

Probing the size of single-particle orbitals in neutron-rich calcium isotopes

Mădălina Enciu

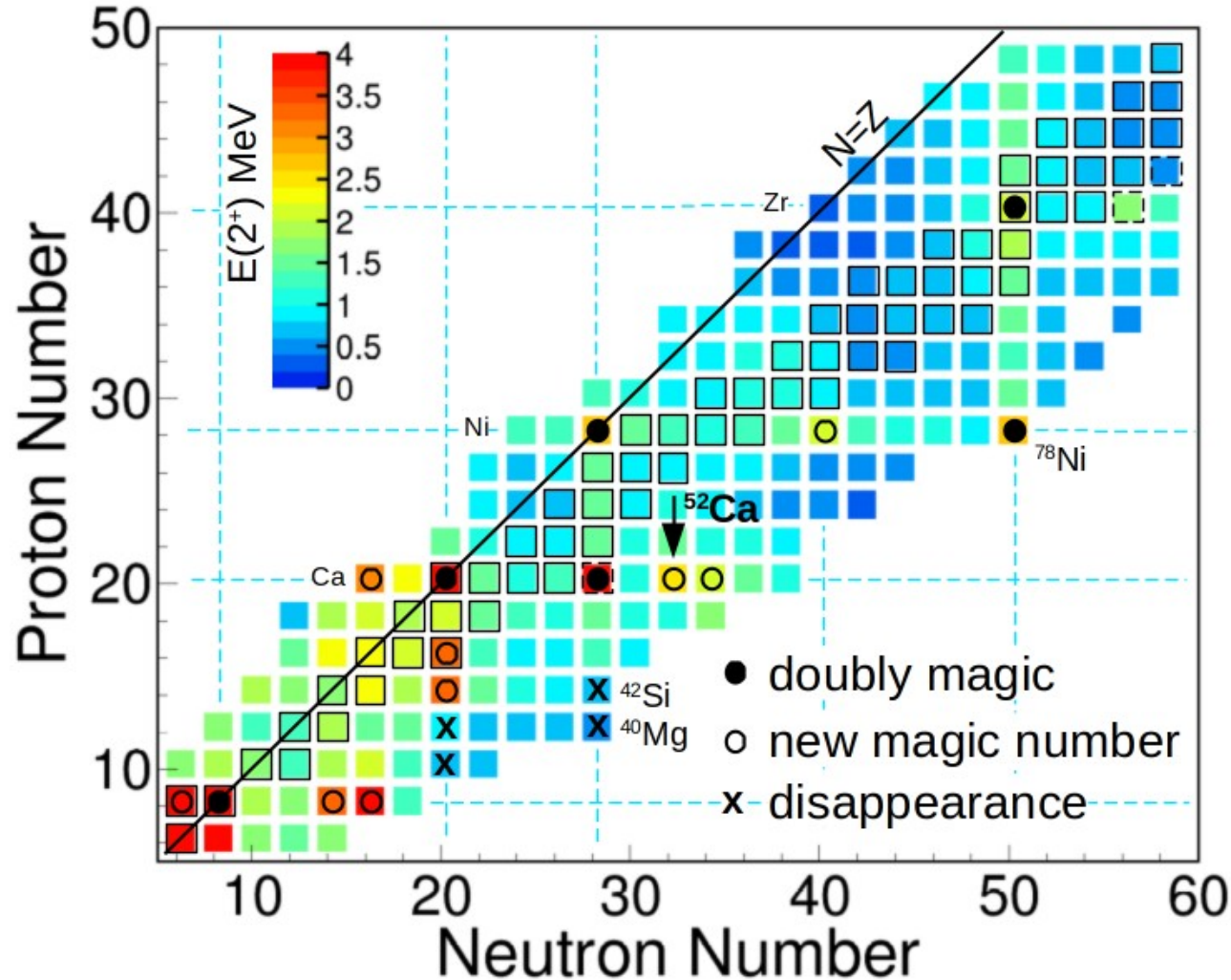
IKP, TU Darmstadt

SAMURAI International Workshop

Daejeon, Korea, July 14th - 16th 2024



The landscape of neutron-rich Ca isotopes

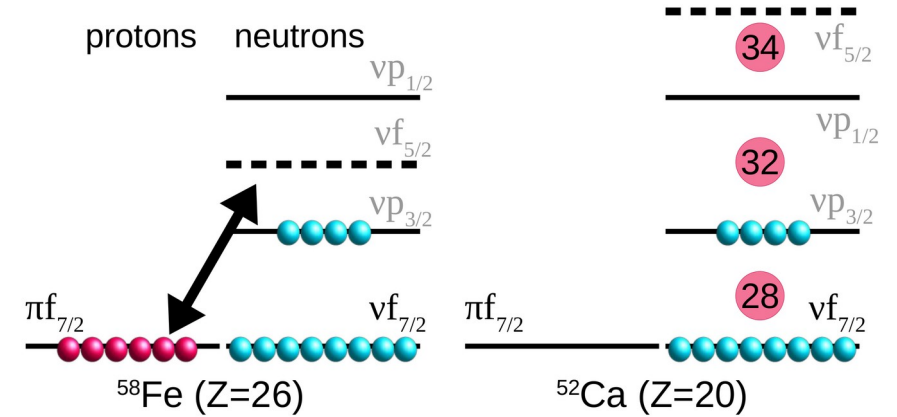


N=32 and N=34 shell closures emerging in neutron-rich Ca

- Mass
- $E(2^+)$ systematics
- Transition Probability
- **neutron knockout cross section**

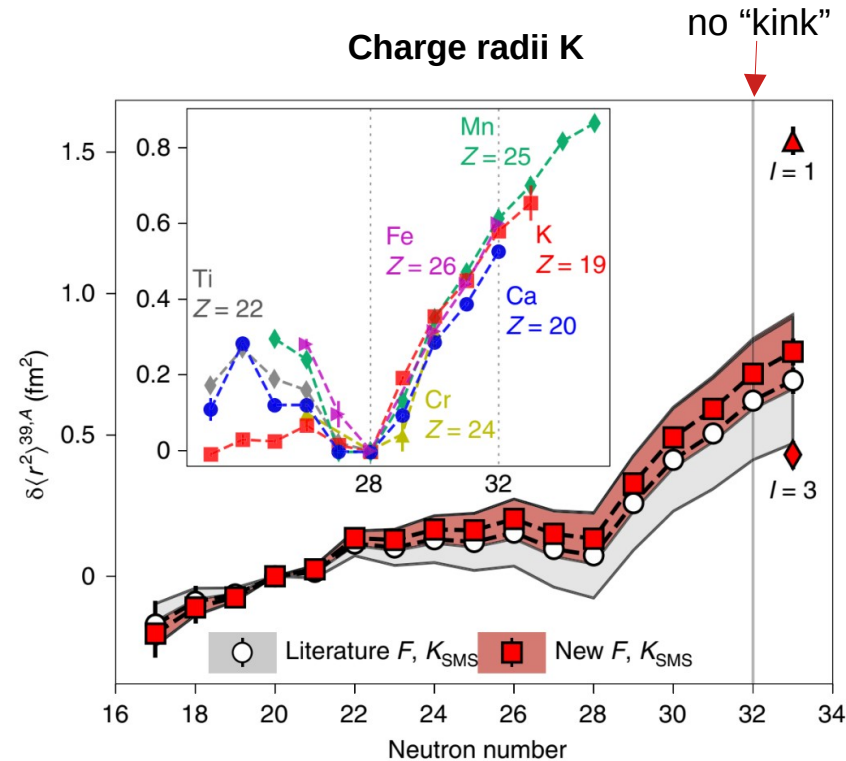
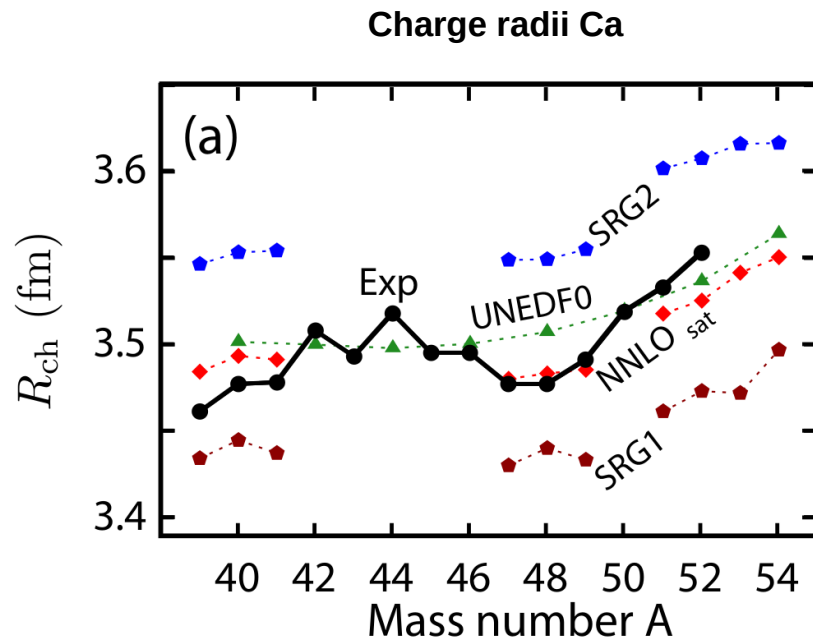
M. Enciu et al., PRL 129 (2022)

S. Chen et al., PRL 123 (2019)

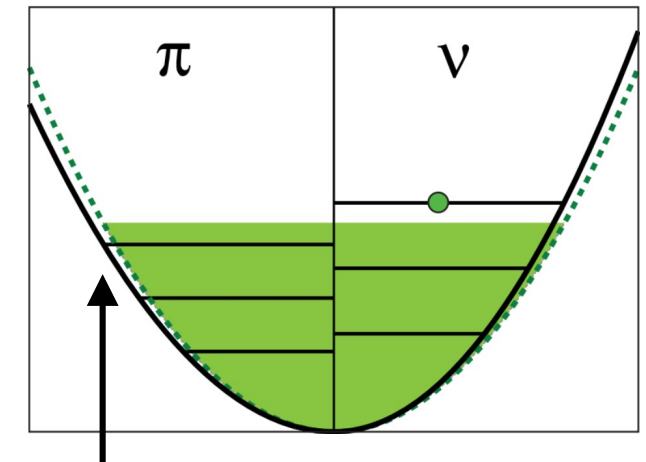


The charge radii of neutron-rich Ca isotopes

- Charge radii measurements via isotope shift method
- Steep linear increase from $N = 28$ to $N=32$



Bonnard et al. proposed a **0.7 fm larger $1p_{3/2}$ ($1p_{1/2}$) neutron orbital** than $0f_{7/2}$ ($0f_{5/2}$)



Charge radius increases as neutrons are added

References

- R. F. Garcia Ruiz et al., Nature Physics 12 (2016)
 Á. Koszorús et al., Nature Physics 17 (2021)
 J. Bonnard et al., Phys. Rev. Lett. 116 (2016)

The matter radii of neutron-rich Ca isotopes

- Matter radii measurements via interaction cross section measurements
- Steep linear increase from $N = 28$ to $N=32$
- Neutron radii contribute the most to the increase of matter radii, but protons also present an increase

→ **Swelling core** as one fills a nodal or j-lower orbit such as $1p$ or $0f_{5/2}$

W. Horiuchi et al. (2020)

- Strong interaction between $vp_{1/2}$ orbitals and neutron and proton $s_{1/2}$ orbitals

→ **driving the $s_{1/2}$ orbitals away from center**

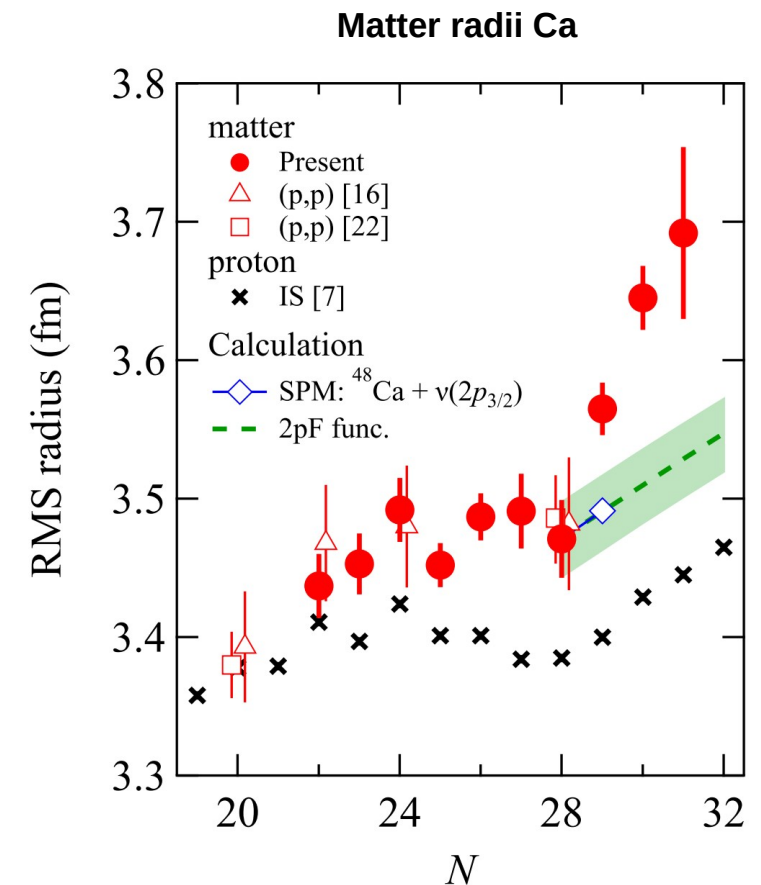
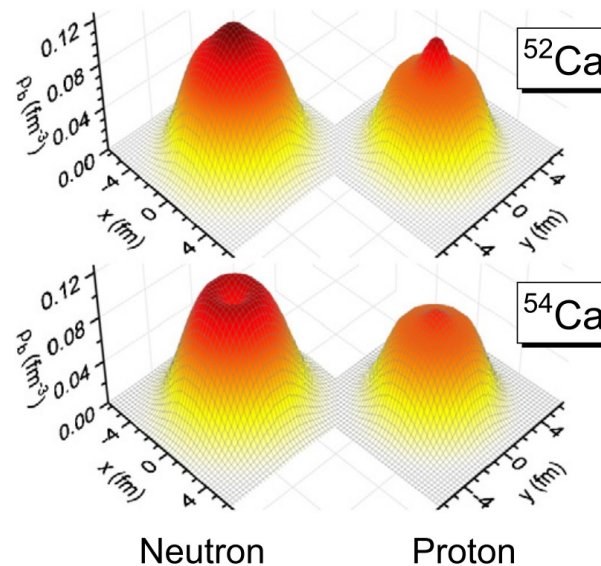
J. Liu et al. (2020)

References

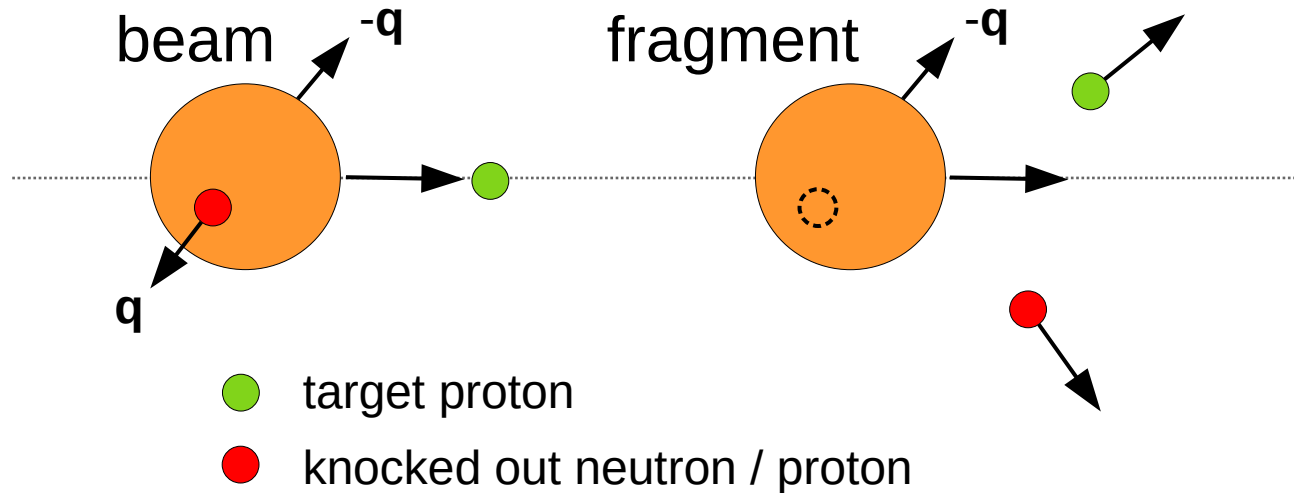
M. Tanaka et al., Phys. Rev. Lett. 124 (2020)

W. Horiuchi et al., Phys. Rev. C 101 (2020)

J. Liu et al., Phys. Lett. B 806 (2020)

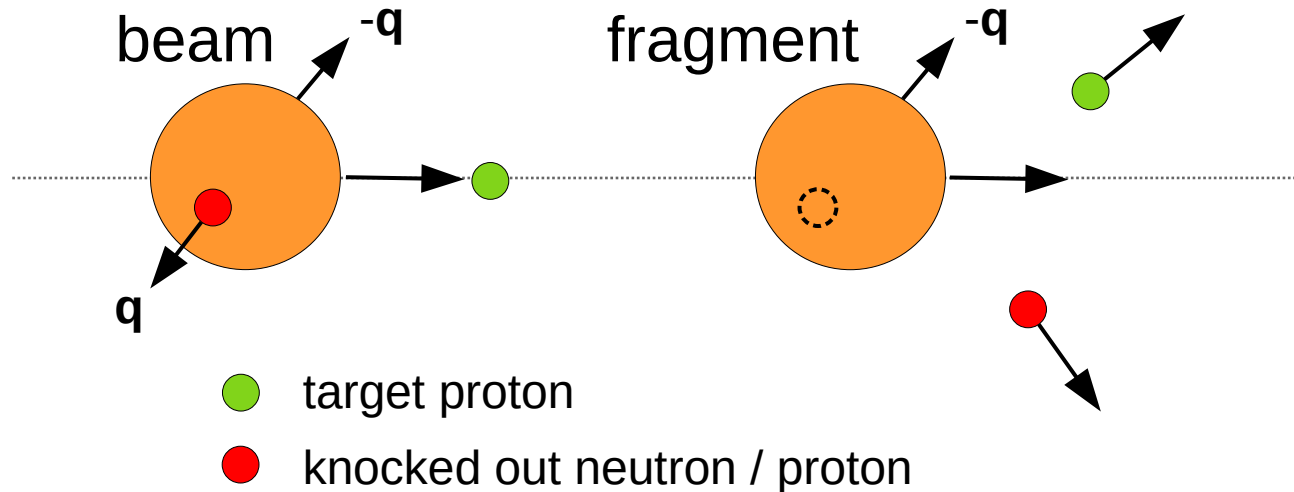


Probing the size of single-particle orbitals in neutron-rich calcium isotopes from quasi-free scattering missing momentum



The momentum distribution of the fragment particles are kinematically linked to the momentum distribution (and wave-function) of the knocked out nucleon

Probing the size of single-particle orbitals in neutron-rich calcium isotopes from quasi-free scattering missing momentum



The momentum distribution of the fragment particles are kinematically linked to the momentum distribution (and wave-function) of the knocked out nucleon

- Interpretation within DWIA framework
- Single-particle wave-function as bound state of Woods-Saxon potential
- **RMS radii of single-particle orbitals**

Acknowledgments:

All DWIA calculations shown in this presentation are performed by **K. Yoshida** and **K. Ogata**

Shell model calculations performed by **F. Nowacki** and **A. Poves**

Ab initio input for comparison by **T. Miyagi** and **M. Heinz**

SEASTAR collaboration for the experimental part

Experimental setup



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- experiment at **RIBF, RIKEN**, SEASTAR3 campaign (2017)
- **^{70}Zn primary beam** at 345 MeV/u
- In-flight **γ -ray spectroscopy** in inverse kinematics
- 15-cm **liquid H_2 target** (MINOS)
- 170 - 270 MeV/u beam energy at vertex
- MINOS TPC for vertex reconstruction
- **DALI2+** γ -ray detector array
- NeuLAND and NEBULA detectors used for subtracting the (p,p') + n-evap.

Studied reactions:

^{52}Ca (p,pn)

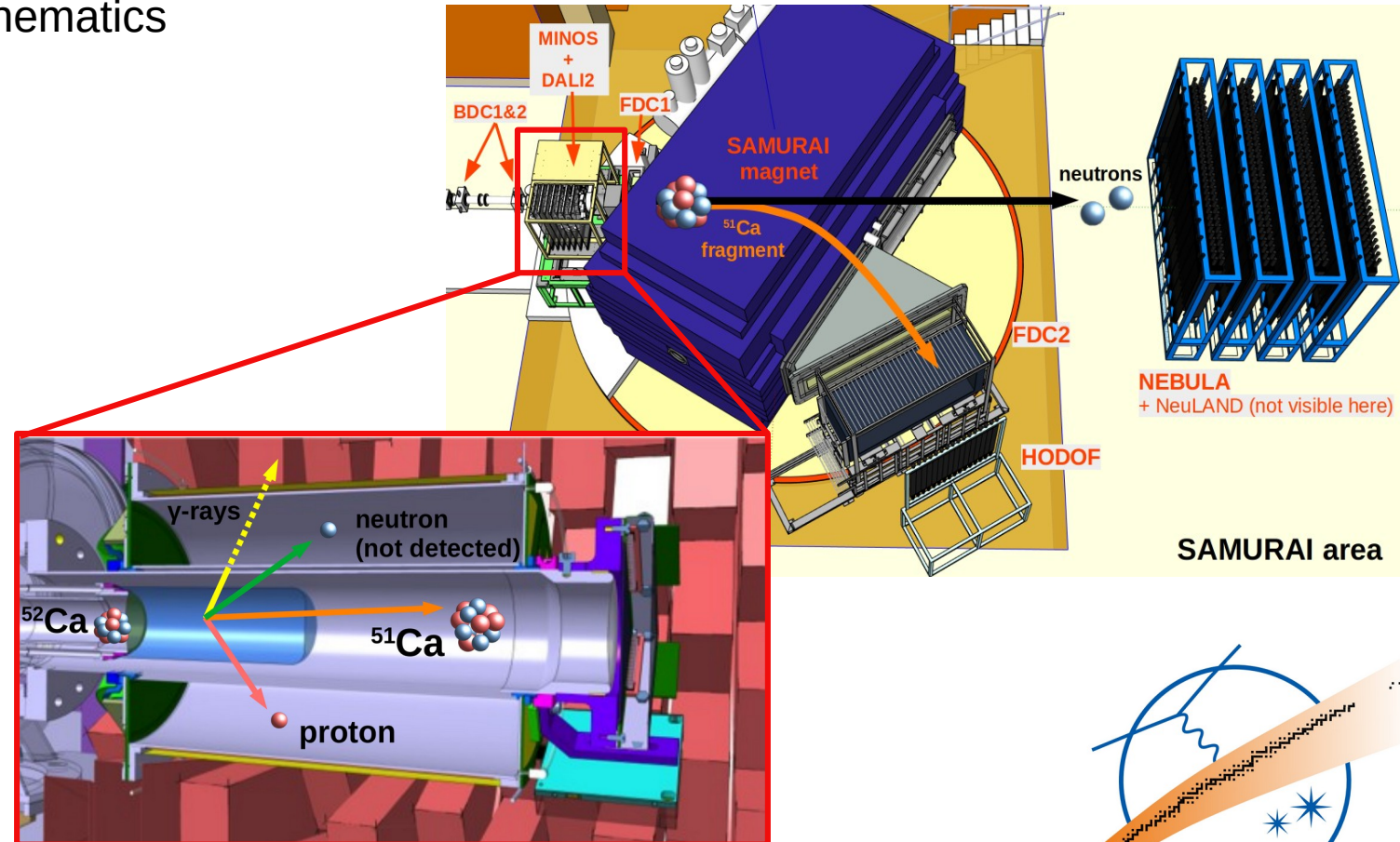
^{53}Ca (p,pn)

^{54}Ca (p,pn)

^{52}Ca (p,2p)

^{53}Ca (p,2p)

^{54}Ca (p,2p)



References

NeuLAND: K. Boretzky, et al., Nucl. Instr. Meth. Phys. Res. A 1014 (2021)

DALI2+: S. Takeuchi, et al., Nucl. Instr. Meth. Phys. Res. A 763 (2014)

MINOS: A. Obertelli et al., Eur. Phys. J. A 50 (2014)

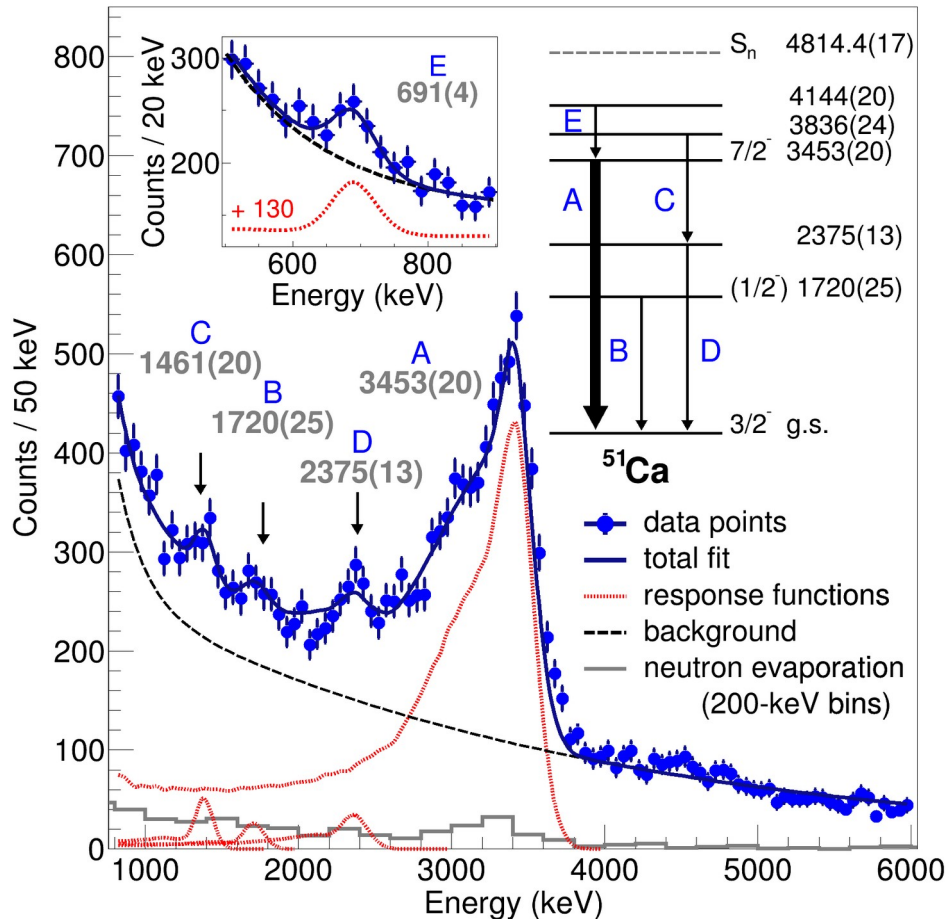


The N=32 shell closure of ^{52}Ca

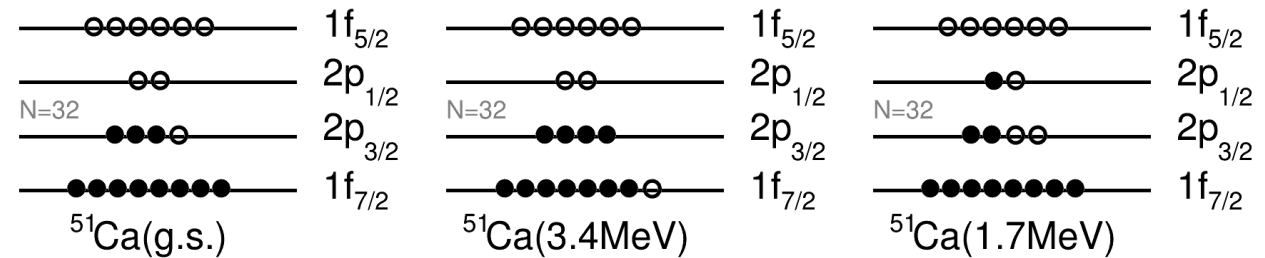


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Doppler-corrected γ -ray spectrum



Single-particle configurations of ^{51}Ca after neutron knockout



- γ -ray spectrum fitted with simulated response functions
- Neutron evaporation subtraction in cross section values
- Observed final states corresponding to the

knockout of $p_{3/2}$, $f_{7/2}$, and $p_{1/2}$ neutrons out of ^{52}Ca

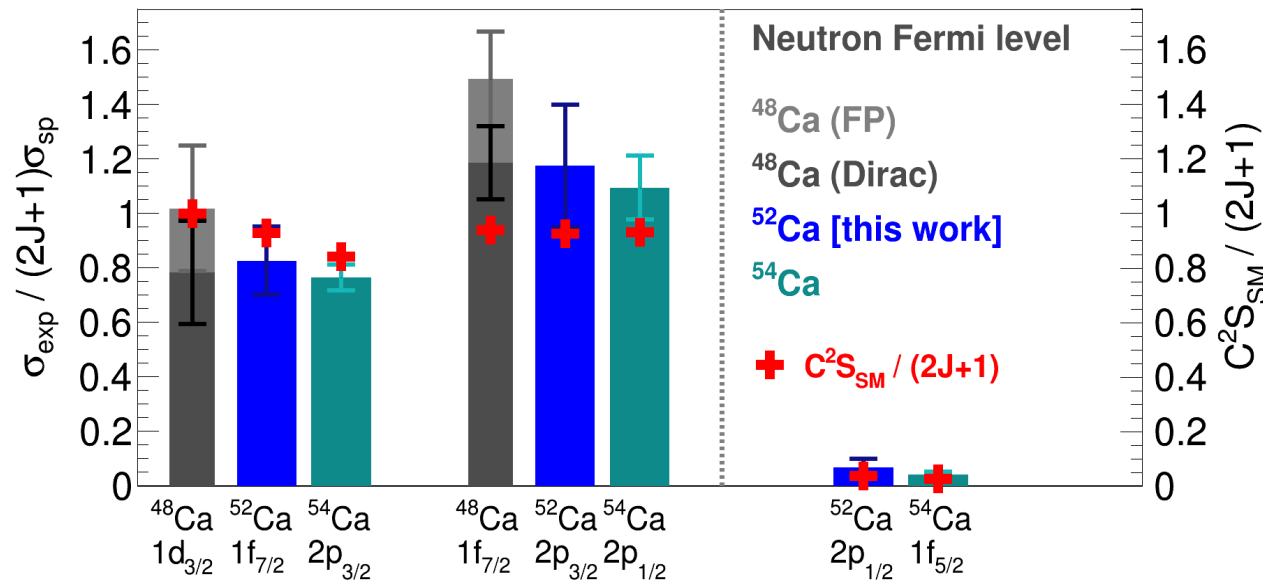
References:

M. Enciu et al., PRL 129 (2022)



The N=32 shell closure of ^{52}Ca

E_{ex}^{exp} (keV)	J^π	-1n	σ_{-1n}^{exp} (mb)	E_{ex}^{SM} (keV)	C^2S	σ_{sp}^{DWIA} (mb)	σ_{-1n}^{th} (mb)
0	$3/2^-$	$p_{3/2}$	30.3(42)	0	3.7	6.5(9)	23.9(32)
1720(25)	$(1/2^-)$	$p_{1/2}$	0.6(3)	1.620	0.1	4.8	0.5
3453(20)	$7/2^-$	$f_{7/2}$	22.3(24)	3.927	7.4	3.4(4)	25.0(27)



- Agreement between σ^{exp} and σ^{th}
Corresponding to a shell closure at N=32
- Systematic comparison between ^{52}Ca and ^{48}Ca and ^{54}Ca
Same behavior observed for all three Ca isotopes

References:

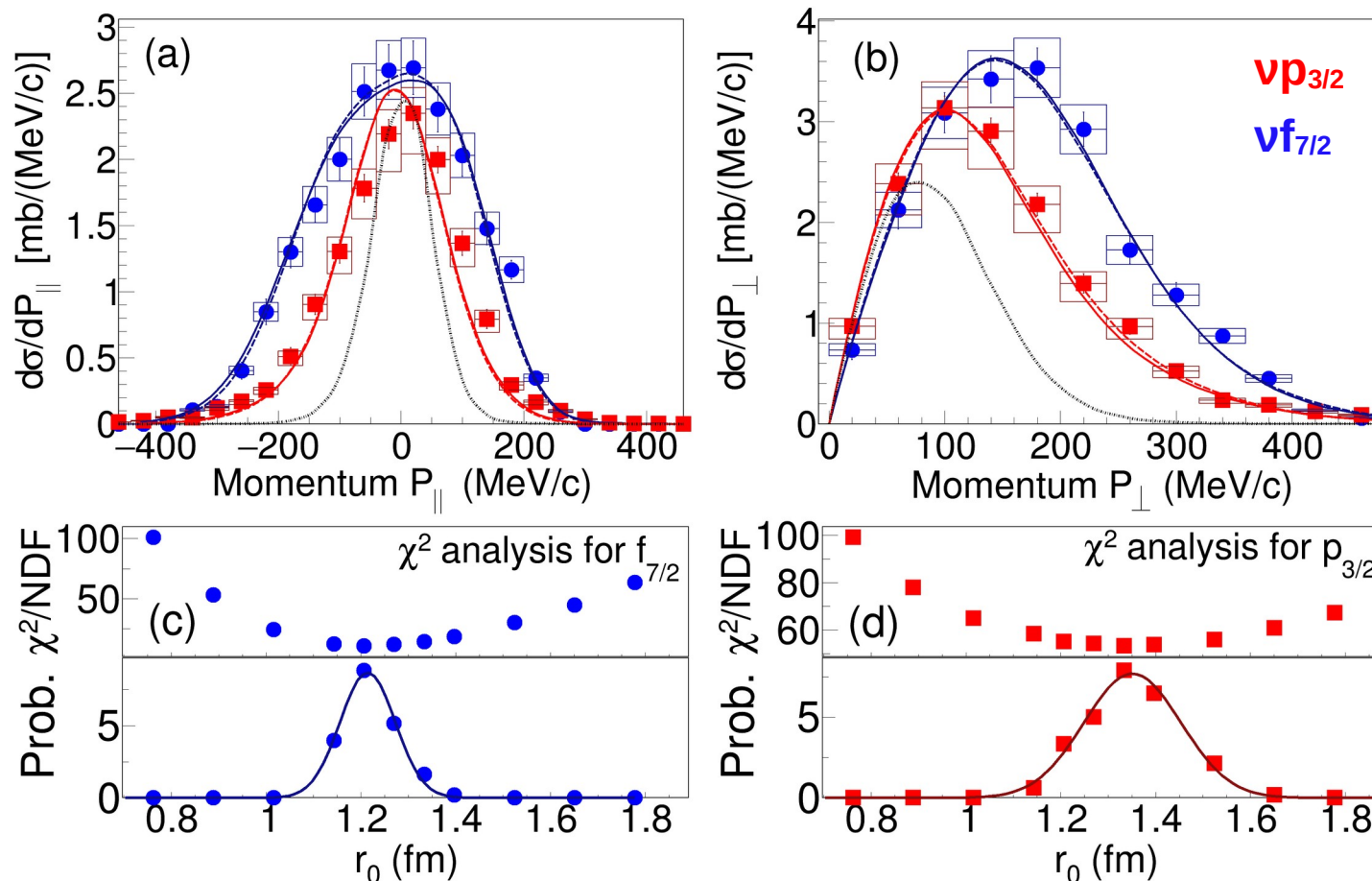
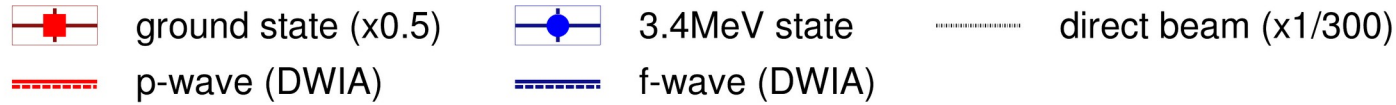
M. Enciu et al., PRL 129 (2022)

^{54}Ca data from S.Chen et al., PRL 123 (2019)

^{48}Ca data from M. Ahmad et al., Nucl. Phys. A 424 (1984)



The pf-shell neutron orbitals of ^{52}Ca



References:
 M. Enciu et al., PRL 129 (2022)

rms radii of single-particle (sp) neutron orbitals obtained by **variation of the radial parameter r_0 and χ^2 minimization**

Resulting rms radii:

$vf_{7/2}$: 4.13(14) fm

$vp_{3/2}$: 4.74(18) fm

Difference: 0.61(23) fm

from HFB, SKM:

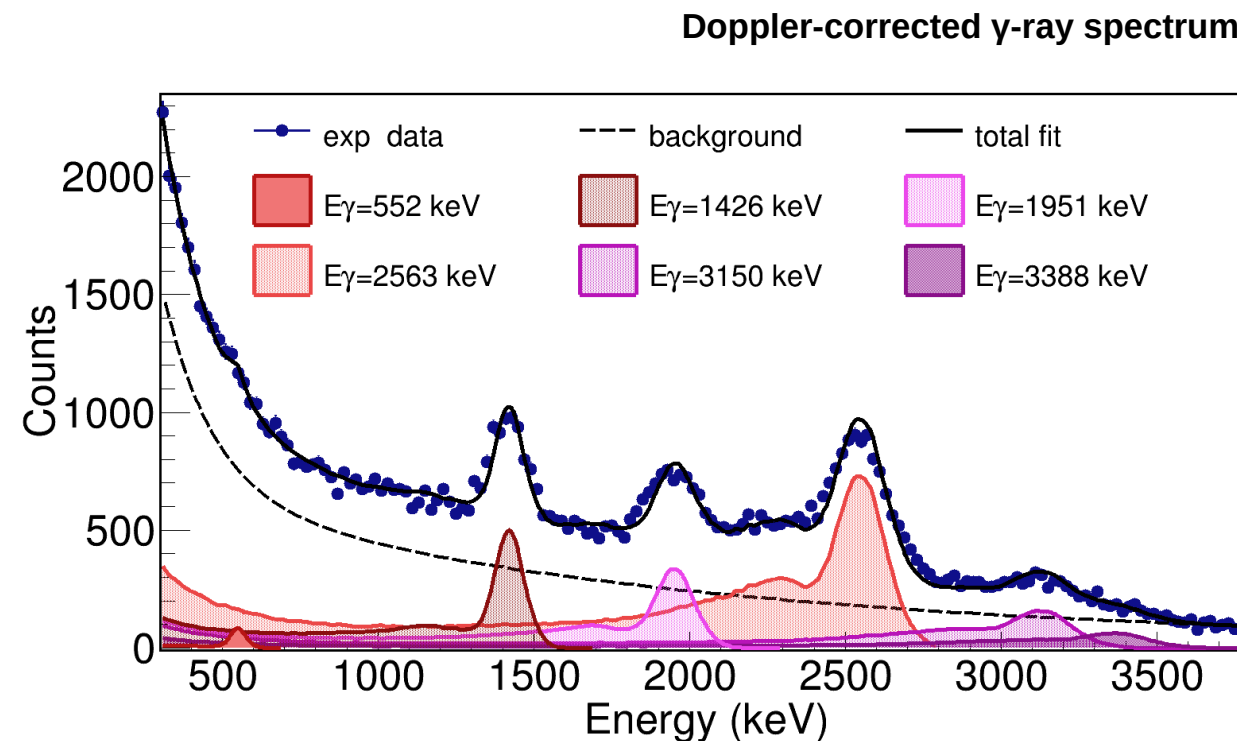
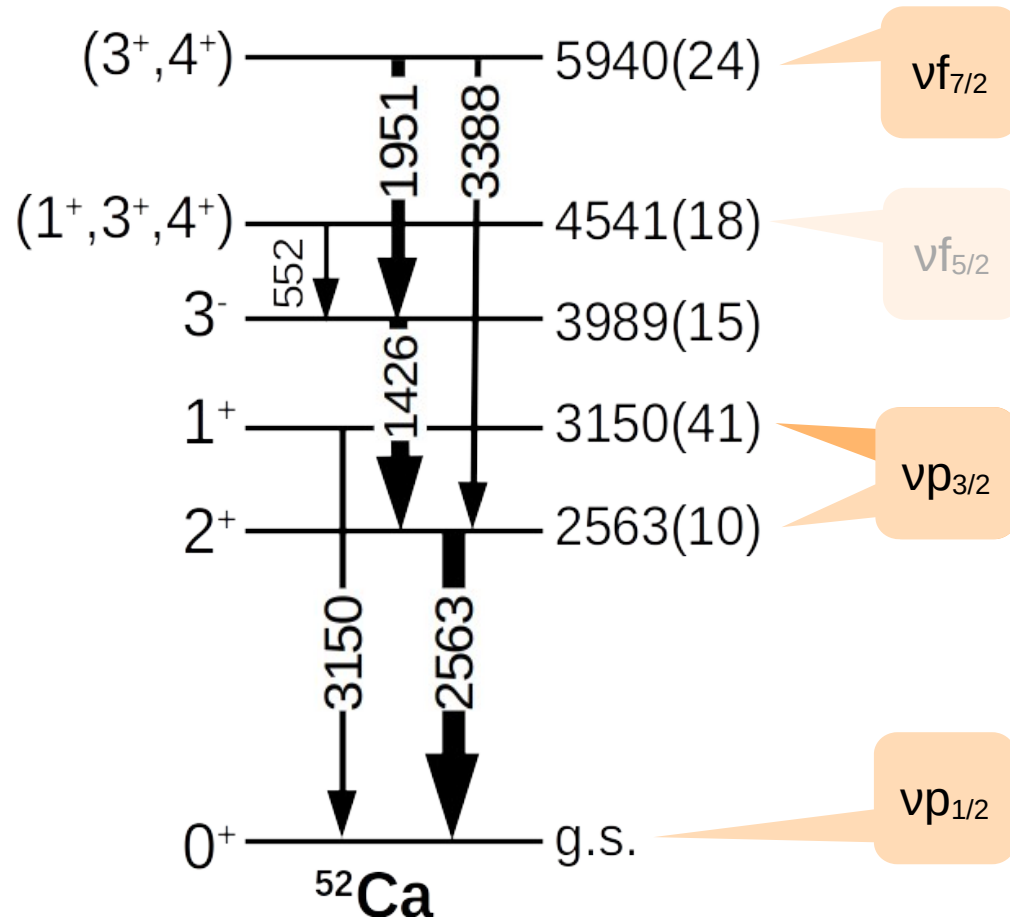
$vf_{7/2}$: 4.12 fm

$vp_{3/2}$: 4.49 fm

Difference: 0.37 fm



The pf-shell neutron orbitals of ^{53}Ca



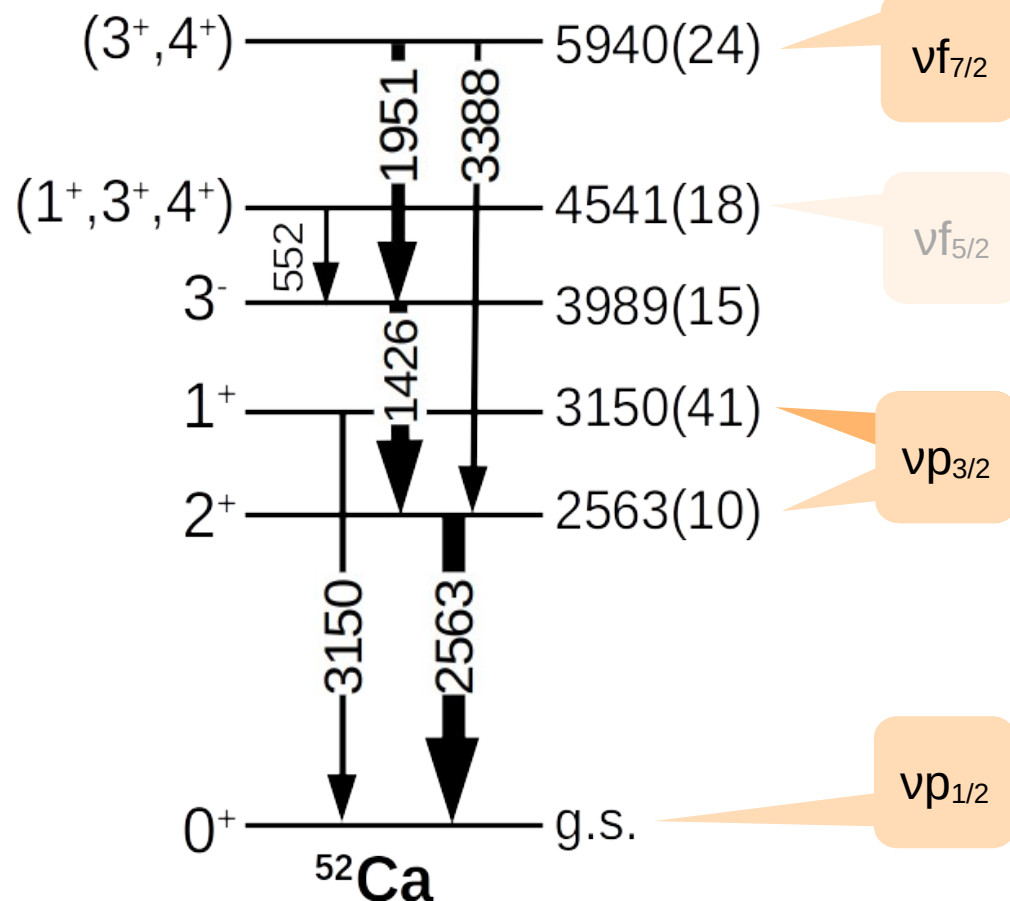
References:

- A. Huck et al., Phys. Rev. C 31, 2226–2237 (1985).
- A. Gade et al., Phys. Rev. C 74, 021302 (2006).
- M. Rejmund et al., Phys. Rev. C 76, 021304 (2007).
- F. Perrot et al., Phys. Rev. C 74, 014313 (2006).

The pf-shell neutron orbitals of ^{53}Ca

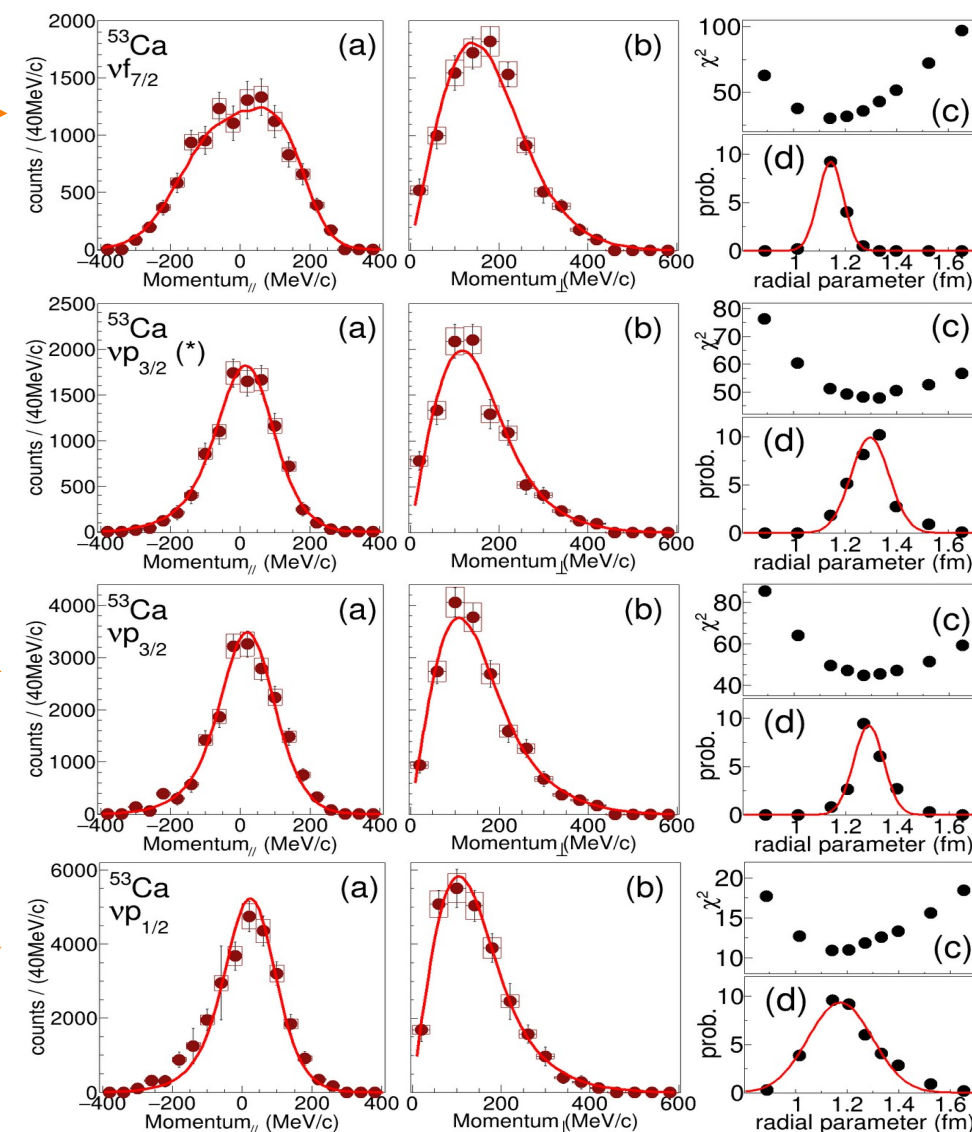


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References:

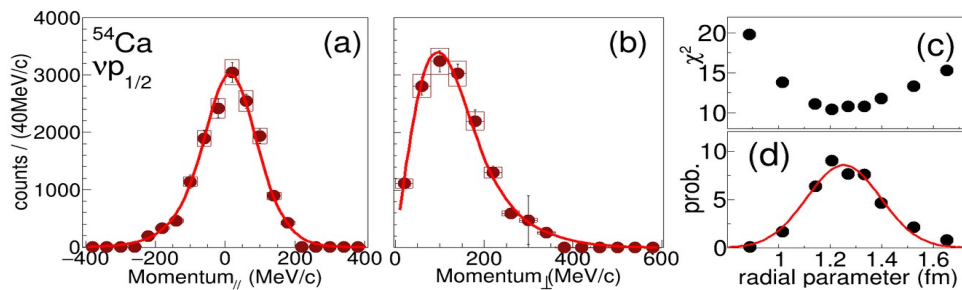
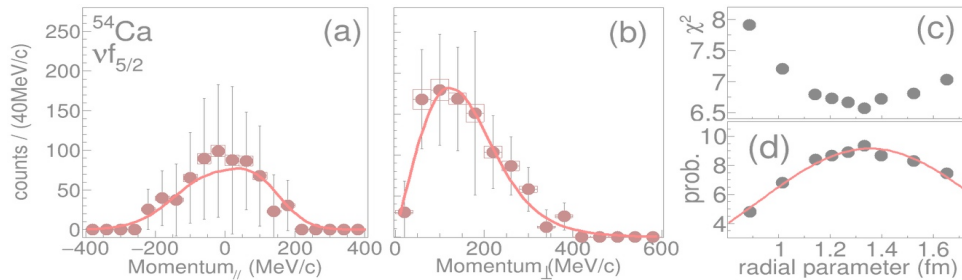
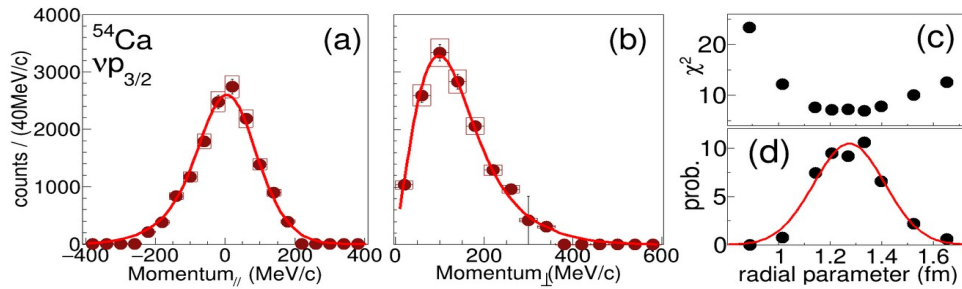
- A. Huck et al., Phys. Rev. C 31, 2226–2237 (1985).
- A. Gade et al., Phys. Rev. C 74, 021302 (2006).
- M. Rejmund et al., Phys. Rev. C 76, 021304 (2007).
- F. Perrot et al., Phys. Rev. C 74, 014313 (2006).



The pf-shell neutron orbitals of ^{54}Ca



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References:

S. Chen et al., Phys. Rev. Lett, 123, 142501 (2019).

$vp_{3/2}$

$3/2^-$ 2220

$vf_{5/2}$

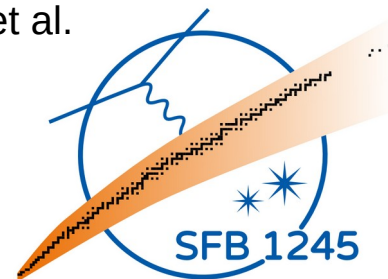
$5/2^-$ 1738

$vp_{1/2}$

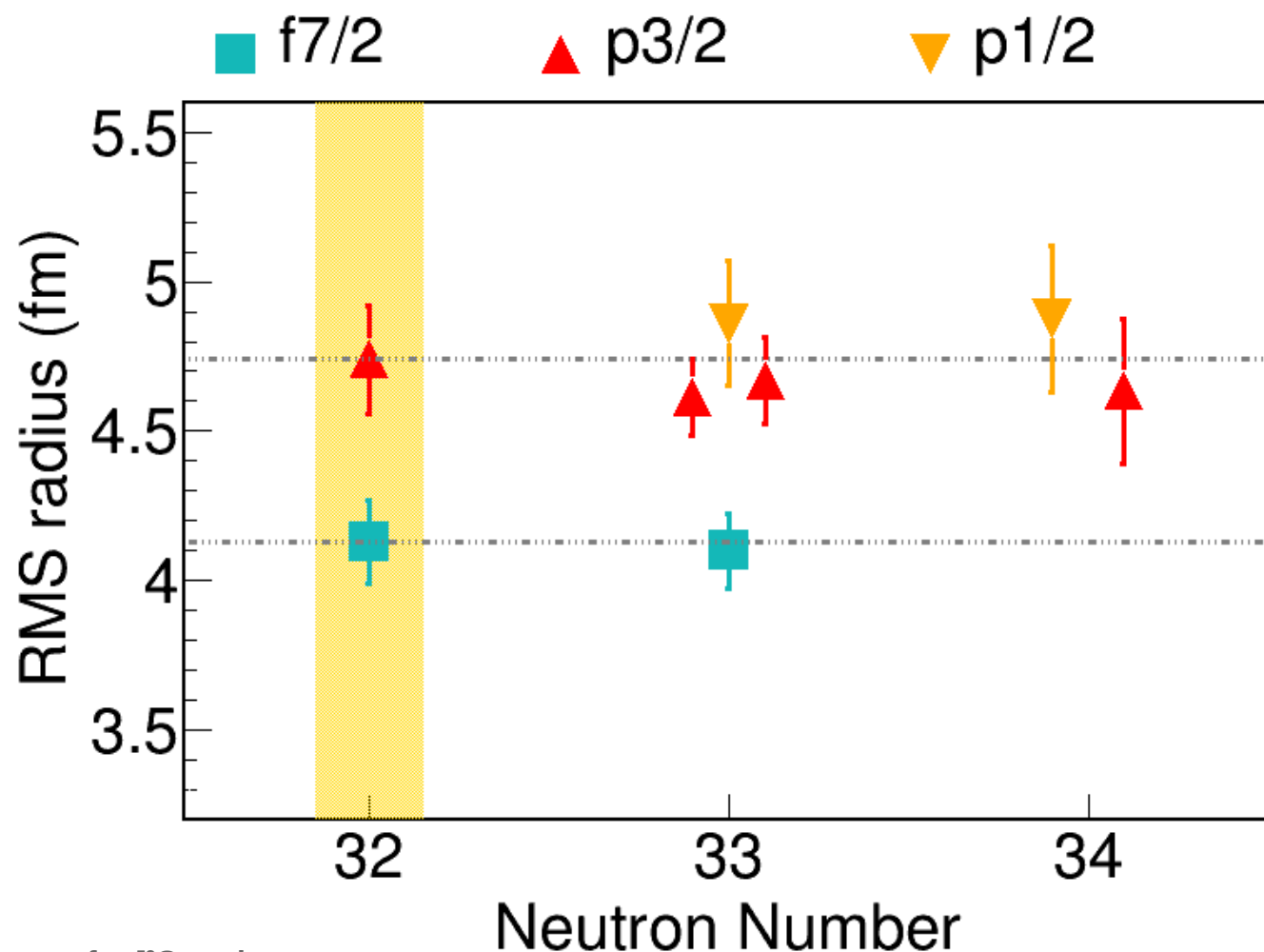
$1/2^-$ g.s.

^{53}Ca

$^{54}\text{Ca}(p,pn)$ already studied by Chen et al.



Rms radii of single-particle neutron orbitals



Reference for ^{52}Ca values:
M. Enciu et al., PRL 129 (2022)

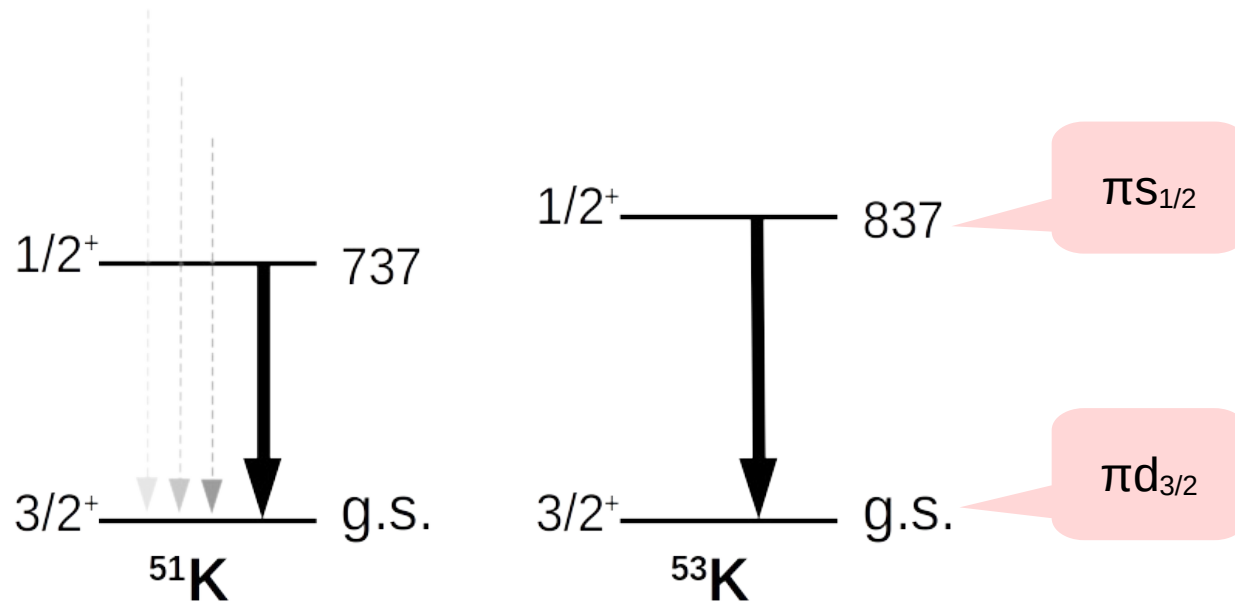
- systematic difference of 0.5 up to 0.75 fm between the $f_{7/2}$ and p orbitals for ^{53}Ca and ^{54}Ca as found for ^{52}Ca

^{52}Ca	$vf_{7/2}$	4.13(14) fm
	$vp_{3/2}$	4.74(18) fm
^{53}Ca	$vf_{7/2}$	4.10(13) fm
	$vp_{3/2}$	4.61(13) fm
	$vp_{1/2}$	4.86(21) fm
^{54}Ca	$vp_{1/2}$	4.67(15) fm
	$vp_{3/2}$	4.64(24) fm
	$vp_{1/2}$	4.88(24) fm

The sd-shell proton orbitals of ^{52}Ca and ^{54}Ca



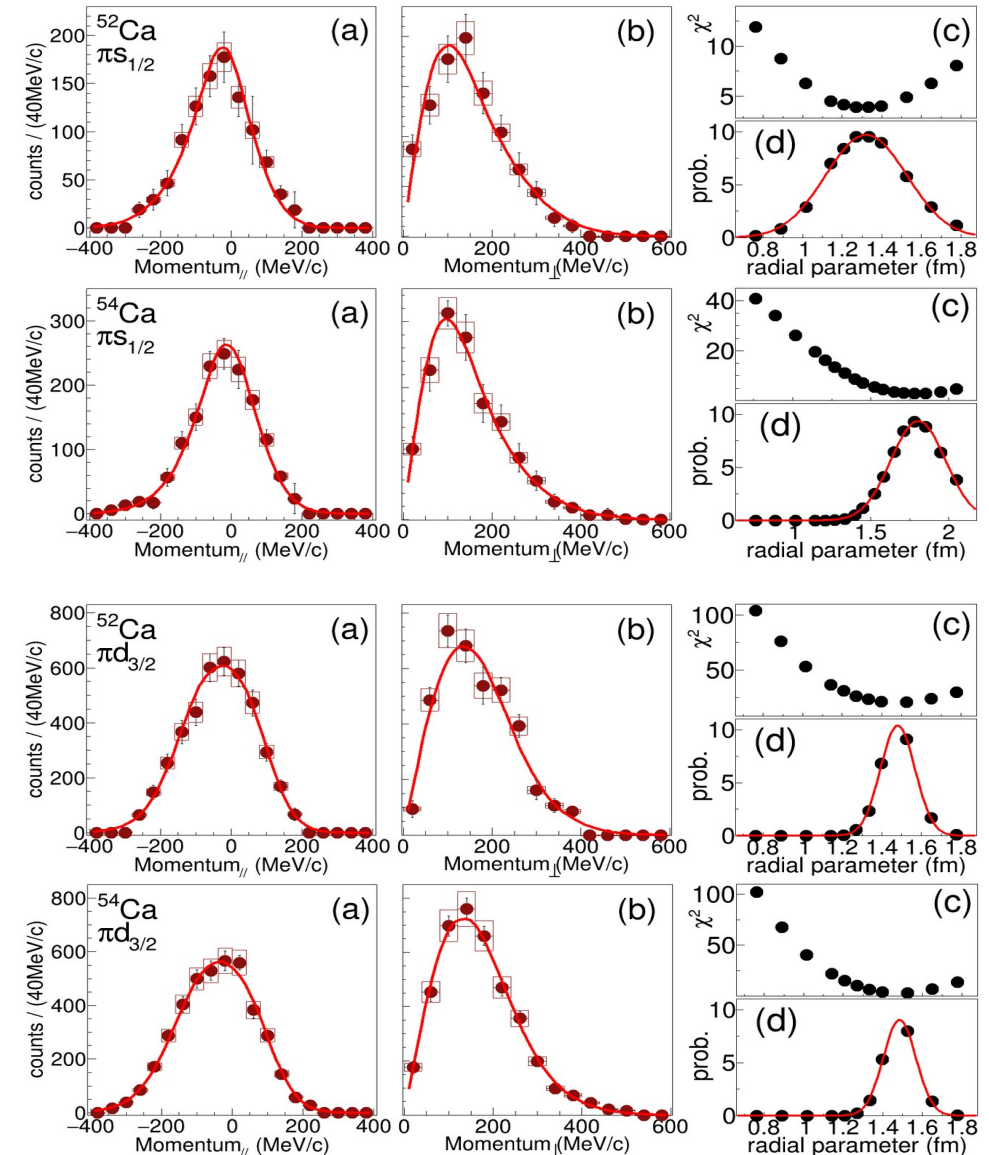
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$^{52}\text{Ca}(p,2p)$ and $^{54}\text{Ca}(p,2p)$ already studied by Sun et al.

References:

Y.L. Sun et al., Phys. Lett. B 802 (2020).

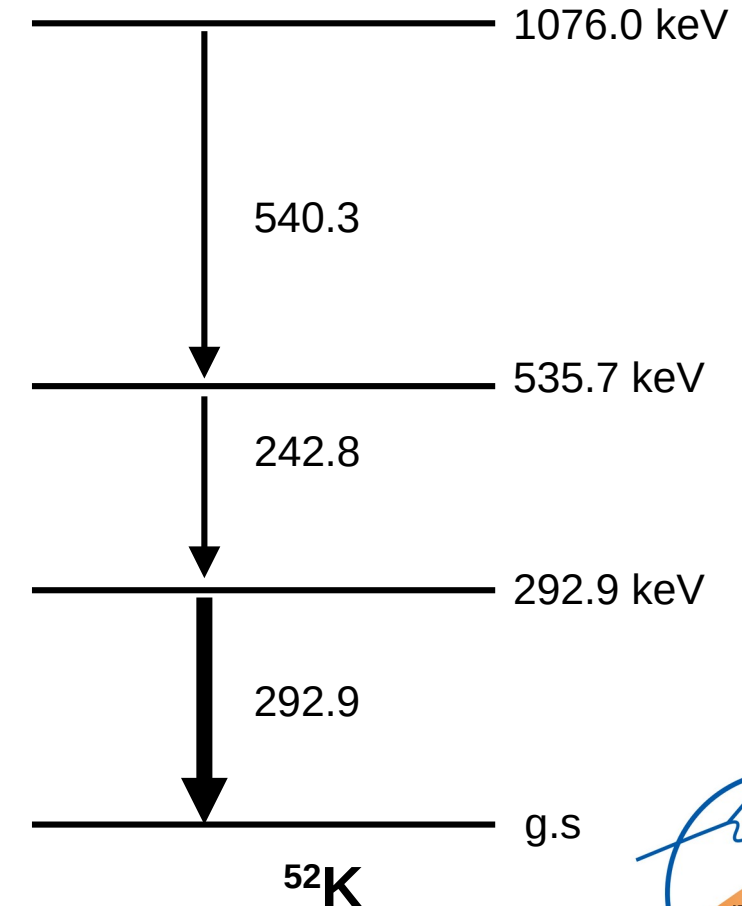
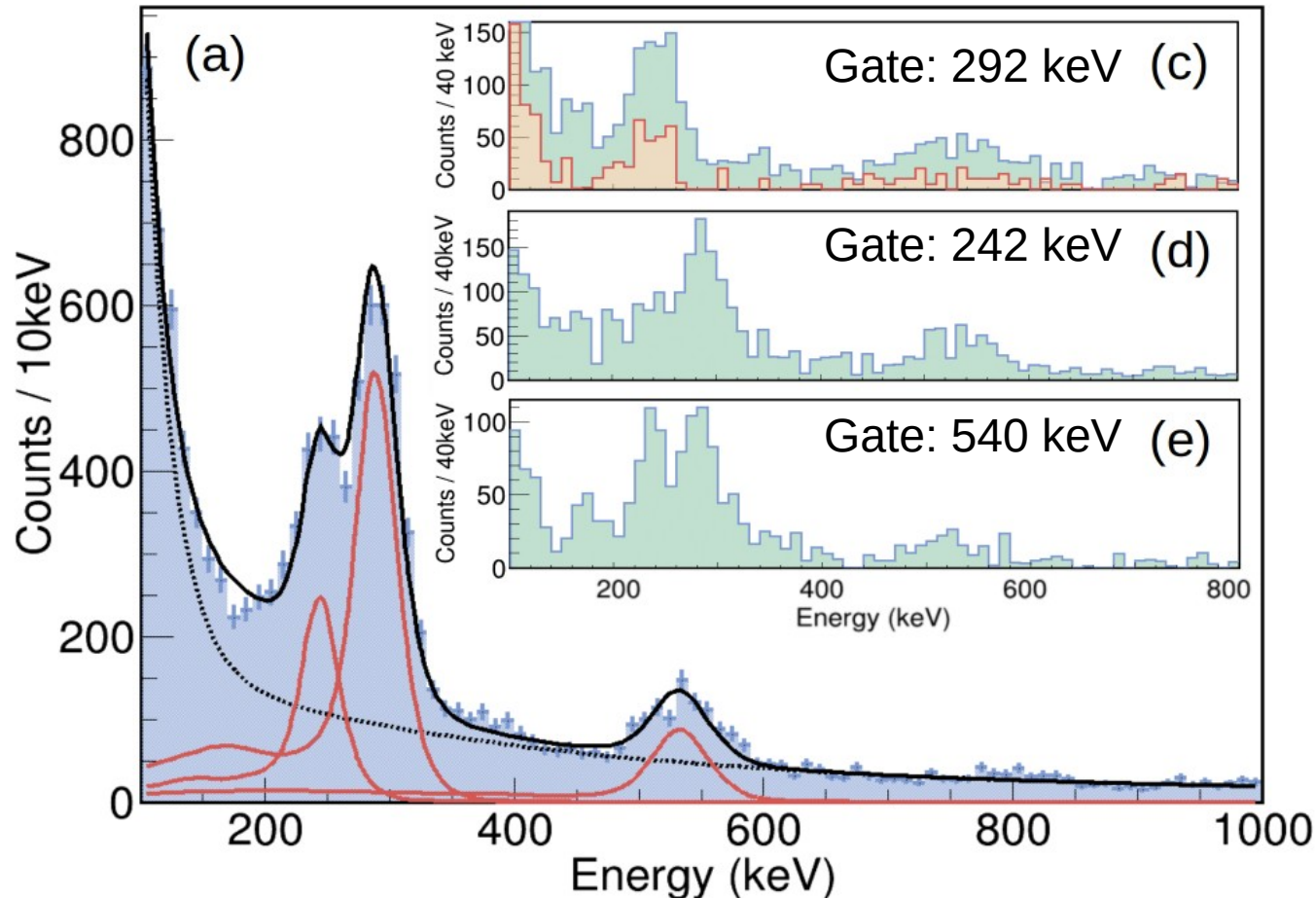


The sd-shell proton orbitals of ^{53}Ca



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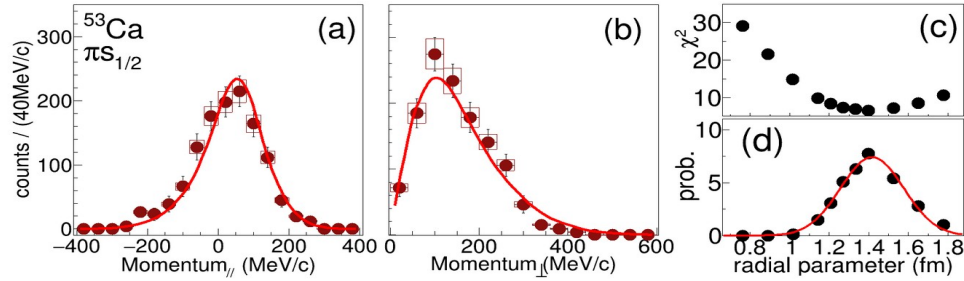
First spectroscopy of ^{52}K



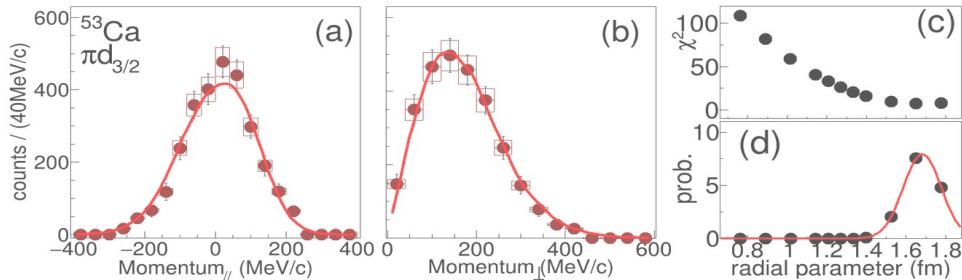
The sd-shell proton orbitals of ^{53}Ca



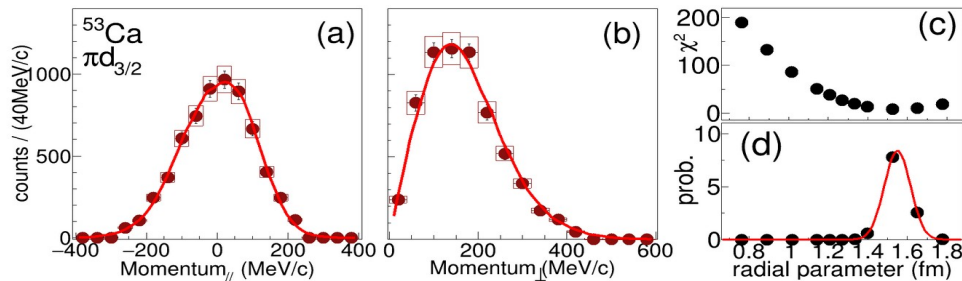
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$\pi s_{1/2}$

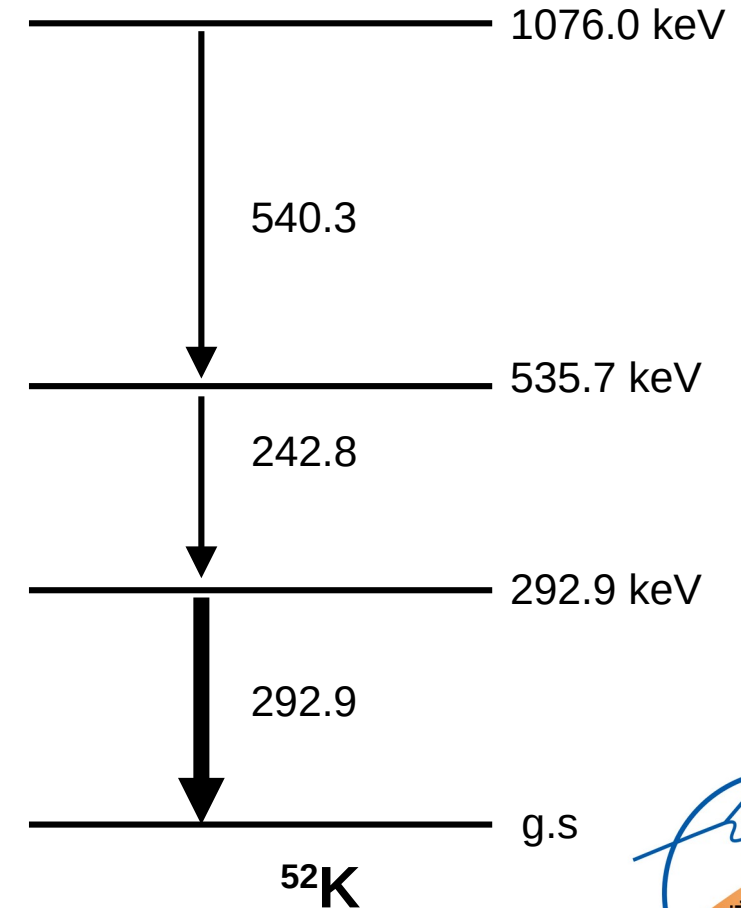


$\pi d_{3/2}$

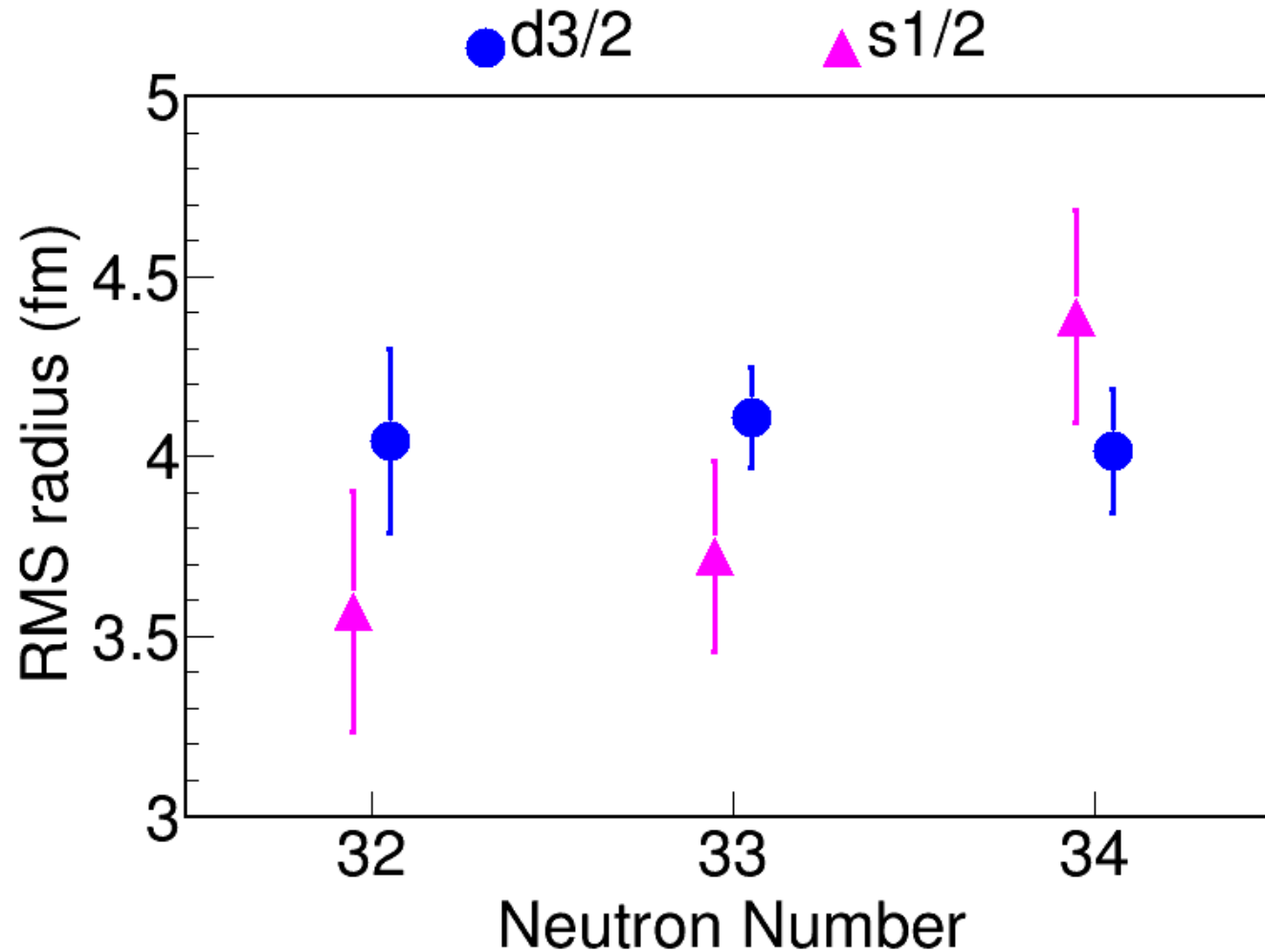


$\pi d_{3/2}$

First spectroscopy of ^{52}K

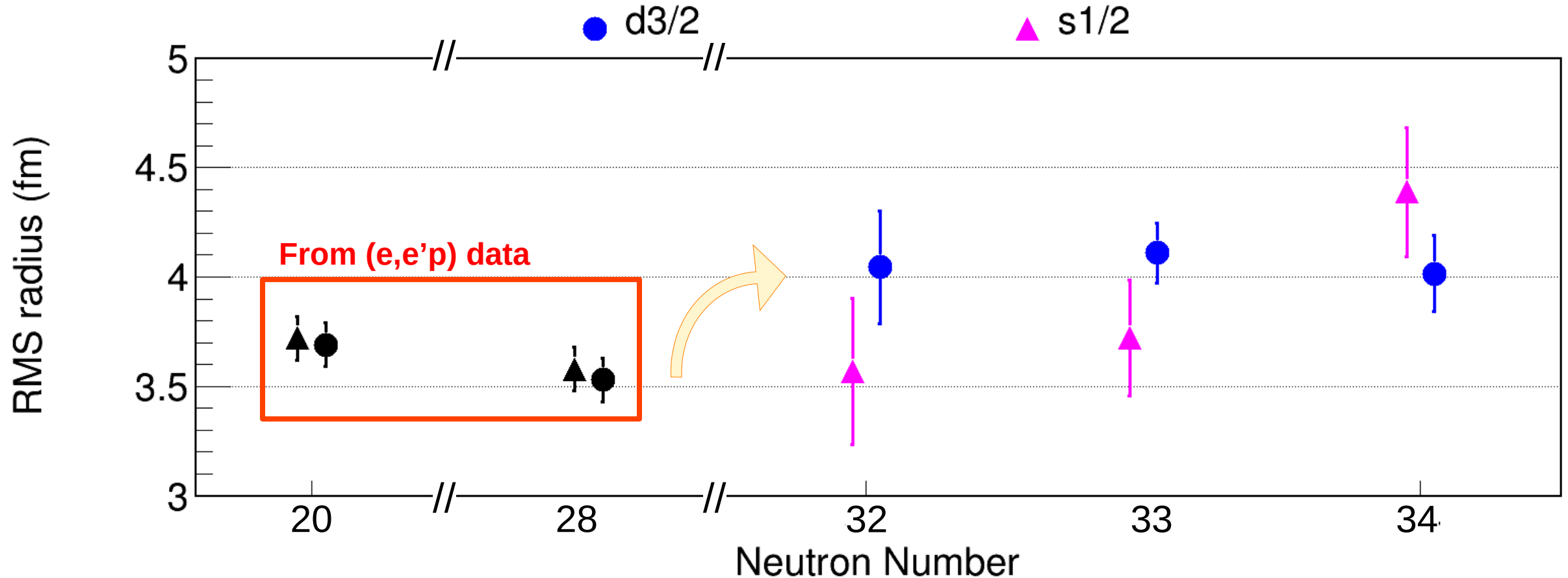


Rms radii of single-particle proton orbitals



⁵² Ca	$\pi d_{3/2}$	: 4.04(26) fm
	$\pi s_{1/2}$	: 3.57(33) fm
⁵³ Ca	$\pi d_{3/2}$	: 4.10(13) fm
	$\pi s_{1/2}$	: 3.72(26) fm
⁵⁴ Ca	$\pi d_{3/2}$	: 4.02(17) fm
	$\pi s_{1/2}$	: 4.39(29) fm

Rms radii of single-particle proton orbitals



⁴⁰Ca and ⁴⁸Ca data from:

G. J. Kramer, "The proton spectral function of ⁴⁰Ca and ⁴⁸Ca studied with the (e,e'p) reaction", PhD Thesis, Amsterdam Univ. (Netherlands), 1990

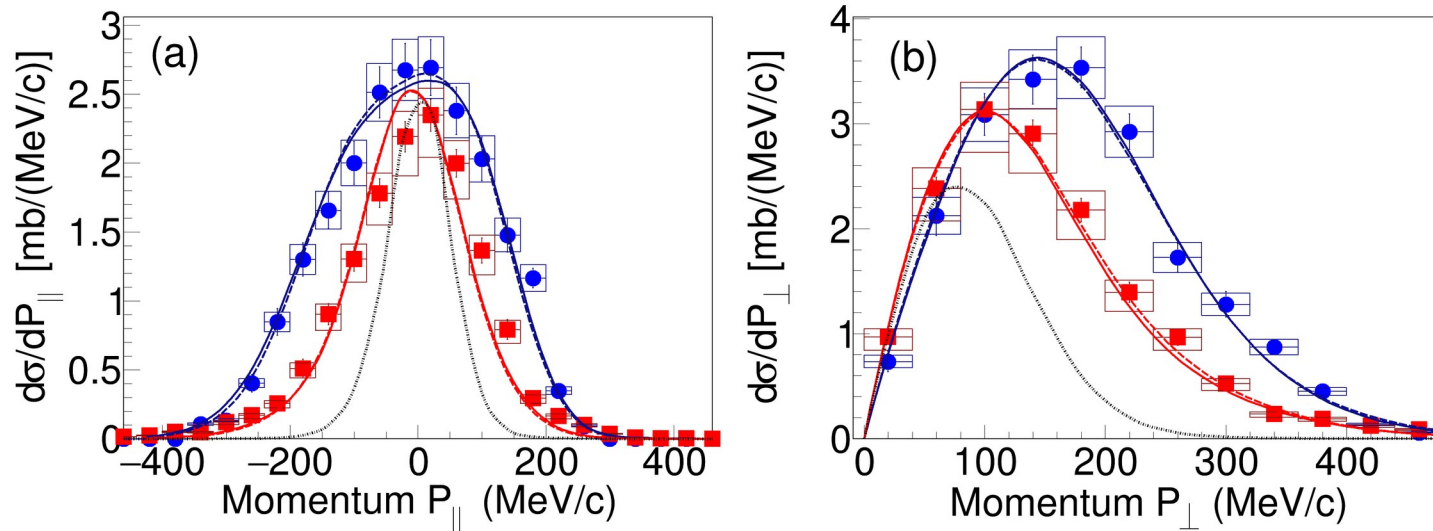
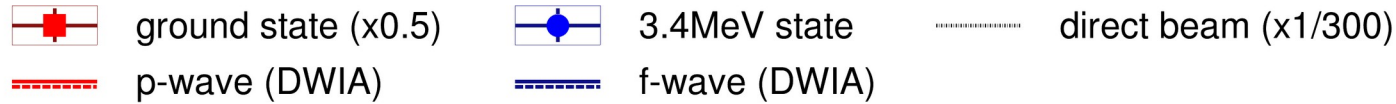
Uncertainty estimations and checks

- **uncertainties of rms radii dominated by experimental statistical uncertainties**
larger uncertainties for (p,2p) with lower statistics
- **choice of potential:**
how the shape of momentum distribution changes
and what is the impact on the determined rms radii?
- **why do we only perform an 1-dimensional r_0 variation?**
comparison 1D r_0 variation vs 2D r_0, a_0 variation of the Woods-Saxon potential parameters
- **how do momentum distributions change when using HFB wavefunction**
or **transition amplitudes from state-of-the-art ab initio calculations?**



Choice of optical potential

used as input for the DWIA calculations



Plots: ^{52}Ca $vp_{3/2}$ and $vf_{7/2}$

Folding (solid line) and Dirac (EDAD1, dashed line)

M. Enciu et al., PRL 129 (2022)

From ^{52}Ca $vp_{3/2}$ and $vf_{7/2}$:

- **4.5% relative difference** for the momentum distributions for the considered energy range and (r_0, a_0) combinations

From ^{53}Ca $vp_{3/2}$:

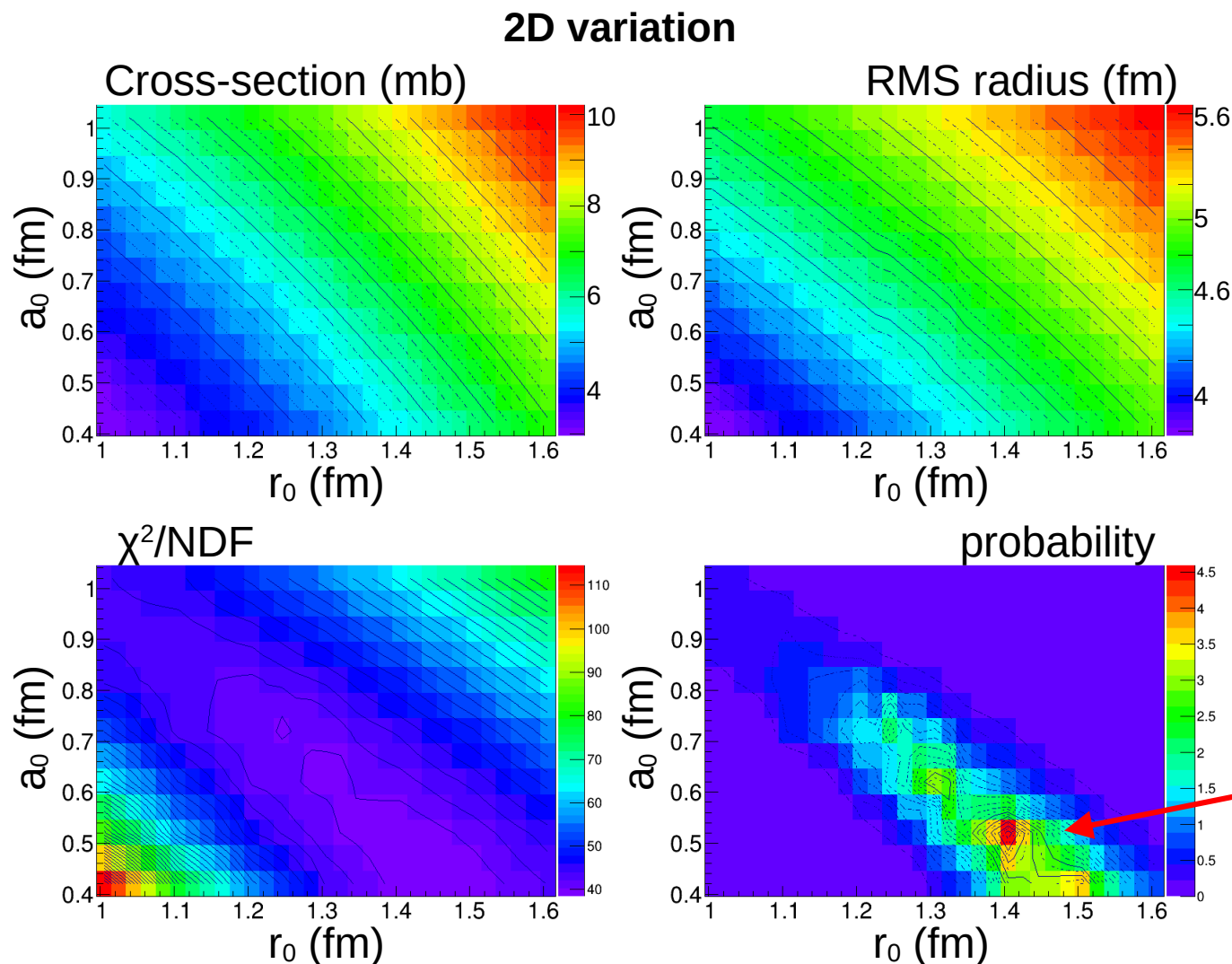
impact on r_0 and rms radii evaluated

- **0.10 fm for r_0 (8.1%)**
- **0.16 fm for rms radius (3.5%)**

1D vs 2D variation: r_0 and a_0 for the WS potential

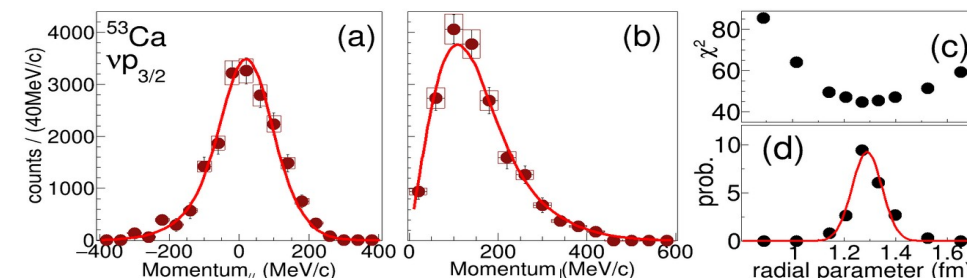


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1D variation

^{53}Ca (p,pn) [$-v_{p_{3/2}}$, 2.5MeV]



(r_0, a_0) pairs not unique

RMS radius results:

1D variation **4.67 fm**

for $r_0 = 1.29$ fm

and $a_0 = 0.67$ fm (fixed)

2D variation **4.663 fm**

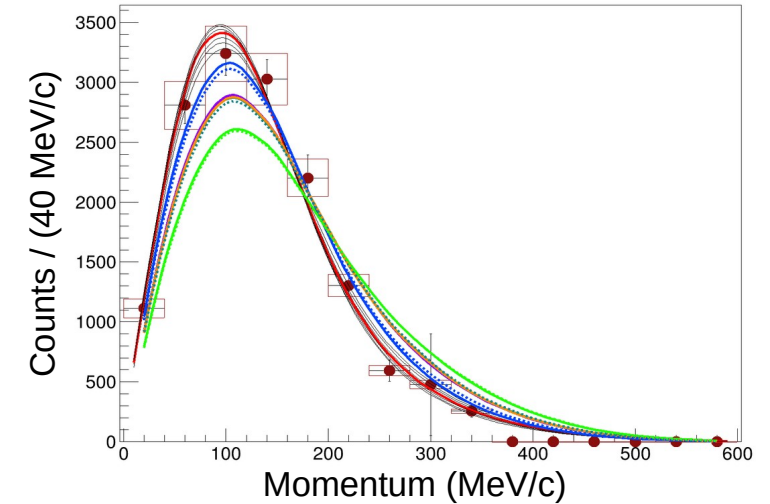
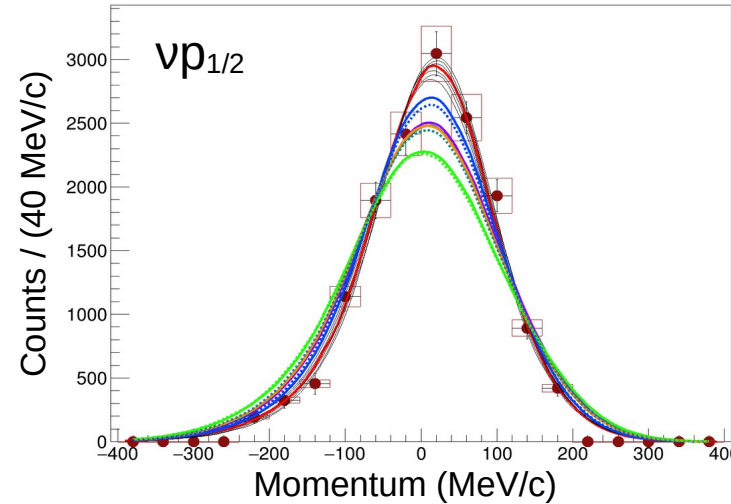
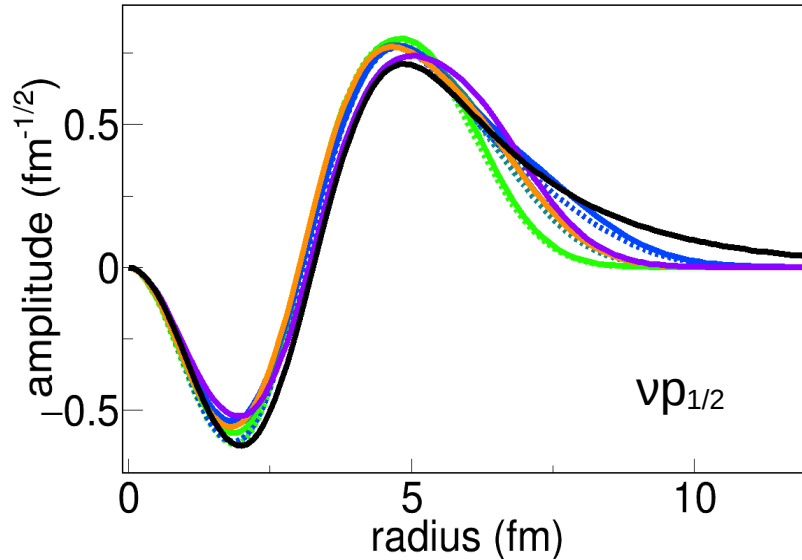
for $r_0 = 1.406$ fm

and $a_0 = 0.52$ fm



Woods-Saxon wavefunction vs ab initio input

ab initio amplitudes: $\langle {}^{54}\text{Ca}(0^+) | a_{n\ell j}^+ | {}^{53}\text{Ca}(J^\pi) \rangle$ used for the DWIA calculations for ${}^{54}\text{Ca}(p,pn)$



Legend:

— Woods-Saxon

DN2LOGO394hw16

EM1.8-2.0hw12 (HF)

EM1.8-2.0hw16 (HF)

EM1.8-2.0hw20 (HF)

N3LOEM500lnlhw16

-- EM1.8-2.0hw12 (NAT)

-- EM1.8-2.0hw16 (NAT)

-- EM1.8-2.0hw20 (NAT)

Similar results for ${}^{54}\text{Ca}$ $vp_{3/2}$ and $vp_{1/2}$ orbitals

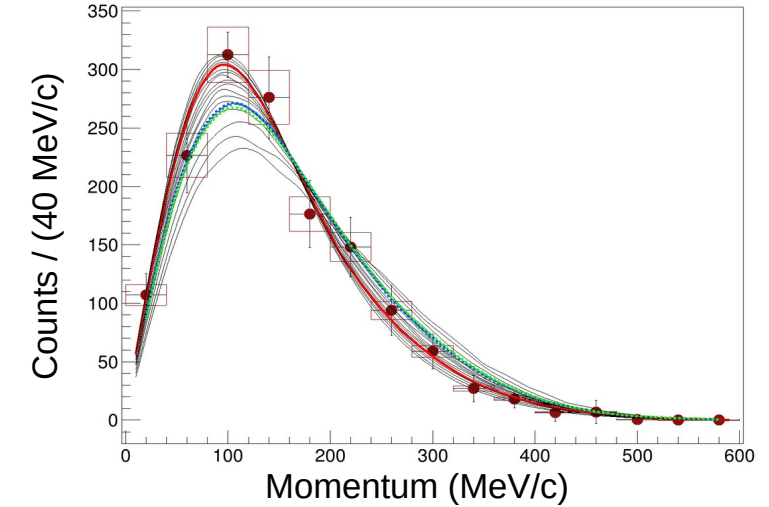
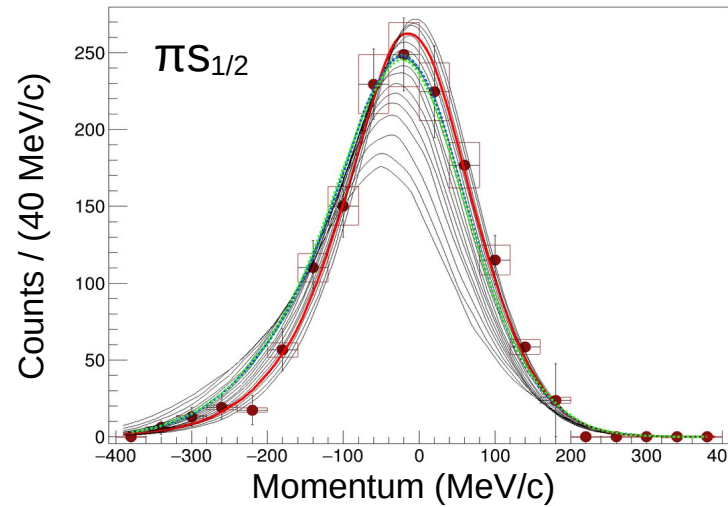
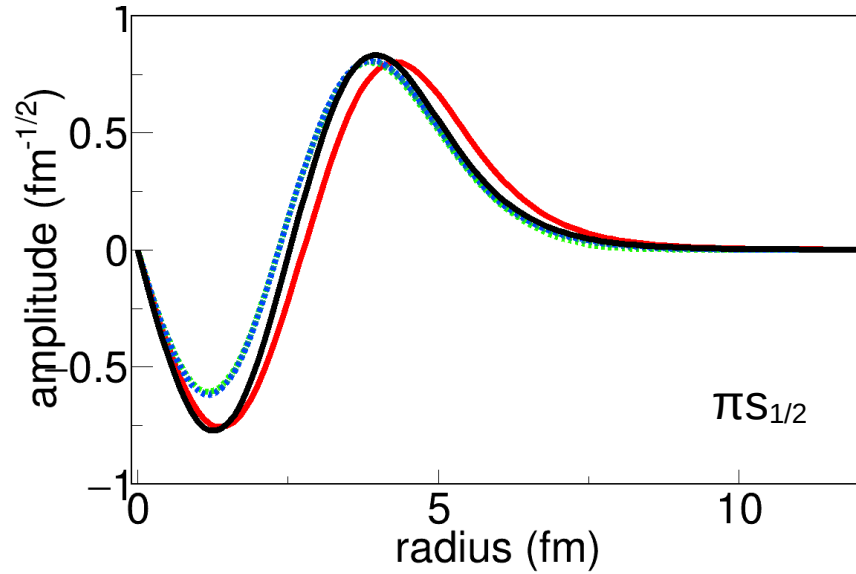
- frequency dependence
- wider momentum distributions
- lower rms radii
- poor fit to experimental data



Woods-Saxon wavefunction vs ab initio input

ab initio amplitudes: $\langle {}^{54}\text{Ca}(0^+) | a_{n\ell j}^+ | {}^{53}\text{K}(J^\pi) \rangle$

used for the DWIA calculations for ${}^{54}\text{Ca}(p,2p)$



Legend:

— Woods-Saxon

-- EM1.8-2.0hw12 (NAT)

-- EM1.8-2.0hw16 (NAT)

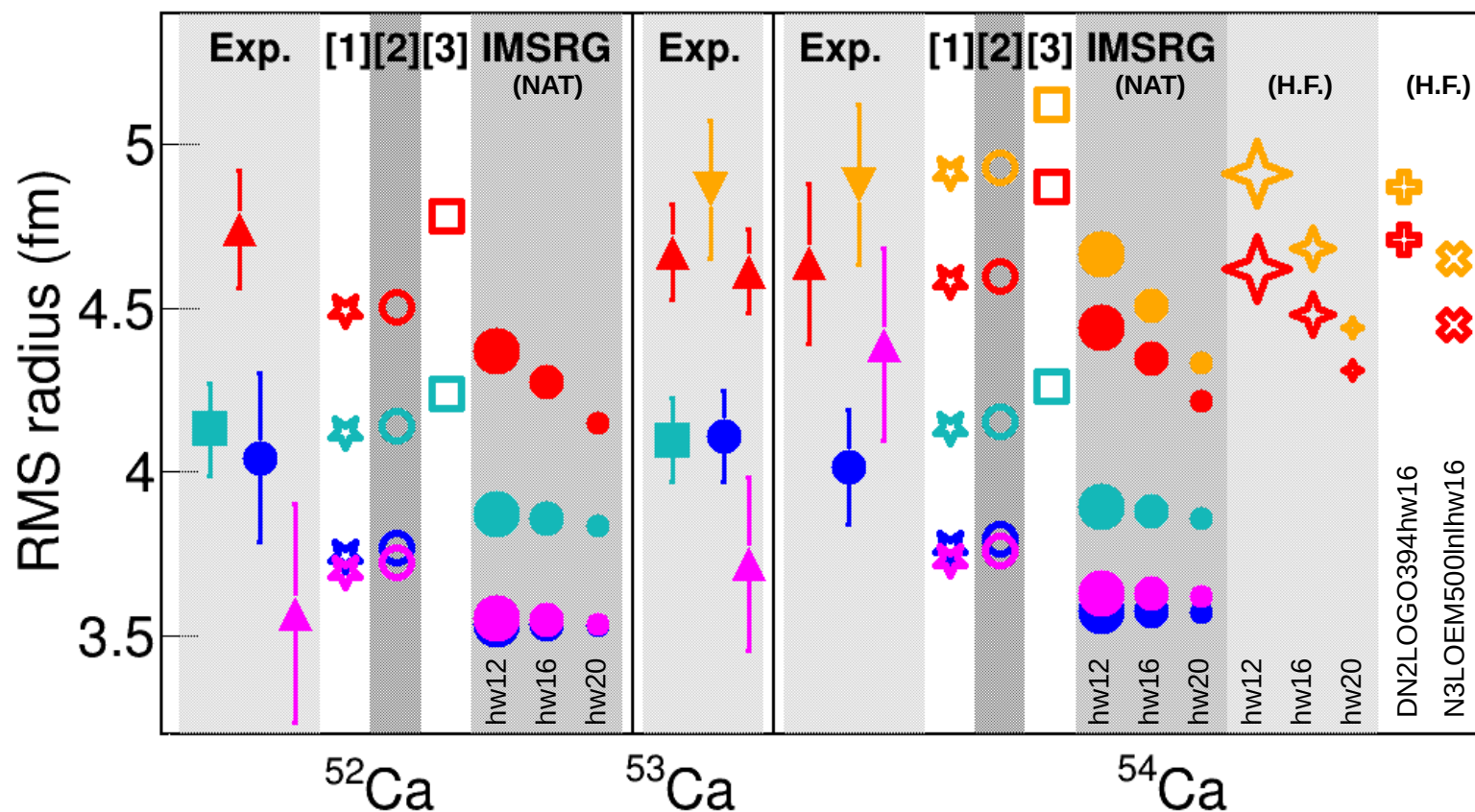
-- EM1.8-2.0hw20 (NAT)

Similar results for ${}^{52,54}\text{Ca} \pi d_{3/2}$ and $\pi S_{1/2}$ orbitals

- no frequency dependence
- wider momentum distributions
- lower rms radii
- poor fit to experimental data



Rms radii of sp proton and neutron orbitals



[1] HFB calculations (HFBRAD code)
with the SKM interaction

[2] Results from private communication with
W. Horiuchi based on published work:
W. Horiuchi et al., Phys. Rev. C 101 (2020)
[Mean Field calculations]

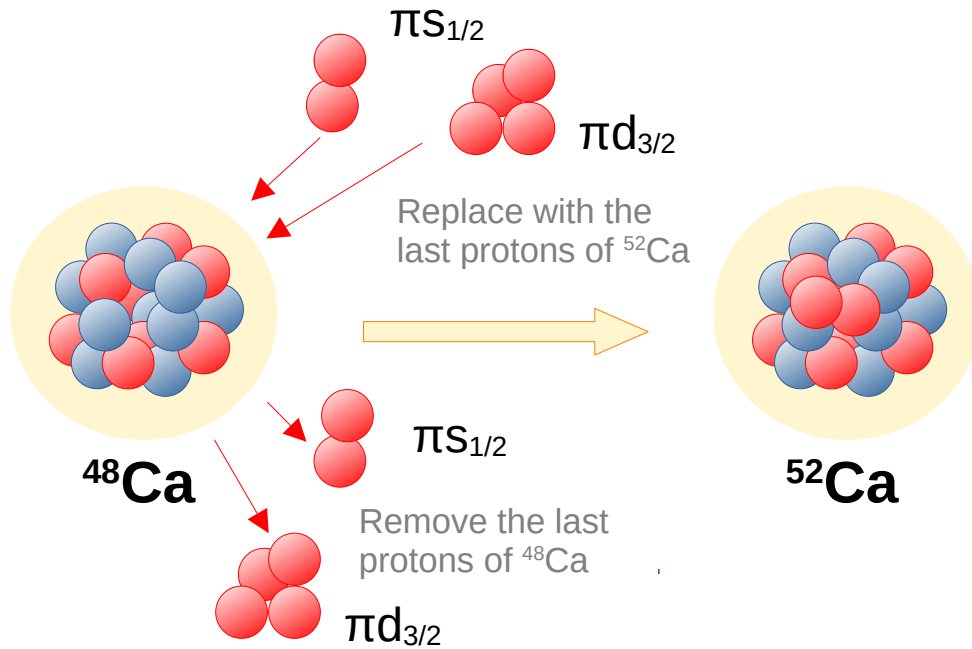
[3] Results from private communication with
J. Liu (via H. Liu) based on published work:
J. Liu et al., Phys. Lett. B 806 (2020)
[Relativistic Hartree-Fock calculations]

IMSRG (full) one-body level: preliminary, ongoing work M. Heinz, T. Miyagi, A. Schwenk, A. Tichai



Charge radii of Calcium isotopes

What is does the size of the $s_{1/2}$ and $d_{3/2}$ proton orbitals of $^{52,53,54}\text{Ca}$ mean for the total charge radius?

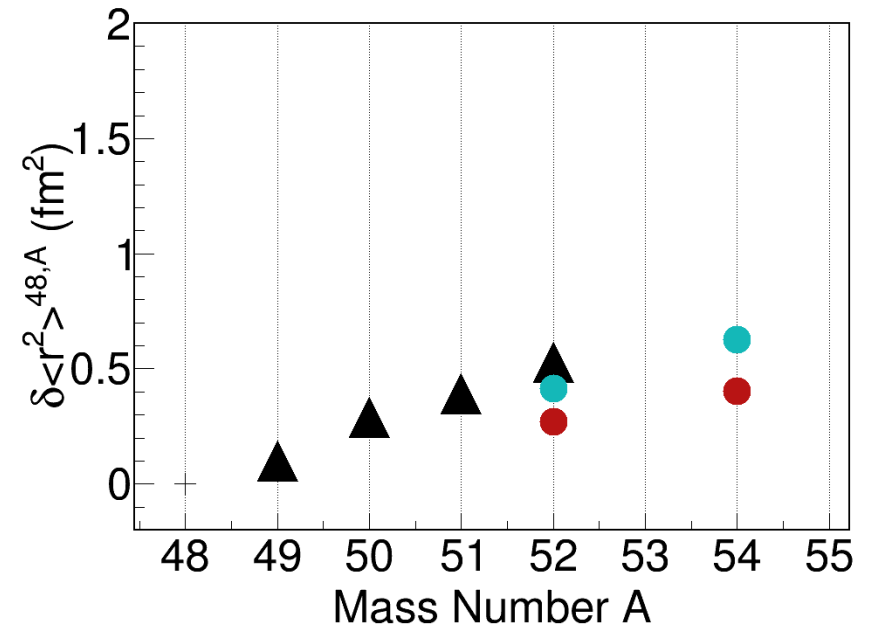
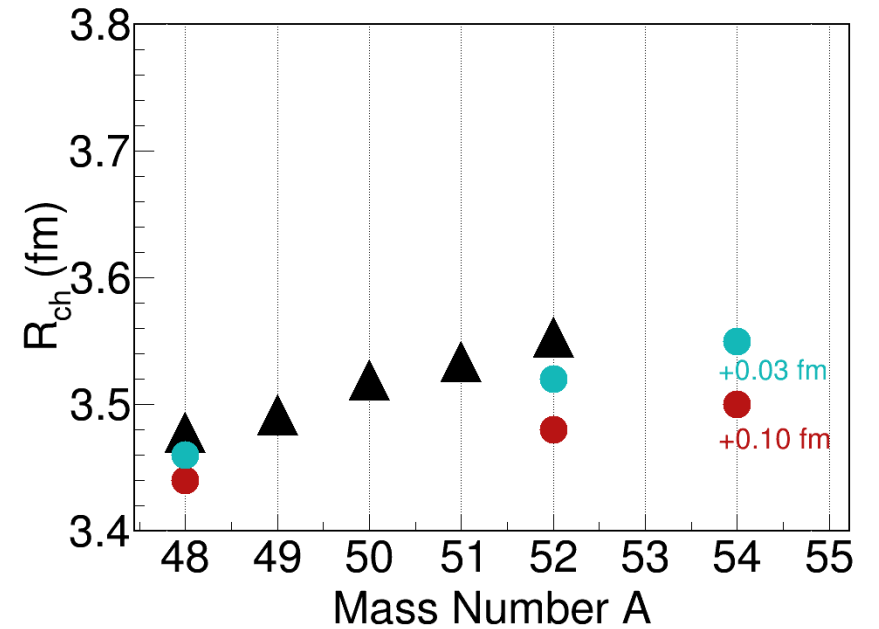


^{48}Ca charge radii: experimental (IS) / calculated

^{48}Ca s and d proton orbitals: (e,e'p) / calculated

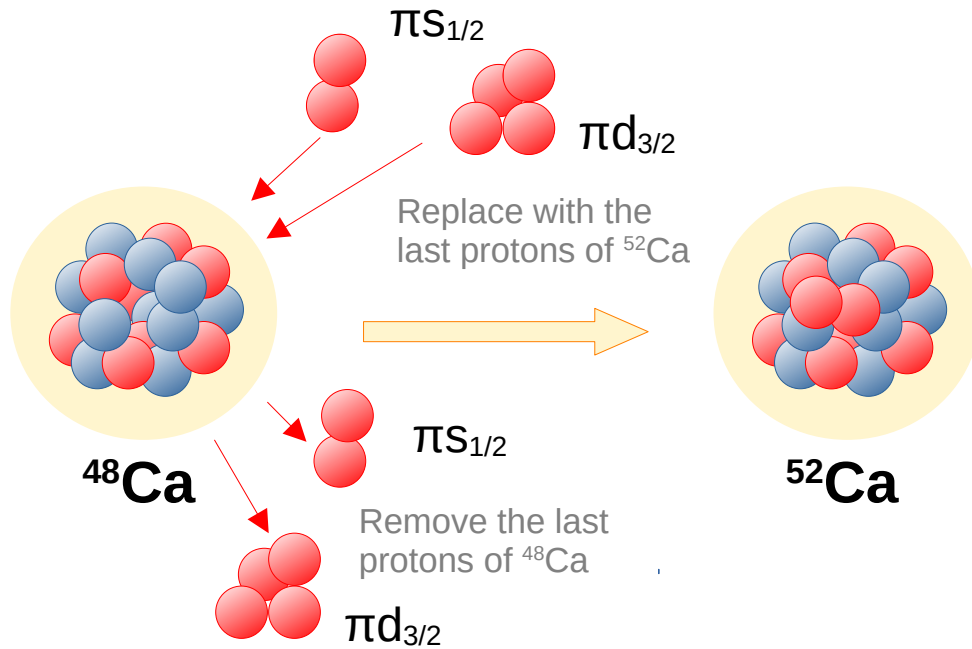
^{52}Ca s and d proton orbitals: from this work

- ▲ Isotope Shift Measurements
- ○ Evaluations w/ sp rms radii
- ■ DN2LOGO hw16
- ● EM1.8-2.0 hw16



Charge radii of Calcium isotopes

What is does the size of the $s_{1/2}$ and $d_{3/2}$ proton orbitals of $^{52,53,54}\text{Ca}$ mean for the total charge radius?

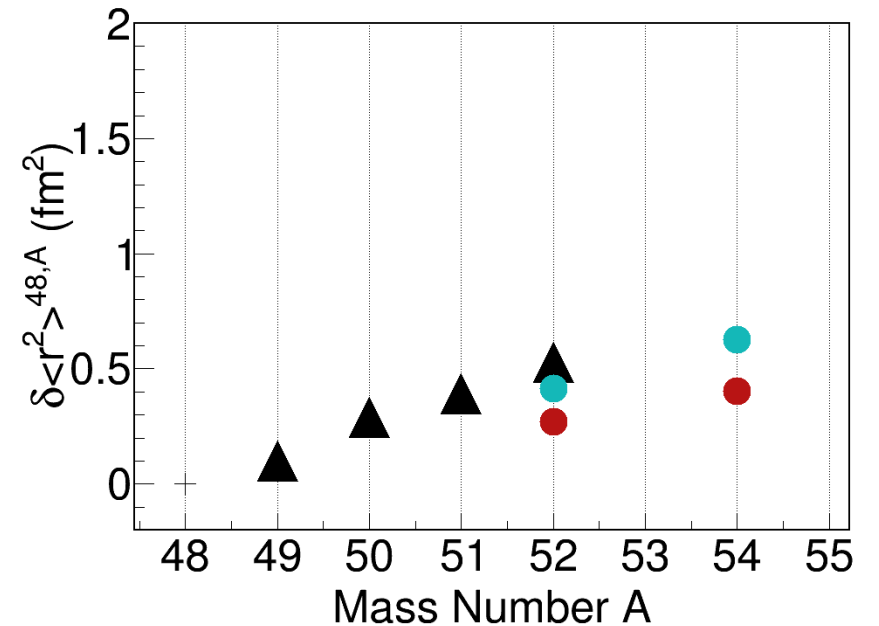
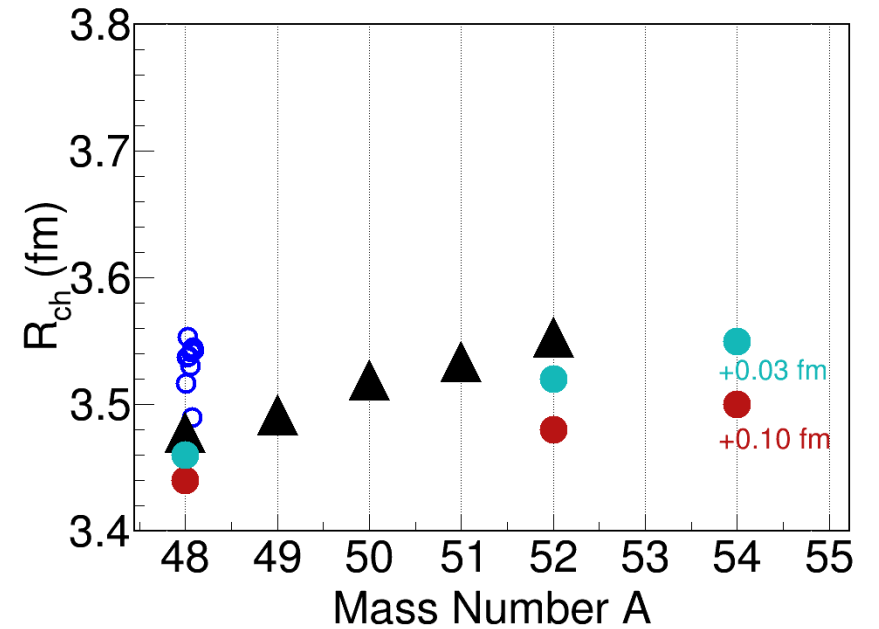


^{48}Ca charge radii: experimental (IS) / calculated

^{48}Ca s and d proton orbitals: (e,e'p) / calculated

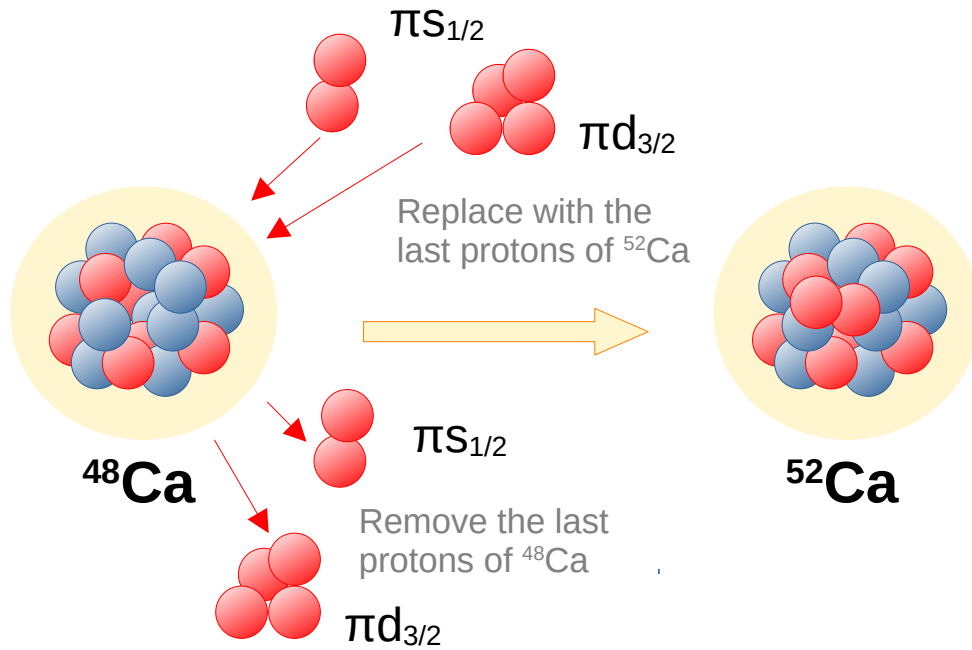
^{52}Ca s and d proton orbitals: from this work

- ▲ Isotope Shift Measurements
- ○ Evaluations w/ sp rms radii
- DN2LOGO hw16
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Charge radii of Calcium isotopes

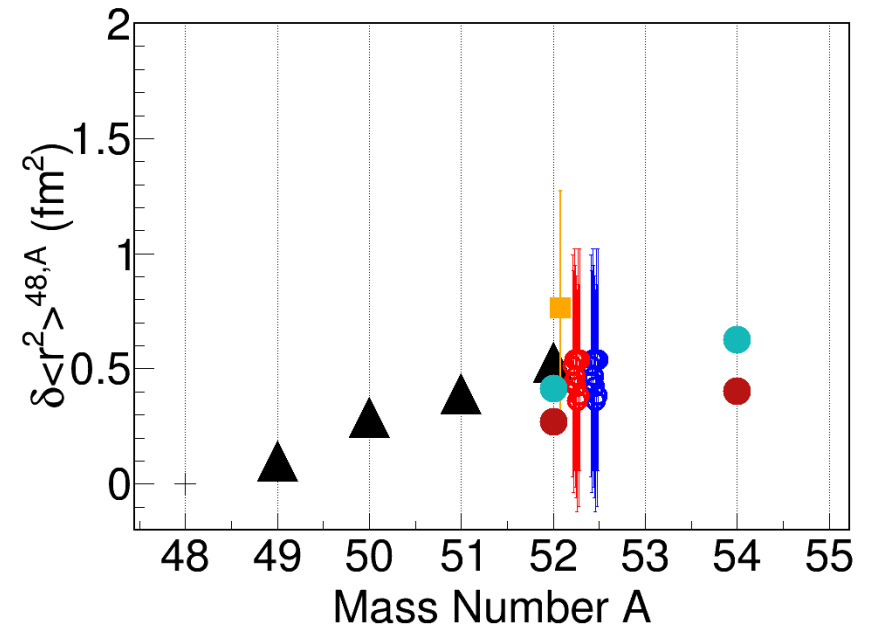
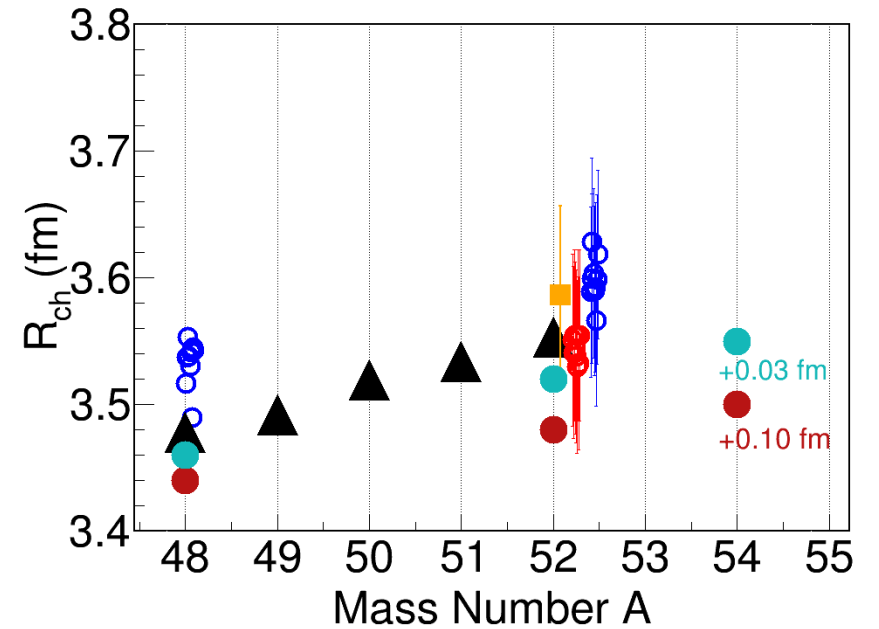
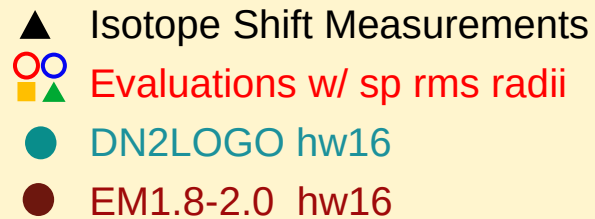
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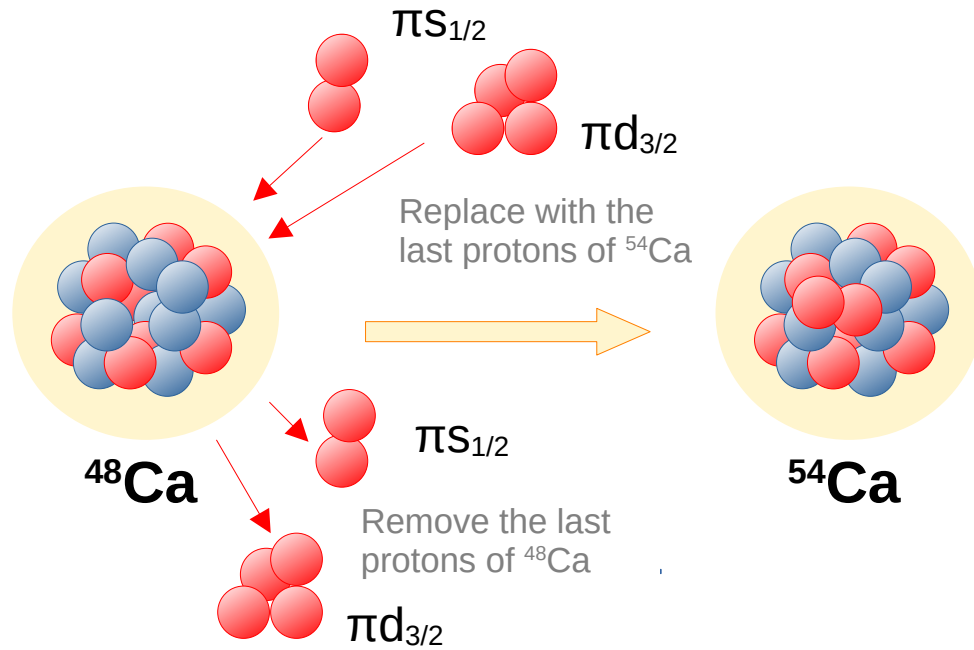
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Charge radii of Calcium isotopes

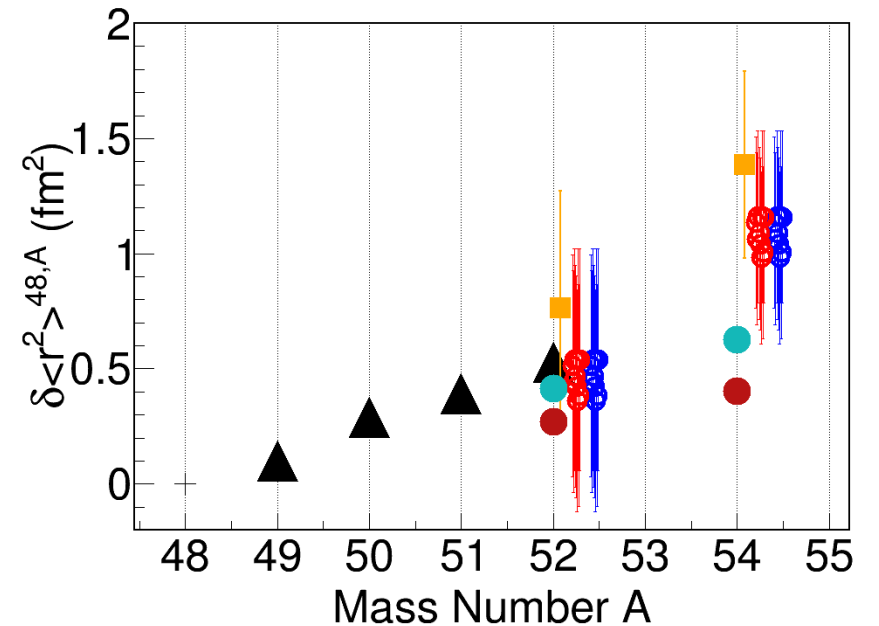
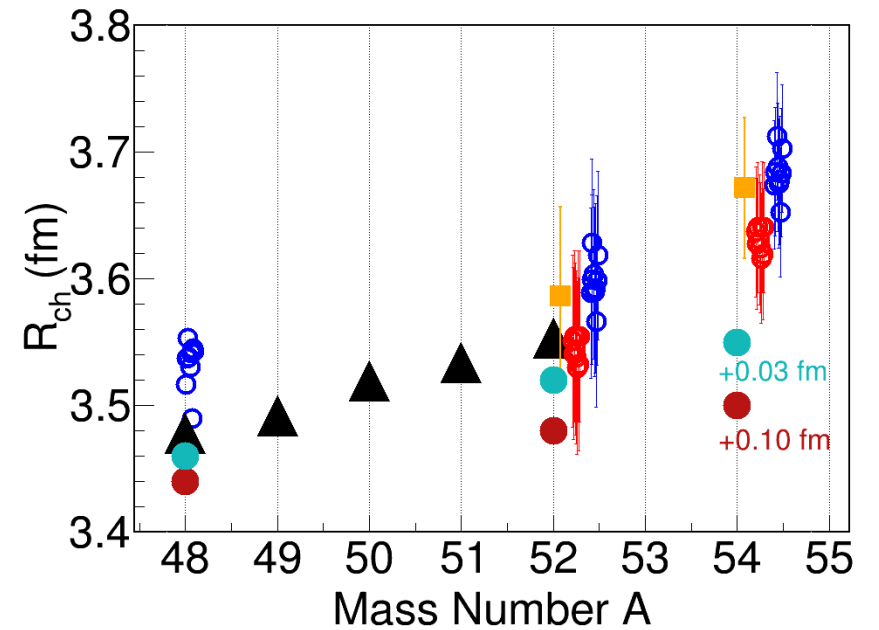
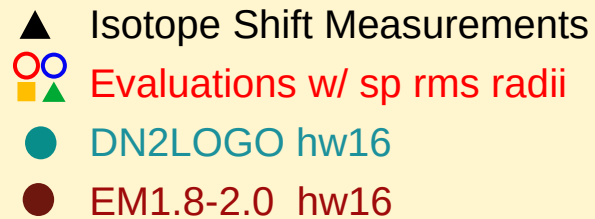
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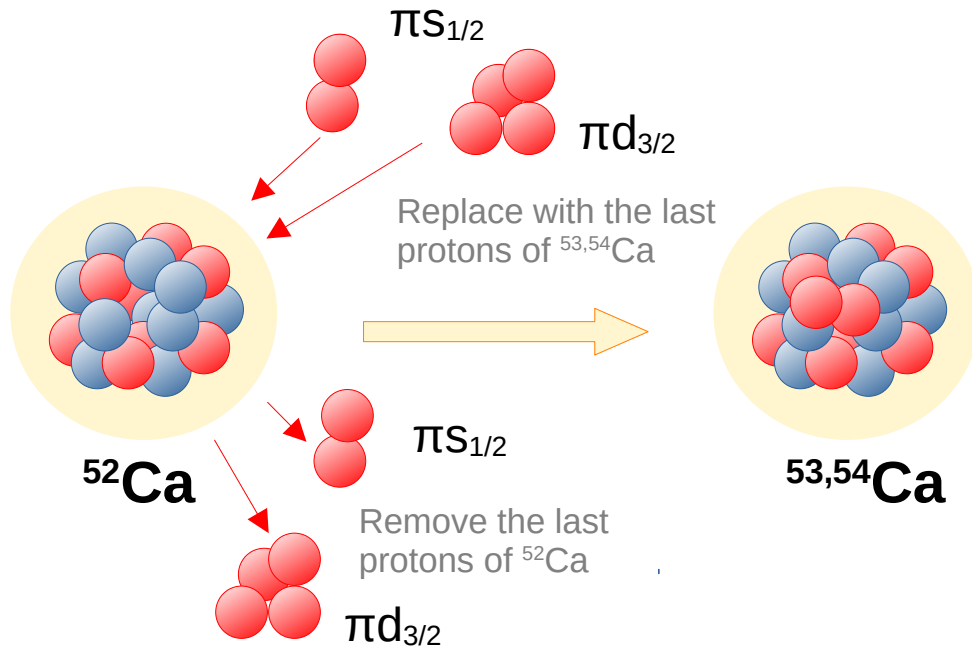
^{48}Ca s and d proton orbitals: (e,e'p) / calculated

^{52}Ca s and d proton orbitals: from this work



Charge radii of Calcium isotopes

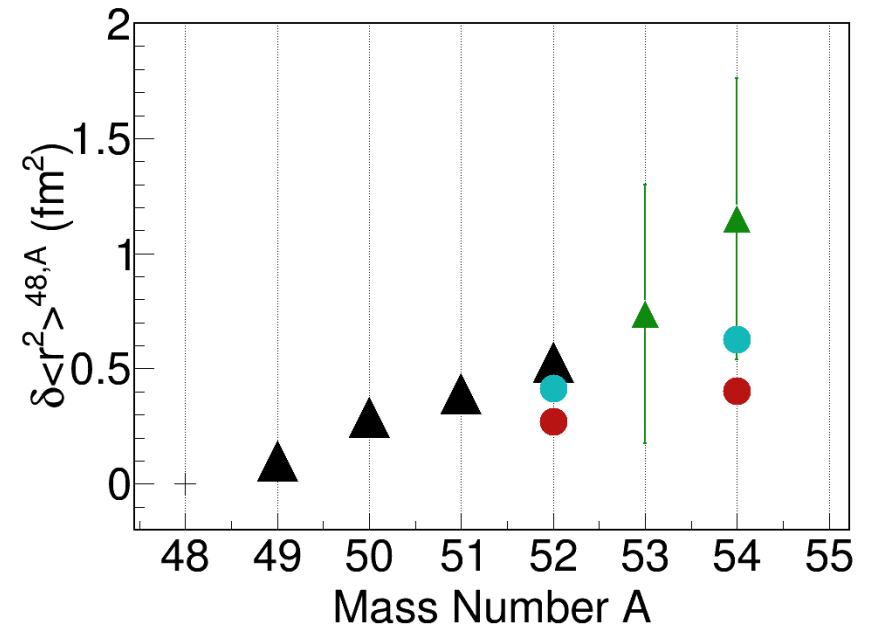
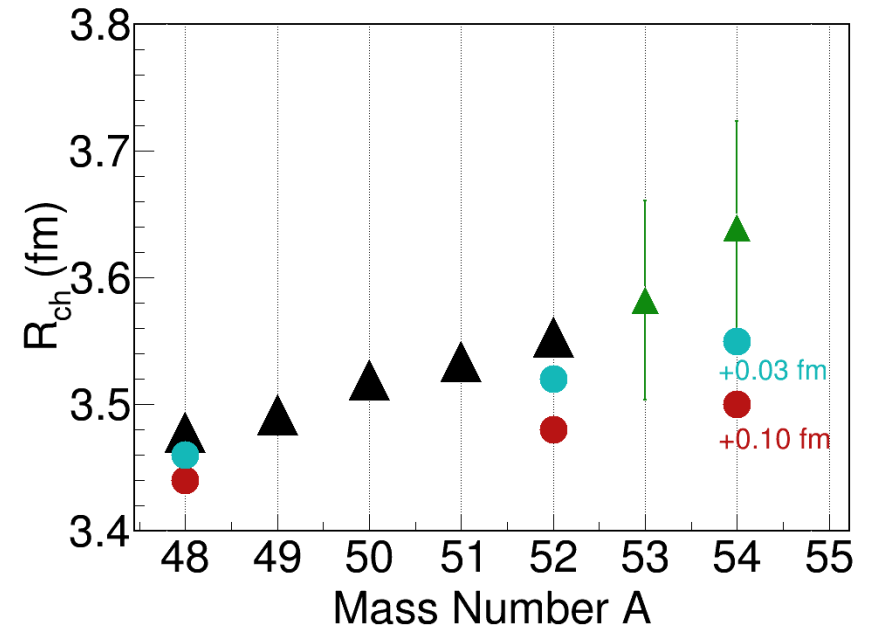
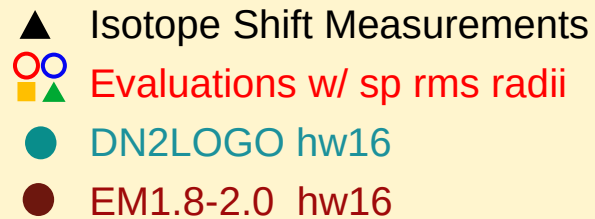
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^{48}Ca s and d proton orbitals: (e,e'p) / calculated

^{52}Ca s and d proton orbitals: from this work



Summary and Conclusions

- **Calcium charge radii puzzle: steep increase from N=28 to N=32** (R. F. Garcia-Ruiz et al. 2016)

matter radii present the same behavior (M. Tanaka et al. 2020)

J. Bonnard et al. 2016: **vp orbitals larger than vf orbitals by 0.7fm** for explaining the charge radii

W. Horiuchi et al. 2020: **core swelling** as one fills the $vp_{1/2}$, $vp_{3/2}$ and $vf_{5/2}$ orbitals

J. Liu et al. 2023: density evolution from ^{52}Ca to ^{54}Ca

- **Rms radii of proton and neutron orbitals in $^{52,53,54}\text{Ca}$:**

Neutrons: large difference between **vp** and **vf** orbitals

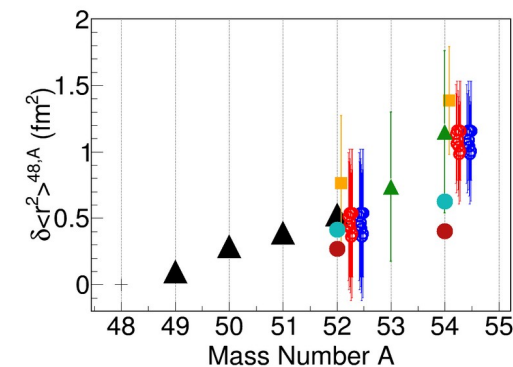
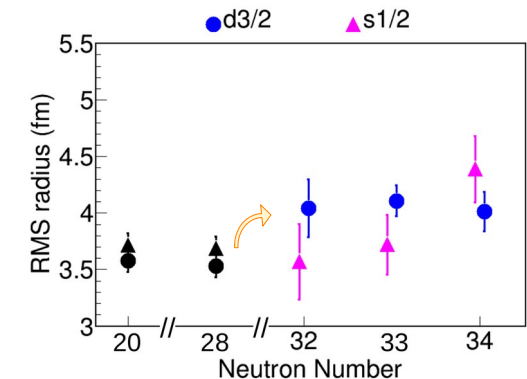
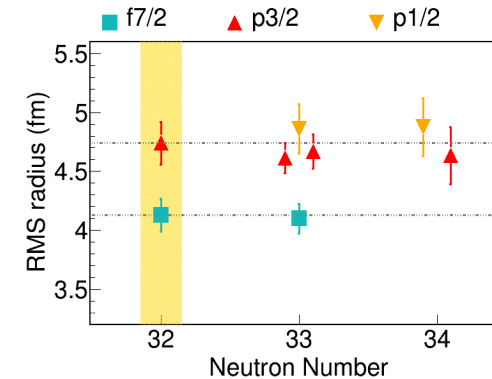
Protons: an **increase in πsd** orbitals compared to $^{40,48}\text{Ca}$

- **From single-particle orbital rms radii to charge radii:**

agreement with the prediction of Bonnard for vp-vf

the $\delta\langle r^2 \rangle_{48,52}$ as well as $R_{\text{ch}}^{52\text{Ca}}$ was reproduced

evaluations for ^{53}Ca and ^{54}Ca charge radii



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