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Baker-Bowler theory for Lagrangian Grassmannians

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Baker and Bowler (2019) showed that the Grassmannian can be defined over a tract, a field-like structure generalizing both partial fields and hyperfields.
This notion unifies theories for matroids over partial fields, valuated matroids, and oriented matroids.
We extend Baker-Bowler theory to the Lagrangian Grassmannian which is the set of maximal isotropic subspaces in a $2n$-dimensional symplectic vector space.
By Boege et al. (2019), the Lagrangian Grassmannian is parameterized into the projective space of dimension $2^{n-2}(4+\binom{n}{2})-1$ and its image is cut out by certain quadrics.
We simplify a list of quadrics so that these are apparently induced by the Laplace expansions only concerning principal and almost-principal minors of a symmetric matrix.
From the idea that the strong basis exchange axiom of matroids captures the combinatorial essence of the Grassmann-Pl\"{u}cker relations, we define matroid-like objects, called antisymmetric matroids, from the quadrics for the Lagrangian Grassmannian.
We also provide its cryptomorphic definition in terms of circuits capturing the orthogonality and maximality of vectors in a Lagrangian subspace.
We define antisymmetric matroids over tracts in two equivalent ways, which generalize both Baker-Bowler theory and the parameterization of the Lagrangian Grassmannian.
It provides a new perspective on the Lagrangian Grassmannian over hyperfields such as the tropical hyperfield and the sign hyperfield.
Our proof involves a homotopy theorem for graphs associated with antisymmetric matroids, generalizing Maurer's homotopy theorem for matroids.
We also prove that if a point in the projective space satisfies the 3-/4-term quadratic relations for the Lagrangian Grassmannian and its supports form the bases of an antisymmetric matroid, then it satisfies all quadratic relations, which is motivated by the earlier work of Tutte (1958) for matroids and the Grassmannian.

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