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Bounds on the number of cells of the Dressian

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A *valuation* of a matroid M with set of bases $\mathcal{B}$ is a function $\nu : \mathcal{B} \to \mathbb{R}$ so that

- **(V)** for all $B, B' \in \mathcal{B}$ and all $e \in B \setminus B'$ there exists an $f \in B' \setminus B$ such that $\nu(B) + \nu(B') \geq \nu(B - e + f) + \nu(B' + e - f)$.

The *Dressian* $\mathcal{D}(M)$ is defined as the set of all valuations of M. The Dressian of M is the support of a polyhedral complex in $\mathbb{R}^{\mathcal{B}}$ whose cells are polyhedral cones. We investigate $\#\mathcal{D}(M)$, the number of cells, and $\dim \mathcal{D}(M)$, the maximum dimension of any cell.

We show that if M is a matroid on an n-element groundset of rank $r \geq t \geq 3$, then

$$\frac{\dim \mathcal{D}(M)}{\binom{n}{r}} \leq \frac{\max\{\dim \mathcal{D}(M/S),\ S \in \binom{E}{r-t} \text{ independent}\}}{\binom{n-r+t}{t}} \leq \frac{3}{n-r+3}$$

and

$$\frac{\log_2 \#\mathcal{D}(M)}{\binom{n}{r}} \leq \frac{\max\{\log_2 \#\mathcal{D}(M/S)\colon S \in \binom{E}{r-t} \text{ independent }\}}{\binom{n-r+t}{t}} O(\ln(n)) \leq \frac{O\big(\ln(n)^2\big)}{n}.$$

For uniform matroids $M = U(r, n)$ these upper bounds may be compared to the lower bounds

$$\frac{1}{n} \leq \frac{\dim \mathcal{D}(M)}{\binom{n}{r}}, \qquad \frac{1}{n} \leq \frac{\log_2 \#\mathcal{D}(M)}{\binom{n}{r}}$$

that follow a previously known construction of valuations of $U(r, n)$ from matroids of rank r on n elements. For non-uniform matroids, we also obtain somewhat tighter upper bounds in terms of the number of free generators of the Tutte group of M.

Our methods include an analogue of Shearers' Entropy Lemma for bounding the dimension of a linear subspace, and a container method for collections of linear subspaces.

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