Consistency of the pion form factor and unpolarized TMDs beyond leading twist in the light-front quark model

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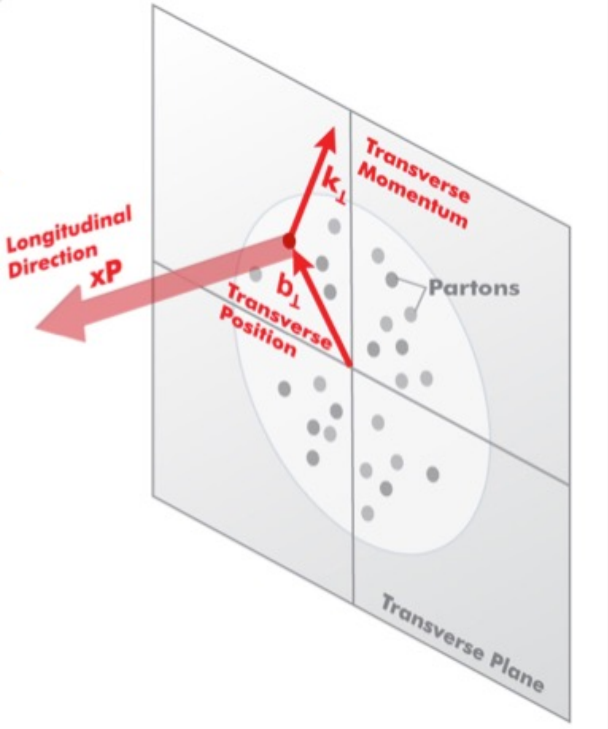
2. Light-Front Quark Model(LFQM)
   - New Development of self-consistent LFQM
   - Pion Form Factor

3. TMDs and PDFs of pion

4. Conclusions

1. Motivation

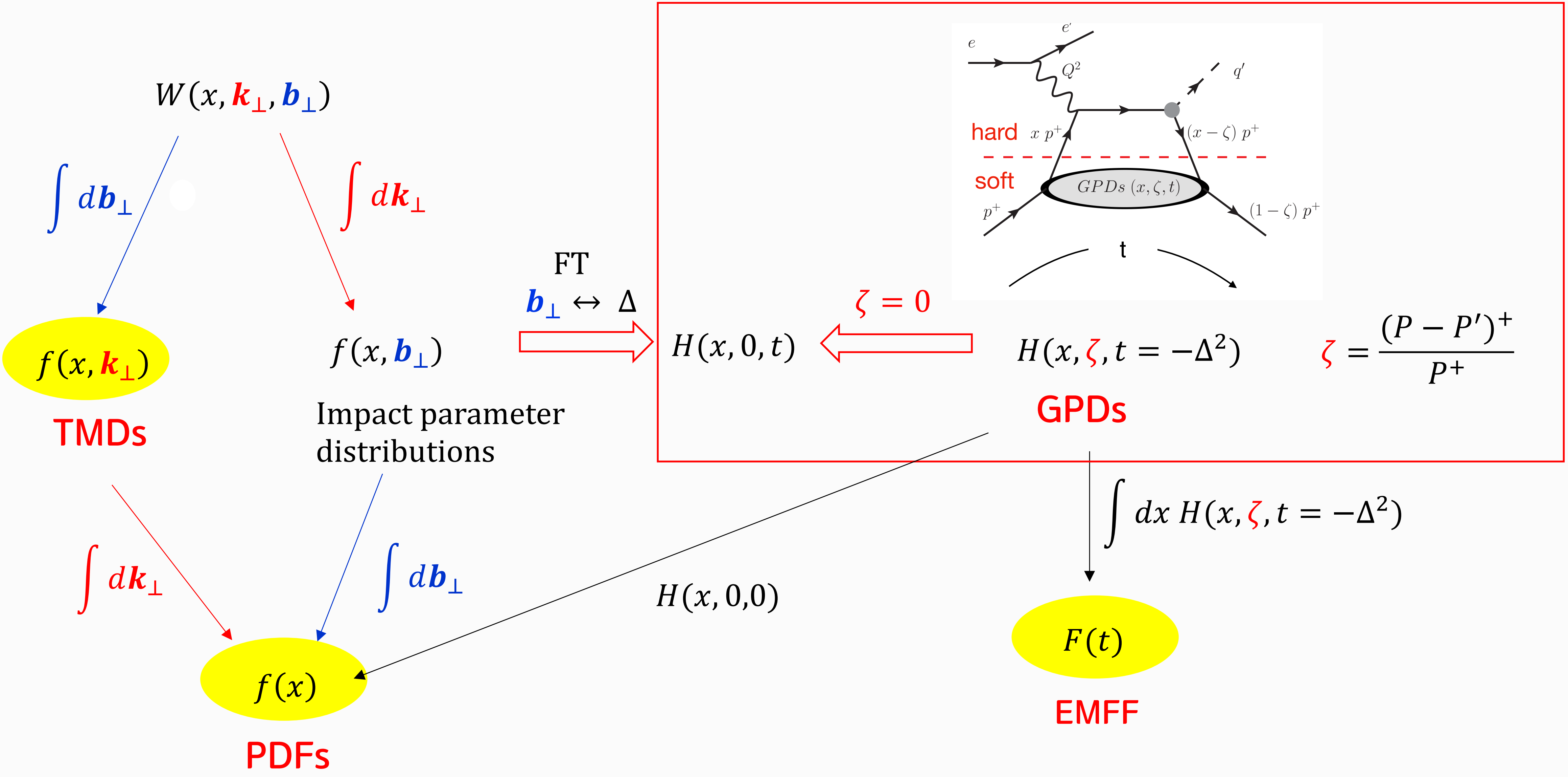
For precision 3D imaging of hadrons, it is essential to access x of partons, along with their $\boldsymbol{k}_\perp$ and $\boldsymbol{b}_\perp$, relative to the hadron's direction of motion.



Wigner distribution $W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$ unifies $(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$ into a 5D phase-space representation of parton structure.

Distribution	Projection from $W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$
TMDs	Integrate over $\boldsymbol{b}_\perp$
GPDs	Integrate over $\boldsymbol{k}_\perp$
PDFs	Integrate over both $\boldsymbol{b}_\perp$ and $\boldsymbol{k}_\perp$
EM Form Factor	Lowest x-moment of GPDs

3D hadron structure from 5D tomography



$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$

$\int d\boldsymbol{b}_\perp$

$\int d\boldsymbol{k}_\perp$

$f(x, \boldsymbol{k}_\perp)$

TMDs

$f(x, \boldsymbol{b}_\perp)$

Impact parameter distributions

FT

$\boldsymbol{b}_\perp \leftrightarrow \Delta$

$H(x, 0, t)$

$\zeta = 0$

$H(x, \zeta, t = -\Delta^2)$

$$\zeta = \frac{(P - P')^+}{P^+}$$

GPDs

$\int d\boldsymbol{k}_\perp$

$\int d\boldsymbol{b}_\perp$

$H(x, 0{,}0)$

$\int dx\ H(x, \zeta, t = -\Delta^2)$

$f(x)$

PDFs

$F(t)$

EMFF

☞ We investigate the interplay among the pion's **EMFF, TMDs, and PDFs** in the **Light-Front Quark Model (LFQM).**

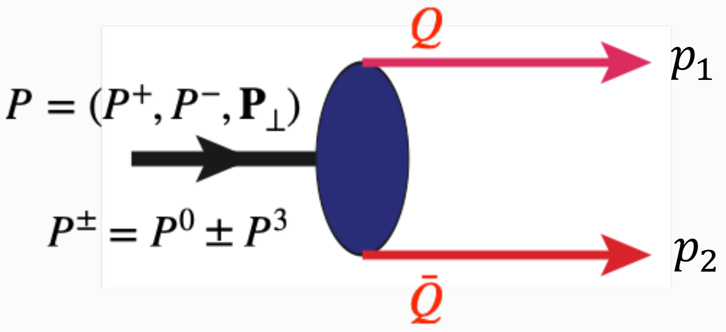
TMD	**Twist**	**Dirac Structure**	**Interpretation**
$f_1^\pi(x, \boldsymbol{k}_\perp)$	2	γ^+	Probability density for unpolarized quarks
$e^\pi(x, \boldsymbol{k}_\perp)$	3	$\mathbb{I}$(scalar)	Sensitive to quark mass and chiral sym. breaking
$f_3^\pi(x, \boldsymbol{k}_\perp)$	3	γ^i	Related to transvers motion; contributes to azimuthal asymmetries
$f_4^\pi(x, \boldsymbol{k}_\perp)$	4	γ^-	Subleading in $1/Q$; relevant for power-suppressed contributions

- Three ($f_1^\pi, f_3^\pi, f_4^\pi$) of them are related with the forward matrix elements $\langle P|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle$, i.e., EMFF of the pion.

2. Light-Front Quark Model(LFQM)

☞ Two essential aspects of our LFQM

1) Meson state: Noninteracting "on-mass" shell Q & $\bar{Q}$ representation.



LF energy conservation

$$\boxed{P^- = p_1^- + p_2^-} \longleftrightarrow \boxed{M = M_0}$$

$$M_0^2 = \frac{m_1^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_2^2 + \boldsymbol{k}_\perp^2}{1-x}$$

2) The interaction between $Q\bar{Q}$ is incorporated into the mass operator via $M := M_0 + V_{Q\bar{Q}}$

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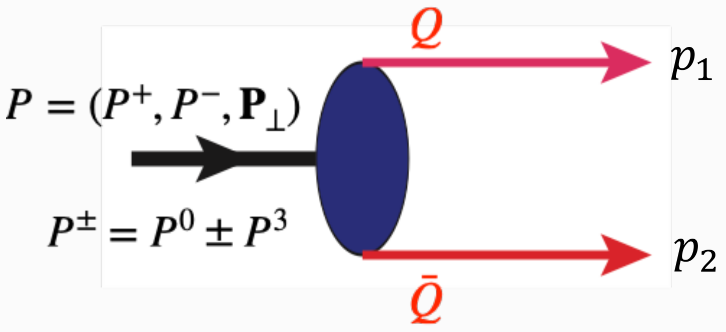

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Bakamjian-Thomas(BT) constuction!

Self-consistent LFQM based on the BT construction

PRD 89, 033011(14); PRD91, 014018(15); PRD95, 056002(17) by HMC and C.-R. Ji   PRD 103, 073004(21); Adv. High Energy Phys., 4277321(21) by HMC

PRD 107, 053003(23); PRD108, 013006(23) by A. J. Arifi, HMC and C.-R. Ji

$$\langle P'|\bar{q}\ \Gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\, \mathcal{F}$$

$\mathcal{F}$: Physical observables $\wp^\mu$: Lorentz factors

- Apply BT ($M \rightarrow M_0$) **equally to both sides**

$$\boxed{M \rightarrow M_0}$$

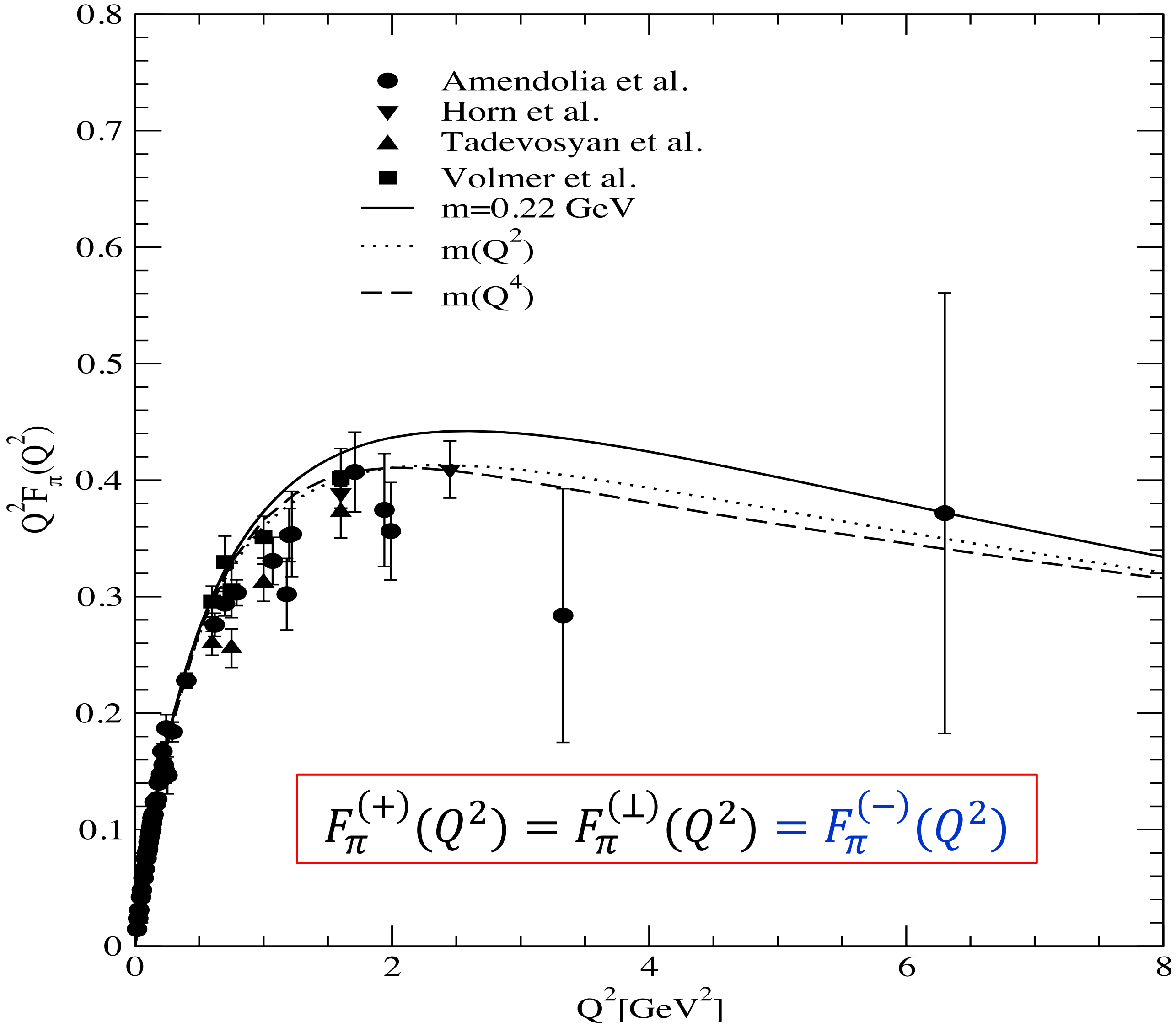
$$\langle P'|\bar{q}\ \Gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\, \mathcal{F}$$

$$\left.\begin{aligned} \wp^\mu &= (P+P')^\mu - q^\mu \frac{(P+P')\cdot q}{q^2} \\ q^\mu &= (P-P')^\mu \end{aligned}\right\}\ \wp\cdot q = 0$$

$$\boxed{\mathcal{F} = \left\langle P'\left|\frac{\bar{q}\ \Gamma^\mu q}{\wp^\mu}\right|\mathrm{P}\right\rangle_{\mathrm{BT}}}$$

becomes independent of the current components!

Current-component independent EMFF



$$F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$$

$f_\pi^{LFQM} = 130$ MeV

(Exp.=131 MeV)

$r_\pi^{LFQM} = 0.654$ fm

(Exp.=0.659(4)fm)

We have demonstrated, for the first time, the **current-component independence** of the FF within the LFQM.

3. TMDs and PDFs of pion

C. Lorce, B. Pasquini, P. Schweitzer; EPJC76,415(2016)

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^+\psi(z)|P\rangle|_{z^+=0} = f_1^q(x, p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma_T^j\psi(z)|P\rangle|_{z^+=0} = \frac{p_T^j}{P^+} f_3^q(x, p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^-\psi(z)|P\rangle|_{z^+=0} = \left(\frac{m_\pi}{P^+}\right)^2 f_4^q(x, p_T),$$

In the forward limit

$$\begin{aligned} 2P^+ \int dx\, f_1^q(x) &= \langle P|\bar{\psi}(0)\gamma^+\psi(0)|P\rangle, \\ 2p_T \int dx\, f_3^q(x) &= \langle P|\bar{\psi}(0)\gamma^\perp\psi(0)|P\rangle, \\ 4P^- \int dx\, f_4^q(x) &= \langle P|\bar{\psi}(0)\gamma^-\psi(0)|P\rangle, \end{aligned}$$

PDF

TMD

$$f(x) = \int d^2p_T\, f(x, p_T).$$

Schematic descriptions of EMFF, TMDs, and PDFs of the Pion

$$\langle P'|\, J^\mu |\mathrm{P}\rangle = \wp^\mu\, F_\pi(q^2) \qquad\qquad \wp \cdot q = 0$$

EMFF $$F_\pi^{(\mu)}(Q^2) = \left\langle P'\middle|\, \frac{J^\mu}{\wp^\mu}\, \middle|\mathrm{P}\right\rangle_{\mathrm{BT}} = \iint dx\, d^2\boldsymbol{k}_\perp\; f^{(\mu)}(x, \boldsymbol{k}_\perp, Q^2)$$

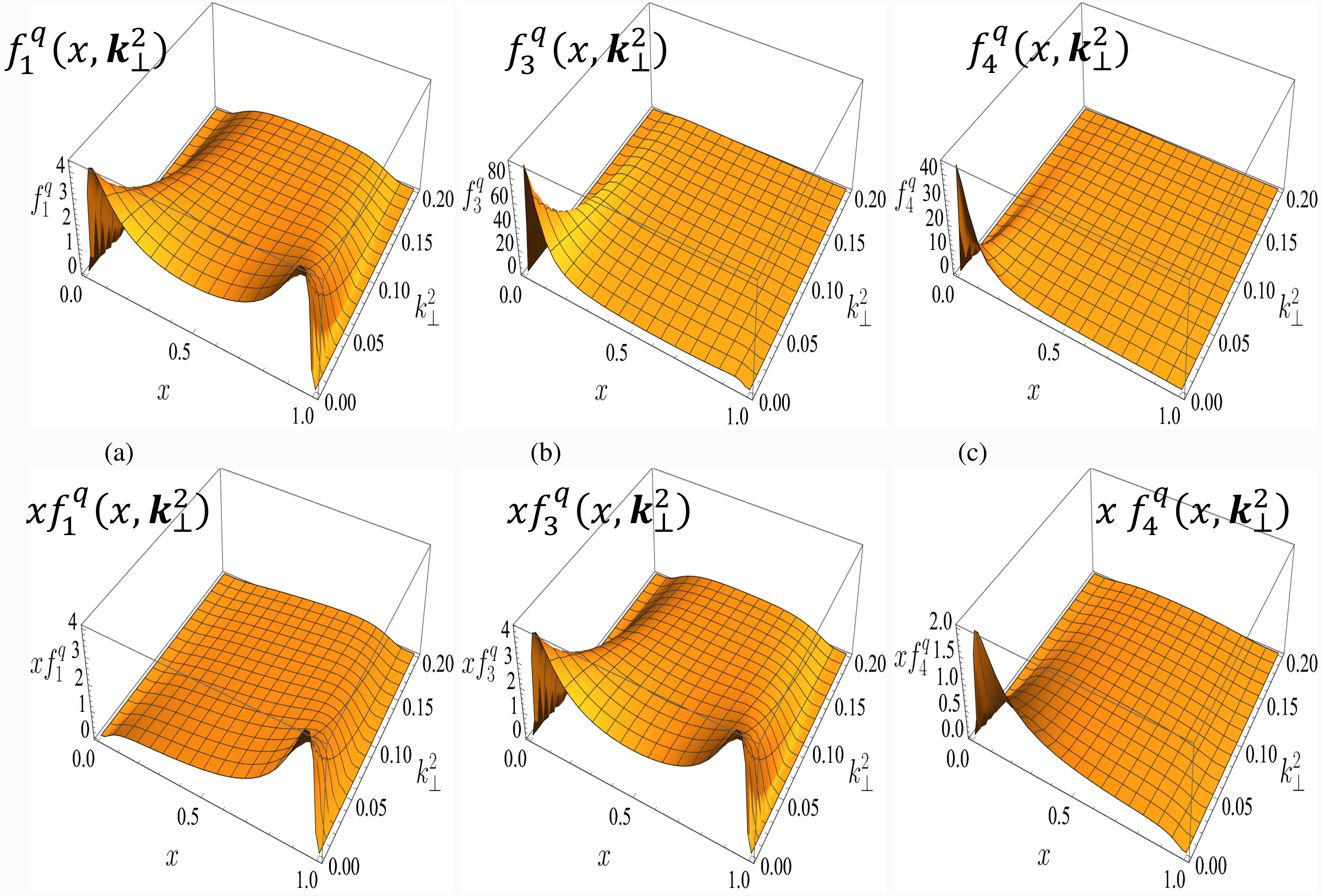
as $Q^2 \to 0$

$$F_\pi^{(\mu)}(0) = 1 = \lim_{Q\to 0} \left\langle P'\middle|\, \frac{J^\mu}{\wp^\mu}\, \middle|\mathrm{P}\right\rangle_{\mathrm{BT}} = \iint dx\, d^2\boldsymbol{k}_\perp\; f^{(\mu)}(x, \boldsymbol{k}_\perp)$$ TMDs

$$= \int dx\; f^{(\mu)}(x)$$ PDFs

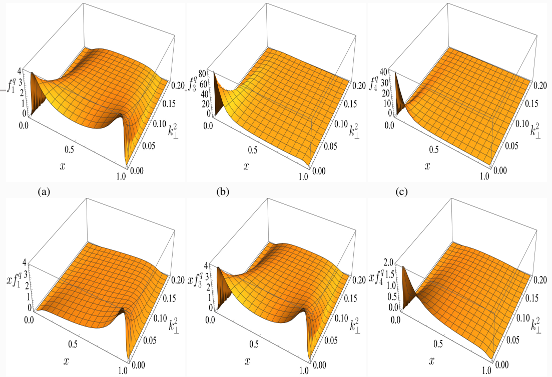
Precise extraction of EMFF is crucial for correctly determining TMDs and PDFs!

Unpolarized TMDs for Pion



(a) (b) (c)

$$xf_3^q(x, \boldsymbol{k}_\perp^2) = f_1^q(x, \boldsymbol{k}_\perp^2)$$

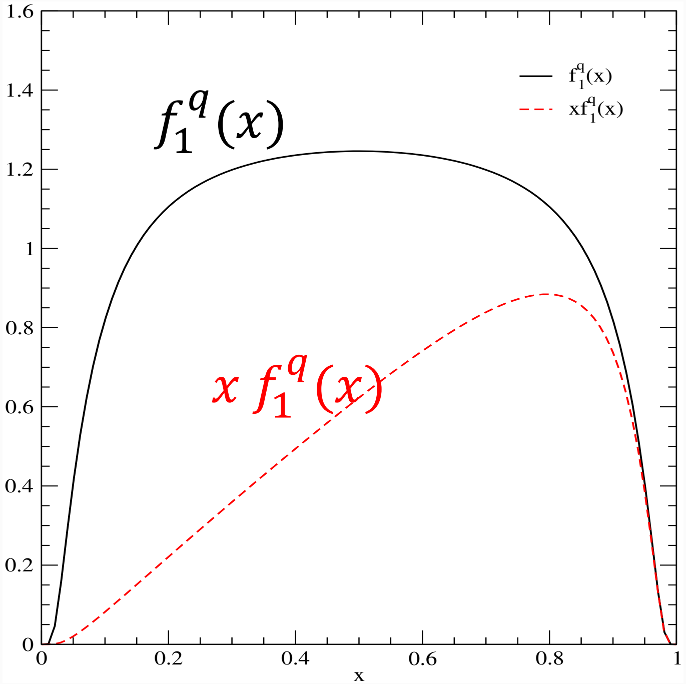


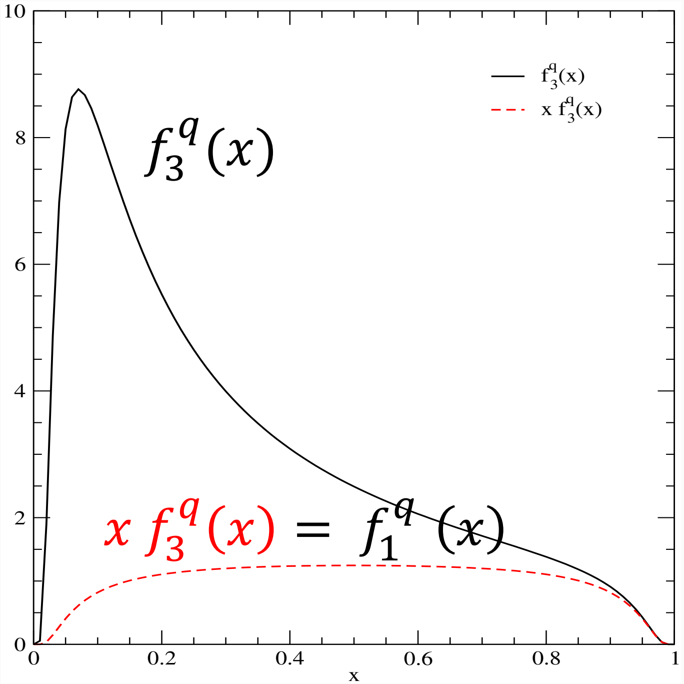
$$\int d^2\boldsymbol{k}_\perp$$

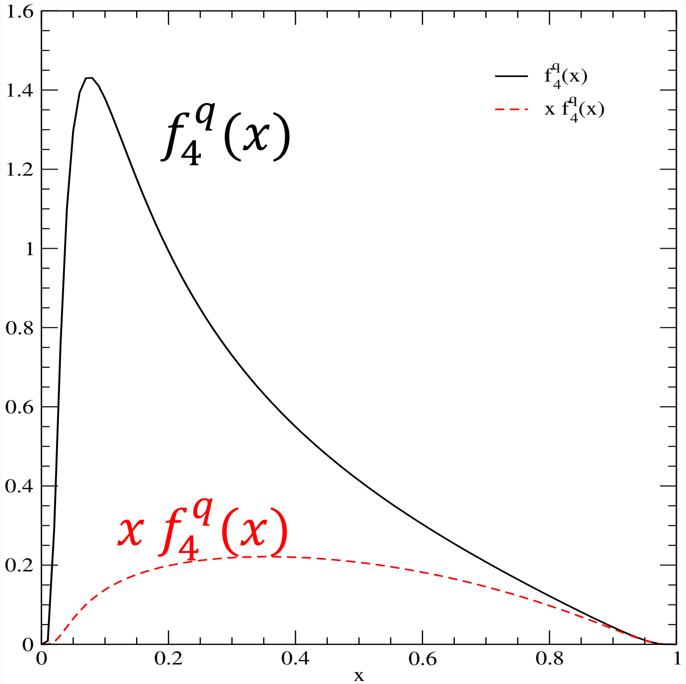
We prove this, for the first time.

$$\boxed{2\int dx\ f_4^q(x) = 1}$$

$$\int dx\ f_1^q(x) = 1$$

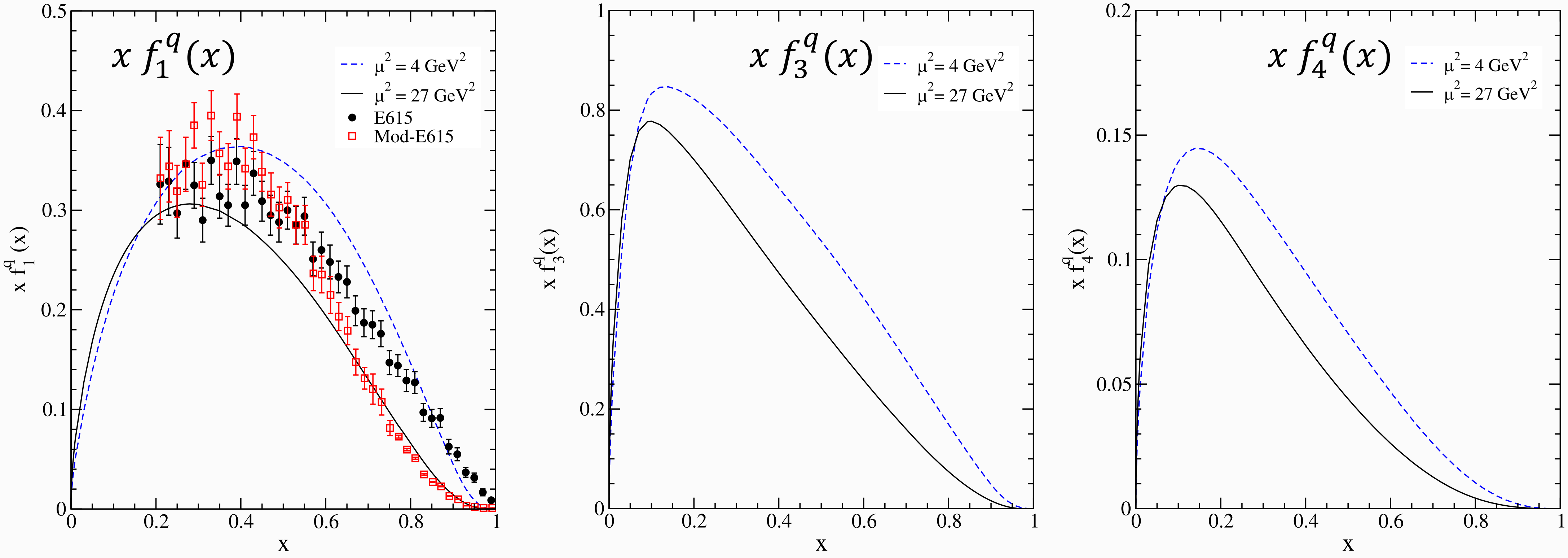






QCD Evolution of Pion PDFs

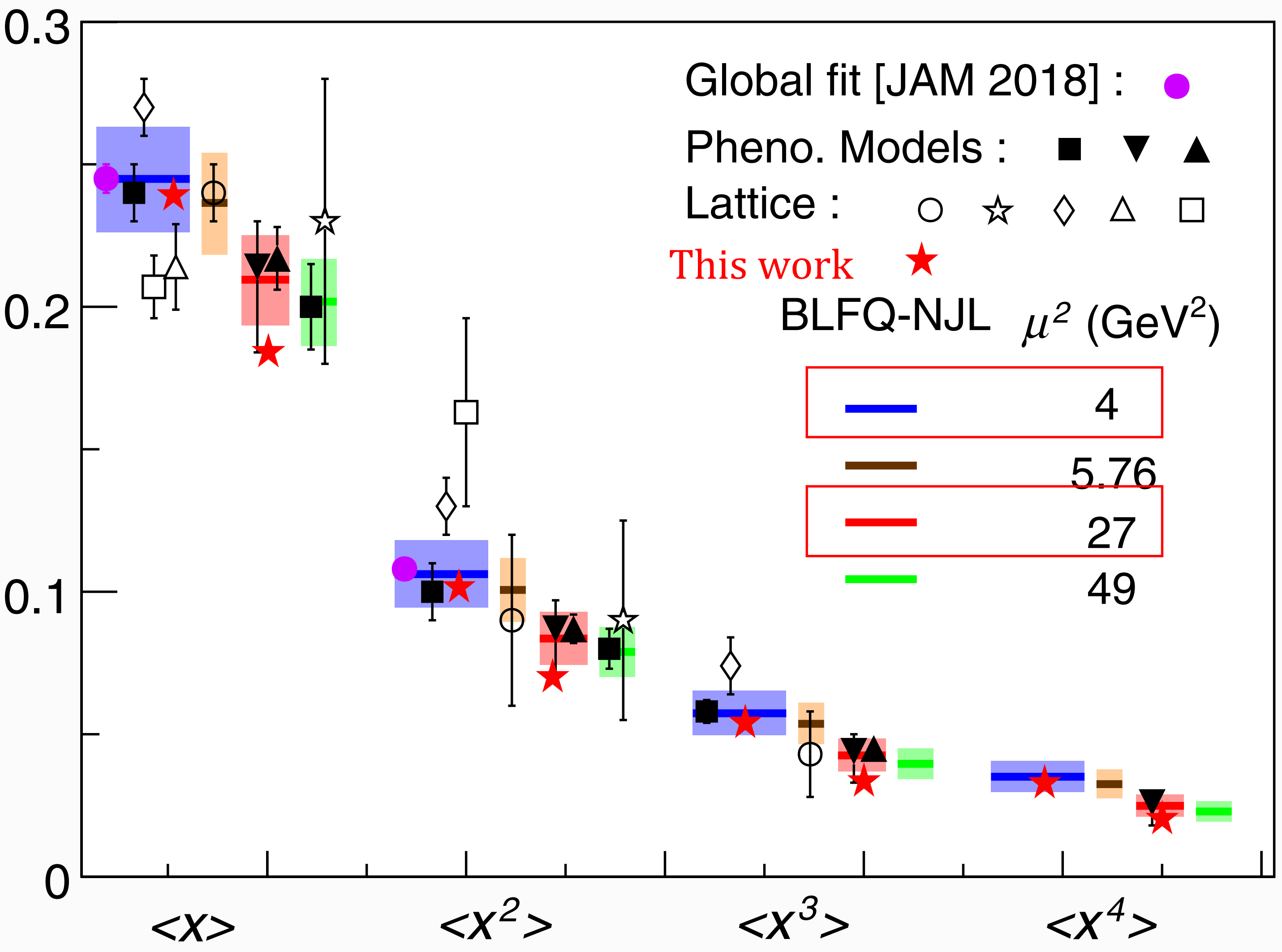
Evolved from $\mu_0^2 = 1\ \text{GeV}^2$ to $\mu^2 = 4$ and $27\ \text{GeV}^2$



We use the **Higher Order Perturbative Parton Evolution toolkit (HOPPET)** to solve the NNLO DGLAP equation.

Lowest four moments of pion valence PDF

Mellin moments: $\langle x^n \rangle = \int_0^1 dx\ x^n f(x)$



Adopted from J. Lan et al. (BLFQ Collab.), PRL 122, 172001 (2019)

4. Conclusions

- We developed a new method for ensuring self-consistency in the LFQM.

Our LFQM: Noninteracting Q & $\bar{Q}$ representation consistent with the Bakamjian-Thomas(BT) constuction!

$$P^- = p_q^- + p_{\bar{q}}^- \text{ , i. e. } M^2 \rightarrow M_0^2$$

$$\langle 0|\bar{q}\,\Gamma^\mu q|\mathrm{P}\rangle = \mathfrak{F}\,\wp^\mu$$

$\mathfrak{F}$: physical observables

$\wp^\mu$: Lorentz factors

$$\downarrow$$

$$\mathfrak{F} = \left\langle 0 \left| \frac{\bar{q}\Gamma^\mu q}{\wp^\mu} \right| \mathrm{P} \right\rangle = \iint dx\, d^2\mathbf{k}_\perp \cdots \boxed{\left(\frac{\Gamma^\mu}{\wp^\mu}\right)} \cdots$$

Constrained by BT construction!

⇨ This allows one to obtain the physical observables independent of the current components !

Partial Extractions of TMD, PDF, GPD from Pion Form Factor

Form factor: $F^{(\mu)}(t) \equiv \iint dx\, d\boldsymbol{k}_\perp\, f^{(\mu)}(x, \boldsymbol{k}_\perp, t)$ Note) $Q^2 \rightarrow -t$

$f^{(\mu)}(x, \boldsymbol{k}_\perp, t \rightarrow 0)$

$\int d\boldsymbol{k}_\perp$

TMD $f(x, \boldsymbol{k}_\perp)$

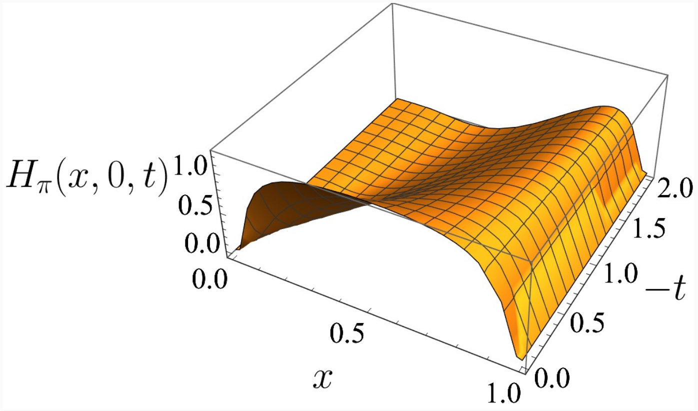
GPD $H(x, 0, t)$

$\boxed{f_1^q(x, \boldsymbol{k}_\perp) \leftrightarrow f^{(+)}(x, \boldsymbol{k}_\perp, 0)}$

$\boxed{2f_4^q(x, \boldsymbol{k}_\perp) \leftrightarrow f^{(-)}(x, \boldsymbol{k}_\perp, 0)}$

$H(x, 0, t) = \int d\boldsymbol{k}_\perp\, f^{(+)}(x, \boldsymbol{k}_\perp, t)$

GPD at $\zeta = 0$



$\int d\boldsymbol{k}_\perp$

$H(x, 0, 0)$

PDFs $f_1^q(x)$: twist-2 PDF

$f_4^q(x)$: twist-4 PDF