

# Towards a classification of 1-homogeneous graphs with positive intersection number $a_1$

Wednesday, 28 August 2024 17:00 (30 minutes)

Let  $\Gamma$  be a graph with diameter at least two. Then  $\Gamma$  is said to be 1-homogeneous (in the sense of Nomura) whenever for every pair of adjacent vertices  $x$  and  $y$  in  $\Gamma$ , the distance partition of the vertex set of  $\Gamma$  with respect to both  $x$  and  $y$  is equitable, and the parameters corresponding to equitable partitions are independent of the choice of  $x$  and  $y$ . Assume  $\Gamma$  is 1-homogeneous distance-regular with intersection number  $a_1 > 0$  and diameter  $D$

Define  $b = b_1/(\theta_1 + 1)$ , where  $b_1$  is the intersection number and  $\theta_1$  is the second largest eigenvalue of  $\Gamma$ . In this talk, we show that if intersection number  $c_2$

then  $b$

and one of the following (i)–(vi) holds: (i)  $\Gamma$  is a regular near  $2D$ -gon, (ii)  $\Gamma$  is a Johnson graph  $J(2D, D)$ , (iii)  $\Gamma$  is a halved  $\ell$ -cube where  $\ell \in \{2D, 2D + 1\}$ , (iv)  $\Gamma$  is a folded Johnson graph  $\bar{J}(4D, 2D)$ , (v)  $\Gamma$  is a folded halved  $(4D)$ -cube, (vi) the valency of  $\Gamma$  is bounded by a function of  $b$ . Moreover, we characterize 1-homogeneous graphs with classical parameters and  $a_1 > 0$ , as well as tight distance-regular graphs. This is a joint work with J. Koolen, M. Abdullah, B. Gebremichel.

**Primary authors:** KOOLEN, Jack (University of Science and Technology of China); ABDULLAH, Mamoon (University of Science and Technology of China); GEBREMICHEL, Brhane (University of Science and Technology of China); LEE, Jae-Ho (University of North Florida & POSTECH)

**Presenter:** LEE, Jae-Ho (University of North Florida & POSTECH)

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