Systematic approach to calculations in EFT

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Summary

Introduction

Since $\sim$ '1970, Standard Model (SM) works very well up to energy scales of few TeV (LHC).

Theory of Everything? Probably not:

— dark matter and dark energy?

— origin of matter-antimatter asymmetry?

— the stability of the Higgs boson mass?

— number of fermion generations - why 3 (light) generations?

— hierarchy of fermion masses, $m_\nu / m_{top} \sim \mathcal{O}\left(10^{-13}\right)$ and flavor structure of SM interactions? ...

How to search for “Beyond the SM” physics?

Two main paths, in the past leading to alternating discoveries:

First approach – “direct” searches. Experiments look for new previously **unknown** particles (usually manifesting themselves as new resonances in cross section spectra) and their interactions.

- discovery = direct indisputable proof of “New Physics” (NP) existence (and usually Nobel prize in due time)
- basic characteristics of new object can be eventually measured - mass, width, charge, parity and eventually other quantum numbers

But: to produce real new particle, we need collider with sufficient energy (and luminosity) - turns out to be almost impossibly difficult these days, every new machine requires tens of years of development and tens of billions of USD!

Second approach – “indirect” searches. Experiments focus on increasingly precise measurements of interactions of **particles**.

- comparison with theoretical predictions of the SM may reveal disagreements, hinting to corrections from exchanges of virtual (off-shell) NP particles
- experiments need more precision rather than more energy, easier to achieve

But: interpretation of results is not unique - observed anomalies usually can be explained by many different BSM models.

Both approaches are complementary - in the past often “indirect” searches helped to focus “direct” experiments on most promising processes and lead to new particle discoveries.

Examples: c and t quark masses and Higgs boson mass was well estimated long before actual discoveries.

Current stage of High Energy Physics - “direct” searches stagnated due to enormous challenges of new collider construction.

We are in the era of

- non-collider experiments - astronomical observations, gravitational wave detectors, DM detectors, …
- precision measurements of SM particles interaction - precision collider physics (HL LHC, FCC project), neutrino experiments, meson factories, …

Two major problems

- deviations from SM predictions annoyingly hard to discover in experiments
- when such anomalies are found, how to classify them against huge variety of theoretical BSM models?

This talk: systematic approach to the last question using “Effective Field Theory” (EFT) formalism.

EFT description of SM extensions

Lets assume that SM is embedded in some larger theory ("Ultraviolet Completion" or UV model), with typical mass scales $\Lambda \gg v = 246$ GeV.

Using the Appelquist-Carazzone theorem, at the energy scales $E \ll \Lambda$, effects of UV theory can be parametrized as new interaction terms of SM fields

$$L_{EFT} = L_{SM}^{(4)} + \frac{1}{\Lambda} L^{(5)} + \frac{1}{\Lambda^2} L^{(6)} + \dots$$

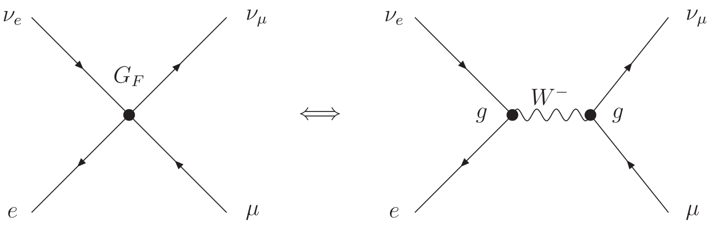
Upper index — "the mass dimension" of terms in the Lagrangian:

$\dim[\phi] = m$	scalar fields
$\dim[V] = m$	vector fields
$\dim[\psi] = m^{3/2}$	spin $\frac{1}{2}$ fermion fields

Example: $\dim[\bar{\psi}\gamma_\mu\psi\ \phi\partial^\mu\phi] = m^6$ ("dimension-6 term").

SM Lagrangian has dimension-4 (required by renormalizability).

EFT description well-known and successful since Fermi theory:



Before SM: weak interactions described as contact 4-fermion interactions of dimension-6.
After SM construction: 2 separate vertices of dimension-4

$$2 \times \frac{-ig}{\sqrt{2}}\bar{\nu}^I\gamma_\mu P_L l^J W^{+\mu} \quad \longrightarrow \quad 2i\sqrt{2}G_F(\bar{\nu}^I\gamma_\mu P_L l^J)(\bar{\nu}^K\gamma_\mu P_L l^L)$$

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2}$$

M_W plays the role of scale Λ for interactions at energies $\mathcal{O}(1)$ GeV.

Systematic setup for EFT calculations

Recent years (especially after 2010): thousands of EFT analyses published. Systematic approach needed to develop calculational techniques, especially at loop level.

Required steps:

- choose operator basis – usually defined in terms of fields before Spontaneous Symmetry Breaking
- perform the SSB and Higgs mechanism; find physical (mass basis) fields and their interactions for easier direct comparison with experiments
- define “input parameters”
- quantise the theory - select gauge fixing
- derive the Feynman rules
- for loop calculations: choose renormalization scheme
- calculate amplitudes and observables
- automate calculations whenever possible!

SMEFT Lagrangian and operator basis

What is “the most general possible Lagrangian”?

Denoting by C_i the dimensionless “Wilson coefficients” (WCs) one has:

$$L_{EFT} = L_{SM}^{(4)} + \frac{1}{\Lambda} C_{\nu\nu}^{(5)} O_{\nu\nu}^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} O_i^{(6)} + \dots$$

Example: Fermi theory - SM $Wl\nu$ interactions at low energy are described as effective contact 4-fermion $ll\nu\nu$ vertex (dimension-6 operator). Other SM interactions can produce other structures, e.g. photon-fermion magnetic coupling $\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$

Matching SM→Fermi model can generate any gauge invariant combination of (low energy) fields $\Longrightarrow$ so-called “Green basis”.

“Green basis” of dimension N - set of ALL possible gauge invariant operators built of model fields and their derivatives up to total mass dimension N.

Green basis is not linearly independent – but often useful in the intermediate step of various calculations (matching to UV models, derivations of RGE equations etc.).

Basis reduction to irreducible set of operators via

- integration by parts
- fermion identities (Fierz, Gordon etc.)
- decomposing field products into irreducible representations
- Equations of Motions (EOMs) or alternatively field redefinitions

Some very subtle problems here: proof that only covariant derivatives should be used for basis, effects of fermion identities in $d \neq 4$ dimensions, EOM reduction for beyond the mass shell amplitudes etc.

EFT base construction became a small industry - many sophisticated theoretical techniques developed for operator classification.

Explicit operators lists and software codes generating operator bases up to dimension-12 at least published – nice playground, but hardly useful for phenomenology beyond dimension-8?

Lowest order SM EFT expansion - operators up to dimension 6.

First attempt of full dim-6 basis construction : Buchmüller, Wyler, NPB 268 (1986) 621. Not fully irreducible, their basis contains 80 operators.

First complete irreducible basis (nicknamed **"Warsaw basis"**): Grządkowski, Iskrzyński, Misiak, JR 1008.4884. Contains 59 operators plus 4 additional ones violating baryon number.

Both papers became very famous, both have over 2000 citations in SPIRES.

Construction of "Warsaw basis" in 2010 revived and streamlined the whole field of EFT analyses - gradually became accepted as an universal parametrization and allowed for immediate comparison between analyses of various authors.

Structure of Warsaw basis

Warsaw basis is constructed in terms of fields in unbroken SM phase, before SSB - gauge invariance can be fully exploited.

Some alternatives considered in literature: “HEFT” (or Higgs EFT) is SM EFT limit in broken phase (more operators allowed in basis as only $U(1)_{em} \times SU(3)_c$ symmetry survives SSB).

Notation: φ — Higgs doublet, B, W, G — $U(1), SU(2), SU(3)$ gauge bosons, $q_{LI}, u_{RI}, d_{RI}, l_{LI}, e_{RI}$ - left and right fermion fields (I is the generation index).

dimension-5: only 1 operator allowed (L-violating, Weinberg 1979, leads to non-vanishing neutrino masses):

$$O_{\nu\nu}^{(5)IJ} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m l_{LI}^k \, \mathbf{C} \, l_{LJ}^n$$

Dimension-6: 10 bosonic and 24 2-fermion operator classes:

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Dimension-6: (29+4) 4-fermion operator classes:

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$		

Fermionic operators Q_X and Wilson coefficients C_X carry flavor indices $\Longrightarrow$ 3×3 or $3\times 3\times 3\times 3$ matrices in flavor space.

“Warsaw basis” has (a bit accidentally, not fully realised when constructed in 2010) some unique advantages:

- Warsaw basis is “flavour-covariant”. Rotations in flavor space may be necessary to diagonalize mass matrices - they change the values of WCs but does not change operator structure of the basis. Example:

$$
\begin{aligned}
e_R^I &\rightarrow (V_R^e)_{IJ} e_R^J \\
C_{\varphi e}^{pr} (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r) &\rightarrow C_{\varphi e}^{\prime\, pr} (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r) \\
C_{\varphi e}^{\prime} &= V_R^{e\dagger} C_{\varphi e} V_R^e
\end{aligned}
$$

Some other bases used earlier e.g. for processes with Higgs boson had more bosonic operators - at the cost of removing by EOMs one flavor only of fermionic operators, like e.g. setting $C_{\varphi e}^{11} = 0$ - not invariant under rotations in flavor space.

- Warsaw basis has minimal possible number of derivatives.

Example: $\phi^\dagger \partial^4 \phi$ is also a dimension-6 term - it produces p^4 term in denominator of scalar propagator and many theoretical problems.

Bilinears with higher order derivatives potentially very dangerous!

If treated perturbatively they will appear as 2-point vertices, resummed at tree-level they

- affect the form of the propagators
- can cause problems with S-matrix unitarity (cross sections fast growing with energy)
- cause problems with locality

$$\phi(x)\left(1+\frac{1}{\Lambda}\frac{\partial}{\partial x}+\frac{1}{2!\Lambda^2}\frac{\partial^2}{\partial x^2}+\ldots\right)\phi(x)=\phi(x)\phi\left(x+\frac{1}{\Lambda}\right)$$

Misiak, Suxho, JR, Zglinicki 1812.11513: **after SSB bosonic bilinear operators in any EFT with linearly realised gauge symmetry may contain at most 2 derivatives.**

Assumptions and notation:

- let's assume an arbitrary but *fixed* order N in the $1/\Lambda^k$-expansion, neglecting higher order terms.
- let's allow only fields with spin $S = 0, \frac{1}{2}, 1$.
- let's use compact notation: put all scalar, fermion and vector fields into 3 big - reducible of course! - gauge multiplets Φ, Ψ and A_μ.
- let's use real representations of gauge group only (eventually doubling the multiplet size by expanding $\phi \to \mathrm{Re}\phi, \mathrm{Im}\phi$.

Before SSB bosonic operators are constructed by adding to dim-4 Lagrangian all gauge invariant combinations of basic "objects": scalar fields Φ, field strength tensors $F_{\mu\nu}$ and covariant derivatives D_μ.

Symbolically higher order operators with n scalar fields, m (dual) field strength tensors, and k covariant derivatives are denoted by (after SSB scalar multiplet is shifted by VEV: $\varphi = \Phi - v$)

$$\Phi^n F^m D^k$$

How can higher derivatives be removed? Use EOMs:

$$D^\mu D_\mu \Phi = \boxed{HL} \qquad\qquad D^\mu F_{\mu\nu} \;=\; \boxed{HL}$$

$\boxed{HL}$ - terms higher in $1/\Lambda$ expansion or with lower number of covariant derivatives. SMEFT example:

$$(D^\mu D_\mu \phi)^j = \underbrace{m^2\phi^j - \lambda(\phi^+\phi)\phi^j - \bar{e}\, Y_e\, l^j - \bar{d}\, Y_d\, q^j + \varepsilon_{jk}\bar{q}^k\, Y_u\, u}_{L} + \underbrace{\mathcal{O}\left(\frac{C}{\Lambda^2}\right)}_{H}$$

Bilinears reduction:

- make list of all possible bilinears to order $1/\Lambda^N$
- start from operators of lowest dimension and highest number of derivatives
- use EOMs, Bianchi identity, commutations, integration by part to reduce unwanted bilinears to others of higher dimension or lower derivatives
- repeat, truncate terms shifted beyond maximal assumed order $1/\Lambda^N$

Conclusion (1812.11513): up to $1/\Lambda^{N+1}$, only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ operators remain for the scalar and gauge boson bilinear terms.

$$L \supset \frac{1}{2}(D_\mu \Phi)_i K^{ij}(\Phi)(D^\mu \Phi)_j - \frac{1}{4} F^a_{\mu\nu} J^{ab}(\Phi) F^{b\,\mu\nu} - V(\Phi)$$

- maximum 2 derivatives in bosonic sector bilinears
- EFT can be formulated as gauge theory with "field dependent" kinetic term metrics $K^{ij}(\Phi)$, $J^{ab}(\Phi)$
- bosonic bilinears reduction crucial for EFT quantization and gauge fixing.

Further classification of higher order operators in SMEFT:

- dim-6 level: fermionic operators Q_X and Wilson coefficients C_X carry flavor indices → 3×3 or $3 \times 3 \times 3 \times 3$ matrices in flavor space – 2499 free real parameters in dim-6 SMEFT
- 1410.4193: dim-7 operators are always B or L violating and not numerous (20 structures).
- 2005.00008, 2005.00059: 993 classes of dim-8 operators, not counting flavor structure ...

To compare with experiment, one should calculate observables in physical field basis → mass eigenstates basis after the SSB.

Physical fields basis in SMEFT

Field rescaling required to get canonically normalised kinetic terms (Λ scale included in coupling definitions $\frac{C_X}{\Lambda^2} \to C_X$).

Example – Higgs doublet decomposition:

$$\varphi = \begin{pmatrix} Z_{G^+}^{-1} G^+ \\ \frac{1}{\sqrt{2}}(v + Z_h^{-1} h + i Z_{G^0}^{-1} G^0) \end{pmatrix}$$

Goldstone fields $G^0, G^\pm$ and physical Higgs field h become canonically normalised if:

$$\begin{aligned} Z_h &\equiv 1 + \frac{1}{4} C^{\varphi D} v^2 - C^{\varphi \Box} v^2 \\ Z_{G^0} &\equiv 1 + \frac{1}{4} C^{\varphi D} v^2 \\ Z_{G^+} &\equiv 1 \end{aligned}$$

Similar corrections to gauge boson mixing, Weinberg angle and fermion masses and mixings.

SM couplings shifted by corrections from higher order WCs:

$$\begin{aligned}
\bar{v} &= \sqrt{\frac{2m^2}{\lambda} + \frac{3m^3}{\sqrt{2}\lambda^{5/2}} C^{\varphi}} \\
\bar{g} &= (1 + C^{\varphi W} v^2) g \\
\bar{g}' &= (1 + C^{\varphi B} v^2) g' \\
\bar{M}_Z &= \frac{1}{2}\sqrt{\bar{g}^2 + \bar{g}'^2}\, v \left(1 + \frac{\bar{g}\bar{g}' v^2 C^{\varphi WB}}{\bar{g}^2 + \bar{g}'^2} + \frac{1}{4} C^{\varphi D} v^2\right) \\
\bar{M}_h^2 &= \lambda v^2 - (3\, C^{\varphi} - 2\,\lambda\, C^{\varphi\Box} + \frac{\lambda}{2}\, C^{\varphi D}) v^4 \\
&\dots
\end{aligned}$$

General corrections to the SM interactions can be split into 2 classes:

- modifications of dimension-4 vertices from shifts in the fields and parameters ("oblique" corrections)
- genuine new vertices from higher order operators $\delta L_{SM}^{(6)}$

$$L_{SM}(g, g', M_z, \dots, \Phi, \dots) = L_{SM}(\bar{g}, \bar{g}', \bar{M}_z, \dots, h, \dots) + \delta L_{SM}^{(6)}$$

“Input scheme” dependence

New problem – determination of SM parameters is affected by presence of new operators!

Consistent calculation to a given maximal EFT order highly non-trivial.

Usual SMEFT parametrization - set of SM parameters + WCs:

- gauge and Higgs couplings g, g', g_s, λ, v (eventually rescaled to absorb oblique corrections)
- fermion masses (quark, lepton and neutrino) masses
- CKM and PMNS matrix elements
- Wilson coefficients of higher order operators in a chosen basis

Standard approach to calculations within SMEFT - find first an expression for an observable in terms of “standard” SM parameters, expanding it to an assumed $1/\Lambda$ order:

$$\mathcal{O} = \mathcal{O}(g, g', \ldots, \mathrm{WCs})$$

But SM parameters cannot be taken from PDG tables - they depend on WCs, too!

SM parameters need to be related to some measured quantities $\mathcal{O}_1, \mathcal{O}_2, \ldots$ (nicknamed "choice of input scheme")

$$\begin{aligned} g &= g(\mathcal{O}_1, \mathcal{O}_2, \ldots) \\ g' &= g'(\mathcal{O}_1, \mathcal{O}_2, \ldots) \\ & \ldots \end{aligned}$$

re-inserted to calculated observable

$$\mathcal{O} = \mathcal{O}(g(\mathcal{O}_1, \mathcal{O}_2, \ldots), g'(\mathcal{O}_1, \mathcal{O}_2, \ldots), \ldots, \mathrm{WCs})$$

the result must be again re-expanded in $1/\Lambda$ powers and truncated to an assumed $1/\Lambda$ order.

Consistent and error-free expansion is crucial - results of calculations in SMEFT are gauge-invariant only when all terms up to the assumed $1/\Lambda$ order are included.

Procedure straightforward but technically complicated and error-prone (especially for dim-6^2 terms) $\Longrightarrow$ use of software packages becomes unavoidable, especially at higher EFT orders and/or loop calculations.

Freedom of reasonable “input scheme” choice actually quite limited.

- electroweak sector - usually input parameters are
  - “GF scheme”: M_Z, M_W, M_h, G_F (G_F from muon lifetime)
  - “AEM scheme”: $M_Z, M_W, M_h, \alpha_{em}$
- quark sector – only one well defined proposition 1812.08163: quark masses + 4 CKM parameters derived from $Br(B \to \tau\nu_\tau), \Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu), \Delta m_{B_d}, \Delta m_{B_s}$
- lepton sector: physical masses and PMNS elements, the latter not measured yet accurate enough to worry about corrections from WCs.

Including physical masses as input parameters hardly avoidable - otherwise dependence on WCs appears in denominators of particle propagators, disaster for efficient calculations.

In some cases not clear what to choose: reported value of QCD coupling α_S is an average from many measurements. SMEFT corrections to each measurement are different - it affects averaging procedure! No consistent reanalysis in the literature yet.

Quantization of SMEFT

Lets assume that the initial steps are done:

- irreducible operator basis known to a required order (in an unbroken gauge phase)
- physical fields identified
- set of input physical observables selected

Can we already calculate amplitudes and observables in SMEFT?

At tree-level, may be, assuming without proof that equivalent of SM “unitary gauge” exists.

SMEFT is a gauge theory - for quantization (either in operator or path integral method) gauge fixing is required!

Again highly non-trivial in the presence of new operators.

Gauge fixing procedures in the SM long established:

- tree level calculations - unitary gauge, no Goldstone fields
- QCD - usually Feynman gauge, sometimes more exotic choices for special kinds of calculations
- Electroweak loop calculations - linear R_ξ gauges (often just simplest Feynman gauge with $\xi = 1$). Goldstone and ghost fields explicitly present

Electroweak R_ξ gauge fixing terms in the SM:

$$\begin{aligned}\mathcal{L}_{GF} = & - \frac{1}{\xi_W}(\partial^\mu W_\mu^+ + i\xi_W M_W G^+)(\partial^\nu W_\nu^- - i\xi_W M_W G^-) \\ & - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu + \xi_Z M_Z G^0)^2 - \frac{1}{2\xi_A}(\partial^\mu A_\mu)^2\end{aligned}$$

Chosen to cancel "unwanted terms" in the SM Lagrangian

$$\mathcal{L}_{SM} \supset - iM_W(W_\mu^+ \partial^\mu G^- - W_\mu^- \partial^\mu G^+) - M_Z Z_\mu \partial^\mu G^0$$

In SMEFT unwanted terms are modified $\Longrightarrow$ also gauge fixing terms must include corrections from dim-6 operators!

Long standing problem in the literature: first proposition of linear R_ξ gauge fixing for dim-6 SMEFT: Dedes, Materkowska, Paraskevas, JR, Suxho 1704.03888.

Based on important observation in dim-6 SMEFT: in terms of physical (canonical) fields and physical masses, the gauge-Goldstone Lagrangian has the form identical as in the "pure" SM:

$$\begin{aligned}\mathcal{L}_{G-EW} = & - \bar{M}_W^2 \bar{W}_\mu^+ \bar{W}^{-\mu} + i\bar{M}_W (\bar{W}_\mu^+ \partial^\mu \bar{G}^- - \bar{W}_\mu^- \partial^\mu \bar{G}^+) \\ & - \frac{1}{2}\bar{M}_Z^2 \bar{Z}_\mu \bar{Z}^\mu - \bar{M}_Z \bar{Z}_\mu \partial^\mu \bar{G}^0\end{aligned}$$

At first sight, looks miraculous:

- correction to M_W, M_Z and gauge/Goldstone field normalisation come from different operators.
- why coefficients of "unwanted" terms are exactly equal to effective gauge boson masses?

Miracles rarely happen: perhaps consequence of gauge invariance?

Valid in SMEFT only? How about HEFT, 2HDM-EFT, other EFT-s?

General answer again in Misiak, Suxho, JR, Zglinicki 1812.11513:, follows from the reduction of bosonic operators producing (after SSB) bilinears to maximum 2 derivatives:

$$L \supset \frac{1}{2}(D_\mu \Phi)_i K^{ij}(\Phi)(D^\mu \Phi)_j - \frac{1}{4} F^a_{\mu\nu} J^{ab}(\Phi) F^{b\,\mu\nu} - V(\Phi)$$

After SSB vector and scalar kinetic terms are modified only by two constant matrices $K(v)$ and $J(v)$, where $v = \langle \Phi \rangle$.

Denoting $A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ and putting all fields into real representations one has:

$$L_{bil} = -\frac{1}{4} A^T_{\mu\nu}\, J(v)\, A^{\mu\nu} + \frac{1}{2}(D_\mu \Phi)^T\, K(v)\,(D^\mu \Phi)$$

Valid to any EFT order, with WC corrections hidden in J and K matrices.

After SSB, "unwanted" terms are (T^a are gauge group generators):

$$L_{A\varphi} = -i\left(\partial^\mu A_\mu^a\right)\left[\varphi^T K T^a v\right]$$

Fields $v^T T^a K\varphi$ define massless "would be Goldstone" bosons (up to canonicalization procedure), including EFT corrections.

Physical scalars are orthogonal to $T^a v$ vectors with matrix K defining the scalar product.

R_ξ gauge fixing conditions can be now generalised to work in any EFT!

Let's define:

$$L_{GF} = -\frac{1}{2\xi}\mathcal{G}^a J^{ab}\mathcal{G}^b$$

with

$$\mathcal{G}^a = \partial^\mu A_\mu^a - i\xi(J^{-1})^{ac}\left[\varphi^T K T^c v\right]$$

"Unwanted" terms cancel out. Rescaling

$$\tilde{\varphi}_i = (K^{\frac{1}{2}})_{ij}\varphi_j, \qquad \tilde{A}^a_\mu = (J^{\frac{1}{2}})^{ab} A^b_\mu$$

one gets canonical form of kinetic terms

$$\begin{aligned} L_{bil} + L_{GF} &= -\frac{1}{4}\tilde{A}^T_{\mu\nu}\tilde{A}^{\mu\nu} + \frac{1}{2}\tilde{A}^T_\mu (M^T M)\tilde{A}^\mu + \frac{1}{2}(\partial_\mu \tilde{\varphi})^T(\partial^\mu \tilde{\varphi}) \\ &\quad -\frac{\xi}{2}\tilde{\varphi}^T(MM^T)\tilde{\varphi} - \frac{1}{2\xi}(\partial^\mu \tilde{A}_\mu)^T(\partial^\nu \tilde{A}_\nu) \end{aligned}$$

with

$$M_j{}^b \equiv [K^{\frac{1}{2}}(iT^a)v]_j\,(J^{-\frac{1}{2}})^{ab}$$

Note: no physical scalar masses included here, they are defined by other terms in the Lagrangian, bilinear part of $V(\Phi)$.

Diagonalization via Singular Value Decomposition (SVD):

$$M = U^T \Sigma V$$

U, V are unitary, Σ is rectangular-diagonal, $\Sigma_j{}^b = 0$ when $j \neq b$.

Physical gauge bosons and canonical Goldstone fields given by redefinition $\phi_i = U_{ij}\tilde{\varphi}_j$, $W^a_\mu = V^{ab}\tilde{A}^b_\mu$:

$$\begin{aligned} L_{\text{kin,mass}} &= -\frac{1}{4}W^T_{\mu\nu}W^{\mu\nu} + \frac{1}{2}W^T_\mu m^2_W W^\mu + \frac{1}{2}(\partial_\mu \phi)^T(\partial^\mu \phi) \\ & -\frac{1}{2\xi}(\partial^\mu W_\mu)^T(\partial^\nu W_\nu) - \frac{\xi}{2}\phi^T m^2_\phi \phi \end{aligned}$$

Gauge boson and Goldstone masses identical up to multiplication by ξ parameter, $p \times p$ diagonal sub-block D_p is common:

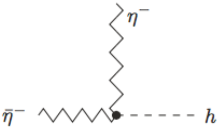
$$m^2_\phi = \Sigma\Sigma^T = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{m\times m} \qquad m^2_W = \Sigma^T\Sigma = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{n\times n}$$

SMEFT miracle explained!

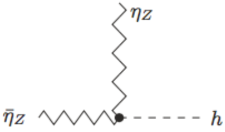
Final step of quantization: adding ghosts interactions and proving BRST invariance of the full Lagrangian - for details see 1812.11513.

Conclusion: standard R_ξ-like gauge fixing is possible in any EFT with fields up to spin 1 and linearly realised gauge symmetry!

Ghost propagators identical as in SM → corrections from dimension-6 operators appear in ghost vertices.



$$+\frac{1}{4}i\bar{g}^2 v\xi_W + \frac{1}{4}i\bar{g}^2 v^3\xi_W C^{\varphi\Box} - \frac{1}{16}i\bar{g}^2 v^3 \xi_W C^{\varphi D}$$



$$+\frac{1}{4}iv\xi_Z\left(\bar{g}^2+\bar{g}'^2\right) + \frac{1}{4}iv^3\xi_Z\left(\bar{g}^2+\bar{g}'^2\right)C^{\varphi\Box} + \frac{1}{16}iv^3\xi_Z\left(\bar{g}^2+\bar{g}'^2\right)C^{\varphi D} + \frac{1}{2}i\bar{g}\bar{g}'v^3\xi_Z C^{\varphi WB}$$

Last step before diagram calculations: derive the Feynman rules.

General EFT theory elegant, but practice in SMEFT less rosy. Already at the minimal dim-6 level:

- ~ 80 primary vertices in unitary gauge, including ghosts and Goldstone bosons ~ 400 vertices in R_ξ gauges
- up to 6-tuple field interactions
- lengthy expressions in general case, especially for non-minimal flavor structure – some primary vertices like 6-gluon interactions are pages long.

For analytical calculations: **full ready-to-use list of Feynman rules in the physical field basis at dim-6 level in 1704.03888**

For most realistic SMEFT loop calculations using symbolic software packages is basically unavoidable $\rightarrow$ discussed later in this talk.

Example of loop calculations

$h \to \gamma\gamma$ **decay in SMEFT** (Dedes, Paraskevas, JR, Suxho, Trifyllis 1805.00302)

SM prediction: finite and precise, small theoretical uncertainties.

Experiments report results relative to SM prediction:

$$\mathcal{R}_{h\to\gamma\gamma} \equiv 1 + \delta\mathcal{R}_{h\to\gamma\gamma} = \frac{\Gamma(\mathrm{EXP}, h \to \gamma\gamma)}{\Gamma(\mathrm{SM}, h \to \gamma\gamma)} .$$

2020 constraints (to follow original paper):

$$\begin{aligned} \text{ATLAS:} \qquad \mathcal{R}_{h\to\gamma\gamma} &= 0.99^{+0.15}_{-0.14} , \\ \text{CMS:} \qquad \mathcal{R}_{h\to\gamma\gamma} &= 1.18^{+0.17}_{-0.14} , \end{aligned}$$

Radiative process, small in SM - sensitive to contributions from new operators.

Earlier calculation by Hartmann and Trott, 1505.02646, 1507.03568: done in complicated Background Field gauge fixing, some typos and overall 1/4 factor missed.

Technically quite complicated (as for 1-loop calculation!):

- 17 CP conserving dim-6 operators contribute, not counting flavor and H.c. (neglected: 10 CP violating operators, strongly suppressed by CP observables like EDM etc.)
- complicated structure of interaction vertices, 3-, 4- and 5-tuple, some momentum dependent, many include scalar and tensor Dirac structures.
- non-trivial multi-parameter renormalization procedure.
- calculation performed in general R_ξ gauges with independent ξ_W, ξ_Z parameters - ξ cancellation provides very strict cross checks of the results and gives analytic proof of gauge invariance.

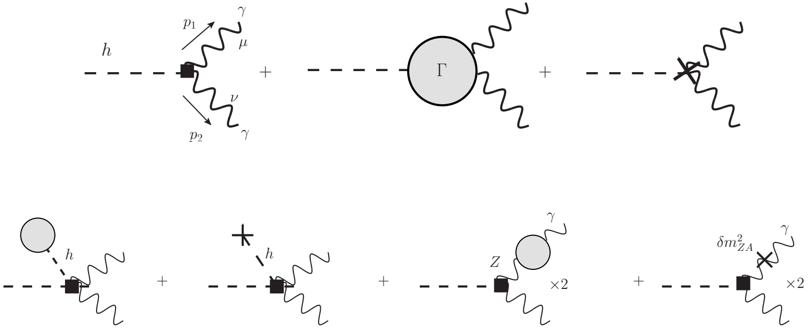
Contributing CP-conserving operators:

$Q_W = \varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{e\varphi} = (\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\varphi\Box} = (\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi} = (\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
$Q_{\varphi D} = \left(\varphi^\dagger D^\mu \varphi\right)^* \left(\varphi^\dagger D_\mu \varphi\right)$	$Q_{d\varphi} = (\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\varphi B} = \varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	$Q_{ll} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$Q_{\varphi W} = \varphi^\dagger \varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	$Q_{\varphi l}^{(3)} = (\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi WB} = \varphi^\dagger \tau^I \varphi\, W_{\mu\nu}^I B^{\mu\nu}$	
$Q_{eB} = (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{eW} = (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$
$Q_{uB} = (\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{uW} = (\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{\varphi}\, W_{\mu\nu}^I$
$Q_{dB} = (\bar{q}_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{dW} = (\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$

Q_φ - present in vertices but cancels out completely in the amplitude.

Q_{ll} and $Q_{\varphi l}^{(3)}$ enter indirectly through corrections to Fermi constant (input scheme dependence).

On-shell S-matrix element:



In addition external wave function renormalisation for the photon and the Higgs required by the LSZ reduction formula.

Renormalization and input parameters

Renormalization performed at 1-loop and up to $1/\Lambda^2$ in EFT expansion.

- integrals regularised with DR-scheme
- hybrid renormalization scheme: on-shell for SM-quantities (a la Sirlin 1980) and $\overline{MS}$ for Wilson coefficients
- result: a closed expression for the amplitude that respects the Ward identities

The renormalized SM parameters are translated to well measured ones (so-called "GF" input scheme):

$$\{g', g, v, \lambda, y_t\} \longrightarrow \{M_Z, M_W, M_h, m_t, G_F\}$$

Relation of g and Fermi constant G_F derived from the muon lifetime at the tree level:

$$\frac{G_F}{\sqrt{2}} = \frac{\bar{g}^2}{8\sqrt{2}M_W^2}\left[1 + v^2(C_{11}^{\varphi l(3)} + C_{22}^{\varphi l(3)}) - v^2 C_{1221}^{ll}\right]$$

Wilson coefficients renormalized in MS-bar scheme $\Longrightarrow$ they become renormalization scale dependent.

$$C - \delta C = \bar{C}(\mu) - \delta\bar{C}$$

Full renormalization scheme - complicated (as the model itself) but fairly standard procedure, in spite of working with non-renormalizable theory.

Nothing special w.r.t. textbook renormalization techniques !!

We checked that process amplitude $\mathcal{A}_{h\to\gamma\gamma}$ is:

- finite
- gauge invariant (ξ parameter independent)
- renormalisation scale invariant, in the sense $\frac{d}{d\mu}\mathcal{A}_{h\to\gamma\gamma}(\mu) = 0$ (proven using RGE for WCs by Alonso, Jenkins, Manohar, Trott 1308.2627, 1310.4838, 1312.2014).

Renormalized amplitude:

$$i\mathcal{A}^{\mu\nu}(h \to \gamma\gamma) = \langle \gamma(\epsilon^{\mu}, p_1),\, \gamma(\epsilon^{\nu}, p_2)\,|\,S\,|\,h(q)\,\rangle = 4i\,\left[\,p_1^{\nu}\,p_2^{\mu} - (p_1 \cdot p_2)\,g^{\mu\nu}\,\right]\,\mathcal{A}_{h\to\gamma\gamma}$$

where

$$\begin{aligned}
\mathcal{A}_{h\to\gamma\gamma} \;=\; & \Bigg\{ c^2\,v\,\bar{C}^{\varphi B}(\mu)\left[1 + \Gamma^{\varphi B} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) + 2\tan\theta_W\,\frac{\Pi_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right] \\
+ \;& s^2\,v\,\bar{C}^{\varphi W}(\mu)\left[1 + \Gamma^{\varphi W} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan\theta_W}\,\frac{\Pi_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right] \\
- \;& s\,c\,v\,\bar{C}^{\varphi WB}(\mu)\left[1 + \Gamma^{\varphi WB} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan 2\theta_W}\,\frac{\Pi_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right] \\
+ \;& \frac{1}{M_W}\overline{\Gamma}^{\mathrm{SM}} + \sum_{X \neq \varphi B, \varphi W, \varphi WB} v\,C^{X}(\mu)\,\Gamma^{X}\Bigg\}_{\text{finite}}
\end{aligned}$$

Semi-analytic formula

$$
\begin{aligned}
\delta\mathcal{R}_{h\to\gamma\gamma} \quad \simeq \quad & 0.06\left(\frac{C^{\ell\ell}_{1221}-C^{\varphi\ell(3)}_{11}-C^{\varphi\ell(3)}_{22}}{\Lambda^2}\right)+0.12\left(\frac{C^{\varphi\Box}-\frac{1}{4}C^{\varphi D}}{\Lambda^2}\right) \\
- \quad & 0.01\left(\frac{C^{e\varphi}_{22}+4C^{e\varphi}_{33}+5C^{u\varphi}_{22}+2C^{d\varphi}_{33}-3C^{u\varphi}_{33}}{\Lambda^2}\right) \\
- \quad & \left[48.04-1.07\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi B}}{\Lambda^2}-\left[14.29-0.12\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi W}}{\Lambda^2} \\
+ \quad & \left[26.62-0.52\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi WB}}{\Lambda^2}+\left[0.16-0.22\log\frac{\mu^2}{M_W^2}\right]\frac{C^{W}}{\Lambda^2} \\
+ \quad & \left[2.11-0.84\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uB}_{33}}{\Lambda^2}+\left[1.13-0.45\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uW}_{33}}{\Lambda^2} \\
- \quad & \left[0.03+0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uB}_{22}}{\Lambda^2}-\left[0.01+0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uW}_{22}}{\Lambda^2} \\
+ \quad & \left[0.03+0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{dB}_{33}}{\Lambda^2}-\left[0.02+0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{dW}_{33}}{\Lambda^2} \\
+ \quad & \left[0.02+0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{eB}_{33}}{\Lambda^2}-\left[0.01+0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{eW}_{33}}{\Lambda^2}+\ldots,
\end{aligned}
$$

Only 5 operators contribute significantly and can be bounded by the LHC experimental measurement. Taking $\mu = M_W$, one has

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.003}{(1\ \text{TeV})^2}\,, \qquad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.011}{(1\ \text{TeV})^2}\,,$$

$$\frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.006}{(1\ \text{TeV})^2}\,,$$

$$\frac{|C^{uB}_{33}|}{\Lambda^2} \lesssim \frac{0.071}{(1\ \text{TeV})^2}\,, \qquad \frac{|C^{uW}_{33}|}{\Lambda^2} \lesssim \frac{0.133}{(1\ \text{TeV})^2}\,.$$

Competing constraints on $C^{\varphi B}, C^{\varphi W}, C^{\varphi WB}$ from EW precision measurements - similar order of magnitude.

Constraints on C^{33}_{uB} and c^{33}_{uW} the $\bar{t}tZ$ and single top production measurements at LHC: *more than an order of magnitude weaker*.

Dedes, Suxho, Trifyllis 1903.12046: even more complicated calculation of $\mathrm{Br}(h \to Z\gamma)$.

Analogous expression for $\delta\mathcal{R}_{h\to Z\gamma}$:

$$\begin{aligned}\delta\mathcal{R}_{h\to Z\gamma} &\simeq \left[14.99 - 0.35\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi B}}{\Lambda^2} - \left[14.88 - 0.15\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi W}}{\Lambda^2} \\ &+ \left[9.44 - 0.26\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi WB}}{\Lambda^2} + (\textit{smaller terms})\end{aligned}$$

In $\delta R_{h\to Z\gamma}$ coefficients of all C_i smaller than in $\delta R_{h\to\gamma\gamma}$ – barring fine-tuning, unlikely to observe $h \to Z\gamma$ decay but no $h \to \gamma\gamma$ decay.

$\delta R_{h\to Z\gamma}$ constrains different combination of WCs than $\delta R_{h\to\gamma\gamma}$ – combining many such analyses one can perform multi-dimensional numerical fits to SMEFT WC values (or their upper bounds).

Wide literature exists on SMEFT fitting already!

Mini-summary – EFT status 2024

One of the most commonly used techniques for NP searches – thousands of analyses published, both theoretical and experimental. Many calculational tools developed and publicly available.

Increasingly difficult issues attacked, corresponding progress in relevant computational techniques required – SMEFT is a very complicate model!

Current attempts:

- "going beyond the leading order" - either including loop corrections or terms beyond the dimension-6 order in SMEFT expansion
- various studies on intrinsic EFT theoretical structure ("geometric" EFT structure, high energy scattering amplitude behaviour and so on).
- multi-parameter fits of SMEFT WCs to experimental data
- ...

Why need we go beyond the leading EFT order?

Terms of higher order in $1/\Lambda$ could be potentially important, both from theoretical and experimental point of view:

- theoretical issues - positivity problems for SMEFT when limited to dim-6 terms
- 2499 WCs in the dim-6 SMEFT Lagrangian - many of them still weakly constrained, dim-6^2 effects not necessarily small
- scattering amplitudes are growing with collision energy, amplifying higher dimension operator effects.

  Example: unitarity violation of the S matrix in the Vector Boson Scattering. Existing analyses for LHC explicitly show importance of dim-6^2 and dim-8 terms.

Calculations above the leading dim-6 order technically a complicated problem - for consistency both dim-6^2 and dim-8 contributions should be simultaneously included.

At current stage of EFT phenomenology use of software tools unavoidable

Many public codes available: *Flavio, FlavorKit, DSixTools, wilson, SMEFT@NLO, SMEFTSim, MatchingTools, Dim6Top . . .*

Various goals and purposes:

- Matching UV models to SMEFT (or further to LEFT, "Low energy" EFT)
- RGE evolution of SMEFT coefficients
- general purpose SMEFT diagram calculations and interfaces to MC generators
- numerical fitting of SMEFT parameters
- codes specialised to various sectors of the theory (top physics, flavor physics etc.)

Our project – SmeftFR library (Dedes, Ryczkowski, JR, Suxho, Trifyllis 2302.01353): www.fuw.edu.pl/smeft

SmeftFR Mathematica package

SmeftFR purpose: generate SMEFT interaction vertices (Feynman rules) up to maximal dimension-8 of EFT expansion in formats compatible with computational tools most commonly used in HE physics: FeynArts/FeynCalc, MadGraph and other Monte Carlo generators etc.

Main features:

- based on FeynRules Mathematica engine (1310.1921)
- generates interaction only for the user-defined subset of operators, relevant for chosen process. It speeds up calculations by orders of magnitude!
- interactions are given in physical field basis
- vertices can be calculated in unitary or R_ξ gauges
- for loop calculation generated R_ξ gauge fixing terms and ghost interactions valid to any SMEFT order (1812.11513)
- Feynman rules can be expressed directly in terms of physical observables, avoiding the need of reparametrization and re-expanding of the amplitudes!

Main assumptions:

- Operator basis choice:
  - for dim-5 and -6 operators SmeftFR uses the "Warsaw basis".
  - all dim-7 operators are lepton and/or baryon number violating - they are not included
  - bosonic dim-8 operators are implemented from basis defined in Murphy 2005.00059. No fermionic dim-8 operators are included by default, can be added if necessary.

  Including *all* dim-8 fermionic operators currently does not seem neither plausible (993 operators **before** flavour expansion, CPU time will exceed thermal death of Universe) nor necessary - fermionic parameters are typically known with lower accuracy the quantities in the EW sector, no need (yet?) for higher order fermionic operators.
- SM parameter choice options:
  - "Default" – based on natural SM parametrization: $g, g', g_s, \lambda,$ vev, fermion masses, CKM and PMNS elements.
  - "User-defined" – can be any set of quantities sufficient to uniquely express "default" parameters in terms of them and the WCs. Assumption: user-defined parameters are chosen as experimental observables, independent of values of WCs.

Predefined input schemes included in SmeftFR v3 distribution:

- EW sector, input scheme G_F, M_W, M_Z, M_h: G_F and vev calculated from muon lifetime up to $1/\Lambda^4$
- EW sector, alternative input scheme $\alpha_{em}, M_W, M_Z, M_h$, again related to other SM parameters up to $1/\Lambda^4$
- QCD sector - $\alpha_s(M_Z)$ used as input, no $1/\Lambda$ corrections included yet!
- quark sector - input scheme based on physical quark masses and implementation of 1812.08163, CKM Wolfenstein parametrization related to measurements of $Br(B \to \tau\nu_\tau)$, $\Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu)$, Δm_{B_d}, Δm_{B_s}. Expansion up to $1/\Lambda^2$, more accurate formulae not available in the literature.
- lepton sector - input scheme based on masses and PMNS elements, no $1/\Lambda$ corrections to PMNS included

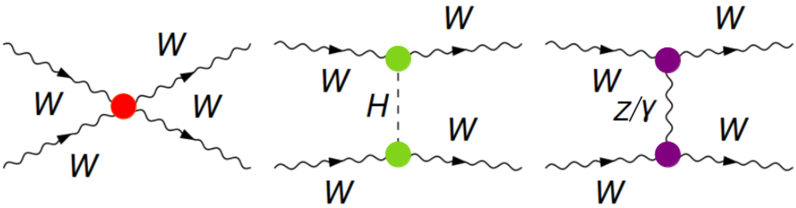
To save CPU time, expansion order can be done to $1/\Lambda^2$ or $1/\Lambda^4$ terms.

SmeftFR by example: Vector Boson Scattering

Vector Boson Scattering (VBS):

(i) important in understanding EW sector

(ii) potential sensitivity to the BSM



diagrams in Unitary gauge

Step #1: choice of SMEFT operators

Chosen SMEFT operators modifying VBS processes (example only, in general more operators contribute):

- dimension-6:

X^3		$\varphi^4 D^2$	
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi\Box}$	$\left(\varphi^\dagger\varphi\right) \Box \left(\varphi^\dagger\varphi\right)$

- dimension-8:

$\varphi^6 D^2$		$\varphi^4 D^4$	
$Q_{\varphi^6\Box}$	$\left(\varphi^\dagger\varphi\right)^2 \Box \left(\varphi^\dagger\varphi\right)$	$Q_{\varphi^4 D^4}^{(1)}$	$\left(D_\mu\varphi^\dagger D_\nu\varphi\right)\left(D^\nu\varphi^\dagger D^\mu\varphi\right)$

Step #2: load packages and operators

Open `SmeftFR-init.nb` notebook provided with SmeftFR distribution, then:

i) set proper installation paths of `FeynRules` and SmeftFR and load both codes
ii) choose set of operators, in our case:

OpList6={"phiBox","W"};

OpList8={"phi6Box","phi4D4n1"};

Operator naming in the code: $Q_{\varphi\Box} \to$ "`phiBox`", $Q^{(1)}_{\varphi^4 D^4} \to$ "`phi4D4n1`", ...

Step #3: choose options and generate of `FeynRules` model files

If necessary modify options in initialisation routine (for all options see manual):

SMEFTInitializeModel[Operators→OpList,
Gauge→"Rxi",
ExpansionOrder→2,
InputScheme→"GF",
MaxParticles→4,
...];

and rerun the full content of notebook `SmeftFR-init.nb`

At this stage all interactions of physical fields are calculated in `FeynRules/Mathematica` internal format.

Step #4: expand & print obtained vertices

In default "smeft" scheme

```
In[7]:= SMEFTExpandVertices[Input→"smeft",ExpOrder→2];
        SelectVertices[GaugeHiggsVerticesExp,SelectParticles →{H,W,Wbar}]
Out[7]= {{{{H,1},{W,2},{W+,3}},
```

$$\frac{1}{2}\ i\ G_W^2\ vev\ \eta_{\mu_2,\mu_3}+\frac{1}{2}\ i\ C^{\phi Box}\ G_W^2\frac{1}{\Lambda^2}vev^3\eta_{\mu_2,\mu_3}+\frac{1}{4}\ i\ (2\ C^{\phi 6Box}+3\ (C^{\phi Box}))^2 G_W^2(\frac{1}{\Lambda^2})^2\ vev^5\ \eta_{\mu_2,\mu_3}\}\}$$

in G_F "user" scheme:

```
In[8]:= SMEFTExpandVertices[Input→"user",ExpOrder→2];
        SelectVertices[GaugeHiggsVerticesExp,SelectParticles →{H,W,Wbar}]
Out[8]= {{{{H,1},{W,2},{W+,3}},
```

$$2\ i\ 2^{1/4}\sqrt{G_F}\ M_W^2\eta_{\mu_2,\mu_3}+\frac{i\ 2^{3/4}\ C^{\phi Box}\ \frac{1}{\Lambda^2}\ M_W^2\eta_{\mu_2,\mu_3}}{\sqrt{G_F}}+\frac{i\ (2\ C^{\phi 6Box}\ +\ 3\ (C^{\phi Box}))^2\ (\frac{1}{\Lambda^2})^2\ M_W^2\eta_{\mu_2,\mu_3}}{2^{3/4}\ G_F{}^{3/2}}\}\}$$

Step #5: interfacing SmeftFR to other formats and codes

Available formats of output:

- *Mathematica/FeynRules* syntax
- WC numerical values may be dumped to file in the *WCxf* format (allows to transfer numerical values of WCs to other compatible SMEFT public packages, *Flavio, FlavorKit, Spheno, DSixTools, wilson, FormFlavor, SMEFTSim, . . .*)
- *Latex* (dedicated Latex generator optimised to make output more readable, draw diagrams etc.)
- *UFO* ("Unified Format Output") format ($\Longrightarrow$ interface to Monte Carlo generators)
- *FeynArts* format

`FeynRules` package offers other types of interface routines and formats – good chance that they work properly, too, but have not been tested by us.

To generate LaTeX, `UFO` and `FeynArts` output one needs to:

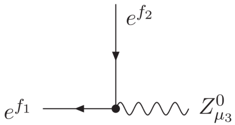
(i) `Quit[]` `Mathematica` kernel, open the `SmeftFR_interfaces.nb` notebook and rerun it (may be time consuming!):

(ii) call the required output routine (eventually with options, see manual):

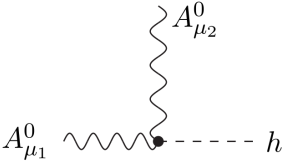
- SMEFTToWCXF[input_file, output_file];
- SMEFTToLatex[ ];
- SMEFTToUFO[SMEFT$MBLagrangian];
- WriteFeynArtsOutput[SMEFT$MBLagrangian];
- ...

For readability, Latex output shows only terms up to dim-6

Vertices in "default" parametrization (Z_X shifts expanded to $1/\Lambda^2$ order):



$$-\frac{i}{2\sqrt{\bar{g}'^2+\bar{g}^2}}\delta_{f_1f_2}\left(\left(\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L+2\bar{g}'^2\gamma^{\mu_3}P_R\right)$$
$$+\frac{i\bar{g}'\bar{g}v^2}{2\left(\bar{g}'^2+\bar{g}^2\right)^{3/2}}\delta_{f_1f_2}C^{\varphi WB}\left(\left(\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L-2\bar{g}^2\gamma^{\mu_3}P_R\right)$$
$$+\frac{1}{2}iv^2\sqrt{\bar{g}'^2+\bar{g}^2}C^{\varphi l1}_{f_1f_2}\gamma^{\mu_3}P_L+\frac{1}{2}iv^2\sqrt{\bar{g}'^2+\bar{g}^2}C^{\varphi l3}_{f_1f_2}\gamma^{\mu_3}P_L$$



$$+\frac{4i\bar{g}^2v}{\bar{g}'^2+\bar{g}^2}C^{\varphi B}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)$$
$$+\frac{4i\bar{g}'^2v}{\bar{g}'^2+\bar{g}^2}C^{\varphi W}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)$$
$$-\frac{4i\bar{g}'\bar{g}v}{\bar{g}'^2+\bar{g}^2}C^{\varphi WB}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)$$

SmeftFR `FeynArts` output

It can be used for further analytical calculations with `FeynArts` or other compatible codes - `FeynCalc`, `FormCalc` etc.

Various cross-checks performed: e.g. calculation of $W_L^+ W_L^+ \to W_L^+ W_L^+$ process based on SmeftFR `FeynArts` output.

$$
\begin{aligned}
\mathcal{M}(W_L^+ W_L^+ \to W_L^+ W_L^+)(s,\theta) \overset{s \gg M_W^2}{=} & -2\sqrt{2} G_F M_H^2 \left[1 - \frac{M_Z^2}{M_H^2}\left(1 - \frac{4}{\sin^2\theta}\right)\right] & \textbf{(SM)} \\
& + (2C_{\varphi\Box} + C_{\varphi D})\,\frac{s}{\Lambda^2} & \textbf{(dim}-\textbf{6)} \\
& + \Big[8C_{\varphi^6\Box} + 2C_{\varphi^6 D^2} + 16(C_{\varphi\Box})^2 + (C_{\varphi D})^2 - 8C_{\varphi\Box} C_{\varphi D} & \\
& - 16(C^{(1)}_{\varphi^4 D^4} + 2C^{(2)}_{\varphi^4 D^4} + C^{(3)}_{\varphi^4 D^4}) G_F M_W^2\Big]\,\frac{\sqrt{2}}{8\,G_F \Lambda^2}\,\frac{s}{\Lambda^2} & \textbf{(dim}-\textbf{8)} \\
& + \left[(3 + \cos 2\theta)(C^{(1)}_{\varphi^4 D^4} + C^{(3)}_{\varphi^4 D^4}) + 8C^{(2)}_{\varphi^4 D^4}\right]\frac{s^2}{8\Lambda^4} &
\end{aligned}
$$

For comparison: Goldstone boson scattering amplitude:

$$\mathcal{M}(G^+G^+ \to G^+G^+)(s,\theta) = -2\sqrt{2}G_F M_H^2 \left[1 - \frac{M_Z^2}{M_H^2}\left(1 - \frac{4}{\sin^2\theta}\right)\right] \qquad \textbf{(SM)}$$

$$+ (2C_{\varphi\Box} + C_{\varphi D})\,\frac{s}{\Lambda^2} \qquad \textbf{(dim}-\textbf{6)}$$

$$+ \Big[8C_{\varphi^6\Box} + 2C_{\varphi^6 D^2} + 16(C_{\varphi\Box})^2 + (C_{\varphi D})^2 - 8C_{\varphi\Box}C_{\varphi D}$$

$$- 16(C^{(1)}_{\varphi^4 D^4} + 2C^{(2)}_{\varphi^4 D^4} + C^{(3)}_{\varphi^4 D^4})G_F M_W^2\Big]\,\frac{\sqrt{2}}{8\,G_F\Lambda^2}\,\frac{s}{\Lambda^2} \qquad \textbf{(dim}-\textbf{8)}$$

$$+ \Big[(3+\cos 2\theta)(C^{(1)}_{\varphi^4 D^4} + C^{(3)}_{\varphi^4 D^4}) + 8C^{(2)}_{\varphi^4 D^4}\Big]\,\frac{s^2}{8\Lambda^4}$$

Results identical

$$\mathcal{M}(G^+G^+ \to G^+G^+) = \mathcal{M}^{s\gg M_W^2}(W_L^+W_L^+ \to W_L^+W_L^+)$$

in agreement with Goldstone Boson Equivalence Theorem!

Another SmeftFR → `FeynArts` test: "elastic positivity bounds"

Using analyticity of the amplitude, the Froissart bound, and the optical theorem, any $2 \to 2$ elastic scattering amplitude satisfies:

$$\frac{d^2}{ds^2}\mathcal{M}(ij \to ij)(s, t=0) \geq 0$$

Applying it to:

$$\mathcal{M}\left(W_L^+ W_L^+ \to W_L^+ W_L^+\right)$$

we get (in agreement e.g. with 1908.09845.2009.04490):

$$C^{(1)}_{\varphi^4 D^4} + 2C^{(2)}_{\varphi^4 D^4} + C^{(3)}_{\varphi^4 D^4} \geq 0$$

Other tested processes include $hh \to hh$, $Z_L h \to Z_L h$ and $W_L^+ h \to W_L^+ h$.

Final example SmeftFR `UFO` output

`UFO` model files can be used for MC simulations e.g. in `MadGraph 5` or other similar MC generators.

Example: `p p > w+ w+ j j` process at the LHC

	`SmeftFR` $\mathcal{O}(\Lambda^{-2})$	`SmeftFR` $\mathcal{O}(\Lambda^{-4})$
`p p > w+ w+ j j QCD=0`		
SM	0.12456 ± 0.00029	
C_W	8.564 ± 0.020	37161 ± 83
$+C_{\varphi\Box}$	0.13387 ± 0.00032	0.20981 ± 0.00059
$-C_{\varphi\Box}$	0.14670 ± 0.00043	0.12511 ± 0.00035
$C_{\varphi 6\Box}$	-	0.12868 ± 0.00031
$C^{(1)}_{\varphi^4 D^4}$	-	10.891 ± 0.024

Table: Simulation results with $\sqrt{s} = 13$ TeV and (G_F, M_H, M_W, M_Z) input scheme. For each run, only one Wilson coefficient has non-zero value $\frac{C_i}{\Lambda^2} = \frac{4\pi}{\mathrm{TeV}^2}$ for dim-6 and $\frac{C_i}{\Lambda^4} = \frac{(4\pi)^2}{\mathrm{TeV}^4}$ for dim-8 operators.

SmeftFR is thoroughly tested!

Analytical tests (SmeftFR $\to$ FeynArts $\to$ FormCalc/FeynCalc):

- Goldstone Boson Equivalence Theorem (GBET)
- elastic positivity bounds
- S-matrix gauge-fixing parameter independence, various Ward identities ...

Numerical tests: SmeftFR $\to$ UFO $\to$ MadGraph 5 – almost perfect agreement with SMEFT@NLO, Dim6Top, SMEFTsim and AnomalousGaugeCoupling (1604.03555), for full list of processes see code's web page.

	SMEFT@NLO $\mathcal{O}\left(\Lambda^{-2}\right)$	SmeftFR $\mathcal{O}\left(\Lambda^{-2}\right)$	SmeftFR $\mathcal{O}\left(\Lambda^{-4}\right)$
	$\mu^+\mu^- \to t\bar{t}$		
SM	0.16606 ± 0.00026	0.16608 ± 0.00024	-
C_{uW}^{33}	0.41862 ± 0.00048	0.41816 ± 0.00047	-
$C_{\varphi u}^{33}$	0.16725 ± 0.00027	0.16730 ± 0.00025	-
C_{lu}^{2233}	6.488 ± 0.016	6.491 ± 0.014	-
$C_{\varphi WB}$	0.21923 ± 0.00032	0.21940 ± 0.00030	0.22419 ± 0.00030
$C_{\varphi D}$	0.18759 ± 0.00030	0.18759 ± 0.00027	0.18829 ± 0.00027

Summary

1. Effective Field Theory is an increasingly important tool for the New Physics searches.

2. SMEFT is a complicated model, calculation of physical observables requires systematic step by step development: basis construction, physical field identification, input parameter selection, gauge fixing and proper quantization, derivation of Feynman rules, diagram selection and evaluation, if necessary renormalization scheme choice etc.

3. Higher order SMEFT calculations technically quite involved. Various software tools published, advertised in this talk: *Mathematica* SmeftFR package calculating Feynman rules directly in terms of physical observables available (`www.fuw.edu.pl/smeft`).