6d Exotic Supermultiplets

Symmetries, Anomalies and (Swampland) Conjectures

Yi Zhang

Peking University
Center for High Energy Physics

Seminar talk at CTPU-PTC
IBS Daejeon
17 April, 2024

Outline

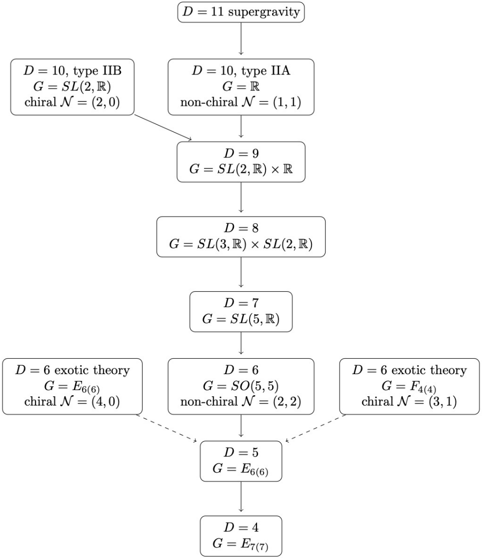
1. Exotic supermultiplets in six dimensions
   - What are those?
   - Why are they interesting?
2. Symmetries and anomalies
   - Review of gravitational anomalies
   - Mixed-Anomaly Cancellation and the Gravitational Remainder
3. Generalized global symmetries and Swampland conjecture
   - No-global Symmetry Conjecture
   - Fermionic Higher-form Symmetries
   - A bosonic global symmetry in $\mathcal{N} = (4, 0)$
4. Summary and discussion

Exotic supermultiplets in six dimensions

What are exotic supermultiplets?

- Maximal supersymmetry consists of 32 real supercharges.
- In six dimensions they are made up of four $SO(5,1)$ symplectic-Majorana-Weyl spinors Q.
- Up to interchange of chirality, the possible combinations of chiralities are $\mathcal{N}=(4,0)$, $(3,1)$ or $(2,2)$.
- There exists non-chiral 6d supergravity theory with $\mathcal{N}=(2,2)$.
  What about the other possibilities, i. e. $\mathcal{N}=(3,1)$ and $\mathcal{N}=(4,0)$?
- Similar situations already happen in 10d, with $\mathcal{N}=(1,1)$ corresponding to TYPE IIA and $\mathcal{N}=(2,0)$ associated to TYPE IIB SUGRA/String theory.

Maximal SUGRA family tree [Lekeu '18]



• In all these cases except for $d = 6$, there is a single massless super Poincaré multiplet whose top sitting spin-2 components is a usual metric tensor g. These are ordinary maximal SUGRA multiplets.

• In $d = 6$, both for $\mathcal{N} = (4, 0)$, $\mathcal{N} = (3, 1)$, the graviton $g_{\mu\nu}$ is replaced by more complicated bosonic tensor fields and there is also an exotic fermionic tensor field other than the usual gravitino ψ_μ.

• We will refer to these as exotic supergravity (or exotic tensor) multiplets and let us have a closer look at them.

Massless representations in six dimensions

Massless multiplets [Strathdee'87]

The massless physical states of the $\mathcal{N} = (p, q)$ SUSY form representations of the little group $G_{\text{little}} = SU(2) \times SU(2) \times G^R_{(p,q)}$, where $G^R_{(p,q)} = Sp(2p) \times Sp(2q)$ is the R-symmetry group.

For example, the $\mathcal{N} = (2, 2)$ graviton $g_{\mu\nu}$ is in the $(\mathbf{3}, \mathbf{3}; \mathbf{1}, \mathbf{1})$.

$\mathcal{N} = (4, 0)$ Fields

$C_{\mu\nu\rho\sigma}$	$B^+_{\mu\nu}$	ϕ	$\psi^R_{\mu\nu}$	λ^R
$(\mathbf{5}, \mathbf{1}; \mathbf{1})$	$(\mathbf{3}, \mathbf{1}; \mathbf{27})$	$(\mathbf{1}, \mathbf{1}; \mathbf{42})$	$(\mathbf{4}, \mathbf{1}; \mathbf{8})$	$(\mathbf{2}, \mathbf{1}; \mathbf{48})$

$\mathcal{N} = (3, 1)$ Fields (only exotic fields)

$D_{\mu\nu\rho}$	$\psi^R_{\mu\nu}$
$(\mathbf{4}, \mathbf{2}; \mathbf{1}, \mathbf{1})$	$(\mathbf{4}, \mathbf{1}; \mathbf{1}, \mathbf{2})$

Remark: Similar multiplets with less supersymmetry containing these exotic fields also exist. [de Wit'02]

Exotic fields [Hull'00]

Exotic graviton
(Self-dual Weyl tensor [Anastasiou, Borsten, Duff, Hughes, Nagy'13])

- $C_{\mu\nu\rho\sigma}$ has index symmetry of Riemann tensor $\ydiagram{2,2}$, i.e.

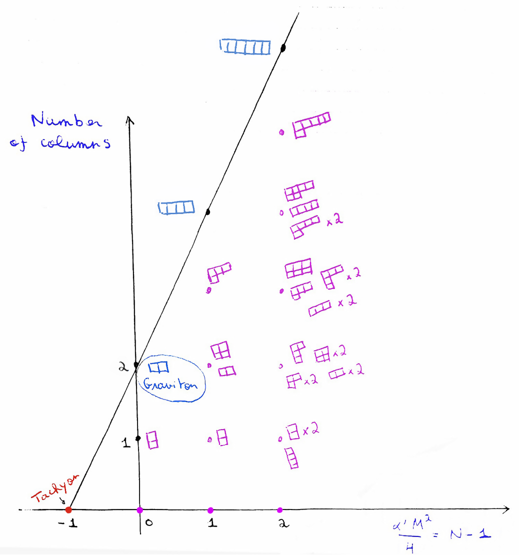
$$C_{\mu\nu\rho\sigma} = C_{\rho\sigma\mu\nu} = C_{[\mu\nu]\rho\sigma} = C_{\mu\nu[\rho\sigma]}\,,$$

$$C_{[\mu\nu\rho]\sigma} = 0\,.$$

- Its field strength $G_{\mu\nu\rho\sigma\tau\kappa} = \partial_{[\mu} C_{\nu\rho][\sigma\tau,\kappa]}$, represented by $\ydiagram{2,2,2}$.
- $G[C]$ is invariant under $\delta C_{\mu\nu\rho\sigma} = \eta_{\mu\nu[\rho,\sigma]} + \eta_{\rho\sigma[\mu,\nu]}$ with the parameter $\eta \sim \ydiagram{2,1}$.
- G satisfies the **self-dual** condition on both the first three and the last three indices $G = \star G = G\star$ (as equation of motion)

$$\begin{aligned} G_{\mu\nu\rho\sigma\tau\kappa} &= (\star G)_{\mu\nu\rho\sigma\tau\kappa} \equiv \frac{1}{3!}\epsilon_{\mu\nu\rho\alpha\beta\gamma} G^{\alpha\beta\gamma}{}_{\sigma\tau\kappa} \\ &= (G\star)_{\mu\nu\rho\sigma\tau\kappa} \equiv \frac{1}{3!} G_{\mu\nu\rho}{}^{\alpha\beta\gamma}\epsilon_{\alpha\beta\gamma\sigma\tau\kappa}. \end{aligned}$$

Exotic fields with mixed young symmetry (closed string spectrum)



Exotic fields [Hull'00]

Exotic gravitino appears in both $(4,0)$ and $(3,1)$ multiplet

- $\psi^R_{\mu\nu}$ belongs to the tensor product $\yng(1,1) \otimes S^+$.
- Field strength $H_{\mu\nu\rho} \equiv 3\partial_{[\mu}\psi_{\nu\rho]}$ is invariant under $\delta\psi_{\mu\nu} = 2\partial_{[\mu}\epsilon_{\nu]}$ with an arbitrary vector-spinor ϵ_μ.
- The field equation leading to $(\mathbf{4}, \mathbf{1})$ is $\gamma^\mu H_{\mu\nu\rho} = 0$, which implies the self-duality $H = \star H$. [Henneaux et al.'2017] [YZ'2021]

Exotic bosonic field in the $(3,1)$ multiplet

- $D_{\mu\nu\rho}$ as represented by the Young tableau $\yng(2,1)$.
- Field strength $S_{\mu\nu\rho\sigma\kappa} = \partial_{[\mu}D_{\nu\rho][\sigma,\kappa]}$ as $\yng(2,2,1)$, invariant under

  $\delta D_{\mu\nu\rho} = \partial_{[\mu}\alpha_{\nu]\rho} + \partial_{[\mu}\beta_{\nu]\rho} + \partial_\rho\beta_{\mu\nu}$, with $\alpha \sim \yng(2)$ and $\beta \sim \yng(1,1)$.
- Self-duality $S_{\mu\nu\rho\sigma\kappa} = (\star S)_{\mu\nu\rho\sigma\kappa} = \frac{1}{6}\epsilon_{\mu\nu\rho\alpha\beta\gamma}S^{\alpha\beta\gamma}{}_{\sigma\kappa}$.

Why should we look at these exotic multiplets?

• Conjecture [Hull' 00]: Strong coupling limit of $5d$ $\mathcal{N} = 8$ supergravity could give a $6d$ superconformal phase of M-theory based on the $\mathcal{N} = (4, 0)$ multiplet. $6d$ back to $5d$ as S^1 reduction.

• If the above is true, then we find a gravity theory formulated NOT in terms of the familiar spin-two graviton and its physics will provide novel insights on interactions of M-theory.

Some evidence

- Free $6d$ $(4, 0)$ theory exists and its circle reduction yields 5d $\mathcal{N} = 8$ linearised SUGRA. [Hull' 00] ("gravitational triality")
- Non-linear $5d$ $\mathcal{N} = 8$ SUGRA is non-renormalisable with a $5d$ Planck length l_p. Conjectural $(4, 0)$ UV-complete theory superconformal and its circle reduction sets the 5d Planck scale by $R = l_p$?
- Similar to that $6d$ $(2, 0)$ non-lagrangian superconformal field theories (SCFTs) reduces to conventional super Yang-mills theories (SYMs) in 5d with coupling constant $g^2_{\mathrm{YM}} = R$.

Linearised gravity in five dimensions

Dual formulations of linearised gravity

- **Linearised graviton** $h_{\hat\mu\hat\nu} \sim \begin{array}{|c|c|}\hline \ & \ \\ \hline \end{array}$; field strength
  $R_{\hat\mu\hat\nu\hat\rho\hat\sigma} \equiv \partial_{[\hat\mu} h_{\hat\nu][\hat\rho,\hat\sigma]} \sim \begin{array}{|c|c|}\hline \ & \ \\ \hline \partial & \partial \\ \hline \end{array}$; invariant under $\delta \begin{array}{|c|c|}\hline \ & \ \\ \hline \end{array} = \begin{array}{|c|c|}\hline \ & \partial \\ \hline \end{array}$;
  field equation $R_{\hat\nu\hat\sigma} \equiv \eta^{\hat\mu\hat\rho} R_{\hat\mu\hat\nu\hat\rho\hat\sigma} = 0\,.$

- **Dual graviton** [Curtright'85] $d_{\hat\mu\hat\nu\hat\lambda} \sim \begin{array}{|c|c|}\hline \ & \ \\ \hline \multicolumn{1}{|c|}{\ } \\ \cline{1-1} \end{array}$; field strength
  $S_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\kappa} = \partial_{[\hat\mu} d_{\hat\nu\hat\rho][\hat\sigma,\hat\kappa]} \sim \begin{array}{|c|c|}\hline \ & \ \\ \hline \ & \partial \\ \hline \multicolumn{1}{|c|}{\partial} \\ \cline{1-1} \end{array}$; invariant under $\delta \begin{array}{|c|c|}\hline \ & \ \\ \hline \multicolumn{1}{|c|}{\ } \\ \cline{1-1} \end{array} = \begin{array}{|c|c|}\hline \ & \partial \\ \hline \multicolumn{1}{|c|}{\ } \\ \cline{1-1} \end{array} + \begin{array}{|c|c|}\hline \ & \ \\ \hline \multicolumn{1}{|c|}{\partial} \\ \cline{1-1} \end{array}$;
  field equation $S^{\hat\mu}{}_{\hat\nu\hat\rho\hat\mu\hat\sigma} = 0\,.$

- **Double dual graviton** $c_{\hat\mu\hat\nu\hat\rho\hat\sigma} \sim \begin{array}{|c|c|}\hline \ & \ \\ \hline \ & \ \\ \hline \end{array}$; field strength
  $G_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\tau\hat\kappa} = \partial_{[\hat\mu} c_{\hat\nu\hat\rho][\hat\sigma\hat\tau,\hat\kappa]} \sim \begin{array}{|c|c|}\hline \ & \ \\ \hline \ & \ \\ \hline \partial & \partial \\ \hline \end{array}$; invariant under $\delta \begin{array}{|c|c|}\hline \ & \ \\ \hline \ & \ \\ \hline \end{array} = \begin{array}{|c|c|}\hline \ & \ \\ \hline \ & \partial \\ \hline \end{array}$;
  field equation $G^{\hat\mu\hat\nu}{}_{\hat\sigma\hat\mu\hat\nu\hat\rho} = 0\,.$

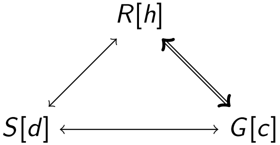
Gravitational dualities (triality)

The triality relation [Hull'00,'01; de Medeiros, Hull'02]
One starts with h with its field strength $R[h]$, we define the tensor S and the tensor G by

$$S = \star R$$
$$G = \star R \star ,$$

$$R_{[\hat{\mu}\hat{\nu}\hat{\rho}]\hat{\sigma}} = 0 \Longrightarrow S^{\hat{\mu}}{}_{\hat{\nu}\hat{\rho}\hat{\mu}\hat{\sigma}} = 0 \qquad R_{[\hat{\mu}\hat{\nu}\hat{\rho}]\hat{\sigma}} = 0 \Longrightarrow G_{[\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}]\hat{\tau}\hat{\kappa}} = 0$$
$$R^{\hat{\mu}}{}_{\hat{\nu}\hat{\mu}\hat{\rho}} = 0 \Longrightarrow S_{[\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}]\hat{\kappa}} = 0 \qquad R^{\hat{\mu}}{}_{\hat{\nu}\hat{\mu}\hat{\rho}} = 0 \Longrightarrow G^{\hat{\mu}\hat{\nu}}{}_{\hat{\sigma}\hat{\mu}\hat{\nu}\hat{\rho}} = 0$$

Generalised Poincaré lemma [Dubois-Violette, Henneaux'99,'01; Bekaert, Boulanger'02]:
S and G can be solved by gauge potentials d and c of type $[2,1]$ and $[2,2]$ up to gauge transformations, i.e. $S = S[d]$ and $G = G[c]$.



[Chatzistavrakidis, Karagiannis'19]

- Only two independent sources [Hull'01]
- h and c are algebraically related [Henneaux, Lekeu, Leonard'19]
- generalise to $[1,1]$, $[n-3,1]$ and $[n-3,n-3]$ tensor fields in n-dimensions

Reduction of $(4,0)$ free fields to five dimensions [Hull]

Circle reduction of the SD-Weyl C field

$$h_{\hat{\mu}\hat{\nu}} \equiv C_{\hat{\mu}5\hat{\nu}5}\,, \quad d_{\hat{\mu}\hat{\nu}\hat{\lambda}} \equiv C_{\hat{\mu}5\hat{\nu}\hat{\lambda}}\,, \quad c_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \equiv C_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\,.$$

$6d$ self-dualities $G[C] = \star G[C] = G[C]\star$ imply duality conditions between $5d$ field strengths. In addition, it fixes the trace of $G[c]$ in terms of $R[h]$

$$G_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}{}^{\hat{\rho}} \propto R_{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\tau}} \Longrightarrow G_{\mu\nu\rho}{}^{\alpha\beta\gamma} \propto R_{[\mu\nu}{}^{[\alpha\beta}\delta_{\rho]}{}^{\gamma]}\,.$$

- C^{6d} yields 5 physical degrees of freedom in $5d$ and described by one of the 3 dual formulations of linearised gravity.
- 27 self-dual $B^{+}_{\mu\nu} \Longrightarrow$ 27 vector $A_{\hat{\mu}}$
- 42 scalar fields $\phi \Longrightarrow$ 42 scalars ϕ in $5d$
- 8 exotic gravitini $\psi^{R}_{\mu\nu} \Longrightarrow$ 8 gravitini $\psi^{R}_{\hat{\mu}}$
- 48 chiral fermions $\lambda^{R} \Longrightarrow$ 48 chiral fermions λ^{R} in $5d$

These are the spectrum of linearised five dimensional $\mathcal{N} = 8$ SUGRA!

Recent studies on the multiplets and Hull's conjecture

- Due to dualities, there are no manifestly Lorentz covariant actions for the complete $(4, 0)$ or $(3, 1)$ multiplets. One can still build free actions with different noncovariant formalism. [Henneaux et al.'18, Samtleben et al.'20 '22]
- $6d$ ordinary local Lagrangian interactions are not sufficient to generate the non-polynomial gravitational interactions in $5d$. [Saliu et al.'04, Boulanger et al.'05]
- Chern-Simons coupling of $5d$ maximal SUGRA.

$$S_{CS} = \int \kappa_{abc} A^a \wedge F^b \wedge F^c$$

with κ_{abc} being the $E_{6(6)}$ cubic invariant. Can not be generated with $6d$ manifest Lorentz symmetry and $E_{6(6)}$ global symmetry. [YZ et al.'21]

- Gravitational anomalies non-vanishing, while vanishing pure R and mixed R/Gravitational-anomalies. [YZ et al.'21 '22, Piljin Yi and YZ' 24]
- Generalized symmetries and Swampland conjecture. [Yi-Nan Wang and YZ '23, Montero and Tartaglia '24]
- Double copy? Matrix model? String/M-theory realisations?...

Symmetries and anomalies

Symmetries and anomalies

- The interaction among the chiral fields therein are poorly understood.
- One universal aspect of quantum theories for which the complete detail of interactions are not needed is the anomaly.
- Three classes of perturbative anomalies relevant here: gravitational anomalies, R-symmetry anomalies, and those of the gauge symmetries of the chiral tensors ($C_{\mu\nu\rho\sigma}$).
- The last is quite essential if one is to investigate the above $5d/6d$ connection, however still unknown.
- We focus on the first two.

Generalities about anomalies

- Remark: no ordinary graviton in exotic theories, however one can think about local Lorentz. [Minasian, Papadimitriou and Yi '83]
- Consider the partition function $Z[A]$

$$Z[A] = e^{-\Gamma[A]} = \int \mathcal{D}\Phi \, e^{-S[\Phi, A]}$$

  with Φ denoting the collection of fields sensitive to anomalies and A is the "external" gauge field to which the fields in Φ are coupled.
- In the cases of gravitational anomalies, A can be taken as the spin connection for local Lorentz symmetry or the Christoffel connection for diffeomorphisms.
- Anomaly

$$\delta_\epsilon \Gamma[A] = \Gamma[A'] - \Gamma[A] = \int -\epsilon(x) \cdot \mathcal{A}(x) \, d^{2n}x$$
$$= \int_{M_{2n}} I^1_{2n}(\epsilon, A) \, .$$

Descent equations, anomaly polynomial and index density

Chain of descent equations

$$I_{2n+2} = d\, I_{2n+1}$$
$$s\, I_{2n+1} = d\, I^1_{2n}$$

- s is the BRST-operator
- Replace ϵ by ghost parameter v
- $I^1_{2n}(v, A)$ is determined up to a d-exact term and an s-exact term

$$I^1_{2n}(v, A) \simeq I^1_{2n}(v, A) + dG^1_{2n-1} + sF_{2n}$$

Anomaly polynomial: $I_{2n+2} = [\mathrm{Ind}(\not{D})]_{2n+2}$, where $\not{D}$ is a certain Dirac operator $\not{D}$ in $2n+2$ dimensions. [O. Alvarez, Singer, Zumino '84]

$$\not{D} : \mathcal{C}^\infty(S^+ \otimes \mathcal{V}_G \otimes \mathcal{V}_R) \longrightarrow \mathcal{C}^\infty(S^- \otimes \mathcal{V}_G \otimes \mathcal{V}_R)$$
$$\mathrm{Ind}(\not{D}) = \mathbb{A}(\mathcal{R}) \wedge \mathrm{ch}_{\mathbf{R}_G}(\mathcal{R}) \wedge \mathrm{ch}_{\mathbf{R}_R}(\mathcal{F})$$

• $\mathbf{R}_G$ collection of $SO(d)$ representations, $\mathbf{R}_R$ relevant R-symmetry representation.

Useful characteristic classes

- **Roof genus** $\mathbb{A}(\mathcal{R})$ is given in terms of the curvature 2-form $\mathcal{R}^a{}_b$ of M by (traces are taken in the fundamental of $SO(d)$)

$$\mathbb{A}(\mathcal{R}) = 1 + \tfrac{1}{(4\pi)^2}\tfrac{1}{12}\,\mathrm{tr}(\mathcal{R}^2) + \tfrac{1}{(4\pi)^4}\left[\tfrac{1}{288}\,\mathrm{tr}(\mathcal{R}^2)^2 + \tfrac{1}{360}\,\mathrm{tr}(\mathcal{R}^4)\right] + \ldots$$

- **Chern character** $\mathrm{ch}(\mathcal{V})$ is defined for a vector bundles $\mathcal{V}$ over $\mathcal{M}$.
- Using the curvature two-form $\mathcal{F}$ of a connection on the bundle:

$$\mathrm{ch}(\mathcal{V}) = \mathrm{tr}_{\mathcal{V}}\left(\exp\left(\frac{i}{2\pi}\mathcal{F}\right)\right) = \mathrm{rk}(\mathcal{V}) + \frac{i}{2\pi}\,\mathrm{tr}_{\mathcal{V}}\,\mathcal{F} + \ldots + \frac{i^k}{k!(2\pi)^k}\,\mathrm{tr}_{\mathcal{V}}\,\mathcal{F}^k + \ldots$$

- **Pontryagin classes** $\mathbf{p}_i$, they are defined by the expansion

$$\det\left(1 - \frac{\mathcal{R}}{2\pi}\right) = 1 + \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_4 + \ldots$$

with each $\mathbf{p}_i$ a differential form of degree $4i$.

Anomalies via field strengths for C and D

- In [AG, Witten '83], a potential A and its chiral field strength F are viewed as independent variables in the path-integral, and one integrates over both of them by using a first order action.
- In the path-integral measure, the self-dual F^+ part and anti-self-dual part F^- both appear. Anomaly for the (anti-)self-dual part can be extracted alone as the Jacobian generated by it under transformations of the Lorentz group in the corresponding (anti-)self-dual representation.
- Since there is no gauge freedom in F^+ or F^-, there is no need to subtract ghost contributions.
- Come back to (4,0) and (3,1) fields, anomalies of bosonic exotic fields $C_{\mu\nu\rho\sigma}$, $D_{\mu\nu\rho}$ are computed via field strengths $G[C]$ and $S[D]$, we need to determine in which representation of the Lorentz group (orthogonal group) they transform.
- They are both singlets under R-symmetry, i. e. $\mathrm{ch}_{\mathbf{R}_R}(\mathcal{F}) = 1$.

Field strengths and representations

In our convention, the Dynkin label of the negative chiral spinor representation of $\mathfrak{su}^*(4)$ is $[0,0,1]$.

Exotic graviton $C_{\mu\nu\rho\sigma}$

$G[C]$ transforms in $[0,0,4]$ contained in $(S^+)^{\otimes 4}$, the relevant quantities is

$$G \in \mathcal{C}^\infty\left(S^+ \otimes [S^+ \otimes S^+ \otimes S^+ - (S^+ \otimes T^*\mathcal{M})^{\oplus 3} + (S^-)^{\oplus 2}]\right)$$

$$\mathsf{ch}_{\mathbf{R}_G}(\mathcal{R}) = (\mathsf{ch}_{+\mathrm{s}}(\mathcal{R}))^3 - 3\,\mathsf{ch}_{+\mathrm{s}}(\mathcal{R})\,\mathsf{ch}_{\mathrm{def}}(\mathcal{R}) + 2\,\mathsf{ch}_{-\mathrm{s}}(\mathcal{R})\,.$$

Exotic graviton $D_{\mu\nu\rho}$

$S[D]$ which is in the $[1,0,3]$ representation, we compute

$$S \in \mathcal{C}^\infty\left(S^+ \otimes [S^+ \otimes S^+ \otimes S^- - (S^- \otimes T^*\mathcal{M})^{\oplus 2} - (S^+)^{\oplus 2}]\right)$$

$$\mathsf{ch}_{\mathbf{R}_G}(\mathcal{R}) = (\mathsf{ch}_{+\mathrm{s}}(\mathcal{R}))^2\,\mathsf{ch}_{-\mathrm{s}}(\mathcal{R}) - 2\,\mathsf{ch}_{-\mathrm{s}}(\mathcal{R})\,\mathsf{ch}_{\mathrm{def}}(\mathcal{R}) - 2\,\mathsf{ch}_{+\mathrm{s}}(\mathcal{R})\,.$$

What about exotic gravitino $\psi_{\mu\nu}$? Fermionic p-forms?

- There is a Lagrangian for the free $\psi_{\mu\nu}$ in exotic multiplets

$$S = \int d^6x\, \bar{\psi}_{\mu\nu} \Gamma^{\mu\nu\rho\sigma\tau} \partial_\rho \psi_{\sigma\tau}$$

- Special case of rank p antisymmetric tensor spinors $\psi^\alpha_{\mu_1\mu_2\ldots\mu_p}$ with $2p < D$ in D dimensions, [Zinoviev '09, Campoleoni '09]

$$S_0[\psi] = -(-1)^{\frac{p(p-1)}{2}} \int d^Dx\, \bar{\psi}_{\mu_1\mu_2\ldots\mu_p}\, \gamma^{\mu_1\mu_2\ldots\mu_p\nu\rho_1\rho_2\ldots\rho_p}\, \partial_\nu \psi_{\rho_1\rho_2\ldots\rho_p}$$

- Gauge symmetries

$$\delta\psi^\alpha_{\mu_1\mu_2\ldots\mu_p} = p\, \partial_{[\mu_1} \Lambda^{(p-1)\alpha}{}_{\mu_2\ldots\mu_p]}\,,$$

- $(p-1)$-stage reducible: Chain of gauge-for-gauge transformations

$$\delta\Lambda^{(p-1)\alpha}{}_{\mu_2\ldots\mu_p} = (p-1)\, \partial_{[\mu_2} \Lambda^{(p-2)\alpha}{}_{\mu_3\ldots\mu_p]}$$

$$\vdots$$

$$\delta\Lambda^{(1)\alpha}{}_{\mu} = \partial_\mu \Lambda^{(0)\alpha}\,.$$

Quantisation and spectrum of fermionic p-forms

- $(p-1)$-**stage reducible gauge system** in the language of Batalin-Vilkovisky (BV) formalism. [Batalin, Vilkovisky '81, '84]
- Quantisation and gauge-fixing need to introduce various ghost fields (fields with wrong spin-statistics) and auxiliary fields.
- As for the fermionic 2-form [Lekeu, YZ '21]

	$\psi_{\mu\nu}$	C_μ	C'_μ	c	c'	b_μ	η	π	π'
S_{G} chirality	$+$	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$-$
Parity	1	0	0	1	1	1	1	0	0

- Find $\not{D} : \mathcal{C}^\infty(S^+ \otimes \mathcal{V}_G) \longrightarrow \mathcal{C}^\infty(S^- \otimes \mathcal{V}_G)$

$$\int \mathcal{D}\psi_{\mu\nu}\, \mathcal{D}\hat{b}_\mu\, \mathcal{D}\hat{C}_\mu\, \mathcal{D}\hat{C}'_\mu\, \mathcal{D}c\, \mathcal{D}c' \Longrightarrow \mathcal{V}_G = \Lambda^2 T^*\mathcal{M} - T^*\mathcal{M} + 1\,.$$

Anomalies of fermionic p-forms

- For p-forms the ghosts structure is quit involve, we give here only the d.o.f.'s $N_p = \frac{(d-2)!(d-2p-1)}{p!(D-p-1)!} 2^{[\frac{d-1}{2}]}$.
- For p-forms the result is $\not{D}_p : \mathcal{C}^\infty(S^+ \otimes \mathcal{V}_{G,p}) \longrightarrow \mathcal{C}^\infty(S^- \otimes \mathcal{V}_{G,p})$. we can find the complex $\mathcal{C}^\infty(S^+ \otimes \mathcal{V}_{G,p})$, on which the Dirac operator $\not{D}_p$ acts and we get

$$\begin{aligned} \mathcal{V}_{G,p} &= \Lambda^p T^*\mathcal{M} - \Lambda^{p-1} T^*\mathcal{M} + \Lambda^{p-2} T^*\mathcal{M} - \Lambda^{p-3} T^*\mathcal{M} + \cdots \pm 1\,, \\ &= \sum_{k=0}^{p} (-1)^k \Lambda^{p-k}\, T^*\mathcal{M}\,. \end{aligned}$$

- Index theorem provides the anomaly polynomial $I^{(p)}_{2n+2}$

$$I^{(p)}_{2n+2} = \left[\mathrm{Ind}(\not{D}_p)\right]_{2n+2} = \left[\mathbb{A}(\mathcal{R}) \left(\sum_{k=0}^{p} (-1)^k \,\mathrm{ch}\,(\Lambda^{p-k}\, T^*\mathcal{M})\right)\right]_{2n+2} .$$

Mixed gravitational and R-anomalies of exotic multiplets

For the $\mathcal{N} = (4, 0)$ multiplet, the complete list is

$$
\begin{aligned}
(\mathbf{5}, \mathbf{1}; \mathbf{1}) \quad &\rightarrow \quad -\frac{1}{2}\mathbb{A}(\mathcal{R}) \left((\mathrm{ch}_{+\mathrm{s}}(\mathcal{R}))^3 - 3\, \mathrm{ch}_{+\mathrm{s}}(\mathcal{R})\, \mathrm{ch}_{\mathrm{def}}(\mathcal{R}) + 2\, \mathrm{ch}_{-\mathrm{s}}(\mathcal{R}) \right) \ , \\
(\mathbf{4}, \mathbf{1}; \mathbf{8}) \quad &\rightarrow \quad +\frac{1}{2}\mathbb{A}(\mathcal{R}) \left(\mathrm{ch}_{[2]}(\mathcal{R}) - \mathrm{ch}_{\mathrm{def}}(\mathcal{R}) + 1 \right) \mathrm{ch}_{[1}(\mathcal{F}_{\mathfrak{sp}(4)}) \ , \\
(\mathbf{3}, \mathbf{1}; \mathbf{27}) \quad &\rightarrow \quad -\frac{1}{4}\mathbb{A}(\mathcal{R})\, \mathrm{ch}_{\mathrm{s}}(\mathcal{R})\, \mathrm{ch}_{[2]}(\mathcal{F}_{\mathfrak{sp}(4)}) \ , \\
(\mathbf{2}, \mathbf{1}; \mathbf{48}) \quad &\rightarrow \quad +\frac{1}{2}\mathbb{A}(\mathcal{R})\, \mathrm{ch}_{[3]}(\mathcal{F}_{\mathfrak{sp}(4)}) \, .
\end{aligned}
$$

These together give an 8-form pure gravitational anomaly only

$$
P_8^{\mathcal{N}=(4,0)} = \frac{1}{72}(23\mathbf{p}_1^2 - 17\mathbf{p}_2) \neq 0 \ , \qquad (1)
$$

Mixed gravitational and R-anomalies of exotic multiplets

The computation for the $\mathcal{N} = (3,1)$ multiplet is analogous. The index density contribution are listed as

$$\begin{aligned}
(\mathbf{4},\mathbf{2};\mathbf{1},\mathbf{1}) &\to -\frac{1}{2}\mathbb{A}(\mathcal{R})[(\mathrm{ch}_{+\mathrm{s}}(\mathcal{R}))^2\,\mathrm{ch}_{-\mathrm{s}}(\mathcal{R}) - 2\,\mathrm{ch}_{-\mathrm{s}}(\mathcal{R})\,\mathrm{ch}_{\mathrm{def}}(\mathcal{R}) - 2\,\mathrm{ch}_{+\mathrm{s}}(\mathcal{R})] \\
(\mathbf{4},\mathbf{1};\mathbf{1},\mathbf{2}) &\to +\frac{1}{2}\mathbb{A}(\mathcal{R})\left(\mathrm{ch}_{[2]}(\mathcal{R}) - \mathrm{ch}_{\mathrm{def}}(\mathcal{R}) + 1\right)\mathrm{ch}_{[1]}(\mathcal{F}_{\mathfrak{sp}(1)}) , \\
(\mathbf{3},\mathbf{2};\mathbf{6},\mathbf{1}) &\to +\frac{1}{2}\mathbb{A}(\mathcal{R})(\mathrm{ch}_{\mathrm{def}}(\mathcal{R}) - 1)\,\mathrm{ch}_{[1]}(\mathcal{F}_{\mathfrak{sp}(3)}) , \\
(\mathbf{3},\mathbf{1};\mathbf{6},\mathbf{2}) &\to -\frac{1}{4}\mathbb{A}(\mathcal{R})\,\mathrm{ch}_{\mathrm{s}}(\mathcal{R})\,\mathrm{ch}_{[1]}(\mathcal{F}_{\mathfrak{sp}(3)})\,\mathrm{ch}_{[1]}(\mathcal{F}_{\mathfrak{sp}(1)}) , \\
(\mathbf{2},\mathbf{1};\mathbf{14},\mathbf{2}) &\to +\frac{1}{2}\mathbb{A}(\mathcal{R})\,\mathrm{ch}_{[2]}(\mathcal{F}_{\mathfrak{sp}(3)})\,\mathrm{ch}_{[1]}(\mathcal{F}_{\mathfrak{sp}(1)}) , \\
(\mathbf{1},\mathbf{2};\mathbf{14}',\mathbf{1}) &\to -\frac{1}{2}\mathbb{A}(\mathcal{R})\,\mathrm{ch}_{[3]}(\mathcal{F}_{\mathfrak{sp}(3)}) .
\end{aligned}$$

Again, again purely gravitational

$$P_8^{\mathcal{N}=(3,1)} = \frac{1}{180}(-61\mathbf{p}_1^2 - 293\mathbf{p}_2) \neq 0 . \qquad (2)$$

Cancellation of anomalies

- All standard anomaly polynomials for ordinary maximally supersymmetric gravitational theories either cancel away or handled by Green-Schwarz type topological terms.
- Previous investigation of only the gravitational anomalies of $(4,0)$ and $(3,1)$ multiplets shows a nonvanishing results. [Minasian, Strickland-Constable and YZ '21]
- This does not need to discourage us, since the proposed $5d/6d$ connection does not imply $d=6$ general covariance in the conventional sense.
- Unlike $d=5$ diffeomorphisms, R-symmetries uplift to $d=6$ verbatim straightforwardly, so there is no room to wiggle out if the R-symmetry proves to be anomalous.
- And they are not! [Piljin Yi, YZ '24]
- Pure R-symmetry anomalies and the mixed anomalies involving the diffeomorphism and the R-symmetry cancel out entirely. Why? We don't have an answer.

Generalized global symmetries and Swampland conjecture

Swampland Conjecture

• Swampland Program aims to identify those effective field theories that can never arise as the low-energy limit of consistent quantum gravity. [Review see Palti '19, van Beest et al. '22]

• "String Universality" is achieved when the list of EFT's allowed by Swampland principles exactly matches those which can be obtained from string compactifications.

• This had been put into tests in various dimensions in case of different supercharges.

• For maximal SUSY with 32 supercharges, this has not been achieved because of the existence of the $6d$ $\mathcal{N} = (4, 0)$ and $\mathcal{N} = (3, 1)$ multiplets.

Swampland Conjecture

• Due to Hull's proposal, the conjecture $6d$ $\mathcal{N} = (4,0)$ theory arising as UV-completion of $5d$ maximal SUGRA (as low-energy limit of M-theory on T^6), can provide new phase of gravity other than perturbative strings or Einsteinian. [S.-J. Lee, Lerche and Weigand '22]

• The question is whether $6d$ $\mathcal{N} = (4,0)$ (as well as $\mathcal{N} = (3,1)$) is in the Swampland.

• There are many conjectures constraining quantum gravity in the swampland program.

• We want to look at the "No Global Symmetries" conjecture.

• It states: there are no global symmetries in quantum gravity (i.e. any symmetry is either broken or gauged).

• Study possible global symmetries of these exotic theories.

Fermionic Higher-form Symmetries [Yi-Nan Wang, YZ '23]

- Motivated by the existence of the fermionic two-form $\psi_{\mu\nu}$ with Lagrangian description, we propose Fermionic Higher-form Symmetries (FHFS).
- An *invertible fermionic p-form symmetry* is generated by topological operator $U_\epsilon(M^{(d-p-1)})$, defined on a $(d-p-1)$-dimensional submanifold $M^{(d-p-1)} \subset \mathbb{R}^{1,d-1}$.
- The symmetry parameter $\epsilon \in \mathbb{R}^{0|s}$ is a fermionic spinor, where s is the number of spinor components.
- $\mathbb{R}^{0|s}$ is the odd part of superspace $\mathbb{R}^{r|s}$.
- $U_\epsilon(M^{(d-p-1)})$ acts on p-dimensional operators $V_i(\mathcal{C}^{(p)})$ as

$$\langle U_\epsilon(M^{(d-p-1)})V_i(\mathcal{C}^{(p)})\rangle = \left(R_i^{\ j}\right)^{\langle \mathcal{C}^{(p)}, M^{(d-p-1)}\rangle} \langle V_j(\mathcal{C}^{(p)})\rangle\,.$$

  $\langle \mathcal{C}^{(p)}, M^{(d-p-1)}\rangle$ is the linking number between $\mathcal{C}^{(p)}$ and $M^{(d-p-1)}$. $R_i^{\ j}$ is a representation of the additive abelian group $\mathbb{R}^{0|s}$.

Fermionic 0-form symmetry

Global supersymmetry

Constant spinor ϵ as the symmetry parameter. The $(d-1)$-dimensional topological operator is

$$U_\epsilon(M^{(d-1)}) = \exp\left(\int_{M^{(d-1)}} i(\bar{\epsilon}\mathcal{J} + \bar{\mathcal{J}}\epsilon)\right) = e^{i(\bar{\epsilon}Q + \bar{Q}\epsilon)},$$

with $(d-1)$-form supercurrent $\mathcal{J}$ and supercharge Q.

Fermionic shift supersymmetry

The action

$$S = \int -\bar{\psi}\gamma^\mu \partial_\mu \psi d^d x$$

is invariant under

$$\psi \to \psi + \epsilon$$

with spinor parameter ϵ satisfies $d\epsilon = 0$ and corresponding

$$U_\epsilon(M^{(d-1)}) = \exp\left(i\int_{M^{(d-1)}} \star\left[\bar{\epsilon}\gamma_{(1)}\psi - \bar{\psi}\gamma_{(1)}\epsilon\right]\right)$$

Fermionic 1-form symmetry

- Lets consider free Rarita-Schwinger Lagrangian in $d \geq 3$ dimensions

$$S[\psi_\mu] = \int -\bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho \, d^d x \, .$$

- Equation of motion

$$\gamma^{\mu\nu\rho} \partial_\nu \psi_\rho = 0$$

- Equivalent to

$$\gamma^\mu H_{\mu\nu} = 0$$

and again we define the field strength as $H_{\mu\nu} = 2\partial_{[\mu} \psi_{\nu]}$.

- The action has the ordinary fermionic 0-form gauge symmetry

$$\psi_\mu \to \psi_\mu + \partial_\mu \lambda$$

Fermionic 1-form symmetry

- Gauge invariant 1-dimensional topological operator

$$V_\eta(\mathcal{C}) = \exp\left(i \oint_{\mathcal{C}} (\bar{\eta}\psi_\mu + \bar{\psi}_\mu \eta)\, dx^\mu\right) = \exp\left(i \int_{\mathcal{C}} (\bar{\eta}\psi_{(1)} + \bar{\psi}_{(1)}\eta)\right)$$

where $\psi_{(1)} = \psi_\mu \, dx^\mu$.

- 2-form $\mathcal{J}_{(2)} = \frac{1}{2}\, \mathcal{J}_{\mu\nu}\, dx^\mu \wedge dx^\nu$ whose components are given as

$$\mathcal{J}_{\mu\nu} \equiv \gamma_{\mu\nu\rho}\psi^\rho$$

- The conservation law follows from e.o.m.

$$\partial^\mu \mathcal{J}_{\mu\nu} = -\gamma_{\nu\mu\rho}\partial^\mu \psi^\rho = 0$$

- We then define a $(d-2)$-dimensional operator

$$U_\epsilon(M^{(d-2)}) = \exp\left(i \int_{M^{(d-2)}} \left[\bar{\epsilon}\,(\star\mathcal{J})_{(d-2)} + (\star\bar{\mathcal{J}})_{(d-2)}\,\epsilon\right]\right)$$

Fermionic 1-form symmetry

- The action of $U_\epsilon(M^{(d-2)})$ on $V_\eta(\mathcal{C})$ can be computed as

$$\langle U_\epsilon(M^{(d-2)}) V_\eta(\mathcal{C})\rangle = \exp\left(i(\bar{\epsilon}\eta + \bar{\eta}\epsilon)\langle \mathcal{C}, M^{(d-2)}\rangle\right)\langle V_\eta(\mathcal{C})\rangle$$

- Fermionic 1-form symmetry on the original Rarita-Schwinger field ψ_μ is the shift by a fermionic 1-form ξ_μ with $\partial_{[\nu}\xi_{\rho]} = 0$

$$\psi_\mu \longrightarrow \psi_\mu + \xi_\mu$$

- The previous consideration generalised to the fermionic p-form case.
- Along with the above electric like 1-form symmetry, is there also magnetic fermionic $(d-3)$-form symmetry?

$$U_\theta(M^{(2)}) = \exp\left(i\int_{M^{(2)}}\left[(\bar{\theta}H)_{(2)} + \text{c.c.}\right]\right)$$

H satisfies the Bianchi identity $dH = 0$.

EM-like duality of fermions

- Dualisation of fermionic fields has been a long-standing question since [Townsend '80, Deser, Townsend and Siegel '81].
- They showed the equivalence between a Dirac field ψ and a fermionic 2-form $\chi_{\mu\nu}$ in four dimensions.

$$\mathcal{L}_{\text{parent}} = \varepsilon^{\mu\nu\alpha\beta}\bar{\chi}_{\mu\nu}\left(\partial_\alpha\psi_\beta + \gamma_\alpha\phi_\beta\right) + \bar{\xi}^{\mu\nu}\partial_\mu\phi_\nu + \bar{\psi}_\mu\gamma^{\mu\nu}\phi_\nu + \text{c.c.}$$

- Integrating out the two fields $\chi_{\mu\nu}$ and $\xi_{\mu\nu}$, one enforces constraints that are solved by: $\psi_\mu = \partial_\mu\alpha + \gamma_\mu\psi$, $\phi_\mu = \partial_\mu\psi$ for two Dirac spinors α and ψ. Inserting back yields a free Dirac Lagrangian only for ψ.
- Integrating out the field ψ_μ and ϕ_μ gives the equivalent *magnetic frame* description of the free Dirac fermion only in terms of $\chi_{\mu\nu}$:

$$\begin{aligned}\mathcal{L}_{\text{dual}} = & -6\partial_{[\mu}\bar{\chi}_{\alpha\beta]}\gamma^\mu\chi^{\alpha\beta} - \frac{2}{3}\partial_\mu\bar{\chi}_{\alpha\beta}\gamma^{\mu\alpha\beta}\gamma^{\rho\sigma}\chi_{\rho\sigma} \\ & - \bar{\chi}_{\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\partial^\lambda\xi_{\lambda\nu} + \text{c.c.}\end{aligned} \quad (3)$$

EM-like duality of fermions

$$\mathcal{L}_{\rm dual} = -6\partial_{[\mu}\bar{\chi}_{\alpha\beta]}\gamma^{\mu}\chi^{\alpha\beta} - \frac{2}{3}\partial_{\mu}\bar{\chi}_{\alpha\beta}\gamma^{\mu\alpha\beta}\gamma^{\rho\sigma}\chi_{\rho\sigma}$$
$$- \bar{\chi}_{\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}\partial_{\mu}\partial^{\lambda}\xi_{\lambda\nu} + \text{c.c.}$$

- The Lagrangian has gauge symmetry: $\delta\chi_{\alpha\beta} = 2\partial_{[\alpha}\gamma_{\beta]}\epsilon$.
- And global symmetry: $\delta\chi_{\mu\nu} = 2\gamma_{[\mu}\epsilon_{\nu]}$, with constant ϵ_{ν}.
- This theory has two conserved currents:

$$\mathcal{J}^{(e)}_{\mu} = \varepsilon_{\mu\nu\alpha\beta}\partial^{\nu}\chi^{\alpha\beta}$$

$$\mathcal{J}^{(m)}_{\mu\nu\rho} = 6\gamma_{[\mu}\chi_{\nu\rho]} + \frac{2}{3}\gamma_{\mu\nu\rho}\gamma_{\sigma\tau}\chi^{\sigma\tau} - \varepsilon_{\mu\nu\rho\sigma}\partial_{\lambda}\xi^{\lambda\sigma}$$

EM-like duality of fermions

- We construct topological operators [Yi-Nan Wang, YZ '23]

$$U_\lambda(\mathcal{S}^1) = \exp(\int_{\mathcal{S}^1} (\bar{\lambda} \star \mathcal{J}^{(m)}_{(3)} + \star \bar{\mathcal{J}}^{(m)}_{(3)} \lambda))$$

  acting on

$$T_\theta(\mathcal{M}^{(2)}) = \exp(i \int_{\mathcal{M}^{(2)}} (\bar{\theta}\chi_{(2)} + \bar{\chi}_{(2)}\theta))\,.$$

- In general d, a fermionic p-form $(d \geq 2p+1)$ is dual to a fermionic $d-p-2$-form. [V. Lekeu and J. O'Connor 'Forthcoming]
- Gauge of the electric like fermionic higher-form symmetry will be discussed. [YZ et al. 'Forthcoming]
- Both $(4,0)$ and $(3,1)$ have fermionic electric and magnetic 2-form symmetries in the IR, due to $\psi_{\mu\nu}$.
- The electric one can be gauged in a non-trivial way. [YZ et al. 'Forthcoming] Breaking of these are still open questions.

The KK-reduction

- A very recent article pointed out that under some assumptions, there is a $U(1)$ global symmetry in $\mathcal{N} = (4,0)$ theory. [Monetro, Tartaglia '24]
- Kaluza-Klein compactification means we make one coordinate periodic, e.g. $x_5 \sim x_5 + 2\pi R$ such that $p_5 = \frac{n}{R}$.
- From $5d$ point of view, each massless field in $6d$ would provide a tower of massive states with mass $m_n = \frac{n}{R}$. (KK-tower)
- In ordinary theories, every field couples minimally to the metric $g_{\mu\nu}$.
- Under KK-compactification, $g_{\mu\nu}$ yields a massless photon in lower dimensions $A_{\hat{\mu}} = g_{\hat{\mu}5}$. (called KK-photon or graviphoton)
- The minimal coupling in $6d$ requires that KK-tower all couple to $A_{\hat{\mu}}$ in $5d$ and the labelling integer n is (proportional) to their $U(1)$ charges under $A_{\hat{\mu}}$.

The absence of KK-photon and global symmetry

- [Monetro, Tartaglia '24] suggest one should take the possible $\mathcal{N} = (4,0)$ theory to satisfy

Assumptions

• $6d$ $(4,0)$ theory is Lorentz invariant. (Lorentz-invariant S-matrix)

• One can take $6d$ flat space states and study the KK-reduction thereof.

- The point is that KK-tower will always be present, irrespective of the form of the $6d$ interactions, as long as Lorentz symmetry is preserved.
- From $5d$ point of view, KK-charge still exist.
- But there is not gravitphoton field there, since $C_{\mu\nu\rho\sigma}$ only reduce to the metric in $5d$.
- $\exists$ conserved KK-charge, $\nexists$ gauge boson couple to it (the would be KK-photon is absent). This is a global symmetry and $\mathcal{N} = (4,0)$ in the swampland.

What about other exotic theories?

- $D_{\mu\nu\rho}$ in the $\mathcal{N} = (3,1)$ multiplet would give a vector field $A_{\hat{\mu}} = D_{\hat{\mu}55}$.
- One cannot exclude that this vector couples to KK-charge, as the KK-photon does in ordinary gravity theories. (This statement appears in version 2 of [Monetro, Tartaglia '24] came on arXiv yesterday.)
- If this is the case, the corresponding symmetry would be gauged, and the (3,1) theory is not excluded by the no global symmetries conjecture.
- In parallel, there are also exotic multiplets with less super symmetry, i.e. $\mathcal{N} = (2,0)$, $\mathcal{N} = (1,0)$ or even $\mathcal{N} = (2,1)$ and $\mathcal{N} = (3,0)$. Similar arguments also apply there.

What about other exotic theories?

- Example $\mathcal{N}=(2,0)$ [Minasian, Strickland-Constable and YZ '21]

$V=(\mathbf{1},\mathbf{3},\mathbf{1})$	(Gravity)				
Field			$B^+_{\mu\nu}$	ψ^-_μ	$g_{\mu\nu}$
G_{little} rep			$(\mathbf{1},\mathbf{3},\mathbf{5})$	$(\mathbf{2},\mathbf{3},\mathbf{4})$	$(\mathbf{3},\mathbf{3},\mathbf{1})$
$V=(\mathbf{2},\mathbf{1},\mathbf{1})$	(Exotic Gravitino)				
Field	ϕ	λ^+	$B^-_{\mu\nu}$	$\psi^+_{\mu\nu}$	
G_{little} rep	$(\mathbf{1},\mathbf{1},\mathbf{4})$	$(\mathbf{2},\mathbf{1},\mathbf{5}+\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{4})$	$(\mathbf{4},\mathbf{1},\mathbf{1})$	
$V=(\mathbf{3},\mathbf{1},\mathbf{1})$	(Exotic Gravity)				
Field	ϕ	λ^+	$B^-_{\mu\nu}$	$\psi^+_{\mu\nu}$	$C_{[\mu\nu][\lambda\kappa]}$
G_{little} rep	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{2},\mathbf{1},\mathbf{4})$	$(\mathbf{3},\mathbf{1},\mathbf{5}+\mathbf{1})$	$(\mathbf{4},\mathbf{1},\mathbf{4})$	$(\mathbf{5},\mathbf{1},\mathbf{1})$

- $\mathcal{N}=(4,0)$ decomposes into $\mathcal{N}=(2,0)$ as:
  Exotic Gravity $\rightarrow$ Exotic Gravity + 4 Exotic Gravitino + 5 Tensor
- Exotic multiplets with less SUSY should also served as UV-completion for $5d$ SUGRAs with corresponding amount of SUSY.

Also $\mathcal{N} = (1, 0)$ exotic multiplets.

$6d$ $\mathcal{N} = (1, 0)$ [Minasian, Strickland-Constable and YZ '21]

$V = (\mathbf{1}, \mathbf{1}, \mathbf{1})$	(Tensor)				
Field	ϕ	λ^+	$B^-_{\mu\nu}$		
G_{little} rep	$(\mathbf{1}, \mathbf{1}, \mathbf{5})$	$(\mathbf{2}, \mathbf{1}, \mathbf{4})$	$(\mathbf{3}, \mathbf{1}, \mathbf{1})$		
$V = (\mathbf{1}, \mathbf{2}, \mathbf{1})$	(Gravitino$^+$)				
Field		λ^-	A_μ	ψ^+_μ	
G_{little} rep		$(\mathbf{1}, \mathbf{2}, \mathbf{5})$	$(\mathbf{2}, \mathbf{2}, \mathbf{4})$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$	
$V = (\mathbf{1}, \mathbf{3}, \mathbf{1})$	(Gravity)				
Field			$B^+_{\mu\nu}$	ψ^-_μ	$g_{\mu\nu}$
G_{little} rep			$(\mathbf{1}, \mathbf{3}, \mathbf{5})$	$(\mathbf{2}, \mathbf{3}, \mathbf{4})$	$(\mathbf{3}, \mathbf{3}, \mathbf{1})$
$V = (\mathbf{2}, \mathbf{1}, \mathbf{1})$	(Exotic Gravitino)				
Field	ϕ	λ^+	$B^-_{\mu\nu}$	$\psi^+_{\mu\nu}$	
G_{little} rep	$(\mathbf{1}, \mathbf{1}, \mathbf{4})$	$(\mathbf{2}, \mathbf{1}, \mathbf{5} + \mathbf{1})$	$(\mathbf{3}, \mathbf{1}, \mathbf{4})$	$(\mathbf{4}, \mathbf{1}, \mathbf{1})$	
$V = (\mathbf{3}, \mathbf{1}, \mathbf{1})$	(Exotic Gravity)				
Field	ϕ	λ^+	$B^-_{\mu\nu}$	$\psi^+_{\mu\nu}$	$C_{[\mu\nu][\lambda\kappa]}$
G_{little} rep	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(\mathbf{2}, \mathbf{1}, \mathbf{4})$	$(\mathbf{3}, \mathbf{1}, \mathbf{5} + \mathbf{1})$	$(\mathbf{4}, \mathbf{1}, \mathbf{4})$	$(\mathbf{5}, \mathbf{1}, \mathbf{1})$

Summary and discussion

Summary and discussion

Conclusion

- Exotic super Poincaré multiplet exist in 6*d* dimensions with chiral supersymmetry.
- Maximal SUSY case $\mathcal{N} = (4, 0)$ conjectured by Hull as UV-completion of 5*d* maximal SUGRA.
- It has nonvanishing gravitational anomalies while the pure R-symmetry anomalies and R/gravitational-mixed anomalies vanish.
- There exist fermionic higher form symmetries associated to $\psi_{\mu\nu}$.
- $\mathcal{N} = (4, 0)$ has global symmetries under assumptions and in the Swampland. Same test on $\mathcal{N} = (3, 1)$ remains open.

Outlook

- New generalization of anomalies for the gauge symmetry of $C_{\mu\nu\rho\sigma}$?
- 't Hooft anomalies of these fermionic higher form symmetries?
- Can one define theories based on exotic multiplets from top-down?
- Superconformal aspect of exotic theories less studied.

Further evidence and questions

- Similarly, one expects in the strong coupling limit of the classical non-linear 5d $\mathcal{N} = 8$ SUGRA an extra dimension opens up, and supersymmetry as well as the $USp(8)$ R-symmetry are preserved.
- 6d $(4, 0)$ theories can be the candidate, since it suggests global $E_{6(6)}$ symmetry while the scalars fit into the coset $E_{6(6)}/USp(8)$, which is the same as that parametrised by the scalars of five-dimensional maximal supergravity.
- One also expects that BPS states are protected in the strong coupling limit. Investigations of BPS spectrum and matching the states carrying central charges appearing in the SUSY algebra would provide further tests on the conjecture.
  See [arXiv:2209.11716] for a very recent review.

Chern-Simons terms

- On the $5d$ maximal SUGRA side, there is a topological term

$$S_{\rm CS} = \int k_{\Lambda\Sigma\Delta}\, A^\Lambda \wedge F^\Sigma \wedge F^\Delta$$

  where $k_{\Lambda\Sigma\Delta}$ is constant and the Λ, Σ, Δ are E_6 indices running from 1 to 27. There is a E_6 singlet in the cubic tensor product of the fundamentals $\mathbf{27} \otimes \mathbf{27} \otimes \mathbf{27} = \mathbf{1} \oplus \cdots$.
- There is refined structures - under $E_{6(6)} \longrightarrow SL(6) \times SL(2)$, $\mathbf{27} = (\mathbf{15}, \mathbf{1}) + (\mathbf{6}, \mathbf{2})$ and the only allowed trilinear couplings involve either three fields in $\mathbf{15}$ of $SL(6)$ that are $SL(2)$ singlets or a single vector field in $\mathbf{15}$ and a doublet of $SL(2)$ in $\mathbf{6}$ of $SL(6)$.

Chern-Simons terms

- Chern-Simons terms may appear as KK reduction of anomalies. But the anomalies are pure gravitational (contains only Pontryagin classes). Only terms like $A \wedge p_1$ are expected, however there is no room for those to be present in the Lagrangian.
- Chern-Simons terms can be by integrated massive modes coming from the KK reduction. [Bonetti, Grimm, Hohenegger '12, '13]

$$
\begin{aligned}
& iq\bar{\psi}\gamma^{\mu}A_{\mu}\psi \\
& iq\bar{\psi}_{\rho}\gamma^{\rho\mu\nu}A_{\mu}\psi_{\nu} \\
& \pm\frac{1}{4}iq\epsilon^{\mu\nu\rho\sigma\tau}\bar{B}_{\mu\nu}A_{\rho}B_{\sigma\tau}.
\end{aligned}
$$

Chern-Simons terms

The chiral fermions in the **48** of E_6, in order to get a minimal coupling in 5d we need

$$iqc_{\alpha ij}\bar{\psi}^i\gamma^\mu A_\mu^\alpha\psi^j,$$

here i, j are **48** indices and $c_{\alpha ij}$ is a constant. For this term to be non-vanishing there must be a E_6 singlet contained in $\mathbf{48}\otimes\mathbf{27}\otimes\mathbf{48}$, this is true. So we can write down a 5d coupling, but in order to lift it to 6d we must complete the term

$$iqc_{\alpha ij}\bar{\psi}^i\gamma^\mu B_{\mu 5}^\alpha\psi^j$$

to a 6d Lorentz scalar. The easiest way is to put a derivative on B and we arrive

$$iqc_{\alpha ij}\bar{\psi}^i\Gamma^M\partial^N B_{MN}^\alpha\psi^j.$$

But this is ruled out for the reason that we recognize $\partial^N B_{MN}^\alpha = 0$ serves like the Lorenz gauge just as in the case for Abelian vector field.

Anomaly polynomials

p	$I_4^{(p)}$
0	$-\frac{1}{24}\,p_1$
1	$\frac{23}{24}\,p_1$
2	$-p_1$
I_4^A	$-\frac{1}{24}\,p_1$

The anomaly polynomials for chiral fermionic p-forms in $D = 2$.

p	$I_8^{(p)}$
0	$\frac{1}{5\,760}\left(7\,p_1^2 - 4\,p_2\right)$
1	$\frac{1}{5\,760}\left(275\,p_1^2 - 980\,p_2\right)$
2	$\frac{1}{5\,760}\left(790\,p_1^2 + 2\,840\,p_2\right)$
3	$\frac{1}{5\,760}\left(790\,p_1^2 + 2\,840\,p_2\right)$
4	$\frac{1}{5\,760}\left(275\,p_1^2 - 980\,p_2\right)$
5	$\frac{1}{5\,760}\left(7\,p_1^2 - 4\,p_2\right)$
6	0
I_8^A	$\frac{1}{5\,760}\left(16\,p_1^2 - 112\,p_2\right)$

The anomaly polynomials for chiral fermionic p-forms in $D = 6$.

Anomaly polynomials of chiral p-forms in $D = 10$

p	$I_{12}^{(p)}$
0	$\frac{1}{967\,680}\left(-31\,p_1^3 + 44\,p_1 p_2 - 16\,p_3\right)$
1	$\frac{1}{967\,680}\left(225\,p_1^3 - 1\,620\,p_1 p_2 + 7\,920\,p_3\right)$
2	$\frac{1}{967\,680}\left(2\,412\,p_1^3 + 27\,792\,p_1 p_2 - 186\,048\,p_3\right)$
3	$\frac{1}{967\,680}\left(7\,980\,p_1^3 + 162\,960\,p_1 p_2 - 73\,920\,p_3\right)$
4	$\frac{1}{967\,680}\left(13\,734\,p_1^3 + 338\,184\,p_1 p_2 + 764\,064\,p_3\right)$
5	$\frac{1}{967\,680}\left(13\,734\,p_1^3 + 338\,184\,p_1 p_2 + 764\,064\,p_3\right)$
6	$\frac{1}{967\,680}\left(7\,980\,p_1^3 + 162\,960\,p_1 p_2 - 73\,920\,p_3\right)$
7	$\frac{1}{967\,680}\left(2\,412\,p_1^3 + 27\,792\,p_1 p_2 - 186\,048\,p_3\right)$
8	$\frac{1}{967\,680}\left(225\,p_1^3 - 1\,620\,p_1 p_2 + 7\,920\,p_3\right)$
9	$\frac{1}{967\,680}\left(-31\,p_1^3 + 44\,p_1 p_2 - 16\,p_3\right)$
10	0
I_{12}^{A}	$\frac{1}{967\,680}\left(-256\,p_1^3 + 1\,664\,p_1 p_2 - 7\,936\,p_3\right)$

Examples and anomaly cancellations

$$D : \mathcal{C}^\infty(S^+ \otimes V) \longrightarrow \mathcal{C}^\infty(S^- \otimes V)$$

Rarita-Schwinger complex for chiral gravitini ψ_μ

$V = T^*M$ The WRONG guess!

There is gauge symmetry $\delta\psi_\mu = \partial_\mu \epsilon$ for free Rarita-Schwinger fields.

Considering the ghost spectrum, the correct *Rarita-Schwinger* complex is

$$V = T^*M - 1$$

$$\Longrightarrow \quad I_{3/2} = \frac{1}{967\,680}\left(225\, p_1^3 - 1\,620\, p_1 p_2 + 7\,920\, p_3\right)$$

Examples and anomaly cancellations

$$I_{1/2} = \frac{1}{967\,680}\left(-31\,p_1^3 + 44\,p_1 p_2 - 16\,p_3\right)$$

$$I_A = \frac{1}{967\,680}\left(-256\,p_1^3 + 1\,664\,p_1 p_2 - 7\,936\,p_3\right)$$

$$I_{3/2} = \frac{1}{967\,680}\left(225\,p_1^3 - 1\,620\,p_1 p_2 + 7\,920\,p_3\right)$$

Anomaly cancellation [Alvarez-Gaumé, Witten '83]:

$$-I_{1/2} + I_{3/2} + I_A = 0\,!$$

It follows that a $10d$ theory with one anti-chiral complex spinor, one chiral complex gravitino and one real self-dual antisymmetric tensor field (modulo fields not contributing to anomalies) is anomaly free.

This is exactly the field contents of the $\mathcal{N} = (2,0)$ IIB SUGRA multiplet!

Examples and anomaly cancellations

Pure gravitational anomalies only arise in spacetime dimension $2n = 4k + 2$. Let us look at the 10-dimensional cases, i.e. $k = 2$.

$$D : \mathcal{C}^\infty(S^+ \otimes V) \longrightarrow \mathcal{C}^\infty(S^- \otimes V)$$

Dirac complex for Weyl spinors ψ

$$V = 1 \quad \Longrightarrow \quad I_{\frac{1}{2}} = \tfrac{1}{967\,680}\left(-31\,p_1^3 + 44\,p_1 p_2 - 16\,p_3\right)$$

Self-dual complex for 4-form gauge fields $A_{\mu\nu\rho\sigma}$ (with 5-form self-dual field strength $F_{\mu\nu\rho\sigma\kappa}$)

$$V = S^+ \quad \Longrightarrow \quad I_A = \tfrac{1}{967\,680}\left(-256\,p_1^3 + 1\,664\,p_1 p_2 - 7\,936\,p_3\right)$$