Shadow formation in gravitational collapse and effect of redshift by spacetime dynamics

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1. Introduction

Observation of BH:

- GW wave during a binary inspiral and merger.
- X-ray emission from a BH jet or accreting matter.
- BH shadow observation.

BH shadow:

- A distant observer observes a BH shadow when a light source exists behind/around the BH.
- Carries information of the immediate vicinity of the BH.
- Very important for testing the theory of gravity in the strong regime.

https://eventhorizontelescope.org

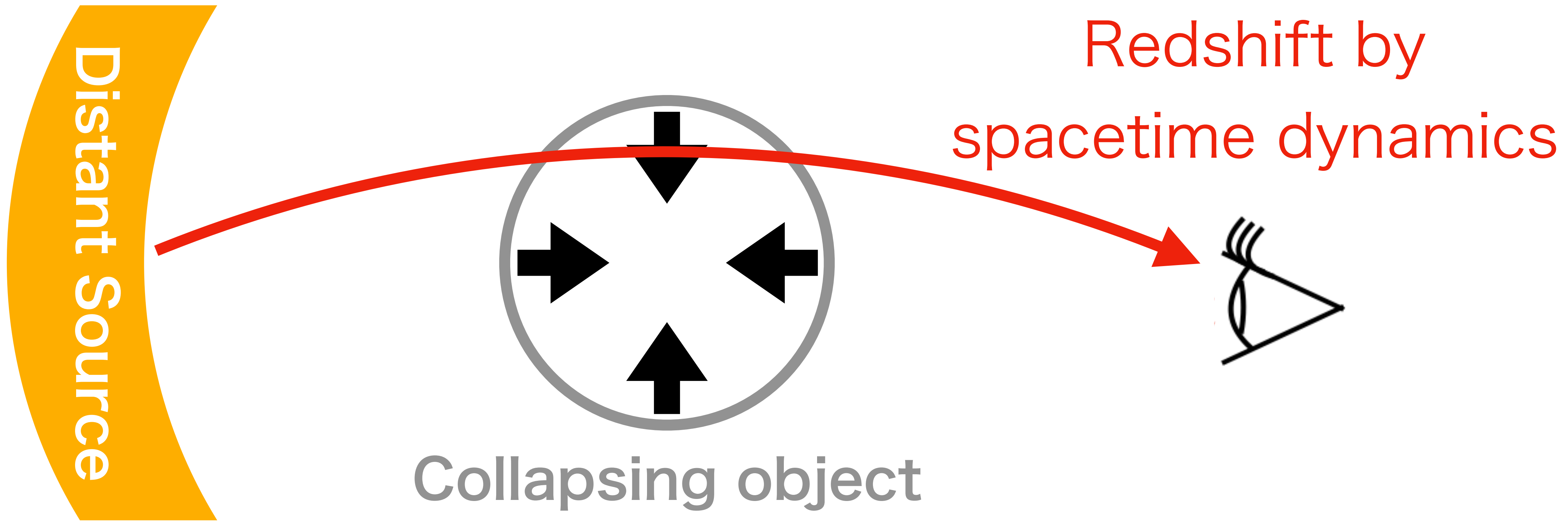
1. Introduction

Study of BH shadow:

- Static/stationary cases have been extensively investigated.
- However, almost no work on the case of gravitational collapse.
- In the case, the effect of redshift by spacetime dynamics is expected to be crucial.

In this talk:

- We investigate redshift of light and shadow formation in gravitational collapse.
- We focus on the case where the collapsing object is transmissive s.t. only the spacetime dynamics causes the redshift of light.



Outline

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1. Introduction
2. Brief review of static case
3. Thin shell model
4. General spherically symmetric case
5. Summary

2. Brief review of static case

2. Brief review

Shadow in Schwarzschild spacetime

- Schwarzschild spacetime:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2, \quad f(r) = 1 - \frac{2M}{r}$$

- Null geodesic eq.:

$$k^\nu \nabla_\nu k^\mu = 0$$

- Conserved quantities:
  - Energy: $E := -g(k, \xi_t)$ — ξ_t: timelike Killing vector
  - Angular momentum: $L := g(k, \xi_\phi)$ — ξ_ϕ: Killing vector for the spherical sym.

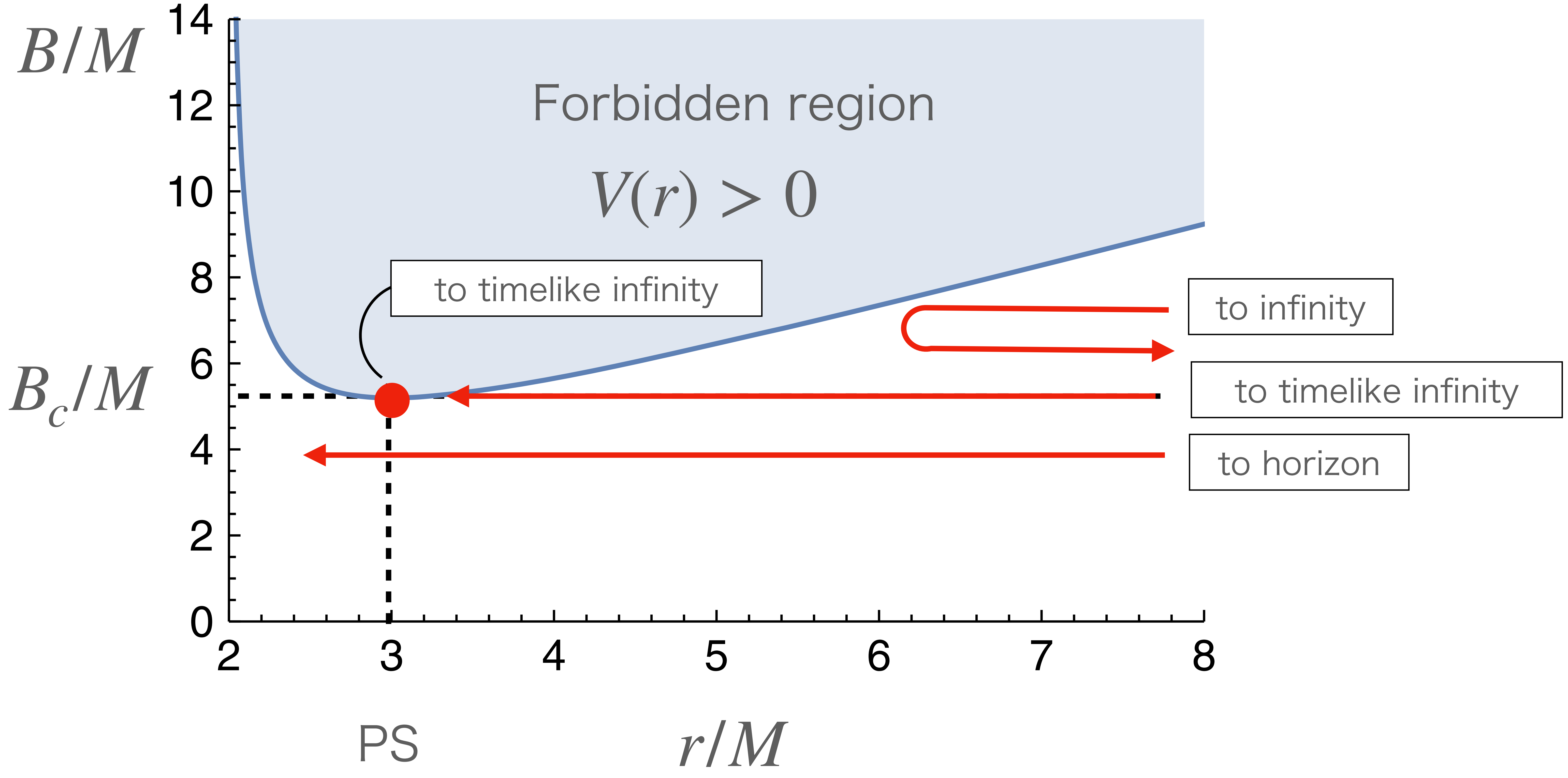
- The radial equation:

$$\dot{r}^2 + V(r) = 0, \quad V(r) = B^2 \left(1 - \frac{2M}{r}\right) r^{-2} - 1.$$

$B := L/E$: conserved impact param.

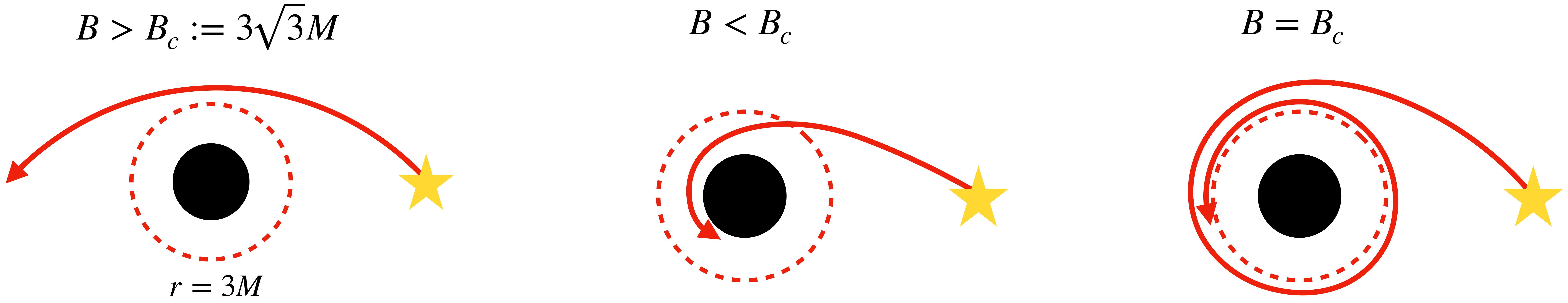
2. Brief review

Analysis of light orbits in $r - B$ plane



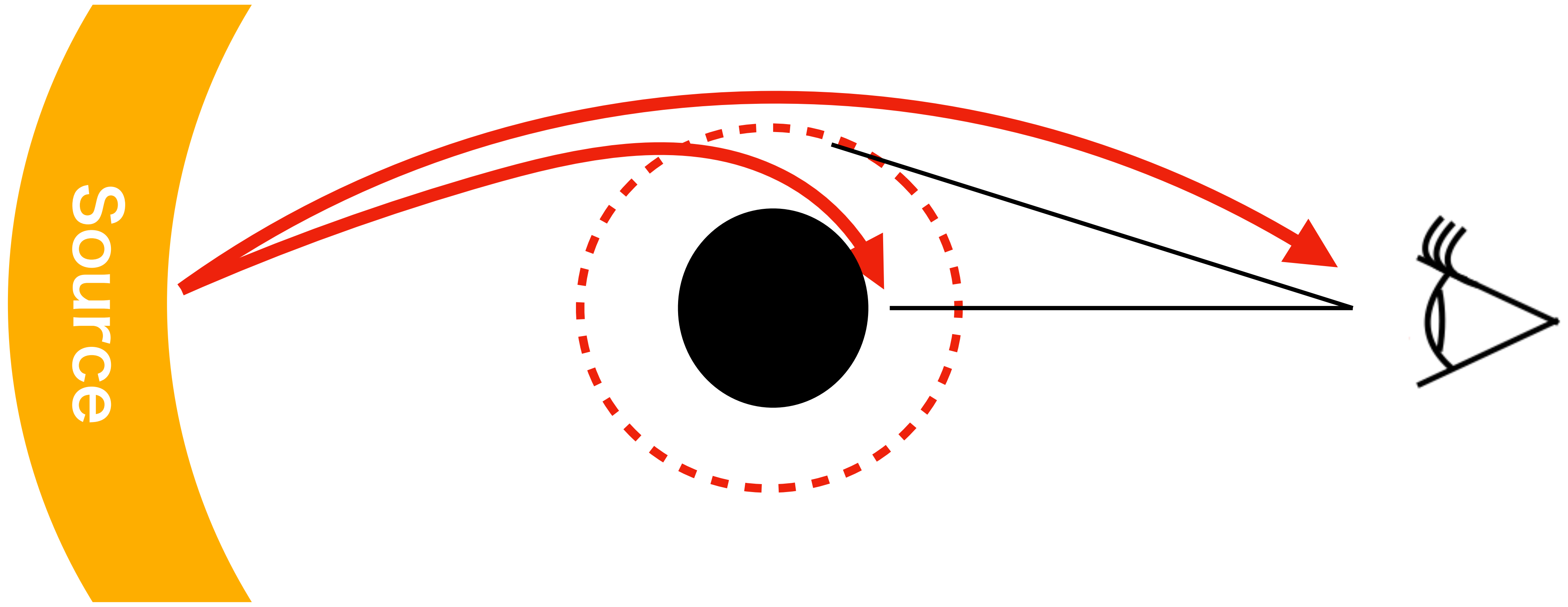
2. Brief review

3 types of light orbit from infinity:



(wind around the Photon sphere)

BH shadow with distant light source:



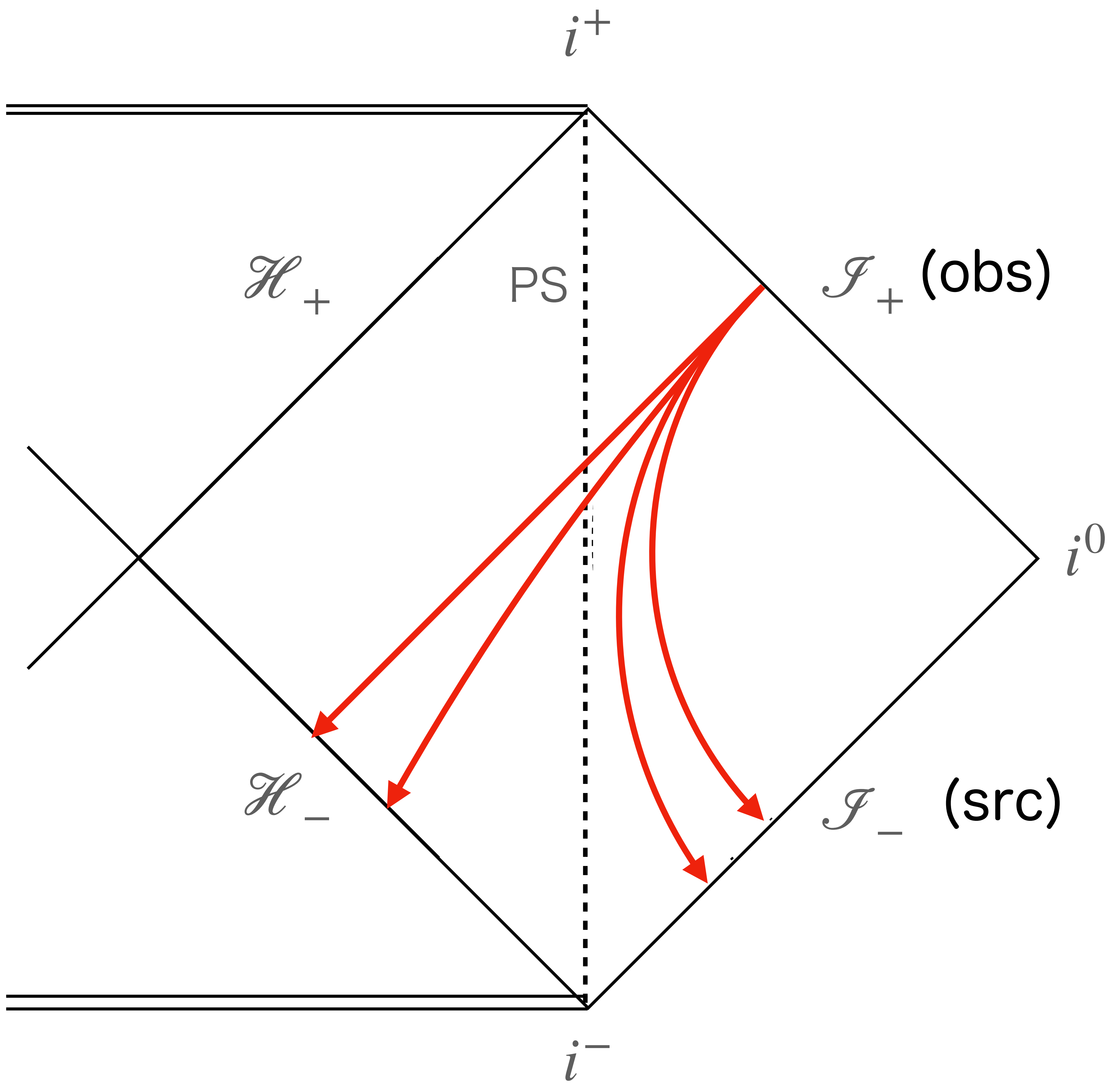
The apparent angular size of the shadow is determined by that of PS (or B_c)

2. Brief review

Causal viewpoint

- Imaging is ray-tracing backward in time.
- Past-directed light from the obs.:
  - For $B > B_c$, from $\mathscr{I}^+$ to $\mathscr{I}^-$ (i.e. light source).
  - For $B < B_c$, from $\mathscr{I}^+$ to $\mathscr{H}^-$ (white hole horizon).

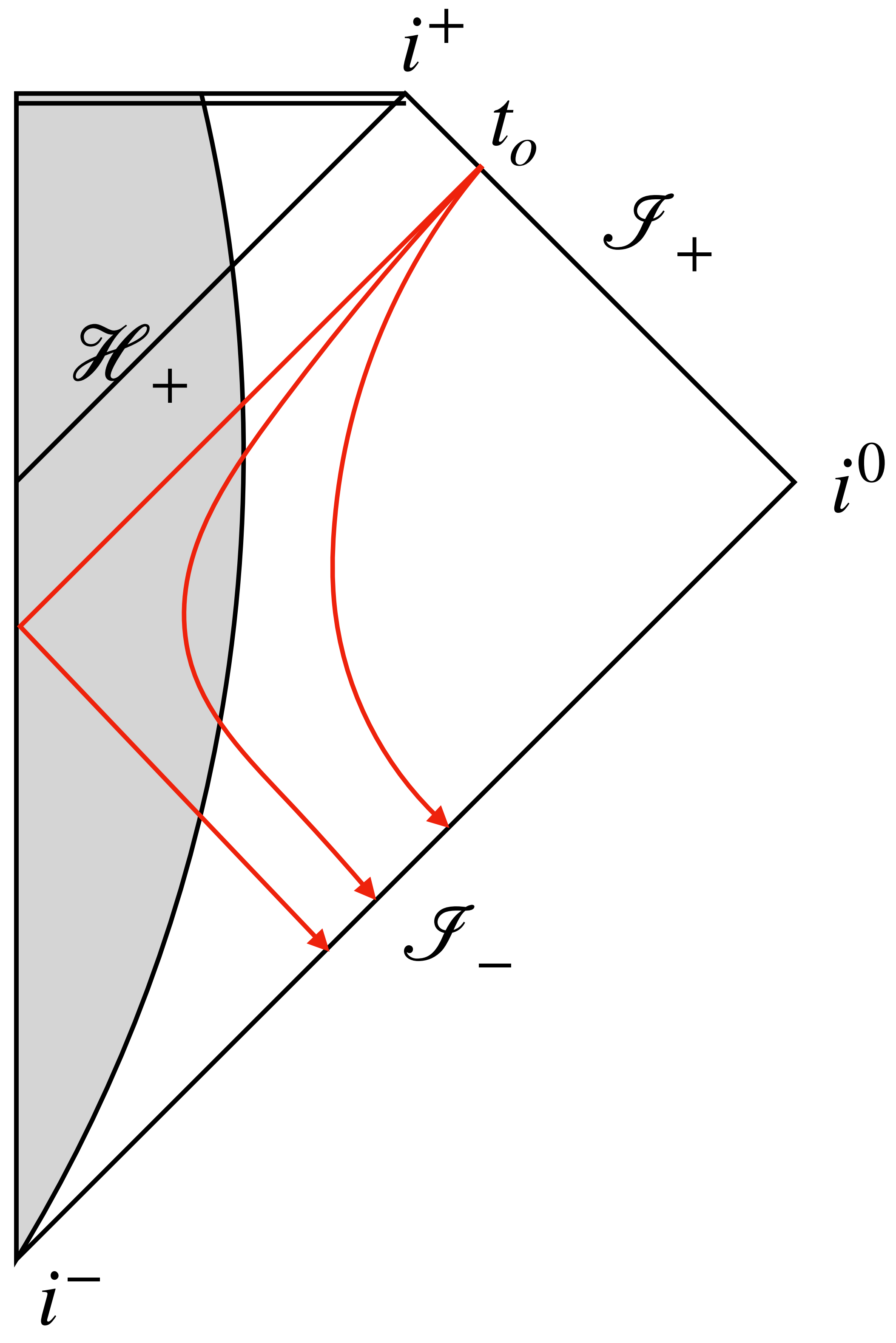
- In an eternal BH spacetime, the shadow is a complemental image of the white hole. cf. [YK, Asaka, Kimura, Okabayashi (2022)]



2. Brief review

Shadow in gravitational collapse:

- All the past-directed null geodesic from $\mathscr{I}^+$ go to $\mathscr{I}^-$ (no white hole).
- This fact does not mean that the observer never observes shadow.
- The shadow (dark) image is formed by the effect of redshift of light.



Def. of "redshift factor"

$$\alpha = \frac{E|_{\mathscr{I}^+}}{E|_{\mathscr{I}^-}}, \qquad E := -g(k, \partial_t), \quad k\text{: null geodesic tangent.}$$

- ∂_t : static Killing vector in the asymptotic region.
- Trivially, $\alpha = 1$ in static spacetime.

3. Thin shell model

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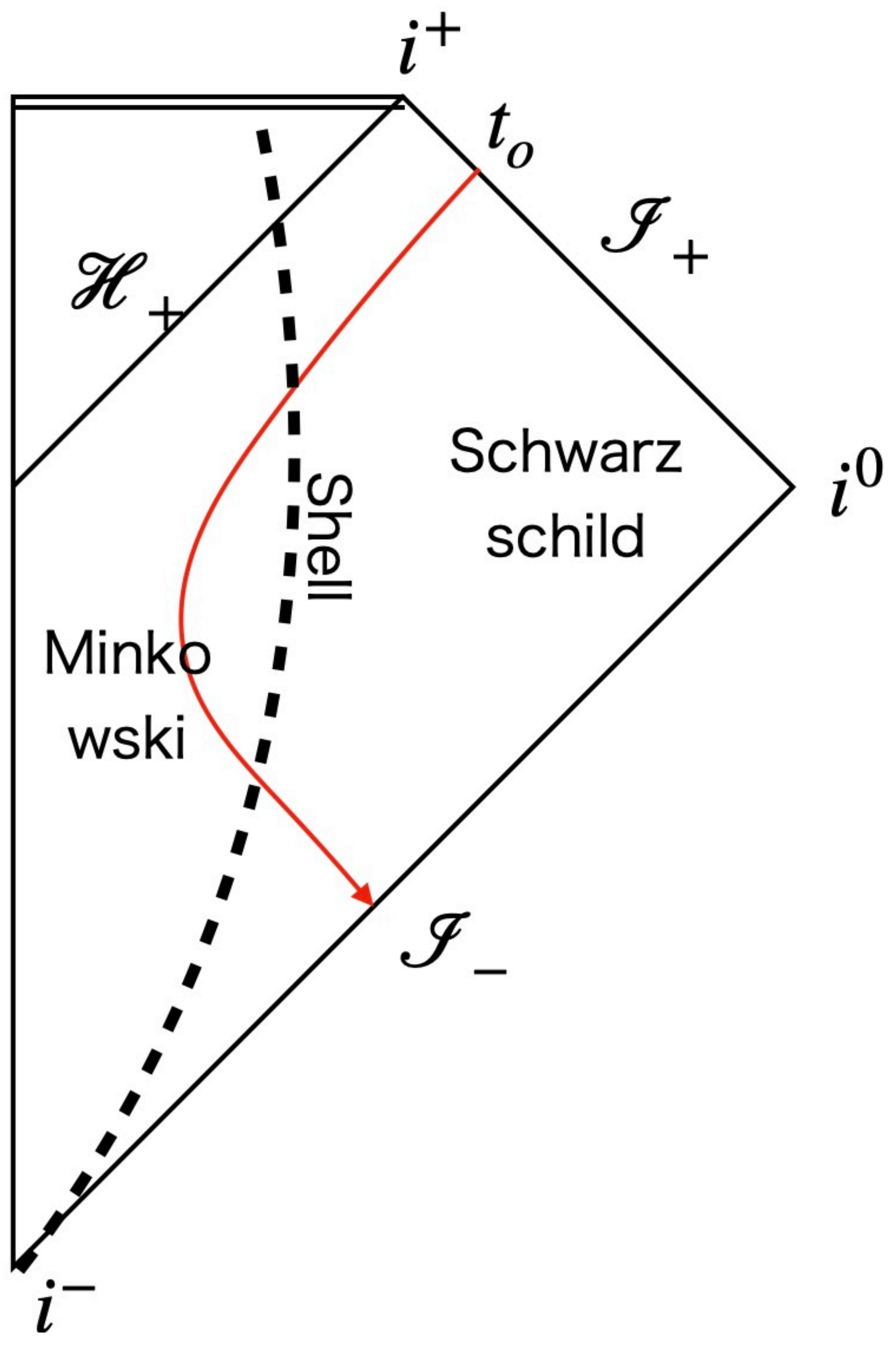
A collapsing thin shell model:



- Exterior: Schwarzschild spacetime

$$ds_{\mathrm{ex}}^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2, \;\; f(r) = 1 - \frac{2M}{r}$$

- Interior: Minkowski spacetime

$$ds_{\mathrm{in}}^2 = -dT^2 + dr^2 + r^2 d\Omega^2$$

- Boundary: a thin shell

$$\Sigma := \partial M_1 \equiv \partial M_2 = \{r = R(\tau)\}$$

τ: proper time of the shell

- Impose 1st junction condition (equivalence of induced metrics on Σ) & specify $R(\tau)$ by hand.

=> spacetime is fixed & coord. trans $t \leftrightarrow T$ is obtained.

Energy of null geodesic

Null geodesic motion:

- In the Schwarzschild region:

$$E := -g(\partial_t, k),\quad L := g(\partial_\phi, k),\quad \dot{r}^2 + V(r) = 0,\quad V(r) := E^2 - f(r)r^{-2}L^2\,.$$

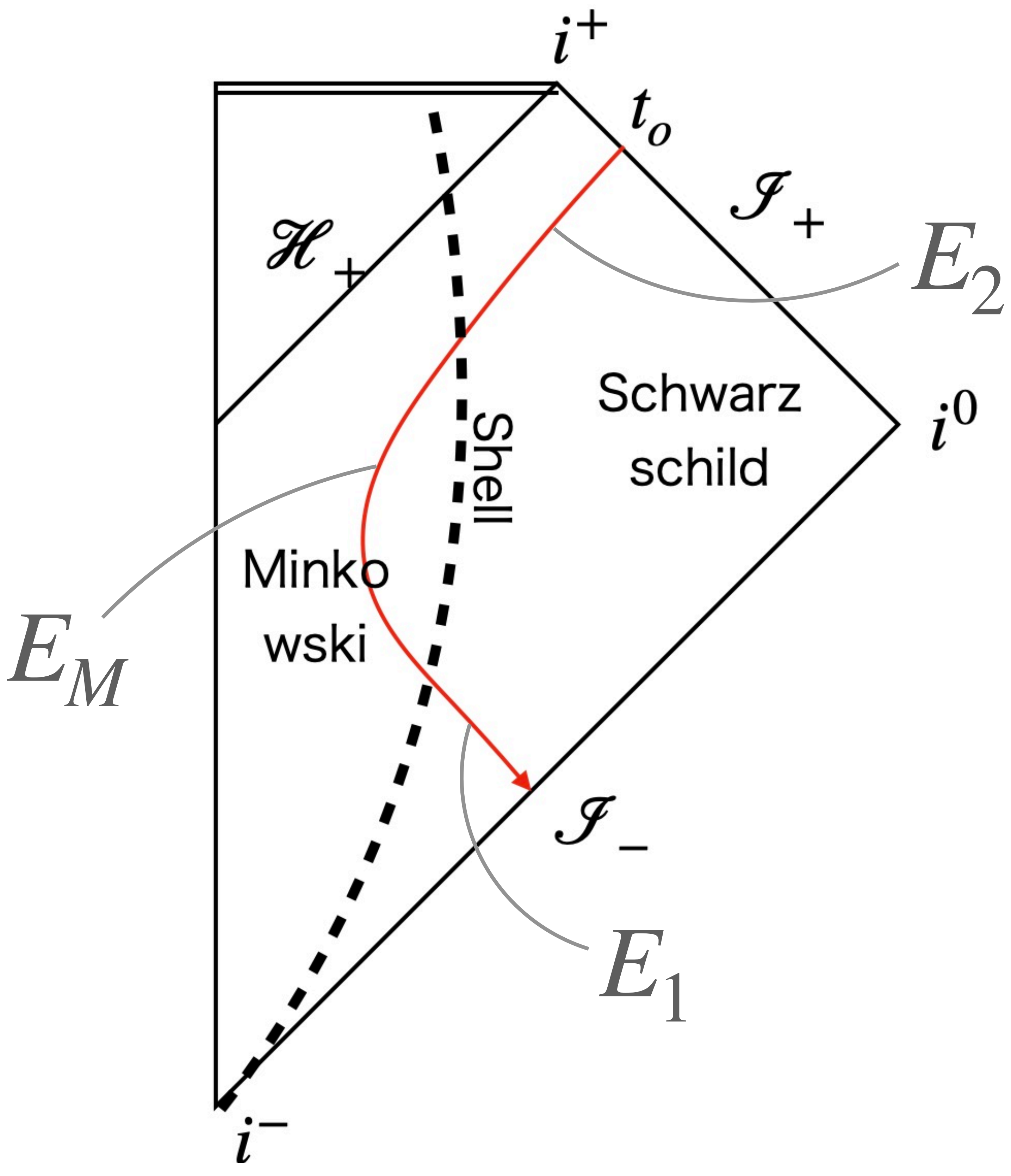
- In the Minkowski region:

$$E_M := -g(\partial_T, k),\quad L := g(\partial_\phi, k),\quad -(T-T_0)^2 + R^2 == L^2/E_M^2\,.$$

- Relation btwn energies, $E \leftrightarrow E_M$:

$$E = -g(\partial_t, k) \qquad E_M = -g(\partial_T, k)$$

coord. transformation on Σ



Energy of null geodesic

Energy of null geodesic

- Eliminating E_M,

$$\alpha = \frac{E|_{\mathscr{I}^+}}{E|_{\mathscr{I}^-}} = \frac{E_1}{E_2} = \dots$$

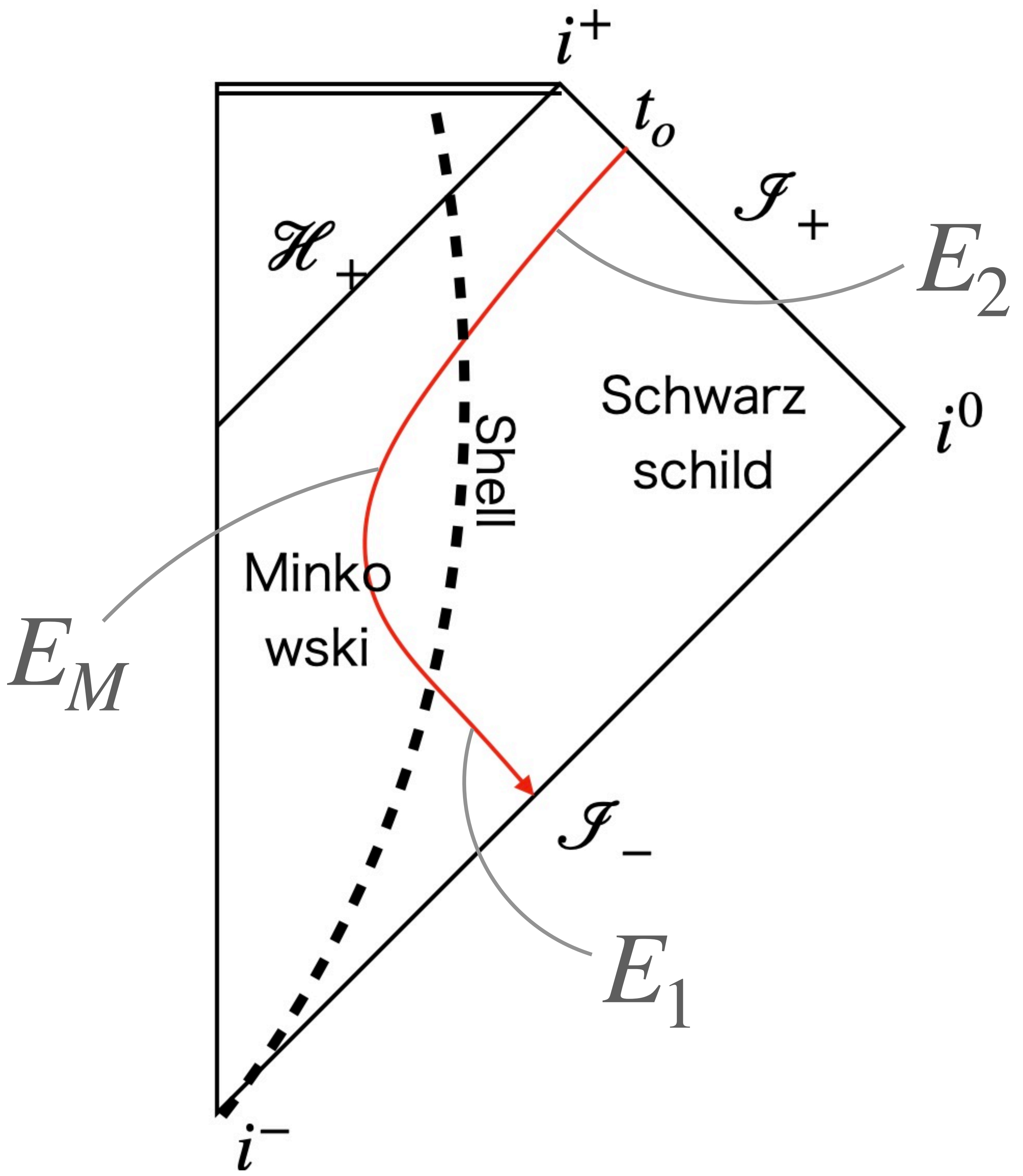
$$= \frac{f_2}{f_1} \frac{A_1\left(A_2 - C_2\sqrt{1 - B_2^2 f_2 r_2^{-2}}\right) + C_1\sqrt{\left(A_2 - C_2\sqrt{1 - B_2^2 f_2 r_2^{-2}}\right)^2 - B_2^2 f_2^2 r_1^{-2}}}{\left(A_2 - C_2\sqrt{1 - B_2^2 f_2 r_2^{-2}}\right)^2 + C_1^2 b B_2^2 f_2^2 f_1^{-1} r_1^{-2}} .$$

$$= \alpha(r_1, r_2, R_1', R_2', \sigma_1, \sigma_2, B_2) .$$

only 7 parameters determines α

$$A = \sqrt{1 + (R')^2}\sqrt{f + (R')^2} - (R')^2,$$

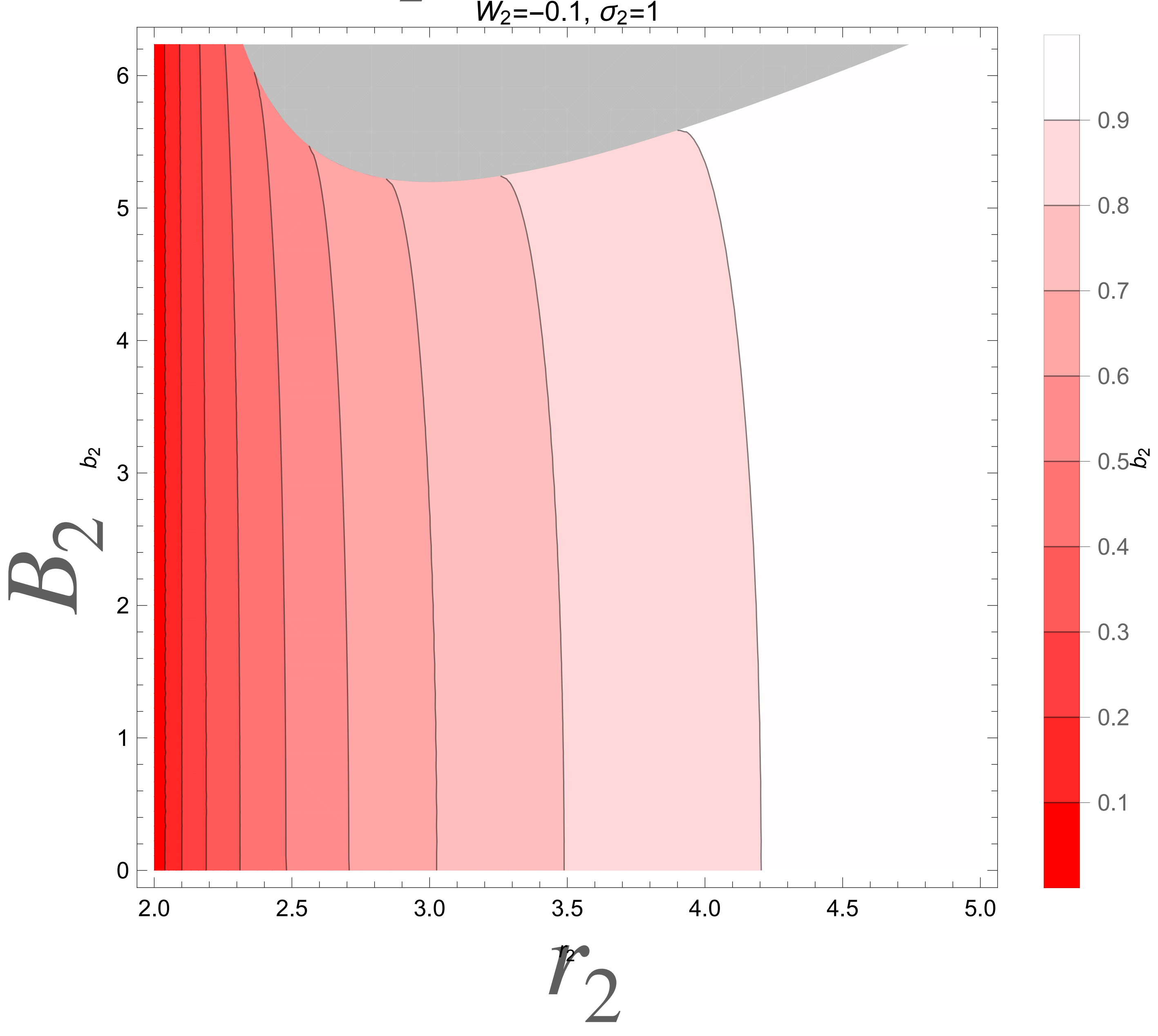
$$C = \sigma(R')\left(\sqrt{1 + (R')^2} - \sqrt{f + (R')^2}\right), \qquad \sigma = \mathrm{Sign}(\dot{r})$$



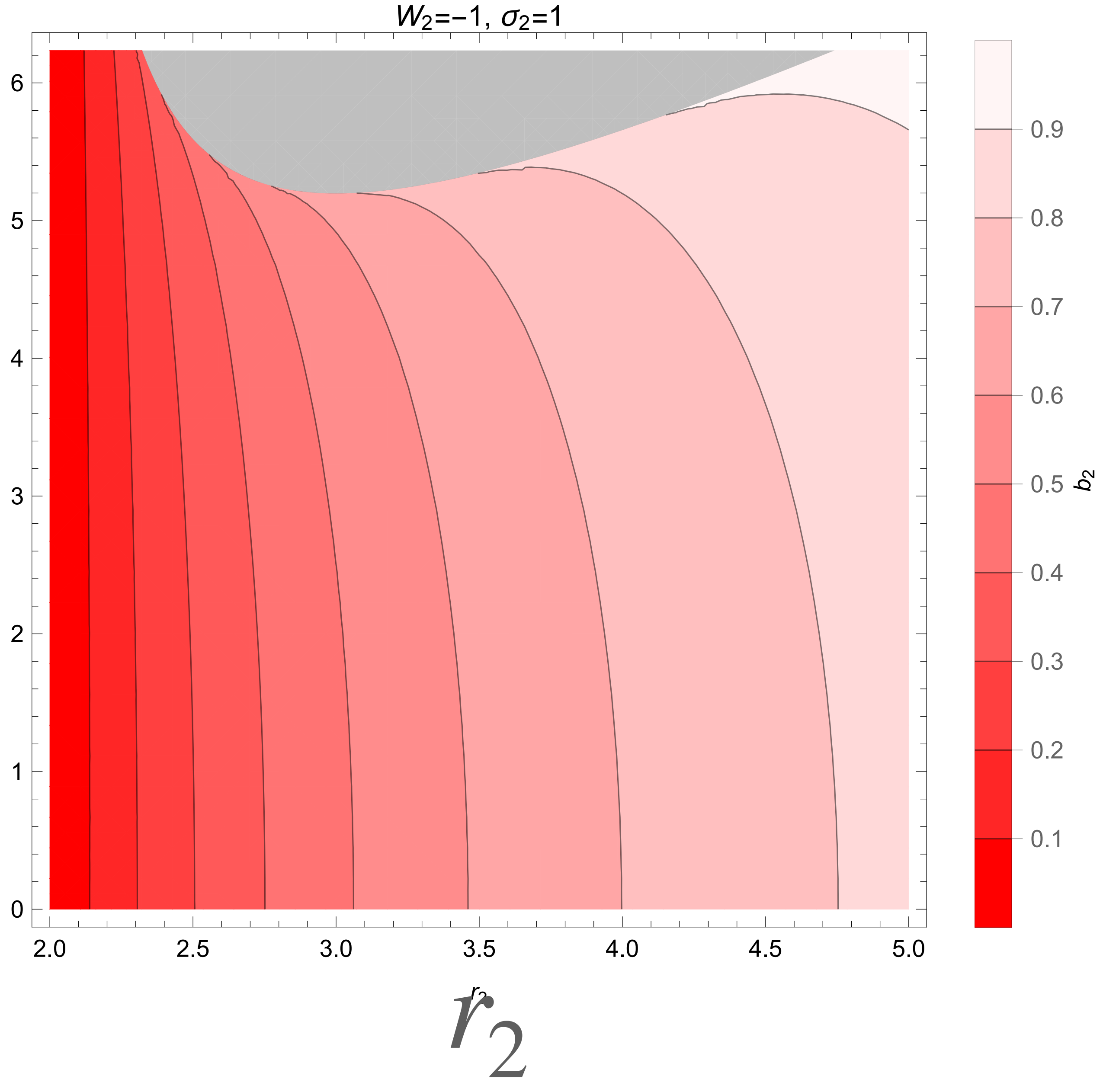
Redshift in monotonic collapse

Redshift Factor for $(r_1, r_2, R'_1, R'_2, \sigma_1, \sigma_2, B_2) = (5M, r_2, 0, R'_2, -1, +1, B_2)$

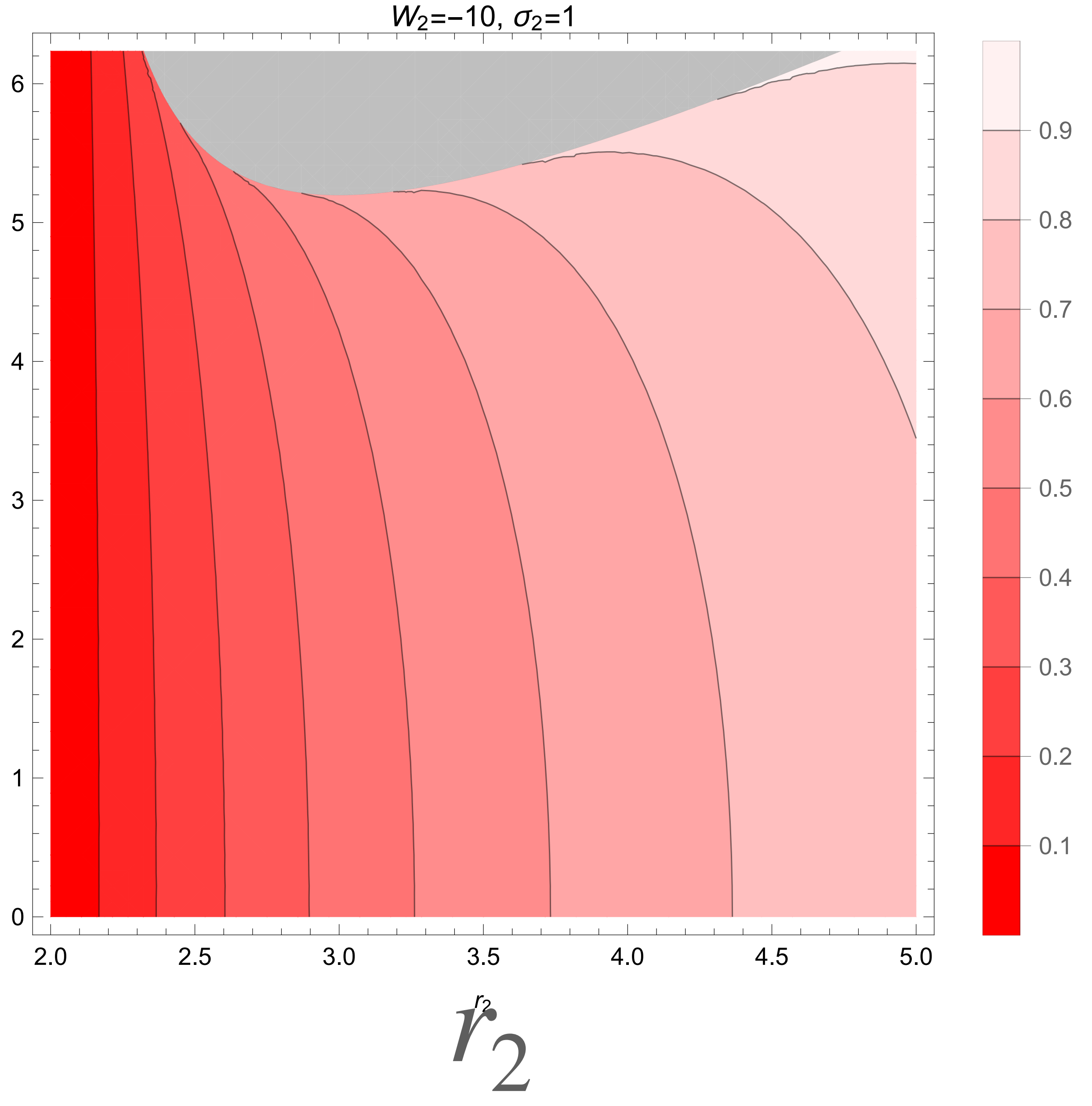
$R'_2 = -0.1$

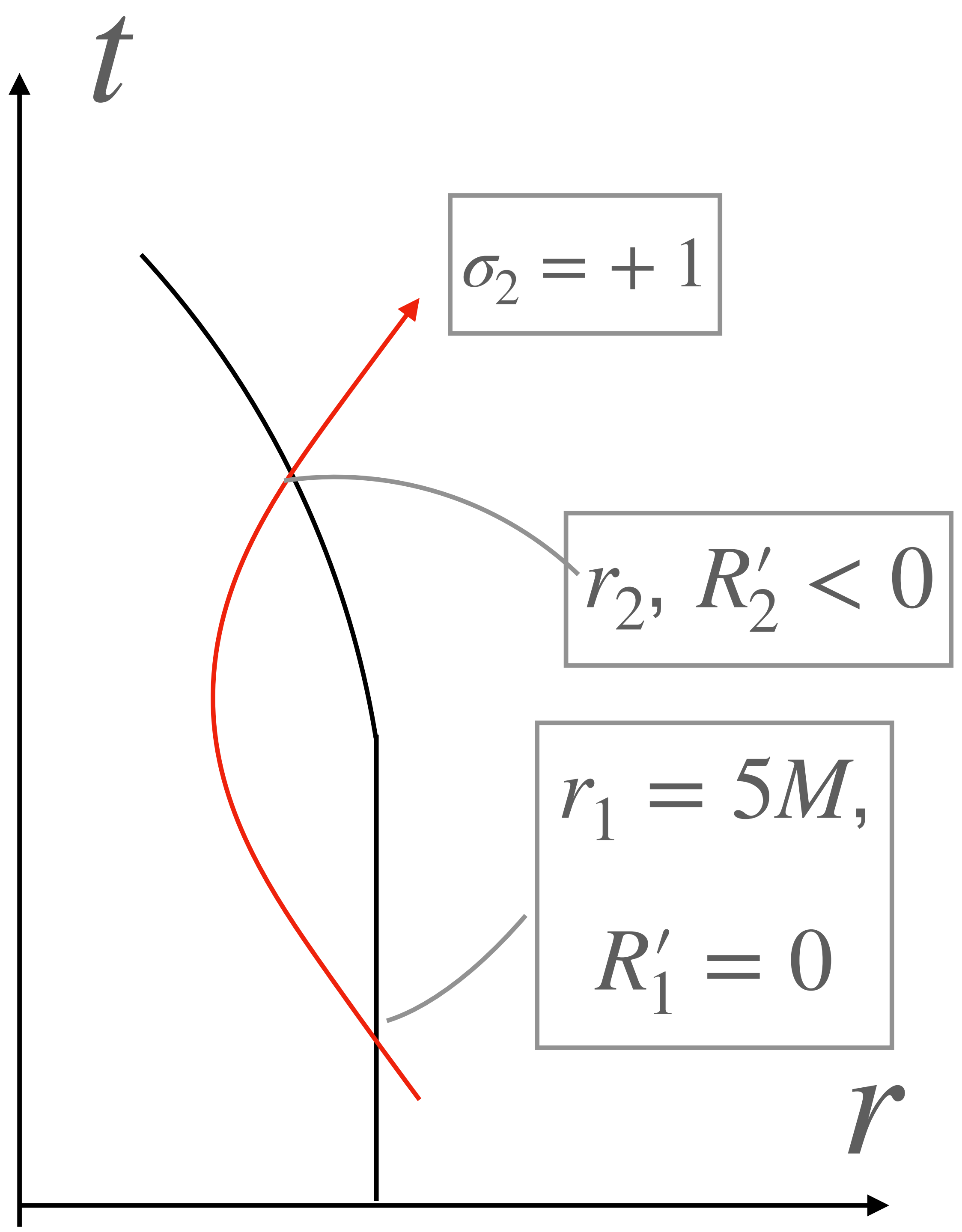


$R'_2 = -1$



$R'_2 = -10$





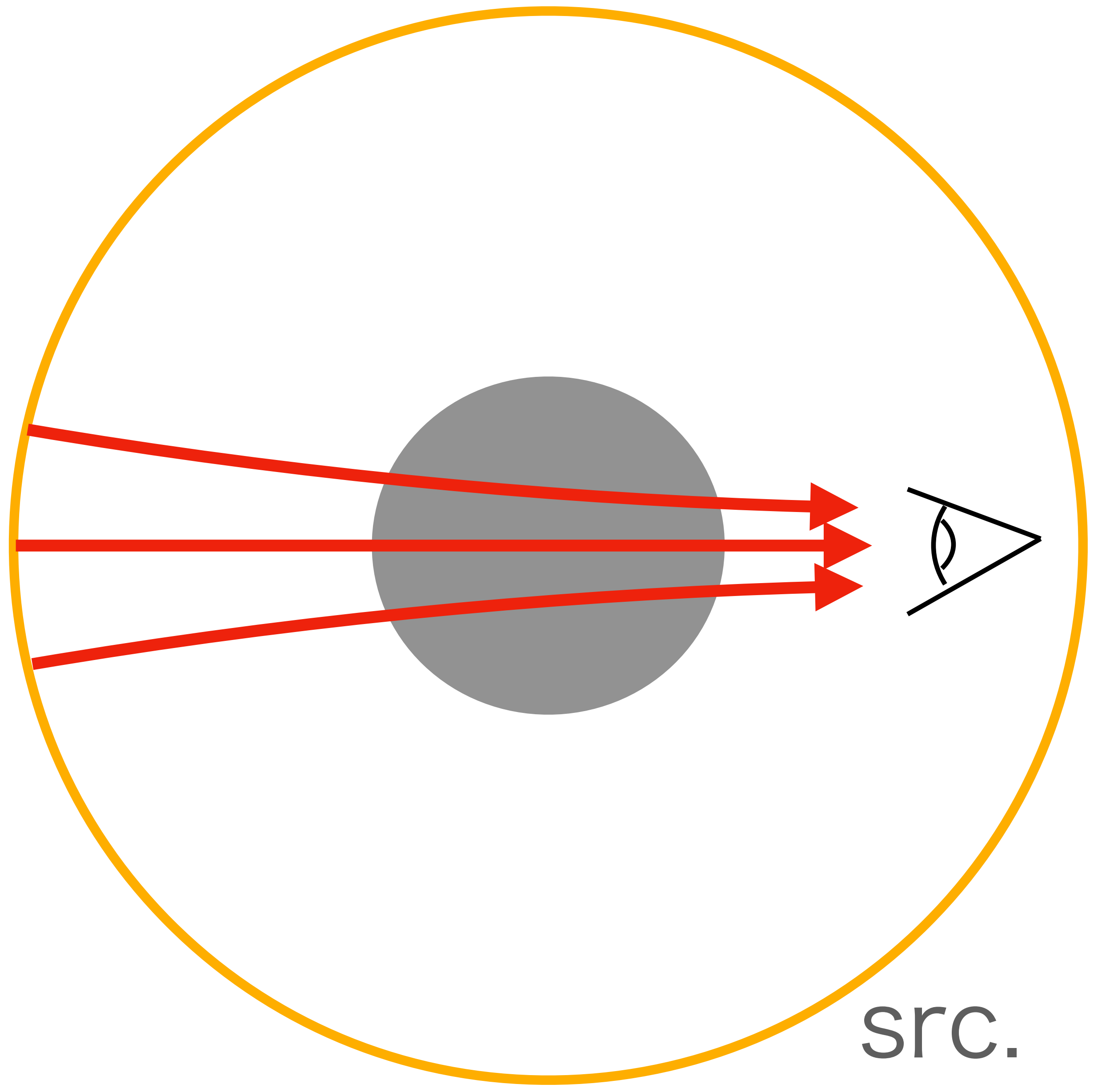
Shell

- Every light is redshifted $\alpha < 1$ if the shell is shrinking and $\sigma_2 > 0$. (no blueshift)
- High redshift (small α) for rapid collapse (large $|R'|$) & in the late stage ($r_2 \to 2M$).

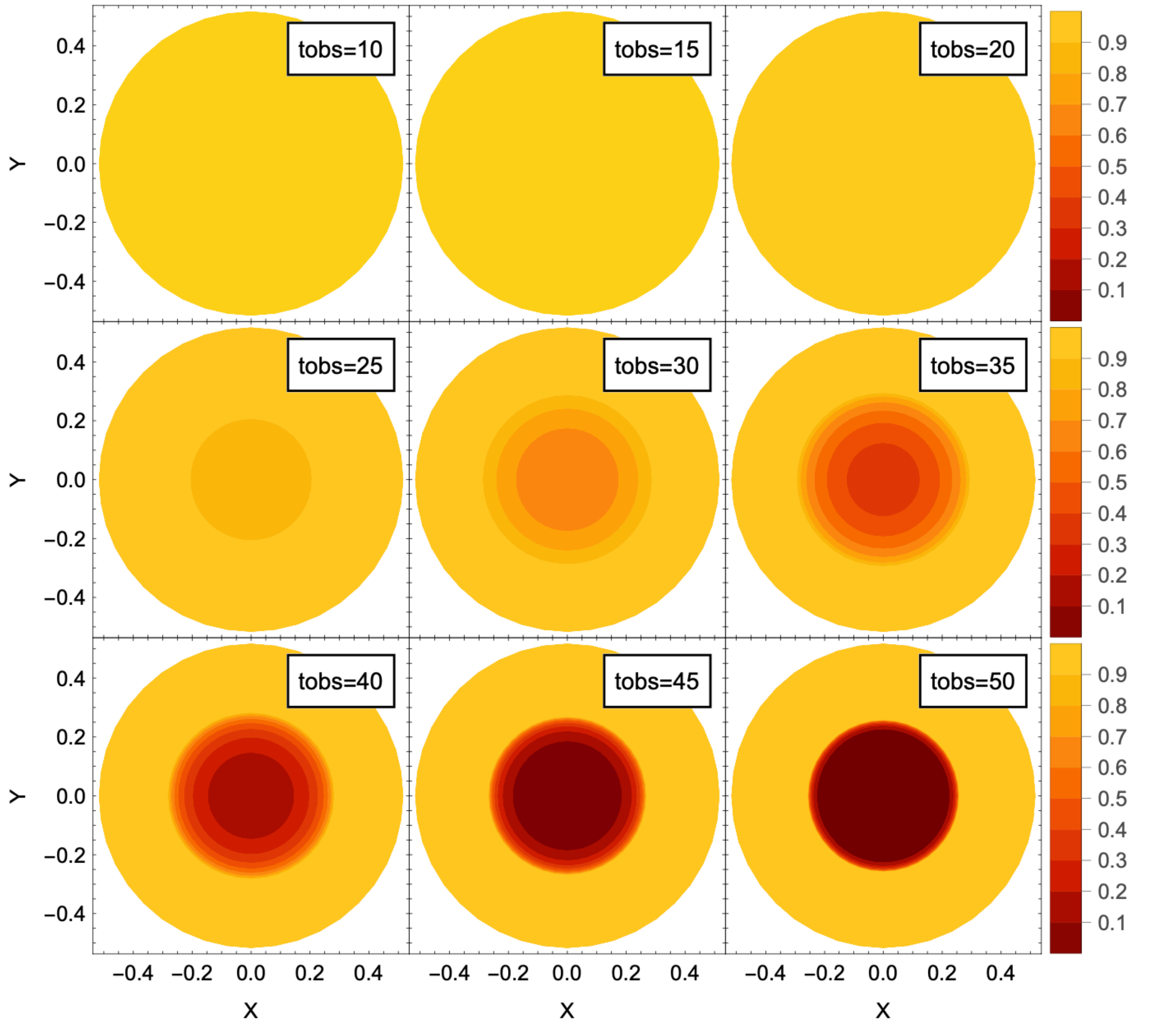
Demostration of Shadow formation

For simplicity, suppose

- A distant light source distributed on a large sphere.
- The shell dynamics:
  - static at $r = 5M$ for $t < 0$,
  - collapses with zero internal pressure for $t \geq 0$ (dust shell).



Dust case



tobs: observation time

X, Y: coords on the celestial sphere

Color: Energy flux

- $r_o = 20M$
- $r_s = 100M$
- $\mathcal{N} J_s \omega_0 = 1$

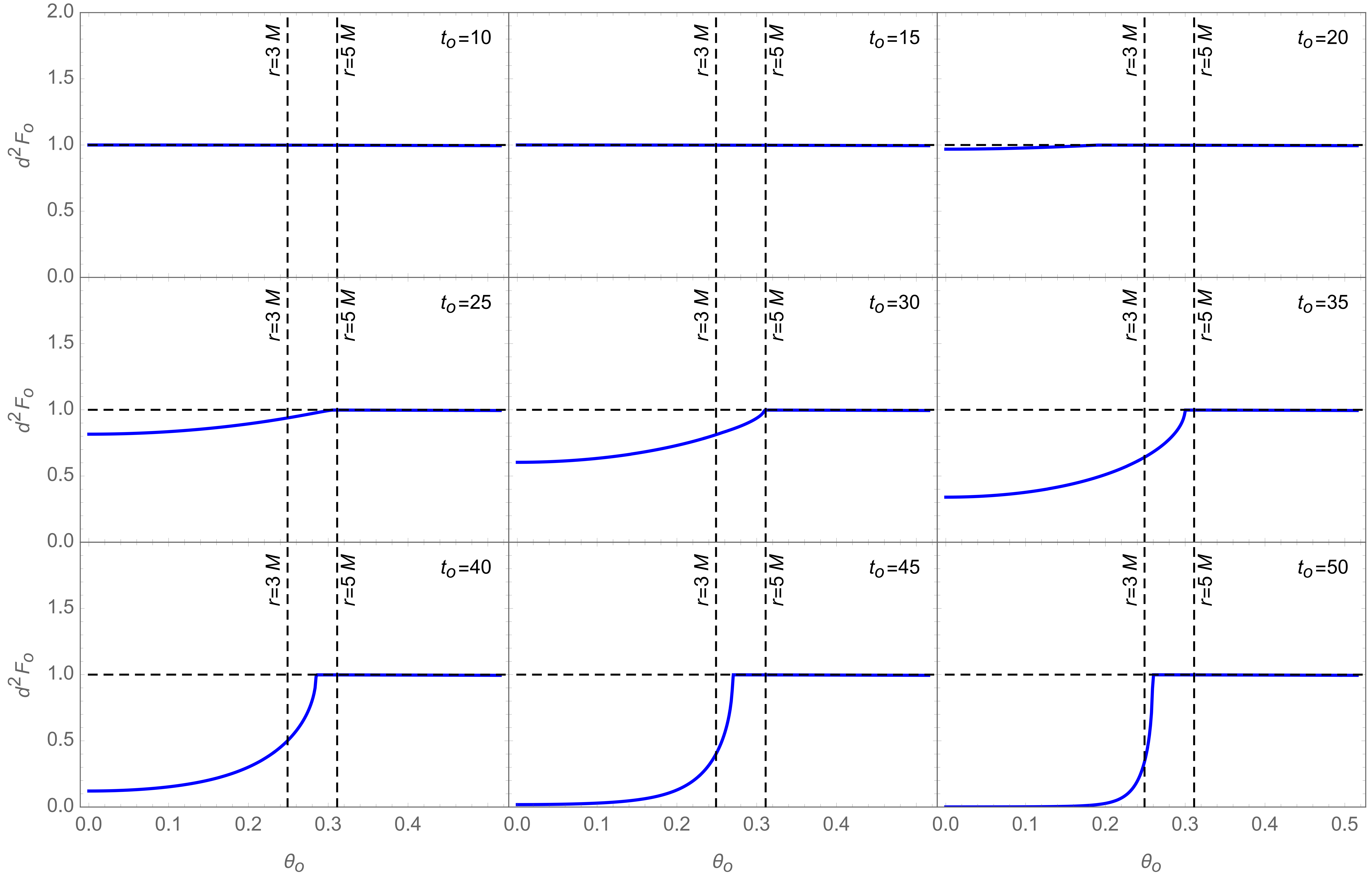
Dust case

t_o: observation time

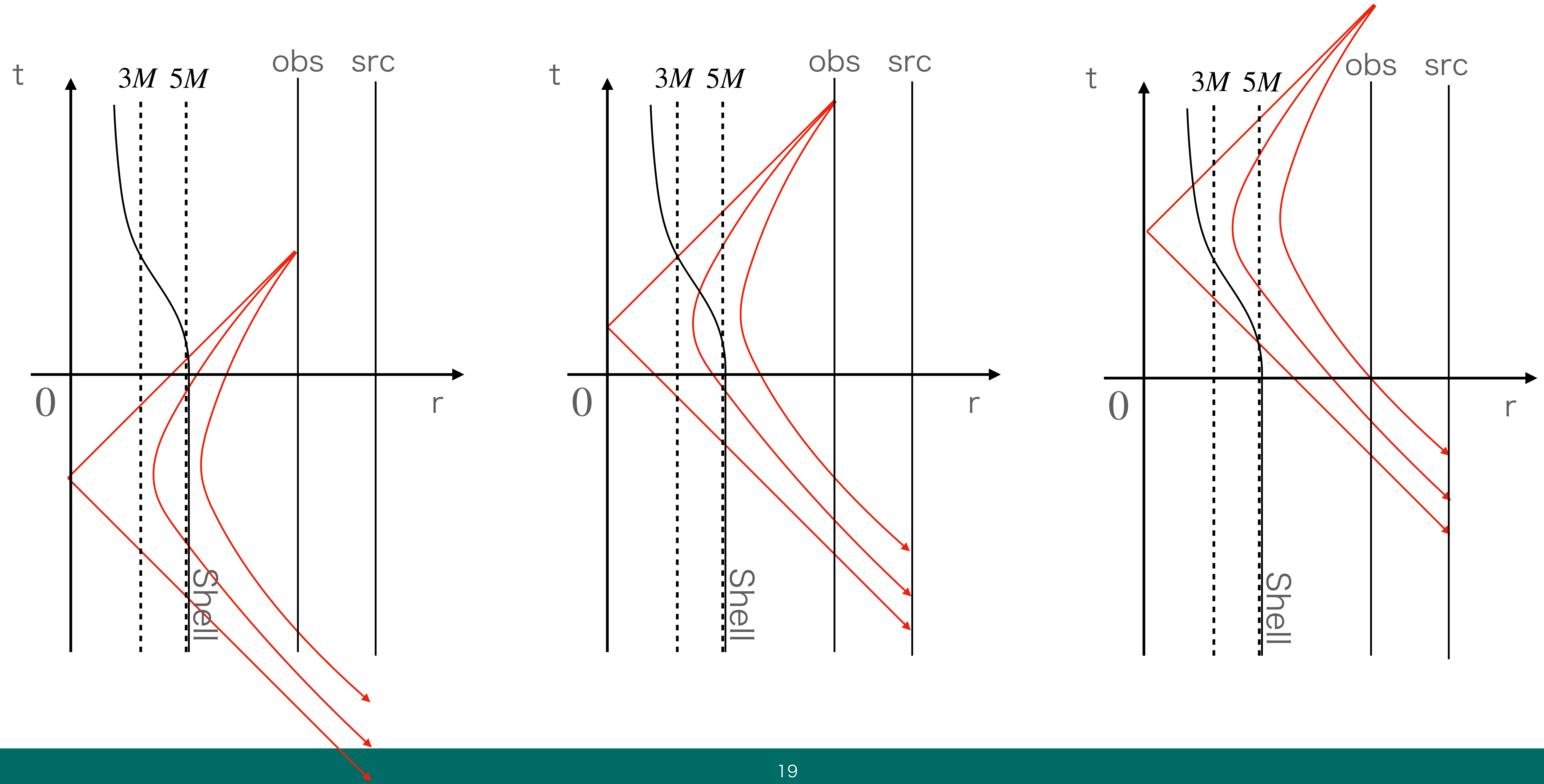
$\theta_o \sim \sqrt{X^2+Y^2}$: incident angle

dFo: Energy flux

- $r_o = 20M$
- $r_s = 100M$
- $\mathcal{N} J_s \omega_0 = 1$



Dust shell


t
3M 5M
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Shell
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src
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r
Shell

4. Generic spacetime and new formula

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Question:

- The study in the shell model implies that gravitational collapse leads to redshift of lights.
- How can we characterize the relation between redshift of lights and spacetime dynamics?

In the following,

- We define the energy of a null geodesic by taking the “Kodama vector” as the reference of time.
- We derive a covariant formula that relates redshift and spacetime dynamics.

Reference of time:

- If the spacetime is static, the static Killing vector ξ_t is the unique reference of time. And the energy of a null geodesic w.r.t ξ_t, $E = -g(\xi_t, k)$, is conserved.
- If the spacetime is dynamical, there is no unique reference of time.

4. Generic spacetime and new formula

Kodama vector

- Suppose general, asymptotically flat, spherically symmetric spacetime $(\mathscr{M}, g)$:

$$g = \underbrace{h_{AB}(x^C)dx^A dx^B}_{\text{submanifold } (\mathscr{N}, h)} + R^2(x^C)(d\theta^2 + \sin^2\theta d\phi^2), \qquad x^C = t, r\,.$$

- Def. of Kodama vector:

$$K := \mathrm{curl}R = -(\epsilon^{AB}\nabla_B R)\partial_A$$

ϵ^{AB}: the totally anti-sym tensor of $(\mathscr{N}, h)$

- Properties:
  - $K \cdot \nabla R = 0$,
  - If static, $K \propto \xi_t$. In vacuum and thus in the asymptotic region, $K = \xi_t$ (can be natural extension of the static KV).
  - $g(K, K) = 0$ on a trapping horizon.
  - Conserved current: $\nabla_a J^a = 0$, where $J^a := T^a{}_b K^b$.
  - Conservation of the quasi-local mass, "Kodama mass" $E(t, r)$, defined by $g(K, K) = 2E/r - 1$.

4. Generic spacetime and new formula

- The energy of light associated with K:

$$E := -g(k, K),$$ k: a null (/timelike) geodesic tangent

- Redshift factor:

$$\alpha := \frac{E|_{\mathscr{I}^+}}{E|_{\mathscr{I}^-}} = \frac{\int_{-\infty}^{+\infty} \nabla_k E d\lambda + E|_{\mathscr{I}^-}}{E|_{\mathscr{I}^-}}$$

- The derivative:

$$\nabla_k E = -\nabla_k g(k, K) = -g(\nabla_k k, K) - g(k, \nabla_k K) = -\nabla_{(a} K_{b)} k^a k^b$$

∵ geodesic eq.

Symmetric derivative of the Kodama vector is a geometrical quantity that characterizes redshift

4. Generic spacetime and new formula

Symmetric derivative of Kodama vector:

Proposition: In a 4-dim spherically symmetric spacetime, the Kodama vector satisfies

$$\nabla_{(a}K_{b)} = 4\pi R \widetilde{\mathcal{T}}_{ab},$$

where

$$\widetilde{\mathcal{T}}_{ab} := \epsilon_a{}^c \mathcal{T}_{cb}$$

is the dual of $\mathcal{T}$,

$$\mathcal{T}_{ab} = T^{\mathcal{N}}_{ab} - \frac{1}{2} h_{ab} h^{cd} T^{\mathcal{N}}_{cd}$$

is the trace-free part of $T^{\mathcal{N}}_{ab}$, and $T^{\mathcal{N}}_{ab}$ is the restriction of the energy momentum tensor T_{ab} onto $(\mathcal{N}, h)$.

Proof: Einstein equation. ∎

4. Generic spacetime and new formula

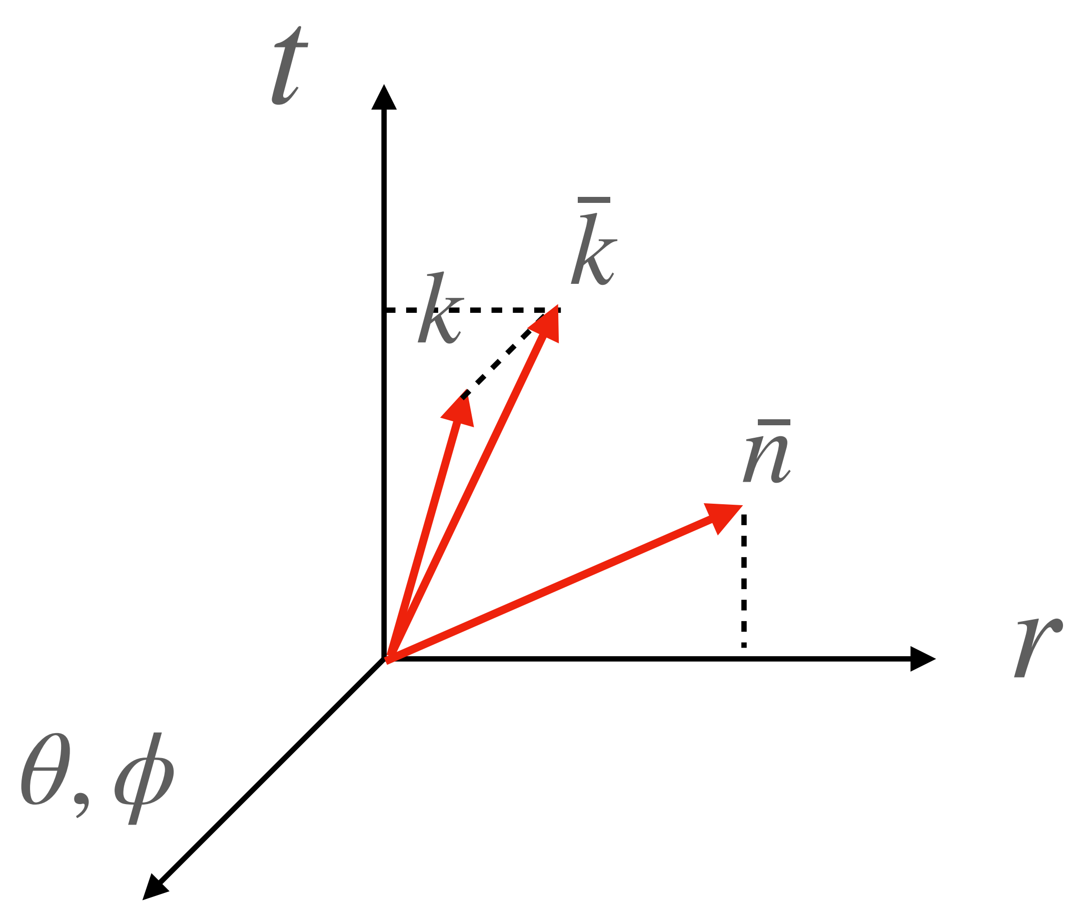
Derivative of the energy:

Theorem: The local redshift of energy of a light associated with Kodama vector is given by

$$\nabla_k E = -4\pi R \widetilde{\mathscr{T}}(k, k)\,.$$

Note this is valid for a timelike geodesic tangent u instead of null k.

4. Generic spacetime and new formula



Meaning of the theorem:

- Rewrite the energy momentum as

$$\widetilde{\mathcal{T}}(k,k) = \widetilde{\mathcal{T}}(\bar{k},\bar{k}) = \mathcal{T}(\bar{k},\tilde{\bar{k}}) = T_{\mathcal{N}}(\bar{k},\tilde{\bar{k}}) = -T(\bar{k},\bar{n})$$

$\bar{k}$: projection of k onto $(\mathcal{N},h)$, $\tilde{\ }$: contraction with $\epsilon^a{}_b$,

$\bar{n} := -\tilde{\bar{k}} = -\epsilon^a{}_b\bar{k}^b\partial_a$: radial outward vector orthogonal to $\bar{k}$.

➡ The energy current in the direction of $\bar{n}$.

- Newtonian analogy:

$$\nabla_u E = -4\pi R\widetilde{\mathcal{T}}(k,k) = -\frac{-4\pi R^2 T(\bar{k},\bar{n})}{R} =: -\frac{\delta M}{R}$$

➡ Loss of the potential energy due to the increase of the mass inside the sphere of R at the moment.

4. Generic spacetime and new formula

The thin shell case:

- The dual energy momentum tensor:

$$\widetilde{\mathcal{T}}_{ab} = -\delta(l)\rho U_{(a}N_{b)}$$

U: shell's 4-velocity, N: orthonomal vector to U,

$\rho = S(U, U)$: shell's rest mass energy,

l: a radial coordinate s.t. $l = 0$ on Σ and $g(dN, dN) = 1$.

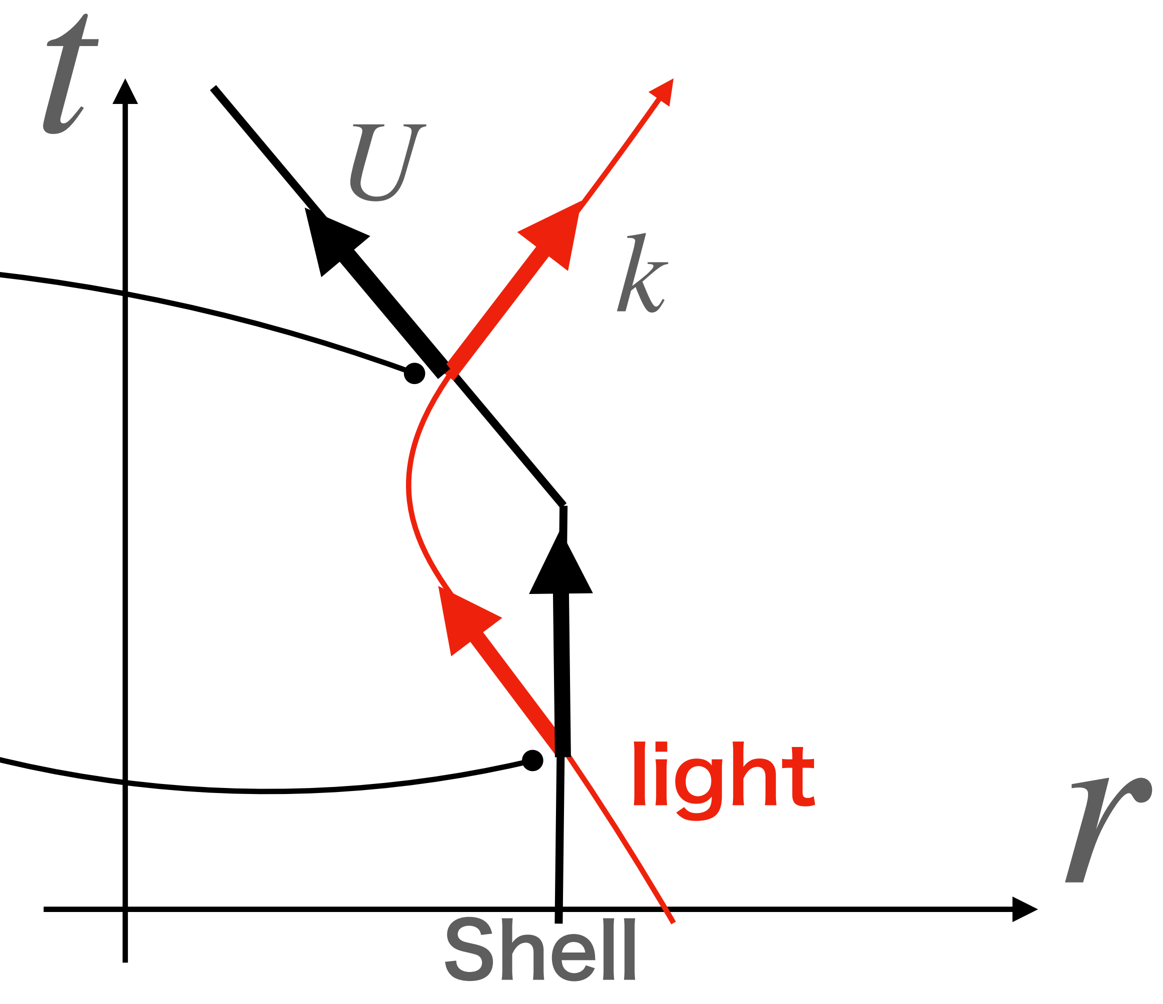
- The redshift when crossing the shell:

Redshift at exit

$$\Delta E|_{\text{exit}} = -4\pi R\rho|g(k, U)|$$

Blueshift at entry

$$\Delta E|_{\text{entry}} = +4\pi R\rho|g(k, U)|$$



Redshift is more efficient if the shell is collapsing

5. Summary

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Summary:

- Tendency of redshift of lights is confirmed in gravitational collapse of a thin shell model.
- Proposed the general covariant formula $\nabla_k E = -4\pi R \widetilde{\mathcal{T}}(k, k)$ of redshift by taking the Kodama vector as a reference.
- The formula gives a very clear interpretation of redshift due to the spacetime dynamics.

Discussion:

- Can we prove the generality of redshift in gravitational collapse?
- Beyond spherical symmetry?
- Application to other gravitational or astrophysical phenomena?