N-$c\bar{c}$ interaction from lattice QCD

Yan Lyu (吕岩)
iTHEMS, RIKEN
May 27, 2025

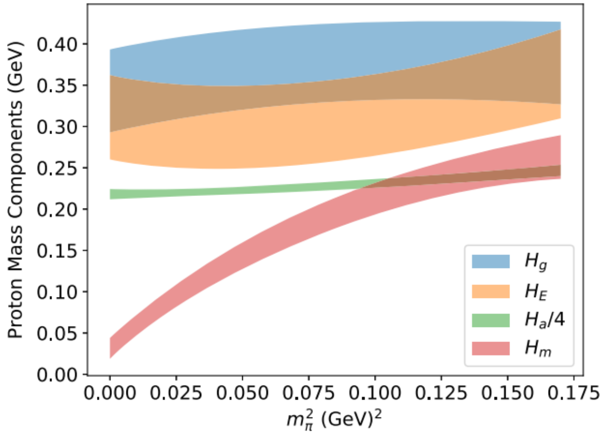
Based on: YL T. Doi, T. Hatsuda, and T. Sugiura, Phys. Lett. B 860 139178 (2025)

N-$c\bar{c}$ interaction and hadron masses

➢ The proton mass

M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Phys. Lett. B 78 (1978)
X.-D. Ji, Phys. Rev. Lett. 74 (1995)

- $c\bar{c}$ is an ideal probe to detect gluonic structure of proton
- The N-J/ψ forward scattering amplitude gives an access to the trace anomaly contribution of proton mass, as containing $\langle N|G_{\mu\nu}G^{\mu\nu}|N\rangle$



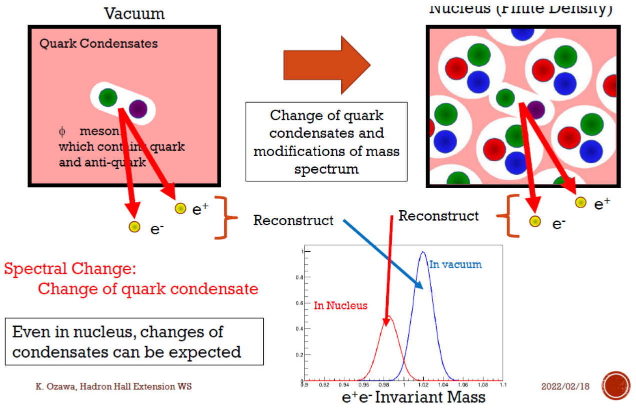
Y.-B. Yang *et. al.*, Phys. Rev. Lett. 121 212001 (2018)

➢ The J/ψ mass modification in nuclear medium

A. Hayashigaki, Prog. Theor. Phys. 101 (1999), T. Hatsuda, and S. H. Lee, Phys. Rev. C 46 R34 (1992)

- To be measured via e^+e^- decay at J-PARC
- Closely related to the charm quark condensate
- Proportional to N-J/ψ scattering length

$$\boxed{\delta m_{J/\psi} \simeq -\frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a_{J/\psi}^{\text{spin-av}} \rho_{\text{nm}}}$$

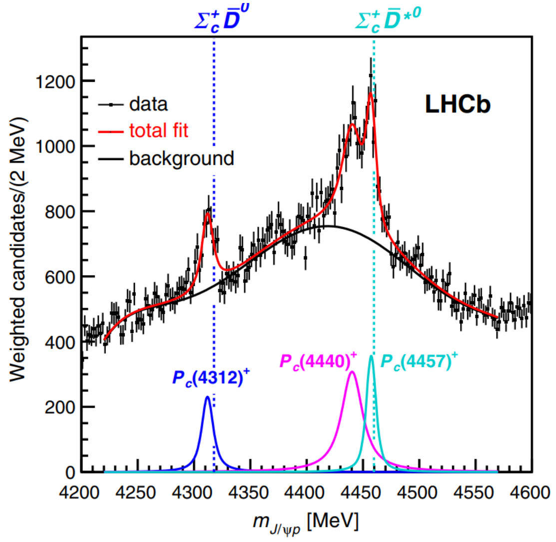


N-$c\bar{c}$ interaction and multi-quark/hadron state

- The pentaquark states P_c

R. Aaij, *et al.* [LHCb Coll.], Phys. Rev. Lett. 115 072001 (2015); Phys. Rev. Lett. 122 (22) 222001 (2019)

- Appear as peaks in the $p\,J/\psi$ spectrum
- Lots of discussions on their quantum number, production/decay properties…
- A full coupled-channel study needs N-$c\bar{c}$ interaction



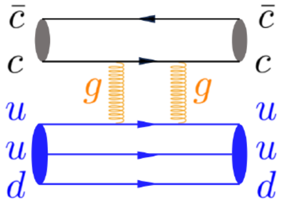
- Predicted charmonium-nucleus bound states (A-$c\bar{c}$)

- The existence of A-$c\bar{c}$ is in agreement among theories
- Predicted binding energies are disparate
- A reliable prediction calls for accurate N-$c\bar{c}$ interaction

S. Brodsky, I. Schmidt, and G. Teramond, Phys. Rev. Lett. 64 (1990)
J.-J. Wu, T.-S. H. Lee, Phys. Rev. C 86 (2012)
A. Yokota, E. Hiyama, M. Oka, PTEP 2013 (11) (2013)
G. Krein, A. Thomas, K. Tsushima, Prog. Part. Nucl. Phys. 100 (2018)

N-$c\bar{c}$ interaction

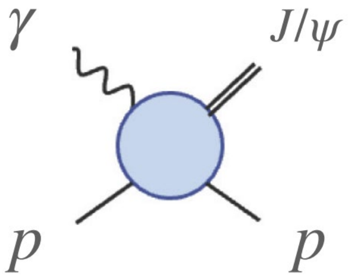
- Theoretically, a characteristic long-range behavior is expected
  - Multiple-gluon exchange is believed to manifest as two-pion exchange



J. Castella and G. Krein, Phys. Rev. D **98**, 014029 (2018)
H. Fujii and D. Kharzeev, Phys. Rev. D **60**, 114039 (1999)
G. Bhanot and M. E. Peskin, Nucl. Phys. B **156**, 391(1979)

- Experientially, large uncertainties remain
  - Vector-meson dominance: $a_{NJ/\psi} \sim O(1-10) \times 10^{-3}$ fm
  - Low-energy unitary: $a_{NJ/\psi} \sim O(1)$ fm

L. Pentchev, and I. I. Strakovsky, Eur. Phys. J. A 57 (2) 56 (2021)
D. Winney, *et al.* (Joint Physics Analysis Center), Phys. Rev. D 108 (5) 054018 (2023)



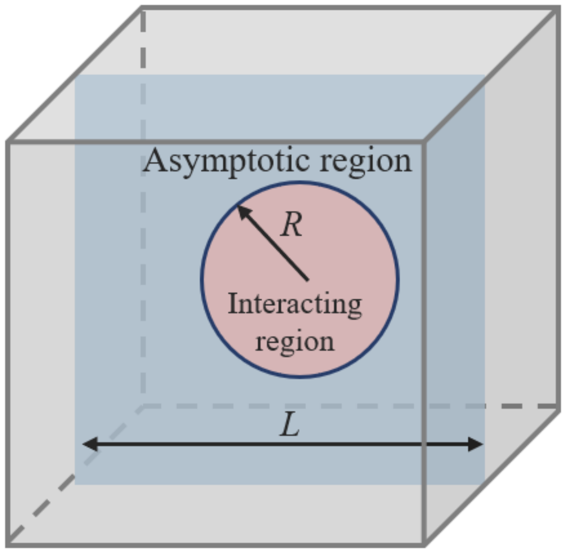
- Previous lattice calculations have been limited to heavy pion masses

K. Yokokawa, S. Sasaki, T. Hatsuda, A. Hayashigaki, Phys. Rev. D 74 (2006); L. Liu, H.-W. Lin, K. Orginos, PoS LATTICE2008 112 (2008)
T. Kawanai, S. Sasaki, Phys. Rev. D 82 (2010); T. Sugiura, Y. Ikeda, N. Ishii, PoS LATTICE2018, 093 (2019)
U. Skerbis, S. Prelovsek, Phys. Rev. D 99, (2019)

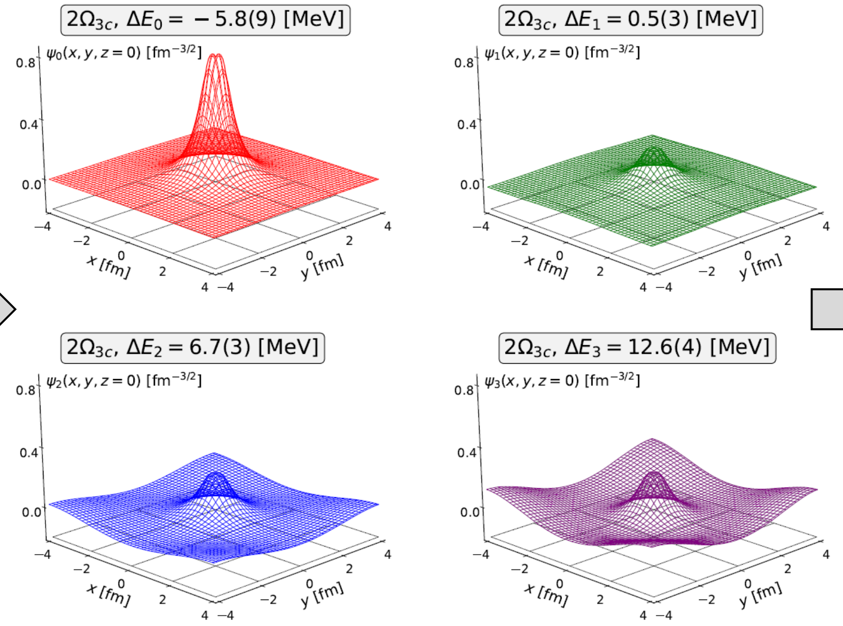
- This talk is going to present a first realistic lattice study on the low-energy N-$c\bar{c}$ interaction
  - Examine the expected behavior of the long-range potential
  - Determine near-threshold scattering properties

HAL QCD method

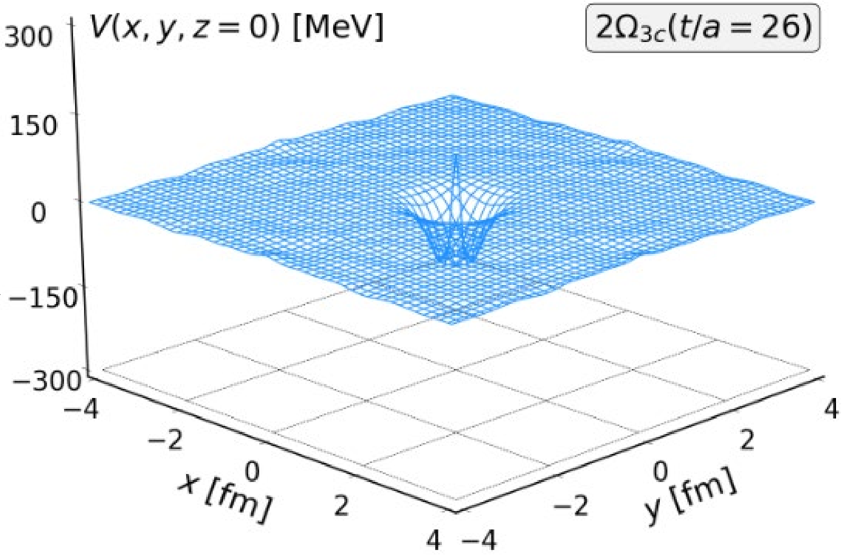
N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)
N. Ishii, *et al.* [HAL QCD Coll.], Phys. Lett. B 712, 437 (2012)



Lattice simulation

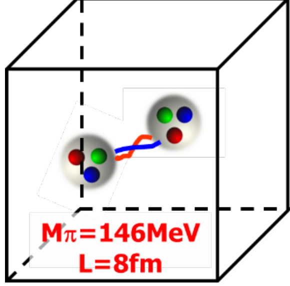


NBS amplitude

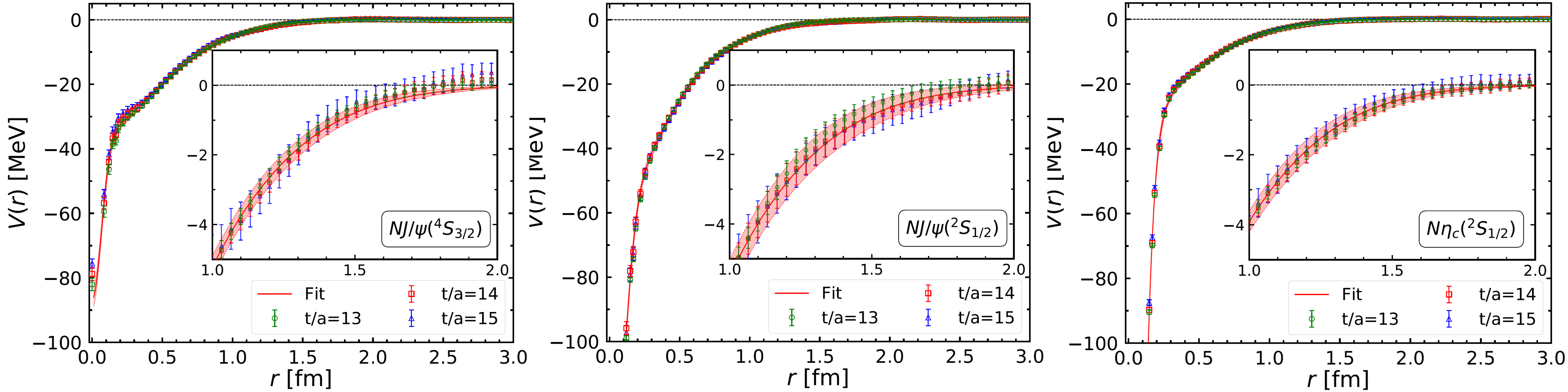


Potential

Potential



- S-wave N-$c\bar{c}$ potential at $t/a = 14 \pm 1$ ($t \simeq 1.2$ fm)

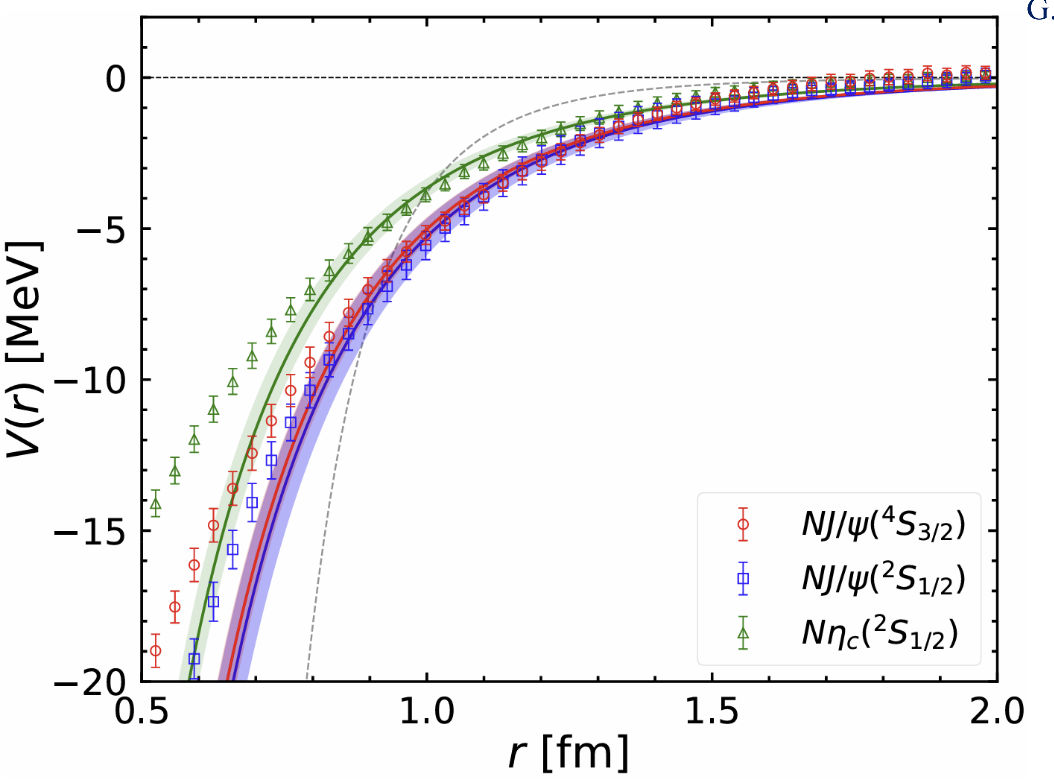


- The attractive potentials weakly depend on t→small systematic errors
- Attractive core, partially because of no Pauli exclusion, similar as $N\phi$, $N\Omega$
- The long-range potentials exhibit similar decreasing behavior→a common mechanism operates at long distances
- For later use, a puly phenomenological multiple Gaussian can describe the lattice data with $\chi^2/\mathrm{dof} \simeq 0.5$ (red bands)

Long-range potential

- The $N\text{-}Q\bar{Q}$ potential is believed to be induced by multiple gluon exchange
  - A QCD analogy of the van der Waals interaction between atoms/molecules
  - Photon can propagate over long distances, leading to $V(r) \sim -\frac{\alpha}{r^7}$ (the dashed line)
  - Gluon can NOT propagate over long distances, where color-neutral hadrons involves especially the lightest two-pion state, leading to $V(r) \sim -\alpha \frac{e^{-2m_\pi r}}{r^2}$ (bands)

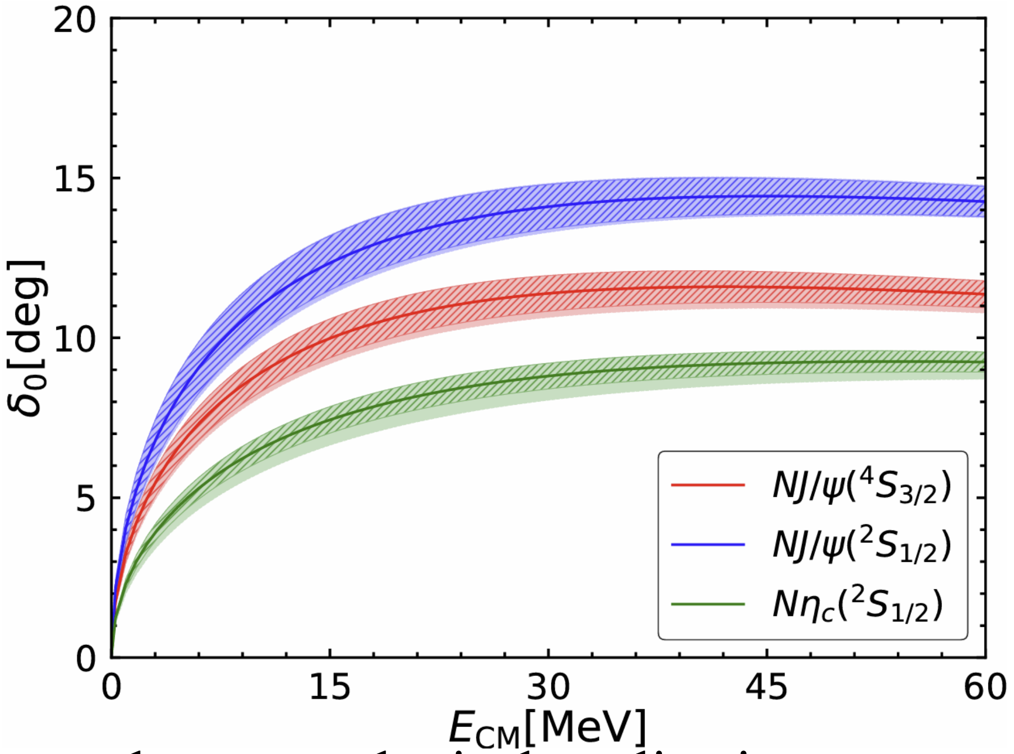
J. Castella and G. Krein, Phys. Rev. D **98**, 014029 (2018)
H. Fujii and D. Kharzeev, Phys. Rev. D **60**, 114039 (1999)
G. Bhanot and M. E. Peskin, Nucl. Phys. B **156**, 391(1979)



  - The long-range $N\text{-}c\bar{c}$ potential is consistent with the two-pion exchange interaction

Physical observables

- Scattering phase shifts



$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2$$

channel	a_0 [fm]	$r_{\rm eff}$ [fm]
N-$J/\psi(^4S_{3/2})$	$0.30(2)\left(^{+0}_{-2}\right)$	$3.25(12)\left(^{+6}_{-9}\right)$
N-$J/\psi(^2S_{1/2})$	$0.38(4)\left(^{+0}_{-3}\right)$	$2.66(21)\left(^{+0}_{-10}\right)$
$N\eta_c(^2S_{1/2})$	$0.21(2)\left(^{+0}_{-1}\right)$	$3.65(19)\left(^{+0}_{-6}\right)$

- A direct phenomenological application
  - The J/ψ mass modification in nuclear medium is related to the spin-averaged scattering length of N-J/ψ scattering

A. Hayashigaki, Prog. Theor. Phys. 101 (1999)

$$\delta m_{J/\psi} \simeq -\frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a_{J/\psi}^{\text{spin-av}} \rho_{\rm nm} = -19(3)\ \text{MeV}$$

Summary & Outlook

- **Summary:** we present a first realistic study on low-energy N-$c\bar{c}$ interaction based on (2+1)-flavor LQCD with nearly physical light quark masses and physical charm quark mass
  - The potential is found to be attractive at all distances
  - Long-range potential→consistent w/ TPE
  - Detailed analysis for near-threshold scattering properties

- **Outlook:**
  - The physical point simulations
  - The long-range potential for other hadron pairs

Thank for your attention

Comparison

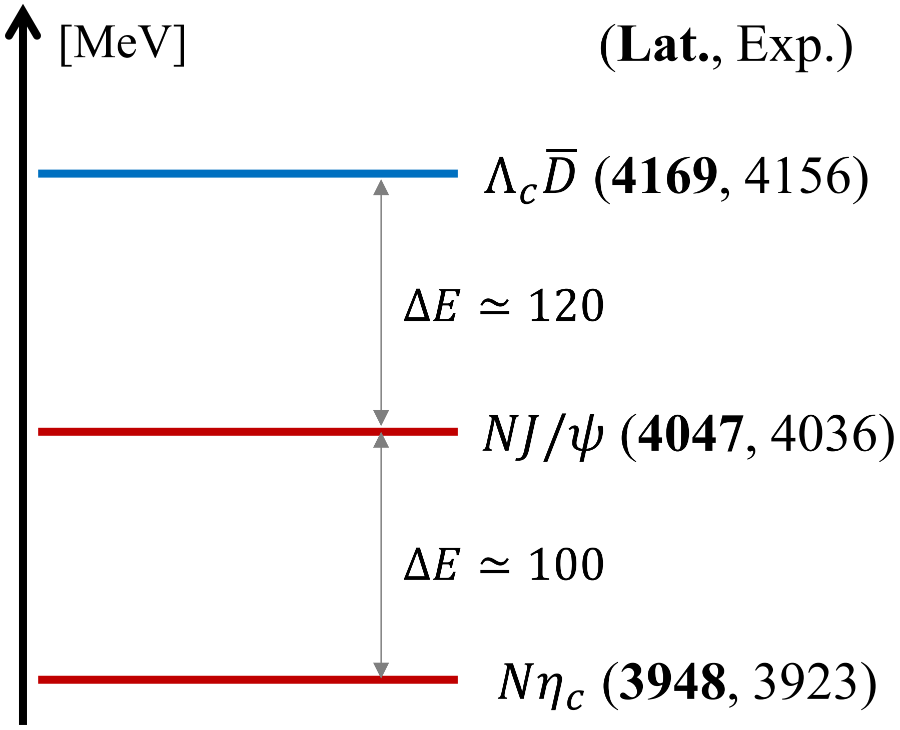
- The spin averaged N-J/ψ scattering length

a [fm]	Year	Author	Method
0.046(5)	2016	Gryniuk-Gryniuk	Photoproduction (VMD)
$3\sim25\cdot10^{-3}$	2021	Pentchev-Strakovsky	Photoproduction (VMD)
$O(1)$	2023	JPAC	Photoproduction (unitarity)
0.05	1992	Kaidalov-Volkovitsky	QCD multipole expansion
0.24	1997	Brodsky-Miller	QCD multipole expansion
≥0.37	2005	Sibirtsev-Voloshin	QCD multipole expansion
0.10(2)	1999	Hayashigaki	QCD sum rule
$0.2\sim3\cdot10^{-3}$	2020	Du-Baru-Guo et. al.	Coupled channel
0.71(48)	2006	Yokokawa-Sasaki etal	LQCD (quenched)
0.1(7)	2008	Liu-Lin-Orginos	LQCD (full, extrapolate to phys. Pt.)
0.33(5)	2010	Kawanai-Sasaki	LQCD (quenched)
0.47(1)	2019	Sugiura-Ikeda-Ishii	LQCD ($m_\pi=700$)
$\simeq 0$	2019	Skerbis-Prelovsek	LQCD ($m_\pi=266$)
0.33(4)	**2024**	**The present work**	**LQCD ($m_\pi=146$)**

Energy levels

- Relevant energy levels

  - The inelastic $\Lambda_c \bar{D}$ is around 120 MeV above, and is suppressed at large t
  - The transition between NJ/ψ to $N\eta_c$ is suppressed by heavy quark spin symmetry, and is further suppressed kinematically for the highest spin state
  - Decay through $c\bar{c}$ annihilation is OZI suppressed

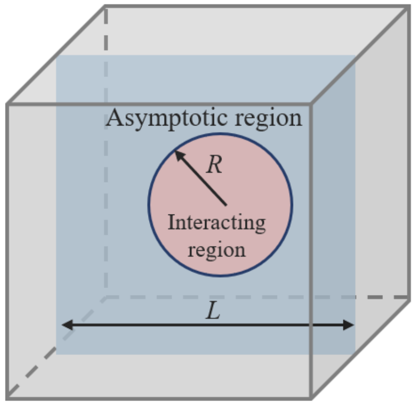


- As we focus on near-threshold scatterings, single-channel calculations are expected to be sufficient

HAL QCD method

- Nambu-Bethe-Salpeter (NBS) amplitude

$$\boxed{\psi^k(\boldsymbol{r},t) = \langle 0|N(\boldsymbol{r},t)O_{c\bar{c}}(\boldsymbol{0},t)|N(\boldsymbol{k}), c\bar{c}(-\boldsymbol{k}); E\rangle}$$



  - Asymptotic region: $\psi^k(r) \simeq A\,\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$
  - Interacting region: define potential

$$\boxed{(\nabla^2 + k^2)\psi^k(\boldsymbol{r}) = 2\mu \int d\boldsymbol{r}'\, U(\boldsymbol{r},\boldsymbol{r}')\psi^k(\boldsymbol{r}')}$$

- R correlator (superposition of NBS amplitudes) $R(\boldsymbol{r},t) = \sum_n a_n \psi_n(\boldsymbol{r}) e^{-\Delta E_n t}$

$$\boxed{\left(\frac{1}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R(\boldsymbol{r},t) = \int d\boldsymbol{r}'\, U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t)}$$

  - Derivative expansion: $U(\boldsymbol{r},\boldsymbol{r}') = \sum V_i(\boldsymbol{r})\,\nabla^i \delta(\boldsymbol{r} - \boldsymbol{r}')$

$$\boxed{V(r) = R(\boldsymbol{r},t)^{-1}\left(\frac{1}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R(\boldsymbol{r},t)}$$

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)
N. Ishii, *et al.* [HAL QCD Coll.], Phys. Lett. B 712, 437 (2012)