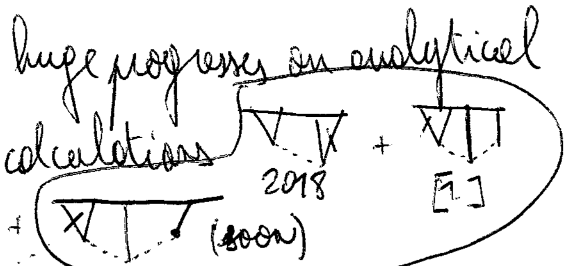
The effective field theory of multi-field inflationary fluctuations

[LP, 2405.02190]

I. Introduction

* Amazing workshop: – huge progress on analytical calculations



– we can compute many things

$$\int - (\tau_i)^{p_i} H_\nu(-k\tau_i)\,\theta(\tau_i-\tau_j)\ldots$$

- any p_i
- spins
- chemical potential
- classical back. osc.

also loops (?) → unclear to me in terms of observables.

* What should we compute?

- Encyclopedia Inflationaris: 283 SF models
  ↳ how many MF? Xi's question
- Model-ind. framework in SF: EFToI [2.]
- In MF, 2 important works [3.]: $\sigma S^\alpha \to \dot{S}+c$
  [4] EFToOSI: S

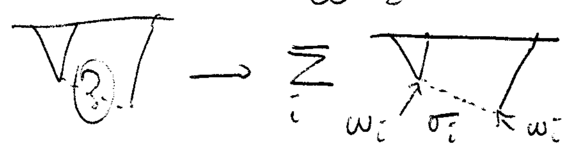
"We lack of unifying description"
↳ this work: ~~θ~~ S^α w.o. symm.

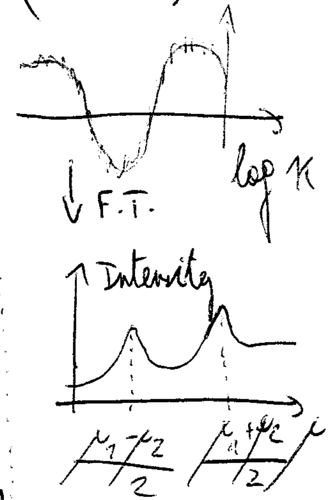
* not just for fun: many-field effects important.
(≥ 2)

↳ inflationary flavor oscillations [5.]

(CC) $M^2_{\alpha\beta} S^\alpha S^\beta \to m_i^2 \sigma_i^2$ ↗ missing angles like CKM, PMNS

$\omega \dot{\xi} \dot{S}^1 \to \Sigma (\omega \times R^1{}_i)\,\dot{\xi}\,\dot{\sigma}_i$





* This was actually done using NLSM interactions:

$$-\frac{1}{2} g^{\mu\nu} G_{IJ}(\vec{\phi})\,\partial_\mu \phi^I \partial_\nu \phi^J - V(\vec{\phi})$$

[6] ↳ $\mathcal{L}^{(2)}(\xi, S^\alpha) + \mathcal{L}^{(3)}(\xi, S^\alpha)$

$$\omega \left[\dot{\xi}^2 - \frac{(\partial\xi)^2}{a^2}\right] S^1$$

⇒ p_i's, diagrams, etc. motivated

but is that all?

$\boxed{\text{Multi-field inflation beyond the lamppost}}$

[1] Cheung et al. 07 [0709.0293]
[2] Senatore et al. 10 [1009.2093]
[3] Noumi et al. 12 [1211.1624]
[4] Aoki, LP et al. 24 [2404.09547]
[5.] LP et al. 21 [2112.05710]; [6.] LP [2011.05930]

II. Single-field EFT of inflationary fluctuations

* [2] $\mathcal{L} = \frac{M_p^2}{2} R - \Lambda(t) - c(t) g^{00} + F^{(2)}[\delta g^{00}, \delta K_{ij}, \delta R_{\mu\nu\rho\sigma} + g_{\mu\nu}, \partial_\mu, f(t)]$ (Einstein frame)

Theory only invariant under spatial diffeos

special time direction $\hat{t}(x)$: $n_\mu \sim \partial_\mu \hat{t}$, $t = \hat{t}(x)$, $\hat{t}$ hyp, axes t, x

unitary gauge : pick $\hat{t}$ as the time : $\partial_\mu \hat{t} \Rightarrow \delta_\mu^0$

1. Einstein eq. $\rightarrow G_{\mu\nu} M_p^2 + (1-c) g_{\mu\nu} - 2c\, \delta_\mu^0 \delta_\nu^0 = 0$, with $(1-c) \to -P$, $2c \to -(\rho + P)$

$\Rightarrow \frac{M_p^2}{2} R - M_p^2 (3H^2 + \dot{H}) + \dot{H} M_p^2 g^{00}$ universal part.

2. simplifications : $\delta R_{\mu\nu\rho\sigma} \rightarrow \left\{ \begin{array}{l} {}^{(3)}\delta R_{ijkl} \rightarrow {}^{(3)}\delta R_{ij} \\ \partial_k K_{ij}, \partial_t K_{ij}, \partial_i \partial_j \delta g^{00} \text{ etc.} \end{array} \right.$

$\underbrace{\qquad\qquad}_{\text{redundant}}$

* $t \rightarrow \tilde{t} = t + \pi(t, \vec{x})$ restores diffeo invariance if $t + \pi$ is covariant.

$g^{\mu\nu} \rightarrow \tilde{g}^{\mu\nu} = \frac{\partial \tilde{x}^\mu}{\partial x^\rho} \frac{\partial \tilde{x}^\nu}{\partial x^\sigma} g^{\rho\sigma}$

$\tilde{g}^{00} = g^{00}(1+\dot{\pi})^2 + 2g^{0i}(1+\dot{\pi})\partial_i \pi + g^{ij} \partial_i \pi \partial_j \pi.$

↳ include metric fluctuations

gauge	π	g^{00}	g^{0i}	g^{ij}
flat	π	$-1+\delta g^{00}_{flat}$	δg^{0i}_{flat}	$a^{-2}(\delta_{ij} + \gamma_{ij})$
com	0	$-1+\delta g^{00}_{com}$	δg^{0i}_{com}	$a^{-2} e^{-2\zeta} (\delta_{ij} + \gamma_{ij})$
long.	π_e	Φ	0	$a^{-2}[(1+2\Psi)\delta_{ij} + \gamma_{ij}]$

e.g. $ds^2 = -N^2 dt^2 + \bar{g}_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$, $\bar{g}_{ij} \to a^2 \delta_{ij}$, $N = 1 + \delta N$

$\left. \begin{array}{l} \delta g^{00}_{flat} = 2\delta N_{flat}, \text{ etc.} \\ \delta g^{0i}_{flat} = N^i_{flat} \end{array} \right\}$ constraints $\rightarrow \left\{ \begin{array}{l} \delta N_{flat} = \ldots \\ N^i_{flat} = \ldots \end{array} \right.$

$\delta g^{00} \rightarrow -2\dot{\pi} - \dot{\pi}^2 + \frac{(\partial \pi)^2}{a^2} + \underbrace{\text{metric fluct.}}_{\text{neglected in the decoupling limit}}$

* lowest order in derivatives
  - of the metric : $\sum_{n=2}^{+\infty} \frac{M_n^4(t)}{n!} (\delta g^{00})^n$
  - of the NG boson : odd $\sum_{n=1} \frac{1}{2\, n!} (\delta g^{00})^n \left[\bar{M}_n^3 \delta K + m_n^2\, {}^{(3)}\delta R \right]$ $+ \sum_{n=0} \frac{\tilde{m}_n^2}{n!} (\delta K_{ij} \delta K^{ij} - \delta K^2)$

$\rightarrow \mathcal{L}^{(2)} =$ linearized Horndeski, beyond when $m_1^2 \neq \tilde{m}_0^2$

$\frac{M_2^4}{2} (\delta g^{00})^2 \rightarrow \left(\frac{1}{c_s^2} - 1\right) \left[\dot{\pi}^2 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right]$ " fixed by non-linearly realised symm."

III. Multi-field EFT

$*$ extra ingredient : $\{S^\alpha(t,\vec{x})\}_{\alpha=1\dots N}$ a set of scalar d.o.f. : $S^\alpha \to S^\alpha$ under $t \to t+\pi$

"matter fields" in the EFT.

⚠ Price to pay for working in the Einstein frame

↳ matter fields coupled to gravity

$\rightsquigarrow F^{(2)}[\delta g^{00}, K_{ij}, \delta^{(3)}R_{ij}, S^\alpha ; t]$

$*$ order in derivatives :

$$\mathcal{L}_{mod,0} = \sum_{n,\ell} \frac{b^{(n,\ell)}_{\alpha_1\dots\alpha_\ell}}{n!\,\ell!} (\delta g^{00})^n S^{\alpha_1} \dots S^{\alpha_\ell}$$

$$\mathcal{L}_{mod,1} = \sum_{n,\ell} \frac{c^{(n,\ell)}_{\alpha\alpha_1\dots\alpha_\ell}}{n!\,\ell!} (\delta g^{00})^n g^{0\mu}\partial_\mu S^\alpha S^{\alpha_1}\dots S^{\alpha_\ell}$$

$$\mathcal{L}_{mod,2} = \sum_{n,\ell} \frac{(\delta g^{00})^n}{2n!\,\ell!}\Big[\bar{d}^{(n,\ell)}_{\alpha\beta\alpha_1\dots\alpha_\ell}\, g^{0\mu} g^{0\nu} - d^{(n,\ell)}_{\alpha\beta\alpha_1\dots\alpha_\ell}\, g^{\mu\nu}\Big]\partial_\mu S^\alpha \partial_\nu S^\beta S^{\alpha_1}\dots S^{\alpha_\ell}$$

- ~~[illegible]~~ : - $b^{(n,0)}$ ~~degenerate~~ degenerate with $M_n \to b^{(n,0)} = 0$

- no tadpole : $b^{(0,1)}_\alpha = 0$, $c^{(0,0)}_\alpha = 0$

Rk: also 3d curvatures, higher-order derivatives.

$*$ Breaking degeneracies for $\mathcal{L}^{(2)}$

$$-\frac{1}{2}\Big[\bar{d}^{(0,0)}_{\alpha\beta}\, g^{0\mu} g^{0\nu} - \underbrace{d^{(0,0)}_{\alpha\beta}}_{\geq 0 \text{ or } > 0}\Big]\partial_\mu S^\alpha \partial_\nu S^\beta \quad \begin{cases} - \text{ no grad. int.} \\ - \text{ neglect higher-order der.}\end{cases}$$

~~sym.~~ symm. and real

↳ $d^{(0,0)}_{\alpha\beta} \to \delta_{\alpha\beta}$, $\bar{d}^{(0,0)}_{\alpha\beta} \to \left(\frac{1}{c_\alpha^2} - 1\right)\delta_{\alpha\beta}$ (no sum.)

- missing $-c^{(0,1)}_{\alpha\beta}\dot{S}^\alpha S^\beta \xrightarrow[\text{I.B.P.}]{} -c^{(0,1)}_{[\alpha\beta]}\dot{S}^\alpha S^\beta + \frac{1}{a^3}\frac{d}{dt}\left(\frac{a^3}{2}c^{(0,1)}_{\alpha\beta}\right)S^\alpha S^\beta$

- mass matrix : $\underbrace{\left[\frac{b^{(0,2)}_{\alpha\beta}}{2} + \frac{1}{a^3}\frac{d}{dt}\left(\frac{a^3}{2}c^{(0,1)}_{\alpha\beta}\right)\right]}_{-m^2_{\alpha\beta}/2} S^\alpha S^\beta$

$b^{(1,1)}_\alpha \delta g^{00} S^\alpha$ $\qquad c^{(1,0)}_\alpha \delta g^{00} g^{0\mu}\partial_\mu S^\alpha$

- mixing with $\pi \to$ ~~[illegible]~~ $\dot{\pi} S^\alpha$, $\dot{\pi}\dot{S}^\alpha$

$$\Rightarrow \mathcal{L}^{(2)}_{mod} = \sum_\alpha \frac{1}{2c_\alpha^2}\left[(\dot{S}^\alpha)^2 - c_\alpha^2\frac{(\partial S^\alpha)^2}{a^2}\right] - \frac{m^2_{\alpha\beta}}{2}S^\alpha S^\beta + \Omega_{\alpha\beta}\dot{S}^\alpha S^\beta$$

$$-2f_\pi^2\omega_\alpha \dot{\pi} S^\alpha - 2\left(f_\pi^2/H\right)\bar{\omega}_\alpha \dot{\pi}\dot{S}^\alpha + \frac{f_\pi^4}{2c_s^3}\left[\dot{\pi}^2 - c_s^2\frac{(\partial\pi)^2}{a^2}\right]$$

$*$ decoupling limit : $\delta N, \delta N^i \Rightarrow \dot{\pi} \to \dot{\pi} + \varepsilon H \pi = -\dot{\zeta}/H$

$*$ NL-realized symm. $b^{(1,1)}_\alpha \delta g^{00} S^\alpha \to f_\pi^2 \omega_\alpha \frac{(\partial_i \pi)^2}{a^2} S^\alpha$

$c^{(0,1)}_{\alpha\beta}\, g^{0\mu}\partial_\mu S^\alpha S^\beta \longrightarrow -\Omega_{\alpha\beta}\frac{\partial_i\pi\,\partial_i S^\alpha}{a^2} S^\beta$

time-dep. : $\omega_\alpha \to \omega_\alpha(t+\pi) \Rightarrow \dot{\omega}_\alpha \dot{\pi}\pi S^\alpha$ ⚠ features!

Appendix A. decoupling limit

I neglected metric fluctuations so far.

$\delta g^{00} \to \underbrace{\delta g^{00}_{flat}}_{2\delta N_{flat}} - 2\dot{\pi} + \dots$, no δg^{0i}_{flat} at quadratic order

Universal part $\to$ $\delta N_{flat} = \varepsilon H \pi \quad \leftarrow \quad \frac{\delta \mathcal{L}}{\cancel{\delta N^i_{flat}}} = 0$

$\delta N^i_{flat} = (\dots) \quad \leftarrow \quad \frac{\delta \mathcal{L}}{\delta N} = 0$

$$\mathcal{L} \sim \underbrace{\frac{\partial_i N^i}{a^2} \times f[\delta N] + A\,\delta N^2}_{\text{universal part}} + B\delta N + 2 f_\pi^2 \omega_\alpha \cancel{\dots}\, \delta N\, S^\alpha + 2\frac{f_\pi^2}{H}\bar{\omega}_\alpha\, \delta N\, \dot{S}^\alpha$$

$\frac{\delta \mathcal{L}}{\delta N^i} = 0 \to f[\delta N] = 0$ i.e. $\delta N = \varepsilon H \pi$

$\frac{\delta \mathcal{L}}{\delta N} = 0 \to \cancel{\dots} \times \cancel{\dots} \; \frac{\delta f}{\delta N} \times \frac{\partial_i N^i}{a^2} + A\delta N + B(\pi)$

$+ 2 f_\pi^2 \omega_\alpha S^\alpha + 2\frac{f_\pi^2}{H}\omega_\alpha \dot{S}^\alpha = 0$

$\Rightarrow \mathcal{L} \sim 0 + A(\varepsilon H \pi)^2 + B(\pi)\,\varepsilon H \pi + 2 f_\pi^2 \omega_\alpha S^\alpha \varepsilon H \pi$

$+ 2 f_\pi^2/H\, \omega_\alpha \dot{S}^\alpha \varepsilon H \pi$

$$\hookrightarrow -2 f_\pi^2 \omega_\alpha \underbrace{(\dot{\pi} - \varepsilon H \pi)}_{-\dot{\xi}/H} S^\alpha - 2\frac{f_\pi^2}{H}\bar{\omega}_\alpha \underbrace{(\dot{\pi} - \varepsilon H \pi)}_{-\dot{\xi}/H} \dot{S}_\alpha$$

Appendix B: Adding 3d-curvatures

$* \; K_{ij} = \frac{1}{2}\sqrt{-g^{00}}\left(\partial_0 g_{ij} - \partial_i g_{0j} - \partial_j g_{0i}\right)$

$\hookrightarrow \tilde{K}_{ij} = K_{ij} - \partial_i\partial_j \pi \; ; \; \bar{K}_{ij} = H\, g_{ij} \quad \hookrightarrow \tilde{\bar{K}}_{ij} = \dot{H}\pi\, g_{ij}$

$\Rightarrow \delta \tilde{H}_{ij} = \delta K^{flat}_{ij} + a^2 \varepsilon H^2 \pi\, \delta_{ij} - \partial_i\partial_j \pi$.

exercise: $\tilde{R}^{(3)}_{ij} = \underbrace{R^{(3)\,flat}_{ij}}_{0} + H\left(\partial_i\partial_j \pi + \delta_{ij}\partial^2\pi\right)$

$*$ adiabatic: $\frac{\bar{M}_1^3}{2}\delta g^{00}\delta K \to \bar{M}_1^3 \left(\delta N_{flat} - \dot{\pi}\right)\left(\delta K_{flat} - \frac{\partial^2\pi}{a^2}\right)$

$\delta K_{flat} = -3H\delta N - \frac{\partial_i N^i}{a^2} \to$ affects the lapse: $\delta N = \frac{\varepsilon H \pi}{1+\alpha} + \frac{\alpha}{1+\alpha}\dot{\pi}$

with $\alpha = \frac{\bar{M}_1^3}{2HM_p^2}$

$\Rightarrow -\frac{\alpha}{1+\alpha}\dot{\pi} \times \left(-\frac{3H\alpha}{1+\alpha}\dot{\pi} - \frac{\partial^2\pi}{a^2}\right) \to \left(\frac{\alpha}{1+\alpha}\right)^2 \dot{\pi}^2$

$+ \frac{\alpha}{1+\alpha}\underbrace{\dot{\pi}\partial^2\pi/a^2}_{\text{I.B.P.}\; (\partial\pi)^2/a^2}$

$\to$ either $\alpha \ll 1$ and no effect at all

either $\alpha \sim 1$ and decoupling limit not justified

$\hookrightarrow c_s^2$ not directly dictates $\dot{\pi}(\partial\pi)^2/a^2$

$*$ non-adiabatic sector: $\bar{b}^{(0,1)}_\alpha \delta K S^\alpha \to \bar{b}^{(0,1)}_\alpha \left(\delta K_{flat} - \frac{\partial^2\pi}{a^2}\right)S^\alpha$, etc.

$\to \delta N = \varepsilon H \pi + \beta_\alpha S^\alpha + \gamma_\alpha \dot{S}^\alpha$

$$\Rightarrow \Delta\mathcal{L}^{(2)} = -\underbrace{f_\alpha \frac{f_\pi^2}{H}\frac{\partial^2\pi}{a^2} S^\alpha}_{\dim 4} - \underbrace{\bar{f}_\alpha \frac{f_\pi^2}{H^2}\frac{\partial^2\pi}{a^2}\dot{S}^\alpha}_{\dim 5}$$