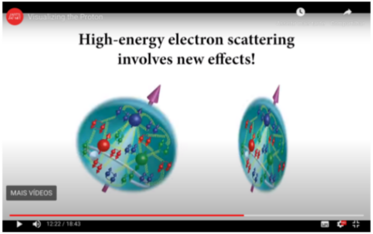
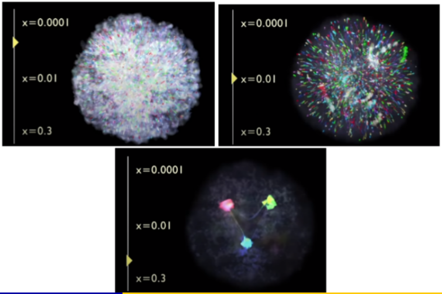


Figure: https://www.nanotechnologyworld.org/post/visualizing-the-proton-through-animation-and-film

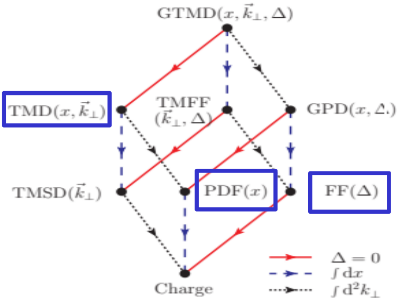


High resolution
short-distances

GTMD

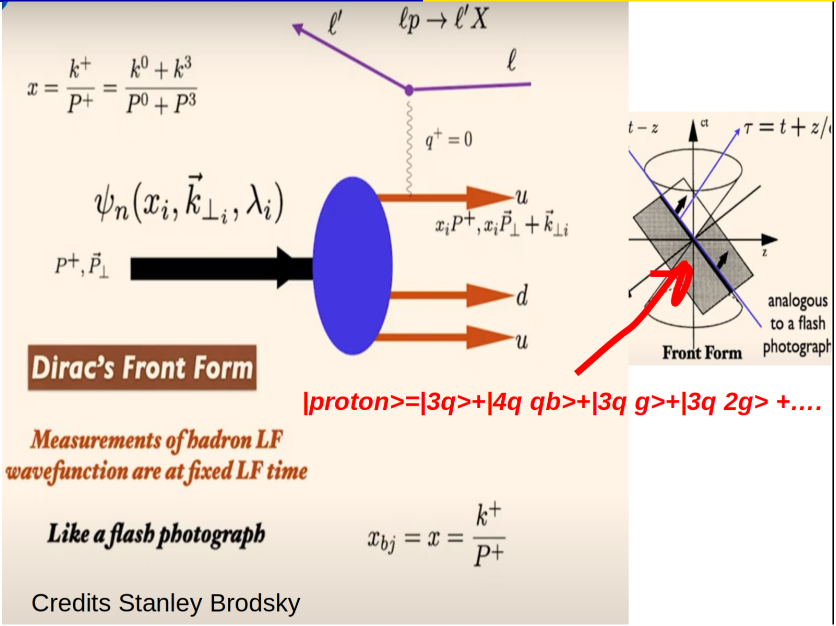
How to get the details?

Observables associated with the hadron structure



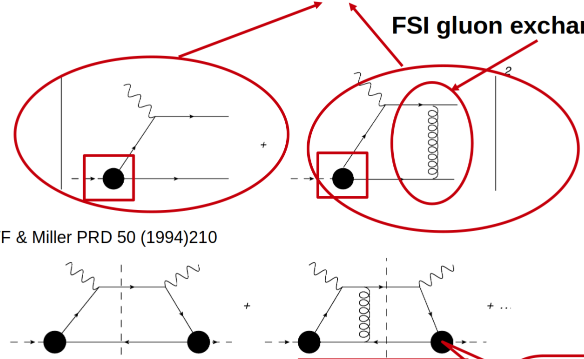
Lorcé, Pasquini, Vanderhaeghen JHEP05(2011)041

- SL form factor, PDF, TMD & 3D image


ℓ'
$\ell p \to \ell' X$
ℓ
$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$
$q^+ = 0$
$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$
u
$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$
$P^+, \vec{P}_\perp$
d
u
Dirac's Front Form
$t - z$
ct
$\tau = t + z/c$
z
analogous to a flash photograph
Front Form
|proton>=|3q>+|4q qb>+|3q g>+|3q 2g> +….
Measurements of hadron LF wavefunction are at fixed LF time
Like a flash photograph
$x_{bj} = x = \frac{k^+}{P^+}$
Credits Stanley Brodsky

DIS - I

TMDs & PDFs



TF & Miller PRD 50 (1994)210

$$q^2 = q^+q^- - q^2_T$$

$$q^+ = q^0+q^3 \qquad q^- = q^0-q^3$$

$q^- \rightarrow$ *infty*
DIS

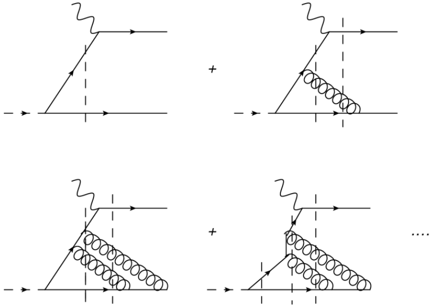
Bethe-Salpeter Amplitude @ x^+=0

DIS - II

Beyond the valence ….

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001)

Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010 (2008)



Population of lower x, due to the gluon radiation! (LF initial state interaction)

Proton mass radius: hint for higher Fock components?

- "a mass radius that is notably smaller than the electric charge radius", Duran et al, Nature 615, 813 (2023)

- Pion charge radius $r^\pi_{val} > r^\pi_{exp}$ & $r^\pi_{nval} < r^\pi_{exp}$
  Ydrefors et al., PLB 820, 136494 (2021)

- Higher Fock-components:
  large virtualities $\rightarrow$ more compact configurations

How to access the LF Fock-space dynamically?

- Define the Light-cone Hamiltonian and diagonalize

  Brodsky, Vary, Zhao...Basis Light-Ftont Quantization

- Start with a four-dimensional model: Bethe-Salpeter equation (pion/nucleon)

- Continuum methods

  Aguilar, El-Bennich, Cloet, de Paula, Eichmann, Fischer, Pelaez, Salme, Roberts, . . .

- Minkowski space representation – 4D equation
  – project onto the Light-Front

  Sales, Sauer, Marinho, Salme, TF

- Caveat: **simple kernels**

Nucleon schematic model

- Spin and color factorized
- Constituent Quarks (totally symmetric state)
- Contact interaction & diquark

What physics remains?

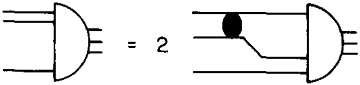
- **4D structure / LF Fock-state decomposition**

What we learn?

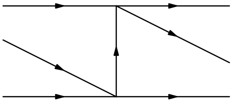
- **Three-body description in Minkowski space**

Faddeev-Bethe-Salpeter equation

- Basic building block is the two-body graph (contact interaction) [TF, PLB 282 (1992) 409]



- The iteration of the BSE gives rise to multiple exchanges



Faddeev-Bethe-Salpeter equation

- The three-body BS amplitude is:

$$i\Phi(k_1,k_2,k_3;p)=i^3\frac{v(k_1)+v(k_2)+v(k_3)}{(k_1^2-m^2+i\epsilon)(k_2^2-m^2+i\epsilon)(k_3^2-m^2+i\epsilon)},$$

- Faddeev-Bethe-Salpeter (FBS) equation (p nucleon momentum):

$$v(q,p)=2\mathrm{i}\mathcal{F}(M_{12}^2)\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{i}}{k^2-m^2+\mathrm{i}\epsilon}\frac{\mathrm{i}}{(p-q-k)^2-m^2+i\epsilon}v(k,p)$$

- $M_{12}^2=(p-q)^2$

$$\mathcal{F}(M_{12}^2)=\frac{\Theta(-M_{12}^2)}{\frac{1}{16\pi^2 y}\log\frac{1+y}{1-y}-\frac{1}{16\pi ma}}+\frac{\Theta(M_{12}^2)\,\Theta(4m^2-M_{12}^2)}{\frac{1}{8\pi^2 y'}\arctan y'-\frac{1}{16\pi ma}}+\frac{\Theta(M_{12}^2-4m^2)}{\frac{y''}{16\pi^2}\log\frac{1+y''}{1-y''}-\frac{1}{16\pi ma}-\frac{iy''}{16\pi}}$$

$$y=\frac{M_{12}}{\sqrt{4m^2-M_{12}^2}}\quad,\ y'=\frac{\sqrt{-M_{12}^2}}{\sqrt{4m^2-M_{12}^2}}\quad,\ y''=\frac{\sqrt{M_{12}^2-4m^2}}{M_{12}}.$$

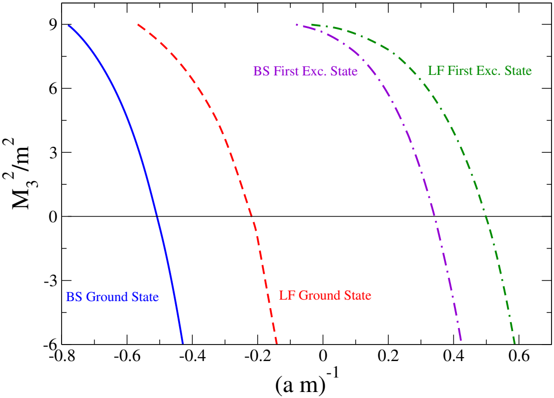
- Bound ($a>0$)/virtual ($a<0$) state (a scatt. length)

Universal low-energy physics

- Non-relativistic limit reduces to the 3-body Skorniakov and Ter-Martirosian equation with contact interactions [1956];
- Contact interaction leads to Thomas collapse of the 3-boson system [1935];
- FBS model: Efimov effect but not Thomas collapse!

Euclidean FBS equation: 3B bound states

- Wick-rotation of the FBS equation

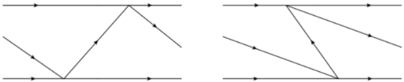


- Low-lying unphysical solution with $M_N^2 < 0$.
- No Thomas collapse.

|proton>=|3q>+|4q qb>+|5q 2qb > +....

E. Ydrefors et al. / Physics Letters B 770 (2017) 131–137

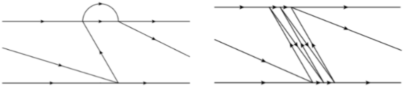
|3q>



|4q qb>

Fig. 2. The three-body LF graphs obtained by time-ordering of the Feynman graph shown in right panel of Fig. 1.

|4q qb>



....

Fig. 3. Examples of many-body intermediate state contributions to the LF three-body forces.

- Valence truncation - the full BS solution has contributions coming from the coupling of the valence state with an infinite number of Fock states: effective three-body force.

FBS in Minkowski space ($a < 0$)

E. Ydrefors et al. PLB 791 (2019) 276 & PRD 101 (2020) 096018

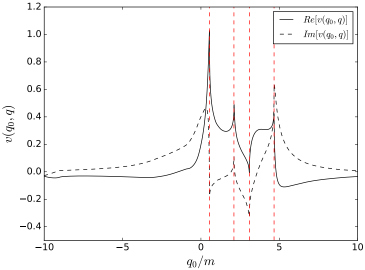
- By adopting a similar method as the one introduced by Carbonell and Karmanov[1] to solve the two-body problem (finite-range interaction) the FBS equation for the equation for the vertex function, can be written in the form

$$v(q_0, q_v) = \frac{\mathcal{F}(M_{12})}{(2\pi)^4} \int_0^\infty k_v^2 dk_v \Big\{ i \frac{[\Pi(q_0, q_v; \varepsilon_k, k_v) v(\varepsilon_k, k_v) + \Pi(q_0, q_v; -\varepsilon_k, k_v) v(-\varepsilon_k, k_v)]}{2\varepsilon_k}$$
$$- 2 \int_{-\infty}^0 dk_0 \left[\frac{\Pi(q_0, q_v; k_0, k_v) v(k_0, k_v) - \Pi(q_0, q_v; -\varepsilon_k, k_v) v(-\varepsilon_k, k_v)}{k_0^2 - \varepsilon_k^2} \right]$$
$$- 2 \int_0^\infty dk_0 \left[\frac{\Pi(q_0, q_v; k_0, k_v) v(k_0, k_v) - \Pi(q_0, q; \varepsilon_k, k_v) v(\varepsilon_k, k_v)}{k_0^2 - \varepsilon_k^2} \right] \Big\},$$

- Singularities at $k_0 = \pm\epsilon_k = \pm\sqrt{k^2 + m^2}$, eliminated by subtractions.
- Remains only weak logarithmic singularities to be handled numerically.

[1] PRD 90 (2014) 056002

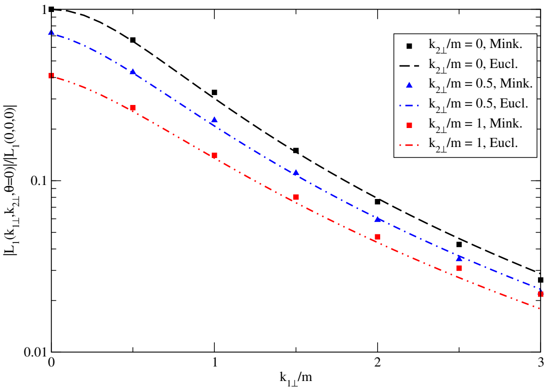
Three-body vertex function in Minkowski space



- The figure shows the real and imaginary parts of $v(q_0, q_v)$ at fixed $q_v/m = 0.5$, for the case $B_3/m = 0.395$ for $a\,m = -1.5$.
- It is seen that there are four peaks (either singularities or branch cuts). It turns out that they have the positions $q_0 = M_3 \pm \sqrt{q_v^2 + 4m^2}$ and $q_0 = M_3 \pm q_v$, shown by red dashed lines. These are thus moving peaks depending on q_v.
- The non-smooth behavior of v makes the solution of this problem numerically very challenging.

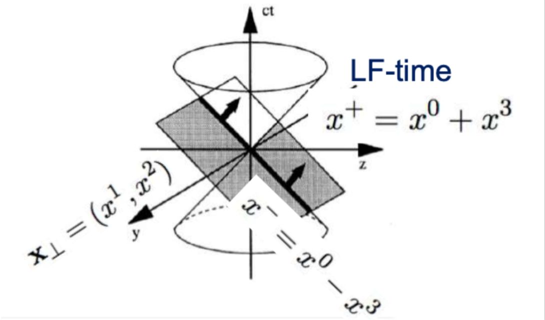
Transverse amplitudes Minkowski vs. Euclidean

$$L(\vec{k}_{1\perp}, \vec{k}_{2\perp}) = \int_{-\infty}^{\infty} dk_{10} \int_{-\infty}^{\infty} dk_{1z} \int_{-\infty}^{\infty} dk_{20} \int_{-\infty}^{\infty} dk_{2z} i\Phi_M(k_{10}, k_{1z}, k_{20}, k_{2z}; \vec{k}_{1\perp}, \vec{k}_{2\perp})$$



- $a\,m = -1.5, \; B_3/m = 0.395$

Light-front dynamics and Fock-space @ $x^+ = 0$



- Fock-components: prob. ampl. invariant under kinematical boosts

$$P_n = \left[\prod_{i=1}^{n} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i\right] \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta\left(\sum_{i=1}^{n} \vec{k}_{i\perp}\right) |\Psi_n(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, ...)|^2,$$

where $n \geqslant 3$. Ψ_n has a probability amplitude interpretation.

Valence truncation

- Valence LF wave function:

$$\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}) = \frac{\Gamma(x_1,\vec{k}_{1\perp})+\Gamma(x_2,\vec{k}_{2\perp})+\Gamma(x_3,\vec{k}_{3\perp})}{\sqrt{x_1x_2x_3}(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))},$$

$$x_3 = 1-x_2-x_3,\ \vec{k}_{3\perp} = -\vec{k}_{1\perp}-\vec{k}_{2\perp},\ M_0^2(x_1,\vec{k}_{1\perp}...) = \sum_{i=1}^3(k_{i\perp}^2+m^2)/x_i$$

- Projection of the BS equation onto the LF:

$$\Gamma(x,k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3}\int_0^{1-x}\frac{dx'}{x'(1-x-x')}\int_0^\infty d^2k'_\perp\frac{1}{M_0^2-M_N^2}\Gamma(x',k'_\perp)$$

$$M_0^2 = (k'^2_\perp+m^2)/x' + (k_\perp^2+m^2)/x + ((\vec{k}'_\perp+\vec{k}_\perp)^2+m^2)/(1-x-x')$$

TF, PLB 282 (1992) 409; Carbonell, Karmanov, PRC 67 (2003) 037001; Ydrefors, TF, PRD 104 (2021) 114012

LF proj.: Sales, TF, Carlson, Sauer, PRC 61 (2000) 044003; J A O Marinho and TF, J. Phys.: Conf. Ser. 110 (2008) 122009

Valence LF dynamical model: QCD scale

E. Ydrefors and TF, PLB 838 (2023) 137732

- Valence three-body regularized LF equation:

$$\Gamma(x, k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3} \int_0^{1-x} \frac{dx'}{x'(1-x-x')} \int_0^\infty d^2k'_\perp \left[\frac{1}{M_0^2 - M_N^2} - \frac{1}{M_0^2 + \mu^2}\right] \Gamma(x', k'_\perp)$$

$$M_0^2 = (k'^2_\perp + m^2)/x' + (k_\perp^2 + m^2)/x + ((\vec{k}'_\perp + \vec{k}_\perp)^2 + m^2)/(1 - x - x')$$

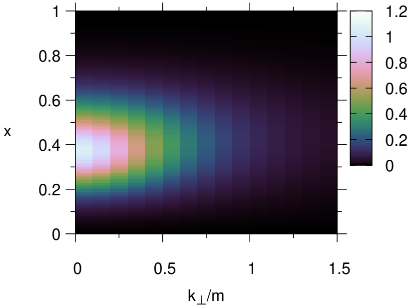
- Cut-off: no unphysical solution $M_3^2 < 0$, and enhances IR vs. UV.
- The quark-quark transition amplitude has a pole: s-wave diquark.

Application: Proton model on the light-front

- Dirac EM form factor
- Unpolarized quark distribution function
- Double parton scattering cross section enters the double parton distribution function (DPDF) [Blok et al, PRD83 (2011) 071501(R)]
- Transverse momentum distribution (SIDIS)
- Image on the null-plane

Mass and vertex function

Model	m [MeV]	a [m^{-1}]	μ/m	M_{dq} [MeV]
(a)	366	2.70	1	644
(b)	362	3.60	∞	682
(c)	317	-1.84	∞	-

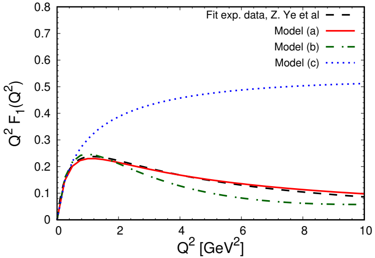


- Vertex function $\Gamma(x, k_\perp)$

Dirac EM form factor: valence

$$F_1(Q^2) = \left[\prod_{i=1}^{3} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i\right] \delta\left(1 - \sum_{i=1}^{3} x_i\right) \delta\left(\sum_{i=1}^{3} \vec{k}^{\mathrm{f}}_{i\perp}\right) \times \Psi_3^\dagger(x_1, \vec{k}^{\mathrm{f}}_{1\perp}, \ldots)\Psi_3(x_1, \vec{k}^{\mathrm{i}}_{1\perp}, \ldots),$$

$$\left|\vec{k}^{\mathrm{f(i)}}_{i\perp}\right|^2 = \left|\vec{k}_{i\perp} \pm \frac{\vec{q}_\perp}{2} x_i\right|^2 \quad \text{and} \quad \left|\vec{k}^{\mathrm{f(i)}}_{3\perp}\right|^2 = \left|\pm \frac{\vec{q}_\perp}{2}(x_3 - 1) - \vec{k}_{1\perp} - \vec{k}_{2\perp}\right|^2$$



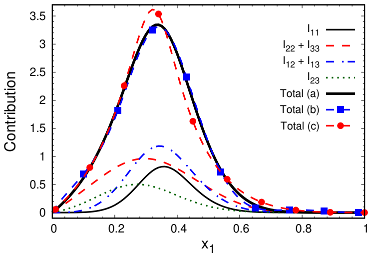
Note: QCD scaling laws are not built-in!

Valence Proton PDF @ initial and experimental scale

$$f_1(x_1) = \frac{1}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} |\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp})|^2 = I_{11}+I_{22}+I_{33}+I_{12}+I_{13}+I_{23}$$

$$I_{ii} = \frac{1}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma^2(x_i,\vec{k}_{i\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))^2}$$

$$I_{ij} = \frac{2}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma(x_i,\vec{k}_{i\perp})\Gamma(x_j,\vec{k}_{j\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))^2}; \quad i \neq j.$$



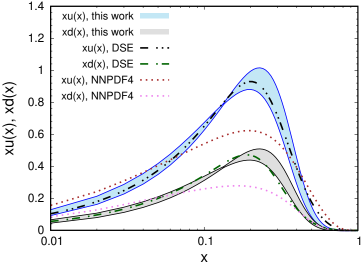
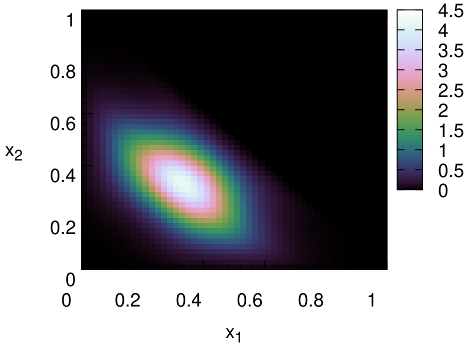


Figure: [Left] proton PDF @ initial scale. [Right] Valence u-quark and d-quark PDFs @ $Q = 3.097\,\text{GeV}$ vs. DSE from Lu et al (2203.00753 [hep-ph]) and the results of the NNPDF4 global fit. The shaded areas indicate the uncertainty with respect to the initial scale $Q_0 = 0.33 \pm 0.03\,\text{GeV}$.

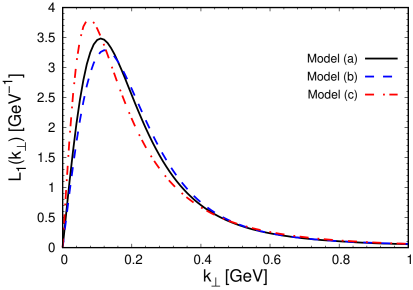
Valence Proton Double PDF

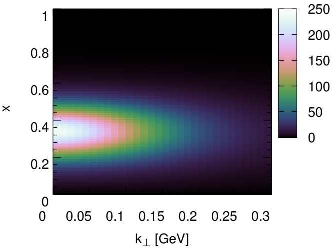
$$D_3(x_1, x_2; \vec{\eta}_\perp) = \frac{1}{(2\pi)^6} \int d^2k_{1\perp} d^2k_{2\perp} \Psi_3^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp; x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp; x_3, \vec{k}_{3\perp}) \Psi_3(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}; x_3, \vec{k}_{3\perp}).$$

- Fourier transform in $\vec{\eta}_\perp$: probability of quarks 1 and 2 for x_1 and x_2 with a separation in the transverse direction $\vec{y}_\perp$.
- $D_3 = 0$ for $x_1 + x_2 > 1$ - momentum conservation. (Below $\vec{y}_\perp = \vec{0}_\perp$)



Transverse momentum distribution



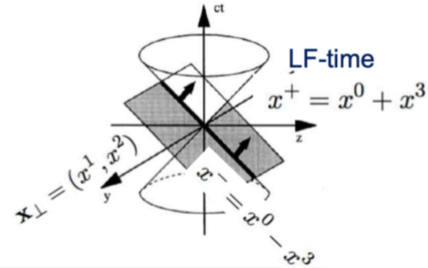


- Quark transverse momentum distribution

$$\tilde{f}_1(k_\perp, x) = \int_0^1 dx_1 \delta(x - x_1) \int \frac{dk_{1\perp}}{(2\pi)^2} \delta(k_\perp - k_{1\perp}) \int_0^{2\pi} d\theta_1 \int \frac{d^2 k_{2\perp}}{(2\pi)^2} \int_0^{1-x} dx_2 \, |\Psi_3(\{x, \vec{k}_\perp\})|^2 \, , \tag{1}$$

- Integrated TMD $L_1(k_\perp) = k_\perp \int_0^1 dx \tilde{f}(k_\perp, x)$,

3D Hadron Image on the null plane



The Ioffe-time is useful for studying the relative importance of short and long light-like distances. It is defined as:

$$\tilde{z} = x \cdot P_{target} = x^- P^+_{target}/2 \quad \text{on the hyperplane } x^+ = 0$$

Miller & Brodsky, PRC 102, 022201 (2020)

Ioffe-time image - valence state

NS FROM … PHYS. REV. D **104,** 114012 (2021)

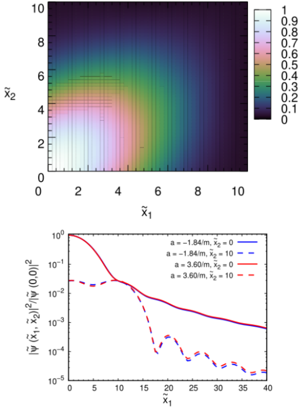
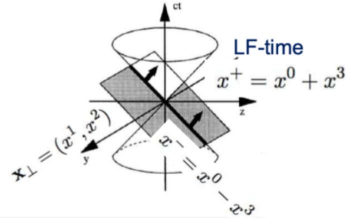


FIG. 3. Upper panel: squared modulus of the Ioffe-time distribution as a function of $\tilde{x}_1$ and $\tilde{x}_2$, for the model I. Lower panel: squared modulus of the Ioffe-time distribution as a function of $\tilde{x}_1$ for two fixed values of $\tilde{x}_2$, namely $\tilde{x}_2 = 0$ (solid line) and $\tilde{x}_2 = 10$ (dashed line). Results shown for the model I (blue line) and model II (red line).



$$\Phi(\tilde{x}_1,\tilde{x}_2) \equiv \tilde{\Psi}_3(\tilde{x}_1,\vec{0}_\perp,\tilde{x}_2,\vec{0}_\perp)$$
$$= \int_0^1 dx_1 e^{i\tilde{x}_1 x_1} \int_0^{1-x_1} dx_2 \int_0^1 dx_3$$
$$\times \delta(1-x_1-x_2-x_3) e^{i\tilde{x}_2 x_2} \phi(x_1,x_2,x_3).$$

First step: quark-diquark in Minkowski space

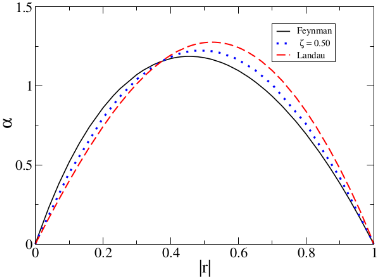
- Fermion $(1/2^+)$-scalar model (0^+) - "gluon" exchange
  - Nakanishi integral representation

    Alvarenga Nogueira et al PRD100, 016021 (2019)
  - Covariant gauges & UV limit

    Aline Noronha et al PRD107, 096019 (2023)

Fermion-scalar bound system in the UV limit

Aline Noronha et al, PRD **107**, 096019 (2023)



Coupling constant of the fermion-boson system in the chiral limit vs. the power r ($g_i(\gamma, z) = \gamma^r f_{i,r}(z)$). Solid line: Feynman gauge ($\zeta = 1$). Dotted line: $\zeta = 0.5$ gauge. Dashed line: Landau gauge ($\zeta = 0$).

- Miransky scaling [1985](Kaplan, Lee, Son, Stephanov [2009])

Credits to Giovanni Salmè

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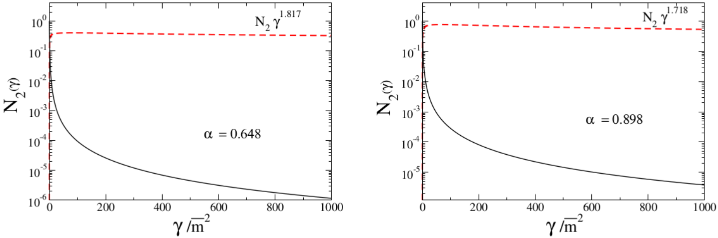


The normalized LF amplitude $N_2(\gamma) = \psi_2(\gamma, \xi_0 = 0.5)/\psi_2(0, \xi_0 = 0.5)$ as a function of the transverse momentum square $\gamma = |\vec{k}_\perp|^2$ in the Feynman gauge,with $m_s/m_f = 2$ and binding energy ratio $B/\bar{m} = 0.1$ (i.e. $M/\bar{m} = 1.9$). Left panel: $\mu/\bar{m} = 0.15$ and $\alpha = 0.648$. Solid line: $N_2(\gamma)$. Dashed line: $N_2(\gamma) \times \gamma^{1.817}$. Right panel: $\mu/\bar{m} = 0.5$ and $\alpha = 0.898$. Solid line: $N_2(\gamma)$. Dashed line: $N_2(\gamma) \times \gamma^{1.718}$.

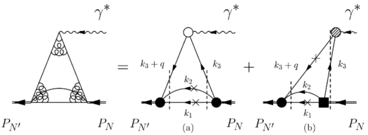
Credits to Giovanni Salmè

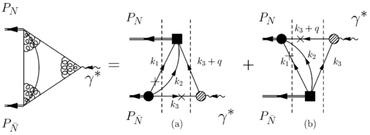
Time-like and space-like nucleon EM factors

de Melo, TF, Pace, Pisano, Salmè, PLB 671 (2009) 153

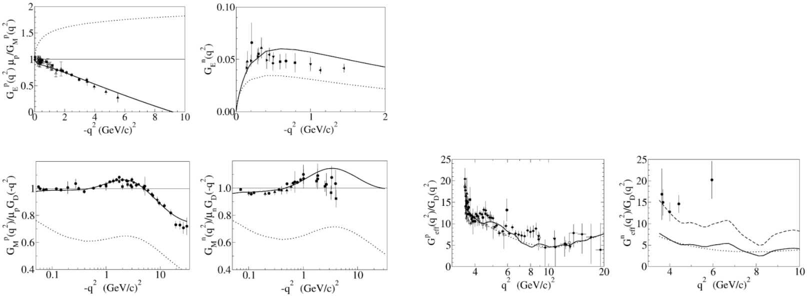
TL nucleon EM FF

J.P.B.C. de Melo et al. / Physics Letters B 671 (2009) 153–157





J.P.B.C. de Melo et al. / Physics Letters B 671 (2009) 153–157



Summary and Prospects

- Relativistic Three-boson system for zero-range interaction:
  - Bethe-Salpeter 4D framework in Euclidean, Minkowski and LF
- Application: EM form factor, PDF, TMD and Ioffe-time image.
- Fermion-scalar model in Minkowski space: UV properties

Prospects:

- Quark spin & dressed constituents [DC Duarte et al PRD105 (2022)114055]
- Gluon exchange & dressed vertices [de Paula et al EPJC83 (2023)985]
- Integral representation to solve the FBS equation (ongoing)
- Quantum algorithm for FBS equation [Fornetti, et al, PRD110 (2024) 056012]

How much exotic is the nucleon itself? THANK YOU!