2024 Combinatorics Workshop

Report of Contributions

Contribution ID: **1** Type: **Invited talk**

Extensions of the colorful Helly theorem for d-collapsible and d-Leray complexes

Thursday, 29 August 2024 09:30 (1 hour)

We present extensions of the colorful Helly theorem for d-collapsible and d-Leray complexes, providing a common generalization to the topological colorful Helly theorem by Kalai and Meshulam, the very colorful Helly theorem by Arocha et al., and the semi-intersecting colorful Helly theorem by Karasev and Montejano. As an application, we obtain a strengthened version of Tverberg's theorem. This is joint work with Alan Lew.

Primary authors: KIM, Minki (GIST); LEW, Alan (Carnegie Mellon University)

Presenter: KIM, Minki (GIST)

Session Classification: Invited Talk

Contribution ID: **2** Type: **not specified**

Alternating $\mathcal{B}$-permutations arising from toric topology

Thursday, 29 August 2024 15:00 (30 minutes)

In this talk, we focus on the rational Betti numbers of real toric manifolds associated with chordal nestohedra. We introduce an explicit description for the Betti numbers using alternating $\mathcal{B}$-permutations for a chordal building set $\mathcal{B}$. We provide detailed computations for interesting cases of chordal nestohedra, including permutohedra, associahedra, stellohedra, Stanley-Pitman polytopes, and Hochschild polytopes.
This is joint work with Suyoung Choi.

Primary authors: Prof. CHOI, Suyoung (Ajou university); YOON, Younghan (Ajou University)

Presenter: YOON, Younghan (Ajou University)

Session Classification: Contributed Talk

Contribution ID: **3** Type: **not specified**

Towards a classification of 1-homogeneous graphs with positive intersection number a_1

Wednesday, 28 August 2024 17:00 (30 minutes)

Let Γ be a graph with diameter at least two. Then Γ is said to be 1-homogeneous (in the sense of Nomura) whenever for every pair of adjacent vertices x and y in Γ, the distance partition of the vertex set of Γ with respect to both x and y is equitable, and the parameters corresponding to equitable partitions are independent of the choice of x and y. Assume Γ is 1-homogeneous distance-regular with intersection number $a_1 > 0$ and diameter D
$geqslant 5$. Define $b = b_1/(\theta_1 + 1)$, where b_1 is the intersection number and θ_1 is the second largest eigenvalue of Γ. In this talk, we show that if intersection number c_2
$geqslant 2$, then b
$geqslant 1$ and one of the following (i)–(vi) holds: (i) Γ is a regular near $2D$-gon, (ii) Γ is a Johnson graph $J(2D, D)$, (iii) Γ is a halved ℓ-cube where $\ell \in \{2D, 2D+1\}$, (iv) Γ is a folded Johnson graph $\bar{J}(4D, 2D)$, (v) Γ is a folded halved $(4D)$-cube, (vi) the valency of Γ is bounded by a function of b. Moreover, we characterize 1-homogeneous graphs with classical parameters and $a_1 > 0$, as well as tight distance-regular graphs. This is a joint work with J. Koolen, M. Abdullah, B. Gebremichel.

Primary authors: KOOLEN, Jack (University of Science and Technology of China); ABDULLAH, Mamoon (University of Science and Technology of China); GEBREMICHEL, Brhane (University of Science and Technology of China); LEE, Jae-Ho (University of North Florida & POSTECH)

Presenter: LEE, Jae-Ho (University of North Florida & POSTECH)

Session Classification: Contributed Talk

Contribution ID: **4** Type: **not specified**

Two ways to generalize matroids with coefficients

Friday, 30 August 2024 14:30 (30 minutes)

Dress (1986) introduced matroids with coefficients offering a unified approach to ordinary matroids, representations of matroids over fields, and oriented matroids. Baker and Bowler (2019) extended this theory, whose result includes a partial field representation by Semple and Whittle (1996).

I will present two generalizations of matroids with coefficients. One is about skew-symmetric matrices and even delta-matroids, based on joint work with Tong Jin. We deduce several results on the representability of even delta-matroids as applications. The other concerns symmetric matrices and new matroid-like objects called antisymmetric matroids. It extends old results on the representability of matroids by Tutte (1958) and basis graphs of matroids by Maurer (1973). These two generalizations involve an interesting interplay between Lagrangian orthogonal/symplectic Grassmannians and combinatorics.

Primary author: KIM, Donggyu (KAIST & IBS DIMAG)

Presenter: KIM, Donggyu (KAIST & IBS DIMAG)

Session Classification: Contributed Talk

Contribution ID: **5** Type: **Contributed talk**

Homotopy Types of Vietoris-Rips Complexes and Their Connection to Hyperconvexity

Thursday, 29 August 2024 16:30 (30 minutes)

The Vietoris-Rips complex, originally introduced by Leopold Vietoris in the early 1900s to develop a homology theory for metric spaces, has since found applications in various areas of mathematics. Eliyahu Rips and Mikhail Gromov further utilized it in their studies of hyperbolic groups. More recently, classifying the homotopy types of Vietoris-Rips complexes has become a significant problem in Topological Data Analysis and Global Metric Geometry. Understanding these complexes can enhance our grasp of the persistence barcode's strength and provide lower bounds for the Gromov-Hausdorff distance between manifolds. In this talk, we will delve into these motivations and introduce the precise connections between Vietoris-Rips complexes, hyperconvex metric spaces, and their homotopy types.

Primary author: LIM, Sunhyuk (Sungkyunkwan University)

Presenter: LIM, Sunhyuk (Sungkyunkwan University)

Session Classification: Contributed Talk

Contribution ID: **6** Type: **Contributed talk**

On the extremal number of face-incidence graphs

Thursday, 29 August 2024 17:00 (30 minutes)

The (k, r)-incidence graph of a regular polytope $\mathcal{P}$ is the bipartite incidence graph between k-faces and r-faces of $\mathcal{P}$. We obtain a general upper bound and a corresponding supersaturation result for the extremal number of the (k, r)-incidence graph of any regular polytope.
This generalises recent results of Janzer and Sudakov, who obtained the same bound for hypercubes and bipartite Kneser graphs, and confirms the conjecture of Conlon and Lee on the extremal number of $K_{d,d}$-free bipartite graphs for certain (k, r)-incidence graphs.

Our proof, based on the reflection group method developed by Conlon and Lee, presents the method in a purely algebraic manner.
As a consequence, this puts a number of results, including the Janzer-Sudakov theorem, the Conlon-Lee theorem on weakly norming graphs, and Coregliano's theorem on Sidorenko's conjecture, in the unified framework and simplifies all the proofs.

Joint work with David Conlon and Joonkyung Lee.

Primary authors: Prof. CONLON, David (California Institute of Technology); BAEK, Jisun (Yonsei University); Prof. LEE, Joonkyung (Yonsei University)

Presenter: BAEK, Jisun (Yonsei University)

Session Classification: Contributed Talk

Contribution ID: **7** Type: **not specified**

Random matchings in linear hypergraphs

Wednesday, 28 August 2024 16:30 (30 minutes)

For a given hypergraph H and a vertex $v \in V(H)$, consider a random matching M chosen uniformly from the set of all matchings in H. In 1995, Kahn conjectured that if H is a d-regular linear k-uniform hypergraph, the probability that M does not cover v is $(1+o_d(1))d^{-1/k}$ for all vertices $v \in V(H)$. This conjecture was proved for $k = 2$ by Kahn and Kim in 1998.
We disprove this conjecture for all $k \geq 3$. For infinitely many values of d, we construct d-regular linear k-uniform hypergraph H containing two vertices v_1 and v_2 such that $\mathcal{P}(v_1 \notin M) = 1 - \frac{(1+o_d(1))}{d^{k-2}}$ and $\mathcal{P}(v_2 \notin M) = \frac{(1+o_d(1))}{d+1}$. The gap between $\mathcal{P}(v_1 \notin M)$ and $\mathcal{P}(v_2 \notin M)$ in this H is best possible. In the course of proving this, we also prove a hypergraph analog of Godsil's result on matching polynomials and paths in graphs, which is of independent interest.

Primary author: LEE, Hyunwoo (KAIST & IBS ECOPRO)

Presenter: LEE, Hyunwoo (KAIST & IBS ECOPRO)

Session Classification: Contributed Talk

Contribution ID: **8** Type: **not specified**

102-avoiding Inversion Sequences

Wednesday, 28 August 2024 16:00 (30 minutes)

A sequence $(e_1, e_2, \cdots, e_n)$ is an inversion sequences if $0 \leq e_i < i$ for all $i = 1, \ldots, n$. We say that an inversion sequences $e = (e_1, e_2, \cdots, e_n)$ \emph{contains} the pattern 102 if there exist some indices $i < j < k$ such that $e_j < e_i < e_k$. Otherwise, e is said to \emph{avoid} the pattern 102.

In this talk, we will construct a correspondence between the set of 2-Schröder paths without peaks and valleys ending with a diagonal step and the set of 102-avoiding inversion sequences.
This is the joint work with JiSun Huh, Sangwook Kim, and Seunghyun Seo.

Primary authors: HUH, JiSun (Ajou University); KIM, Sangwook (Chonnam National University); SEO, Seunghyun (Kangwon National University); SHIN, Heesung (Inha University)

Presenter: SHIN, Heesung (Inha University)

Session Classification: Contributed Talk

Contribution ID: **9** Type: **not specified**

Partitions of ordered partitions and Bott manifolds

Thursday, 29 August 2024 15:30 (30 minutes)

Bott manifolds are smooth projective toric varieties providing interesting avenues among topology, geometry, representation theory, and combinatorics. They are used to understand the geometric structure of Bott-Samelson-Demazure-Hansen (BSDH) varieties, which provide desingularizations of Schubert varieties. However, not all Bott manifolds originate from BSDH varieties. Those that do are specifically referred to as Bott manifolds *of Bott-Samelson-Demazure-Hansen type*. In this talk, we explore a relationship between Bott manifolds of BSDH type and partitions of ordered partitions. This talk is based on joint work with Jang Soo Kim and Eunjeong Lee.

Primary authors: JEONG, Junho (Chungbuk National University); Prof. KIM, Jang Soo (Sungkyunkwan University); LEE, Eunjeong (Chungbuk National University)

Presenter: JEONG, Junho (Chungbuk National University)

Session Classification: Contributed Talk

Contribution ID: **10** Type: **not specified**

Transversal Hamilton paths and cycles of arbitrary orientations in tournaments

Thursday, 29 August 2024 11:00 (30 minutes)

It is well-known that a tournament always contains a directed Hamilton path. Rosenfeld conjectured that if a tournament is sufficiently large, it contains a Hamilton path of any given orientation. This conjecture was approved by Thomason, and Havet and Thomassé completely resolved it by showing there are exactly three exceptions.

We generalized this result into a transversal setting. Let $\mathbf{T} = \{T_1, \ldots, T_{n-1}\}$ be a collection of tournaments on a common vertex set V of size n. We showed that if n is sufficiently large, there is a Hamilton path on V of any given orientation which is obtained by collecting exactly one arc from each T_i. Such a path is said to be *transversal*.

It is also a folklore that a strongly connected tournament always contains a directed Hamilton cycle. Rosenfeld made a conjecture for arbitrarily oriented Hamilton cycles in tournaments as well, which was approved by Thomason (for sufficiently large tournaments) and Zein (by specifying all the exceptions). We also showed a transversal version of this result. Together with the aforementioned result, it extends our previous research, which is on transversal generalizations of existence of directed paths and cycles in tournaments.

This is a joint work with Debsoumya Chakraborti, Jaehoon Kim, and Hyunwoo Lee.

Primary authors: LEE, Hyunwoo (KAIST & IBS ECOPRO); CHAKRABORTI, Debsoumya (University of Warwick); Prof. KIM, Jaehoon (KAIST); SEO, Jaehyeon (Yonsei University)

Presenter: SEO, Jaehyeon (Yonsei University)

Session Classification: Contributed Talk

Contribution ID: **13** Type: **Contributed talk**

Combinatorics of orthogonal polynomials on the unit circle

Friday, 30 August 2024 11:00 (30 minutes)

Orthogonal polynomials on the unit circle (OPUC) are a family of polynomials orthogonal with respect to integration on the unit circle in the complex plane. The values of these integrals can be obtained by calculating moments. Numerous combinatorial studies have explored the moments of various types of orthogonal polynomials, including classical orthogonal polynomials, Laurent biorthogonal polynomials, and orthogonal polynomials of type R_I.

In this talk, we first explain how OPUC relate to these other variations. Next, we study the moments of OPUC from a combinatorial perspective, providing three path interpretations: Łukasiewicz paths, gentle Motzkin paths, and Schröder paths. Using these combinatorial interpretations, we derive explicit formulas for the generalized moments of some examples of OPUC, including the circular Jacobi polynomials and the Rogers–Szegő polynomials. Furthermore, we introduce several types of generalized linearization coefficients and provide combinatorial interpretations for each of them.

Primary authors: Dr JANG, Jihyeug (Sungkyunkwan University); SONG, Minho (Sungkyunkwan University)

Presenter: SONG, Minho (Sungkyunkwan University)

Session Classification: Contributed Talk

Contribution ID: **14** Type: **Invited talk**

Enumeration of multiplex juggling card sequences using generalized q-derivatives

Friday, 30 August 2024 13:30 (1 hour)

In 2019, Butler, Choi, Kim, and Seo introduced a new type of juggling card that represents multiplex juggling patterns in a natural bijective way. They conjectured a formula for the generating function for the number of multiplex juggling cards with capacity.
In this paper we prove their conjecture. More generally, we find an explicit formula for the generating function with any capacity. We also find an expression for the generating function for multiplex juggling card sequences by introducing a generalization of the q-derivative operator. As a consequence, we show that this generating function is a rational function.

Primary authors: Ms CHO, Yumin (Gyeonggi Science High School for the Gifted); Mr KIM, Jaehyun (Gyeonggi Science High School for the Gifted); KIM, Jang Soo (Sungkyunkwan University); Ms LEE, Nakyung (Gyeonggi Science High School for the Gifted)

Presenter: KIM, Jang Soo (Sungkyunkwan University)

Session Classification: Invited Talk

Contribution ID: **16** Type: **Invited talk**

Toric Colorability of Graphs of Simplicial d-Polytopes with $+4$ vertices

Thursday, 29 August 2024 13:30 (1 hour)

The 1-skeleton of a convex polytope P is called the graph of P.
A graph of a simplicial d-polytope is said to be toric colorable if there is a vertex coloring $\lambda\colon V(G) \to \mathbb{Z}^d$ such that $\{v_1, \ldots, v_d\}$ forms a face of P implies that $\{\lambda(v_1), \ldots, \lambda(v_d)\}$ is unimodular.
In this talk, we discuss the toric colorability of graphs of simplicial d-polytopes with $d+4$ vertices.

Primary author: CHOI, Suyoung (Ajou University)

Presenter: CHOI, Suyoung (Ajou University)

Session Classification: Invited Talk

Contribution ID: **19** Type: **Invited talk**

Asymptotic bounds of Ramsey Numbers

Wednesday, 28 August 2024 14:30 (1 hour)

Ramsey numbers, denoted as R(s,t), are fundamental in graph theory, representing the smallest number of vertices n such that every graph on n vertices either contains a clique of size s or an independent set of size t. Recent developments in Ramsey theory have focused on finding asymptotic bounds for Ramsey numbers. In this talk, we survey asymptotic bounds of Ramsey Numbers R(3,t) and R(4,t), including significant contributions of Sam Mattheus and Jacques Verstraete on R(4,t).

Primary author: KIM, Jeong Han (KIAS)

Presenter: KIM, Jeong Han (KIAS)

Session Classification: Invited Talk

Contribution ID: **20** Type: **Invited talk**

Lusztig q weight multiplicities via affine crystals

Friday, 30 August 2024 09:30 (1 hour)

Lusztig q weight multiplicity is a polynomial in q whose positivity has been verified by linking it to a specific affine Kazhdan-Lusztig polynomial. However, a combinatorial formula beyond type A has not been known until recently.

In 2019, Lee proposed a combinatorial formula for type C using a novel combinatorial concept known as semistandard oscillating tableaux. We will outline the proof of Lee's conjecture and discuss how it can be extended to type B spin weights case.

Based on joint work with Hyeonjae Choi and Seung Jin Lee.

Primary authors: CHOI, Hyeonjae; KIM, Donghyun; LEE, Seung Jin

Presenter: KIM, Donghyun

Session Classification: Invited Talk