Contribution ID: **13** Type: **Presentation (25 min)**

Problems on Group-labeled Matroid Bases

Consider a matroid equipped with a labeling of its ground set to an abelian group. We define the label of a subset of the ground set as the sum of the labels of its elements. We study a collection of problems on finding bases and common bases of matroids with restrictions on their labels. For zero bases and zero common bases, the results are mostly negative. While finding a non-zero basis of a matroid is not difficult, it turns out that the complexity of finding a non-zero common basis depends on the group. Namely, we show that the problem is hard for a fixed group if it contains an element of order two, otherwise it is polynomially solvable.

As a generalization of both zero and non-zero constraints, we further study F-avoiding constraints where we seek a basis or common basis whose label is not in a given set F of forbidden labels. Using algebraic techniques, we give a randomized algorithm for finding an F-avoiding common basis of two matroids represented over the same field for finite groups given as operation tables. The study of F-avoiding bases with groups given as oracles leads to a conjecture stating that whenever an F-avoiding basis exists, an F-avoiding basis can be obtained from an arbitrary basis by exchanging at most $|F|$ elements. We prove the conjecture for the special cases when $|F| \leq 2$ or the group is ordered. By relying on structural observations on matroids representable over fixed, finite fields, we verify a relaxed version of the conjecture for these matroids. As a consequence, we obtain a polynomial-time algorithm in these special cases for finding an F-avoiding basis when $|F|$ is fixed.

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