Combinatorics of orthogonal polynomials on the unit circle

Friday, 30 August 2024 11:00 (30 minutes)

Orthogonal polynomials on the unit circle (OPUC) are a family of polynomials orthogonal with respect to integration on the unit circle in the complex plane. The values of these integrals can be obtained by calculating moments. Numerous combinatorial studies have explored the moments of various types of orthogonal polynomials, including classical orthogonal polynomials, Laurent biorthogonal polynomials, and orthogonal polynomials of type R_I .

In this talk, we first explain how OPUC relate to these other variations. Next, we study the moments of OPUC from a combinatorial perspective, providing three path interpretations: Łukasiewicz paths, gentle Motzkin paths, and Schröder paths. Using these combinatorial interpretations, we derive explicit formulas for the generalized moments of some examples of OPUC, including the circular Jacobi polynomials and the Rogers–Szegő polynomials. Furthermore, we introduce several types of generalized linearization coefficients and provide combinatorial interpretations for each of them.

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