

2024 Combinatorics Workshop

Wednesday, 28 August 2024 - Friday, 30 August 2024

Bldg. S1-6 (자연대 6 호관)

Book of Abstracts

Contents

Asymptotic bounds of Ramsey Numbers	1
102-avoiding Inversion Sequences	1
Random matchings in linear hypergraphs	1
Towards a classification of 1-homogeneous graphs with positive intersection number a_1	2
Extensions of the colorful Helly theorem for d -collapsible and d -Leray complexes	2
Transversal Hamilton paths and cycles of arbitrary orientations in tournaments	2
Toric Colorability of Graphs of Simplicial d -Polytopes with $+ 4$ vertices	3
Alternating \mathcal{B} -permutations arising from toric topology	3
Partitions of ordered partitions and Bott manifolds	4
Homotopy Types of Vietoris-Rips Complexes and Their Connection to Hyperconvexity .	4
On the extremal number of face-incidence graphs	4
Lusztig q weight multiplicities via affine crystals	5
Combinatorics of orthogonal polynomials on the unit circle	5
Enumeration of multiplex juggling card sequences using generalized q -derivatives	6
Two ways to generalize matroids with coefficients	6

Invited Talk / 19**Asymptotic bounds of Ramsey Numbers****Author:** Jeong Han Kim¹¹ *KLAS***Corresponding Author:** jhkim@kias.re.kr

Ramsey numbers, denoted as $R(s,t)$, are fundamental in graph theory, representing the smallest number of vertices n such that every graph on n vertices either contains a clique of size s or an independent set of size t . Recent developments in Ramsey theory have focused on finding asymptotic bounds for Ramsey numbers. In this talk, we survey asymptotic bounds of Ramsey Numbers $R(3,t)$ and $R(4,t)$, including significant contributions of Sam Mattheus and Jacques Verstraete on $R(4,t)$.

Contributed Talk / 8**102-avoiding Inversion Sequences****Authors:** JiSun Huh¹; Sangwook Kim²; Seunghyun Seo³; Heesung Shin⁴¹ *Ajou University*² *Chonnam National University*³ *Kangwon National University*⁴ *Inha University***Corresponding Author:** shin@inha.ac.kr

A sequence (e_1, e_2, \dots, e_n) is an inversion sequences if $0 \leq e_i < i$ for all $i = 1, \dots, n$. We say that an inversion sequences $e = (e_1, e_2, \dots, e_n)$ contains the pattern 102 if there exist some indices $i < j < k$ such that $e_j < e_i < e_k$. Otherwise, e is said to avoid the pattern 102.

In this talk, we will construct a correspondence between the set of 2-Schröder paths without peaks and valleys ending with a diagonal step and the set of 102-avoiding inversion sequences. This is the joint work with JiSun Huh, Sangwook Kim, and Seunghyun Seo.

Contributed Talk / 7**Random matchings in linear hypergraphs****Author:** Hyunwoo Lee¹¹ *KAIST & IBS ECOPRO***Corresponding Author:** hyunwo9216@gmail.com

For a given hypergraph H and a vertex $v \in V(H)$, consider a random matching M chosen uniformly from the set of all matchings in H . In 1995, Kahn conjectured that if H is a d -regular linear k -uniform hypergraph, the probability that M does not cover v is $(1 + o_d(1))d^{-1/k}$ for all vertices $v \in V(H)$. This conjecture was proved for $k = 2$ by Kahn and Kim in 1998.

We disprove this conjecture for all $k \geq 3$. For infinitely many values of d , we construct d -regular linear k -uniform hypergraph H containing two vertices v_1 and v_2 such that $\mathcal{P}(v_1 \notin M) = 1 - \frac{(1+o_d(1))}{d^{k-2}}$ and $\mathcal{P}(v_2 \notin M) = \frac{(1+o_d(1))}{d+1}$. The gap between $\mathcal{P}(v_1 \notin M)$ and $\mathcal{P}(v_2 \notin M)$ in this H is

best possible. In the course of proving this, we also prove a hypergraph analog of Godsil's result on matching polynomials and paths in graphs, which is of independent interest.

Contributed Talk / 3

Towards a classification of 1-homogeneous graphs with positive intersection number a_1

Authors: Jack Koolen¹; Mamoon Abdullah¹; Brhane Gebremichel¹; Jae-Ho Lee²

¹ *University of Science and Technology of China*

² *University of North Florida & POSTECH*

Corresponding Author: jaeho.lee@unf.edu

Let Γ be a graph with diameter at least two. Then Γ is said to be 1-homogeneous (in the sense of Nomura) whenever for every pair of adjacent vertices x and y in Γ , the distance partition of the vertex set of Γ with respect to both x and y is equitable, and the parameters corresponding to equitable partitions are independent of the choice of x and y . Assume Γ is 1-homogeneous distance-regular with intersection number $a_1 > 0$ and diameter D

Define $b = b_1/(\theta_1 + 1)$, where b_1 is the intersection number and θ_1 is the second largest eigenvalue of Γ . In this talk, we show that if intersection number c_2

then b

and one of the following (i)–(vi) holds: (i) Γ is a regular near $2D$ -gon, (ii) Γ is a Johnson graph $J(2D, D)$, (iii) Γ is a halved ℓ -cube where $\ell \in \{2D, 2D + 1\}$, (iv) Γ is a folded Johnson graph $J(4D, 2D)$, (v) Γ is a folded halved $(4D)$ -cube, (vi) the valency of Γ is bounded by a function of b . Moreover, we characterize 1-homogeneous graphs with classical parameters and $a_1 > 0$, as well as tight distance-regular graphs. This is a joint work with J. Koolen, M. Abdullah, B. Gebremichel.

Invited Talk / 1

Extensions of the colorful Helly theorem for d -collapsible and d -Leray complexes

Authors: Minki Kim¹; Alan Lew²

¹ *GIST*

² *Carnegie Mellon University*

Corresponding Author: minkikim@gist.ac.kr

We present extensions of the colorful Helly theorem for d -collapsible and d -Leray complexes, providing a common generalization to the topological colorful Helly theorem by Kalai and Meshulam, the very colorful Helly theorem by Arocha et al., and the semi-intersecting colorful Helly theorem by Karasev and Montejano. As an application, we obtain a strengthened version of Tverberg's theorem. This is joint work with Alan Lew.

Contributed Talk / 10

Transversal Hamilton paths and cycles of arbitrary orientations in tournaments

Authors: Hyunwoo Lee¹; Debsoumya Chakraborti²; Jaehoon Kim³; Jaehyeon Seo⁴

¹ KAIST & IBS ECOPRO

² University of Warwick

³ KAIST

⁴ Yonsei University

Corresponding Author: jaehyeonseo@yonsei.ac.kr

It is well-known that a tournament always contains a directed Hamilton path. Rosenfeld conjectured that if a tournament is sufficiently large, it contains a Hamilton path of any given orientation. This conjecture was approved by Thomason, and Havet and Thomassé completely resolved it by showing there are exactly three exceptions.

We generalized this result into a transversal setting. Let $\mathbf{T} = \{T_1, \dots, T_{n-1}\}$ be a collection of tournaments on a common vertex set V of size n . We showed that if n is sufficiently large, there is a Hamilton path on V of any given orientation which is obtained by collecting exactly one arc from each T_i . Such a path is said to be *transversal*.

It is also a folklore that a strongly connected tournament always contains a directed Hamilton cycle. Rosenfeld made a conjecture for arbitrarily oriented Hamilton cycles in tournaments as well, which was approved by Thomason (for sufficiently large tournaments) and Zein (by specifying all the exceptions). We also showed a transversal version of this result. Together with the aforementioned result, it extends our previous research, which is on transversal generalizations of existence of directed paths and cycles in tournaments.

This is a joint work with Debsoumya Chakraborti, Jaehoon Kim, and Hyunwoo Lee.

Invited Talk / 16

Toric Colorability of Graphs of Simplicial d -Polytopes with $d + 4$ vertices

Author: Suyoung Choi¹

¹ Ajou University

Corresponding Author: schoi@ajou.ac.kr

The 1-skeleton of a convex polytope P is called the graph of P .

A graph of a simplicial d -polytope is said to be toric colorable if there is a vertex coloring $\lambda: V(G) \rightarrow \mathbb{Z}^d$ such that $\{v_1, \dots, v_d\}$ forms a face of P implies that $\{\lambda(v_1), \dots, \lambda(v_d)\}$ is unimodular.

In this talk, we discuss the toric colorability of graphs of simplicial d -polytopes with $d + 4$ vertices.

Contributed Talk / 2

Alternating \mathcal{B} -permutations arising from toric topology

Authors: Suyoung Choi¹; Younghun Yoon²

¹ Ajou university

² Ajou University

Corresponding Author: younghan300@ajou.ac.kr

In this talk, we focus on the rational Betti numbers of real toric manifolds associated with chordal nestohedra. We introduce an explicit description for the Betti numbers using alternating \mathcal{B} -permutations for a chordal building set \mathcal{B} . We provide detailed computations for interesting cases of chordal nestohedra, including permutohedra, associahedra, stellohedra, Stanley-Pitman polytopes, and Hochschild polytopes.

This is joint work with Suyoung Choi.

Contributed Talk / 9

Partitions of ordered partitions and Bott manifolds

Authors: Junho Jeong¹; Jang Soo Kim²; Eunjeong Lee¹

¹ *Chungbuk National University*

² *Sungkyunkwan University*

Corresponding Author: junhojeong@chungbuk.ac.kr

Bott manifolds are smooth projective toric varieties providing interesting avenues among topology, geometry, representation theory, and combinatorics. They are used to understand the geometric structure of Bott-Samelson-Demazure-Hansen (BSDH) varieties, which provide desingularizations of Schubert varieties. However, not all Bott manifolds originate from BSDH varieties. Those that do are specifically referred to as Bott manifolds of *Bott-Samelson-Demazure-Hansen type*. In this talk, we explore a relationship between Bott manifolds of BSDH type and partitions of ordered partitions. This talk is based on joint work with Jang Soo Kim and Eunjeong Lee.

Contributed Talk / 5

Homotopy Types of Vietoris-Rips Complexes and Their Connection to Hyperconvexity

Author: Sunhyuk Lim¹

¹ *Sungkyunkwan University*

Corresponding Author: lsh3109@skku.edu

The Vietoris-Rips complex, originally introduced by Leopold Vietoris in the early 1900s to develop a homology theory for metric spaces, has since found applications in various areas of mathematics. Eliyahu Rips and Mikhail Gromov further utilized it in their studies of hyperbolic groups. More recently, classifying the homotopy types of Vietoris-Rips complexes has become a significant problem in Topological Data Analysis and Global Metric Geometry. Understanding these complexes can enhance our grasp of the persistence barcode's strength and provide lower bounds for the Gromov-Hausdorff distance between manifolds. In this talk, we will delve into these motivations and introduce the precise connections between Vietoris-Rips complexes, hyperconvex metric spaces, and their homotopy types.

Contributed Talk / 6

On the extremal number of face-incidence graphs

Authors: David Conlon¹; Jisun Baek²; Joonkyung Lee²

¹ *California Institute of Technology*

² *Yonsei University*

Corresponding Author: baek_jisun@yonsei.ac.kr

The (k, r) -incidence graph of a regular polytope \mathcal{P} is the bipartite incidence graph between k -faces and r -faces of \mathcal{P} . We obtain a general upper bound and a corresponding supersaturation result for the extremal number of the (k, r) -incidence graph of any regular polytope.

This generalises recent results of Janzer and Sudakov, who obtained the same bound for hypercubes and bipartite Kneser graphs, and confirms the conjecture of Conlon and Lee on the extremal number of $K_{d,d}$ -free bipartite graphs for certain (k, r) -incidence graphs.

Our proof, based on the reflection group method developed by Conlon and Lee, presents the method in a purely algebraic manner.

As a consequence, this puts a number of results, including the Janzer-Sudakov theorem, the Conlon-Lee theorem on weakly norming graphs, and Coregliano's theorem on Sidorenko's conjecture, in the unified framework and simplifies all the proofs.

Joint work with David Conlon and Joonkyung Lee.

Invited Talk / 20

Lusztig q weight multiplicities via affine crystals

Authors: Hyeonjae Choi^{None}; Donghyun Kim^{None}; Seung Jin Lee^{None}

Corresponding Author: hyun920310@snu.ac.kr

Lusztig q weight multiplicity is a polynomial in q whose positivity has been verified by linking it to a specific affine Kazhdan-Lusztig polynomial. However, a combinatorial formula beyond type A has not been known until recently.

In 2019, Lee proposed a combinatorial formula for type C using a novel combinatorial concept known as semistandard oscillating tableaux. We will outline the proof of Lee's conjecture and discuss how it can be extended to type B spin weights case.

Based on joint work with Hyeonjae Choi and Seung Jin Lee.

Contributed Talk / 13

Combinatorics of orthogonal polynomials on the unit circle

Authors: Jihyeug Jang¹; Minho Song¹

¹ *Sungkyunkwan University*

Corresponding Author: smh3227@skku.edu

Orthogonal polynomials on the unit circle (OPUC) are a family of polynomials orthogonal with respect to integration on the unit circle in the complex plane. The values of these integrals can be obtained by calculating moments. Numerous combinatorial studies have explored the moments of various types of orthogonal polynomials, including classical orthogonal polynomials, Laurent biorthogonal polynomials, and orthogonal polynomials of type R_I .

In this talk, we first explain how OPUC relate to these other variations. Next, we study the moments of OPUC from a combinatorial perspective, providing three path interpretations: Łukasiewicz paths, gentle Motzkin paths, and Schröder paths. Using these combinatorial interpretations, we derive explicit formulas for the generalized moments of some examples of OPUC, including the circular Jacobi polynomials and the Rogers–Szegő polynomials. Furthermore, we introduce several types of generalized linearization coefficients and provide combinatorial interpretations for each of them.

Invited Talk / 14

Enumeration of multiplex juggling card sequences using generalized q -derivatives

Authors: Yumin Cho¹; Jaehyun Kim¹; Jang Soo Kim²; Nakyung Lee¹

¹ *Gyeonggi Science High School for the Gifted*

² *Sungkyunkwan University*

Corresponding Author: jangsookim@skku.edu

In 2019, Butler, Choi, Kim, and Seo introduced a new type of juggling card that represents multiplex juggling patterns in a natural bijective way. They conjectured a formula for the generating function for the number of multiplex juggling cards with capacity.

In this paper we prove their conjecture. More generally, we find an explicit formula for the generating function with any capacity. We also find an expression for the generating function for multiplex juggling card sequences by introducing a generalization of the q -derivative operator. As a consequence, we show that this generating function is a rational function.

Contributed Talk / 4

Two ways to generalize matroids with coefficients

Author: Donggyu Kim¹

¹ *KAIST & IBS DIMAG*

Corresponding Author: donggyu@kaist.ac.kr

Dress (1986) introduced matroids with coefficients offering a unified approach to ordinary matroids, representations of matroids over fields, and oriented matroids. Baker and Bowler (2019) extended this theory, whose result includes a partial field representation by Semple and Whittle (1996).

I will present two generalizations of matroids with coefficients. One is about skew-symmetric matrices and even delta-matroids, based on joint work with Tong Jin. We deduce several results on the representability of even delta-matroids as applications. The other concerns symmetric matrices and new matroid-like objects called antisymmetric matroids. It extends old results on the representability of matroids by Tutte (1958) and basis graphs of matroids by Maurer (1973). These two generalizations involve an interesting interplay between Lagrangian orthogonal/symplectic Grassmannians and combinatorics.